Inefficient Job Destructions and Training with Hold-up

Arnaud Chéron — Bénédicte Rouland

Abstract. This paper develops an equilibrium search model with endogenous job destructions and where firms decide at the time of job entry how much to invest in match-specific human capital. We first show that job destruction and training investment decisions are strongly complementary. It is possible that there are no firings at equilibrium. Further, training investments are confronted to a hold-up problem making the decentralized equilibrium always inefficient. We show therefore that both training subsidies and firing taxes must be implemented to bring back efficiency.

1. Introduction

The normative analysis of the link between investment in specific human capital and labor market outcomes dates back to Becker's contribution: within the context of standard competitive theory, workers will not pay for specific training but firms will. However, as Becker (1962) also pointed out, firms might let workers share in the returns (and the costs as well) to reduce both inefficient turnover and investments.[1] The sharing decision supposes that the worker and the firm write a non-renegotiable contract specifying a fixed wage, set in such a way to maximize the expected total surplus. But sharing the costs is possible only if training investments can be preceded by contract negotiations specifying that workers agree to take part in the costs through lower wages. Otherwise, only firms pay all the costs leading to a hold-up problem as they cannot get all the returns on their investment.[2] This finally results in under-investments and the decentralized equilibrium is always inefficient.

New developments focus on interactions between employers and employees within the framework of labor market imperfections [see Acemoglu and Pischke (1998) for a survey]. In particular, when wages are determined by an *ex post* bargaining, contracts are not enforceable, and there is potential for hold-ups: the Nash assumption implies that a fraction of the expected investment cost, which the firm saves when the worker stays in the match, is actually captured by the worker through a higher wage. As the equilibrium is then always inefficient, there is room for labor market policy. For instance, Acemoglu and Shimer (1999) and Sato and Sugiura (2003) consider *ex ante* investments that take place before production begins. Acemoglu and Shimer (1999) analyse the potential for hold-up in case of physical capital investments. Sato and Sugiura (2003) consider workers investments in general human capital and investigate the effects of labor market policies both on human capital accumulation and on the hold-up problem. Chéron (2005) adds match-specific costs

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in the standard matching model that can be only partially protected from hold-up. This allows the author to re-examine welfare effects of a decrease in equilibrium unemployment.

In this paper, we extend those works to account for endogenous job destructions. More precisely, our model is characterized by endogenous hiring and firing decisions and by training investment decisions of firms in specific human capital. We first emphasize the interplay between job destructions and training. In particular, we show that job destructions and training investments are highly complementary as firms have strong incentives to invest in training to protect matches from idiosyncratic productivity shocks. Expected productivity gains due to training investments rise the job tenure, which in turn encourages firms to invest more. Therefore, there might be no firings at equilibrium.

Second, this paper shows how the efficient allocation can be reached in this framework where a hold-up problem may arise. In particular, we focus on a hold-up problem that results from an 'insider wage structure' (Mortensen and Pissarides, 1999).[3] Assuming a training investment at the time of job entry (wasted if the negotiation fails) typically comes to introduce a fixed job creation cost. Reducing the expected job value, hold-up then results in an excess of job destructions at equilibrium and on the contrary in a lack of training investments. Hence, we show that it is optimal to implement both training subsidies and firing taxes to achieve efficiency.

More generally, several papers have studied the positive link between employment protection and training investments. Fostering long-term employment, employment protection may promote investments in human capital as longer-lasting employment will increase the expected returns to training. The empirical contributions of Bishop (1991) and Pierre and Scarpetta (2006) put the emphasis on the fact that firms react to strict employment protection by investing more in training. From a natural experiment in France, Messe and Rouland (2010) show that stricter employment protection for older workers rises firms incentives to train them. In a theoretical perspective, Fella (2005) explains that large enough conditional termination penalties improve employers investments in general training if the latter is not directly contractible. The reason is that separation payments ensure firms to capture a positive share of the return as training is vested in the worker on separation. Closer to our paper are Lechthaler (2009) and Belot *et al.* (2007) who investigate the impact of firings costs on equilibrium unemployment and welfare in a matching model with training. Lechthaler (2009) considers firms investments in general training. Therefore, inefficiencies stem from the fact that training firms do not take into account that fired workers are more productive in their following relationships as well. Firing costs are useful as mean to raise training investments. Hold-up is another potential source of inefficiency, as explored in Belot *et al.* (2007) who focus on workers investments in firm-specific knowledge. Firing costs work as a commitment device of the employer and workers react with higher investments in firm-specific knowledge. In comparison with those contributions, we consider in this paper specific training investments provided by firms to their workers and show that employment protection is not enough to cover inefficiencies due to hold-up in a context of firms investments in specific training. In addition to firing costs, we stress the need for training subsidies to restore social efficiency. Firing taxes and training subsidies are mostly studied separately but here, it is definitively the combination of those two parameters that achieves efficiency. We show that both policy instruments are not unconnected because of the strong complementarity between firing and training decisions. Therefore, our paper suggests that the right design of firing costs should account for the fact that training investments are suboptimal.

The paper is organized as follows. The next section presents the decentralized partial equilibrium with first a two-tier (efficient) wage contract and second, an insider wage structure. We then examine the optimal design of labor market policy. A last section concludes.

2. Labor market equilibrium

2.1 Model environment and labor market flows

We consider a continuous-time equilibrium search model at steady state with endogenous job destruction. The population of workers is a continuum mass. Workers look for jobs and are randomly matched with employers looking for workers to fill vacant units of production. A productive unit is the association of one worker and one firm. Matches are randomly formed according to a constant return to scale matching function $M(V, U)$ that gives the number of hirings (the job creation flows) as a function of the number of vacancies V and the number of unemployed workers U. Each worker matches with a firm with probability $\theta q(\theta) \equiv M(v, u)/u$ where $q(\theta) \equiv M(v, u)/v$ defines the probability to fill a vacancy for a firm, and with $\theta \equiv v/u$ the labor market tightness.

The time of events and of decisions is as follows. First, at the time of match formation, firms decide on the investment in continuing training[4] k, resulting in the human capital of workers $y(k)$.[5] The firm and its worker then bargain over the wage. We assume that the human capital level of a worker is determined at the entry into the job and is constant for all the job tenure. Training investments increase the output of the worker only if he or she stays with the training firm. In this way, training is assumed to be specific in Becker's (1962) sense as it is fully lost on separation.[6] The productivity of a worker is the sum of a random component ε and a deterministic one $y(k)$.[7] The random component is related to shocks that occur at Poisson rate λ, and where the cumulative distribution function is $G(\varepsilon)\ \forall\ \varepsilon \in [0, \bar{\varepsilon}]$.

Second, a new value of ε is randomly drawn from its distribution. The worker and the firm then bargain over a new wage if there is a positive surplus to share. In the opposite case, they optimally separate. Job destructions arise when ε falls below an endogenous threshold that depends on the invested amount in continuing training. We denote this threshold $R(k)$. We assume that jobs can also be destroyed exogenously at rate s in the form of voluntary quits of workers,[8] so that $s + \lambda G(R(k))$ gives the overall destruction rate of a job matched with a worker who has received a training investment k.

Lastly, whenever an idiosyncratic shock arrives, the firm either accounts for this new value of ε in a new wage negotiation or destroys the job for a zero return.

2.2 Firms decisions

2.2.1 Hiring and firing decisions. For a firm, the intertemporal value of a filled job depends both on worker's human capital $y(k)$ and on the idiosyncratic component ε. We denote this value by $J(k, \varepsilon)$. Following Mortensen and Pissarides (1994), we assume that all new jobs are created with maximum productivity, $\bar{\varepsilon}$. We also assume that firms pay all the training cost $C(k)$[9] at the time of match formation (before the wage bargaining). The asset value of a vacancy then writes:

$$rV = -c + q(\theta)\left[\bar{J}(k) - C(k) - V\right]$$

with r the discount factor, $c \geq 0$ the flow cost of recruiting a worker and where the corresponding Bellman equations for new and continuing jobs respectively satisfy:[10]

$$r\bar{J}(k) = y(k) + \bar{\varepsilon} - \bar{w}(k) + \lambda \int_{R(k)}^{\bar{\varepsilon}} J(k, x)\,\mathrm{d}G(x) - (\lambda + s)\bar{J}(k)$$

$$rJ(k, \varepsilon) = y(k) + \varepsilon - w(k, \varepsilon) + \lambda \int_{R(k)}^{\bar{\varepsilon}} J(k, x)\,\mathrm{d}G(x) - (\lambda + s)J(k, \varepsilon)$$

where $w(k, \varepsilon)$ denotes the real wage.

As firms open vacancies until all rents from vacant jobs are exhausted, endogenous job creation satisfies the condition:

$$\frac{c}{q(\theta)} = \bar{J}(k) - C(k). \quad [1]$$

Job creation entails both a recruiting cost c, proportional to the probability to fill a vacancy, and a training cost $C(k)$ depending on the invested amount.

In turn, the endogenous job destruction rule $J(k, \varepsilon) \leq 0$ leads to a reservation productivity $R(k)$ defined by $J(k, R(k)) = 0$ and such as:

$$R(k) = -y(k) + w(k, R(k)) - \lambda \int_{R(k)}^{\bar{\varepsilon}} J(k, x)\,\mathrm{d}G(x). \quad [2]$$

On one hand, the higher the wage, the higher the reservation productivity $R(k)$, and hence the higher the job destruction flow. On the other hand, the higher the option value of filled jobs (expected gains in the future) depending on the training investment, the weaker the job destructions. It follows that a firm may be able to afford to lose instantaneous profit, waiting for future gains that may compensate for. In addition, given the wage, training investments improve job tenure increasing both present and future productivity gains.

2.2.2 Human capital investment decision. Firms choose how much specific training they invest in order to maximize the net expected value of a filled job. It follows that the investment decision is stated as:

$$\max_{k \geq 0} \bar{J}(k) - C(k) \Rightarrow C'(k) = \overline{J_1'}(k).$$

In this way, firms decide on the sum they invest in specific training so that the expected marginal return on investment is equaled to its marginal cost. The marginal return particularly depends on the relation between the bargained wage and the investment level, and hence on the potential hold-up problem.

2.3 Equilibrium with a two-tier wage structure

2.3.1 The Nash wage bargaining. Wages are determined by a Nash bargaining. The firm and the worker share the global surplus generated by a job according to their bargaining power. But, following Mortensen and Pissarides (1999), we first assume that the wage structure that arises when firms are liable for hiring costs (a training cost here) is a two-tier one. On one hand, the initial wage reflects the fact that workers share in the initial hiring (training) cost by accepting a lower wage. On the other hand, renegotiated wages subsequent to match productivity shocks no longer include training costs as they are sunk.

Standard function values of employed and unemployed workers are respectively given by:

$$r\bar{W}(k) = \bar{w}(k) + \lambda\int_{R(k)}^{\bar{\varepsilon}} W(k,x)\,\mathrm{d}G(x) + \lambda G(R(k))U - \lambda\bar{W}(k) + s\left[U - \bar{W}(k)\right]$$

$$rU = z + \theta q(\theta)\left[\bar{W}(k) - U\right]$$

where z is home production. The global surplus of a new match $\bar{S}(k) = \bar{J}(k) - C(k) + \bar{W}(k) - U$ is split as follows:

$$\bar{J}(k) - C(k) = (1-\gamma)\bar{S}(k) \quad \text{and} \quad \bar{W}(k) - U = \gamma\bar{S}(k).$$

From the Nash-bargaining rule of the surplus of new matches $(1-\gamma)\left[\bar{W}(k) - U\right] = \gamma\left[\bar{J}(k) - C(k)\right]$ and the job creation condition, we then derive the following expression for the starting wage:[11]

$$\bar{w}(k) = (1-\gamma)z + \gamma\left[y(k) + \bar{\varepsilon} + c\theta\right] - \gamma(r+s+\lambda)C(k).$$

Then, from the Nash-bargaining rule of the surplus generated by a job after a random change in ε, $(1-\gamma)[W(k,\varepsilon) - U] = \gamma[J(k,\varepsilon)]$, and the job creation condition, renegotiated wages write:

$$w^C(k,\varepsilon) = (1-\gamma)z + \gamma[y(k) + \varepsilon + c\theta].$$

First, wages are a weighted average of the reservation wage of the worker (first term in the right-hand side of $\bar{w}(k)$) and second, of the productivity and recruitment costs the firm saves (second term). The last term of stating wages reflects the fact that workers agree to share the training cost with the firm. This is in line with the definition of complete contracts: anyone benefiting from an investment must pay one's share of the cost.

2.3.2 The labor market equilibrium

Definition 1: A labor market equilibrium with a two-tier wage structure is characterized by a triplet $\{\theta^C, R^C(k), k^C\}$ *solving:*[12]

$$\frac{c}{q(\theta)} = \left(\frac{1-\gamma}{r+\lambda+s}\right)[\bar{\varepsilon} - R(k)] - (1-\gamma)C(k) \qquad [3a]$$

$$R(k) = -y(k) + z + \left(\frac{\gamma}{1-\gamma}\right)c\theta - \frac{\lambda}{r+\lambda+s}\int_{R(k)}^{\bar{\varepsilon}}[1-G(x)]\,\mathrm{d}x \qquad [3b]$$

$$C'(k) = \left(\frac{1}{r+s+\lambda G(R(k))}\right)y'(k). \qquad [3c]$$

Proposition 1: Let $C(k) = k$, $y(k) = k^{\alpha}$, $G(x) = x$, $x \in [0, \overline{\varepsilon}]$. *If*

$$r + s > \alpha^{\frac{1}{2-\alpha}} \left[\frac{\lambda(r+\lambda+s)}{1-\alpha} \right]^{\frac{1-\alpha}{2-\alpha}},$$

a unique equilibrium $\{\theta^C, R^C(k), k^C\}$ *with firings exists.*

Proof. See Appendix D.1. ■

Corollary 1: Considering $r \to 0$, *if* $s = 0$, *it comes that* $R^C(k) = 0$ *and* $k = (\alpha/r)^{1/2-\alpha}$.

Proof. Condition

$$r + s > \alpha^{\frac{1}{2-\alpha}} \left[\frac{\lambda(r+\lambda+s)}{1-\alpha} \right]^{\frac{1-\alpha}{2-\alpha}}$$

can never be achieved for $s = 0$. ■

If there are no exogenous breakups ($s = 0$), firms can definitely reap all the benefits of their training investment. Therefore, firms have strong incentives to invest high enough in training to protect matches from idiosyncratic productivity shocks. Training investment, improving both present and future productivity, increases the job value and then reduces the productivity threshold. Consequently, the probability that the random component of productivity falls below this threshold, and the probability that the match endogenously closes, are both smaller. Both probabilities are all the more low than the training investment is high. Anticipating that a higher training investment leads to increase job tenure, firms are finally encouraged to invest more. At the limit, a substantial training investment leads to a so tiny threshold that the job is never destroyed.

However, this is valid as long as there is no risk for the firm to lose the benefits of the training investment. On the opposite, if the exogenous probability of breakups is high enough (in the form of voluntary quits), i.e. if s satisfies Proposition 1, training investment is then low enough so that endogenous firings exist at equilibrium.[13] This actually points out how complementary job destruction and training investment decisions are.

2.4 Equilibrium with insider wage

2.4.1 The Nash wage bargaining. Effects of training investments on wages and job destructions are highly dependent on the wage setting game. To stress that point, we now derive the partial equilibrium with an insider wage structure as proposed by Mortensen and Pissarides (1999). When firms support hiring costs (such as a training cost), a natural hold-up problem may arise. Indeed, new workers have an incentive to renegotiate immediately after been hired as training investments require continuing relationships to be efficient. Starting wages are then not credible. Therefore, second-tier wages apply initially as well as subsequent to any shock to match productivity ('insider wage'). The *ex post* bargaining process increases employees' threat point. Demanding a higher wage, workers capture some of the rents created by the training cost without paying for, leading finally to a hold-up problem.

The global surplus generated by a continuing job $S(k, \varepsilon) = J(k, \varepsilon) + W(k, \varepsilon) - U$ is now split as follows:

$$J(k,\varepsilon)=(1-\gamma)S(k,\varepsilon) \quad \text{and} \quad W(k,\varepsilon)-U=\gamma S(k,\varepsilon).$$

As firms have to pay the training cost in both cases of success and failure of the wage bargaining, wages write:

$$w(k,\varepsilon)=(1-\gamma)z+\gamma[y(k)+\varepsilon+c\theta]+\gamma\theta q(\theta)C(k).$$

The last term of the right-hand side does not appear in renegotiated wages of the two-tier wage structure and refers to hold-up. It depends on the investment level and rises the bargained wage: if the negotiation fails, the firm will have to pay another training cost $C(k)$ when it meets a new worker, an event that takes place at rate $\theta q(\theta)$. So, staying in the match, the worker enables the firm to save the expected cost $\theta q(\theta)C(k)$ and wages increase by a fraction γ of that saving by the Nash assumptions. Workers are all the more in a position to threaten firms than the probability to find a job is high. The hold-up problem becomes then stronger.

2.4.2 The labor market equilibrium

Definition 2: A labor market equilibrium with wage bargaining is characterized by a triplet $\{\theta^I, R^I(k), k^I\}$ *solving:*

$$\frac{c}{q(\theta)}=\left(\frac{1-\gamma}{r+\lambda+s}\right)[\bar{\varepsilon}-R(k)]-C(k) \qquad [4a]$$

$$R(k)=-y(k)+z+\left(\frac{\gamma}{1-\gamma}\right)c\theta+\frac{\gamma}{1-\gamma}\theta q(\theta)C(k)-\frac{\lambda}{r+\lambda+s}\int_{R(k)}^{\bar{\varepsilon}}[1-G(x)]\,\mathrm{d}x \qquad [4b]$$

$$C'(k)=\left(\frac{1-\gamma}{r+s+\lambda G(R(k))+\gamma\theta q\theta}\right)y'(k). \qquad [4c]$$

Proposition 2: Let $C(k)=k,\ y(k)=k^{\alpha},\ G(x)=x,\ x\in[0,\bar{\varepsilon}]$.

$$r+s>\alpha^{\frac{1}{2-\alpha}}\left[\frac{\lambda(r+\lambda+s)}{1-\alpha}\right]^{\frac{1-\alpha}{2-\alpha}}$$

is a sufficient condition for a unique equilibrium $\{\theta^I, R^I(k), k^I\}$ *with some firings to exist.*

Proof. See Appendix D.2. ■

To get back to Corollary 1, hold-up counteracts the positive effects of the training investment on the job tenure by increasing wages. Thereby, the risk firms face to lose all or a part of the training investment rises. Hold-up reduces then the incentives firms have to invest highly in training.

3. Optimal labor market policy

3.1 The efficient allocation

We derive the optimal allocation by maximizing steady-state output with respect to the labor market tightness θ^*, the reservation productivity $R^*(k)$, and the training investment k^*. The problem of the planner is stated as follows:

$$\max_{\{\theta,R(k),k\}}\int_0^\infty e^{-rt}[\bar{y}+uz-c\theta u-\theta q(\theta)uC(k)]\mathrm{d}t \qquad [5]$$

subject to the evolution of u and $\bar{y}$:

$$\dot{u}=(1-u)[\lambda G(R(k))+s]-u\theta q(\theta)$$

$$\dot{\bar{y}}=u\theta q(\theta)[\bar{\varepsilon}+y(k)]+\lambda(1-u)[1-G(R(k))]y(k)+\lambda(1-u)\int_{R(k)}^{\bar{\varepsilon}}\varepsilon\mathrm{d}G(\varepsilon)-(\lambda+s)\bar{y}.$$

Definition 3: Defining $\eta(\theta)=-\theta q'(\theta)/q(\theta)$, the efficient labor market allocation is then characterized by a triplet $\{\theta^*, R^*(k), k^*\}$ solving:

$$\frac{c}{q(\theta^*)}=\left(\frac{1-\eta(\theta^*)}{r+\lambda+s}\right)[\bar{\varepsilon}-R^*(k)]-(1-\eta(\theta^*))C(k^*) \qquad [6a]$$

$$R^*(k)=-y(k^*)+z+\left(\frac{\eta(\theta^*)}{1-\eta(\theta^*)}\right)c\theta^*-\frac{\lambda}{r+\lambda+s}\int_{R^*(k)}^{\bar{\varepsilon}}[1-G(x)]\mathrm{d}x \qquad [6b]$$

$$C'(k^*)=\left(\frac{1}{r+s+\lambda G(R^*(k))}\right)y'(k^*). \qquad [6c]$$

Proposition 3: Let $C(k)=k$, $y(k)=k^\alpha$, $G(x)=x$, $x\in[0,\bar{\varepsilon}]$. If

$$r+s>\alpha^{\frac{1}{2-\alpha}}\left[\frac{\lambda(r+\lambda+s)}{1-\alpha}\right]^{\frac{1-\alpha}{2-\alpha}},$$

a unique efficient allocation $\{\theta^*, R^*(k), k^*\}$ with some firings exists.

Proof. See Appendix D.3. ■

Property 1: The Hosios condition $\gamma=\eta(\theta)$ does not achieve equilibrium efficiency in the insider wage case whereas it does in the two-tier wage structure.

Proof. Straightforward by comparing expressions of $\{\theta^*, R^*(k), k^*\}$ in Definition 3 to $\{\theta^C, R^C(k), k^C\}$ in Definition 1 and to $\{\theta^I, R^I(k), k^I\}$ in Definition 2. ■

Equilibrium choices are not efficient due to hold-up that reduces the expected job value. In order to recognize the mechanisms that make the equilibrium inefficient, we explore separately the free entry condition of firms [4a], the reservation productivity [4b], and the investment decision condition of firms [4c]. This helps us to consider labor policies that remove the distortions.

First, we examine the investment decision condition of firms. Evaluating [4c] at $R^I(k) = R^*(k)$, we obtain the investment level under the assumption that the reservation productivity takes the optimal value $k^I\,|_{R(k)=R^*(k)}$. Comparing $k^I\,|_{R(k)=R^*(k)}$ with k^* leads to the following lemma.

Lemma 1: The investment decision condition of firms generates under-investments: $k^I\,|_{R(k)=R^*(k)} < k^*$.

Proof. See Appendix D.4. ■

Training investments are not enough at equilibrium. With an insider wage structure, not only workers do not contribute to the training cost but they also capture some of the returns on the training investment (hold-up). Therefore, firms under-invest in training their workers because they have to pay all the training cost whereas they get only a fraction of the gains. Thereby, the job value is lowered by hold-up.

Second, we address the reservation productivity of firms [4b]. Evaluating [4b] at $k^I = k^*$ and $\theta^I = \theta^*$ gives the reservation productivity under the assumptions that the investment level and the market tightness are both optimal $R^I\,|_{k=k^*,\theta=\theta^*}$. Comparing $R^I(k)\,|_{k=k^*,\theta=\theta^*}$ with $R^*(k)$ gives the following lemma.

Lemma 2: Under the Hosios condition, the productivity reservation of firms generates too many job destructions: $R^I(k)\,|_{k=k^*,\theta=\theta^*} > R^*(k)$.

Proof. See Appendix D.5. ■

There are too many job destructions at equilibrium. Firms close endogenously too many jobs at equilibrium because hold-up, increasing wages, rises the productivity threshold. Therefore, productivity gains induced by the training investments are not enough to improve job tenure.

Finally, we examine the free entry condition of firms [4a]. Evaluating [4a] at $k^I = k^*$ and $R^I(k) = R^*(k)$ gives the market tightness under the assumptions that the investment level and the reservation productivity are both optimal $\theta^I(k)\,|_{k=k^*,R(k)=R^*(k)}$. Comparing $\theta^I(k)\,|_{k=k^*,R(k)=R^*(k)}$ with $\theta^*(k)$ gives the following lemma.

Lemma 3: Under the Hosios condition, the free entry condition generates too little labor market tightness: $\theta^I(k)\,|_{k=k^*,R(k)=R^*(k)} < \theta^*(k)$.

Proof. See Appendix D.6. ■

Firms do not post enough vacant jobs at equilibrium. Here again, the inefficiency comes from the contract type that allows workers to capture some of the rents following the training investment without contributing to its cost (hold-up). As shown previously, this rises firms' reservation productivity, which in turn decreases both the job tenure and the expected job value. Further, the expected job value is also reduced as firms have to bear all the training cost. Firms then post too few vacant jobs compared with what would be optimal.

3.2 Restoring efficiency

This last section investigates the way to restore the optimality of equilibrium choices. Job destruction decisions and training investment decisions are strongly complementary: a fraction γ of the expected training cost $\theta q(\theta)C(k)$, which the firm saves when the worker stays in the match, is captured by the worker through the wage bargaining (hold-up). This rises the productivity threshold, leading finally to an excess of job destructions. Firing taxes F can be implemented to reach the efficient level of job destructions, together with training subsidies T get at the time of match formation in order to lower the training cost.

As the training subsidy decreases the training cost, the value of a vacancy is now such as:

$$rV = -c + q(\theta)\left[\bar{J}(k) - C(k) + T - V\right].$$

The free entry condition implies $V = 0$ and then:

$$\frac{c}{q(\theta)} = \bar{J}(k) - C(k) + T. \qquad [7]$$

The reservation productivity $R(k)$ is now defined by $J(k, R(k)) = -F$. In the context of an insider wage structure, the surplus sharing rule is now such that $W(a, \varepsilon) - U = \gamma S(a, \varepsilon)$ and $J(a, \varepsilon) + F = (1 - \gamma)S(a, \varepsilon)$ where $S(a, \varepsilon) = J(a, \varepsilon) + F + W(a, \varepsilon) - U$. We therefore derive the following wage expression:

$$w^P(k, \varepsilon) = (1-\gamma)z + \gamma[y(k) + \varepsilon + c\theta] + \gamma(r + s + \theta q(\theta))F + \gamma\theta q(\theta)[C(k) - T].$$

On one hand, the training subsidy reduces the training cost (last term of the right-hand side). But, on the other hand, workers are now in a position to capture also a fraction γ of the firing taxes the firm saves if the negotiation does not fail or when the worker quits voluntary the firm (second term of the right-hand side).

Proposition 4: Assuming $\gamma = \eta(\theta)$, the optimal labor market policy with training subsidies and firing taxes $\{T, F\}$ solves:[14]

$$\begin{cases} T = \gamma C(k^*) + F \\ F = \dfrac{\theta^* q(\theta^*)}{r+s}\gamma C(k^*) \end{cases}$$

where k^* solves the optimal allocation.

Proof. See Appendix C.1. ■

First, firing taxes depend on the expected value of the distortion on job destructions, i.e. hold-up that increases wages. $\gamma\theta q(\theta)C(k)$ defines the instantaneous value of this distortion, whereas $1/(r + s)$ defines the discount factor that depends not only on the interest but also on the probability of voluntary quits: with probability s, firms will not have to pay firing taxes. Therefore, the higher the probability the worker quits voluntary the firm, the lower the distortion, hence firing taxes.

Second, training subsidies result from the distortion on invested amounts in training: workers get a fraction γ of the expected training cost that firms save when they stay in the match. Then, the optimal training subsidy integrates this distortion plus the negative incidence of the firing tax on job creations.

4. Conclusion

This paper mainly emphasized that there exists a strong complementarity between firing and training decisions. This has first allowed to establish under which conditions positive firing decisions occur at equilibrium. We have then stressed the need for both firing taxes and training subsidies in order to restore equilibrium efficiency when a hold-up problem arises. More generally, our work shows that the interplay between firing and training decisions should lead to re-examine the instruments of economic policy used to bring back efficiency. In this way, training subsidies turn out to be a central instrument.

The need for training subsidies could also be relevant within frameworks where inefficient job destructions do not arise as a result of hold-up. For instance, they could be important in the presence of (unobservable) heterogeneity of workers that results in too many job destructions and too few training investments for low-skilled workers at equilibrium (see Chéron and Rouland, 2010).

We have assumed that human capital is purely match-specific for convenience. This is a strong assumption and skills are practically neither purely general nor specific. But, considering general human capital as a part of worker's productivity as well would imply to consider *ex ante* heterogeneous workers: as accumulation (depreciation) of general human capital depends on the length of employment (unemployment) spells, we should determine the steady state of the distribution of general human capital. However, the complementarity between both firms decisions would still stand as firms invest in training to protect from negative shocks. Further, the problem of training under-investments would be stronger with both general and specific human capital because of the transferability of skills. Not only workers would benefit from the investment without bearing the cost (hold-up), but also future employers ['poaching externality', Acemoglu (1997) or Lechthaler (2009)]. The interaction between firing taxes and training subsidies we have emphasized would be even more relevant. Looking at the complementarity between investments in general and specific human capital would be an interesting issue for future work.

Appendix A: Equilibrium equations under two-tier wage contract

Wage setting. First, from the Nash-bargaining rule of the surplus from new matches $(1-\gamma)\left[\bar{W}(k)-U\right]=\gamma\left[\bar{J}(k)-C(k)\right]$, entry wages write $\bar{w}(k)=(1-\gamma)z+\gamma\left(y(k)+\bar{\varepsilon}\right)-\gamma(r+s+\lambda)C(k)-\gamma\theta q(\theta)C(k)+\gamma\theta q(\theta)\bar{J}(k)$. From this Nash-bargaining rule and the job creation condition $c/q(\theta)=\bar{J}(k)-C(k)$, entry wages finally solve:

$$\bar{w}(k)=(1-\gamma)z+\gamma\left[y(k)+\bar{\varepsilon}+c\theta\right]-\gamma(r+s+\lambda)C(k).$$

Then, from the Nash-bargaining rule of the surplus generated by a job after a random change in ε $(1-\gamma)[W(k,\varepsilon)-U]=\gamma[J(k,\varepsilon)]$, and the job creation condition, renegotiated wages write:

$$w^{C}(k,\varepsilon)=(1-\gamma)z+\gamma[y(k)+\varepsilon+c\theta].$$

Job creations. As $J(k, R(k)) = 0$ in $(r+\lambda+s)\left[\bar{J}(k)-J(k,R(k))\right]=\bar{\varepsilon}-R(k)-\bar{w}(k)+w(k,R(k))$ and using wage expressions, it follows $(r+\lambda+s)\bar{J}(k)=(1-\gamma)[\bar{\varepsilon}-R(k)]+\gamma(r+s+\lambda)C(k)$.

The job creation condition $c/q(\theta)=\bar{J}(k)-C(k)$ finally leads to derive job creation equation:

$$\frac{c}{q(\theta)}=\left(\frac{1-\gamma}{r+\lambda+s}\right)[\bar{\varepsilon}-R(k)]-(1-\gamma)C(k).$$

Job destructions. Replacing the renegotiated wage expression in the reservation productivity threshold $R(k)=-y(k)+w(k,R(k))-\lambda\int_{R(k)}^{\bar{\varepsilon}}J(k,x)\,\mathrm{d}G(x)$ leads to the job destruction equation:

$$R(k)=-y(k)+z+\frac{\gamma}{1-\gamma}c\theta-\frac{\lambda}{r+\lambda+s}\int_{R(k)}^{\bar{\varepsilon}}[1-G(x)]\,\mathrm{d}x$$

because integrating by parts $\lambda\int_{R(k)}^{\bar{\varepsilon}}J(k,x)\,\mathrm{d}G(x)=\lambda\int_{R(k)}^{\bar{\varepsilon}}J'(k,x)[1-G(x)]\,\mathrm{d}x$, with

$$J(k,x)=\left(\frac{1-\gamma}{r+\lambda+s}\right)[x-R(k)]+\gamma C(k).$$

Then, it turns:

$$\frac{\partial R(k)}{\partial k}=-\left(\frac{r+\lambda+s}{r+\lambda G(R(k))+s}\right)y'(k).$$

Training investment. As

$$\bar{J}(k)=\left(\frac{1-\gamma}{r+\lambda+s}\right)[\bar{\varepsilon}-R(k)]+\gamma C(k)$$

at equilibrium, the training investment level is determined by:

$$\max_k \bar{J}(k)-C(k)\Leftrightarrow \max_k\left(\frac{1-\gamma}{r+\lambda+s}\right)[\bar{\varepsilon}-R(k)]-(1-\gamma)C(k)$$

where the first-order condition implies

$$C'(k)=-\left(\frac{1}{r+s+\lambda}\right)\frac{\partial R(k)}{\partial k}.$$

The training equation is finally defined by

$$C'(k)=\left(\frac{1}{r+s+\lambda G(R(k))}\right)y'(k).$$

Assuming both $r\to 0$ and $\gamma=\eta(\theta)$, let remark that the general equilibrium with a two-tier wage structure is first-best efficient.

Appendix B: Equilibrium equations with insider wage contract

Job creations As $J(k, R(k)) = 0$ in $(r+\lambda+s)\left[\bar{J}(k) - J(k, R(k))\right] = \bar{\varepsilon} - R(k) - w(k, \bar{\varepsilon}) + w(k, R(k))$ and using the wage expression, it follows $(r+\lambda+s)\bar{J}(k) = (1-\gamma)[\bar{\varepsilon} - R(k)]$. Equation [1] leads then to derive job creation equation:

$$\frac{c}{q(\theta)} = \left(\frac{1-\gamma}{r+\lambda+s}\right)[\bar{\varepsilon} - R(k)] - C(k).$$

Job destructions. We derive job destruction equation from reservation productivity [2] and wage expression $w(k, R(k))$. By integrating by parts, it comes that $\int_{R(k)}^{\bar{\varepsilon}} J(k, x)\,\mathrm{d}G(x) = \int_{R(k)}^{\bar{\varepsilon}} J'(k, x)[1 - G(x)]\,\mathrm{d}x$. Furthermore, noticing that

$$J'(k, x) = \frac{1-\gamma}{r+\lambda+s}$$

as

$$J(k, x) = \left(\frac{1-\gamma}{r+\lambda+s}\right)[x - R(k)],$$

it follows that

$$\int_{R(k)}^{\bar{\varepsilon}} J'(k, x)[1 - G(x)]\,\mathrm{d}x = \frac{1-\gamma}{r+\lambda+s}\int_{R(k)}^{\bar{\varepsilon}} [1 - G(x)]\,\mathrm{d}x.$$

Job destruction equation is finally defined by:

$$R(k) = -y(k) + z + \left(\frac{\gamma}{1-\gamma}\right)c\theta + \frac{\gamma}{1-\gamma}\theta q(\theta)C(k) - \frac{\lambda}{r+\lambda+s}\int_{R(k)}^{\bar{\varepsilon}} [1 - G(x)]\,\mathrm{d}x.$$

Training investment level. As

$$\bar{J}(k) = \left(\frac{1-\gamma}{r+\lambda+s}\right)[\bar{\varepsilon} - R(k)],$$

the training investment level is determined by:

$$\max_k \bar{J}(k) - C(k) \Leftrightarrow \max_k \left(\frac{1-\gamma}{r+\lambda+s}\right)[\bar{\varepsilon} - R(k)] - C(k).$$

First-order condition implies

$$C'(k) = -\left(\frac{1-\gamma}{r+\lambda+s}\right)\frac{\partial R(k)}{\partial k},$$

with

$$\frac{\partial R(k)}{\partial k} = \left(\frac{r+\lambda+s}{r+\lambda G(R(k))+s}\right)\left[-y'(k)+\left(\frac{\gamma}{1-\gamma}\right)\theta q(\theta)C'(k)\right],$$

and then

$$C'(k) = \left(\frac{1-\gamma}{r+s+\lambda G(R(k))+\gamma\theta q(\theta)}\right)y'(k).$$

Appendix C: Equilibrium equations with labor market policy (and insider wage)

C.1 Implementing firing taxes and training subsidies

Wage setting. First, the surplus from a match, $S(k, \varepsilon) = J(k, \varepsilon) + F + W(k, \varepsilon) - U$ shared such as $W(k, \varepsilon) - U = [\gamma/1 - \gamma]J(k, \varepsilon)$, implies $w(k,\varepsilon) = (1-\gamma)z + \gamma[y(k)+\varepsilon] + (1-\gamma)\theta q(\theta)[W(k,\bar{\varepsilon}) - U] + \gamma(r+s)F$.

Then, from the job creation condition $c/q(\theta) = \bar{J}(k) - C(k) + T$ and using the Nash-bargaining rule of the surplus, wages finally write:

$$w^P(k,\varepsilon) = (1-\gamma)z + \gamma[y(k)+\varepsilon+c\theta] + \gamma(r+s+\theta q(\theta))F + \gamma\theta q(\theta)[C(k)-T].$$

Job creations. Endogenous job destruction rule now such as $J(k, \varepsilon) < -F$ leads to a reservation productivity $R(k)$ defined by $J(k, R(k)) = -F$. Therefore, it follows $(r+s+\lambda)\left[\bar{J}(k) - J(k, R(k))\right] = \bar{\varepsilon} - R(k) - w^P(k,\bar{\varepsilon}) + w^P(k, R(k))$. Using the wage expression it comes that:

$$(r+s+\lambda)\left[\bar{J}(k) - J(k,R(k))\right] = (1-\gamma)[\bar{\varepsilon} - R(k)] \Leftrightarrow \bar{J}(k) = \left(\frac{1-\gamma}{r+\lambda+s}\right)[\bar{\varepsilon} - R(k)] - F.$$

Job creations equation is then defined by:

$$\frac{c}{q(\theta^P)} = \left(\frac{1-\gamma}{r+\lambda+s}\right)[\bar{\varepsilon} - R^P(k)] - C(k^P) + T - F.$$

Making job creations optimal when Hosios condition holds implies:

$$\frac{c}{q(\theta^P)} = \frac{c}{q(\theta^*)} \Leftrightarrow T = \gamma C(k^P) + F.$$

Job destructions. The reservation productivity $R(k)$ defined by $J(k, R(k)) = -F$ is such as:

$$R(k) = -y(k) + w^P(k, R(k)) - (r+s)F - \lambda\int_{R(k)}^{\bar{\varepsilon}}[J(k,x)+F]\,\mathrm{d}G(x).$$

As mentioned before, $J(k, R(k)) = -F$ implies

$$\bar{J}(k)=\left(\frac{1-\gamma}{r+\lambda+s}\right)[\bar{\varepsilon}-R(k)]-F.$$

It then comes that

$$\lambda\int_{R(k)}^{\bar{\varepsilon}}[J(k,x)+F]\,\mathrm{d}G(x)=\left(\frac{\lambda(1-\gamma)}{r+\lambda+s}\right)\int_{R(k)}^{\bar{\varepsilon}}[x-R(k)]\,\mathrm{d}G(x).$$

Integrating by parts this term and replacing the wage expression finally leads to the job destruction equation:

$$R^P(k)=-y(k^P)+z+\left(\frac{\gamma}{1-\gamma}\right)c\theta+\left(\frac{\gamma}{1-\gamma}\right)\theta q\theta[C(k^P)-T+F]-(r+s)F$$
$$-\frac{\lambda}{r+\lambda+s}\int_{R^P(k)}^{\bar{\varepsilon}}[1-G(x)]\,\mathrm{d}x.$$

Given that $T=\gamma C(k)+F$, making job destructions optimal when Hosios condition holds implies:

$$R^P(k)=R(k)^*\Leftrightarrow F=\left(\frac{\gamma\theta q(\theta)}{r+s}\right)C(k^P).$$

The job destruction equation at equilibrium is then defined by:

$$R^P(k)=-y(k^P)+z+\left(\frac{\gamma}{1-\gamma}\right)c\theta-\frac{\lambda}{r+\lambda+s}\int_{R^P(k)}^{\bar{\varepsilon}}[1-G(x)]\,\mathrm{d}x.$$

Then it turns

$$\frac{\partial R^P(k)}{\partial k^P}=-\left(\frac{r+\lambda+s}{r+\lambda G(R(k))+s}\right)y'(k^P).$$

Training investment level. As $T=\gamma C(K)+F$ and as

$$\bar{J}(k)=\left(\frac{1-\gamma}{r+\lambda+s}\right)[\bar{\varepsilon}-R(k)]-F$$

at equilibrium, the training investment level is determined by:

$$\max_k \bar{J}(k)-C(k)+T\Leftrightarrow\max_k\left(\frac{1-\gamma}{\lambda+s}\right)[\bar{\varepsilon}-R^P(k)]-(1-\gamma)C(k)$$

where the first-order condition implies

$$C'(k^P)=-\left(\frac{1}{r+\lambda+s}\right)\frac{\partial R^P(k)}{\partial k^P}$$

with

$$\frac{\partial R^P(k)}{\partial k^P} = -\left(\frac{r+\lambda+s}{r+\lambda G(R^P(k))+s}\right) y'(k^P).$$

Training equation is finally defined by

$$C'(k^P) = \left(\frac{1}{r+\lambda G(R^P(k))+s}\right) y'(k^P).$$

C.2 Implementing unemployment benefits and training subsidies

Wage setting. As unemployment benefits b rise the reservation wage of the worker, the value of an unemployed worker satisfies now: $rU = b + z + \theta q(\theta)[W(k, \varepsilon) - U]$. With training subsidies T and from the Nash-bargaining rule of the surplus and the job creation condition $c/q(\theta) = \bar{J}(k) - C(k) + T$, wages finally write:

$$w^B(k, \varepsilon) = (1-\gamma)(z+b) + \gamma[y(k) + \varepsilon + c\theta] + \gamma\theta q(\theta)[C(k) - T].$$

Job creations. As

$$\bar{J}(k) = \left(\frac{1-\gamma}{r+\lambda+s}\right)[\bar{\varepsilon} - R(k)],$$

job creation equation is defined by:

$$\frac{c}{q(\theta^B)} = \left(\frac{1-\gamma}{r+\lambda+s}\right)[\bar{\varepsilon} - R^B(k)] - C(k^B) + T.$$

Making job creations optimal when Hosios condition holds implies:

$$\frac{c}{q(\theta^B)} = \frac{c}{q(\theta^*)} \Leftrightarrow T = \gamma C(k^B).$$

Job destructions. With unemployment benefits, the reservation productivity $R(k)$ defined by $J(k, R(k)) = 0$ is such as:

$$R^B(k) = -y(k^B) + (b+z) + \left(\frac{\gamma}{1-\gamma}\right)c\theta + \left(\frac{\gamma}{1-\gamma}\right)\theta q\theta[C(k^B) - T] - \frac{\lambda}{r+\lambda+s}\int_{R^B(k)}^{\bar{\varepsilon}} [1 - G(x)]\,\mathrm{d}x.$$

Given that $T = \gamma C(k)$, making job destructions optimal when Hosios condition holds implies:

$$R^B(k) = R(k)^* \Leftrightarrow b = -\gamma\theta q(\theta)C(k^B).$$

The job destruction equation at equilibrium is then defined by:

$$R^B(k) = -y(k^B) + z + \left(\frac{\gamma}{1-\gamma}\right)c\theta - \frac{\lambda}{r+\lambda+s}\int_{R^B(k)}^{\bar{\varepsilon}}[1-G(x)]\,\mathrm{d}x.$$

Then it turns

$$\frac{\partial R(k)^B}{\partial k} = -\left(\frac{r+\lambda+s}{r+\lambda G(R(k))+s}\right)y'(k).$$

Training investment level. As $T = \gamma C(K)$ and as

$$\bar{J}(k) = \left(\frac{1-\gamma}{r+\lambda+s}\right)[\bar{\varepsilon} - R(k)]$$

at equilibrium, the training investment level is determined by:

$$\max_k \bar{J}(k) - C(k) + T \Leftrightarrow \max_k \left(\frac{1-\gamma}{\lambda+s}\right)[\bar{\varepsilon} - R(k)] - (1-\gamma)C(k)$$

where the first-order condition implies

$$C'(k) = -\left(\frac{1}{r+\lambda+s}\right)\frac{\partial R(k)}{\partial k}$$

with

$$\frac{\partial R(k)}{\partial k} = -\left(\frac{r+\lambda+s}{r+\lambda G(R(k))+s}\right)y'(k).$$

Training equation is finally defined by

$$C'(k^B) = \left(\frac{1}{r+\lambda G(R(k))+s}\right)y'(k^B).$$

Appendix D: Proofs of propositions

D.1 Two-tier equilibrium existence proof

On one hand, combining equations [4] and [5] of the optimal allocation implies

$$R(k) + \left[\frac{\alpha}{r+s+\lambda R(k)}\right]^{\frac{\alpha}{1-\alpha}} - \frac{1}{2}\left(\frac{\lambda}{r+\lambda+s}\right)\left[(1-\bar{\varepsilon})^2 - (1-R(k))^2\right] = z + \left(\frac{\gamma}{1-\gamma}\right)c\theta.$$

Therefore,

$$\frac{\mathrm{d}R(k)}{\mathrm{d}\theta}=\left(\frac{c\gamma}{1-\gamma}\right)\left\{\frac{(r+\lambda+s)(1-\alpha)}{(1-\alpha)[r+s+\lambda R(k)]-\lambda(r+\lambda+s)\alpha^{\frac{1}{1-\alpha}}[r+s+\lambda R(k)]^{\frac{1}{1-\alpha}}}\right\}.$$

Job destruction equation is then an upward-sloping curve in the reservation productivity-tightness space if the denominator of the second term on the right-hand side is positive, namely if

$$r+s+\lambda R(k)>\left[\frac{\lambda(r+\lambda+s)}{1-\alpha}\right]^{\frac{1-\alpha}{2-\alpha}}\alpha^{\frac{1}{2-\alpha}}.$$

It is then straightforward to see that

$$r+s>\alpha^{\frac{1}{2-\alpha}}\left[\frac{\lambda(r+\lambda+s)}{1-\alpha}\right]^{\frac{1-\alpha}{2-\alpha}}$$

is a sufficient condition.

On the other hand, from equation [3a] and [3c], it follows that

$$\frac{\mathrm{d}R(k)}{\mathrm{d}\theta}=\left[\frac{cq'(\theta)}{(1-\eta(\theta))(q(\theta))^2}\right]\left\{\frac{(r+\lambda+s)(1-\alpha)}{(1-\alpha)-\lambda(r+\lambda+s)\alpha^{\frac{1}{1-\alpha}}[r+s+\lambda R(k)]^{\frac{-1}{1-\alpha}-1}}\right\}.$$

As $q'(\theta)<0$, the job creation equation is a downward-sloping curve if

$$r+s+\lambda R(k)>\left[\frac{\lambda(r+\lambda+s)}{1-\alpha}\right]^{\frac{1-\alpha}{2-\alpha}}\alpha^{\frac{1}{2-\alpha}}.$$

Again,

$$r+s>\alpha^{\frac{1}{2-\alpha}}\left[\frac{\lambda(r+\lambda+s)}{1-\alpha}\right]^{\frac{1-\alpha}{2-\alpha}}$$

is a sufficient condition.

Finally, there exists a unique equilibrium if the intersection of job destruction and job creation curves corresponds with both θ and $R(k)$ positive. This is the case if the job destruction curve evaluated for $\theta=0$ is below the job creation curve also evaluated for $\theta=0$, namely if:

$$2[R(k)-z][r+s+\lambda R(k)]^{\frac{1}{1-\alpha}+\frac{\alpha}{1-\alpha}}[\bar{\varepsilon}-R(k)]+2\alpha^{\frac{\alpha}{1-\alpha}}[\bar{\varepsilon}-R(k)][r+s+\lambda R(k)]^{\frac{1}{1-\alpha}}$$
$$-\lambda\alpha^{\frac{1}{1-\alpha}}[r+s+\lambda R(k)]^{\frac{\alpha}{1-\alpha}}\left[(1-\bar{\varepsilon})^2-(1-R(k))^2\right]>0.$$

As $z<R(k)$ and $\bar{\varepsilon}>R(k)$, this inequality is true.

D.2 Insider equilibrium existence proof

On one hand, combining equations [4b] and [4c] of the decentralized equilibrium implies

$$\underbrace{R(k) - \frac{1}{2}\left(\frac{r+\lambda}{s+\lambda}\right)\left[(1-\bar{\varepsilon})^2 - (1-R(k))^2\right]}_{\psi(R(k))}$$
$$+ \underbrace{\left[\frac{\alpha(1-\gamma)}{r+s+\lambda R(k)+\gamma p(\theta)}\right]^{\frac{\alpha}{1-\alpha}} - \frac{\gamma p(\theta)}{1-\gamma}\left[\frac{\alpha(1-\gamma)}{r+s+\lambda R(k)+\gamma p(\theta)}\right]^{\frac{1}{1-\alpha}}}_{\varphi(R(k),\theta)}$$
$$= z + \frac{\gamma}{1-\gamma} c\theta.$$

Therefore,

$$\frac{\mathrm{d}R(k)}{\mathrm{d}\theta} = \frac{\frac{\gamma}{1-\gamma}c - \varphi_2'(R(k),\theta)}{\psi'(R(k)) + \varphi_1'(R(k),\theta)}$$

with $\psi'(R(k)) > 0$, $\varphi_1'(R(k),\theta) < 0$, and $\varphi_2'(R(k),\theta) < 0$.

First, $[\gamma/(1-\gamma)]c - \varphi_2'(R(k),\theta)$ is positive as $\varphi_2'(R(k),\theta) < 0$. Job destruction equation is then an upward-sloping curve in the reservation productivity-tightness space if

$$\psi'(R(k)) + \varphi_1'(R(k),\theta) > 0 \Leftrightarrow r+s+\lambda R(k)+\gamma p(\theta) > \alpha^{\frac{1}{2-\alpha}}(1-\gamma)^{\frac{\alpha}{2-\alpha}}\left[\frac{\lambda(r+\lambda+s)}{1-\alpha}\right]^{\frac{1-\alpha}{2-\alpha}}.$$

It is then straightforward to see that

$$r+s > \alpha^{\frac{1}{2-\alpha}}\left[\frac{\lambda(r+\lambda+s)}{1-\alpha}\right]^{\frac{1-\alpha}{2-\alpha}}$$

is a sufficient condition.

On the other hand, from equation [4a] and [4c], it follows that:

$$\frac{\mathrm{d}R(k)}{\mathrm{d}\theta} = \left(\frac{r+\lambda+s}{(q(\theta))^2}\right)$$
$$\times \left\{\frac{(q(\theta))^2[\alpha(1-\gamma)]^{\frac{1}{1-\alpha}}\gamma p'(\theta)[r+s+\lambda R(k)+\gamma p(\theta)]^{\frac{-1}{1-\alpha}-1} + cq'(\theta)(1-\alpha)(1-\gamma)}{(1-\alpha)(1-\gamma) - \lambda(r+\lambda+s)[\alpha(1-\gamma)]^{\frac{1}{1-\alpha}}[r+s+\lambda R(k)+\gamma p(\theta)]^{\frac{-1}{1-\alpha}-1}}\right\}.$$

As $q'(\theta) < 0$ and $p'(\theta) > 0$, the job creation equation is a downward-sloping curve either if the numerator is positive and the denominator negative (first case), or if the numerator is negative and the denominator positive (second case). But the numerator is clearly negative if $\gamma = 0$, suggesting that the second case is the most likely. The job creation equation is then a downward-sloping curve if the denominator is positive, which implies

$$r+s+\lambda R(k)+\gamma p(\theta)>\alpha^{\frac{1}{2-\alpha}}(1-\gamma)^{\frac{\alpha}{2-\alpha}}\left[\frac{\lambda(r+\lambda+s)}{1-\alpha}\right]^{\frac{1-\alpha}{2-\alpha}}.$$

Again, it is then straightforward to see that

$$r+s>\alpha^{\frac{1}{2-\alpha}}\left[\frac{\lambda(r+\lambda+s)}{1-\alpha}\right]^{\frac{1-\alpha}{2-\alpha}}$$

is a sufficient condition.

Finally, there exists a unique equilibrium if the intersection of job destruction and job creation curves corresponds with both θ and $R(k)$ positive. This is the case if the job destruction curve evaluated for $\theta = 0$ is below the job creation curve also evaluated for $\theta = 0$, namely if:

$$\begin{aligned}&2[R(k)-z](1-\gamma)[\bar{\varepsilon}-R(k)][s+\lambda R(k)]^{\frac{1}{1-\alpha}+\frac{\alpha}{1-\alpha}}\\&\quad+2[\alpha(1-\gamma)]^{\frac{\alpha}{1-\alpha}}(1-\gamma)[\bar{\varepsilon}-R(k)][s+\lambda R(k)]^{\frac{1}{1-\alpha}}\\&\quad-\lambda[\alpha(1-\gamma)]^{\frac{1}{1-\alpha}}[s+\lambda R(k)]^{\frac{\alpha}{1-\alpha}}\left[(1-\bar{\varepsilon})^2-(1-R(k))^2\right]>0.\end{aligned}$$

As $z < R(k)$ and $(1-\bar{\varepsilon})^2-(1-R(k))^2<0$ (because $\bar{\varepsilon}>R(k)$), this inequality is true.

D.3 Optimum existence proof

On one hand, combining equations [6b] and [6c] of the optimal allocation implies

$$R(k)+\left[\frac{\alpha}{r+s+\lambda R(k)}\right]^{\frac{\alpha}{1-\alpha}}-\frac{1}{2}\left(\frac{\lambda}{r+\lambda+s}\right)\left[(1-\bar{\varepsilon})^2-(1-R(k))^2\right]=z+\left(\frac{\eta(\theta)}{1-\eta(\theta)}\right)c\theta.$$

Therefore,

$$\frac{\mathrm{d}R(k)}{\mathrm{d}\theta}=\left(\frac{c\eta(\theta)}{1-\eta(\theta)}\right)\left\{\frac{(r+\lambda+s)(1-\alpha)}{(1-\alpha)[r+s+\lambda R(k)]-\lambda(r+\lambda+s)\alpha^{\frac{1}{1-\alpha}}[r+s+\lambda R(k)]^{\frac{1}{1-\alpha}}}\right\}.$$

Job destruction equation is then an upward-sloping curve in the reservation productivitytightness space if the denominator of the second term on the right-hand side is positive, namely if

$$r+s+\lambda R(k)>\left[\frac{\lambda(r+\lambda+s)}{1-\alpha}\right]^{\frac{1-\alpha}{2-\alpha}}\alpha^{\frac{1}{2-\alpha}}.$$

It is then straightforward to see that

$$r+s>\alpha^{\frac{1}{2-\alpha}}\left[\frac{\lambda(r+\lambda+s)}{1-\alpha}\right]^{\frac{1-\alpha}{2-\alpha}}$$

is a sufficient condition.

On the other hand, from equation [10] and [12], it follows that:

$$\frac{\mathrm{d}R(k)}{\mathrm{d}\theta}=\left[\frac{cq'(\theta)}{(1-\eta(\theta))(q(\theta))^2}\right]\left\{\frac{(r+\lambda+s)(1-\alpha)}{(1-\alpha)-\lambda(r+\lambda+s)\alpha^{\frac{1}{1-\alpha}}[r+s+\lambda R(k)]^{\frac{-1}{1-\alpha}-1}}\right\}.$$

As $q'(\theta)<0$, the job creation equation is a downward-sloping curve if

$$r+s+\lambda R(k)>\left[\frac{\lambda(r+\lambda+s)}{1-\alpha}\right]^{\frac{1-\alpha}{2-\alpha}}\alpha^{\frac{1}{2-\alpha}}.$$

Again,

$$r+s>\alpha^{\frac{1}{2-\alpha}}\left[\frac{\lambda(r+\lambda+s)}{1-\alpha}\right]^{\frac{1-\alpha}{2-\alpha}}$$

is a sufficient condition.

Finally, there exists a unique equilibrium if the intersection of job destruction and job creation curves corresponds with both θ and $R(k)$ positive. This is the case if the job destruction curve evaluated for $\theta=0$ is below the job creation curve also evaluated for $\theta=0$, namely if:

$$2[R(k)-z][r+s+\lambda R(k)]^{\frac{1}{1-\alpha}+\frac{\alpha}{1-\alpha}}[\overline{\varepsilon}-R(k)]+2\alpha^{\frac{\alpha}{1-\alpha}}[\overline{\varepsilon}-R(k)][r+s+\lambda R(k)]^{\frac{1}{1-\alpha}}$$
$$-\lambda\alpha^{\frac{1}{1-\alpha}}[r+s+\lambda R(k)]^{\frac{\alpha}{1-\alpha}}\left[(1-\overline{\varepsilon})^2-(1-R(k))^2\right]>0.$$

As $z<R(k)$ and $\overline{\varepsilon}>R(k)$, this inequality is true.

D.4 Proof of Lemma 1

Lemma 1: The investment decision condition of firms generates under-investments: $k^I\,|_{R(k)=R^*(k)}<k^*$.

Proof. Assume $R^I(k)=R^*(k)=\overline{R(k)}$. Then, as

$$\frac{r+s+\lambda G\left(\overline{R(k)}\right)+\gamma\theta^I q(\theta^I)}{1-\gamma}>r+s+\lambda G\left(\overline{R(k)}\right),$$

[4c] and [6c] imply that

$$\frac{r+s+\lambda G\left(\overline{R(k)}\right)+\gamma\theta^I q(\theta^I)}{1-\gamma}\frac{C'(k^I)}{y'(k^I)}=r+s+\lambda G\left(\overline{R(k)}\right)\frac{C'(k^*)}{y'(k^*)}.$$

Therefore, it must be that $C'(k^I)$ is smaller than $C'(k^*)$), and thus k^* exceeds k^I.

D.5 Proof of Lemma 2

Lemma 2: Under the Hosios condition, the productivity reservation of firms generates too many job destructions: $R^I(k)\mid_{k=k^*,\theta=\theta^*} > R^*(k)$.

Proof. Assume $k^I = k^* = \bar{k}$ and $\theta^I = \theta^* = \bar{\theta}$. Then, as

$$y(\bar{k})+z+\frac{\gamma}{1-\gamma}c\theta+\frac{\gamma}{1-\gamma}\theta q(\theta)C(\bar{k})>y(\bar{k})+z+\frac{\eta(\theta)}{1-\eta(\theta)}c\theta,$$

[4b] and [6b] imply that:

$$\begin{aligned}&R(k^I)+\frac{\lambda}{r+\lambda+s}\int_{R(k)}^{\bar{\varepsilon}}[1-G(x)]\mathrm{d}x-\left[y(\bar{k})+z+\frac{\gamma}{1-\gamma}c\theta+\frac{\gamma}{1-\gamma}\theta q(\theta)C(\bar{k})\right]\\&=R(k^*)+\frac{\lambda}{r+\lambda+s}\int_{R^*(k)}^{\bar{\varepsilon}}[1-G(x)]\mathrm{d}x-\left[y(\bar{k})+z+\frac{\eta(\theta)}{1-\eta(\theta)}c\theta\right].\end{aligned}$$

Therefore, $R(k^*)$ must be smaller than $R(k^I)$ to ensure this equality.

D.6 Proof of Lemma 3

Lemma 3: Under the Hosios condition, the free entry condition generates too small labor market tightness: $\theta^I(k)\mid_{k=k^*,R(k)=R^*(k)} < \theta^*(k)$.

Proof. Assume $k^I = k^* = \bar{k}$ and $R^I(k) = R^*(k) = \overline{R(k)}$. Then, as

$$\left(\frac{1-\gamma}{r+\lambda+s}\right)\left[\bar{\varepsilon}-\overline{R(k)}\right]-C(\bar{k})<\left(\frac{1-\eta(\theta)}{r+\lambda+s}\right)\left[\bar{\varepsilon}-\overline{R(k)}\right]-(1-\eta(\theta))C(\bar{k}),$$

[4a] and [6a] imply that:

$$\left[\left(\frac{1-\gamma}{r+\lambda+s}\right)\left[\bar{\varepsilon}-\overline{R(k)}\right]-C(\bar{k})\right]\frac{q(\theta^I)}{c}=\left[\left(\frac{1-\eta(\theta)}{r+\lambda+s}\right)\left[\bar{\varepsilon}-\overline{R(k)}\right]-(1-\eta(\theta))C(\bar{k})\right]\frac{q(\theta^*)}{c}.$$

Therefore, θ^I is smaller than θ^* when $\gamma = \eta(\theta)$.

Notes

[1] Hashimoto (1981) first formalized Becker's sharing conjecture in a model with transaction costs related to post-investment uncertainty. Leuven and Oosterbeek (2001) then rigorously considered the role of uncertainty in this model.

[2] See Malcomson (1997) for a survey on 'hold-up' theory.

[3] Pissarides (2009) gives some empirical arguments supporting this wage setting: the author shows that fixed job creation costs (paid before the Nash-bargaining of wages) can raise the volatility of unemployment over business cycles, as found in data.

[4] Continuing training refers to training that occurs after leaving school.

[5] The function $y(k)$ is assumed strictly increasing and concave, with $y(0) = 0$.

[6] Using data from the International Adult Literacy Survey, O'Connell (1999) reports (i) that employed adults are more likely than unemployed adults to participate in training; (ii) that employers are by far the

most common financial sponsors of training; and (iii) that participation in job-related training is substantially higher than participation in training undertaken for personal or other reasons. All in all, most of the training sessions are enrolled while employed and are not only firms-financed but also job-related, making them apparently more specific. This may support our choice of training modeling.

[7] The additive form of the output of the match we assumed between an endogenous component ($y(k)$) and another exogenous one (ε) clearly simplifies calculations but also fits the usual definition of training. Usually, training is considered as a way to improve workers' skills. Without training, workers are still able to produce but at lower productivity levels. To mention only a few, Lechthaler (2009) and Belot *et al.* (2007), within the framework of endogenous human capital and productivity shocks, consider an additive form of the output of the match as well.

[8] We will put the emphasis on the role played by this assumption in the section devoted to the labor market equilibrium analysis.

[9] With $C'(0) = 0$, $C'(k) > 0$, and $C''(k) > 0$.

[10] The upper bar refers to new jobs.

[11] See Appendix A for details on derivation.

[12] Again, see Appendix A for details on derivation.

[13] For $r \to 0$, $\alpha = 0.1$, and $\lambda = 0.1$, the condition

$$r + s > \alpha^{\frac{1}{2-\alpha}} \left[\frac{\lambda(r + \lambda + s)}{1 - \alpha} \right]^{\frac{1-\alpha}{2-\alpha}}$$

is true for $s \geq 0.0417$ whereas for $\alpha = 0.5$ (and again for $\lambda = 0.1$), the condition is true for $s \geq 0.2627$. In other words, the quit rate of workers must be at least 4.17 per cent. According to US data from the Department of Labor (Job Openings and Labor Turnover Survey — JOLTS), the total annual quit rate between 2001 and 2008 fluctuates between 22.6 per cent (for 2008) and 27.6 per cent (for 2001). This means that the condition so that an interior solution exists is typically satisfied.

[14] Similarly, an appropriate combination of training subsidies and unemployment benefits can also be implemented to reach the efficient allocation. On one hand, as the excess of job destructions comes from hold-up that rises wages, negative unemployment benefits can be used to reduce outside options of workers and thus wages. On the other hand, workers do not contribute to the training cost but get a fraction of the gains of the training investment (through the productivity gains). Then, training subsidies can be used to share the training cost between both parties. More precisely, unemployment benefits have to cover the fraction of the expected training cost that the firm saves when the worker stays in the match and that the latter gets while bargaining over the wage (hold-up). In this way, unemployment benefits remove the distortion on job destructions. About training subsidies, they have to cover the fraction of the training cost that the worker should have born as he or she shares in the returns. Training subsidies depend on worker's bargaining power that determines the fraction of the gains he or she gets. See Appendix C.2 for details.

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