New β -delayed proton lines from ^{23}Al

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Abstract. We report on a new measurement of the β -delayed proton spectrum of ^{23}Al . Higher statistics compared to previous measurements allow us to identify new proton lines in the energy range 1–2 MeV. A statistical analysis of the observed β strength shows that the $B(\text{GT})$ values are fully consistent with having a Porter-Thomas distribution. This is indicative of chaotic behaviour and implies that only the average β strength carries physical meaning.

1 Introduction

With a half-life of 446(6) ms [1] and a Q_{EC} value of 12 221.45(84) keV [2], the proton-rich aluminum isotope ^{23}Al provides a good opportunity to study β -decay strength over a wide energy interval. The spin and parity of the ^{23}Al ground state is $5/2^+$ [1,3,4], implying that in allowed decays levels with spins $3/2$, $5/2$, $7/2$ and positive parity are populated in the daughter nucleus, ^{23}Mg . The β -delayed γ spectrum was recently measured by [1,5,6]. β -delayed protons from excited levels in ^{23}Mg above the proton separation energy of 7 580.9(7) keV [2,7,8] have been observed in five previous experiments [5,6,9–11]. Together, the existing measurements of the β -delayed γ and proton spectra map the β strength up to an excitation energy of 8.5 MeV. In the present experiment, we extend the β -strength map to 9.7 MeV by measuring the β -delayed proton spectrum of ^{23}Al with higher statistics compared to previous experiments.

2 Experimental method

The radioactive beams were produced with the Ion Guide Isotope Separation On-Line (IGISOL) technique [12], using a light-ion fusion ion guide [13] and a proton primary beam from the K130 cyclotron of the Accelerator Laboratory of University of Jyväskylä. The ^{23}Al beam was produced through the $^{24}\text{Mg}(p,2n)^{23}\text{Al}$ reaction. The energy

of the proton beam was 40 MeV with a typical intensity of 8–10 μA , and the target was self-supporting 4.3 mg/cm² natural Mg. The mass-separated $A = 23$ beam was implanted in a carbon foil of thickness 26 ± 2 $\mu\text{g}/\text{cm}^2$. The acceleration voltage was 20 kV resulting in the ions, on average, being implanted at a depth of 6.3 $\mu\text{g}/\text{cm}^2$. The average ^{23}Al implantation rate was 2×10^2 per second, though the total $A = 23$ implantation rate was much higher, 5×10^4 per second, because the mass-separated $A = 23$ beam contains vast amounts of ^{23}Mg produced through the $^{24}\text{Mg}(p,pn)^{23}\text{Mg}$ reaction. The present Al-to-Mg ratio is consistent with the ratio of 1:200 reported in [5]. ^{23}Mg is a pure β emitter and gives rise to a large β background below 0.5 MeV in the proton spectrum.

The detection system consisted of four 60 μm thick, double-sided silicon strip detectors (DSSSD) [14], each backed by a 1.5 mm thick, unsegmented silicon detector. The detectors were placed 5 cm from the carbon foil in a rectangular configuration as shown in fig. 1, whereby a solid-angle coverage of 30% was achieved with an angular

Fig. 1. Schematic and simplified illustration of the experimental setup (top view).

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Fig. 2. The β -delayed proton spectrum of ^{23}Al measured during the second run divided into two energy intervals: 0.4–1.0 MeV (a) and 0.8–2.2 MeV (b).

resolution of 3 degrees. The carbon foil was perpendicular to the beam. Data taking was split in two runs, the first lasting 8 hours, the second 22 hours. In between, the setup was used for measuring the decays of ^8B (72 hours) and ^{20}Na (10 hours). At no point during this interval was the vacuum broken. The ^8B and ^{20}Na measurements will be discussed elsewhere [15,16]. The two ^{23}Al runs give consistent results.

The ^{23}Al measurement would have benefited from a γ detector which would have made it possible to observe β -delayed proton branches to excited states in ^{22}Na . However, the detection system was designed with a special view to the ^8B measurement which did not require γ detection.

3 Data analysis and discussion

3.1 Energy calibration

The proton spectrum obtained in the second run is shown in fig. 2 (a) and (b). At the level of sensitivity of the experiment, no proton lines were observed outside the energy interval shown in the figure. The experimental resolution is 26 keV (FWHM). The energy axis was calibrated using the known 557.7(11) keV (see footnote¹) β -delayed proton line of ^{23}Al and the 2153.3(10) and 4433.9(15) keV (see footnote²) β -delayed α lines of ^{20}Na . We take into account the different energy loss of protons and α particles in the carbon foil and detector dead layer using SRIM stopping

¹ Energy deduced from the excitation energy given in [17,18].

² Energies deduced from the excitation energies and α separation energy given in [19].

Fig. 3. Deviations of the present proton energies relative to the evaluated energies [22].

powers [20] and correct for the pulse height defect [21]. The calibration includes a 5 keV quadratic component due to a non-linearity in the electronical response (deduced from measurements with a precision pulse generator) and a 20 keV quadratic component due to the changing pulse shape of physical particles at low energies. See [15] for further details.

3.2 Level energies and intensities

The measured proton energies (E_p) may be converted to ^{23}Mg level energies (E_x) using the relation

$$E_x = \frac{M}{m+M} E_p + S_p, \quad S_p = 7580.9(7) \text{ keV},$$

where m is the proton mass and M is the mass of ^{22}Na . Proton energies and level energies obtained in the present study are given in columns 3 and 5 of table 1. These have been matched with evaluated [22] level energies in ^{23}Mg which are given in column 4. The deviations of the present energies relative to the evaluated energies are shown in fig. 3. The present uncertainties and the uncertainties on the evaluated energies were added in quadrature to obtain the error bars shown. In all cases except one (line 14) the uncertainties on the evaluated energies dominate. The relative intensities obtained in the present study are given in column 7 of table 1. Proton energies and relative intensities obtained in the recent study of [6] are given columns 2 and 6.

A matching level in ^{23}Mg is found in all except two cases, namely, lines 3 and 5 where the best candidates are off by 20 and 30 keV, respectively. For line 13 a matching level is found at $E_x = 9596(7)$ keV but rejected because its spin-parity (tentatively assigned $17/2^+$ based on evidence from a $^{12}\text{C} + ^{12}\text{C}$ reaction study [23]) is incompatible with population in β decay. Most likely, the level seen in the present study and the level seen in the $^{12}\text{C} + ^{12}\text{C}$ reaction study are different but closely spaced levels.

The possibility that lines 3 and 5 result from transitions to *excited* states in ^{22}Na cannot be excluded based on the present study alone, *e.g.*, if one assumes that

Table 1. Energies and relative intensities. E_p is the measured proton energy (*i.e.* in the rest frame of the decaying ^{23}Al nucleus), E_x is the level energy in ^{23}Mg and the relative intensity is with respect to line 2.

No.	E_p (keV)		E_x (keV)		Relative intensity (%)	
	Previous ^(a)	Present	Evaluated ^(b)	Present	Previous ^(a)	Present
...	197(11)	...	7786.9(5) ^(c)	...	34(6)	...
...	255(9)	...	7854.8(12)	...	45(9)	...
...	322(14)	...	7917(15) ^(a)	...	8(3)	...
...	424(14)	...	8017.2(12)	...	4(2)	...
1 ^(d)	554(8)	558.2(12)	8163.9(8) ^(e)	8164.7(12)	69(3)	46.2(8)
2	828(8)	828.8(19)	8453(5)	8448(2)	100	100
3	...	954(2)	...	8579(3)	...	1.55(10)
4	1152(8)	1148(3)	8793(8)	8781(3)	4(1)	3.17(15)
5	...	1204(3)	...	8840(3)	...	1.42(10)
6	1280(9)	1266(3)	8916(6)	8905(3)	6(1)	4.11(16)
7	^(f)	1379(4)	9018(6)	9023(4)	^(f)	0.77(7)
8	...	1454(5)	9103(6)	9102(5)	...	0.78(14)
9	1493(9)	1495(4)	9138(6)	9144(4)	6(1)	4.2(2)
10	1654(24)	1664(4)	9328(8)	9321(4)	4(1)	1.44(10)
11	1763(9)	1760(4)	9421(8)	9422(5)	11(1)	5.21(19)
12	...	1805(5)	9465(6)	9469(5)	...	2.04(13)
13	...	1935(5)	...	9604(5)	...	0.62(7)
14	...	2009(7)	9662(8)	9682(7)	...	0.20(5)

^(a) Ref. [6].^(b) Ref. [22].^(c) Weighted average of [1,22].^(d) Used for calibration.^(e) Value taken from [17,18].^(f) Ref. [6] sees a proton line at 1357(10) keV with relative intensity 5(1)%.

line 5 results from a transition to the 1st excited state at 583.11(9) keV, one obtains a level energy of 9422(3) keV which is consistent with the energy of the 11th level in table 1. Similarly, if one assumes that line 3 results from a transition to the 3rd excited state at 890.81(13) keV, one obtains a level energy of 9468(3) keV which is consistent with the energy of the 12th level in table 1. Interestingly, γ decays from the 1st excited state in ^{22}Na were observed by [1] who estimated that 0.25% of the β -delayed proton decays feed the 1st excited state rather than the ground state. Taking into account the four proton lines below 0.5 MeV identified by [6] (which could not be identified in the present study due to the intense β background), we find that lines 3 and 5 account for 1.1% of the total β -delayed proton branch, which seems to exclude the possibility that they should result from transitions to excited states in ^{22}Na . Nevertheless, the 0.25% branch to the 1st excited state observed by [1] must be present in our spectrum (possibly fragmented in several components) and could easily be hidden under the observed proton lines.

The proton line at 1935(5) keV was also observed by [5] albeit with very few counts. They determine the energy

to 1931(14) keV. Why then, do they not observe the more intense lines between 1 and 2 MeV? Given the number of counts in their 839 keV line, they should have seen a clear signal (13 counts) at 1.78 MeV corresponding to the close-lying proton lines 11 and 12. The two-dimensional inset in their fig. 1 suggests a possible explanation: for proton energies above ≈ 0.9 MeV the ΔE signal is cut off due to detection thresholds. Most likely, the weak signal observed by [5] at 1931 keV is background and the good agreement in energy a pure coincidence.

In the present study, the relative intensity of the two lowest-energy proton lines is determined to 0.462(8) which is significantly lower than the value of 0.69(3) obtained by [6] and the value of 0.7(1) obtained by both [5] and [10]. We think this could be due to a reduction in the intrinsic efficiency of our detection system at low energies not properly accounted for. For proton lines 4, 6, 9 our intensities are in good agreement with those of [6], especially when taking into consideration that the resolution of [6] was insufficient to resolve lines 8 and 9. For lines 10 and 11 there is a significant discrepancy which persists even when taking into consideration that the resolution of [6] was insufficient to resolve lines 11 and 12. We think this

Table 2. Absolute branching ratios, log ft values and spin-parity assignments. The log ft values used for the calculation of the $B(\text{GT})$ distribution are shown in bold.

No.	Absolute branching (%)		log ft		J^π
	Previous ^(a)	Present	Previous ^(a)	Present	Evaluated ^(b)
...	3.95(37) ^(c)	...	3.86(4)	...	(7/2) ⁺
...	0.18(4)	...	5.16(10)	...	(7/2 ⁺)
...	0.03(1)	...	5.90(14)	...	...
...	0.02(1)	...	6.0(2)	...	...
1	0.28(1)	0.190(6)	4.779(17)	4.946(14)	5/2 ⁺
2	0.41(1)	0.41(1)	4.421(12)	4.421(12)	3/2–7/2 ⁺
3	...	0.0064(4)	...	6.14(3)	...
4	0.02(1)	0.0130(7)	5.5(2)	5.67(2)	...
5	...	0.0058(4)	...	5.97(3)	...
6	0.02(1)	0.0168(8)	5.4(2)	5.46(2)	...
7	...	0.0032(3)	...	6.09(4)	...
8	...	0.0032(6)	...	6.01(8)	...
9	0.03(1)	0.0174(9)	5.00(14)	5.24(2)	...
10	0.02(1)	0.0059(4)	5.0(2)	5.54(3)	...
11	0.05(1)	0.0214(9)	4.50(9)	4.87(2)	...
12	...	0.0084(6)	...	5.23(3)	(1/2–9/2) ⁺
13	...	0.0025(3)	...	5.60(5)	...
14	...	0.0008(2)	...	5.99(12)	...

^(a) Ref. [6].^(b) Ref. [22].^(c) Ref. [1].

discrepancy might be an artifact of the poor signal-to-background level of [6].

Finally, we would like to point out that there is a misprint in [22]. The level at 8353(5) keV should be corrected to 8453(5) keV and the level at 8354(5) keV should be corrected to 8334(8) keV [24].

3.3 log ft values

The relative intensities in table 1 have been converted to absolute branching ratios using the 0.41(1)% β -decay branching ratio to the 8448 keV level [6] and further converted to partial half-lives using the 446(6) ms half-life of ^{23}Al [1]. The β -decay phase-space factor has been calculated for each transition using the parametrization of [25]. Following this procedure³, the absolute branching ratios and log ft values given in table 2 result. We have tacitly assumed γ widths to be negligible compared to proton widths. This approximation is justified for $T = 1/2$ levels that are not too close to the $p + ^{22}\text{Na}$ threshold. (For $T = 3/2$ levels, γ decay could potentially compete with

proton decay, even far above threshold, as proton decay is hindered by isospin conservation.) It is not straightforward to quantify the statement “not too close”. We may note, however, that in the studies of [1,5,6,26] no γ decays were observed from the levels listed in tables 1 and 2 except the 7786.9 keV level.

The log ft values are indicative of allowed transitions, implying that the final states have spins 3/2, 5/2, 7/2 and positive parity, consistent with the existing spin-parity information: Level 1 is known to be 5/2⁺. Level 2 is known to have positive parity and spin between 3/2 and 7/2. Level 12 is known to have positive parity with spin probably between 1/2 and 9/2. No previous spin-parity information exists for levels 3–11, 13 and 14.

Figure 4 gives a graphical overview of the β -delayed proton decay of ^{23}Al .

3.4 B(GT) distribution

Using the identity

$$B(\text{GT}) = \frac{g_V^2}{g_A^2} \frac{K}{ft}, \quad (1)$$

where $K = 6144(2)$ s [27] and $g_A/g_V = -1.2694(28)$ [28], we convert the log ft values in table 1 to reduced transi-

³ Except for the 7786.9 keV level, which decays predominantly by γ emission, where we use the β -decay branching ratio given in [1].

Fig. 4. Decay scheme showing β -decay branches of ^{23}Al to proton-unbound levels in ^{23}Mg . Excitation energy, absolute branching ratio and $\log ft$ value, as given in tables 1 and 2, are displayed for each level. For the IAS, the values given in ref. [1] are used. The 7787 keV level mainly decays by γ emission; it has a small proton branch of $\Gamma_p/\Gamma = 3.7\%$ [6]. The IAS decays by γ emission only [6]. The higher-lying levels decay by proton emission only. The first four excited levels in ^{22}Na are shown.

Fig. 5. Comparison of experimental and theoretical $B(\text{GT})$ distributions in the excitation region 7.5–10 MeV. The theoretical $B(\text{GT})$ values already include an overall quenching factor of 0.6. The experimental strength to the isobaric analog state (IAS) at 7802.9 keV is assumed to be pure Fermi strength and, accordingly, is not shown. Theory predicts both Fermi and GT strength to the IAS. Only the GT component is shown.

tion strengths, $B(\text{GT})$, which are shown in fig. 5 (a) along with data from [1] and [6]. In attempt to determine the spin and parity of the observed levels, we compare the experimental $B(\text{GT})$ distribution to the theoretical $B(\text{GT})$ distributions [29] shown in fig. 5 (b) and (c), obtained from shell-model calculations performed with the two new USD Hamiltonians, USDA and USDB [30]. The overall agreement is quite good, but the differences between the two theoretical distributions prevent any firm spin-parity assignments from being made.

In fig. 6 we compare the *cumulative* $B(\text{GT})$ distributions over the entire β -decay window. Within the experimental uncertainties, theory and experiment are in good agreement except for the isobaric analogue state (IAS) at 7802.9 keV, causing the theoretical curves to lie slightly above the experimental curve for $E_x > 8$ MeV. The experimentally determined strength to the IAS, $B(\text{F}) = K/ft = 2.95(15)$ [1], is consistent with being only Fermi strength, $B(\text{F}) = 3$. Theory, on the other hand, suggests that a small GT component, $B(\text{GT}) = 0.14$, is present as well, indicating that the decay of the IAS proceeds through more states than presently established [1, 5]. We note that the theoretical $B(\text{GT})$ values already include an overall quenching factor of 0.6.

Fig. 6. Comparison of cumulative experimental and theoretical $B(\text{GT})$ distributions.

To produce the experimental distribution we used the $B(\text{GT})$ values of [1] up to 7.8 MeV, the $B(\text{GT})$ values of [6] between 7.8 and 8.5 MeV and the $B(\text{GT})$ values of the present study between 8.5 and 11.2 MeV. (We prefer to rely on [6] for the 8165 keV level because the efficiency of our own detection system is ill determined at low energies. See the discussion at the end of sect. 3.2.) Above 9.7 MeV, we do not observe any β strength (dashed red line). The continued increase (solid blue line) represents our estimate of the experimental sensitivity (1 count/keV).

As noted above, the present measurement may have failed to identify a small decay branch to the 1st excited state in ^{22}Na , implying that a small amount of strength is missing from the experimental $B(\text{GT})$ distribution. The amount of missing strength below 9.7 MeV excitation energy is, however, too small to affect the present analysis, nor would it affect the statistical analysis below.

3.5 Statistical analysis

It is noteworthy that the number of states we observe experimentally in the region from 8.1 to 9.7 MeV is the same as predicted in the two shell-model calculations, see fig. 5. Even though a detailed match between experiment and calculations cannot be made it thus seems likely that we have observed all states fed in β decay within our energy window. We shall exploit this to make a further analysis of the $B(\text{GT})$ distribution. Chaotic behaviour in nuclear spectra has been observed at modest excitation energies [31] and a visual comparison of the three panels in fig. 5 suggests that similar fluctuations may also be present here.

The lack of spin-parity information implies that we at most can aim for an analysis of the distribution of $B(\text{GT})$ values. In the chaotic regime, these are expected to follow

a Porter-Thomas distribution [31,32] around the average strength. In principle, each final spin could have a different average strength, but we shall test whether the complete sample can be described in terms of one common average. As alternatives to the Porter-Thomas distribution we shall also test for a Gaussian or an exponential distribution. The tests used are the Empirical Distribution Function goodness-of-fit type [33–35] which are tests free of binning based on the difference between cumulative experimental and theoretical distributions. These tests are known to be more discriminative than, *e.g.*, the χ^2 test, cf. the power studies in [34]. Here, two slightly different tests have been employed, Cramér-von Mises and Anderson-Darling, and give consistent results.

Apart from the 14 experimental $B(\text{GT})$ values, we have also selected 14 theoretical values in the same excitation range for analysis (one set of values for each of the two calculations). In all three cases, the $B(\text{GT})$ values are inconsistent with a Gaussian distribution (probabilities below 10^{-3} for each data set to be described by a Gaussian) while marginally consistent with an exponential distribution (probabilities around 5% for the experimental data set and the USDB set and around 15% for the USDA set). All three sets are perfectly consistent with having a Porter-Thomas distribution with probabilities in the range 25% to 95%. Having a Porter-Thomas distribution implies that only the average strength carries physical meaning, and we conclude that this is the only parameter we can extract from the data and compare to calculations. The average $B(\text{GT})$ strength and its estimated uncertainty, determined by fitting the Porter-Thomas distribution to the data, is 0.026(10) for the experimental data, 0.044(17) for USDA and 0.035(14) for USDB, so experiment lies below theory, but they are consistent within the uncertainties.

A similar analysis has not been made at lower excitation energies where, as already mentioned, it is likely that all GT strength has still not been observed. Correcting for this would complicate the analysis.

4 Conclusion

In summary, we have observed a number of new β -delayed proton lines from ^{23}Al with energies between 1 and 2 MeV. In most cases, these lines can be matched with known levels in ^{23}Mg . Excitation energies are determined with improved precision. The $\log ft$ values obtained are suggestive of allowed transitions, thus constraining the spin-parity of the levels populated in ^{23}Mg to $3/2^+$, $5/2^+$, $7/2^+$. A comparison of the experimental strength distribution to shell-model calculations did not allow for firm spin-parity assignments to be made. A statistical analysis of the β strength between 8.1–9.7 MeV showed that experimental as well as theoretical $B(\text{GT})$ values are fully consistent with having a random distribution around a mean value, so that the only physics parameter one can extract is the mean β strength.

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