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Decay of $^{12}\text{C}^*$ into Three α Particles

Using the 3α Momentum Distribution as a Spectroscopic Tool

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Abstract Complete-kinematics data from a measurement of the decay of $^{12}\text{C}^*$ into three α particles are analyzed. Owing to the symmetries of the three- α final state, the Dalitz plot exhibits nodes (regions of vanishing intensity) characteristic of the spin and parity of the system. Exploiting this property, we argue that a broad unidentified structure in the excitation spectrum around 12.4 MeV is likely to have unnatural spin–parity and $J \geq 4$.

1 Introduction

When the excitation energy imparted to the ^{12}C nucleus exceeds 7.275 MeV, it becomes unstable with respect to decay into three α particles. We thus witness the transformation of a many-body fermionic system consisting of 12 nucleons into a few-body bosonic system consisting of three (greatly separated) α particles. The contrasting natures of the initial and the final state were, in a certain sense, already pointed out by Schrödinger in the late twenties. During a seminar held in Berlin he supposedly remarked,

“Just because you see alpha particles coming out of the nucleus, you should not necessarily conclude that inside they exist as such.” [1]

Schrödinger was right, of course. However, for some excited states in ^{12}C the picture of three, closely separated, α particles is believed to provide a good *approximation* of the nuclear structure. A notable example is the Hoyle state at 7.654 MeV, situated only 379 keV above the triple- α threshold and famous for its important role in the synthesis of carbon in stars [2]. Clustering is a common phenomenon in nuclear physics and generally occurs close to decay thresholds [3, 4].

In experiments, we measure the properties of the *final* state, such as the 3α momentum distribution, but often we are interested in the properties of the *initial* state. The challenge is to *deduce* the latter from the former. This usually involves (though it does not necessarily require) a detailed understanding of the mechanism by which the initial state transforms into the final state.

As discussed below, the spin and parity of the system, J^π , is “encoded” into the 3α momentum distribution in a way that allows us to determine, or at least constrain, J^π without a detailed understanding of the decay process. The 3α momentum distribution thus serves as a spectroscopic tool. In contrast, it is unclear how and to

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which extent structural¹ information can be obtained from the 3α momentum distribution. It might well be that the 3α momentum distribution is sensitive to the properties of the 3α system that *forms* in the decay process while largely *insensitive* to the structure of the initial nuclear state [5]. In a previous paper [6], we tested this hypothesis by comparing the 3α momentum distributions measured for two well-known states in ^{12}C with the predictions of a sophisticated three-body model [7]. Experiment and theory were in reasonable, but not perfect, agreement, indicating that the 3α momentum distribution *is* sensitive to the many-body fermionic structure of the initial nuclear state (which is not properly taken into account in the three-body model).

The question of (in)sensitivity to nuclear structure will not be pursued any further here. Instead, we explore the ability of the 3α momentum distribution to serve as a spectroscopic tool. In our previous paper [6], this ability was successfully exploited to determine the spin and parity of a state at 13.35 MeV. Here, we consider a broad unidentified structure around 12.4 MeV. The experimental data are from a complete-kinematics measurement of the reaction $^{11}\text{B}(^3\text{He}, d\alpha\alpha\alpha)$ at 8.5 MeV. Data from a similar measurement of the reaction $^{10}\text{B}(^3\text{He}, p\alpha\alpha\alpha)$ at 4.9 MeV are used for cross-checking; see Refs. [6, 8, 9] for further details.

2 Nodes of the Dalitz Plot

Assuming an unpolarized initial state, the measurement of two α -particle energies suffices to determine the 3α momentum distribution. In other words, the measurement provides complete kinematic information. The data are best visualized using the symmetric Dalitz plot shown in Fig. 1a which is particularly adapted to the case of three particles of equal mass [10]. A useful property of the Dalitz plot is that pure phase-space decays result in flat distributions. (The intensity is proportional to the transition matrix element squared.)

As first discussed by Zemach [11] in the context of three- π decay and soon after discussed by MacDonald [12] in the context of three- α decay, the Dalitz plot of a decay to three identical spin-zero particles is strongly influenced by the symmetries of the final state. As illustrated in Fig. 1b, the Dalitz plot exhibits nodes (regions of vanishing intensity) that are characteristic of the spin and parity of the system and does so irrespective of the decay mechanism. The nodes thus serve as a model-independent spectroscopic tool to determine, or at least constrain, the spin and parity of states in ^{12}C .

Notice that the mere existence of a node in the Dalitz plot cannot be used as conclusive evidence for a particular spin-parity assignment because there could also be dynamical reasons for the vanishing intensity. (The absence of a node can, however, always be used to falsify a proposed spin-parity assignment.) As an example of dynamical influence, consider a resonant interaction between two α particles. Provided the two-body resonance lives long enough, the decay may be treated as sequential. It follows that the intensity of the Dalitz plot will be confined to bands like those shown in Fig. 1c, the position and width of which reflect the energy and width of the two-body resonance. See Refs. [6, 13] for further discussion of the sequential decay mechanism.

3 The Excitation Region Around 12.4 MeV

In Fig. 2a the centre-of-mass energy of the deuteron is plotted against the centre-of-mass energy of a randomly chosen α particle. Excited states of ^{12}C appear as vertical bands of increased intensity. Reactions passing through the narrow ground state of ^8Be have been removed by placing an anti-gate on the α - α invariant-mass spectrum. There appears to be no significant contribution neither from the $\alpha + ^{10}\text{B}^*$ channel (horizontal bands), nor from the $^6\text{Li}^* + ^8\text{Be}^*$ channel (diagonal bands), except for reactions passing through a narrow state in ^6Li at 2.186 MeV, which were removed by placing an anti-gate on the d - α invariant-mass spectrum. There are two other states in ^6Li that could potentially contribute [14]. They are found at 4.312 and 5.65 MeV and should appear as diagonal bands at $E_d + E_\alpha \approx 5.1$ and 5.7 MeV. However, their broad nature ($\Gamma = 1.3$ and 1.5 MeV) makes them difficult to observe. In the following, we assume their contribution to be negligible. The projection on the deuteron axis is shown in Fig. 2b. The shaded area indicates the excitation region selected for the Dalitz-plot analysis, $E_x = 12.25$ –12.50 MeV. The contribution of the tails of the neighboring states is estimated to be 11 %. The estimate was obtained by fitting the relevant states separately, then integrating the tails in the 12.25–12.50 MeV region. For the 12.71 MeV state, which is much narrower than the experimental resolution, we used a Gaussian. For the 11.83 MeV state, the width of which is similar to the experimental

¹ To nuclear physicists, the term “structure” carries an intuitive meaning (loosely speaking, the spatial distribution and motion of the nucleons and correlations among them), but a precise definition is only possible in the context of a particular nuclear model.

Fig. 1 **a** Dalitz plot particularly adapted to the case of three particles of equal mass; $\varepsilon_{i,j,k}$ are the particle energies normalized to the decay energy, i.e. $\varepsilon_{i,j,k} = E_{i,j,k}/(E_i + E_j + E_k)$. Momentum conservation limits events to within the inscribed circle. The periphery corresponds to collinear momenta. The Dalitz plot exhibits sixfold symmetry. By selecting $\varepsilon_i > \varepsilon_j > \varepsilon_k$ we “collapse” the data into the shaded region. **b** Regions of the Dalitz plot where the intensity is forced to vanish (nodes) owing to the symmetries of the three- α system are shown in black. The vanishing is of higher order where the black lines and dots overlap. The pattern for a spin $J + 2n$ ($J \geq 2$ and $n = 1, 2, 3, \dots$) is identical to the pattern for spin J except that the vanishing at the center is not required for spins ≥ 4 . **c** Band structure expected for the sequential decay of a state at 12.4 MeV through the 2^+ first-excited state in ^8Be . The dotted line indicate the centroid of the band; the gray area the full width at half maximum (color figure online)

Fig. 2 **a** Deuteron centre-of-mass energy plotted against the centre-of-mass energy of a randomly chosen α particle. Excited states of ^{12}C appear as vertical bands of increased intensity. Reactions proceeding via the narrow ground state of ^8Be or/and the narrow 2.186 MeV state in ^6Li have been removed by placing anti-gates on the α - α and d - α invariant-mass spectra. **b** Projection on the deuteron axis. The peaks have been identified with known states in ^{12}C , labelled by their excitation in MeV and spin-parity. The shaded area indicates the excitation region selected for the Dalitz-plot analysis

resolution, we used a Breit–Wigner folded with a Gaussian. For the 13.35 MeV state, which is much wider than the experimental resolution, we used a Breit–Wigner. In each case we used a quadratic function to model the “background”.

In Fig. 3a we show the Dalitz plot of the selected excitation region. Simulations show [9] that the experimental acceptance is fairly uniform except for the top left corner where the ^8Be ground-state cut removes all events. Fig. 3b–d show three different projections of the Dalitz plot. The projected coordinates, ρ , ξ , and η , are given by,

$$(3\rho)^2 = (3\varepsilon_i - 1)^2 + 3(\varepsilon_i + 2\varepsilon_j - 1)^2, \quad 2\sqrt{3}\xi = 1 - 2(\varepsilon_i - \varepsilon_j), \quad 2\sqrt{3}\eta = 3 - 2(\varepsilon_i + 2\varepsilon_j), \quad (1)$$

where $\varepsilon_{i,j,k} = E_{i,j,k}/(E_i + E_j + E_k)$ are the α -particle energies normalized to the decay energy. The area elements are given by,

$$d\rho = \frac{\pi}{3}\rho d\rho, \quad d\xi = \frac{1}{6} \left(2\sqrt{3}x - 1 + \left[4 - 3(2\sqrt{3}x - 1)^2 \right]^{1/2} \right) dx, \quad x = \xi, \eta. \quad (2)$$

Fig. 3 **a** Dalitz plot of the excitation region $E_x = 12.25\text{--}12.50$ MeV. The dashed (red) lines indicate the ${}^8\text{Be}$ ground-state anti-gate. **b–d** Projections of the Dalitz plot. The geometrical interpretations of the projected coordinates are shown to the right

The ρ projection shows that the Dalitz plot has a node at the periphery, but not at the centre. The ξ projection indicates that the Dalitz plot may have a node in the top left corner (care must be paid to the effect of the ${}^8\text{Be}$ ground-state cut). The η projection indicates that the Dalitz plot may have a node in the right corner. It is, however, difficult to say whether the vanishing at $\xi = 0$ and $\eta = 0$ are of higher order or simply reflect the fact that the distribution has a node at the periphery.

The vanishing at the periphery is suggestive of unnatural parity, and the absence of a node at the centre implies $J \geq 4$. The (possible) presence of a node in the right corner would seem to favor a 5^+ assignment over 4^- . However, it must be remembered that there can also be dynamical reasons for the suppressed intensity in the corners. As Fig. 1c shows, a sequential decay via the broad 2^+ first-excited state in ${}^8\text{Be}$ naturally predicts a reduced (though not vanishing) intensity in the corners.

The three-body model of Álvarez-Rodríguez et al. [15] only predicts one 4^- state. Experimentally, a 4^- state is already known at 13.35 MeV. This would seem to favor a 5^+ assignment. However, a preliminary calculation [16] shows that a very strong three-body potential ($S = -92$ MeV) is required to produce a 5^+ state at the observed excitation energy. The same calculation also predicts the state to be relatively narrow, $\Gamma \sim 0.08$ MeV, whereas the experimental data suggest a rather large width of $\Gamma \gtrsim 0.3$ MeV. Given the low excitation energy, spins $J \geq 6$ appear unlikely. Clearly, more experimental data is needed to further constrain the spin and parity of the excitation region around 12.4 MeV.

We note that the Dalitz plot obtained from the ${}^{10}\text{B}({}^3\text{He}, p\alpha\alpha\alpha)$ data (not shown) is consistent with the one obtained from the ${}^{11}\text{B}({}^3\text{He}, d\alpha\alpha\alpha)$ data (Fig. 3). This strengthens our confidence in the result. We also analyzed the Dalitz plot in the excitation region $E_x = 9.9\text{--}10.6$ MeV and found no evidence of nodes, consistent with the dominant 0^+ character of this excitation region [17, 18].

4 A Four-Body Problem?

As we consider ${}^{12}\text{C}$ states of increasing width, and hence shorter life time, we will eventually be confronted with a four-body problem in which dynamical correlations between the deuteron and the three α particles must be taken into account. One indicator of the necessity of a four-body treatment is the Coulomb energy, E_C , of the $d + {}^{12}\text{C}^*$ system at the time of the three- α decay. The separation of the two bodies at the time of the

three- α decay may be estimated as $v\tau$, where v is the relative speed at infinite separation, and τ is the life time of $^{12}\text{C}^*$. For a state at $E_x = 12.4$ MeV, we find $E_C/E_{3\alpha} = 0.11$ (Γ/MeV), where $E_{3\alpha} = E_x - 7.275$ MeV is the decay energy, and $\Gamma = \hbar/\tau$ is the width of the state. If we require $E_C/E_{3\alpha} < 0.1$, we obtain the upper limit $\Gamma < 0.9$ MeV. Another indicator of the necessity of a four-body treatment is the de Broglie wave length, $\lambda = h/p$. If we picture the deuteron as a wave packet, λ gives a measure of the spatial extension. In this naïve picture, a four-body description becomes necessary when λ is comparable to the distance between the deuteron and $^{12}\text{C}^*$ at the time of the three- α decay. We find that this happens when $\Gamma > 0.7$ MeV. Even though these estimates are very crude, they demonstrate that dynamical four-body correlations may become important when Γ exceeds 0.5–1 MeV.

5 Conclusion

We argued that the 3α momentum distribution (in the form of a Dalitz plot) can serve as model-independent spectroscopic tool, because the special symmetries of the three- α final state cause nodes in the Dalitz plot that are characteristic of the spin and parity of the system. Exploiting this property, we argued that a broad unidentified structure in the ^{12}C excitation spectrum around 12.4 MeV is likely to have unnatural spin-parity and $J \geq 4$.

It remains to be determined what, if anything, the 3α momentum distribution can reveal about the *structure* of the initial state. Other observables, such as the deuteron angular distribution, the orientation of the three- α decay with respect to the reaction plane, and (for natural-parity states) the branching ratio to the ^8Be ground state, can provide further insight.

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