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AN INTEGRATED VENDOR-BUYER MODEL WITH STOCK-DEPENDENT DEMAND

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Abstract

An integrated vendor-buyer model for a two-stage supply chain is developed and analyzed. The vendor manufactures the product at a finite rate and delivers it in a number of equal-sized batches to the buyer. The items delivered are presented to the end customers in a display area and/or are kept in the buyer's warehouse. The demand is assumed to be positively dependent on the amount of items shown in the display area. The proposed model determines the buyer's optimal shipment quantity and number of shipments, as well as the vendor's optimal production batch. The objective is to maximize total supply-chain profit. The numerical analysis shows that as long as the maximum display area is not used, it is more valuable for the buyer and the vendor to cooperate in situations when the demand is more stock- dependent. It also shows the effect of double marginalization in this integrated vendor-buyer model.

Keywords:

Buyer; Double Marginalization; Integrated; Inventory; Production; Vendor.

1 INTRODUCTION

In order to satisfy customer demands in today's competitive markets, critical information needs to be shared along the supply chain. A high level of coordination between vendors' and buyers' decision making is also required. The concept of joint economic lot sizing (JELS) has been introduced to refine traditional, independent inventory control methods. The idea is to find the more profitable joint production and inventory policy, as opposed to the policy resulting from independent decision making. The basic JELS models have been extended in several different directions. We refer to [1] for a comprehensive review of the JELS literature. It is beyond the scope of this paper to discuss these contributions in detail.

Empirical evidence from the marketing literature shows that consumer demand might indeed vary with inventory on display or on the shelf at the retailer. The investigation in [2] supports the hypothesis that direct shelf-space elasticities are significantly non-zero for many product categories. In particular, they conclude that product categories typical of impulse buying have higher space elasticities. Ref. [3] provides empirical evidence from magazine retailing and demonstrates that the demand for a specific brand decreases as the inventory on the shelf of that brand decreases.

Ref. [4] was among the first to introduce a class of inventory models in which the demand rate is inventory-dependent. Recently, [5] proposed an inventory model to determine the product assortment, inventory replenishment policy, display area and shelf- space allocation decisions that maximize the retailer's profit. Ref. [6] studied a single-store, multi-product inventory problem in which product sales are a composite function of the shelf space.

Ref. [7] contains models for coordinating decentralized two-stage supply chains when demand is shelf-space dependent. They characterize retailers' Nash equilibrium and explore whether the manufacturer can use incentives to coordinate such supply chains. Ref. [8] considers the coordination issues in a decentralized two-stage supply chain, where the manufacturer follows a lot-for-lot policy, and the demand is dependent on the inventory-level on display. The Stackelberg game structure is discussed. This model also provides the manufacturer with a quantity discount scheme to entice the retailer to increase the order quantity. Also recently, [9] proposes an inventory model with both ordering and transfers, where the demand rate depends on the stock level displayed. However, it determines the ordering and transfer schedule based on the buyer's costs only.

In this paper, a joint economic lot-sizing model for coordinating in a centralized supply chain and determining the optimal vendor and buyer policies is proposed and analyzed. The vendor manufactures the product in batches at a finite rate, and delivers it in equal-sized transfer lots (shipments) to the buyer. Some of the delivered items are displayed on the shelves in the buyer's retail store, while the rest of the items are kept in the buyer's warehouse. Final customer demand is positively dependent on the amount of the items shown in the shelf/display area. The objective is to maximize the total system profit when there is centralized coordination between supply-chain members. The result is then compared to the total profit obtained in the corresponding non-coordinated supply chain. It is also shown how the so-called *double marginalization effect* ([10]) impacts the performance of the supply chain in the non-coordinated case.

2 ASSUMPTIONS AND NOTATION

The following assumptions are used throughout this paper in the models proposed:

1. The supply chain consists of a vendor supplying a single product to a buyer.
2. The buyer faces a deterministic consumer demand rate $D(I)$ which is an increasing function of the stock on display I. It has the polynomial form (see also [1] and [8]): $D(I)=\alpha I^{\beta}$, where $\alpha>0$ and $0<\beta<1$ are the scale and the shape parameters, respectively. The shape parameter, β, reflects the elasticity of the demand rate with respect to the stock level on display.
3. There is a limited capacity C_d of the display area, i.e. $I \le C_d$. This limitation could be interpreted as a given shelf space allocated to the product.
4. The vendor has a finite production rate P, which is greater than the maximum possible demand rate, i.e. $P > \alpha C_d^{\beta}$.
5. Inventory at the buyer's warehouse is continuously reviewed. The buyer orders a lot of size Q when the inventory level reaches the reorder point. The items are transferred from the warehouse to the display area in n_b

equal lots of size q until the inventory level in the warehouse falls to zero and a new lot of size Q is delivered. Hence, $Q=n_bq$, where n_b is an integer.

6. At each setup the vendor manufactures a production batch n_vQ, where n_v is an integer and the size of each shipment to the buyer is Q.
7. Both the lead time between the vendor and the buyer, and the lead time between the buyer's warehouse and the display area are constant. However, as demand is deterministic, we may assume (without loss of generality) that both lead times are zero.
8. Inventory holding costs at the vendor and the buyer stages are independent of the unit prices.
9. Shortages are not permitted to occur.
10. The time horizon is infinite.

The cost parameters are:

A_v Vendor's setup cost

A_b Buyer's fixed ordering cost

S Buyer's fixed transfer cost from the warehouse to the display area

c The net unit purchasing price (charged by the vendor to the buyer, and net of the vendor's, i.e. the supply chain's, acquisition costs)

δ The net unit selling price (charged by the buyer to the consumer, and net of the vendor's, i.e. the supply chain's, acquisition costs)

h_v Inventory holding cost per unit per unit time at the vendor stage

h_w Inventory holding cost per unit per unit time at the buyer's warehouse, $h_w>h_v$.

h_d Inventory holding cost per unit per unit time at the buyer's display area, $h_d>h_w$.

We note that the simple cost structure suggested here is in accordance with the theoretical implications for inventory control derived in [11] from an empirical study of handling operations in grocery retail stores.

3 NON-COORDINATED SUPPLY CHAIN

For comparative purposes, we first obtain the independently optimal policies for the vendor and the buyer, respectively. In this non-coordinated case, each supply chain member tries to maximize its own profit. The result is then compared to the coordinated system (in Section 4), where the two parties cooperate.

3.1 Buyer's optimal policy

The objective of the buyer is now to maximize its own profit. The elements of the buyer's profit are as follows: the revenue from selling the product, the fixed cost of ordering from the vendor, the holding cost at the warehouse, the holding cost at the display area, the fixed transfer cost from the warehouse to the display area, and the variable purchasing cost.

The demand rate at time t is equal to the decrease in the inventory level at that time. Therefore, the inventory dynamics $I(t)$ are described by the differential equation

$$\frac{dI(t)}{dt} = -\alpha I(t)^{\beta}, \qquad 0 \le t \le T_d \ ,$$

where T_d is the cycle time. Thus, the (on-hand) inventory at time t can be obtained by solving $I(t)^{-\beta}dI(t)=-\alpha dt$. By integrating both sides, we have

$$\int_0^t I(t)^{-\beta}\, dI(t) = \int_0^t -\alpha dt \ .$$

Hence,

$$I(t)^{1-\beta} - I(0)^{1-\beta} = -\alpha(1-\beta)t$$

As $I(0)=q$, we get

$$I(t) = \left[-\alpha(1-\beta)t + q^{1-\beta}\right]^{\frac{1}{1-\beta}} .$$

Substituting $I(T_d)=0$ into the above expression, we get T_d based on the transfer quantity q as $T_d=q^{1-\beta}/[\alpha(1-\beta)]$.

The buyer's total cost TC_{bw} at the warehouse is obtained as

$$TC_{bw} = \frac{A_b}{T_w} + \frac{h_w}{T_w}\left(\frac{n_b(n_b-1)qT_d}{2}\right),$$

where T_w is specified by $T_w=n_bT_d$. Substituting this and $T_d=q^{1-\beta}/[\alpha(1-\beta)]$ into the above expression and simplifying, the total cost per unit time at the warehouse is

$$TC_{bw} = \frac{A_b\alpha(1-\beta)}{n_bq^{1-\beta}} + \frac{h_w(n_b-1)q}{2} . \tag{1}$$

The buyer's total cost TC_{bd} at the display area consists of the fixed cost of transfer from the warehouse to the display area, and the holding cost at the display area. Therefore,

$$TC_{bd} = \frac{S}{T_d} + \frac{h_d}{T_d}\int_0^{T_d} I(t)d_t = \frac{S}{T_d} + \frac{h_d}{T_d}\left(\frac{q^{2-\beta}}{\alpha(2-\beta)}\right).$$

Substituting $T_d=q^{1-\beta}/[\alpha(1-\beta)]$ into the above expression and simplifying, the vendor's total cost per unit time at the display area is

$$TC_{bd} = \frac{S\alpha(1-\beta)}{q^{1-\beta}} + \frac{h_d(1-\beta)q}{2-\beta} . \tag{2}$$

Total net revenue per unit time is $TR=(\delta-c)q/T_d = (\delta-c)\alpha(1-\beta)q^{\beta}$, and the buyer's total profit is $TP_b= TR-TC_{bw}-TC_{bd}$. Thus,

$$TP_b(q,n_b) = (\delta-c)\alpha(1-\beta)q^{\beta} - \frac{\alpha(1-\beta)(A_b/n_b+S)}{q^{1-\beta}} - \left(\frac{h_w(n_b-1)}{2} + \frac{h_d(1-\beta)}{2-\beta}\right)q . \tag{3}$$

Taking the second partial derivative of $TP_b(q,n_b)$ with respect to q, we get

$$\frac{\partial^2 TP_b(q,n_b)}{\partial q^2} = -(1-\beta)^2 q^{\beta-2}\left[(\delta-c)\alpha\beta + \alpha(2-\beta)(A_b/n_b+S)q^{-1}\right] < 0 .$$

Hence, $TP_b(q,n_b)$ is concave in the transfer quantity q for a given value of n_b. However, there is no closed-form solution for the optimal q. Therefore, we employ a one-dimensional search algorithm to find its optimal value. First, we assume that n_b is a continuous variable. Then, taking the second partial derivative of $TP_b(q,n_b)$ with respect to n_b, we obtain

$$\frac{\partial^2 TP_b(q,n_b)}{\partial n_b^{\,2}} = -\frac{2\alpha(1-\beta)A_b}{q^{1-\beta}n_b^{\,3}} < 0 .$$

Thus, $TP_b(q,n_b)$ is also concave in n_b for a given value of q. Taking the first partial derivative of $TP_b(q,n_b)$ with respect to n_b, and equalizing it to zero, we have

$$\frac{\partial TP_b(q,n_b)}{\partial n_b} = \frac{\alpha(1-\beta)A_b}{q^{1-\beta}n_b^{\,2}} - \frac{h_wq}{2} = 0 ,$$

from which we obtain $n_b^{\,2} = \dfrac{2\alpha(1-\beta)A_b}{h_w q^{2-\beta}}$.

As expected, there is a negative relation between n_b and q: as the transfer quantity decreases, the number of transfers increases. Although theoretically the transfer quantity is only assumed to be greater than zero, for practical purpose, we can assume that it is not less than one. In other words, there is a finite smallest unit for the product which the transfer is based upon. The upper bound for the optimal number of transfers can then be obtained by considering q=1 as the smallest possible quantity. Hence,

$$n_b^{\max} = \left\lceil \sqrt{\frac{2\alpha(1-\beta)A_b}{h_w}} \right\rceil, \tag{4}$$

where the brackets indicate rounding up to the nearest integer. The lower bound for the optimal number of transfers can be obtained by considering $q=C_d$ as the largest possible transfer quantity. Hence,

$$n_b^{\min} = \max\left\{ \left\lfloor \sqrt{\frac{2\alpha(1-\beta)A_b}{h_w C_d^{2-\beta}}} \right\rfloor, 1 \right\}, \tag{5}$$

where the brackets in this case indicate rounding down to the nearest integer. The bounds in (4) and (5) are used in a solution algorithm for finding the optimal number of transfers to the display area.

3.2 Vendor's optimal policy

When the transfer quantity and the number of transfers have been decided by the buyer, orders are received by the vendor at known intervals T_w. The vendor's average inventory is then obtained as

$$\bar{I}_v = \frac{1}{T_v}\left\{ \begin{bmatrix} \left[n_v Q\left(\frac{Q}{P} + \frac{(n_v-1)}{n_v}T_v\right) - \frac{n_v^{\,2}Q^2}{2P} \right] \\ -\left[\frac{T_v Q}{n_v}(1+2+\ldots+(n_v-1)) \right] \end{bmatrix} \right\}$$
$$= \frac{Q}{2}\left((n_v-1)\left(1-\frac{n_v Q}{T_v P}\right) + \frac{n_v Q}{T_v P} \right),$$

where T_v is a cycle time. The vendor's total profit per unit time can then be expressed as

$$TP_v(n_v) = \frac{c n_v Q}{T_v} - \frac{A_v}{T_v} - h_v \frac{Q}{2}\left((n_v-1)\left(1-\frac{n_v Q}{T_v P}\right) + \frac{n_v Q}{T_v P} \right). \tag{6}$$

Substituting $T_v = n_v n_b T_d$, $T_d = q^{1-\beta}/[\alpha(1-\beta)]$, and $Q = n_b q$ into expression (6) and simplifying, we obtain the total profit per unit time for the vendor as

$$TP_v(n_v) = c\alpha(1-\beta)q^{\beta} - \frac{\alpha(1-\beta)A_v}{n_v n_b q^{1-\beta}} - h_v \frac{n_b q}{2}\left((n_v-1) + \frac{(2-n_v)\alpha(1-\beta)q^{\beta}}{P} \right). \tag{7}$$

Taking the first and second derivatives of $TP_v(n_v)$ with respect to n_v (assumed temporarily to be continuous), we obtain

$$\frac{dTP_v(n_v)}{dn_v} = \frac{\alpha(1-\beta)A_v}{n_b q^{1-\beta} n_v^2} - h_v \frac{n_b q}{2}\left(1 - \frac{\alpha(1-\beta)q^{\beta}}{P}\right),$$

$$\frac{d^2 TP_v(n_v)}{dn_v^{\,2}} = \frac{-2\alpha(1-\beta)A_v}{n_b q^{1-\beta} n_v^3} < 0.$$

Hence, $TP_v(n_v)$ is concave in n_v. Therefore, the following optimality conditions can be obtained for n_v

$$n_v(n_v-1) \le \frac{2\alpha(1-\beta)A_v P q^{\beta}}{h_v n_b^2 q^2 (P - \alpha q^{\beta}(1-\beta))} \le n_v(n_v+1). \tag{8}$$

In the non-coordinated supply chain, the buyer chooses its own optimal policy (q^*, n_b^*), and the vendor then chooses its optimal number of shipments n_v^*. Total system profit per unit time, $TP_N(q^*, n_b^*, n_v^*)$, is thus obtained as $TP_N(q^*, n_b^*, n_v^*) = TP_b(q^*, n_b^*) + TP_v(n_v^*)$.

4 COORDINATED SUPPLY CHAIN

Now, we consider the situation in which the two parties cooperate and agree to follow the jointly optimal policy by maximizing the total profit $TP_C(q, n_b, n_v)$. The optimal policy of this coordinated system is found by solving the following problem:

$$\begin{aligned} \underset{q, n_b, n_v}{Max} \quad & TP_C(q, n_b, n_v) = \delta\alpha(1-\beta)q^{\beta} \\ & - \frac{\alpha(1-\beta)\left[A_v/(n_b n_v) + A_b/n_b + S\right]}{q^{1-\beta}} \\ & - \left(\frac{h_w(n_b-1) + h_v n_b(n_v-1)}{2} + \frac{h_d(1-\beta)}{2-\beta} \right) q \\ & - \frac{h_v n_b (2-n_v)\alpha(1-\beta)q^{1+\beta}}{2P} \\ s.t. \quad & 0 < q \le C_d, \\ & n_b, n_v \ integer. \end{aligned} \tag{9}$$

Taking the second partial derivative of $TP_C(q,n_b,n_v)$ with respect to q, we obtain

$$\frac{\partial^2 TP_C(q, n_b, n_v)}{\partial q^2} = -\delta\alpha\beta(1-\beta)^2 q^{\beta-2}$$
$$-\alpha(1-\beta)^2(2-\beta)\left[A_v/(n_b n_v) + A_b/n_b + S\right]q^{\beta-3}$$
$$-\frac{h_v n_b(2-n_v)\alpha\beta(1-\beta^2)q^{\beta-1}}{2P}.$$

It cannot be concluded that $\partial^2 TP_C(q, n_b, n_v)/\partial q^2$ is necessarily negative. Two cases may occur depending on the number of shipments n_v:

Case 1: $n_v \le 2$

All three terms in $\partial^2 TP_C(q, n_b, n_v)/\partial q^2$ are negative, and therefore total system profit is concave in q for known values of n_b and n_v. However, there is no closed-form solution for the transfer quantity q. We then employ a one-dimensional search algorithm to find its optimal value.

Case 2: $n_v > 2$

The first two terms in $\partial^2 TP_C(q, n_b, n_v)/\partial q^2$ are negative. However, the third term is positive.
Rewriting $\partial^2 TP_C(q, n_b, n_v)/\partial q^2$, we have

$$\frac{\partial^2 TP_C(q,n_b,n_v)}{\partial q^2} = q^{\beta-3}(\vartheta_3 q^2 - \vartheta_1 q - \vartheta_2),$$

where

$\vartheta_1 = \delta\alpha\beta(1-\beta)^2$,

$\vartheta_2 = \alpha(1-\beta)^2(2-\beta)\left[A_v/(n_b n_v) + A_b/n_b + S\right]$, and

$\vartheta_3 = h_v n_b (n_v - 2)\alpha\beta(1-\beta^2)/(2P)$.

As can be seen directly, ϑ_1, ϑ_2, and ϑ_3 are all positive. Setting the second partial derivative of $TP_C(q, n_b, n_v)$ with respect to q equal to zero, and solving, we obtain the two saddle points

$$q_1 = \frac{\vartheta_1 - \sqrt{\vartheta_1^2 + 4\vartheta_2\vartheta_3}}{2\vartheta_3},$$

$$q_2 = \frac{\vartheta_1 + \sqrt{\vartheta_1^2 + 4\vartheta_2\vartheta_3}}{2\vartheta_3}.$$

Therefore

$$\frac{\partial^2 TP_C(q,n_b,n_v)}{\partial q^2}\begin{cases} < 0 & if\ q_1 < q < q_2, \\ > 0 & otherwise. \end{cases}$$

Thus, the total profit function is concave between the two saddle points, and convex when $q \le q_1$ or $q \ge q_2$. Moreover, as $\sqrt{\vartheta_1^2 + 4\vartheta_2\vartheta_3} > \vartheta_1$ and $\vartheta_3 > 0$, it follows that $q_1<0$ and $q_2>0$. Therefore, the optimal transfer quantity for any given values of n_b and n_v is the smallest of the local optimum point and the maximum capacity of the display area, C_d.

Taking the second partial derivatives of $TP_C(q, n_b, n_v)$ with respect to n_b and n_v (assuming continuous variables), we obtain

$$\frac{\partial^2 TP_C(q,n_b,n_v)}{\partial n_v^2} = -\frac{2\alpha(1-\beta)A_v}{n_v^3 n_b q^{1-\beta}} < 0,$$

$$\frac{\partial^2 TP_C(q,n_b,n_v)}{\partial n_b^2} = -\frac{2\alpha(1-\beta)\left[A_b + A_v/n_v\right]}{n_b^3 q^{1-\beta}} < 0.$$

Hence, $TP_C(q, n_b, n_v)$ is concave in n_v for given values of q and n_b, and concave in n_b for given values of q and n_v. Taking the first partial derivative of $TP_C(q, n_b, n_v)$ with respect to n_v, and setting it equal to zero, we obtain

$$n_v^{\,2} = \frac{2\alpha(1-\beta)A_v}{h_v n_b^2 q^{2-\beta}\left[1 - q^\beta \alpha(1-\beta)/P\right]},$$

which shows that there is a negative relation between n_b and the optimal n_v. Thus, the maximal optimuml value of n_v for a given q is obtained when n_b=1.

In order to find the maximal optimum value of n_v, we also need to find the minimum of $\gamma = q^{2-\beta}\left(1 - q^\beta \alpha(1-\beta)/P\right)$.

Taking the second derivative of γ with respect to q, we find the saddle point q' to be

$$q' = \left(\frac{(2-\beta)P}{2\alpha}\right)^{1/\beta},$$

where γ is convex when $q \le q'$, and concave when $q > q'$. Taking the first derivative of γ with respect to q, and equalizing it to zero, we obtain the local minimum and maximum as

$$q_{Local\min} = 0,$$

$$q_{Local\max} = \left(\frac{(2-\beta)P}{2\alpha(1-\beta)}\right)^{1/\beta}.$$

As $P > \alpha C_d^\beta \ge \alpha q^\beta$, γ is a positive function. The minimum of γ is obtained at q=0 or $q=C_d$. However, the minimum transfer quantity is *1*. Therefore, the minimum of γ will be

$$\gamma_{\min} = \min\left\{1 - \alpha(1-\beta)/P,\, C_d^{2-\beta}(1 - C_d^\beta\,\alpha(1-\beta)/P)\right\}.$$

Consequently, we have

$$n_v^{\max} = \left\lceil \sqrt{\frac{2\alpha(1-\beta)A_v}{h_v \gamma_{\min}}} \right\rceil. \qquad (10)$$

Similarly, taking the first partial derivative of $TP_C(q,n_b,n_v)$ with respect to n_b, and equalizing it to zero, we have

$$n_b^2 = \frac{2\alpha(1-\beta)(A_b + A_v/n_v)}{h_v q^{2-\beta}\left[1 - q^\beta\alpha(1-\beta)/P\right]n_v + (h_w - h_v)q^{2-\beta} + 2h_v\alpha(1-\beta)q^2/P}.$$

As shown above, $\gamma = q^{2-\beta}\left(1 - q^\beta\alpha(1-\beta)/P\right)$ is positive by assumption. Hence, there is also a negative relation between the optimal n_b and any given n_v. The maximal optimum value of n_b is then obtained for n_v=1. Assuming n_v=1, the above expression reduces to

$$n_b^2\left(n_v = 1\right) = \frac{2\alpha(1-\beta)(A_b + A_v)}{h_w q^{2-\beta} + h_v\,\alpha(1-\beta)q^2/P}.$$

Note, that there is also a negative relation between the optimal n_b and any given q. The maximal optimum value of n_b is then obtained at q=1. Consequently, we have

$$n_b^{\max} = \left\lceil \sqrt{\frac{2\alpha(1-\beta)(A_b + A_v)}{h_w + h_v\,\alpha(1-\beta)/P}} \right\rceil. \qquad (11)$$

Together these results can be used to specify an algorithm that terminates with the optimal solution of the coordinated problem in a finite number of iterations.

5 NUMERICAL STUDY

We first consider a base case with the following data: α=100, β=0.2, P=4500/year, S=\$25/transfer, A_b=\$100/order, A_v=\$400/setup, h_d=\$20/unit/year, h_w=\$5/unit/year, h_v=\$4/unit/year, δ=\$30/unit, c=\$20/unit, and C_d=500. Most of these parameter values have been collected from earlier studies. In order to analyze the effect of stock-dependent demand on the benefits of supply chain coordination, we specify twelve levels for the parameter β. Specifically, $\beta \in [0.00, 0.05, ..., 0.55]$ is used. To represent gains in coordinated versus non-coordinated policies, we define the percentage gain PG as $PG = (TP_C - TP_N)/TP_N \times 100$.

First, for the different values of the demand parameter β in Table 1 we compare the optimal values of the decision variables in the coordinated and non-coordinated supply chain. The buyer's optimal order quantity $Q=n_b q$ is always smaller in the non-coordinated case compared to the coordinated case. However, there are no such general relations between the optimal transfer quantities q, or between the vendor's optimal production batches $n_v Q$ in the coordinated and non-coordinated cases.

Moreover, the percentage gains exhibit an almost bell curve as β increases. Initially, as demand becomes more sensitive to the stock level in the display area, coordination becomes more valuable. However, because of the limited capacity of the display area, the transfer quantity cannot increase further for values of β above a certain level. The decision variables then remain the same. Consequently, the percentage gains start decreasing and eventually vanish.

Second, we examine the effect on the gains of coordination of the unit (net) purchasing price *c* paid by the buyer to the vendor. For this purpose, we use different levels of the purchasing price *c* expressed by the ratio c/δ (for a given value of δ). Specifically, we consider $c/\delta \in [-25\%, 0\%, 25\%, 50\%, 75\%, 100\%]$. However, the purchasing price paid is an internal transfer from one supply chain member (the buyer) to another (the vendor). Hence, it is not a cost for the supply chain as a whole. Therefore, total system profit does not change by the unit purchasing price *c* when the control of the supply chain is coordinated.

Table 2 shows how the absolute difference $D = TP_C - TP_N$ and the relative difference *PG* between total system profits under coordinated and non-coordinated regimes varies by the ratio c/δ. The non-coordinated supply chain's performance is exactly the same as in the coordinated case when the ratio c/δ is approximately -25% (more precisely: -24.2%). In this case, the buyer's optimal order quantity $Q = n_b q$ in the non-coordinated supply chain equals the quantity in the coordinated case. Hence, the net unit purchasing price *c* can be employed as a coordination tool in a supply chain dyad with stock-dependent demand. Furthermore, as c/δ increases, the buyer's marginal profit from selling the product, δ-*c* decreases. The optimal transfer quantity from the warehouse to the display area then becomes smaller when the buyer maximizes its own profit. Therefore, the number of products in the display area is reduced, and as a result, demand as well as total system profit in the non-coordinated case decreases.

Although costless coordination is always beneficial for both supply chain parties, there may be cases when the parties are not inclined to share information. In general, finding the precise value of the net unit purchasing price that achieves perfect coordination between the supply chain members requires information sharing about cost and other parameters similarly to what is required in order to obtain the full coordination solution directly. Because both *PG* and its sensitivity with respect to variations of the ratio c/δ are quite low around zero, we suggest that by choosing *c*=0 as the transfer price, it will be possible to capture most of the potential gains of supply-chain coordination in the case of stock-dependent demand. Using *c*=0 corresponds to a unit transfer price that equals the vendor's unit acquisition cost. The advantage of this arrangement is that the buyer and the vendor can obtain some coordination without further costly sharing of information.

However, Table 2 also shows that the distribution of profit between the vendor and the buyer depends heavily on the ratio c/δ. In particular, the profit for the buyer and for the vendor is negative for large and small unit purchasing prices, respectively. In order to obtain profit levels that are acceptable to both parties a profit sharing mechanism needs to be established For example, a side payment may be agreed upon between the buyer and the vendor in order to share the total profits, and in particular the gains obtained from a more coordinated solution. Again, however, reaching a conceded profit-sharing agreement may require a fair amount of information sharing to motivate the agreement. Note, though, that whenever there is a need for coordination between the supply chain parties, any coordinated solution will require some kind of profit sharing mechanism.

It should be observed that if β=0, then TC_N and therefore *D* and *PG* do not change with *c*. Thus, if there is no relation between the stock in the display area and the consumer demand, then the net unit purchasing price *c* will have no effect on coordination in the supply chain dyad. This demonstrates that it is the stock-dependent demand that creates the *double marginalization effect* discussed above. Ref. [10] originally identified the problem of double marginalization; in a supply chain dyad there is coordination failure if each member of the dyad only considers its own net profit margin when making decisions about quantities that affect profits. We refer to Chapters 6-7 in [12] for further discussions about this problem. In the case treated in this paper, the double marginalization effect also involves the production- and inventory-related setup, ordering, transfer, and holding costs included in the model.

6 CONCLUSIONS

This paper deals with developing an integrated production-inventory model for a two-stage supply chain. The contribution of the paper to the joint economic lot-sizing literature is to add the stock-dependency of demand to existing integrated vendor-buyer models. Hence, a more general model is built in which we assume that demand is not constant but sensitive to the amount of inventory displayed on the retailer's shelves. The main findings can be summarized as follows.

Taking the relationship between demand and stock into consideration, we find a further advantage of supply chain coordination in the form of an increase in the total selling amount (demand). When both parties cooperate or are centrally coordinated, the optimal amount of product in the display area is higher than in the non-coordinated case when the parties optimize individually. As a result, end customer demand, and consequently total system profit, is higher for the coordinated supply chain. Assuming that the capacity limitation of the display area is not binding, the numerical results show a strong relation between the improvements obtained with coordination and the stock-sensitivity of demand. This implies that it is particularly beneficial for supply chain parties to cooperate in cases when consumer demand is increased as more of the product is displayed on the retailer's shelves.

The transfer price agreed between the supply chain parties was also found to be a useful tool for partially coordinating the supply chain with only limited sharing of information. This shows the presence of a double marginalization problem in this model. The vendor and the buyer can improve the coordination of their production and inventory decision variables by appropriately choosing the transfer price. However, in addition they also need to agree on a suitable profit sharing mechanism in order to allocate the net benefits that can be obtained from the improved coordination.

A future research direction might be to extend this study to other product shipment policies, e.g., the geometric shipment policy. It might also be useful to compare coordinated and non-coordinated supply chains under other types of stock-dependency functions for demand and in stochastic demand settings.

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Table 1 : Decision variables of non-coordinated versus coordinated optimization

Parameters:		Non-coordinated optimization:				Coordinated optimization:				Result:	
α	β	q	n_b	n_v	TP_N	q	n_b	n_v	TP_C	$TP_C - TP_N$	PG
100	0.00	19.7	3	2	1952.0	17.7	8	1	2012.6	60.6	3.10
	0.05	22.4	3	2	2245.3	30.4	5	1	2309.4	64.1	2.86
	0.10	32.7	2	3	2624.4	55.8	3	1	2756.9	132.5	5.05
	0.15	39.4	2	2	3150.6	98.1	2	1	3437.8	287.2	9.11
	0.20	49.3	2	2	3855.1	207.1	1	1	4521.9	666.8	17.30
	0.25	95.2	1	2	5222.6	316.2	1	1	6208.1	985.5	18.87
	0.30	135.1	1	2	7033.3	500.0	1	1	8856.9	1823.6	25.93
	0.35	207.8	1	2	9913.8	500.0	1	1	12498.8	2585.0	26.07
	0.40	351.7	1	1	15111.9	500.0	1	1	16953.4	1841.5	12.19
	0.45	500.0	1	1	22345.7	500.0	1	1	22345.7	0.0	0.00
	0.50	500.0	1	1	28785.3	500.0	1	1	28785.3	0.0	0.00
	0.55	500.0	1	1	36337.6	500.0	1	1	36337.6	0.0	0.00

Table 2 :. Effect of purchasing price on non-coordinated and coordinated optimization

Coordinated optimization:			
q	n_b	n_v	TP_C
207.05	1	1	4521.9

Parameters:			Non-coordinated optimization:					Result:	
δ	c/δ	q	n_b	n_v	TP_b	TP_v	TP_N	$TP_C - TP_N$	PG
30	-25%	209.03	1	1	6735.7	-2213.9	4521.8	0.1	0.00
	0%	165.08	1	1	5028.6	-554.6	4474.1	47.8	1.07
	25%	125.30	1	2	3406.5	990.5	4397.0	124.9	2.84
	50%	90.94	1	2	1878.2	2342.0	4220.2	301.7	7.15
	75%	43.12	2	2	487.3	3254.9	3742.2	779.7	20.84
	100%	18.73	4	2	-690.7	3778.9	3088.2	1433.7	46.43

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