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Hendrik Kaufmann and Robinson Kruse

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Department of Economics and Business
Aarhus University
Fuglesangs Allé 4
DK-8210 Aarhus V
Denmark

Email: oekonomi@au.dk
Tel: +45 8716 5515

Bias-corrected estimation in potentially mildly explosive autoregressive models*

Hendrik Kaufmann[‡], Robinson Kruse[§]

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Abstract

This paper provides a comprehensive Monte Carlo comparison of different finite-sample bias-correction methods for autoregressive processes. We consider classic situations where the process is either stationary or exhibits a unit root. Importantly, the case of mildly explosive behaviour is studied as well. We compare the empirical performance of an indirect inference estimator (Phillips, Wu, and Yu, 2011), a jackknife approach (Chambers, 2013), the approximately median-unbiased estimator by Roy and Fuller (2001) and the bootstrap-aided estimator by Kim (2003). Our findings suggest that the indirect inference approach offers a valuable alternative to other existing techniques. Its performance (measured by its bias and root mean squared error) is balanced and highly competitive across many different settings. A clear advantage is its applicability for mildly explosive processes. In an empirical application to a long annual US Debt/GDP series we consider rolling window estimation of autoregressive models. We find substantial evidence for time-varying persistence and periods of explosiveness during the Civil War and World War II. During the recent years, the series is nearly explosive again. Further applications to commodity and interest rate series are considered as well.

JEL-Codes: C13, C22, H62

Keywords: Bias-correction · Explosive behavior · Rolling window estimation

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‡Leibniz University Hannover, School of Economics and Management, Institute of Statistics, Königsworther Platz 1, D-30167 Hannover, Germany. E-Mail: `kaufmann@statistik.uni-hannover.de`. Phone: +49-511-762-5636.

§Leibniz University Hannover, School of Economics and Management, Institute of Statistics, Königsworther Platz 1, D-30167 Hannover, Germany. E-mail address: `kruse@statistik.uni-hannover.de` and CREATES, Aarhus University, Department of Economics and Business, Fuglesangs Allé 4, DK-8210 Aarhus V, Denmark. E-mail address: `rkruse@creates.au.dk`.

1 Introduction

Measuring and estimating the persistence of time series is a long standing issue in econometrics. The most common framework for assessing the persistence is the autoregressive model. But, a major practical problem is the inherent bias of the conventional OLS estimator. Its bias increases amongst two dimensions: a small sample size and a true autoregressive parameter in the vicinity of unity are disadvantageous. Given a relatively small sample size, it is a complicated task to estimate the persistence if the process is (i) either stationary, but highly persistent, (ii) exhibits a unit root or (iii) is mildly explosive. As we argue, these situations are likely to occur in practice.

In economics, it is a well established fact that most time series are characterized by high persistence or even stochastic trends, see e.g. Nelson and Plosser (1982) and Schotman and Van Dijk (1991). Another important empirical issue is the instability of parameters, which is often observed and documented (see e.g. Stock and Watson, 1994). During the past decade, a literature on structural changes in persistence emerged, see e.g. Chong (2001), Kim (2000), Leybourne, Taylor, and Kim (2007) and Harvey, Leybourne, and Taylor (2006) amongst many others. In order to cope with potential time-variation in the parameters, users often apply the popular rolling window technique. Under these empirically relevant circumstances, the issue of unbiased and efficient estimation of persistence becomes particularly important: Typically, a relatively small window size is chosen. If a bubble or a crisis occurs in this particular window, some economic time series are likely to exhibit explosive behaviour. Leading examples for time series with at least local explosive roots are stock prices (as caused by bubbles, see Diba and Grossman, 1988), price-dividend and price-earnings ratios (as caused by a dominant regime of chartist traders, see Lof, 2012), house and oil prices (due to speculation, see Homm and Breitung, 2012, Clark and Coggin, 2011 and Shi and Arora, 2012), hyperinflation (due to a collapse of a country's monetary system, see Casella, 1989), exchange rates (due to speculation van Norden, 1996 and Pavlidis, Paya, and Peel, 2012) and the US Debt/GDP ratio (due to unsustainable fiscal policies, see Yoon, 2011) amongst others.

The complicated estimation of autoregressive processes in finite-samples sparked a fruitful area of research. Kendall (1954), Shaman and Stine (1988), Tjøstheim and Paulsen (1983), Tanaka (1984) and Abadir (1993) provide analytic derivations of asymptotic expansions which can be used for bias-correction. Approximately median-unbiased estimation is proposed in e.g. Andrews (1993), Andrews and Chen (1994) and Roy and Fuller (2001). Restricted maximum likelihood estimation is considered in Cheang and Reinsel (2000). Bootstrap-based bias-correction procedures have been suggested by e.g.

Hansen (1999) and Kim (2003). Recently, Engsted and Pedersen (2011) compare analytical bias formulas and bootstrapping for stationary VAR models. Indirect inference has been put forward in MacKinnon and Smith (1998) and Gouriéroux, Renault, and Touzi (2000). Jackknifing based on Efron (1979) is recently studied in Chambers (2013). Importantly, we note that the main body of the literature focusses on stationary autoregressive models and on the unit root case while the case of (mildly) explosive behaviour has received less attention.

This work compares the analytic median-bias-correction by Roy and Fuller (2001), the bootstrap technique by Kim (2003) and the Jacknife approach by Chambers (2013) to the indirect inference approach by Phillips et al. (2011), who propose a technique for autoregressive processes, based on the work of MacKinnon and Smith (1998) and Gouriéroux et al. (2000). Indirect inference estimators to correct the small sample bias have a long tradition, e.g. see Gouriéroux, Monfort, and Renault (1993) and Smith (1993). In a recent contribution, Gouriéroux, Phillips, and Yu (2010) extend this principle to dynamic panel data models. The indirect inference estimator allows for explosiveness in addition to highly persistent and unit root behavior, see also Phillips (2012) for a recent contribution on its limit theory. Most competing methods rule out explosive behaviour by construction (i.e. Roy and Fuller (2001) and Kim (2003)). This feature renders the indirect inference estimation approach to autoregressive models particularly attractive. However, the finite-sample properties of the indirect inference estimator are not fully explored and a comprehensive comparison to other popular and successful bias-correction techniques has not been conducted yet.

In our Monte Carlo study, we consider various sample sizes, normal and fat-tailed innovations, ARCH disturbances and misspecification of the autoregressive lag structure. Furthermore, we also study the case where a linear deterministic trend is included in the autoregressive model. We evaluate the performance of the estimators by means of bias and root mean squared errors (RMSE). Our results suggest that all procedures lead to a substantial bias-reduction in most non-explosive cases. The best procedure in terms of bias-reduction is the jackknife, but comes with the costs of an increase in the variance. The indirect inference estimator provides almost the same level of bias-reduction with a remarkably low variance.

We provide a detailed empirical application to a long annual US Debt/GDP ratio from 1791-2011, where we use rolling window estimation to investigate potential instabilities. Our results suggest that persistence is characterized by strong time-variation. Episodes of stationarity, unit root and explosive behaviour are observed. These episodes are related to major wars, peace movements during the Sixties and Seventies, and recent activities in the aftermath of 9/11. Moreover, we consider three further applications to Oil prices, Gold prices and the spread between long-term interest rates in Germany and Greece. All applications stress the importance of bias-correction. In addition, accounting for locally explosive behaviour is relevant in all cases, too.

The paper is organized as follows. Section 2 briefly describes the different estimation techniques. Our simulation results are presented in Section 3. The empirical applications are located in Section 4 while conclusions are drawn in Section 5. The Appendix contains further simulation results.

2 Bias-correction procedures

Point of departure is the inherent bias of the OLS estimator. In order to illustrate the problem, we simulate the empirical performance of the OLS estimator. Therefore, we focus on finite samples and the possibility of mild explosiveness in a simple autoregressive framework:

$$y_t = \mu + \rho y_{t-1} + \varepsilon_t \, . \tag{1}$$

We consider the cases of stationarity and unit roots, i.e. $|\rho| < 1$ and $\rho = 1$, and the case where ρ satisfies $\rho = 1 + c/k_T$, with $c > 0$ and k_T being a sequence tending to infinity such that $k_T = o(T)$ as $T \to \infty$. In the latter case, the autoregressive parameter is local-to-unity in the sense that $\rho \to 1$ as $T \to \infty$. For finite T (as considered in this work), ρ deviates moderately from unity. Asymptotic theory for this case is developed in Phillips and Magdalinos (2007).

The left panel of Figure 1 shows the AR(1) case as in equation (1) for four different sample sizes, i.e. $T = \{30, 60, 120, 240\}$. The true autoregressive parameter ρ (on the x-axis) ranges from 0.6 to 1.2 which measures the persistence of the process. The bias is given on the y-axis. The results confirms the theoretical finding that the bias depends on the true value of the autoregressive parameter. The smaller the sample size, the more severe is the bias. The vicinity of unity is the region where the bias is strongest. Furthermore, it can be seen that the bias reduces for explosive processes and approaches zero at some point, but that the estimation of mildly explosive processes is still heavily biased.

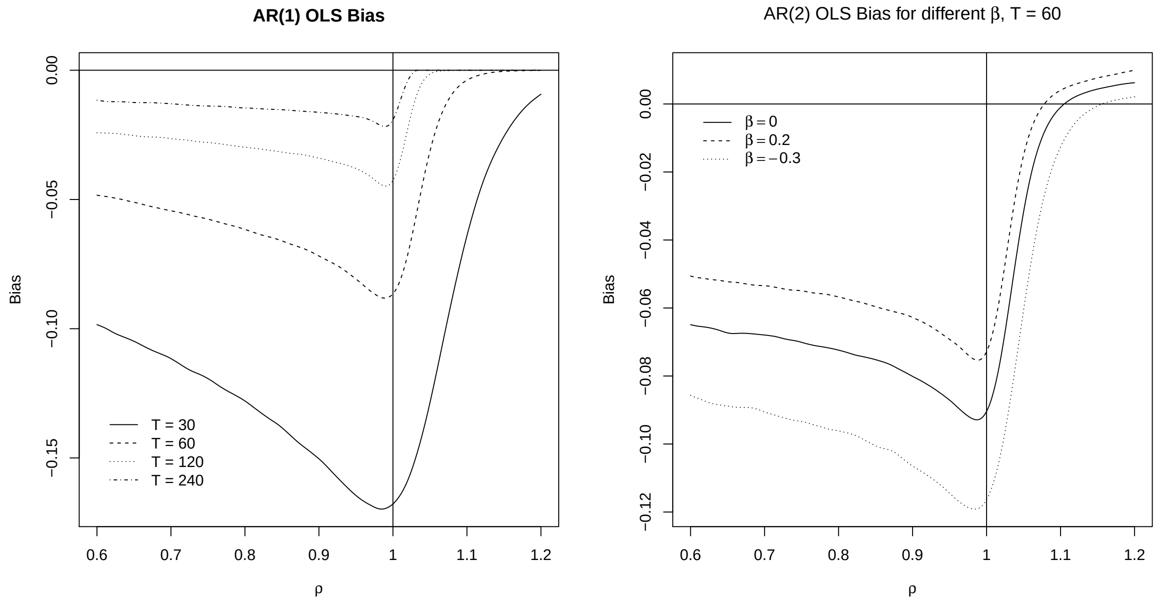

Figure 1: OLS Bias for different values of ρ, β and sample sizes for AR(1) and AR(2) processes (constant included).

As expected, the bias problem persists if we consider the AR(2) process, i.e.

$$y_t = \mu + \phi_1 y_{t-1} + \phi_2 y_{t-2} + \varepsilon_t \ . \tag{2}$$

Since our primary interest is the persistance of the time series, we work with an alternative representation which gathers the persistence in the parameter ρ:

$$y_t = \mu + \rho y_{t-1} + \beta \Delta y_{t-1} + \varepsilon_t \ , \tag{3}$$

where $\rho = \phi_1 + \phi_2$ and $\beta = -\phi_2$. The usefulness of this approach stems from the fact that a direct relationship to the cumulative impulse response $(1/(1-\rho))$ exists (for stationary autoregressive processes). Moreover, it is also directly connected to the spectrum at frequency zero which measures the low-frequency autocovariance. It is given by $var(\varepsilon_t)/(1-\rho)^2$.[1] The right panel of Figure 1 shows the bias for ρ for three different values of β: -0.2, 0 and 0.3. The bias depends substantially on the value of β. Positive values decrease the bias and vice versa. A comparison between the AR(1) case for $T = 60$ and the AR(2) case for $T = 60$ and $\beta = 0$ shows that the estimation of an additional, but unnecessary, parameter increases the bias slightly. These results motivate the development of bias-correction techniques. Four different methods are briefly discussed in the following.

[1] Alternative measures of persistence are the largest autoregressive root, see Stock (1991) for its median-unbiased estimation, and the half life of a unit shock, see Rossi (2005).

2.1 Roy-Fuller median-unbiased estimator

The first bias-correction method we consider is the approximately median-unbiased[2] Roy-Fuller estimator which has been proven to be of empirical usefulness (see Kim, 2003). The Roy and Fuller (2001) estimator provides an analytic modification of the OLS estimator for the persistence parameter ρ. Let $\widehat{\rho}$ denote the OLS estimator for ρ in $\bar{y}_t = \rho\bar{y}_{t-1} + \beta\Delta\bar{y}_{t-1} + \varepsilon_t$, where $\bar{y}_t$ is the previously de-meaned time series y_t, i.e., $\bar{y}_t \equiv y_t - (1/T)\sum_{t=1}^{T} y_t$. Furthermore, $\widehat{\sigma}$ denotes the standard error of $\widehat{\rho}$ and $\widehat{\lambda} = (\widehat{\rho} - 1)/\widehat{\sigma}$ is the usual Dickey and Fuller (1979) unit root test statistic. The Roy-Fuller estimator[3] $\widehat{\rho}^{RF}$ is now given by $\widehat{\rho}^{RF} = \min(\widetilde{\rho}, 1)$, where

$$\widetilde{\rho} = \widehat{\rho} + C(\widehat{\lambda})\widehat{\sigma}\,.$$

Related to the asymptotic bias of the OLS estimator, the function $C(\widehat{\lambda})$ is constructed to make $\widetilde{\rho}$ approximately median-unbiased at $\rho = 1$. The function is given by

$$C(\widehat{\lambda}) \quad = \quad \begin{cases} 0, & \text{if } \widehat{\lambda} \leq -\sqrt{2T} \\ T^{-1}\widehat{\lambda} - 2\widehat{\lambda}^{-1}, & \text{if } -\sqrt{2T} < \widehat{\lambda} \leq -K \\ T^{-1}\widehat{\lambda} - 2\Big[\widehat{\lambda} + k(\widehat{\lambda} + K)\Big]^{-1}, & \text{if } -K < \widehat{\lambda} \leq \lambda_{0.5} \\ -\lambda_{0.5} + d_n(\widehat{\lambda} - \lambda_{0.5}), & \text{if } \widehat{\lambda} > \lambda_{0.5}\,, \end{cases}$$

where $\lambda_{0.5} = -1.57$ denotes the median of the limiting distribution of $\widehat{\lambda}$ if $\rho = 1$ and data is demeaned prior to testing, K is some fixed number (set to 5), d_n is a slope parameter (set to 0.1111) and $k = \Big(2 - T^{-1}\lambda_{0.5}^2\Big)\Big[(1 + T^{-1})\lambda_{0.5}(\lambda_{0.5} - K)\Big]^{-1}$. The function $C\Big(\widehat{\lambda}\Big)$ accounts for different asymptotics and convergence rates for different persistence levels of ρ. Further details can be found in Roy and Fuller (2001). After the bias-corrected estimation of ρ the other parameters of the process, μ in the AR(1) case given in equation (1) and μ, ϕ_1 and ϕ_2 in the AR(2) case given in equation (2), can be estimated subject to the restriction $\rho = \widehat{\rho}^{RF}$.

2.2 Bootstrap bias-corrected estimator

The second competitor is the bootstrap-based procedure by Kim (2003). This method involves the generation of a large number of pseudo-data sets using the estimated coefficients and re-sampled residuals. Pseudo-data sets shall resemble the dependence

[2] An estimator $\widetilde{\rho}$ for ρ is said to be median-unbiased if $P(\widetilde{\rho} \geq \rho) \geq 1/2$ and $P(\widetilde{\rho} \leq \rho) \geq 1/2$.

[3] The original Roy-Fuller estimator corrects positive and negative autocorrelation bias in AR(p) processes. In this work only substantial positive autocorrelations of AR(1) and AR(2) processes are considered. The given formulas are simplified for this case.

structure that is present in the original data set. The bias of the OLS estimator can be estimated as follows: Estimate the model via OLS and obtain the estimates $\widehat{\theta} = \left(\widehat{\mu}, \widehat{\rho}, \widehat{\beta}\right)'$. Generate a pseudo-data set $\{y_t^b\}_{t=1}^T$ based on these estimates according to

$$y_t^b = \widehat{\mu} + \widehat{\rho} y_{t-1}^b + \widehat{\beta} \Delta y_{t-1}^b + u_t^b \; ,$$

where u_t^b is a random draw with replacement from the OLS residuals $\{\widehat{u}_t\}_{t=1}^T$. B sets of pseudo-data are generated. Each pseudo-data set gives a bootstrap parameter estimate $\widehat{\theta}^b = (\widehat{\mu}^b, \widehat{\rho}^b, \widehat{\beta}^b)'$ by estimating the model $y_t^b = \mu + \rho y_{t-1}^b + \beta \Delta y_{t-1}^b + v_t$, $b = 1, \ldots, B$. We obtain the sequence $\{\widehat{\theta}^b\}_{b=1}^B$ and the average bias of $\widehat{\theta}^b$ is estimated as $\widetilde{\theta} - \widehat{\theta}$, where $\widetilde{\theta}$ is the sample average of $\{\widehat{\theta}^b\}_{i=1}^B$, i.e.

$$\widetilde{\theta} \equiv \frac{1}{B} \sum_{b=1}^{B} \widehat{\theta}^b \; .$$

Using this bootstrap-based estimator for the bias, a bias-correction for $\widehat{\theta}$ can be directly obtained via

$$\widehat{\theta}^{KIM} = \widehat{\theta} - \left(\widetilde{\theta} - \widehat{\theta}\right) = 2\widehat{\theta} - \widetilde{\theta} \; .$$

If $\widehat{\theta}^{KIM}$ does not fulfill the stationarity condition $\widehat{\rho} < 1$, the iterative filter

$$\widehat{\theta}_i^{KIM} = \widehat{\theta} - \prod_{j=1}^{i} (1 - 0.01i) \left(\widetilde{\theta} - \widehat{\theta}\right), \; i = 1, 2, 3, \ldots,$$

is applied until $\widehat{\rho} < 1$ is ensured. Denote by $\bar{i}$ the index where the iteration stops. Thus, $\widehat{\theta}^{KIM} = \widehat{\theta}_{\bar{i}}^{KIM}$. For further details regarding this estimator, the interested reader is referred to Kim (2003). This estimator computes the OLS estimation bias for a process with parameter values $\widehat{\theta}$ and uses this bias as approximation for the true bias of $\widehat{\theta}$. In contrast to the former procedure all parameters of the model are estimated simultaneously.

2.3 Indirect inference estimator

We now turn to a simulation-based estimator relying on the concept of indirect inference. The following exposition draws heavily from Phillips et al. (2011). The basic idea of this simulation-based estimator is to consider initially the OLS estimator labeled as $\widehat{\rho}$. Consider a set of simulated series with AR(1) coefficient equal to some ρ, i.e. $\{y_t^h(\rho)\}_{h=1}^H$, $h = 1, 2, \ldots, H$. H denotes the total number of available simulation paths.[4] For each

[4]In order to generate $\{y_t^h(\rho)\}_{h=1}^H$, we assume normal errors in the following. The importance of this assumption is investigated later in Section 3.2.

single $h \in 1,2,...,H$, we obtain an OLS estimate denoted as $\widehat{\rho}^h(\rho)$. The indirect inference estimator (which belongs to the class of extremum estimators) is given by

$$\widehat{\rho}_H^{II} = \arg\min_{\rho \in \Theta} \left\| \widehat{\rho} - \frac{1}{H} \sum_{h=1}^{H} \widehat{\rho}^h(\rho) \right\| ,$$

where Θ is a compact parameter space and $\| \cdot \|$ is a distance metric. For $H \to \infty$ one obtains

$$\widehat{\rho}^{II} = \arg\min_{\rho \in \Theta} \| \widehat{\rho} - q(\rho) \| ,$$

where $q(\rho) = E\left(\widehat{\rho}^h(\rho)\right)$ is the so-called binding function. Given invertibility of q, the indirect inference estimator results as

$$\widehat{\rho}^{II} = q^{-1}(\widehat{\rho}) .$$

So the idea of this estimator is to have a grid of possible true values for ρ and the corresponding average OLS estimates $(1/H)\sum_{h=1}^{H} \widehat{\rho}^h(\rho)$. The estimate $\widehat{\rho}$ is compared to the average OLS estimates. $\widehat{\rho}^{II}$ is now the value which leads to the average OLS estimate with the minimal distance to $\widehat{\rho}$. The finite-sample bias-correction stems from the simulation of $q(\rho)$. Precision is naturally expected to be increased with rising H, although it can be computationally costly. Nonetheless, the binding function has to be simulated only once and can thus be applied afterwards without any further simulation or re-sampling. This is a fundamental difference to the bootstrap approach. Furthermore, the indirect inference estimator is applicable even for mildly explosive processes. This is not the case for the Roy-Fuller and the bootstrap-based estimator by Kim (2003). Estimation of all other parameters of the process can be done analogously to the Roy-Fuller estimator.

2.4 Jackknife estimators

In general, Bao and Ullah (2007) show that the expected value of the OLS estimator $\widehat{\theta} = \left(\widehat{\mu}, \widehat{\rho}, \widehat{\beta}\right)$ has the form

$$E\left(\widehat{\theta}\right) = \theta + \frac{a}{T} + O\left(T^{-2}\right) .$$

Shaman and Stine (1988) show that the vector $a = -(1+3\rho)$ for $\widehat{\rho}$ in the AR(1) process and $a = -(1+\rho, 2-4\beta)'$ for $\left(\widehat{\rho}, \widehat{\beta}\right)'$ in the AR(2) process. If the full sample y is divided into m sub-samples Y_j of same length l, $j = 1, \ldots, m$, and $\widehat{\theta}^j$ is the OLS estimate for θ in sub-sample Y_j then the jackknife statistic

$$\widehat{\theta}^J = \left(\frac{T}{T-l}\right)\widehat{\theta} - \left(\frac{l}{T-l}\right)\widetilde{\theta}$$

with $\widetilde{\theta} = \frac{1}{m}\sum_{j=1}^{m}\widehat{\theta}^j$ satisfies $E(\widehat{\theta}^J) = \theta + O(T^{-2})$ and is thus able to reduce the bias. Chambers (2013) proposes and compares various jackknife techniques to reduce the small sample bias. In this work we focus on one of the methods in the comparison of Chambers (2013): the non-overlapping sub-samples jackknife. This estimator has good bias-correction properties without the considerable increase of the RMSE of higher order jackknife estimators. Here the time series is splitted in m non-overlapping sub-samples,

$$Y_j = (y_{[(j-1)T/m+1]}, \ldots, y_{[jT/m]})', \qquad j = 1, \ldots, m.$$

In the following we work with $m = 2$ sub-samples, because the procedure with this particular choice of m has the best bias-correction properties according to Chambers (2013) (Table 1). This simplifies the jackknife statistic to

$$\widehat{\theta}^J = 2\widehat{\theta} - \widetilde{\theta}\,.$$

The intuition behind this approach is almost the same as in the bootstrap approach of Kim (2003). The average bias in the sub-samples is higher because of the smaller sample size and therefore a bias-reduction is induced. The difference to the bootstrap procedure is that the average bias is calculated on sub-samples of the true process and not on pseudo-data. In the following we abbreviate this procedure as J(2). It should be noted that the introduced jackknife procedure is only valid as long as the process is stationary, see Chambers (2013). The unit root case is tackled in Chambers and Kyriacou (2012). To our best knowledge, the (mildly) explosive case has not been under consideration so far.

3 Finite-sample properties

In this section we investigate the properties of various bias-correction methods via Monte Carlo simulation. The foci of this analysis are the bias-reduction and the RMSE of these estimators for AR(1) and AR(2) models in various settings. The simulation setup is as follows: We consider autoregressive models of the structure

$$y_t = \mu + \rho y_{t-1} + \beta \Delta y_{t-1} + \varepsilon_t$$

with $\varepsilon_t \sim N(0,1)$. Non-normal and heteroscedastic errors are studied in Section 3.2. The case of a linear deterministic trend in addition to the intercept μ is located in Section 3.3. The autoregressive parameter ρ measures the persistence of y_t and takes values $\rho = \{0.85, 0.9, 0.95, 0.99, 1, 1.01, 1.02\}$. The considered samples sizes are $T = \{30, 60, 120, 240\}$. The mildly autoregressive process is characterized by $\rho = 1 + \frac{c}{T^\gamma}$ with $0 < \gamma < 1$ and $c > 0$. Following Breitung and Kruse (2013) [5], $\gamma = 0.75$ corresponds to $c = \{0.13, 0.22, 0.36, 0.61\}$ and $c = \{0.26, 0.43, 0.73, 1.22\}$ for $\rho = 1.01$ and for $\rho = 1.02$, respectively. Thus, the degree of explosiveness is in fact very mild in our setup. The intercept μ is set equal to zero without loss of generality. If the data is generated by an AR(2) process, β is set to $\beta = \{-0.2, 0.3\}$. The number of Monte Carlo repetitions is set to $10,000$ for each single experiment. The number of bootstrap repetitions for the procedure of Kim (2003) is set to 499. The binding function for the indirect inference estimator was simulated with $\rho = \{0.60, 0.61, \ldots, 1.20\}$ and $\beta = \{-0.90, -0.89, \ldots, 0.90\}$. The number of simulation paths H equals $10,000$ in the AR(1) case and $H = 100$ for AR(2) models. In an unreported comparison between different values for H, we find that there are only marginal changes in the results as long as $H \geq 100$. That means that the indirect inference procedure can be applied at low computational costs with negligible loss of precision.

Summary results are reported in Section 3.5. Detailed results are reported in Tables 1-7. Table 1 shows the results for the case where the estimated model coincides with the true DGP which is an AR(1). The next subsection discusses the performance for GARCH and heavy-tailed innovations (see Tables 2 and 6). Results for processes with deterministic trends are given in Table 3. Finally Tables 4, 5 and 7 contain results for correctly specified AR(2) models, under-fitted AR(2) models and over-fitted AR(1) models.

3.1 First-order autoregressive model with i.i.d. Normal innovations

Our benchmark case is the AR(1) process with constant as in equation (1). The left-hand side of Table 1 provides the average bias of the OLS estimator and all discussed bias-correction procedures. Every procedure leads to a substantial bias-reduction compared to the OLS estimator. For $T = 60$, the jackknife estimator J(2) has the best bias-correction capabilities in nearly all cases. The indirect inference estimator is second-best followed by the approximately median-unbiased Roy-Fuller estimator and the bootstrap-based approach (Kim). In smaller samples ($T = 30$), the jackknife is still the best procedure for unit root and explosive cases, but the results for stationary autoregressive

[5] Breitung and Kruse (2013) consider values for c in the range of one half to five when simulating the empirical performance of Chow-type tests for bursting bubbles.

T	ρ	Bias					RMSE				
		OLS	II	RF	Kim	J(2)	OLS	II	RF	Kim	J(2)
30	0.85	-0.135	0.012	**0.000**	-0.028	-0.002	0.201	**0.145**	0.165	0.162	0.218
	0.90	-0.148	**-0.002**	-0.018	-0.044	-0.010	0.206	**0.141**	0.153	0.155	0.217
	0.95	-0.162	**-0.019**	-0.043	-0.067	-0.020	0.213	**0.135**	0.143	0.153	0.217
	0.99	-0.168	-0.029	-0.065	-0.089	**-0.021**	0.214	**0.128**	0.138	0.155	0.214
	1.00	-0.166	-0.030	-0.069	-0.094	**-0.018**	0.212	**0.125**	0.136	0.156	0.211
	1.01	-0.163	-0.030	-	-	**-0.014**	0.209	**0.122**	-	-	0.207
	1.02	-0.157	-0.028	-	-	**-0.009**	0.204	**0.117**	-	-	0.203
60	0.85	-0.066	0.004	0.007	-0.006	**0.003**	0.113	**0.097**	0.104	0.098	0.123
	0.90	-0.072	**0.001**	0.004	-0.011	0.002	0.113	0.093	0.095	**0.091**	0.121
	0.95	-0.081	-0.006	-0.010	-0.023	**-0.003**	0.114	0.084	**0.081**	0.083	0.119
	0.99	-0.088	-0.016	-0.029	-0.041	**-0.007**	0.114	0.072	**0.070**	0.078	0.113
	1.00	-0.086	-0.016	-0.033	-0.046	**-0.005**	0.111	0.068	**0.068**	0.078	0.111
	1.01	-0.081	-0.015	-	-	**0.000**	0.106	**0.063**	-	-	0.107
	1.02	-0.071	-0.012	-	-	**0.004**	0.099	**0.058**	-	-	0.101
120	0.85	-0.032	**0.000**	0.001	-0.002	0.002	0.065	0.059	0.061	**0.059**	0.069
	0.90	-0.034	**0.001**	0.003	-0.002	0.002	0.062	0.055	0.057	**0.054**	0.066
	0.95	-0.038	**0.000**	0.003	-0.005	0.002	0.059	0.048	0.048	**0.046**	0.063
	0.99	-0.045	-0.007	-0.012	-0.018	**-0.003**	0.059	0.038	**0.037**	0.040	0.059
	1.00	-0.044	-0.009	-0.017	-0.023	**-0.002**	0.058	0.036	**0.036**	0.040	0.057
	1.01	-0.036	-0.006	-	-	**0.003**	0.051	**0.031**	-	-	0.052
	1.02	-0.021	**-0.002**	-	-	0.009	0.037	**0.023**	-	-	0.043
240	0.85	-0.015	0.000	0.000	**0.000**	0.001	0.040	0.038	0.038	**0.038**	0.042
	0.90	-0.016	0.000	0.001	**0.000**	0.001	0.036	0.033	0.034	**0.033**	0.038
	0.95	-0.018	0.001	0.002	**0.000**	0.001	0.032	0.028	0.029	**0.027**	0.034
	0.99	-0.022	-0.002	-0.003	-0.006	**-0.001**	0.030	0.020	**0.019**	0.020	0.030
	1.00	-0.022	-0.004	-0.008	-0.011	**-0.001**	0.029	0.018	**0.017**	0.020	0.029
	1.01	-0.011	**-0.001**	-	-	0.005	0.019	**0.011**	-	-	0.022
	1.02	-0.002	**0.000**	-	-	0.008	0.009	**0.006**	-	-	0.017

Table 1: Bias and RMSE for OLS, indirect inference (II), Roy-Fuller (RF), Kim and jackknife (J(2)) estimation procedures for different AR(1) processes and sample sizes (constant included).

models are mixed. In larger samples ($T = 120$), the indirect inference estimator is the best method for stationary processes whereas the jackknife wins for $\rho = 1$ and $\rho = 1.01$. Interestingly, for $\rho = 1.02$ the bias of the J(2) approach changes its sign and yields a very small, but positive bias. While this behavior may not seem striking at first sight, it becomes more important when $\rho > 1.02$ (not reported). The higher ρ, the more obvious is the overcorrection even in small samples. For $T = 240$, the OLS bias is quite small and the need for bias-correction procedures becomes less important. Nevertheless, a reduction of the bias to levels very close to zero is possible with any method.

The second important statistic we investigate is the RMSE. It is reported at the right-hand side of Table 1. For $T = 60$, the bootstrap procedure has the highest RMSE reduction for stationary cases, the Roy-Fuller method for processes close to and at the unit root and the indirect inference estimator for explosive cases. All three techniques are highly competitive in terms of variance reduction whereas the J(2) causes an increase in the variance compared to the OLS estimator. This pattern remains the same for larger samples. For $T = 30$, the indirect inference estimator is always the best procedure in terms of the RMSE. This shows that the jackknife estimator provides the best bias-correction on average, but comes along with a fairly large variance. This result is in line with Chambers (2013) where only stationary autoregressive models are considered. Our results indicate that the general conclusion remain to hold for unit root and mildly explosive autoregressive models as well. On the contrary, the indirect inference estimator offers a similar performance in terms of bias-reduction (even though somewhat less effective) and does not suffer from an increased variance.

3.2 Heteroscedastic and heavy-tailed innovations

So far all results are based on $\varepsilon_t \sim N(0,1)$ innovations. As a robustness check on the normality assumption we also investigate the performance of the bias-reduction methods under heteroscedasticity and heavy-tailed error distributions. In order to investigate the influence of heteroscedasticity we generate highly persistent GARCH disturbances as follows:

$$\varepsilon_t = \sigma_t z_t$$
$$\sigma_t = a_0 + a_1 \varepsilon_{t-1}^2 + b_1 \sigma_{t-1}^2,$$

where $z_t \sim N(0,1)$ and the parameters are set equal to $a_0 = 0.05$, $a_1 = 0.1$ and $b_1 = 0.85$. The simulation results for those DGPs are given in Table 2. For $T = 60$, the OLS bias is slightly higher (in absolute value) than in the standard *iid* case. All procedures still offer a substantial bias-reduction, and the remaining bias is usually smaller than in the benchmark case. This means that the GARCH disturbances affect all estimators in a similar way. The general ranking of the bias-correction methods stays the same as in the benchmark case. For all other sample sizes in this setup, the jackknife estimator has the best bias-correction abilities. The RMSE is on average slightly higher than in the benchmark case, but the pattern remains exactly the same.

In order to investigate whether heavy-tailed innovations may lead to problems, we use stable distributed errors which are generated as $\varepsilon_t \sim S(\alpha = 1.85, \beta = 0, \gamma = 1, \delta = 0)$. This distribution exhibits much fatter tails than the standard Normal distribution: $P(|\varepsilon_t| > 2.5758) = 8.6\%$ instead of 1% as for the $N(0,1)$ distribution. Remarkably, the change in the error distribution has hardly any impact on the bias and RMSE results compared to the benchmark case. Therefore, the corresponding Table 6 is located in the Appendix.

		Bias					RMSE				
T	ρ	OLS	II	RF	Kim	J(2)	OLS	II	RF	Kim	J(2)
30	0.85	-0.141	0.007	-0.007	-0.035	**-0.007**	0.208	**0.147**	0.170	0.168	0.227
	0.90	-0.153	**-0.007**	-0.024	-0.051	-0.014	0.213	**0.144**	0.158	0.162	0.226
	0.95	-0.167	-0.024	-0.049	-0.074	**-0.024**	0.220	**0.139**	0.149	0.160	0.225
	0.99	-0.173	-0.034	-0.070	-0.095	**-0.026**	0.221	**0.133**	0.144	0.163	0.220
	1.00	-0.171	-0.035	-0.074	-0.100	**-0.023**	0.220	**0.130**	0.143	0.165	0.218
	1.01	-0.168	-0.034	-	-	**-0.019**	0.216	**0.127**	-	-	0.215
	1.02	-0.162	-0.032	-	-	**-0.015**	0.212	**0.123**	-	-	0.210
60	0.85	-0.068	0.002	0.005	-0.008	**-0.001**	0.116	**0.099**	0.107	0.101	0.126
	0.90	-0.074	**-0.001**	0.002	-0.013	-0.002	0.115	0.095	0.097	**0.094**	0.123
	0.95	-0.083	-0.008	-0.012	-0.025	**-0.007**	0.117	0.086	**0.084**	0.086	0.121
	0.99	-0.091	-0.018	-0.032	-0.044	**-0.011**	0.117	0.076	**0.075**	0.083	0.117
	1.00	-0.089	-0.019	-0.036	-0.049	**-0.007**	0.115	**0.072**	0.072	0.082	0.116
	1.01	-0.083	-0.017	-	-	**-0.003**	0.110	**0.068**	-	-	0.111
	1.02	-0.073	-0.013	-	-	**0.002**	0.101	**0.061**	-	-	0.103
120	0.85	-0.033	-0.001	**0.000**	-0.003	0.000	0.069	0.063	0.064	**0.062**	0.074
	0.90	-0.036	-0.001	0.002	-0.004	**0.000**	0.064	0.057	0.059	**0.056**	0.069
	0.95	-0.040	-0.001	0.001	-0.006	**0.000**	0.061	0.050	0.050	**0.048**	0.065
	0.99	-0.046	-0.008	-0.013	-0.019	**-0.004**	0.061	0.040	**0.039**	0.042	0.062
	1.00	-0.045	-0.009	-0.018	-0.024	**-0.001**	0.059	0.037	**0.036**	0.042	0.060
	1.01	-0.037	-0.007	-	-	**0.003**	0.051	**0.031**	-	-	0.053
	1.02	-0.022	**-0.002**	-	-	0.009	0.038	**0.023**	-	-	0.045
240	0.85	-0.017	-0.001	-0.002	-0.002	**-0.001**	0.044	0.042	0.042	**0.042**	0.047
	0.90	-0.018	-0.001	-0.001	-0.002	**0.000**	0.040	0.036	0.037	**0.036**	0.042
	0.95	-0.019	-0.001	0.001	-0.002	**0.000**	0.034	0.030	0.031	**0.029**	0.036
	0.99	-0.022	-0.003	-0.004	-0.007	**-0.001**	0.031	0.022	**0.021**	0.022	0.032
	1.00	-0.023	-0.005	-0.009	-0.012	**0.000**	0.030	0.019	**0.018**	0.021	0.031
	1.01	-0.011	**-0.001**	-	-	0.005	0.019	**0.012**	-	-	0.023
	1.02	-0.002	**0.000**	-	-	0.008	0.009	**0.006**	-	-	0.018

Table 2: Bias and RMSE for OLS, indirect inference (II), Roy-Fuller (RF), Kim and jackknife (J(2)) estimation procedures for different AR(1) processes and sample sizes (constant included) with GARCH(1,1) errors.

3.3 Inclusion of a linear deterministic trend and misspecified AR(1)

In this subsection we study autoregressive models with an additional linear trend term of the form

$$y_t = \mu + \delta t + \rho y_{t-1} + \varepsilon_t \,.$$

In all simulations we set $\delta = 0$ (in addition to $\mu = 0$) without loss of generality. Table 3 shows that the additional uncertainty about the trend parameter causes a rise of the OLS bias. As expected, all procedures perform worse than in the benchmark case (see Table 1). Further deviations from the benchmark case are the better overall performance of the Roy-Fuller estimator in stationary setups and the superior performance of the J(2) procedure in small samples ($T = 30$). An interesting development is the reduction of

		Bias					RMSE				
T	ρ	OLS	II	RF	Kim	J(2)	OLS	II	RF	Kim	J(2)
30	0.85	-0.227	0.027	-0.033	-0.074	**0.002**	0.282	**0.183**	0.200	0.200	0.278
	0.90	-0.247	0.008	-0.056	-0.096	**-0.005**	0.297	**0.178**	0.195	0.203	0.283
	0.95	-0.271	-0.018	-0.086	-0.124	**-0.018**	0.317	**0.175**	0.197	0.212	0.291
	0.99	-0.300	-0.048	-0.117	-0.156	**-0.040**	0.342	**0.178**	0.208	0.229	0.297
	1.00	-0.309	-0.057	-0.127	-0.165	**-0.049**	0.350	**0.180**	0.214	0.236	0.299
	1.01	-0.320	-0.068	-	-	**-0.059**	0.359	**0.184**	-	-	0.302
	1.02	-0.331	-0.079	-	-	**-0.072**	0.369	**0.188**	-	-	0.305
60	0.85	-0.109	0.010	**0.001**	-0.018	0.009	0.148	0.119	0.119	**0.111**	0.150
	0.90	-0.119	**0.007**	-0.008	-0.028	0.008	0.153	0.116	0.109	**0.107**	0.152
	0.95	-0.134	-0.005	-0.027	-0.046	**0.004**	0.163	0.107	**0.101**	0.105	0.155
	0.99	-0.155	-0.026	-0.053	-0.072	**-0.008**	0.179	**0.104**	0.104	0.114	0.160
	1.00	-0.163	-0.034	-0.062	-0.081	**-0.016**	0.187	**0.106**	0.108	0.119	0.161
	1.01	-0.174	-0.045	-	-	**-0.027**	0.196	**0.109**	-	-	0.162
	1.02	-0.183	-0.053	-	-	**-0.034**	0.204	**0.113**	-	-	0.164
120	0.85	-0.049	0.002	**0.002**	-0.002	0.007	0.078	0.066	0.067	**0.064**	0.080
	0.90	-0.054	0.004	**0.004**	-0.005	0.008	0.078	0.065	0.064	**0.060**	0.080
	0.95	-0.062	0.004	**-0.002**	-0.012	0.008	0.080	0.060	0.056	**0.054**	0.081
	0.99	-0.075	-0.009	-0.021	-0.030	**0.002**	0.089	0.053	**0.050**	0.055	0.083
	1.00	-0.083	-0.017	-0.030	-0.039	**-0.005**	0.096	**0.054**	0.054	0.059	0.084
	1.01	-0.093	-0.027	-	-	**-0.014**	0.105	**0.057**	-	-	0.085
	1.02	-0.063	**-0.013**	-	-	0.037	0.081	**0.043**	-	-	0.099
240	0.85	-0.025	**0.000**	-0.002	-0.001	0.002	0.046	0.040	0.040	**0.040**	0.046
	0.90	-0.026	**0.000**	-0.001	-0.001	0.003	0.043	0.036	0.037	**0.036**	0.044
	0.95	-0.029	0.001	**0.001**	-0.003	0.005	0.041	0.033	0.033	**0.031**	0.042
	0.99	-0.036	**-0.002**	-0.008	-0.012	0.004	0.044	0.027	**0.026**	0.027	0.043
	1.00	-0.042	-0.009	-0.015	-0.020	**-0.001**	0.049	**0.027**	0.028	0.030	0.044
	1.01	-0.032	**-0.007**	-	-	0.020	0.041	**0.022**	-	-	0.052
	1.02	-0.004	**0.001**	-	-	0.028	0.014	**0.008**	-	-	0.040

Table 3: Bias and RMSE for OLS, indirect inference (II), Roy-Fuller (RF), Kim and jackknife (J(2)) estimation procedures for different AR(1) processes and sample sizes (constant and trend included).

the variance of the indirect inference, Roy-Fuller and Kim's bootstrap estimator. The performance in terms of RMSE is not as convincingly good as in the benchmark case, but the average raise of the RMSE for the OLS estimator is higher than for the bias-correction procedures. Even the J(2) estimator is now able to a lower RMSE than the OLS estimator in most cases, although not in a competitive way.

Almost the same pattern is visible if the AR(1) process is misspecified as an AR(2) process. This means that the data is generated as in the benchmark case, but an AR(2) model with the additional parameter β is estimated. Instead of the trend parameter δ an additional autoregressive parameter adds uncertainty to the estimation. All the effects caused by the inclusion of a linear trend are also visible in the misspecified case, but in a much milder form. The detailed results are gathered in Table 7 in the Appendix.

			Bias					RMSE				
T	β	ρ	OLS	II	RF	Kim	J(2)	OLS	II	RF	Kim	J(2)
60	0.2	0.85	-0.059	**0.001**	0.004	-0.004	0.010	0.102	**0.089**	0.095	0.089	0.117
		0.90	-0.063	-0.001	0.002	-0.008	**0.009**	0.099	**0.083**	0.086	0.081	0.112
		0.95	-0.069	-0.006	-0.008	-0.017	**0.005**	0.099	**0.074**	0.073	0.072	0.109
		0.99	-0.075	-0.014	-0.025	-0.035	**-0.001**	0.098	**0.063**	0.062	0.068	0.104
		1.00	-0.073	-0.014	-0.029	-0.039	**0.004**	0.095	**0.059**	0.059	0.067	0.102
		1.01	-0.067	-0.012	-	-	**0.006**	0.090	**0.055**	-	-	0.097
		1.02	-0.054	**-0.007**	-	-	0.010	0.079	**0.047**	-	-	0.089
	-0.3	0.85	-0.099	**0.002**	0.005	-0.012	0.010	0.155	**0.119**	0.133	0.126	0.172
		0.90	-0.106	**-0.006**	-0.005	-0.022	0.007	0.154	**0.115**	0.119	0.116	0.168
		0.95	-0.113	-0.015	-0.021	-0.037	**0.002**	0.153	0.105	**0.103**	0.106	0.163
		0.99	-0.117	-0.024	-0.041	-0.057	**-0.002**	0.152	**0.095**	0.095	0.104	0.158
		1.00	-0.114	-0.023	-0.044	-0.060	**0.002**	0.147	**0.090**	0.091	0.103	0.154
		1.01	-0.111	-0.024	-	-	**0.004**	0.146	**0.089**	-	-	0.151
		1.02	-0.102	**-0.020**	-	-	0.009	0.139	**0.083**	-	-	0.144

Table 4: Bias and RMSE for OLS, indirect inference (II), Roy-Fuller (RF), Kim and jackknife (J(2)) estimation procedures for different AR(2) processes (constant included).

3.4 Higher-order and misspecified autoregressive models

Finally, we extend our analysis to the AR(2) model as in equation (3). As visible in Figure 1, the OLS bias depends on the value of β. We work with $\beta = \{0.2, -0.3\}$, typical values in macroeconomic time series. In order to save space only results for $T = 60$ are reported in Table 4, results for all other sample sizes can be found in Table 8 in the Appendix. All procedures are able to reduce the OLS bias for higher order models and the order in terms of bias-correction does not deviate from the AR(1) case. The jackknife is the best method, in particular for the unit root and stationary near unit root setups. The procedure is also the only one which does not depend on β. All other methods gain strictly better results for $\beta = 0.2$. The same pattern appears if $T = 30$. For larger sample sizes the results are more mixed in favor of the indirect inference estimator.

The RMSE results show that the indirect inference estimator has the highest RMSE reduction for most cases. In comparison to the benchmark case, the typical pattern appears only for samples sizes of $T = 120$ or larger, in smaller samples the indirect inference estimator is the best procedure in terms of RMSE reduction. It is also notable that the J(2) estimator leads to a significant raise in the variance compared to the OLS estimator in all setups.

This result changes if the order of the model is underestimated. The results for a simulated AR(2) process but an estimated AR(1) model are given in Table 5 and for other sample sizes in Table 9 in the Appendix. For $T = 60$, the jackknife is the best

			Bias					RMSE				
T	β	ρ	OLS	II	RF	Kim	J(2)	OLS	II	RF	Kim	J(2)
60	0.2	0.85	**-0.023**	0.051	0.056	0.040	0.030	**0.073**	0.093	0.097	0.085	0.097
		0.90	-0.034	0.043	0.044	0.029	**0.023**	0.073	0.082	0.081	**0.073**	0.095
		0.95	-0.048	0.027	0.019	**0.007**	0.013	0.078	0.066	0.058	**0.057**	0.095
		0.99	-0.062	0.008	-0.013	-0.024	**-0.002**	0.084	0.051	**0.047**	0.056	0.094
		1.00	-0.062	0.005	-0.020	-0.033	**-0.002**	0.084	0.047	**0.046**	0.061	0.092
		1.01	-0.058	0.004	-	-	**0.002**	0.079	**0.043**	-	-	0.087
		1.02	-0.047	**0.005**	-	-	0.006	0.071	**0.039**	-	-	0.082
	-0.3	0.85	-0.189	-0.108	-0.129	-0.137	**-0.086**	0.236	**0.156**	0.202	0.203	0.206
		0.90	-0.179	-0.105	-0.113	-0.124	**-0.069**	0.224	**0.161**	0.188	0.189	0.197
		0.95	-0.172	-0.099	-0.103	-0.115	**-0.055**	0.212	**0.157**	0.171	0.175	0.186
		0.99	-0.161	-0.087	-0.095	-0.108	**-0.036**	0.197	**0.144**	0.153	0.158	0.174
		1.00	-0.154	-0.081	-0.091	-0.104	**-0.027**	0.191	**0.139**	0.147	0.153	0.169
		1.01	-0.145	-0.073	-	-	**-0.016**	0.182	**0.131**	-	-	0.163
		1.02	-0.132	-0.064	-	-	**-0.007**	0.171	**0.122**	-	-	0.155

Table 5: Bias and RMSE for OLS, indirect inference (II), Roy-Fuller (RF), Kim and jackknife (J(2)) estimation procedures for different AR(2) processes when the model is misspecified as AR(1).

bias-correction method. In particular, if $\beta = -0.3$ it is significantly better than its competitors. Although the ranking of the other bias-correction procedures remains the same, it is not as obvious as before. All methods perform worse than in the correctly specified model. In one setup, $\beta = 0.2$ and $\rho = 0.85$, all bias-corrected estimators have a higher bias than the OLS estimator. For larger samples the results depend on the value of β. If $\beta = 0.2$, no best procedure can be identified but in more and more setups bias-correction is not successful at all. If $\beta = -0.3$, the J(2) estimator offers the highest bias-reduction.

In terms of RMSE the standard pattern from the AR(1) case is visible for $\beta = 0.2$, whereas for $\beta = -0.3$ the indirect inference estimator is the best procedure. But, no procedure is able to offer a constant reduction of the RMSE and if this reduction is much less than in the correctly specified model. These results lead to the recommendation to choose the model order with a parameter friendly information criterion like the AIC when bias-correction should be applied.

3.5 Summary of simulation results

Our main results are as follows: (i) bias-correction plays an important role for all considered levels of persistence (i.e. stationarity, unit roots and explosive behaviour), in particular for samples sizes up to $T = 120$, (ii) the most effective bias-correction is obtained when applying the jackknife estimator for small and moderate sample sizes; in terms of RMSE, the indirect inference approach is generally recommendable. It performs particulary well for small sample sizes and explosive processes. (iii) Under the presence of a unit root, the Roy-Fuller and the indirect inference estimator perform best in terms of RMSE, while the bootstrap-based estimator by Kim (2003) performs well for stationary models. (iv) Heteroscedastic and heavy-tailed errors hardly affect the former conclusions. (v) In case of correct specification, the exact order of the autoregressive model does alter our main findings. Overfitting of the autoregressive model is not harmful, while underfitting turns out to be an important issue. Therefore, the lag length shall be carried out on the basis of liberal selection procedures like the AIC. (vi) The performance of all estimators weakens when a deterministic trend in addition to an intercept is included. But, the ranking of estimators remains unaffected.

4 Empirical applications

We apply the different bias-correction methods to four economic time series using the popular rolling window technique. In Section 4.1 we analyze a long annual ratio of the US Debt/GDP series in detail. Recently, there has been an extensive discussion on lifting the US government debt ceiling. The sustainability of US fiscal policy hinges on the persistence properties of the US Debt/GDP series: only when the series exhibits stationarity, fiscal policies are sustainable.

Further empirical applications are considered in Section 4.2 where the following three series are studied: (1) log Oil price, (2) log Gold price and (3) spread between long-term interest rates in Germany and Greece. Figure 2 contains time series plots of all four variables. All series are strongly autocorrelated. The first three series are even likely to exhibit locally explosive behaviour due to expansions during war times (US Debt) and speculation (Oil and Gold). The situation is different for the interest rate series whose persistence properties have not been studied extensively yet. Data for the debt series is available at http://www.econ.ucsb.edu/~bohn/data.html while the remaining data has been obtained from the FRED and the ECB database. Bias-corrected rolling window estimation (with 60 observations per window) is compared to classic OLS estimation. The lag length is chosen via the Akaike information criterion as underfitting is a problematic issue. For each series, an intercept is included in the autoregressive model due to a non-zero mean.

4.1 US Debt/GDP ratio

The US Debt/GDP ratio series is measured in percent. The sample ranges from 1791-2011, yielding 221 annual observations. Given a window size of 60, we obtain the first

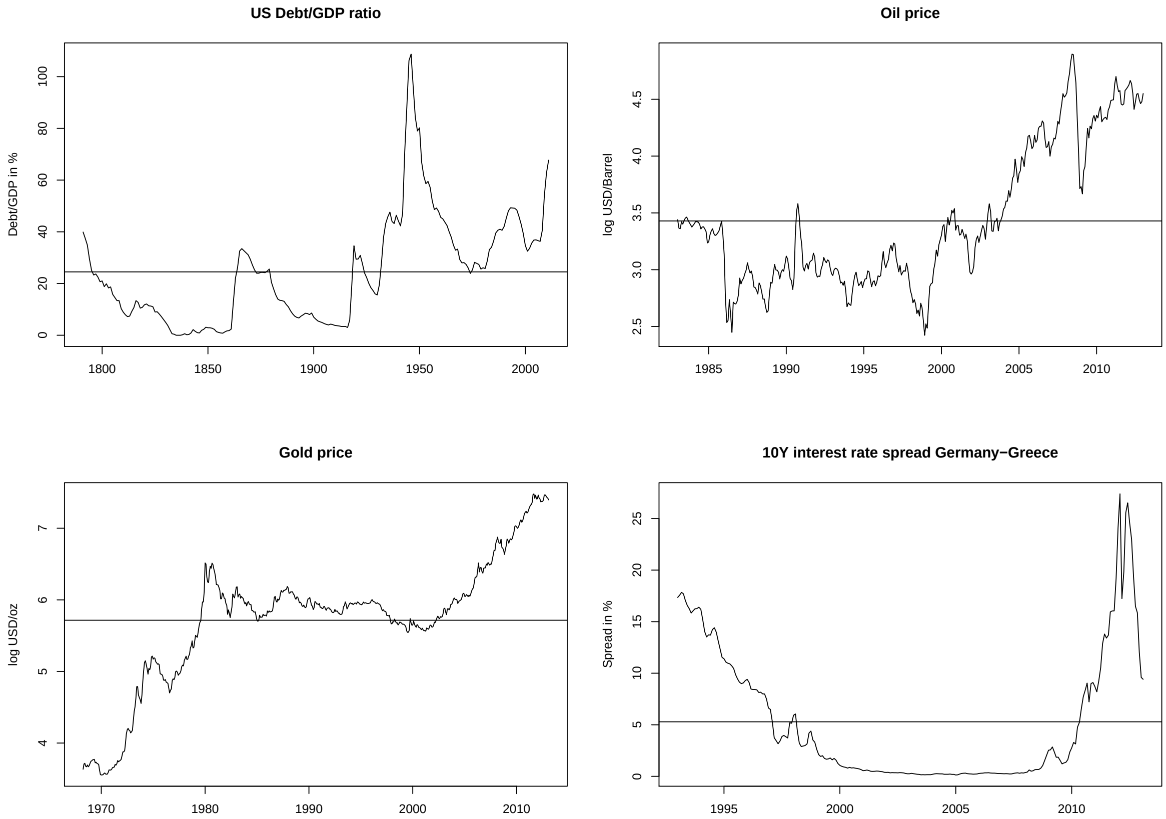

Figure 2: Time series under consideration.

estimates for the period from 1791 to 1850.[6] The second estimates are based on the sample ranging from 1792-1851 and so on. The last estimates use the sample from 1952 to 2011. According to the AIC, an AR(2) model is fitted to the data.

The estimated values of ρ for the different bias-correction techniques are given in Figure 3, each in comparison to the OLS estimator. First, bias-correction obviously plays an important role in this application as differences between OLS and bias-corrected estimates are clearly visible. Second, the Roy-Fuller, bootstrap and indirect inference estimator agree on the general evolution of the persistence over time, whereas the jackknife estimator shows a more volatile behavior. An obvious shortcoming of the Roy-Fuller bias-correction and the bootstrap technique by Kim (2003) is their limitation to the parameter space $\widehat{\rho} \leq 1$. The results for the OLS, indirect inference and jackknife estimator clearly suggest the need of relaxing this restriction for obtaining meaningful estimates of the persistence. Therefore, we focus on the indirect inference estimator in comparison

[6]The choice of 60 observations has been made in accordance to the simulations in the previous section. However, our calculations for 50 observations (half a century of data per window) lead to very similar conclusions.

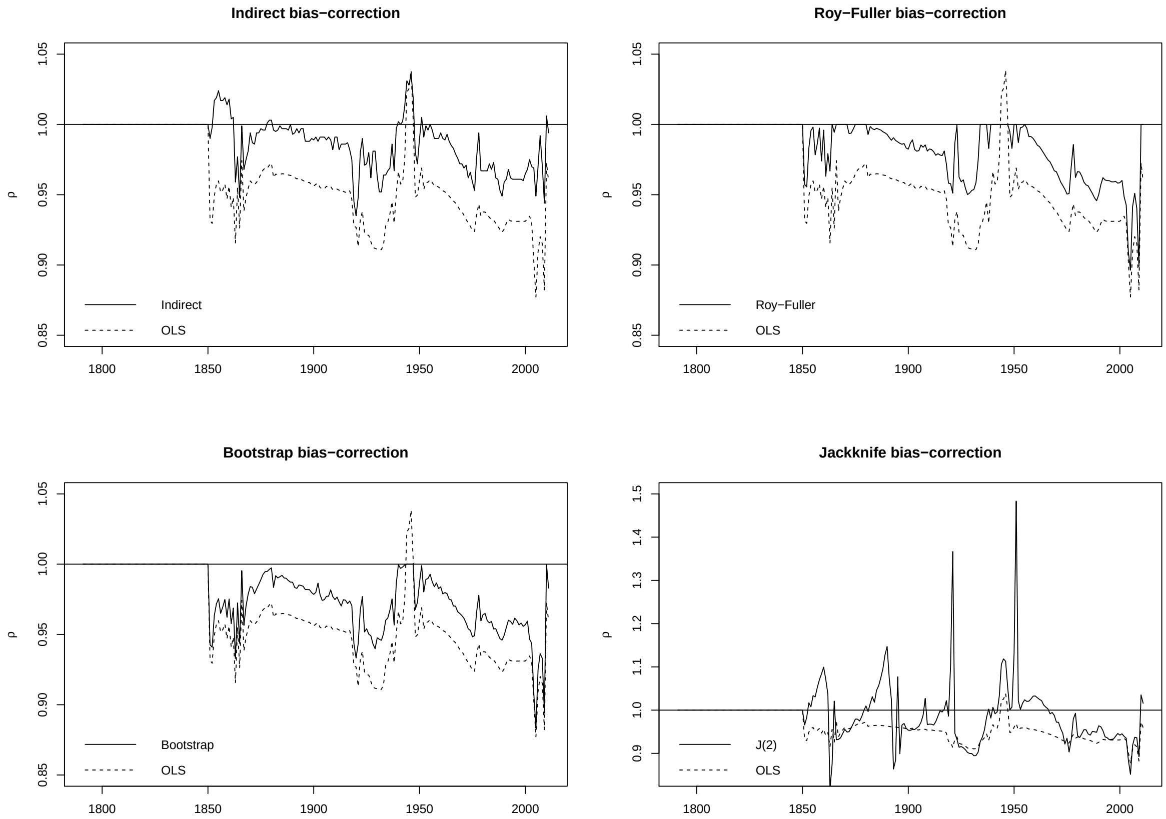

Figure 3: Rolling window AR(2) estimation for the US Debt/GDP series with different bias-correction methods.

to the jackknife estimator.

The indirect inference estimator displays explosiveness during major wars (Civil War and World War II), where the autoregressive parameter estimates reach a maximum of $\widehat{\rho}^{II} = 1.036$. After 1950, persistence dropped remarkably, recovering during the recent years since 2001 possibly in response to the patriot act and related policies after 9/11. The very last point estimates indicate a high persistence and a possible unit root. Parameter estimates for the J(2) bias-correction method show explosive behavior during the Civil War, in the late 18th century and both World Wars. Estimated persistence is relatively close around the unit root with an interval from $\widehat{\rho}_{J,2} = [0.818, 1.483]$.[7] These results support the Monte Carlo analysis, where the jackknife estimators show a very good bias-reduction but at the costs of high standard errors. The OLS estimation results only indicate only a single period of explosive behaviour, i.e. the second World War. Moreover, the OLS results for the Civil War period are in clear discrepancy to the ones

[7] The higher-order J(2,3) bias-correction (not reported to conserve some space) yields very volatile results with many highly explosive phases but also some major drops down to $\widehat{\rho}_{J(2,3)} = 0.558$.

obtained by bias-corrected estimators.

Our results suggest that a lifting of the US government debt ceiling may easily end up in unsustainable fiscal policies as the persistence of the series is non-stationary and nearly explosive during the most recent years. In general, our findings are in line with Yoon (2011) who applies the recursive right-tailed unit root test of Phillips et al. (2011) to test the hypothesis of a unit root against explosive behavior. His main result is that the US Debt/GDP ratio is explosive and that the explosiveness is linked to the high increase in the ratio during and after the World War II. Our study complements Yoon (2011) as the author did not consider bias-corrected estimation for the series.

4.2 Further applications: Oil, Gold and European interest rates

In this subsection we analyze some further time series which potentially exhibit phases of explosiveness due to pronounced growth rates. We start with the spot oil price series (West Texas Intermediate), which is measured in US Dollars per barrel. Episodes of explosive behaviour hint at strong speculation activities in the market. The sample ranges from 1983:01 to 2013:01 ($T = 361$). An AR(2) model is fitted to the data. The window size equals 60 months (5 years).

The estimated values of ρ for the different bias-correction techniques are given in Figure 4, each in comparison to the OLS estimator. The general evolution of all estimators suggests that persistence has undergone remarkable changes. Bias-correction is of importance in this application, too. The OLS estimates do not indicate explosive behaviour (and thus phases of pronounced speculation) at all. When looking at the results for the indirect inference estimator, one observes that oil prices have been much less persistent (and presumably stationary) during the Nineties. Persistence increased towards the year 2000 and stayed above, but close to, unity. Around 2004, persistence dropped again whilst recovering quickly to high levels indicating mild explosiveness. Interestingly, there has been another drop to values around 0.9 in the recent years. The rolling window estimation results reflect the movements in the series, see Figure 2 (upper right panel). The Roy-Fuller and the bootstrap bias-correction techniques suggest similar findings expect of the important periods of explosiveness. The jackknife estimator provides results which are in general accordance to the ones for the indirect inference estimator. However, estimated persistence is much higher in explosive phases and the persistence path is more volatile. This behaviour is confirmed by our simulation results which show that the jackknife estimator has a fairly large variance.

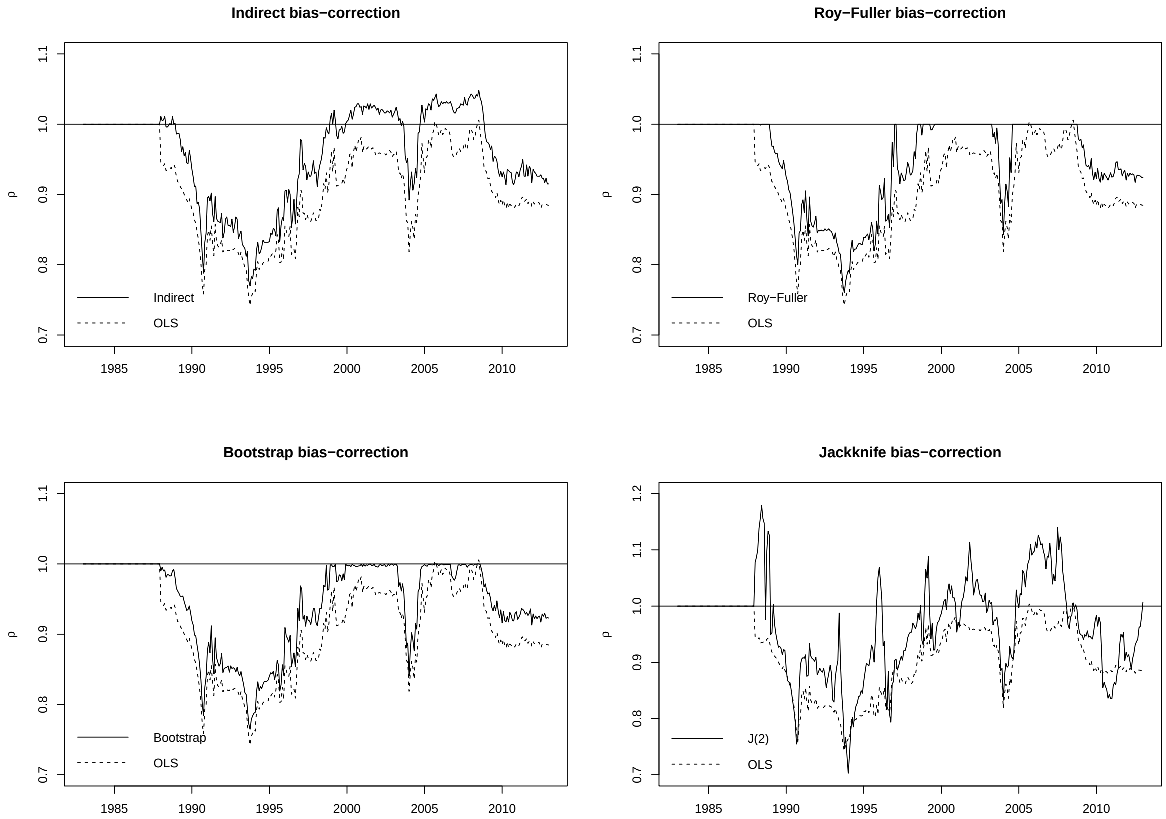

Figure 4: Rolling window AR(2) estimation for the log Oil price series with different bias-correction methods.

Next, we study another important commodity series. The presence of bubbles (characterized by explosive price paths) in gold prices (measured in US Dollars per ounce) has implications with respect to its safe haven property, see Baur, Dimpfl, and Jung (2012) and Baur and McDermott (2010). During periods of explosive behaviour, the stabilizing effect of Gold vanishes which may endanger the financial system to a certain extent. Monthly data is sampled from 1968:04 to 2013:01, yielding 539 observations. An AR(1) model is fitted to the data.

The results are reported in Figure 5. As a first clear result, the series is strongly persistent and exhibits many and long phases of mild explosiveness. Even the rolling window OLS estimates clearly indicate two such phases in the beginning of the Seventies and the Eighties, respectively. When comparing different bias-correction techniques, we find a similar picture as for the previous applications. The importance of bias-correction and the simultaneous allowance for explosive behaviour is further underlined.

Finally, we consider the spread between long-term interest rates in Germany and Greece.

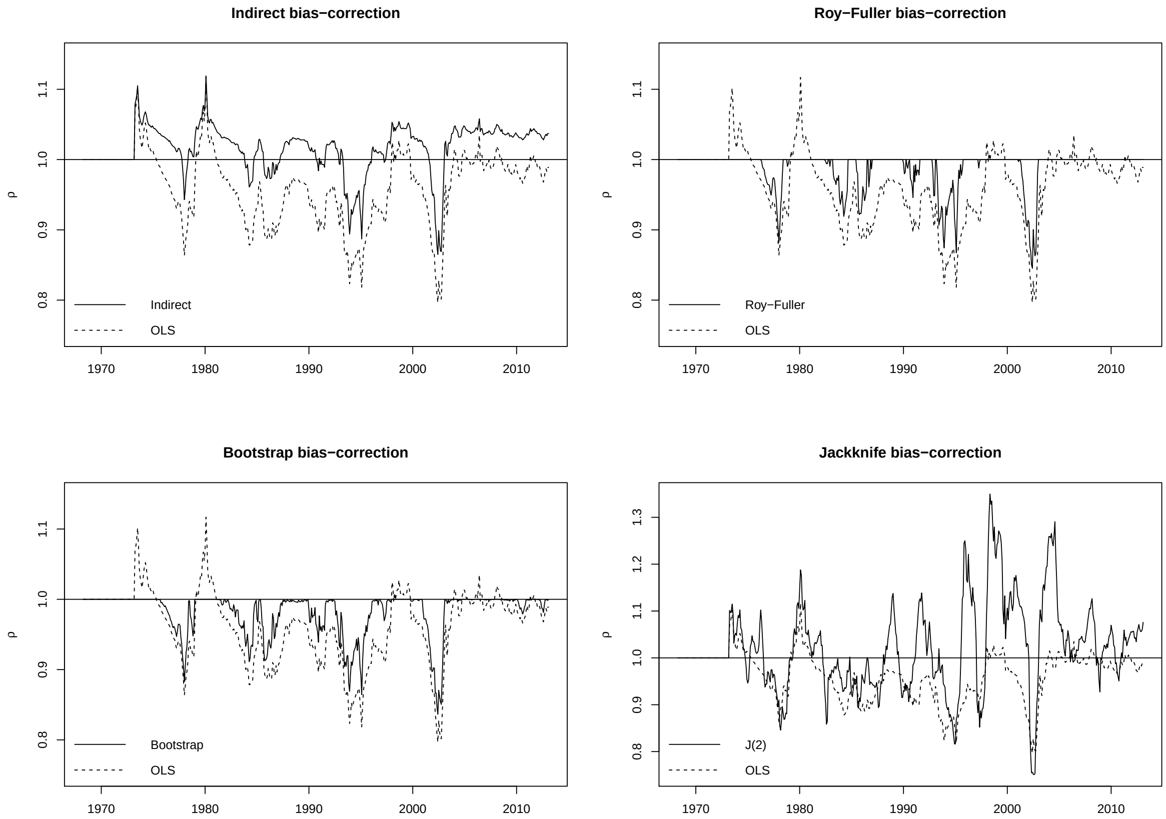

Figure 5: Rolling window AR(1) estimation for the log Gold price series with different bias-correction methods.

The series spans 1993:01–2013:02, thereby giving a total number of 242 observations. The selected lag length equals one. The spread has remarkably declined during the European economic integration and reached levels near zero after the Euro introduction. During the following years (up to 2007), long-term interest rates remained nearly the same in Germany and Greece and only a minor risk premium for investing in Greece has been paid. After the beginning of the financial crisis, however, the spread reached historic values above 25% reflecting the increased default risk. Results for bias-corrected estimation of persistence in this series are reported in Figure 6. In the beginning of the sample, estimated persistence indicate a unit root followed by lower persistence caused by European monetary integration efforts. But, the results also show a dramatic increase in persistence at the beginning of the global financial crisis and even the OLS estimates take values above 1.3 which is remarkably high. Obviously, it is of major importance to allow for explosiveness in this application. Towards the end of the sample, persistence lowered considerably to values near unity indicating one of the outcomes of the European Stability Mechanism. The indirect inference estimator and the jackknife estimator yield similar results as they agree on the general evolution of persistence.

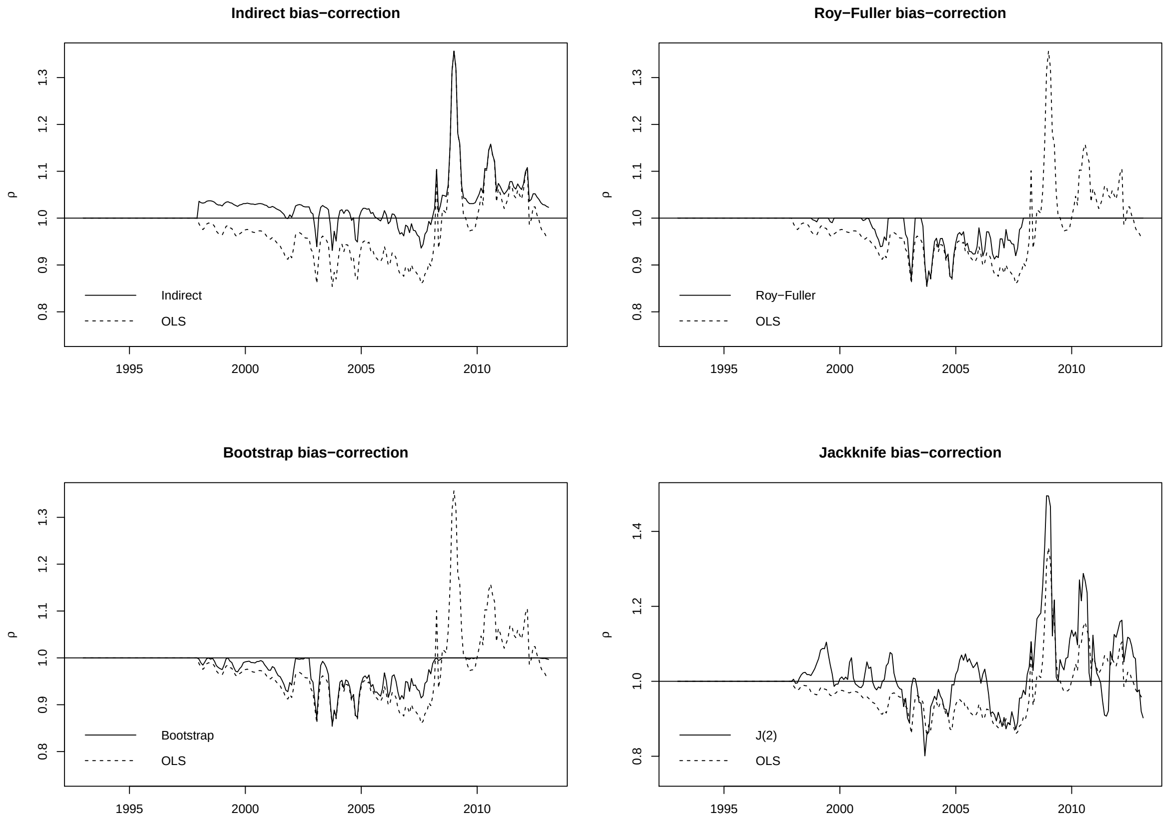

Figure 6: Rolling window AR(1) estimation for the interest rate spread series with different bias-correction methods.

5 Conclusions

This paper compares four different bias-correction techniques for autoregressive processes. Among these are the approximately median-unbiased estimator by Roy and Fuller (2001), a bootstrap-based estimator by Kim (2003), an indirect inference estimator by Phillips et al. (2011) and a jackknife estimator suggested in Chambers (2013). We thus compare established techniques to newly proposed procedures in a comprehensive way. In particular, we focus on situations where the sample size is relatively small and data is highly persistent, exhibits a unit root or is even mildly explosive. When the popular rolling window framework is applied for assessing the possibly time-varying persistence of a time series, sample sizes are typically small. Moreover, it is reasonable to expect that time series undergo changes in persistence during different regimes and episodes. These changes can be either driven by episodes of speculation (leading to temporary bubbles) or policy induced (typically leading to a reduction in persistence). Therefore, we study an empirically relevant situation and provide practical recommendations for further applications.

A large-scale simulation study of bias and root mean squared errors of estimators reveals the following results: The substantial bias of the OLS estimator can be remarkably reduced across the whole range of considered autoregressive parameter values. The most promising approaches are the indirect inference estimator and the jackknife estimator. The indirect inference estimator provides excellent bias-correction in various settings (i.e. heavy-tailed errors, GARCH errors, linear trend and misspecified autoregression) together with a reasonably low variance, while the jackknife estimator performs often best in terms of bias-correction, but has a clearly larger variance rendering this estimator less recommendable in terms of RMSE.

As the main empirical application, we consider a long annual US Debt/GDP series in a rolling window estimation framework. Remarkable evidence for time-varying persistence and periods of explosiveness during the Civil War and World War II are documented. The results clearly suggest substantial differences for various estimation techniques and thus, different policy implications. Further empirical applications are made to Oil prices, Gold prices and the spread between long-term interest rates in Germany and Greece. In all cases, the importance of bias-correction and the simultaneous allowance for locally explosive behaviour is further stressed.

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A Appendix

A.1 Stable errors

		Bias					RMSE				
T	ρ	OLS	II	RF	Kim	J(2)	OLS	II	RF	Kim	J(2)
30	0.85	-0.135	0.012	**0.000**	-0.030	-0.008	0.199	**0.144**	0.163	0.160	0.220
	0.90	-0.148	**-0.002**	-0.017	-0.046	-0.014	0.205	**0.140**	0.150	0.155	0.221
	0.95	-0.162	**-0.018**	-0.042	-0.069	-0.023	0.213	**0.134**	0.142	0.154	0.222
	0.99	-0.167	-0.028	-0.064	-0.090	**-0.022**	0.215	**0.127**	0.139	0.160	0.221
	1.00	-0.166	-0.029	-0.068	-0.095	**-0.019**	0.213	**0.125**	0.137	0.160	0.219
	1.01	-0.162	-0.029	-	-	**-0.015**	0.210	**0.123**	-	-	0.215
	1.02	-0.157	-0.028	-	-	**-0.012**	0.206	**0.120**	-	-	0.210
60	0.85	-0.064	0.006	0.010	-0.005	**0.002**	0.108	0.094	0.100	**0.093**	0.119
	0.90	-0.070	0.004	0.007	-0.009	**0.001**	0.108	0.089	0.091	**0.087**	0.118
	0.95	-0.078	**-0.003**	-0.006	-0.021	-0.003	0.109	0.080	**0.077**	0.079	0.116
	0.99	-0.086	-0.014	-0.028	-0.041	**-0.008**	0.112	0.071	**0.071**	0.079	0.116
	1.00	-0.085	-0.015	-0.033	-0.046	**-0.006**	0.111	**0.068**	0.070	0.081	0.115
	1.01	-0.079	-0.013	-	-	**-0.001**	0.104	**0.062**	-	-	0.107
	1.02	-0.069	-0.010	-	-	**0.003**	0.095	**0.056**	-	-	0.099
120	0.85	-0.030	0.002	0.003	**0.000**	0.002	0.062	0.057	0.059	**0.057**	0.068
	0.90	-0.033	0.002	0.005	**-0.001**	0.002	0.059	0.053	0.055	**0.052**	0.065
	0.95	-0.037	0.002	0.004	-0.004	**0.002**	0.057	0.047	0.047	**0.045**	0.062
	0.99	-0.044	-0.006	-0.011	-0.018	**-0.003**	0.059	0.038	**0.037**	0.040	0.061
	1.00	-0.044	-0.008	-0.016	-0.022	**-0.002**	0.057	**0.035**	0.035	0.040	0.059
	1.01	-0.035	-0.005	-	-	**0.003**	0.049	**0.028**	-	-	0.051
	1.02	-0.021	**-0.002**	-	-	0.009	0.037	**0.022**	-	-	0.044
240	0.85	-0.015	0.000	**0.000**	0.000	0.001	0.039	0.036	0.037	**0.036**	0.041
	0.90	-0.016	0.001	0.001	**0.000**	0.001	0.035	0.032	0.032	**0.032**	0.038
	0.95	-0.017	0.001	0.003	**0.000**	0.002	0.031	0.027	0.028	**0.026**	0.034
	0.99	-0.021	-0.001	-0.002	-0.006	**0.000**	0.029	0.020	**0.019**	0.020	0.031
	1.00	-0.022	-0.004	-0.008	-0.011	**0.000**	0.030	**0.019**	0.019	0.022	0.032
	1.01	-0.010	**-0.001**	-	-	0.005	0.018	**0.011**	-	-	0.022
	1.02	-0.002	**0.000**	-	-	0.008	0.009	**0.006**	-	-	0.017

Table 6: Bias and RMSE for OLS, indirect inference (II), Roy-Fuller (RF), Kim and jackknife (J(2)) estimation procedures for different AR(1) processes and sample sizes (constant included) with stable error distribution.

A.2 Misspecified AR(1) process

		Bias					RMSE				
T	ρ	OLS	II	RF	Kim	J(2)	OLS	II	RF	Kim	J(2)
30	0.85	-0.162	**0.003**	-0.021	-0.044	0.024	0.237	**0.151**	0.190	0.186	0.280
	0.90	-0.173	-0.014	-0.038	-0.061	**0.013**	0.240	**0.149**	0.178	0.180	0.273
	0.95	-0.183	-0.030	-0.060	-0.083	**0.003**	0.242	**0.145**	0.167	0.176	0.269
	0.99	-0.186	-0.039	-0.078	-0.105	**0.002**	0.241	**0.140**	0.160	0.179	0.262
	1.00	-0.182	-0.037	-0.079	-0.106	**0.009**	0.236	**0.136**	0.155	0.177	0.259
	1.01	-0.178	-0.036	-	-	**0.009**	0.232	**0.134**	-	-	0.255
	1.02	-0.170	-0.033	-	-	**0.016**	0.226	**0.130**	-	-	0.254
60	0.85	-0.075	**0.002**	0.004	-0.007	0.008	0.125	**0.104**	0.113	0.106	0.141
	0.90	-0.080	-0.003	**-0.001**	-0.013	0.007	0.123	0.099	0.102	**0.098**	0.137
	0.95	-0.086	-0.010	-0.013	-0.025	**0.004**	0.121	0.087	**0.085**	0.086	0.130
	0.99	-0.094	-0.020	-0.034	-0.045	**-0.003**	0.122	0.079	**0.078**	0.085	0.127
	1.00	-0.090	-0.018	-0.036	-0.048	**0.003**	0.116	**0.072**	0.073	0.083	0.123
	1.01	-0.086	-0.018	-	-	**0.004**	0.114	**0.070**	-	-	0.118
	1.02	-0.074	-0.013	-	-	**0.009**	0.103	**0.062**	-	-	0.112
120	0.85	-0.035	**-0.001**	0.001	-0.002	0.003	0.069	0.063	0.064	**0.062**	0.075
	0.90	-0.037	**0.000**	0.002	-0.003	0.003	0.065	0.057	0.059	**0.056**	0.071
	0.95	-0.041	-0.001	**0.001**	-0.006	0.003	0.062	0.050	0.050	**0.048**	0.066
	0.99	-0.046	-0.008	-0.013	-0.019	**-0.001**	0.061	0.040	**0.038**	0.041	0.063
	1.00	-0.045	-0.009	-0.017	-0.023	**0.000**	0.059	0.036	**0.036**	0.040	0.060
	1.01	-0.037	-0.006	-	-	**0.005**	0.051	**0.031**	-	-	0.055
	1.02	-0.022	**-0.002**	-	-	0.010	0.039	**0.024**	-	-	0.046
240	0.85	-0.017	**0.000**	0.000	0.000	0.001	0.043	0.040	0.040	**0.040**	0.044
	0.90	-0.018	0.000	**0.000**	-0.001	0.001	0.038	0.034	0.035	**0.034**	0.040
	0.95	-0.019	**0.000**	0.002	-0.001	0.002	0.033	0.029	0.030	**0.028**	0.035
	0.99	-0.022	-0.003	-0.003	-0.007	**0.000**	0.031	0.021	**0.020**	0.021	0.031
	1.00	-0.022	-0.004	-0.008	-0.011	**0.000**	0.029	0.017	**0.017**	0.020	0.030
	1.01	-0.011	**-0.001**	-	-	0.005	0.019	**0.012**	-	-	0.023
	1.02	-0.002	**0.000**	-	-	0.008	0.009	**0.006**	-	-	0.018

Table 7: Bias and RMSE for OLS, indirect inference (II), Roy-Fuller (RF), Kim and jackknife (J(2)) estimation procedures for different AR(1) processes when the model is misspecified as AR(2) (constant included).

A.3 AR(2) process

			Bias					RMSE				
T	β	ρ	OLS	II	RF	Kim	J(2)	OLS	II	RF	Kim	J(2)
30	0.2	0.85	-0.134	**-0.001**	-0.015	-0.032	0.023	0.199	**0.138**	0.165	0.160	0.239
		0.90	-0.142	**-0.012**	-0.028	-0.045	0.017	0.201	**0.136**	0.153	0.152	0.235
		0.95	-0.151	-0.025	-0.047	-0.065	**0.012**	0.201	**0.128**	0.140	0.146	0.232
		0.99	-0.153	-0.030	-0.063	-0.084	**0.008**	0.200	**0.120**	0.132	0.147	0.224
		1.00	-0.150	-0.030	-0.066	-0.089	**0.011**	0.196	**0.118**	0.129	0.150	0.222
		1.01	-0.144	-0.027	-	-	**0.018**	0.191	**0.113**	-	-	0.219
		1.02	-0.139	-0.026	-	-	**0.018**	0.188	**0.111**	-	-	0.214
	-0.3	0.85	-0.217	**-0.001**	-0.040	-0.072	0.008	0.304	**0.167**	0.232	0.232	0.335
		0.90	-0.229	-0.022	-0.063	-0.093	**-0.007**	0.309	**0.167**	0.222	0.229	0.329
		0.95	-0.240	-0.043	-0.087	-0.119	**-0.016**	0.311	**0.168**	0.213	0.228	0.328
		0.99	-0.237	-0.048	-0.101	-0.135	**-0.007**	0.303	**0.163**	0.202	0.226	0.320
		1.00	-0.231	-0.045	-0.101	-0.137	**0.003**	0.297	**0.158**	0.196	0.227	0.317
		1.01	-0.228	-0.046	-	-	**0.005**	0.295	**0.157**	-	-	0.313
		1.02	-0.224	-0.046	-	-	**0.007**	0.293	**0.157**	-	-	0.309
120	0.2	0.85	-0.029	-0.001	**-0.001**	-0.002	0.001	0.060	0.055	0.057	**0.055**	0.064
		0.90	-0.029	**0.000**	0.001	-0.002	0.002	0.055	0.049	0.051	**0.048**	0.059
		0.95	-0.032	**0.000**	0.002	-0.003	0.003	0.050	0.042	0.043	**0.041**	0.055
		0.99	-0.037	-0.006	-0.009	-0.014	**-0.001**	0.050	0.033	**0.032**	0.034	0.051
		1.00	-0.036	-0.007	-0.014	-0.019	**0.002**	0.047	0.029	**0.029**	0.033	0.050
		1.01	-0.027	-0.004	-	-	**0.004**	0.040	**0.024**	-	-	0.042
		1.02	-0.012	**0.000**	-	-	0.011	0.027	**0.017**	-	-	0.036
	-0.3	0.85	-0.048	**-0.001**	0.002	-0.004	0.003	0.087	0.076	0.080	**0.076**	0.092
		0.90	-0.050	**-0.001**	0.004	-0.005	0.004	0.082	0.070	0.073	**0.069**	0.088
		0.95	-0.054	-0.004	-0.002	-0.011	**0.002**	0.080	0.061	0.060	**0.059**	0.084
		0.99	-0.059	-0.011	-0.017	-0.025	**0.000**	0.078	0.051	**0.048**	0.053	0.081
		1.00	-0.059	-0.012	-0.022	-0.030	**-0.001**	0.076	0.047	**0.047**	0.053	0.078
		1.01	-0.050	**-0.009**	-	-	0.005	0.069	**0.042**	-	-	0.072
		1.02	-0.037	**-0.005**	-	-	0.009	0.058	**0.035**	-	-	0.064
240	0.2	0.85	-0.013	0.000	0.000	**0.000**	0.001	0.037	0.035	0.035	**0.035**	0.038
		0.90	-0.013	0.000	0.000	**0.000**	0.001	0.032	0.030	0.030	**0.030**	0.034
		0.95	-0.015	**0.000**	0.001	0.000	0.002	0.028	0.025	0.025	**0.024**	0.029
		0.99	-0.017	-0.001	-0.002	-0.004	**0.000**	0.024	0.017	**0.016**	0.017	0.025
		1.00	-0.018	-0.003	-0.007	-0.009	**0.000**	0.023	0.014	**0.014**	0.016	0.024
		1.01	-0.006	**0.000**	-	-	0.006	0.013	**0.008**	-	-	0.017
		1.02	0.000	**0.000**	-	-	0.006	0.005	**0.004**	-	-	0.013
	-0.3	0.85	-0.023	-0.001	-0.001	-0.002	**0.000**	0.051	0.047	0.047	**0.047**	0.053
		0.90	-0.024	-0.001	0.000	-0.001	**0.000**	0.046	0.041	0.042	**0.041**	0.048
		0.95	-0.024	**0.001**	0.003	-0.001	0.002	0.041	0.034	0.036	**0.033**	0.043
		0.99	-0.029	-0.004	-0.006	-0.010	**-0.001**	0.040	0.027	**0.025**	0.027	0.041
		1.00	-0.029	-0.005	-0.011	-0.015	**0.000**	0.037	0.023	**0.022**	0.025	0.038
		1.01	-0.019	**-0.003**	-	-	0.004	0.029	**0.017**	-	-	0.031
		1.02	-0.006	**0.000**	-	-	0.011	0.016	**0.010**	-	-	0.026

Table 8: Bias and RMSE for OLS, indirect inference (II), Roy-Fuller (RF), Kim and jackknife (J(2)) estimation procedures for different AR(2) processes (constant included).

A.4 Misspecified AR(2) process

			Bias					RMSE				
T	β	ρ	OLS	II	RF	Kim	J(2)	OLS	II	RF	Kim	J(2)
30	0.2	0.85	-0.081	0.066	0.054	**0.027**	0.028	0.147	0.142	0.140	**0.131**	0.184
		0.90	-0.097	0.047	0.027	**0.002**	0.015	0.154	0.127	0.119	**0.119**	0.185
		0.95	-0.116	0.024	-0.009	-0.032	**-0.002**	0.164	0.112	**0.105**	0.116	0.187
		0.99	-0.126	**0.007**	-0.040	-0.065	-0.009	0.169	**0.100**	0.101	0.128	0.186
		1.00	-0.126	**0.005**	-0.046	-0.075	-0.008	0.168	**0.097**	0.101	0.136	0.185
		1.01	-0.123	**0.003**	-	-	-0.005	0.166	**0.094**	-	-	0.182
		1.02	-0.118	0.003	-	-	**-0.001**	0.162	**0.091**	-	-	0.177
	-0.3	0.85	-0.314	**-0.122**	-0.202	-0.223	-0.145	0.382	**0.187**	0.325	0.329	0.340
		0.90	-0.313	-0.136	-0.197	-0.219	**-0.134**	0.378	**0.205**	0.315	0.320	0.334
		0.95	-0.309	-0.143	-0.193	-0.215	**-0.120**	0.372	**0.217**	0.301	0.309	0.327
		0.99	-0.298	-0.141	-0.186	-0.209	**-0.098**	0.360	**0.218**	0.286	0.296	0.316
		1.00	-0.292	-0.137	-0.182	-0.205	**-0.090**	0.354	**0.216**	0.280	0.291	0.311
		1.01	-0.285	-0.133	-	-	**-0.080**	0.348	**0.213**	-	-	0.306
		1.02	-0.277	-0.128	-	-	**-0.070**	0.340	**0.209**	-	-	0.300
120	0.2	0.85	**0.002**	0.036	0.038	0.034	0.026	**0.043**	0.058	0.061	0.056	0.058
		0.90	**-0.007**	0.030	0.034	0.027	0.018	**0.039**	0.052	0.055	0.049	0.052
		0.95	-0.017	0.023	**0.024**	0.016	0.011	0.038	0.042	0.041	**0.037**	0.048
		0.99	-0.029	0.008	-0.001	-0.006	**0.001**	0.041	0.026	**0.022**	0.026	0.046
		1.00	-0.031	0.003	-0.009	-0.015	0.000	0.041	0.022	**0.021**	0.030	0.045
		1.01	-0.023	**0.003**	-	-	0.003	0.035	**0.019**	-	-	0.041
		1.02	-0.008	**0.006**	-	-	0.014	0.023	**0.015**	-	-	0.035
	-0.3	0.85	-0.124	-0.092	-0.095	-0.097	**-0.069**	0.153	**0.125**	0.134	0.134	0.130
		0.90	-0.108	-0.076	-0.076	-0.079	**-0.047**	0.136	**0.113**	0.116	0.116	0.116
		0.95	-0.095	-0.060	-0.058	-0.063	**-0.029**	0.120	**0.098**	0.099	0.099	0.105
		0.99	-0.086	-0.049	-0.049	-0.056	**-0.016**	0.107	0.083	**0.083**	0.085	0.096
		1.00	-0.080	-0.043	-0.046	-0.052	**-0.007**	0.100	**0.075**	0.076	0.079	0.091
		1.01	-0.068	-0.033	-	-	**0.002**	0.089	**0.066**	-	-	0.084
		1.02	-0.051	-0.024	-	-	**0.007**	0.075	**0.055**	-	-	0.072
240	0.2	0.85	**0.014**	0.030	0.030	0.030	0.025	**0.031**	0.041	0.042	0.041	0.040
		0.90	**0.006**	0.023	0.023	0.022	0.018	**0.025**	0.033	0.034	0.033	0.033
		0.95	**-0.003**	0.016	0.018	0.014	0.010	**0.020**	0.026	0.028	0.025	0.026
		0.99	-0.012	0.007	0.004	**0.001**	0.003	0.020	0.015	**0.013**	0.013	0.023
		1.00	-0.015	0.002	-0.004	-0.007	**0.000**	0.020	0.011	**0.010**	0.015	0.022
		1.01	-0.004	**0.003**	-	-	0.007	0.012	**0.007**	-	-	0.018
		1.02	**0.004**	0.005	-	-	0.010	0.006	**0.006**	-	-	0.015
	-0.3	0.85	-0.093	-0.079	-0.079	-0.079	**-0.066**	0.110	0.099	0.100	0.100	**0.094**
		0.90	-0.074	-0.059	-0.059	-0.059	**-0.044**	0.091	0.080	0.080	0.080	**0.076**
		0.95	-0.056	-0.039	-0.039	-0.040	**-0.022**	0.071	**0.060**	0.060	0.060	0.061
		0.99	-0.046	-0.026	-0.026	-0.029	**-0.008**	0.057	0.045	**0.045**	0.046	0.051
		1.00	-0.041	-0.022	-0.023	-0.026	**-0.002**	0.052	**0.039**	0.039	0.040	0.048
		1.01	-0.026	-0.012	-	-	**0.004**	0.038	**0.028**	-	-	0.038
		1.02	-0.012	**-0.007**	-	-	0.008	0.022	**0.016**	-	-	0.028

Table 9: Bias and RMSE for OLS. indirect inference (II), Roy-Fuller (RF), Kim and jackknife (J(2)) estimation procedures for different AR(2) processes when the model is misspecified as AR(1) (constant included).

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