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A semi-analytical computation of the theoretical uncertainties of the solar neutrino flux

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ABSTRACT

We present a comparison between Monte Carlo simulations and a semi-analytical approach that reproduces the theoretical probability distribution functions of the solar neutrino fluxes, stemming from the *pp*, *pep*, *hep*, ^{7}Be, ^{8}B, ^{13}N, ^{15}O and ^{17}F source reactions. We obtain good agreement between the two approaches. Thus, the semi-analytical method yields confidence intervals that closely match those found, based on Monte Carlo simulations, and points towards the same general symmetries of the investigated probability distribution functions. Furthermore, the negligible computational cost of this method is a clear advantage over Monte Carlo simulations, making it trivial to take new observational constraints on the input parameters into account.

Key words: neutrinos.

1 INTRODUCTION

Over the last century, the progress in particle and nuclear physics has contributed to a thorough understanding of the interior structure of the Sun, and well-constrained solar models, on the other hand, have been used to shed light on the employed input physics. Thus, solar models and observations have led to a better understanding of neutrinos and vice versa (cf. Christensen-Dalsgaard et al. 1996; Bahcall, Basu & Pinsonneault 1998).

To compare model predictions with observations, it is essential to establish theoretical parameter estimates as well as thorough uncertainties for the investigated parameters. One way to obtain these uncertainties, in the case of the theoretical solar neutrino fluxes from different source reactions, is to map the associated probability distribution functions (PDFs) based on a Monte Carlo simulation. Such an analysis has been performed by Bahcall, Serenelli & Basu (2006), including 10 000 standard solar models and 21 relevant input parameters. Recently, Vinyoles et al. (2017) have published yet another Monte Carlo analysis based on yet another 10 000 standard solar models, including updated solar input physics.

While a Monte Carlo analysis is reliable, it is also cumbersome and requires large amounts of computing time. In this paper, we present a semi-analytical method that is computationally light and capable of reproducing the correct theoretical PDFs.

Our work will be presented in the following order: in Section 2, we elaborate on the method used, and specify both the considered neutrino source reactions and the input physics employed. In Section 3, we present the results of our analysis and compare these to the results of the Monte Carlo analyses by Bahcall et al. (2006) and Vinyoles et al. (2017).

2 METHOD

As pointed out by Bahcall & Serenelli (2005), Peña-Garay & Serenelli (2008) and Serenelli, Penã-Garay & Haxton (2013), the logarithm of the relative change in the predicted neutrino flux of any source reaction depends approximately linearly on the logarithm of the relative change in the input parameters. In other words, the predicted neutrino flux shows a power-law dependence on the input parameters:

$$\frac{\phi_i}{\phi_i(0)} = \prod_{j=1}^{N} \left(\frac{\beta_j}{\beta_j(0)} \right)^{\alpha(i,j)}, \quad \alpha(i,j) = \frac{\partial \ln \phi_i}{\partial \ln \beta_j}. \tag{1}$$

Here, ϕ_i denotes the neutrino flux of the *i*th source reaction, β_j refers to the *j*th input parameter and $\phi_i(0)$ and $\beta_j(0)$ are the corresponding values of the flux and the input parameter, respectively, for the chosen reference model.

We have tested the applicability of the approximation given by equation (1), using the Aarhus Stellar evolution code, ASTEC (cf. Christensen-Dalsgaard 2008). We find the approximation to hold true, even when the input parameters are changed by several standard deviations, and obtain results that are in good agreement with the literature (cf. Jørgensen 2015).

Having established an analytical expression for the neutrino flux of each source reaction, and knowing the uncertainties of the input parameters, it is possible to reproduce the PDF of each of the

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considered neutrino fluxes, as we will show in the following. The idea is to select values of β_j, based on the corresponding probability distributions of the input parameters, and to calculate ϕ_i for each selected set of β_j, using equation (1). If the number of sets of β_j drawn from the multivariate distribution function of the input parameters is sufficiently large, the number density of sets drawn from a given region of the corresponding parameter space will reflect the probability density of the said region. Just as in a Monte Carlo analysis, the resulting distribution of ϕ_i will consequently reflect the PDF of the neutrino flux. However, as opposed to a Monte Carlo analysis, no further solar models have to be computed to obtain the fluxes. A handful of models suffices to establish $\alpha(i, j)$.

Assuming that all input parameters are uncorrelated and normally distributed, the obtained 68.27 per cent confidence intervals for the neutrino fluxes will be well approximated by the law of propagation of error:

$$\sigma\left(\frac{\phi_i}{\phi_i(0)}\right) \approx \sqrt{\sum_j \left(\alpha(i, j)\sigma(\beta_j)\right)^2}. \qquad (2)$$

This procedure has been used as the standard approach by other authors as suggested by Villante et al. (2014). However, since the relevant composition variables follow lognormal distributions, as discussed in Section 2.2, the assumptions underlying equation (2) are not fulfilled.

Other authors have suggested to estimate the uncertainties of the neutrino fluxes based on fractional uncertainties in the input parameters (cf. Bahcall & Serenelli 2005):

$$\frac{\Delta\phi_{i,j}}{\phi_i} = \left(1 + \frac{\Delta\beta_j}{\beta_j}\right)^{\alpha(i,j)} - 1. \qquad (3)$$

However, as opposed to the method elaborated in this paper, no clear statistical interpretation of the uncertainty given by equation (3) exists.

2.1 Output: neutrino fluxes

In accordance with Bahcall et al. (2006) and Vinyoles et al. (2017), we distinguish between eight different neutrino fluxes: the fluxes from five neutrino source reactions in the PP chain and three neutrino source reactions that are involved in the CNO cycle. As regards the PP-chain, we look at neutrinos stemming from the $p(p, \mathrm{e}^+\nu_\mathrm{e})^2$H reaction, the $p(e^-p, \nu_\mathrm{e})^2$H reaction, the ^{3}He$(p, \mathrm{e}^+\nu_\mathrm{e})^4$He reaction, the ^{7}Be$(\mathrm{e}^-, \nu_\mathrm{e})^7$Li reaction and the ^{8}B$(, \mathrm{e}^+\nu_\mathrm{e})2^4$He reaction. Neutrinos stemming from these reactions will be referred to as *pp*, *pep*, *hep*, ^{7}Be and ^{8}B neutrinos, respectively. Regarding the CNO cycle, we include the ^{13}N$(, \mathrm{e}^+\nu_\mathrm{e})^{13}$C reaction, the ^{15}O$(, \mathrm{e}^+\nu_\mathrm{e})^{15}$N reaction and the ^{17}F$(, \mathrm{e}^+\nu_\mathrm{e})^{17}$O reaction. Neutrinos stemming from these reactions will be referred to as ^{13}N, ^{15}O and ^{17}F neutrinos, respectively.

2.2 Input parameters

In order to facilitate an easy comparison with Vinyoles et al. (2017), we employ 22 input parameters to characterize the models: age, surface luminosity, the element diffusion rate, two parameters (a and b) parametrizing the opacity (cf. Vinyoles et al. 2017), S-factors for eight nuclear reactions (S_{11}, S_{33}, S_{33}, S_{34}, S_{e7}, S_{17}, S_hep, $S_{1,\,14}$, $S_{1,\,16}$) and the abundances of nine elements (C, N, O, Ne, Mg, Si, S, Ar, Fe).

Using ASTEC, we have also investigated the influence of four additional reactions[1] in the CNO cycle, but we found the respective $\alpha(i, j)$ to be negligibly small, which was to be expected, as ^{14}N$(p, \gamma)^{15}$O is the bottleneck reaction and in accordance with Bahcall et al. (2006) (cf. Jørgensen 2015).

For the sake of an easy comparison, we use the values for $\alpha(i, j)$ that were extracted from the models that enter the Monte Carlo analysis published[2] by Vinyoles et al. (2017). We also use the corresponding uncertainties listed in the bulk text and tables 1 – 3 of the same paper.

The large discrepancies between the solar compositions suggested by different authors have led to debate regarding the associated uncertainties. Bahcall et al. (2006) therefore distinguish between the so-called optimistic uncertainties recommend by Asplund, Grevesse & Sauval (2005) and the so-called conservative (historical) uncertainties. These conservative uncertainties are based on the difference in the abundances presented by Grevesse & Sauval (1998) and Asplund, Grevesse & Sauval (2005), and hence include both statistical and systematic errors. The discrepancies between the determination of the solar surface composition, leading to the conservative uncertainties employed by Bahcall et al. (2006), are clearly a serious concern for determinations of the model neutrino fluxes and their PDFs. However, we note that since this discrepancy is not of a statistical nature it cannot, strictly speaking, be included in the assumed probability distribution of the input parameters. A more rigorous approach is probably to regard models based on the different composition determinations as distinct groups of models, each of which can be compared with the observations. However, a detailed discussion of this matter is beyond the scope of this paper, as our purpose is to merely compare our method with Monte Carlo simulations. Based on the arguments above, we simply employ the spectroscopic uncertainties recommended by Asplund et al. (2009) and Grevesse & Sauval (1998).

To compute the eight investigated neutrino fluxes, it is necessary to know the PDFs of each of the varied input parameters. For simplicity, we assume all input parameters to be normally distributed and thus interpret the uncertainties listed in Vinyoles et al. (2017) as standard deviations. It is worth to stress that the validity of the method presented in this paper does not depend on the assumption that the input parameters are normally distributed. Other probability distributions from which to draw the samples of input parameters may be chosen.

Furthermore, we assume all estimates of $\beta_j(0)$ and the corresponding standard deviations to be uncorrelated, as they follow from different observations and experiments. Note that we do not state that the model parameters are uncorrelated. Obviously, they are correlated. However, the observational constraints are not.

As regards the solar composition, the logarithms of the abundances that are generally quoted in the literature are assumed to follow Gaussian distributions. Consequently, the relative changes in the individual abundances, to which the corresponding values of $\alpha(i, j)$ refer, follow lognormal distributions. We therefore draw the logarithms of the abundances from normal distributions as described above and convert these into relative changes in the abundances before computing the associated changes in the neutrino fluxes.

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[1] The ^{12}C$(p, \gamma)^{13}$N reaction, the ^{13}C$(p, \gamma)^{14}$N reaction, the ^{15}N$(p, \alpha)^{12}$C reaction and the ^{15}N$(p, \gamma)^{16}$O reaction.

[2] Tables containing the derived values of $\alpha(i, j)$ can be found at http://www.ice.csic.es/personal/aldos/Solar_Data.html.

Table 1. Uncertainties of the eight investigated neutrino fluxes, using the abundances by GS98 and AGSS09. Columns labelled 'Percentiles' list the associated 15.87th, 50th and 84.13th percentiles of $\phi_i/\phi_i(0)$, summarizing the empirical probability distribution. σ_- and σ_+ denote the relative difference between the 15.87th and 50th percentiles and between the 84.13th and 50th percentiles, respectively. Columns labelled 'Distribution' list whether the associated probability distribution is well approximated by a normal distribution or a lognormal distribution, according to a Kolmogorov–Smirnov test, at a 5 per cent significance level. 10^6 combinations of input parameters were employed.

	GS98			AGSS09		
Flux	Percentiles	$[-\sigma_-; \sigma_+]$ (percent)	Distribution	Percentiles	$[-\sigma_-; \sigma_+]$ (percent)	Distribution
pp	[0.994; 1.000; 1.006]	[−0.60; 0.62]	Neither	[0.994; 1.000; 1.006]	[−0.58; 0.59]	Neither
pep	[0.990; 1.000; 1.011]	[−1.00; 1.02]	Neither	[0.991; 1.000; 1.010]	[−0.95; 0.97]	Neither
hep	[0.697; 1.002; 1.306]	[−30.4; 30.4]	Neither	[0.699; 1.001; 1.305]	[−30.2; 30.4]	Neither
^{7}Be	[0.929; 0.997; 1.070]	[−6.87; 7.23]	Lognormal	[0.928; 0.997; 1.070]	[−6.92; 7.31]	Lognormal
^{8}B	[0.880; 0.994; 1.122]	[−11.5; 12.8]	Lognormal	[0.879; 0.994; 1.122]	[−11.6; 12.9]	Lognormal
^{13}N	[0.852; 0.993; 1.157]	[−14.3; 16.4]	Lognormal	[0.867; 0.994; 1.138]	[−12.8; 14.5]	Lognormal
^{15}O	[0.837; 0.992; 1.173]	[−15.7; 18.2]	Lognormal	[0.848; 0.993; 1.158]	[−14.5; 16.6]	Lognormal
^{17}F	[0.808; 0.991; 1.213]	[−18.5; 22.4]	Lognormal	[0.825; 0.992; 1.189]	[−16.9; 19.9]	Lognormal

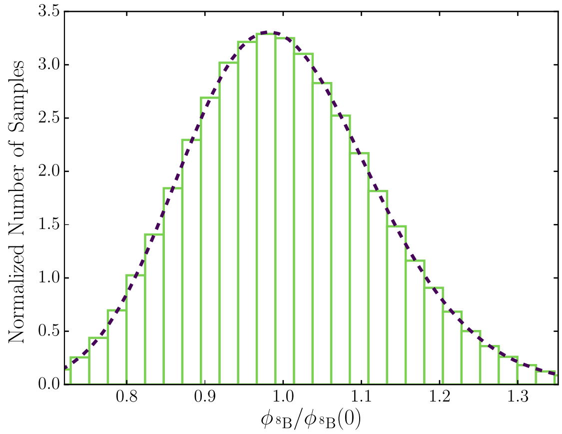

Figure 1. Histograms of the evaluated normalized probability distribution of the relative change in the ^{8}B neutrino flux, based on 10^6 combinations of input parameters drawn randomly from the corresponding normal distributions. We used AGSS09. The plot shows the distribution between the 0.5th and 99.5th percentiles, and includes the best-fitting lognormal distribution.

3 RESULTS

Fig. 1 shows the probability distributions obtained for the ^{8}B neutrino flux, based on 10^6 combinations of the input parameter drawn randomly from the corresponding multivariate Gaussian distribution. Thus, the presented PDF corresponds to the output of a Monte Carlo analysis involving one million standard solar models.

Table 1 summarizes the computed PDFs of the eight investigated neutrino fluxes.

3.1 Comparison with Monte Carlo analyses

As mentioned in the Introduction, Bahcall et al. (2006) and Vinyoles et al. (2017) presented probability distributions for solar neutrino fluxes, based on Monte Carlo analyses, including 10 000 standard solar models. Broadly speaking, both Monte Carlo analyses led to results that are consistent with the confidence intervals summarized in Table 1; a detailed comparison turns out to be fruitful.

First, as can be seen from Table 1, all confidence intervals are asymmetric. Such asymmetries are also found in the cited Monte Carlo analyses. According to Bahcall et al. (2006), the asymmetric probability distributions of the ^{8}B and the CNO neutrino fluxes are more well approximated by lognormal distributions, while the remaining fluxes follow Gaussian distributions. The asymmetries of the former are more pronounced than in this paper. Vinyoles et al. (2017), on the other hand, find the asymmetries to be rather small in all cases, and ascribe Gaussian distributions to all eight neutrino fluxes.

These observed asymmetries of the probability distributions of the neutrino fluxes are largely due to the fact that the composition variables follow lognormal distributions. Thus, in accordance with Bahcall et al. (2006), we find the asymmetries to be most pronounced for the neutrino source reactions that are involved in the CNO cycle.

In order to access the null hypotheses that the empirical probability distributions obtained, using our method, likewise correspond to either a normal or a lognormal distribution, at a 5 per cent significance level, we have employed a Kolmogorov–Smirnov test. In all cases, we have been able to reject the hypothesis that the empirical distribution is well approximated by a normal distribution, while some of the fluxes still seem to follow lognormal distributions. The results are specified in Table 1.This being said, while our analysis is based on 10^6 samples from the relevant parameter space, Bahcall et al. (2006) and Vinyoles et al. (2017) only include 5000 models for two different solar compositions. We have thus rerun our analysis with 5000 samples and could in several cases no longer reject either of the stated null hypotheses (cf. Fig. 2). Thus, our analysis implies that a Monte Carlo analysis, based on a few thousand solar models, may lead to the conclusion that the probability distributions are well approximated by normal and lognormal distributions, while these hypotheses can be discarded in many cases, when including more models.

Secondly, although we find the asymmetries to be more pronounced than Vinyoles et al. (2017), the absolute values of the obtained 15.87th and 84.13th percentiles in Table 1 match the results listed in table 6 of Vinyoles et al. (2017) quite well. This holds true for both compositions considered in this paper: Grevesse & Sauval (1998) and Asplund et al. (2009). In Table 1, these compositions are abbreviated as GS98 and AGSS09, respectively.

In comparison, the confidence intervals listed in Bahcall et al. (2006) are slightly broader, which is largely due to the updated reaction rates, i.e. the fact that the uncertainties on the relevant *S*-factors have been reduced significantly since 2006 (Adelberger et al. 1998, 2011). Thus, when using values of $\alpha(i, j)$ and $\sigma(\beta_j)$ that correspond to the assumptions made in Bahcall et al. (2006), we arrive at confidence intervals that lie close to the results obtained by these authors.

Any changes in $\beta_j(0)$ and $\sigma(\beta_j)$ necessitate a recalculation of the neutrino probability distributions. While a recomputation of a

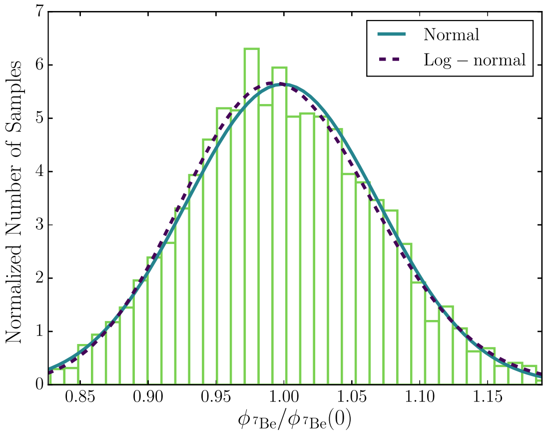

Figure 2. Histograms of the evaluated normalized probability distribution of the relative change in the ^{7}Be neutrino flux, based on 5000 combinations of input parameters drawn randomly from the corresponding normal distributions. We used AGSS09. The plot shows the distribution between the 0.5th and 99.5th percentiles, and includes the best-fitting normal and lognormal distributions.

Monte Carlo analysis is rather cumbersome, updating the probability distributions of the neutrino fluxes, to take new observational constraints into account, can be achieved at a low computational cost, using the method presented in this paper. Severe changes in the input physics that lead to changes in $\beta_j(0)$ may affect $\alpha(i, j)$. Hence, the values listed in the literature differ, depending on the underlying assumptions, such as the composition. However, only a handful of models are needed to reestablish $\alpha(i, j)$.

4 CONCLUSION

We have presented a computationally light semi-analytical approach to evaluate the theoretical probability distributions of the solar neutrino flux for different source reactions. This approach is based on the linear response of the logarithm of the predicted flux to changes in the logarithm of different input parameters. As pointed out by Haxton & Serenelli (2008) and Peña-Garay & Serenelli (2008), this linear relationship can be expressed in a single parameter, $\alpha(i, j)$.

The results obtained from this semi-analytical approach are in good agreement with results obtained from Monte Carlo analyses and reveal the same general symmetries of the PDFs of the neutrino fluxes. Hence, this method reliably provides confidence intervals at any confidence level. Furthermore, the low computational cost of the presented method is a clear advantage over a Monte Carlo analysis. Thus, $\alpha(i, j)$ can be evaluated based on only a handful of solar models. Moreover, as the computational cost of computing the probability distributions is negligible, keeping the uncertainties up to date and including new observational constraints on input parameters are trivial.

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