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How to cite this publication

Please cite the final published version:

Darmawan, A., Wong, H. W., & Thorstenson, A. (2020). Integrated sales and operations planning with multiple products: Jointly optimizing the number and timing of promotions and production decisions. *Applied Mathematical Modelling*, 80, 792-814. <https://doi.org/10.1016/j.apm.2019.12.001>

Publication metadata

Title:	Integrated sales and operations planning with multiple products: Jointly optimizing the number and timing of promotions and production decisions.
Author(s):	Darmawan, Agus ; Wong, Hartanto Wijaya ; Thorstenson, Anders.
Journal:	Applied Mathematical Modelling.
DOI/Link:	10.1016/j.apm.2019.12.001
Document version:	Accepted manuscript (post-print)
Document license:	CC BY-NC-ND

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Integrated sales and operations planning with multiple products: Jointly optimizing the number and timing of promotions and production decisions

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Abstract

This paper presents a modelling framework for sales and operations planning (S&OP) that considers the integration of price promotion and production planning for multiple products. Such a modelling framework takes into account the potential competition and cannibalization between products, as well as the allocation of shared production resources. The demand model that we adopt combines purchase incidence, consumer choice and purchase quantity in a sequential framework to obtain the dynamics and heterogeneity of consumer response to promotions. Due to large problem sizes, we develop a heuristic approach for solving the resulting joint optimization problem. The results of our numerical study show interesting findings on the optimal number and timing of promotions that take into account the mutual dependence of marketing and production related factors.

Keywords: demand model, forward buying, product substitution, cannibalization, promotion

1. Introduction

Nowadays, cross-functional intra-company and supply chain inter-company coordination have become important requirements to obtain competitive advantages. In some industries these are requirements just to stay in the market. One of the typical planning activities that serve to

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synchronize different business functions within firms as well as to integrate supply chain planning processes is sales and operations planning (S&OP). S&OP is a tactical planning process that integrates marketing plans for new and existing products with the management of the supply chain [1].

Data from Gartner shows that firms with a demand-driven S&OP process can increase revenues and reduce inventories significantly [2]. This reference also points out the need for technology and skill sets that share a common set of terms for all business functions. Hinkel et al. [3] report that there is still a widespread use of rough approaches and lack of coordination in S&OP. They emphasize the need for enhanced coordination among cross-functional teams and to improve operational plan stability through a more advanced and integrated planning procedure. In their study focusing on Danish companies, Lund and Raun [4] report that many firms still do not possess a clear S&OP vision, so that the marketing and operations departments develop their own plans, resulting in S&OP that is not truly integrated. All these observations suggest that enhancing the integration in S&OP remains an important and relevant research theme.

Trade promotions play an important role as a marketing-mix tool. According to a study by Nielsen [5], spending on trade promotions in the consumer packaged goods sector is approximately \$1 trillion annually. Gomez et al. [6] report that trade promotions play an extremely important role in the U.S. supermarket industry, as well as in many consumer packaged goods industries. Gedenk et al. [7] report that price promotion may increase total sales by 12-25 % for retailers in Europe, thereby playing an important role in generating revenues.

Yet, despite their importance, several business reports reveal that a great deal of trade promotions fail to grow the brand and the category for both manufacturers and retailers [8]. One major possible challenge is that the data are often too high-level so that it is difficult for

the manufacturers and retailers to assess the effectiveness of a promotion on multiple SKUs at different stores [8, 9]. Dawes [11] puts forward the issue of ‘sibling rivalry’ as many companies too often ignore the impact on other products they sell when offering discounts. Firms producing and selling multiple related products should coordinate the price promotions of their brands. One major decision they must make is whether to offer a promotion on one product at a time or to offer promotions on several products simultaneously. This decision may depend on both marketing related factors such as consumers’ substitution patterns between products and production related factors such as the flexibility of shared production resources. The existing literature offers little help in addressing the challenges discussed above, and our study aims to fill this void in the literature.

Our contributions are threefold. First, we develop a modelling framework for S&OP that enhances the integration of price promotion and production planning and supports joint decision-making in a multiple-product setting. We integrate an existing econometric-based demand model and a standard mixed integer linear programming based aggregate production planning model, allowing a common framework shared by the marketing and production planners. The demand model is able to capture purchase incidence, consumer choice and quantity decisions, and household’s inventory levels dynamically, which allows for decomposition of total sales into consumption, brand switching and forward buying. Furthermore, such a demand model is able to capture the potential competition and cannibalization between products offered by the same manufacturer. This is important because the integration of promotion and production planning becomes even more relevant when considering manufacturing firms that offer a family of similar products or the same product with different package formats. This is so not only because these products consume the same production resources, but also because there is internal competition in the selling of these products. Hence, firms can use the modelling framework that we develop as a basis for

coordinating price promotions of multiple products by simultaneously considering both demand and production related factors.

Second, due to the large size of the solution space of the integrated optimization problem, we develop an easily implementable heuristic based on genetic algorithms (GA) to determine near optimal or good promotion and production plans without requiring excessive computation burdens. We show that this heuristic performs reasonably well in comparison to the brute force enumeration, as well as compared to an alternative meta-heuristic based on Simulated Annealing. The model and solution method presented in this paper can serve as a building block for the development of a more integrated S&OP.

Finally, through our numerical study, we provide a number of important managerial insights into how the joint production and promotion planning decisions should be made, and how the decisions are influenced by the different production and marketing related factors. One notable finding is that, in general, the simultaneous promotions of multiple products are not recommended. The main downside of simultaneous promotions is due to the possible cannibalization between internal products that cancel out the main objective of offering a price discount. In addition, the effectiveness of promotions for increasing profits deteriorates in the case where production capacity changes are costly, i.e. when capacity flexibility is low.

The paper proceeds as follows. Section 2 presents our review of relevant literature. Section 3 consists of two parts. In the first part, we introduce the main elements of the demand model that include purchase incidence, brand choice, and purchase quantity. The second part of this section specifies the integration of the demand and aggregate production planning models. In Section 4, we present the GA heuristic used for solving the integrated optimization problem, and the results of our numerical study for the performance evaluation of the heuristic. In Section 5, we study the integrated promotion and production decisions in the case of multiple

products. In particular, we focus on assessing the timing of promotions of the products. Section 6 summarizes the conclusions of our study and provide some directions for future research.

2. Literature review

The literature addressing the coordination of promotion and operational decisions is rich. Many studies (e.g. [11-18]), however, focus on inventory rather than production planning decisions. More related to our paper is the literature on the coordination of marketing and production decisions in aggregate planning. Martinez-Costa et al. [19] present a comprehensive review of the literature considering the importance of integration in aggregate planning that coordinates marketing and production decisions. Based on the fact that advertising can be used to smooth seasonal product demand, Leitch [20] presents an optimisation model for production and advertising planning in a multi-period setting. Sogomonian and Tang [21] develop a modelling framework for joint optimisation of promotion and aggregate planning within a firm. They use a simple demand function to generate demand in each period, which depends on price, time and level of last promotion. They test the model for both the sequential and integrated planning approaches. Ulusoy and Yazgac [22] consider pricing and advertising in their aggregate planning within a multi-product and multi-period setting. In their simple demand function, demand is assumed proportional to the advertising level but inversely proportional to the price. Feng et al. [23] present a model to integrate sales and production under the assumption that demand and price are both normally distributed. Affonso et al. [24] perform demand perturbations to show the importance of coordination in S&OP. Gonzales-Ramirez et al. [25], Lusa et al. [26], and Bajwa et al. [27] discuss the integration of production and marketing decisions using a simple relation between demand and price. Sodhi and Tang [28] present a stochastic programming model for S&OP that determines the production requirement while optimally trading off risks of unmet demand, excess inventory, and inadequate liquidity in the

presence of demand uncertainty. Although they consider an effective unit price that is realized after various discounts off the list price, the effects of discounts on demand are not considered in their model and promotion is not part of the decision variables.

We concur with Feng et al. [29] and Martinez-Costa et al. [19] in emphasizing the importance of research in S&OP that adopts a rich demand model that is suitable for the purpose of developing decision support tools for the joint planning process. We extend the previous studies by adopting such a rich demand model that captures the possible effects of various marketing factors such as price discount level, seasonality, promotion impact, brand loyalty, etc. Moreover, the demand model adopted in this paper is able to capture the effect of product substitution or cannibalization that is necessary when considering integrated production and promotion planning in a multi-product setting. Recently, Darmawan et al. [30] develop a modelling framework for an integrated S&OP that considers joint promotion and production decisions using the same demand model. However, they only consider the case with a single product and focus on the benefits of adopting an integrated approach over a sequential approach. We extend their modelling framework in two respects. First, while the demand model used in this paper is the same as in theirs with respect to the ability to capture competition among products or brands, extending the model to the multi-product setting for the manufacturer results in a more complex optimization problem regarding promotion and production decisions. One important aspect relevant to examine is whether the promotion for the different products should be carried out simultaneously taking into account the possible effect of product substitution (marketing) and the use of common production resources (production). Second, as a consequence of focusing on the more complex optimization problem, it becomes necessary to develop a heuristic solution approach, which we also address in this paper. Obviously, these two extensions provide an opportunity to investigate the benefits of an integrated S&OP more fully.

Our literature review suggests that research that considers S&OP in a multi-product setting is very scant. Taskin et al. [31] develop a mathematical programming model for developing S&OP based on a real case at a television manufacturer. They minimise production and procurement costs with respect to the resource constraints in a dynamic environment, where sales forecasts are regularly updated. Lim et al. [32] use a simulation-optimization approach for solving the S&OP problem for a case in the automotive industry with multiple parts and distant sourcing. However, these papers consider neither promotions nor how sales of one product might have an impact on other products. Ghasemy Yaghin et al. [33] consider aggregate planning and markdown pricing where price only changes downwards and thus cannot be increased again after a price promotion. They study a production-pricing planning problem in a two-echelon supply chain that serves a demand from two or more market segments. Although they consider the integration of production and pricing decisions, as we do in this paper, our demand model captures the additional aspects of product substitution, forward buying, and brand loyalty. More recently, Ghasemy Yaghin [34] relaxes a restriction in the previous work [33] and also considers cannibalization. In his numerical study, he assumes that cannibalization rates are equal for all products and not directly dependent on the dynamics and heterogeneity of consumers' response. Thus, there is certainly still a gap in the literature to be filled by employing a more sophisticated demand representation. Our model in this paper takes into account the potential competition and cannibalization between products, as well as the allocation of shared production resources.

Promotion planning is one of the topics attracting wide attention in the marketing literature. Promotion is an important element of the marketing mix that can be used to attract potential and current customers to buy the product [35, 36]. A number of authors, e.g., Silva-Risso et al. [37], and Ailawadi et al. [38], present a promotion planning model that is based on a disaggregate consumer response model. This is also the approach adopted in our paper. Fok

et al. [39] study the purchase-timing behaviour of households. Promotion may result in shortened inter-purchase times, but may also result in longer inter-purchase times due to stockpiling. To incorporate the dynamic effect of marketing strategies, they suggest combining purchase incidence with brand choice and purchase quantity decisions.

Simester [40] studies the characteristics of promotion strategies in a multi-product setting. His study suggests that retailers should offer deeper promotions in the case where customers are highly price sensitive and the impact of promotion on product switching is strong. He also points out that deeper promotions are suitable for products that savour complementary relationships and low substitution effects. Srinivasan et al. [41] discuss the effect of cannibalization on marketing strategies. Promotions not only increase the sales of the discounted product but also have an impact on a substituted product due to diverting mechanisms. Ignoring this phenomenon leads to a sub-optimal promotion plan. Gumus et al. [42] further justify why a deeper price promotion is common in the case of expensive products with a low degree of substitution. In relation to promotion timing, their finding is that simultaneous schedules of promotions are more suitable in the case of high substitution effects, while sequential schedules are more suitable in the case of low substitution effects. All the above mentioned studies published in the marketing literature seem to disregard the impact of promotions on production-related factors and costs. Hence, even though supported by rich demand models, the resulting promotion plans are most likely sub-optimal when viewed from an integrated planning perspective.

To the best of our knowledge, this paper is the first attempt to integrate existing research in both the operations and the marketing literature for the development of integrated S&OP that considers multiple products. We adopt the disaggregate consumer response model widely used in the marketing literature that facilitates the potential substitution (cannibalization) effects between products. The resulting demand forecasts derived from a promotion plan

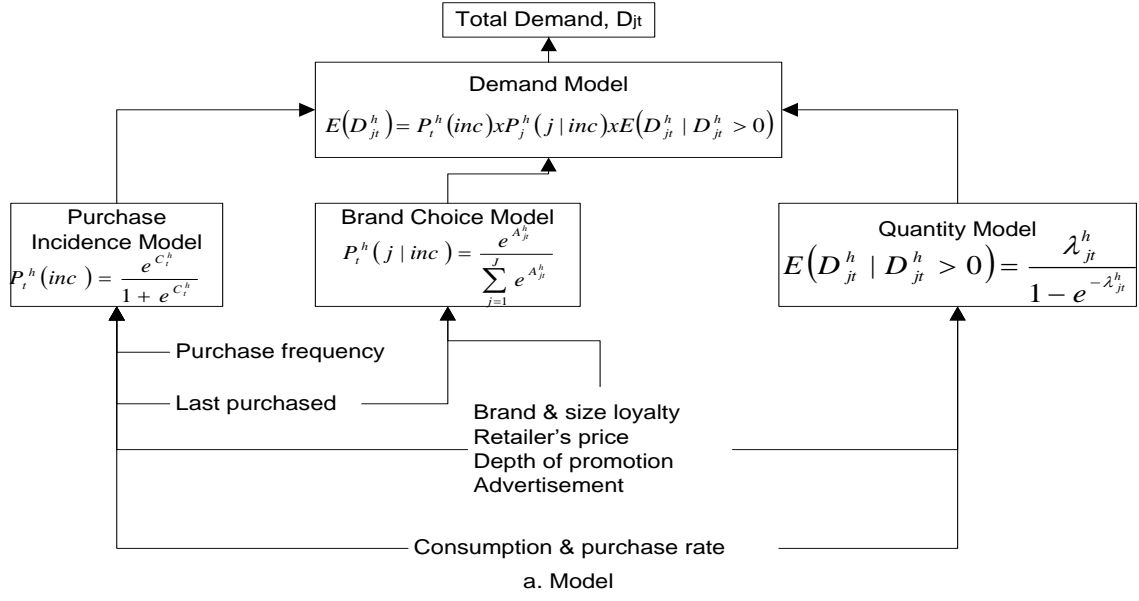
become the main inputs for the development of an aggregate production plan. Thus, this integrated approach takes into account the effect of promotions on the net demand of the products, which also has an impact on the use of production resources to satisfy the demand.

3. The models

In this section, we address two parts of our modelling framework. The first part presents an introduction to the demand model and the second part describes the joint optimization of sales and operations planning by considering promotion and production decisions simultaneously.

3.1 Demand model

The demand model adopted in this paper is based on the incidence-brand choice-purchase quantity model that is widely used in the marketing literature [37, 38, 43, 44]. We refer to Darmawan et al. [30] for the similar model that only considers a single product offered by the manufacturer. The model captures consumers' purchase timing, product or brand choice, and quantity decisions, conditional on shopping trip and store choice. It takes into account household specific variables (e.g. brand loyalty, consumption, and purchase rates), as well as environment variables (e.g. retailer pass through, mark-up, and competition). We summarize the main elements of the demand model in Figure 1 [30]. Note that due to the application of the model in a multi-product setting, we use the terms brand and product interchangeably as appropriate.



The store scanner data

Marketing Information

- Price
- Price at manufacturer
- Depth of promotion
- Pass-through
- Markup
- Advertisement
- Feature
- Display

The panelist data

Shopping trip & histories

- Last brand purchased, last size purchased
- Purchase frequency
- Consumption & purchase rate
- Brand & size loyalty

b. Data

Figure 1. The main elements of the demand model

The expected household demand, given a store visit, is obtained as

$$E(D_{jt}^h) = P_t^h(inc) \times P_t^h(j|inc) \times E(D_{jt}^h | D_{jt}^h > 0) \quad (1)$$

where

$P_t^h(inc)$ The probability that household h makes a purchase in the product category on a store visit in time period t

$P_t^h(j|inc)$ The probability that household h chooses product j , given that household h decides to make a purchase in the product category in time period t

$E(D_{jt}^h | D_{jt}^h > 0)$ The expected quantity that household h will buy of product j , given that household h decides to purchase product j in time period t

The purchase incidence probability takes the following form:

$$P_t^h(inc) = \frac{e^{C_t^h}}{1 + e^{C_t^h}}, \quad (2)$$

where C_t^h is the deterministic component of utility associated with household h in time period t , and is influenced by the proportion of purchase frequency, household inventory, and consumption rate. The probability that household h will choose product j is handled in a multinomial logit framework in the following form:

$$P_t^h(j|inc) = \frac{e^{A_{jt}^h}}{\sum_{j=1}^J e^{A_{jt}^h}}, \quad (3)$$

where A_{jt}^h is the deterministic component of utility associated with product j for household h in time period t , and is a function of price, promotion and consumer-specific variables such as brand loyalty. Next, the expected quantity that household h will buy given that household h decides to purchase product j in time period t is determined from a truncated Poisson distribution:

$$E(D_{jt}^h | D_{jt}^h > 0) = \frac{\lambda_{jt}^h}{1 - e^{-\lambda_{jt}^h}}, \quad (4)$$

where λ_{jt}^h is the purchase rate of household h for product j at time t , which is influenced by the average number of units purchased, household inventory, brand loyalty, and price. To generate the dynamics of consumer response, we use the Monte Carlo technique to simulate the purchase probabilities of a panel of households [38, 45]. This technique allows us to capture the effect of stockpiling and repeat purchases. The output is the demand forecast D_{jt} for brand j in time

period t , which represents the aggregate household demand for each period, i.e., $D_{jt} = \sum_{h=1}^H E(D_{jt}^h)$, where H is the number of households simulated.

To estimate the parameter values of the model, scanner-panel data obtained from sources such as Nielsen Consumer Panels are usually employed. In our numerical study, however, we use the secondary parameter values presented in Silva-Risso et al. [37] for the purpose of forming the basis of a market simulator that allows us to examine how the integrated promotion and production decisions are affected by various production and marketing related factors in a multi-product setting. See Appendix 1 for a more detailed presentation of the demand model and the values of the demand parameters.

3.2 The integrated optimization model

Let L_{jt} ($0 \leq L_{jt} < 1$) denote the level of discount (%) offered in period t for product j ($L_{jt} = 0$ means that there is no promotion offered). We define $P \in \mathbb{P}$ as a promotion plan for the products offered by the manufacturer, where $P = (L_{11} \cdots L_{1T} \cdots L_{j1} \cdots L_{jT})$ and \mathbb{P} is the set of all possible promotion plans. The promotion plan is the main input for the demand model, and we define $D_{jt}|P$ as the resulting demand forecast for product j in period t that corresponds to promotion plan P .

Parameters:

cp_j	Production unit cost (including materials; excluding labour cost) for product j
cl	Regular labour cost per worker and time period
ch	Hiring cost per worker
cf	Firing cost per worker
co	Overtime cost per hour
$cInv_j$	Inventory holding cost per time period for product j

cs_j	Subcontracting cost per unit for product j
LL	Minimum number of workers
UL	Maximum number of workers
wh_t	Number of regular working hours available per worker in time period t
nl_o	Number of workers at the beginning of the planning horizon
I_{j0}	Inventory of product j at the beginning of the planning horizon
nu_j	Number of units produced per hour for product j
O_t	Maximum number of overtime hours per worker in time period t
T	Planning horizon in number of time periods
J	Number of products offered by the manufacturer
SS_{jt}	Safety stock requirement for product j at the end of time period t
K_j	Maximum number of promotions during the planning horizon for product j
Rp_{jt}	Regular price per unit from manufacturer for product j in time period t
V_t	Promotion cost per promotion event
M	Sufficiently large number

Decision variables:

qp_{jt}	Number of units produced during regular time for product j in period t
qo_{jt}	Number of units produced during overtime for product j in period t
qs_{jt}	Number of units produced using subcontracting for product j in period t
nh_t	Number of workers hired at the beginning of time period t
nf_t	Number of workers fired at the beginning of time period t
L_{jt}	Level of discount (in percent) in time period t for product j

Consequential variables:

D_{jt}	Demand forecast for product j in time period t
I_{jt}	Inventory of product j at the end of time period t
nl_t	Number of workers available in period t

Z_{jt}	Binary variable: 1 if promotion with discount is offered for product j in time period t ; 0 otherwise
Y_t	Binary variable: 1 if there is a promotion with discount offered to at least one product j ($j=1, \dots, J$) in period time t ; 0 if there is no promotion in time period t

By integrating the promotion and production planning, the joint optimization problem is formulated as:

$$\begin{aligned}
& \max_{qp_{jt}, qo_{jt}, qs_{jt}, nh_t, nf_t, P \in \mathbb{P}} Profit \\
& = \sum_{t=1}^T \sum_{j=1}^J D_{jt} |P \cdot Rp_{jt} \cdot (1 - L_{jt} |P) \\
& \quad - \sum_{t=1}^T \sum_{j=1}^J \left((qp_{jt} + qo_{jt}) \cdot cp_j + \frac{qo_{jt}}{nu_j} \cdot co + qs_{jt} \cdot cs_j + I_{jt} \cdot cInv_j \right) \\
& \quad - \sum_{t=1}^T (nl_t \cdot cl + nh_t \cdot ch + nf_t \cdot cf + Y_t \cdot V_t) \tag{4}
\end{aligned}$$

Subject to:

$$I_{jt} = I_{jt-1} + qp_{jt} + qo_{jt} + qs_{jt} - D_{jt} |P \quad t = 1, \dots, T; j = 1, \dots, J \tag{5}$$

$$I_{jt} \geq SS_j \quad t = 1, \dots, T; j = 1, \dots, J \tag{6}$$

$$nl_t = nl_{t-1} + nh_t - nf_t \quad t = 1, \dots, T \tag{7}$$

$$nl_0 = nl_T \tag{8}$$

$$LL \leq nl_t \leq UL \quad t = 1, \dots, T \tag{9}$$

$$\sum_{j=1}^J \frac{qp_{jt}}{nu_j} \leq nl_t \cdot wh_t \quad t = 1, \dots, T \quad (10)$$

$$\sum_{j=1}^J \frac{qo_{jt}}{nu_j} \leq nl_t \cdot O_t \quad t = 1, \dots, T \quad (11)$$

$$0 \leq L_{jt}|P < 1 \quad t = 1, \dots, T; j = 1, \dots, J \quad (12)$$

$$L_{jt}|P \leq Z_{jt} \leq M \cdot L_{jt}|P \quad t = 1, \dots, T; j = 1, \dots, J \quad (13)$$

$$Y_t \leq \sum_{j=1}^J Z_{jt} \leq J \cdot Y_t \quad t = 1, \dots, T \quad (14)$$

$$\sum_{t=1}^T Z_{jt} \leq K_j \quad j = 1, \dots, J \quad (15)$$

$$Z_{jt} \text{ binary} \quad t = 1, \dots, T; j = 1, \dots, J \quad (16)$$

$$Y_t \text{ binary} \quad t = 1, \dots, T \quad (17)$$

$$qp_{jt}, qo_{jt}, qs_{jt} \geq 0 \quad t = 1, \dots, T; j = 1, \dots, J \quad (18)$$

$$nh_t, nf_t \geq 0 \text{ and integer} \quad t = 1, \dots, T \quad (19)$$

In the above optimization problem, the objective is to maximize the profit, obtained by subtracting all the costs for material, overtime, subcontracting, inventory, labour, hiring and firing, and promotions from the sales revenue that is affected by price promotions. Constraints (5) and (6) are the inventory balance equations and minimum safety stock levels, respectively. The number of resources and capacity in terms of the size of the work force, and the units produced in regular time and on overtime are represented by Constraints (7) - (11). In (8), following the standard assumption in the S&OP literature (see e.g. [1, 46]), we assume that the size of work force at the end of the planning horizon is the same as the initial size, which allows us to compare alternative plans with the same beginning and ending conditions. The range of the promotion discount levels is given by (12), and constraints in relation to the number of

promotions are given in (13) - (17). Constraints (18)-(19) are the usual non-negativity and integer constraints.

Note that in the above formulation, we capture the economies of scale in implementing a joint promotion by considering a scenario where a fixed promotional cost is incurred per promotional time period regardless of whether the promotional event is only for a single product or for multiple products. We will also focus on a scenario where there are a set of discrete discount levels that the manufacturer may choose from, and once a certain discount level is selected for product j , this level is used throughout the whole planning horizon. Besides reducing the decision space, this allows us to get a clearer understanding regarding the effect of changing the discount level. Choosing from a discrete set and then applying the same discount level to a particular product is a restriction that is commonly observed in practice. In one of the Danish retail chains, for example, the standard price of instant coffee Nescafe Gold Blend 175 gr. is 65 DKK, and the discounted price during a promotion event is always 55 DKK. The same applies to potato chips Pringles Original 165 gr. for which the standard price is 15 DKK while the discounted price is always 10 DKK.

4. The heuristic solution procedure

The integrated optimization problem specified in Subsection 3.2 is a non-linear mixed integer problem. Darmawan et al. [30] use complete enumeration to determine the optimal solution in a single-product setting. When considering all possible combinations of promotion levels and promotion timing in a multi-product setting, the solution space can be quite large so that complete enumeration is not feasible. Therefore, in this paper we develop a heuristic based on genetic algorithms (GA) that will help obtain good solutions with reasonable computation times. In the following, we first outline the structure of the heuristic solution procedure. Then, in the next two subsections, we present results aimed at evaluating the effectiveness of the

suggested procedure.

4.1 Structure of the GA heuristic

GA is a well-known and widely applied method for solving complex optimization problems. Because GA explores a population of solution points in parallel rather than a single solution point at a time, it has the capability to handle large search spaces. Hence, it has relatively good performance in terms of speed [47–49]. In solving our optimization problem, we use the GA heuristic to generate a promotion plan, based on which an aggregate production plan is optimized by solving the corresponding mixed integer linear programming problem. The main purpose of developing the heuristic in this paper is not computational efficiency *per se* but the possibility to produce good quality solutions for making comparisons of the solution characteristics.

GA adopts a mechanism that is an analogy of natural selection by introducing concepts like population, selection, crossover and mutation. Using a directed random search procedure, GA attempts to find a near-optimal solution in multi-dimensional search spaces. The main principle is that GA evaluates multiple chromosomes or solution candidates at each generation, where a chromosome represents a promotion plan $P \in \mathbb{P}$ in our implementation. For each promotion plan $P = (L_{11} \cdots L_{1T} \cdots L_{J1} \cdots L_{JT})$, we run a Monte Carlo simulation of the demand model to compute the corresponding demand forecast $D_{jt}|P$. We then solve the aggregate production planning problem. What'sBest®15.0.1.2 is used for solving the mixed integer programming model of the aggregate production planning problem, i.e., for each promotion plan generated by the GA heuristic, we obtain an optimal production plan. The whole optimization procedure is coded as macros in Visual Basic for Application (VBA) within Microsoft®Excel 2016.

As GA is well-known to be sensitive to parameterization, there are a few approaches to help prevent premature convergence [50, 51]. In this study, we use rank selection and new generation process for the selection procedure, as well as two-point crossover for chromosomes. In the rank selection, we sort the population according to the objective function. Then, we calculate the fitness value for each of the chromosomes and rank them. We allocate a selection probability with respect to the rank. In a two-point crossover, we pick two crossover points randomly from the parent chromosomes and swap values between chromosomes. We refer to Appendix 2 for a more detailed algorithmic specification of the GA heuristic. In the next subsection, we present evaluations of the performance of the heuristic solution procedure.

4.2 Evaluation of the GA heuristic

For evaluation purposes, we compare the results of the GA heuristic to those obtained by using two other approaches, namely complete enumeration and an alternative meta-heuristic technique based on Simulated Annealing (SA). Simulated Annealing is a well-established meta-heuristic (see [51]) that considers random moves in a solution's neighbourhood. If a move results in a better objective function value, then SA will always accept it. However, to avoid premature convergence to a local optimum, SA will also accept a worse solution with a certain probability. A more detailed algorithmic specification of the SA heuristic is provided in Appendix 3. Both complete enumeration and SA serve the same ends as the GA-part in our heuristic procedure, namely to generate promotion plans.

In this evaluation, we consider problem instances with two products. We consider a scenario where a manufacturing firm sells two different products (products A and B) in the same product family, while there is also a related product (product C) offered by a competitor. To avoid excessive computation times, especially in running the complete enumeration technique, we limit the optimization problem by restricting it to the case where promotions are

offered in the first week of every second month. While this may seem restrictive, some manufacturers or retailers indeed offer promotions with minimum time intervals and limit the number of promotions motivated by the fact that they wish to preserve the image of their store and not to train customers to become strategic buyers [18]. However, in the numerical study in Section 5, we consider a more general problem with less restriction on the promotion timing by allowing promotions to take place in any week of the year. In all our numerical studies, the time periods specified in the optimization model presented in Section 3 are interpreted to be represented by work weeks. This interpretation is motivated by our observation that the typical length of a promotion event is a week.

We generate 96 problem instances differentiated by six experimental factors: flexibility in changing production capacity, margin gap, seasonality effect, loyalty gap, promotion impact, and promotion discount level. The flexibility in changing the production capacity is represented by the levels of the hiring and firing costs. We use two levels of gap between product A's and product B's profit margins. This is achieved by differentiating the selling price for product B, while keeping the same unit production cost for both products. Seasonality effects are captured through the scale factor, F^h , in the purchase incidence model (see Appendix 1). We divide the planning horizon into ten segments of (approximately) equal length (5-6 weeks) and vary the scale factor values across these segments to obtain a demand pattern with seasonality. If the scale factor value is set at a constant value, we obtain a demand pattern without seasonality. The total expected demand over the entire planning horizon is assumed to be the same with and without seasonality. There are two levels of loyalty gap between the two products. The brand loyalty parameter (B_j) is set in the brand choice model (see Appendix 1). We vary the temporary price reduction coefficient in the choice model, θ_6 , to capture the effect of different levels of the promotion impact. Furthermore, we examine the results based on three price

discount levels: 10%, 20%, and 30%. Table 1 presents the parameter values used in our study.

The other base-case parameters used are as follows (for $j = A, B$ and $t = 1, 2, \dots, T$):

$cl = 8$; $co = 12$; $cInv_j = 0.092$; $LL = 35$; $UL = 140$; $wh_t = 40$; $nl_o = 50$; $nu_j = 8$; $O_t = 2.5$; $I_{j0} = 4000$; $SS_{jt} = 2000$; $T = 52$; $K_j = 12$; $V_t = 1000$; $H = 121,350$ (Note: subcontracting is excluded in this numerical study). No production parameters are necessary for product C, since it is assumed to be a competitor's product. However, the parameter values of the demand model for product C are presented in Appendix 1.

Table 1. Parameter setting for experimental factors

Factors	# Levels	Values
Flexibility in production capacity	2	High: $c_h = 1000$, $c_f = 2000$ Low: $c_h = 2000$, $c_f = 3000$
Margin gap	2	Low: $Rp_{At} = 12$; $Rp_{Bt} = 12$; $cp_j = 7$; $Rp_{Ct} = 10$ High: $Rp_{At} = 12$; $Rp_{Bt} = 11$; $cp_j = 7$; $Rp_{Ct} = 10$
Seasonality	2	Low: $F^h = 0.81$ (constant) High: $F^h = 0.83, 0.7, 0.58, 0.48, 0.68, 0.85, 0.92, 0.99, 0.98, 0.92$
Promotion impact	2	Low: $\theta_6 = 0.2$ High: $\theta_6 = 0.8$
Loyalty gap	2	Low: $B_A = 0.4$; $B_B = 0.3$ High: $B_A = 0.6$; $B_B = 0.1$
Promotion discount level	3	Low: $L_t = 10\%$ or 0% for $\text{mod}(t, 8) = 0$; otherwise $L_t = 0\%$ Medium: $L_t = 20\%$ or 0% (as above) High: $L_t = 30\%$ or 0% (as above)

In order to make fair comparisons of all the problem instances, we use the same set of random numbers to simulate the forecasted demands in the demand model. For all the 96 problem instances, Table 2 presents the performances of the GA and SA heuristics measured by the relative gaps (in %) of the profits according to (4) in comparison to the profits from the solutions obtained by complete enumeration. Each instance in Table 2 is represented by the levels of the first five experimental factors in Table 1. For example, the first instance is

represented by L-L-L-L-L, i.e., the instance with low capacity flexibility, low margin gap, low seasonality, low promotion impact, and low loyalty gap.

In our experiment, we set a stopping criterion for the two heuristics by allowing a maximum of 100 iterations in total. Preliminary experiments revealed that this stopping criterion is sufficient to reach convergence. As indicated in Table 2, the GA and SA heuristics perform reasonably well, as indicated by the small relative gaps. On average, GA performs better than SA. The average profit gaps for GA are 0.89%, 1.03% and 0.67%, for the discount levels 10%, 20%, and 30%, respectively. In addition to the profit gaps, we also recorded the number of promotions suggested, as presented in Table 3. This table shows that in general the heuristics result in a number of promotions that is close to or the same as the optimal number obtained by complete enumeration. However, the same number of promotions does not necessarily imply the same timing of the promotions, which explains why we observe positive gaps in the profits in Table 2. The average computation time to run the GA and SA heuristic with the above stopping criterion are 2.45 and 2.33 hours respectively, where the complete enumeration needs 5.97 hours. The experiment was run on a computer with the following technical specifications: Intel(R) Core(TM) i5-5200U CPU, 8.00GB (RAM), 64bit, Windows10. This numerical study suggests that the GA heuristic tested in this paper can be used for the purpose of examining the effects of the different marketing and production related factors on the joint promotion and production decision model. Its good performance appears to be consistent when benchmarked against the complete enumeration technique, as well as with the alternative meta-heuristic approach.

Table 2. The profit gap (in %) between the heuristic and the optimal solutions for the discount levels of 10%, 20%, and 30%

No	Instances	Discount level: 10%		Discount level: 20%		Discount level: 30%	
		GA	SA	GA	SA	GA	SA

1	L-L-L-L-L	0.00	2.67	1.55	1.45	0.67	1.25
2	L-L-L-L-H	1.94	0.00	1.78	3.51	0.48	0.65
3	L-L-L-H-L	2.19	1.96	0.31	0.07	0.24	0.47
4	L-L-L-H-H	1.42	4.44	0.92	1.41	0.34	0.32
5	L-L-H-L-L	0.95	1.31	1.38	1.26	1.82	3.54
6	L-L-H-L-H	0.00	0.00	1.03	1.90	1.89	2.21
7	L-L-H-H-L	3.38	2.52	1.69	3.57	1.42	2.61
8	L-L-H-H-H	0.78	0.32	0.82	1.13	0.58	0.79
9	L-H-L-L-L	3.23	3.01	0.29	0.40	0.00	0.00
10	L-H-L-L-H	0.36	0.16	0.00	0.00	0.00	0.00
11	L-H-L-H-L	0.00	0.00	1.91	2.54	0.00	0.00
12	L-H-L-H-H	0.00	0.00	0.00	0.00	0.00	0.00
13	L-H-H-L-L	1.67	2.30	1.21	2.50	2.41	3.61
14	L-H-H-L-H	0.00	0.00	1.82	3.43	0.00	0.00
15	L-H-H-H-L	0.44	3.77	0.24	0.33	0.00	0.00
16	L-H-H-H-H	3.92	5.39	0.04	0.05	0.00	0.00
17	H-L-L-L-L	0.00	0.00	1.39	2.79	2.63	3.32
18	H-L-L-L-H	2.50	3.43	0.00	0.00	0.00	0.00
19	H-L-L-H-L	0.28	1.67	2.03	1.90	2.05	2.81
20	H-L-L-H-H	0.75	1.06	2.01	1.22	0.69	0.95
21	H-L-H-L-L	0.54	0.26	1.60	2.36	0.68	0.93
22	H-L-H-L-H	0.44	1.12	0.25	0.05	2.94	4.05
23	H-L-H-H-L	0.81	0.61	0.89	2.77	1.03	1.73
24	H-L-H-H-H	0.25	0.07	1.72	2.21	0.39	0.00
25	H-H-L-L-L	0.08	0.20	0.00	0.00	0.00	0.00
26	H-H-L-L-H	0.22	0.35	1.33	1.42	0.00	0.53
27	H-H-L-H-L	0.00	0.00	0.06	0.09	0.00	0.00
28	H-H-L-H-H	0.06	0.08	1.18	2.21	0.00	0.00
29	H-H-H-L-L	0.00	0.00	1.56	1.65	0.00	0.00
30	H-H-H-L-H	0.14	1.37	1.65	1.30	0.00	0.01
31	H-H-H-H-L	1.00	0.19	1.20	2.15	1.20	0.00
32	H-H-H-H-H	0.99	1.12	0.94	2.26	0.00	0.00
Average		0.89	1.23	1.03	1.50	0.67	0.93

GA: Genetic algorithm; SA: Simulated annealing

Table 3. The number of promotions for the discount levels of 10%, 20%, and 30%

No	Instances	Discount level: 10%			Discount level: 20%			Discount level: 30%		
		Opt*	GA	SA	Opt*	GA	SA	Opt*	GA	SA
1	L-L-L-L-L	4	6	4	1	1	1	1	1	1
2	L-L-L-L-H	0	0	2	1	1	1	0	1	1
3	L-L-L-H-L	6	5	5	3	3	3	1	1	1
4	L-L-L-H-H	3	3	3	5	4	5	0	1	1

5	L-L-H-L-L	5	5	5	4	4	4	1	1	1
6	L-L-H-L-H	3	3	3	3	3	3	0	1	1
7	L-L-H-H-L	3	3	3	4	4	3	2	2	2
8	L-L-H-H-H	5	4	4	3	3	3	1	1	1
9	L-H-L-L-L	3	3	3	0	1	1	0	0	0
10	L-H-L-L-H	3	3	3	0	0	0	0	0	0
11	L-H-L-H-L	4	4	4	0	1	1	0	0	0
12	L-H-L-H-H	2	2	2	0	0	0	0	0	0
13	L-H-H-L-L	0	0	0	2	2	2	2	3	3
14	L-H-H-L-H	4	4	5	2	2	2	0	0	0
15	L-H-H-H-L	5	4	4	2	2	2	1	1	1
16	L-H-H-H-H	6	5	5	2	2	2	0	0	0
17	H-L-L-L-L	3	3	4	4	4	4	3	3	3
18	H-L-L-L-H	1	1	1	3	3	3	2	2	2
19	H-L-L-H-L	6	5	5	2	1	2	2	2	2
20	H-L-L-H-H	6	5	5	5	4	5	3	3	3
21	H-L-H-L-L	4	4	4	4	5	4	4	4	4
22	H-L-H-L-H	3	3	3	3	3	3	2	2	2
23	H-L-H-H-L	5	5	5	4	4	4	2	2	2
24	H-L-H-H-H	2	3	2	4	4	4	3	3	3
25	H-H-L-L-L	3	3	3	0	0	0	0	0	0
26	H-H-L-L-H	1	2	1	1	1	1	0	0	1
27	H-H-L-H-L	4	4	4	1	1	1	0	0	0
28	H-H-L-H-H	6	6	6	1	2	2	0	0	0
29	H-H-H-L-L	3	4	4	4	4	4	0	0	0
30	H-H-H-L-H	3	3	3	2	2	2	2	2	2
31	H-H-H-H-L	3	3	3	5	5	5	4	3	4
32	H-H-H-H-H	4	4	4	0	1	1	3	3	3
Average		3.53	3.50	3.50	2.34	2.41	2.44	1.22	1.31	1.38

*Opt: Optimal solution; GA: Genetic algorithm; SA: Simulated annealing

The stopping criterion for GA

Further numerical evaluation revealed that the performance of GA is still satisfactory even if the maximum number of iterations is reduced. This finding is particularly useful when considering the idea of developing a decision support model for integrating promotions and production decisions in S&OP, where long computation times should be avoided in order to allow alternative scenarios to be explored. We tested several values of n , where n is the number of consecutive iterations that do not result in solution improvements before the heuristic is stopped. In Figure 2, we show the average profit gap and the average computation time for

several values of n and for the three discount levels. Based on these observations, $n = 10$ appears to be a good choice for the stopping criterion, as the resulting average profit gap is still below 1% while the average computation time is less than 30 minutes. This stopping criterion will be used in Section 5 for the full numerical study that considers multiple products.

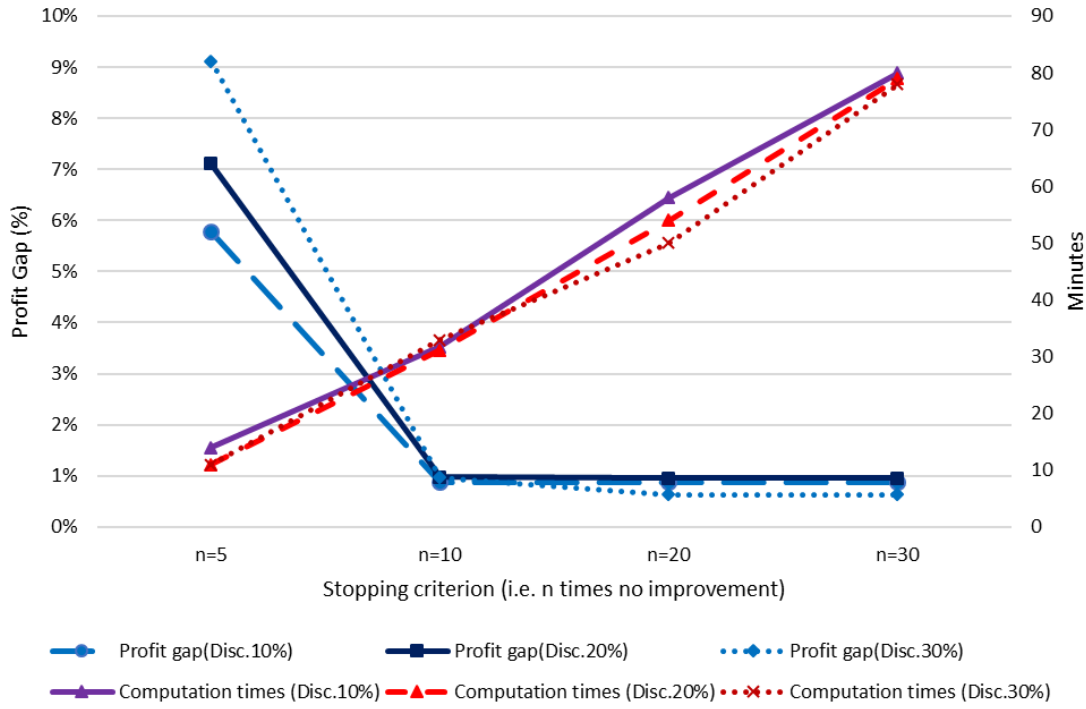


Figure 2. The performance of GA with different threshold values for the stopping criterion

4.3 Sensitivity analysis of promotion timing

Before we present the results of the full numerical study, we discuss some modifications in relation to the promotion timing decisions. Recall that in the numerical study in Subsection 4.2, we restricted promotions to be carried out only in the first week of every second month. One could question whether such a restriction would generate promotion plans that are far from optimal. Therefore, we have undertaken a sensitivity analysis to examine the effect of this promotion timing restriction. Removing the restriction on the promotion timing means that

promotions may be carried out in any week of any month. This results in a significant increase of the solution space from 2^{12} to 2^{104} possible promotion plans (1 year = 52 weeks).

We use the same problem instances as in the first numerical study in Subsection 4.2 and apply the GA heuristic to solve the corresponding optimization problem. Instead of presenting the results for each of the 32 individual instances, in Table 4 we present for each level of the experimental factors, the average relative increases of profit for the 16 problem instances for the three discount levels as the consequence of relaxing the restricted timing of promotions.

Table 4. The average relative increase of profits (in %) by relaxing the restricted timing of promotions

Factors	Level	Promotion discount level		
		10%	20%	30%
Flexibility in production capacity	High	2.96	2.33	2.25
	Low	2.82	2.14	1.99
Margin gap	Low	3.55	2.63	2.50
	High	2.22	1.96	1.62
Seasonality	Low	2.19	1.75	1.61
	High	3.59	2.85	2.50
Promotion impact	Low	2.77	1.82	1.68
	High	3.01	2.65	2.56
Loyalty gap	Low	3.25	2.52	2.19
	High	2.53	2.05	1.94
Overall average		2.89	2.27	2.09

The overall averages of the profit increase after relaxing the restricted promotion timing are 2.89%, 2.27% and 2.09% for the discount levels of 10%, 20% and 30%, respectively. These results show the benefits of having higher flexibility in the promotion timing. The relative benefits decrease when the discount level is higher. Applying a deeper discount for one product would expectedly increase the number of customers switching from the competitor's product (recall that when a product is on promotion, the choice probability of a household for the

discounted product will increase). This, in turn, will lessen the effect of forward buying, so that the outcome becomes less sensitive to the timing of the promotions. Also, as expected, relaxing the restriction on timing of promotions also tends to generate more frequent promotions. This is shown in Table 5. Based on the results of this sensitivity analysis, we relax the promotion timing constraint in the full numerical study presented in Section 5.

Table 5. The average number of promotions with restricted and relaxed promotion timing for the discount level of 10%, 20%, and 30%

Factors	Level	Promotion discount level					
		10%		20%		30%	
		GA1	GA2	GA1	GA2	GA1	GA2
Flexibility in production capacity	High	3.63	7.06	2.75	3.38	1.81	3.13
	Low	3.38	5.69	2.06	3.31	0.81	1.88
Margin gap	Low	3.63	7.88	3.19	4.69	1.88	3.25
	High	3.38	4.88	1.63	2.00	0.75	1.75
Seasonality	Low	3.44	5.00	1.69	2.56	0.88	0.81
	High	3.56	7.75	3.13	4.13	1.75	4.19
Promotion impact	Low	2.94	4.44	2.25	2.94	1.25	2.25
	High	4.06	8.31	2.56	3.75	1.38	2.75
Loyalty gap	Low	3.81	6.63	2.63	3.88	1.44	3.06
	High	3.19	6.13	2.19	2.81	1.19	1.94
Overall average		3.50	6.38	2.41	3.34	1.31	2.50

GA1: Promotion only in the first week of each month

GA2: Relaxed promotion timing

5. Numerical study of integrated S&OP in the case of multiple products

In this section, we present the setup and results of our full numerical study on the integrated promotion and production decisions in the case of multiple products. As in the previous numerical study in Section 4, we consider a scenario where there are two internal products in the same product family and one external, competing product. In such a scenario, in addition

to the competition with products offered by competitors, product substitution or cannibalization within the same product family also occurs. This could be due to differences in customer preferences, prices and marketing strategies [41]. To maximise profit, the existence of product substitution should be recognized when determining the ultimate promotion and production plan. We study this base case scenario in Subsection 5.1. For comparisons, in Subsection 5.2 we consider one example with the special case of two identical internal products and one example with two complementary internal products. Finally, in Subsection 5.3 we extend the problem setting to three internal products and one external product.

5.1 The case with product substitution

We use all the parameters and six experimental factors presented in Section 4 and add retailer's pass through rate (PT) as an experimental factor. A retailer's pass-through rate is defined as the proportion of the manufacturer's discount that the retailer passes on to the consumer. This addition would be useful for generating a more complete insight regarding the effect of marketing-related factors. Two levels of pass through rate ($PT=0.6$ and $PT=0.8$) are considered in this numerical study. Thus, in total there are now 192 ($= 32 \times 3 \times 2$) problem instances to be evaluated.

Table 6 presents the average number of promotions for each of the two products, as well as the average number of simultaneous promotions. For each level of the factors, the average values in this table are calculated from 32 problem instances. For the average number of simultaneous promotions, we record the number of occasions where the promotions of products A and B take place in the same week.

In general, a higher discount level tends to result in a lower average number of promotions. The motivation to offer promotions of the two products at the same time could be supported by the saving in the promotion costs, but hindered by the possible cannibalization between the

two products. Furthermore, carrying out promotions at the same time may result in higher production costs, especially when the flexibility is low. A higher number of promotions tends to give a higher chance of getting the same promotion timing for product A and product B. This explains why we observe that the average number of promotions with the same timing is higher in the case of a low discount level than in the case of a high discount level.

Table 6 also shows that high flexibility of capacity (low cost of hiring/firing) tends to yield more promotions than low flexibility (high cost of hiring/firing). This seems reasonable, because more frequent promotions imply that more frequent adjustments of production capacity are necessary. While this finding is also observed in the single-product problem (see Darmawan et al. [30]), one could envisage that ignoring the interdependence between products would most likely lead to similar frequency and timing of promotions for products A and B. However, as shown in Table 6, our multi-product model prevents that to occur, as evidenced by the relatively low frequency averages of the same promotion timing for the two products compared to the average number of promotions for each individual product. The number of promotions with the same timing is higher in the case of high flexibility than in the case of low flexibility. As product A and product B are produced using the same production resources, simultaneous promotions should particularly be avoided when the flexibility of capacity is low. This effect of flexibility extends the results of the marketing literature on promotions, where the production related factors are absent.

Table 6: The average number of promotions for two products with substitution effects

Factors	Level*	Disc: 10%			Disc: 20%			Disc: 30%		
		A	B	Simul- taneous	A	B	Same timing	A	B	Simul- taneous
Flexibility in production	H	3.88	3.44	1.09	1.84	1.88	0.38	1.41	1.56	0.34
	L	3.72	2.91	0.94	1.69	1.50	0.06	1.09	1.09	0.06

Margin gap	L	4.34	4.38	1.53	2.06	2.75	0.31	1.66	1.84	0.28
	H	3.25	1.97	0.50	1.47	0.63	0.09	0.84	0.81	0.03
Seasonality	L	3.41	2.47	0.59	1.03	1.50	0.06	0.31	0.72	0.03
	H	4.19	3.88	1.44	2.50	1.88	0.38	2.19	1.94	0.25
Promotion impact	L	2.47	2.56	0.75	1.31	1.50	0.19	1.16	1.28	0.09
	H	5.13	3.78	1.28	2.22	1.88	0.31	1.34	1.38	0.13
Loyalty gap	L	3.59	3.06	1.13	1.84	1.84	0.25	1.47	1.44	0.25
	H	3.50	3.28	0.41	1.69	1.53	0.16	1.03	1.22	0.06
Pass through	L	3.50	3.06	1.03	1.50	1.44	0.13	1.16	1.22	0.09
	H	4.09	3.28	1.30	2.03	1.94	0.28	1.34	1.44	0.22
Overall average		3.76	3.17	1.00	1.77	1.69	0.22	1.25	1.33	0.15

* L: Low, H: High

The profit margin of a product also affects the chosen number of promotions. The motivation to offer promotions could be reduced when the product's profit margin is low, and this is especially true when promotions generate a high number of units of forward buying. Fewer promotions are observed in Table 6 in the case of high margin gap and are mainly due to the lower profit margin of product B. It is not uncommon that manufacturing firms offer multiple products with margins that are not level, which could be driven by the differences in production (e.g. material), as well as marketing (e.g. degree of competition intensity) factors. Our results suggest that in general these firms need to focus their time and effort on promoting the products with a high margin while limiting the promotion or even applying the '*everyday low price*' policy for the low-margin products.

The average number of promotions with the same timing is higher in the case of low margin gap than in the case of high margin gap. Offering promotions at the same time for products A and B is less preferable when product B has a narrower profit margin. Figure 3 shows, in further detail, the number of promotions with the same timing for the three discount levels and for low and high margin gap, respectively. The horizontal axis in this figure represents the individual problem instances characterized by the level (low or high) of the experimental factors. For example, we use *L-H-L-L-L-L* to represent the problem instance with

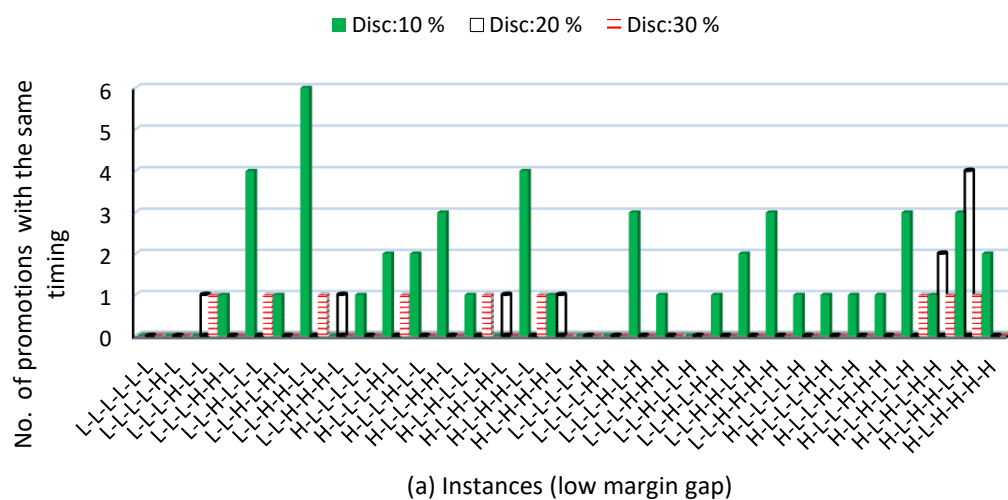
low flexibility, high margin gap, low seasonality, low loyalty, low promotion impact, and low pass through rate.

Table 6 also shows that the average number of promotions with the same timing is higher in the case of low loyalty gap than in the case of high loyalty gap. In the case of high loyalty gap, cannibalization may become more severe such that offering promotions at the same time will not be beneficial for the product with a low loyalty parameter. One of the main aims of offering a promotion is to induce consumers to buy the discounted product instead of the competitor's product. In our numerical examples, a promotion of either product A or B or both is aimed at inducing some of the product C buyers to switch product due to the lower price of A and/or B. In the case of a high loyalty gap, we increase the difference in the brand loyalty parameter values between product A and the other two products. Consequently, offering a promotion on only one of the two products (A and B) may be sufficient in competing with the competitor's product C, because offering a promotion on the second product (B) might just cannibalize the sales of the first discounted product (A).

Our results also show that higher demand seasonality seems to trigger more frequent promotions. In the case of high demand seasonality, offering promotions in the low-demand season will help smooth the demand and production over the planning horizon. As the two products A and B are assumed to have identical expected demand patterns, there is also a higher likelihood to find the same promotion timing for the two products when there is seasonality. Finally, as expected, increasing the promotion impact and pass through rate seems to generate more frequent promotions during the planning horizon. As also observed in the other parameters, more frequent promotions for the two products increase the likelihood of simultaneous promotions. In many practical settings, an actual pass through rate is beyond the manufacturing firms' own control and is often the result of negotiations between manufacturing firms and retailers. Hence, manufacturing firms should carefully consider the

response of retailers to their promotional efforts in order to ensure the effectiveness of their marketing strategies.

The above findings provide useful information for production and marketing planners about aspects they need to consider when deciding on whether joint promotions for products within a product family should be carried out or not. Our numerical study clearly shows that to enhance the effectiveness of a promotion, firms cannot ignore the impact on other products of offering discounts to one product. They should benefit from considering the integrated framework as proposed in this paper, because it enhances profitability by taking into account the issue of ‘sibling rivalry’ in both marketing and production. Our results should also inspire planners on the importance of acquiring and analysing data at a non-aggregate (e.g. SKU) level, without which a good understanding of the underlying factors of promotion effectiveness is hard to obtain.



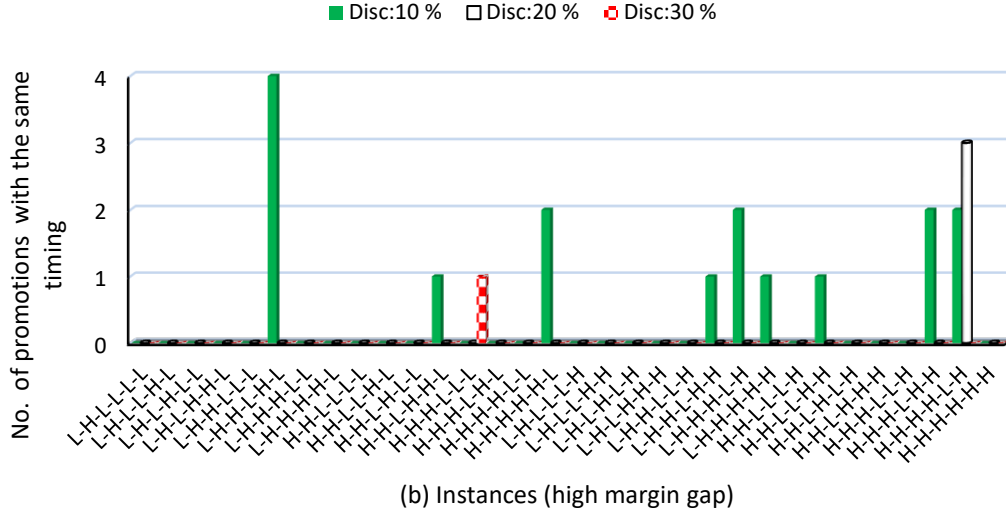


Figure 3. The number of promotions with the same timing with respect to the margin gap

In what follows, we present a more detailed discussion on the effect of the discount level. In Figure 4, we plot the profits for all problem instances differentiated by the three discount levels. The figure shows that in most (but not all) cases a discount level of 20% gives a higher average profit compared to what the other two discount levels provide. In our numerical experiments, we observe that a higher discount level will generate a higher total demand for each particular promotion event. However, since we also have to consider the lower price per unit due to the higher discount level and the promotion and production-related costs, focusing solely on higher total demand for a particular promotion may not necessarily be a good approach. Table 7 presents the averages of total incremental demand for the discount levels of 10%, 20% and 30%, and shows that these averages are highest (in most cases) when the discount level is 20%. Note that the total incremental demand for each discount level is also dependent on the number of promotions carried out throughout the planning horizon. This number is shown in Table 6.

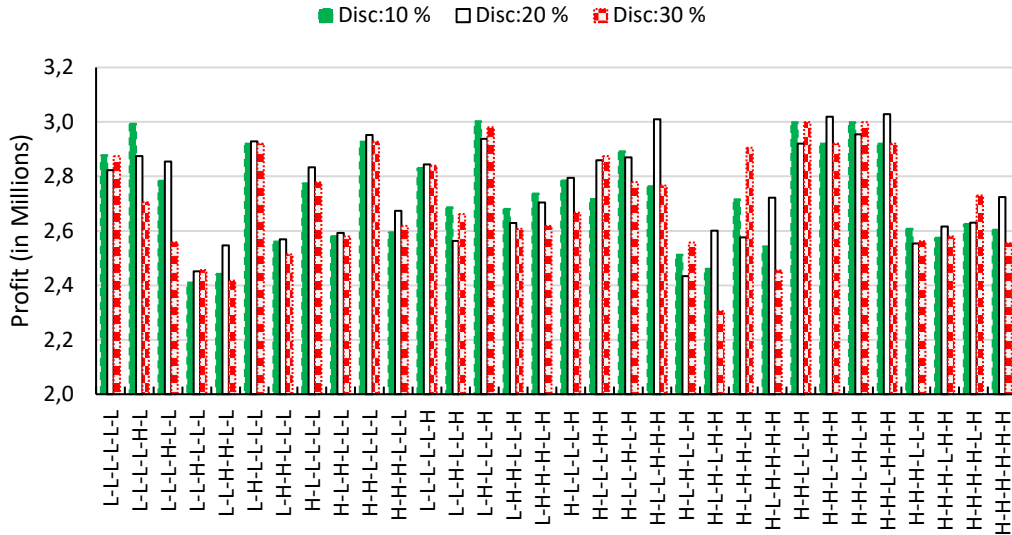


Figure 4 : Profit for each problem instance differentiated by the discount level

Table 7: The average of incremental demand (in %) and its distribution for two products
with substitution effects

Factors	Level*	Disc: 10%				Disc: 20%				Disc: 30%			
		Inc.D	Percentage of Inc.Demand			Inc.D	Percentage of Inc.Demand			Inc.D	Percentage of Inc.Demand		
			Cons.	BS	FB		Cons.	BS	FB		Cons.	BS	FB
Flexibility in production	H	3.62	20.03	47.27	32.71	3.83	28.51	42.64	28.84	3.80	32.68	37.47	29.85
	L	3.47	19.88	48.41	31.71	3.59	27.86	41.89	30.25	2.91	33.64	36.70	29.66
Margin gap	L	4.74	19.10	50.35	30.56	5.38	26.14	43.26	30.61	4.27	32.03	40.44	27.53
	H	2.35	20.87	45.12	34.00	2.05	31.26	36.57	32.18	2.44	35.12	33.18	31.71
Seasonality	L	3.49	21.36	47.22	31.42	3.26	27.79	44.66	27.55	2.58	30.40	45.26	24.35
	H	3.60	18.64	48.38	32.98	4.16	28.52	39.69	31.79	4.13	34.68	32.52	32.80
Promotion impact	L	1.55	11.59	45.36	43.05	1.69	17.14	41.96	40.90	1.76	23.17	41.29	35.54
	H	5.54	28.31	50.28	21.40	5.74	38.52	42.63	18.86	4.95	43.94	36.90	19.16
Loyalty gap	L	3.71	19.52	47.68	32.80	3.75	27.71	42.22	30.07	3.36	34.46	39.27	26.27
	H	3.37	20.42	47.96	31.62	3.67	28.70	42.39	28.92	3.35	31.92	35.11	32.97
Pass through	L	2.70	18.39	48.30	33.30	3.40	26.46	43.74	29.80	2.88	31.84	38.63	29.53
	H	4.39	21.51	47.34	31.15	4.03	29.99	38.88	31.14	3.83	34.45	40.57	24.98
Overall average		3.54	19.97	47.81	32.23	3.71	28.21	41.71	30.07	3.35	33.19	38.11	28.70

* L: Low, H: High

Inc.Demand: Incremental demand; Cons.: Consumption; BS: Brand switching; FB: Forward Buying

As stated above, the demand model we adopt allows us to divide the incremental demand into consumption, brand switching and forward buying. As for illustration, in the problem instances with high flexibility and discount level 20%, the average total incremental demand is 3.83 %, while the averages of increase of consumption, brand switching and forward buying are 28.51%, 42.64%, and 28.84%, respectively. We observe that the distribution of incremental demand is strongly dependant on the discount level.

In Figure 5 we depict incremental demands for products A, B, and C due to promotion for the problem instance with high-flexibility, low-margin gap, high-seasonality, high-promotion effect, high-loyalty gap, high pass through rate, and a 20% discount level. It is interesting to notice that for this specific problem instance, in the high-demand season, promotion is only done once and only for product A. Promotions in the high season would require a higher level of production capacity, and the firm needs to balance all extra costs associated with the changes in capacity against the potential sales increase. In this case, promotion for product B in the high season appears to be undesirable, as it may increase the cost of changing capacity. In addition, as shown in Figure 5, the promotion offered for product A causes a temporary decrease of demand for product B due to brand switching (cannibalization). Thus, the temporary increase of demand of product A is partially levelled out by the temporary decrease of demand of product B, so that the need for a drastic change in production capacity can be avoided. Figure 5 also shows how promotions for product A and/or product B affect the demand of the competitor's product (product C). This type of interaction can only be learned through the consideration of an integrated S&OP in a multi-product setting, which lends support to the need for extension of single-product models such as the one in Darmawan et al. [30].

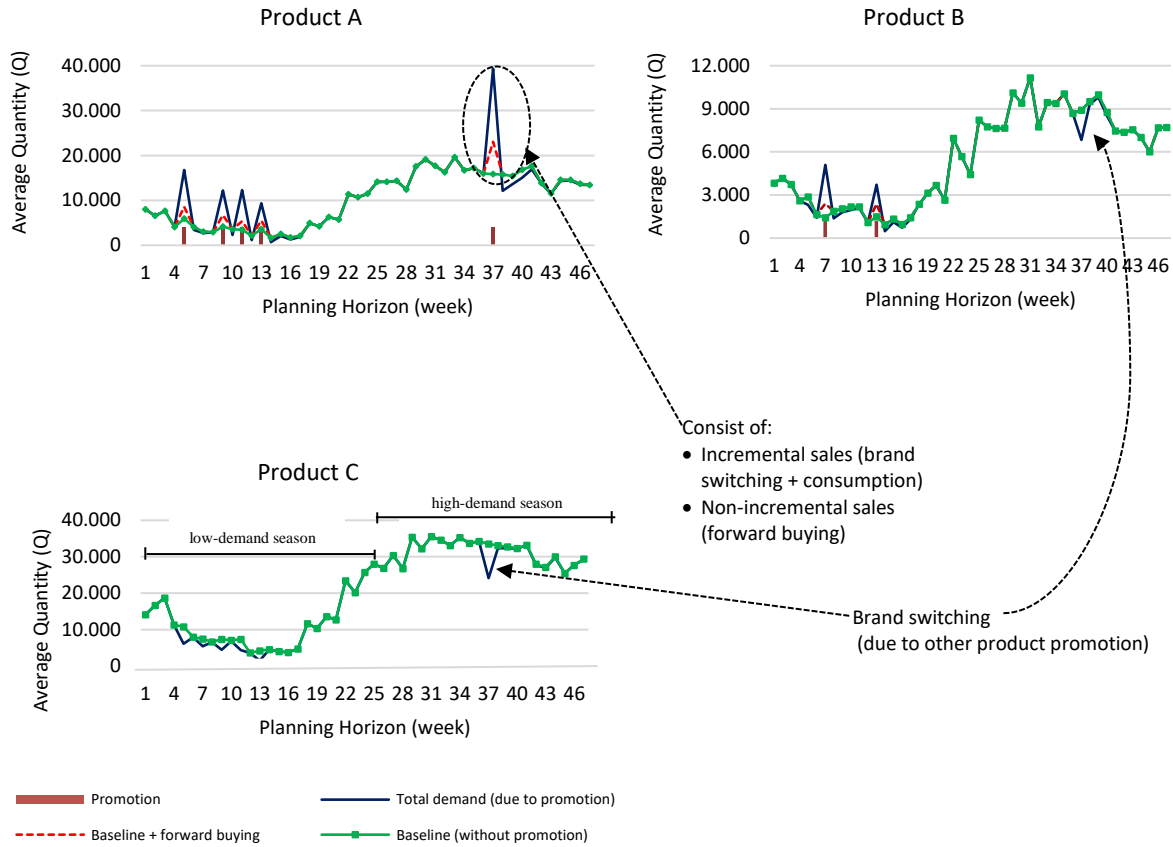


Figure 5. The effect of promotion and cannibalization on demand

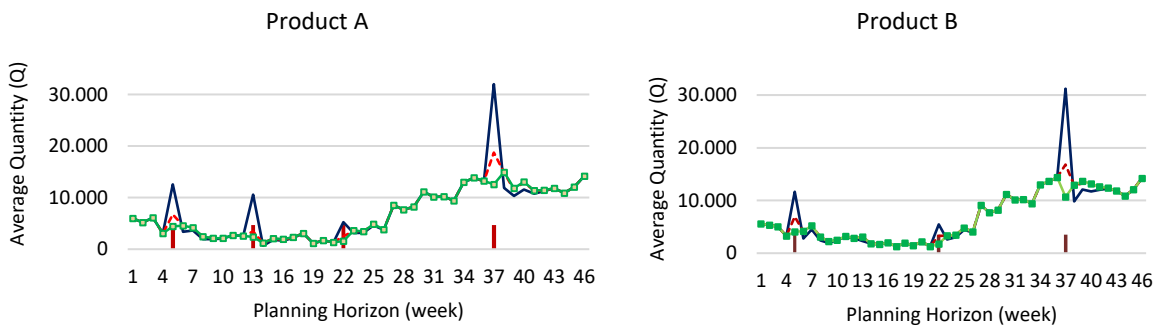
Figure 5 also shows how the incremental demand is obtained from the increase of consumption, brand switching, and forward buying. An illustrative explanation is provided in the figure to show how a promotion event for product A, e.g. in week 37, results in a decrease in demand for products B and C due to brand switching, which contributes to the total increase in demand for product A.

5.2 Insights for cases with identical and complementary products

The main observations discussed above are based on the setting with multiple different and competitive products. In principle, however, due to the general demand model and its ability to accommodate different assumptions, one could also examine a different setting where the two internal products (A and B) are either identical or complementary. In this subsection, we

provide a numerical example for each of these two settings with parameter values modified from the example used in Figure 5.

For the setting with identical products, we take the average values of all the production and marketing related parameters for product A and product B in the current setting, and use those average values as the modified parameter values for the two identical products. The result from using our model in this example is depicted in Figure 6. It shows that the number of simultaneous promotions of the two products is higher in the setting with identical products than in the setting with different products (cf. Figure 5). This observation seems close to the result that would be derived from the single-product setting (Darmawan et al. 2018), where two identical products could be represented as an aggregate single product. The difference between the settings with different and identical products highlights the importance of accurate approximation of the differentiating product parameter values related to both marketing and production. Moreover, the difference in the results also emphasizes that it is important to develop an integrated S&OP model that can accommodate non-identical multiple products, such as the model presented in this paper. Even though the model for multiple products is more complex than the single-product one, the resulting S&OP most likely will provide opportunities for higher profits.



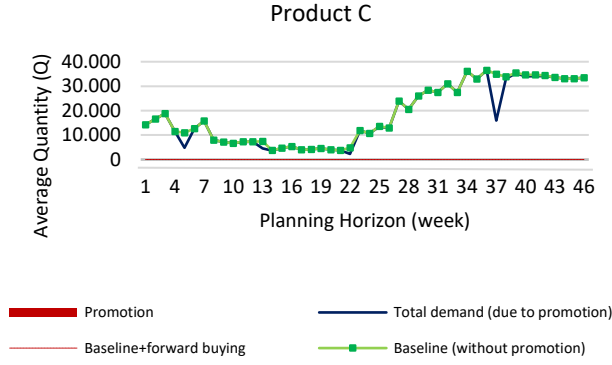


Figure 6. The effect of promotion on demand for two identical products (Products A and B)

For the setting with complementary products, we need to modify the demand model, since the current model considers substitutable products. More specifically, we modify the purchase incidence probability of product B so that it becomes conditional upon demand for product A. In our example, we set the probability equal to 0.8 that a household will buy product B, if it buys product A. In the case with two complementary products, it is expected that the number of simultaneous promotions of the two products will decrease, because promotion of one product will also induce demand of the other. As shown in Figure 7, for this numerical example, there is even no promotion offered for product B in the solution obtained. It can also be noted that the timing of promotions for product A is different from the cases when products are substitutes, although less so when the products are identical (cf. Figure 6). From this example, we conjecture that most of the results presented in this paper are valid only for products that are substitutes. However, there is certainly a need to conduct more thorough research in order to examine integrated S&OP for complementary products.

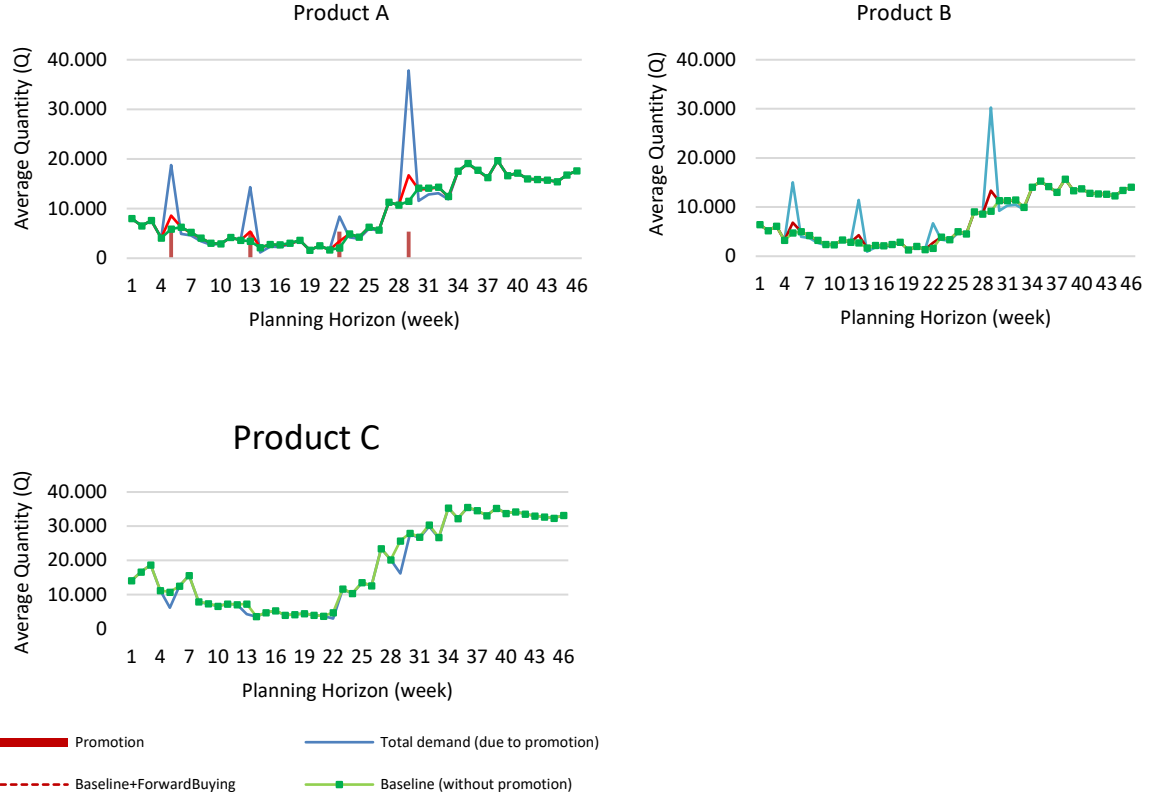


Figure 7. The effect of promotion on demand for two complementary products (A&B)

5.3 Problem size extension

To test how the results may be affected when considering larger problem instances, we have conducted additional numerical experiments in a three-product setting, i.e., the manufacturer offers three product variants (A, B, and C), while there is a fourth product (D) offered by a competitor. We are mainly interested in gaining some further insights regarding the performance of the GA heuristic and the frequency of simultaneous promotions. In these experiments, we compare the results of the GA heuristic with those obtained by the SA

heuristic. We exclude the complete enumeration technique as a benchmark, since the computational burden becomes too heavy.

We include 10 problem instances selected from the previous numerical study with the following modified parameter values. We set $Rp_{At}=12$ (12); $Rp_{Bt}=12$ (11); $Rp_{Ct}=12$ (11); $cp_i=7$ (7); $Rp_{Dt}=10$ (10) for the low (high) margin gap, and $B_A=0.4$ (0.6); $B_B=0.3$ (0.1); $B_C=0.2$ (0.1) for the low (high) loyalty gap. All other parameter values are the same as in Table 1.

Table 8 shows the results for the 10 selected problem instances. We present the profits obtained by both the GA and SA heuristics. As was also seen in the two-product setting (Subsection 4.2), the performance of the GA heuristic appears consistently better than the SA heuristic, though the differences are not significant. Although we cannot compare the results to optimality, the results shown in this numerical study provide further information regarding the consistency of the performance of the GA heuristic benchmarked against the other well-known meta-heuristic in solving the integrated S&OP in a multi-product setting. The average computation times for the GA and SA heuristics are 53.158 minutes and 50.694 minutes for a single instance, respectively, which suggest that for this problem size or larger, there are still computational challenges to overcome in order to be able to solve problems on an operational basis.

Table 8 also shows the frequencies of single promotion for the three products as well as simultaneous promotions. Offering simultaneous promotions of the three products rarely appears in the heuristic solutions. This indicates that simultaneous promotion of all products may generate a higher risk of potential cannibalization between the three products that offsets one of the the main intended effect of promotions, namely to to capture demand from customers who are switching from the competitor's product. Solutions with simultaneous promotions of any two products are also still infrequent.

Table 8: The profit and number of promotions for three products with discount level 20%

Instances	GA								SA							
	Profit	# of promotions							Profit	# of promotions						
		A	B	C	same timing					A	B	C	same timing			
					A&B	A&C	B&C	A,B&C					A&B	A&C	B&C	A,B&C
L-L-L-L-H-H	3,555,416	0	1	3	0	0	1	0	3,508,310	0	1	3	0	0	1	0
L-L-L-L-L-H	3,661,566	0	0	3	0	0	0	0	3,615,979	0	0	3	0	0	0	0
L-L-L-H-H-H	3,668,234	4	5	5	1	1	2	1	3,581,420	3	4	5	1	1	2	1
L-L-L-H-L-H	3,665,782	1	0	1	0	0	0	0	3,637,721	1	0	0	0	0	0	0
L-L-H-L-H-H	3,276,893	0	0	2	0	0	0	0	3,225,180	0	0	2	0	0	0	0
L-L-H-L-L-H	3,351,180	1	1	3	0	0	0	0	3,351,180	1	1	3	0	0	0	0
L-L-H-H-H-H	3,325,744	4	1	0	0	0	0	0	3,299,396	4	1	0	0	0	0	0
L-L-H-H-L-H	3,425,981	4	4	3	1	1	2	1	3,418,471	4	4	2	1	1	1	1
L-H-L-L-H-H	3,517,475	2	1	2	1	1	1	1	3,464,762	2	2	2	1	1	1	1
L-H-L-L-L-H	3,634,371	0	1	1	0	0	0	0	3,634,371	0	1	1	0	0	0	0

6. Conclusions

6.1 Contributions

We have integrated a rich econometric-based demand model and an aggregate production planning model to generate a joint promotion and production plan in an environment, where manufacturers sell a family of separate products that substitute each other and are produced using the same production resources. This multi-product setting constitutes a relevant framework for a sales and operations planning process. The demand model that we have adopted captures the dynamics and heterogeneity of consumer response by combining purchase incidence, consumer choice and purchase quantity. It also allows for taking into account possible substitution among the products. Due to the large problem sizes, we have developed and evaluated a heuristic based on the genetic algorithms for solving the integrated promotion and production planning problem. Our numerical results show that the heuristic performs quite satisfactorily, as indicated by the very small gaps between the heuristic's solutions and the

optimal solutions obtained by complete enumeration. On average, the heuristic also performs better when benchmarked against the simulated annealing heuristic.

We have conducted an extended numerical study to examine how different factors related to marketing and production affect the overall profitability of the promotion and production plan. In particular, we have been interested in understanding if simultaneous promotion events are preferred to sequential promotions in case the manufacturing firm offers multiple products. In general, our results show no evidence of strong preference for implementing simultaneous promotions. The main downside of simultaneous promotions is due to the possible cannibalization between internal products that counteracts the main objective of offering a price discount. In addition, the effectiveness of promotions for increasing profits deteriorates in the case where production capacity changes are costly, i.e. when capacity flexibility is low. However, in the cases with high flexibility, narrow margin gaps, small loyalty gaps, and low discount level, the frequency of simultaneous promotions is higher relative to the other cases.

6.2 Managerial Implications

This paper attempts to address some of the main challenges faced by manufacturing firms producing product substitutes in enhancing the effectiveness of their promotion strategies. The integrated framework presented in this paper can inspire practitioners in developing decision support tools of an integrated S&OP that will help them to coordinate price promotions of their products. Production and marketing planners can use this framework as a basis for negotiations during their planning processes so that the resulting joint decisions would generate greater benefits for the firm as a whole. The possibility of examining the effects of promotion for one product on the demand of other products will most likely be useful for guiding planners to make better decisions. The framework and solution approach we propose address one of the major challenges faced by firms in quantifying and understanding the effect of ‘sibling rivalry’

(Dawes [10]) when developing their promotion strategies. This approach should also motivate them not to focus only on high-level or aggregate data, but also on lower level data representing individual SKUs. With knowledge of the detailed demand structure, it is possible to use the framework presented here to derive reasonably good promotional strategies.

The demand model adopted in this paper can further help production and marketing planners to better understand some of the driving forces of their planning results. In particular, the possibility of decomposing incremental sales into true incremental sales and forward buying helps to provide insightful explanations. This would be very difficult to obtain if one uses a rather simple demand model, as has been the case in most of the previous literature on this topic. The optimal decision on discount level, for example, depends on many inter-related factors. Although our numerical study shows that applying a moderate discount level seems to give the largest net profit in general, there are also cases, where applying a lower or higher discount level is preferable. These cases can be difficult to identify without an integrated decision-support tool for sales and operations planning.

6.3 Future Research

We acknowledge some limitations of this paper and therefore suggest a few topics for future research. First, no strategic interaction between the manufacturer and the retailer is considered in this study. For example, the retailer's pass through rate in a promotion event is assumed to be given. In many realistic settings, however, retailers may respond strategically to the manufacturer's promotion plan by choosing to pass through rates that maximize their own benefits. The existence of such strategic behaviour may have an effect on the optimal production and promotion plans of the manufacturer. Second, the model developed in this paper considers a fixed planning horizon, where demand uncertainties and forecast

inaccuracies have not been incorporated. An interesting research avenue is to extend the modelling framework presented in this paper so that it works based on a rolling instead of fixed planning horizon. This would allow us to further address planning issues met in practice, especially in relation to demand forecast updating. Further analysis of the cost implications of sharing common production resources among products also represents an interesting research avenue. To capture (dis-)economies of scale in production, there is a need to introduce a non-linear (or piecewise linear) model for the production planning problem. Finally, even though the solution method presented in this paper is helpful in providing useful insights into the importance of developing an integrated S&OP in a multi-product setting, it has limitations especially when used for solving large problems, as the computation times are still too long, especially if one considers developing a practical decision support tool. Therefore, designing a more efficient approximation and/or heuristic would certainly represent an important research avenue.

Acknowledgement

The work of the first author is funded by the Directorate General of Resources for Science, Technology and Higher Education (DG-RSTHE) of the Republic of Indonesia.

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Appendix 1: The demand model

Purchase incidence model

The decision that a household will make a purchase in the product category on a store visit is modelled with a binary nested logit model, as presented in (2). The value of the deterministic component of household utility (C_t^h) in the purchase incidence model takes the following form [37,38,43,44]:

$$C_t^h = \beta_0 + \beta_1 F^h + \beta_2 I_t^h + \beta_3 W_t^h \quad h=1, \dots, H; \quad t=1, \dots, T \quad (A1)$$

$$W_t^h = \ln \sum_{j=1}^J e^{A_{jt}^h} \quad h=1, \dots, H; \quad t=1, \dots, T \quad (A2)$$

where

C_t^h The deterministic component of utility related with household h in time period t

F^h Proportion of purchase frequency for household h on store visit

I_t^h Inventory for household h at the end of time period t

$$I_t^h = \text{Max}(0, I_{t-1}^h + \sum_{j=1}^J D_{j,t-1}^h - U_{t-1}^h) \quad (A3)$$

$D_{j,t-1}^h$ Quantity of brand j bought in time period $t-1$ by household h

U_t^h Rate of consumption for household h in time period t

$$U_t^h = I_t^h \left[\frac{\overline{U}^h}{\overline{U}^h + (I_t^h)^\pi} \right] \quad (A4)$$

\overline{U}^h Mean rate of consumption for household h

W_t^h The expected maximum utility from the brand choice decision for household h in time period t .

π Parameter to be estimated

$\{\beta_0, \dots, \beta_3\}$ Parameters to be estimated

Brand choice model

In the brand choice model, we use a multinomial logit form as presented in (3) to calculate probability that a household chooses a particular brand. The value of the deterministic component of brand utility (A_{jt}^h) in (3) is modelled as [37,38,44]

$$A_{jt}^h = \alpha_b + \alpha_s + \theta_1 B_j^h + \theta_2 LB_j^h + \theta_3 S_j^h + \theta_4 LS_j^h + \theta_5 R_{jt} + \theta_6 TC_{jt} + \theta_7 X_{jt} + \theta_8 Y_{jt} \quad (A5)$$

where

A_{jt}^h The deterministic component of utility related with brand j for household h in time period t

B_j^h Brand loyalty of household h to brand j

LB_j^h 1 if j was last brand purchased, 0 otherwise

S_j^h Size loyalty of household h to brand j

LS_j^h 1 if j was last size purchased, 0 otherwise

R_{jt} Regular store price for brand j in time period t

$$R_{jt} = Rp_{jt}(1 + Up) \quad t = 1, \dots, T \quad (A6)$$

Rp_{jt} Regular price from manufacturer of brand j in time period t

Up Store's markup in percent

TC_{jt} Temporary price cut for brand j in time period t

$$TC_{jt} = Rp_{jt}(PT_{jt} \cdot L_{jt}) \quad t = 1, \dots, T \quad (A7)$$

PT_{jt} Store's pass-through in percent

L_{jt} Level of discount in percent for brand j in time period t

X_{jt} $\begin{cases} 1 & \text{if a feature ad is offered for brand } j \text{ in time period } t \\ 0, & \text{otherwise} \end{cases}$

Y_{jt} $\begin{cases} 1 & \text{if a display is offered for brand } j \text{ in time period } t \\ 0, & \text{otherwise} \end{cases}$

α_b	Brand constant to be estimated
α_s	Size constant to be estimated
$\{\theta_1, \dots, \theta_8\}$	Parameters to be estimated

Quantity model

The expected value of the truncated Poisson distribution as presented in (4) is used to calculate the expected quantity purchased by a household. The purchase rate of the household takes the following form [37,38,44]

$$\lambda_{jt}^h = \exp(\mu_b + \mu_s + \omega_1 G^h + \omega_2 I_t^h + \omega_3 B_j^h + \omega_4 S_j^h + \omega_5 R_{jt} + \omega_6 TC_{jt} + \omega_7 X_{jt} + \omega_8 Y_{jt}) \quad (\text{A8})$$

where

λ_{jt}^h	The purchase rate of household h for the brand alternative j in time period t
G^h	Average quantity bought by household h per purchase trip
I_t^h	Inventory for household h at the end of time t
μ_b	Brand constant to be estimated
μ_s	Size constant to be estimated
$\{\omega_1, \dots, \omega_8\}$	Parameters to be estimated

The decomposition of incremental demand.

The expected incremental demand of brand j sold to household h in time period t is obtained by subtracting baseline plus forward buying demand, $E(BFD_{jt}^h)$, from total demand, $E(D_{jt}^h)$,

and adding back borrowed demand that resulted in incremental consumption ($\Delta U_t^h = UBF_t^h - UB_t^h$), as shown in A9 [37]

$$E(\Delta D_{jt}^h) = E(D_{jt}^h) - E(BFD_{jt}^h) + \Delta U_t^h \quad (A9)$$

UBF is the consumption rate for the simulated baseline plus forward buying, and UB is the consumption rate for the baseline. In the baseline plus forward buying model, we remove choice effect such that promotions resulted only in forward buying through purchase acceleration and/or stockpiling by setting no promotion and no purchased feedback in the choice model, and no-incremental consumption as shown in A10 [37]

$$E(BFD_{jt}^h) = P_t^h(inc)_{|I_t^h = IBF_t^h} \times P_t^h(j|inc)_{|No\ promotion, No\ purchased\ feedback} \times E(D_{jt}^h | D_{jt}^h > 0)_{|I_t^h = IBF_t^h} \quad (A10)$$

IBF_t^h is the household's inventory given that promotion effect in the choice model is removed, No purchased feedback eliminates carryover effects (last brand purchased) in the choice model. The expected baseline is given by [37]

$$E(BD_{jt}^h) = P_t^h(inc)_{|I_t^h = IB_t^h, No\ promotion} \times P_t^h(j|inc)_{|No\ promotion, No\ purchased\ feedback} \times E(D_{jt}^h | D_{jt}^h > 0)_{|I_t^h = IB_t^h, No\ promotion} \quad (A11)$$

BD is baseline volume and IB_t^h refers to the household's inventory for the case of no promotions.

Parameter for consumer response model		
Purchase incidence model	Choice model	Quantity model
$\beta_0 = -5.2562$	$\alpha_A = 0.4537$	$\mu_A = 0.0140$
$\beta_1 = 5.2590$	$\alpha_B = 0.8096$	$\mu_B = -0.1356$
$\beta_2 = -0.0201$	$\alpha_C = 0.7432$	$\mu_C = -0.2888$
$\beta_3 = 0.3338$	$\alpha_s = -0.4521$	$\mu_s = -0.0146$
	$\theta_1 = 1.9085$	$\omega_1 = 0.3153$
	$\theta_2 = 0.9154$	$\omega_2 = -0.0097$

$\theta_3 = 2.5672$	$\omega_3 = 0.0428$
$\theta_4 = 0.3876$	$\omega_4 = -0.3135$
$\theta_5 = -4156$	$\omega_5 = -0.0770$
$\theta_6 = 0.4752$	$\omega_6 = 0.3239$
$\theta_7 = 1.2259$	$\omega_7 = 0.5517$
$\theta_8 = 1.1042$	$\omega_8 = -0.0686$

Source: Silva-Risso et al. [37]

Appendix 2: The Algorithms of the GA Heuristic

The specific parameters and variables are:

N	Population size
g	Index for generation ($g = 1, \dots, G$)
$CrossRate$	Crossover rate
$MutRate$	Mutation rate
FV	The fitness value
P_{best}	The best promotion plan so far / personal best solution
$IProfit(P)$	Objective function value when using promotion plan P
$IProfit_{best}$	The best objective function value so far
$IProfit_{lowest}$	The lowest objective function value in each generation

The algorithm consists of the following steps:

Step 1: Choose an initial generation that consists of N chromosomes: $P^{(1)}, P^{(2)}, \dots, P^{(N)}$. Set

$$g = 0.$$

Step 2: Calculate the corresponding demand forecasts $D_{jt}|P^{(i)}$ ($i=1, \dots, N; j=1, \dots, J; t=1, \dots, T$);

Step 3: For each chromosome, solve the aggregate production planning problem; Calculate the objective function value $IProfit(P^{(i)})$, ($i=1, \dots, N$).

Step 4: Find the lowest and best objective function

$$IProfit_{lowest} = \min\{IProfit(P^{(i)})\}$$

$$\text{If } IProfit_{best} = \max\{IProfit(P^{(i)})\} \text{ then } P_{best} = P^{(i)}$$

Step 5: If $g = G$ then select P_{best} and $IProfit_{best}$; otherwise go to Step 6.

Step 6: Calculate the fitness value for each chromosome

$$FV^{(i)} = IProfit(P^{(i)}) - IProfit_{Lowest}$$

Step 7: Selection procedure (combining three ways of picking chromosomes).

7.1 Select the best solution for next generation, $P^{(1)} = P_{best}$

7.2 For each of the existing old chromosomes, calculate the probability

$$Prob^{(i)} = \frac{FV^{(i)}}{\sum_{i=1}^n FV^{(i)}}$$

Generate a random variate $r_1 \leftarrow U(0, 1)$ and pick the chromosome that corresponds to the c.d.f of the fitness value. Repeat this procedure until we pick n_1 chromosomes.

7.3 Generate $N - 1 - n_1$ new chromosomes (to avoid premature convergence)

Step 8 Crossover (two points of crossover)

Form $N/2$ pair of chromosomes. For each pair of chromosomes, e.g.

$$P^{(1)} = (L_{11}^{(1)} \dots L_{1T}^{(1)} \dots L_{j1}^{(1)} \dots L_{jT}^{(1)}) \text{ and}$$

$$P^{(2)} = (L_{11}^{(2)} \dots L_{1T}^{(2)} \dots L_{j1}^{(2)} \dots L_{jT}^{(2)}),$$

Generate $r_2 \leftarrow U(0, 1)$; if $r_2 < CrossRate$ then undergo the following crossover;

Otherwise no crossover

$$\text{Set } Lnew_{jt}^{(1)} = L_{jt}^{(1)} \text{ and } Lnew_{jt}^{(2)} = L_{jt}^{(2)} \quad (j=1, \dots, J; t=1, \dots, T);$$

Generate the borders of cross-over range $x_1 \leftarrow U(0, T)$ and $x_2 \leftarrow U(0, T)$ with $x_1 < x_2$

$$\text{Set } Lnew_{jt}^{(1)} = L_{jt}^{(2)}, \text{ and } Lnew_{jt}^{(2)} = L_{jt}^{(1)} \quad (j=1, \dots, J; t=x_1, \dots, x_2);$$

$$\text{Set } L_{jt}^{(1)} = Lnew_{jt}^{(1)} \text{ and } L_{jt}^{(2)} = Lnew_{jt}^{(2)} \quad (j=1, \dots, J; t=1, \dots, T);$$

Step 9: Mutation (swap mutation)

$$\text{For each chromosome } P^{(i)} = (L_{11}^{(i)} \dots L_{1T}^{(i)} \dots L_{j1}^{(i)} \dots L_{jT}^{(i)})$$

Generate $r_3 \leftarrow U(0, 1)$; if $r_3 < MutRate$ then undergo mutation; Otherwise no mutation

Set $Lnew_{jt}^{(i)} = L_{jt}^{(i)}$

Generate $y_1 \leftarrow U(0, T)$ and $y_2 \leftarrow U(0, T)$

Set $L_{jy_2}^{(i)} = Lnew_{jy_1}^{(i)}$ and $L_{jy_1}^{(i)} = Lnew_{jy_2}^{(i)}$

Step 10: Set $g = g + 1$. Go to Step 2.

Appendix 3: The Algorithm of the SA Heuristic

The specific parameters and variables are:

$Temp^0$	Initial temperature
$Temp^f$	Final temperature
α	Decreasing rate of temperature
$MaxIt$	Maximum number of iterations at each temperature

The algorithm consists of the following steps:

Step 1: Choose an initial promotion plan P_0 , and assign $P_{best} = P_0$,

$Temp = Temp^0$; Calculate the corresponding demand forecasts $D_{jt}|P_0$ ($j=1, \dots, J$; $t=1, \dots, T$) and solve the resulting aggregate production planning problem; Calculate the objective function value $IProfit(P_0)$; Assign $IProfit_{best} = IProfit(P_0)$.

Step 2: Generate a neighbourhood solution, promotion plan P'' .

Step 3: Calculate demand forecasts $D_{jt}|P''$ and solve the aggregate production planning problem;

Calculate the objective function value $IProfit(P'')$.

Step 4: If $IProfit(P'') \geq IProfit(P_{best})$, then $P_{best} = P''$ and $IProfit_{best} = IProfit(P'')$ and go to Step 6; otherwise go to Step 5.

Step 5: Generate $y \leftarrow U(0,1)$. If $y < e^{\frac{-|IProfit(P'') - IProfit(P_{best})|}{Temp}}$, then $IProfit_{best} = IProfit(P'')$, $P_{best} = P''$.

Step 6: Is the number of iterations in temperature $Temp < MaxIt$? If yes, then go to Step 2; otherwise go to Step 7.

Step 7: $Temp = \alpha \cdot Temp$.

Step 8: If $Temp = Temp^f$, then Stop; else go to Step 2.