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ORIGINAL ARTICLE

New roles for mathematics in multi-disciplinary, upper secondary school projects

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Abstract A new concept, compulsory multi-disciplinary courses, was introduced in upper secondary school curriculum as a central part of a recent reform. This paper reports from a case study of such a triple/four-disciplinary project in mathematics, physics, chemistry and 'general study preparation' performed under the reform by a team of experienced teachers. The aim of the case study was to inquire how the teachers met the demands of the introduction of this new concept and, to look for signs of new relations established by the students between mathematics and other subjects, as a result of the multi-disciplinary teaching. The study revealed examples of good practice in planning and teaching. In addition, it served to illuminate interesting aspects of how students perceived the school subject mathematics and its relations to other subjects and to common sense.

1 Multi-disciplinary teaching

A new construct, 'multi-disciplinarity', was prescribed in Danish Upper Secondary Schools' curriculum by governmental regulations in 2005. Multi-disciplinary teaching was to be planned and carried out in projects where teams of subject teachers were able to collaborate. Prior to the reform teachers, specialised in their particular subject and taught all lessons in their subject and there were no obligations for the teacher to collaborate with other teachers. In some schools, experiments were carried out on a voluntary basis by small groups of fiery souls who wanted to explore different models of collaboration but unfortunately, such experiments were seldom publicly reported in detail. Although after the reform, the major part of all lessons continue to be spent on single-subject teaching, these new multi-disciplinary projects may have a significant impact on the students' understanding of mathematics, its roles, relations with other subjects, and its connection with daily life outside school. So, multi-disciplinary teaching is a new challenge for teachers in all subjects, including mathematics, and draws attention to certain aspects of the subjects at the expense of others. The later, major part of this article, which reports the results of a qualitative study of a multi-disciplinary project, is structured in accordance with that double change in upper secondary mathematics.

The new term 'multi-disciplinary' was defined in contrast to cross-disciplinarity or inter-disciplinarity, where the borders between subjects are more or less cancelled, and in contrast to trans-disciplinarity which does not acknowledge the very division of knowledge into subjects. Multi-disciplinary teaching still considers each subject to be delimited from the others, although the subjects collaborate. For example, each subject with its teacher spends lessons in the joint project. This type of teaching is intended to let the individual subjects mutually fertilise and, in parallel, to shape a clear picture in the students' mind the characteristics of each subject.

The setting for multi-disciplinary teaching was formed by the Ministry's reform texts and teacher guidance materials (Curriculum Instruction 797, 27/06/ 2007 and Curriculum Guidelines May 2008), which were based on experienced teachers' and experts' consultancy. The

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expectations and goals of the multi- disciplinary teaching were announced in these texts, and examples of teaching schedules outlined for inspiration. The Ministry's intentions and requirements were centred on applications of and reflections upon each subject. For example, the teaching of mathematics at the highest level (in Danish termed: Mathematics A) intended to support the students' knowledge about 'important aspects of the interplay between mathematics and culture, science and technology'. The students are also supposed to know 'how mathematics adds to understanding, formulating and treating problems in different subject areas' and to know about mathematical reasoning. This knowledge is meant to enable the students to competently take a position on the applications of mathematics and to pass further education involving mathematics.

Another example that illustrates the setting for multi-disciplinary teaching of mathematics are the following three out of nine objectives of teaching high-level mathematics, announced in the regulations:

(The students should be able to:)

A. Use simple statistical or probabilistically models to describe a given set of data or a phenomenon associated with another subject, to ask questions based on models, to have an eye for the expected type of answer and to formulate clear answers
B. Display knowledge about applications, within specific areas, of mathematics including more complex problems
C. Display knowledge about the interplay between development of mathematics and history, science and culture.

2 Models for collaboration between subjects

Two well known models for collaboration between subjects in Danish teaching experiments are functional- and formal cross-disciplinary collaboration. The terms cross-disciplinary and inter-disciplinary are used synonymously. Both types have drawbacks:

> Functional cross-disciplinary collaboration takes a theme or a problem as its starting point and includes the subjects needed for the study. Since the learning goals are commonly very vague and implicit in this form of collaboration there is a risk that common sense rather than profound insight is developed.
> Formal cross-disciplinary collaboration starts with picking out the subjects, whilst the themes are decided and developed subsequently. Studies of the subjects, then, seldom result in new perspectives on the theme.

The introduction of multi-disciplinarity can be seen as an attempt to combine the intentions of the two types of collaboration, avoiding their respective drawbacks.

3 Potentials of multi-disciplinary

The didactical potentials of multi-disciplinary teaching resemble some of the potentials of inter-disciplinary activities, already described and discussed by other authors.

For example, multi-disciplinary teaching should have the potential to establish and strengthen relations between mathematics and other subjects, since, according to Michelsen et al. (2005 p. 33), "the lack of coordination between the curricula of physics and mathematics is in one of the primary cause of students' difficulty of application of mathematics in physics. It is difficult for the students to transfer concepts, ideas and procedures learned in mathematics to a new and unanticipated situation in physics lessons." Michelsen et al. point to an alternative approach that stresses the importance of modelling activities in an inter-disciplinary context between the two subjects. In line with this, we believe that multi-disciplinary projects between mathematics and physics may serve the same purpose.

We also believe that multi-disciplinary projects can stimulate the students' interest and engagement in mathematics in the same way as Filo and Yarkoni reported on a project, which integrated geometry and art, aiming at inter-disciplinary learning of parallel concepts. The concepts had different, but complementary expressions within the two fields. Filo and Yarkoni state that it is "assumed that combining art with math should cause the students to feel more positive towards the study of math. As well as, including art as an integral part of math studies will strengthen and 'elevate the status' of the art classes." (2005). Hence, the authors' assumption in this case was that an enriched concept formation was supplied by an advanced status of both subjects in the students' minds. In Denmark, multi-disciplinary teaching is likely to get a similar effect, because the prevailing norms dictate that students ask, repeatedly, for explanation and demonstration of the use of any topic, concept or relation introduced to them by the teacher. Multi-disciplinary projects can meet the students' request for links between mathematics and their own, experienced world. The chance to give clearer answers to these questions though is a two-edged sword: referring to the abovementioned change of focus, we see a bare technical view on mathematics as a potential danger for multi-disciplinary teaching projects. In some aspects, the impact of multi-disciplinarity on the students' view on mathematics is comparable to the impact of use of computers: the 2005-reform also imposed the introduction of compulsory use of computer algebra systems (CAS) in mathematics. This introduction will, obviously, cause comprehensive changes now and in the future. For the teaching of models and modelling, especially in multi-disciplinary environments, the use of CAS opens up a wider range of topics. Furthermore, it has potential for the significant extension and development of the teaching of ‘authentic’ models and technical modelling in the sense of comparing a number of models and fitting them with a set of data (Andresen 2007 p. 5). CAS also has the potential to support students’ model recognition and capability to understand and criticise authentic use of ready-made models in societal, ethical and economical contexts.

Results from our previous research, though, show that in general, the use of CAS tends to change focus of attention into technical and practical aspects of upper secondary school mathematics. This tendency results from the individual teachers’ choices based on preferences, habits and CAS competencies. In general, teaching with computer is centred upon solving tasks, whereas the reading of proofs and theoretical treatments in general are carried out without use of computer (Andresen 2006 p. 28).

Now, we see a potential danger, that the same trend might direct the multi-disciplinary teaching of models and modelling into a bare application view on mathematics, at the expense of giving the students a more profound insight into mathematical activities, theory and knowledge.

To avoid this, Andresen (2008) argues that the students’ more technical and practical view on models and modelling can and should be balanced out by explicit mathematical reflections. Andresen and Froelund (2008) present a model that combines mathematical reasoning and formation of conceptions with philosophical reflections. This model intends to serve as a basis for such balanced teaching.

In summary, we see good reasons for the introduction of multi-disciplinary teaching into mathematics using a balanced approach. These reasons could be applied as well, to argument for cross-disciplinary teaching. However, we shall see, one main difference between the two in favour of ‘multi-’, when later in this article we try to answer some of the following questions: how can this aspect of the reform be successfully implemented—what demands are made on the teachers, on the students, on the leadership and on the learning environment?

4 The teacher part and the student part of our case study

Based on the above-mentioned considerations we decided to focus one part of our case study, the teacher part, on the teachers’ teamwork as an example of good practice. We found it interesting to inquire how the team of teachers in the study tailored a multi-disciplinary project, which could fit into the new requests, based on their own teaching experiences, and how they included a new, inexperienced colleague in the process.

The other part focuses on some results of the new teaching approach; in the student part we try to identify results of the multi-disciplinary teaching. Formally, the introduction of multi-disciplinary projects aims to provide the students with means and methods to recognise and practice connections and relations between mathematics and other sciences. Therefore, we decided to concentrate our classroom case study on signs of school mathematics and on references to the math lessons during a few of the project’s lessons in chemistry, physics and ‘general study preparation’ [Danish: almen studieforberedelse].

To sum up:

The teacher part of the study should serve to cast light on issues like:

- Can we identify key factors, important for the development of a ‘multi-disciplinary teaching culture’?
- What do teachers need to fulfil the reform’s new demands?
- How can inexperienced teachers be supported?
- Is it possible to identify (any) constitutive elements of ‘good practice’?

In the study, the teachers’ professional culture and background were analysed. We made our interpretention their visions of interconnections between mathematics and other subjects, as they were revealed during the teaching.

The student part of the study aimed to inquire upon the students’ picture of the interplay between mathematics and other school activities, as it was revealed by these activities during the observed lessons. In the student part of the study, our interpretations were based on observations of signs and indications, which appeared in the classroom when students

- Asked clarifying questions to the teacher
- Gave references to previous lessons on mathematics or on other subjects
- Gave answers and explanations to each others questions
- Answered the teachers’ questions
- Took part in the ongoing discussions and negotiations in the classroom by other means

Thus, the student part of the study was designed in accordance with an interactionist’s perspective like Heinrich Bauersfeld presents it in (Bauersfeld 1994 pp. 137–139). This view implies that the “students interactively constitute the culture of the classroom, conventions both for subject matter and social regulations emerge, communication lives from negotiation and taken-as-shared meanings." So, we were looking for indications of a classroom culture where, for example, arguments from one subject (mathematics) were used and accepted in discussions within another subject (chemistry or physics) or used to convince other members of the group in discussions of problem-solving strategies etc. As well, we evaluated signs of the students' formation of conceptions, as it was revealed in our observation.

The rest of this article is structured in accordance with this. Although the study was divided into the teacher part and the student part, these two parts are intertwined. Consequently, some of the following paragraphs are marked teacher part or student part but others are common to both parts. For example, the two episodes with excerpts from the transcription of classroom observations are analysed with the aim to illuminate the student part of the study but they also serve as input to the discussion of the teachers' visions.

5 Successful implementation of multi-disciplinarity

The *teacher part* of the study was inspired by literature on educational change with special reference to top-down initiatives involving change of curriculum:

Victor Freiman and Nicole Lirette-Pitre (2005) argue that the introduction of a new approach to the school curriculum, which emphases interdisciplinary connections and linkage between subjects will require a clear vision by the teachers of such interconnections. According to Freiman and Lirette-Pitre, the teachers' development of a variety of teaching approaches is also prerequisite. These different approaches will facilitate trans-disciplinary learning about communication, information and communication technology (ICT), critical thinking, personal and social development, study and work habits, culture and heritage. We agree with Freiman and Lirette-Pitre and, naturally, extend the requirement for the teachers' clear vision and capability of a variety of teaching methods to the case of multi-disciplinary teaching. We return to the issues of visions and capability in the discussion of the case study.

5.1 Prerequisites for implementation of multi-disciplinarity

In general, for the succes of top-down educational change initiatives, such as the reform described here, there are established pre requistes. Fullan and Haregraeves describe four main elements of a framework for understanding teacher development: (1) the teachers' purpose; (2) the teacher as a person; (3) the real world context in which teachers work and (4) the culture of teaching; the working relationship that the teachers have with their colleagues inside and outside the school (1992 p. 5). Initiatives have to resonate (to some degree!) with all four elements. Linda Darling-Hammond describes this from the local school perspective: "… policy is not so much implemented as it is re-invented at each level of the system. What ultimately happens in schools and classrooms is less related to the intensions of policymakers than it is to the knowledge, beliefs, resources, leadership, and motivations that operate in local contexts. These are among the factors that produce what Richard Elmore (1983) has called 'the power of the bottom over the top.' (…) Studies of change efforts have found that the fate of new programs and ideas rests on the teachers' and administrators' opportunities to learn, experiment, and adapt ideas to their local context." (2005 pp. 366–367).

With regards to the fourth element of Fullan and Hargreaves' framework; the culture of the teaching, multi-disciplinarity has good chance of success, as, the concept of multi-disciplinary teaching will fit better with mathematics teachers' culture than is commonly the case for cross- or trans-disciplinary teaching. We base this claim on the fact that teachers in Danish upper secondary school have to take their degree at masters' level in two scientific subjects and then, afterwards, spend 2 years as a trainee to qualify for teaching. Thus, the educational courses are not integrated in the scientific study. Consequently, to describe the teachers' culture we should say that most teachers would give priority to subject over didactics, rather than vice versa. In line with this, mathematics teachers generally, according to our personal experiences, tend to concentrate on the mathematical content also when collaborating with other subjects. Since teachers are required to qualify in two subjects, a number of mathematics teachers, though, are familiar with such collaborations. Common combinations with mathematics are, beside physics and chemistry, Danish language, societal sciences and others. So, following Fullan and Hargreaves, the second element; the teacher as a person, will also count as a factor with positive impact on successful implementation in the case of multi-disciplinarity. In contrast, these elements do not favour cross-disciplinary collaboration, where the borders between subjects are more or less cancelled and the learning goals are chosen independently of, for instance, the mathematical content.

Before the reform, fiery souls and other motivated experienced mathematics teachers frequently carried out what should be called multi-disciplinary projects on their own accord; Danish teachers autonomously decide about the planning at that level of details. This was also the case for some of the teachers in the case study reported below. So at least partly, the first element (1) the teachers' purpose, fits with implementation of multi-disciplinarity.

The third element (3) the real world context in which teachers work, can be interpreted in terms of formal requests and regulations that direct decisions about syllabus and examinations. In the case of multi-disciplinarity, the reform sets the stage for the teachers work as mentioned above.

5.2 Object of the case study: the rocket project

The school is situated in a suburb of Copenhagen with a diverse population. During the last three decades, the school was a leader at the national level, with regard to teaching and curriculum experiments (Thorup 2003). Three of the teachers in our case study had been employed as teachers at the school for more than a decade; as well, the team included a teacher trainee. In their traditional teaching, the group of physics- and chemistry-teachers, who were also teaching most of the mathematics, stressed the experimental aspects of the students' independent practical, at the expense of ready-made cooking-book exercises. For example, the experiments in physics most often were carried out, based on the students' own written notes, and prepared in advance under the teacher's guidance. In chemistry, one of the teachers throughout the last decade had run an open-lab set-up where students could get help to complete chemical experiments on their own. Some years ago, the teachers in physics and in mathematics had worked together with peer-supervision.

According to our impressions and experiences, the classroom atmosphere in this school was friendly, relaxed and informal. Although the teachers appeared kind and forthcoming when working with students, they also demonstrated engagement and authority. The relationship between students and teachers was confident and respectful.

The project in our case involved a second-year class with 17 students. The project involved the subjects of mathematics, chemistry and physics and it focused on rockets: the theory and practice of rocket construction as well as military and civil use of satellites and rockets. A team of four teachers, namely the class teachers in mathematics, physics and chemistry plus a trainee teacher in mathematics as trainee planned, managed and evaluated the teaching sequence in common. Each subject spent about 3 weeks of lessons on the project. As well, the new school subject 'general study preparation' spent about 25 lessons on the project and in total there was about 70, 50 minute lessons were spent on the project.

5.3 Teacher part: the planning of the project

The team of teachers linked the project with other school activities: a preceding sequence of lessons on pyrotechnics in chemistry led up to the rocket-project. The rocket project was succeeded by a 5-day study visit to Munich by the students; this stay in Munich included a visit to the Technical Museum, where some of the students presented the exhibition's different rockets.

According to the teachers' plan, the content parts of the project were related to the individual subjects like this:

In mathematics,

The rocket equation: $v_{\text{final}} = u \cdot \text{In}\left(\frac{M+m_0}{M}\right)$ was deduced.

Here, the rocket's final velocity v_{final} is expressed by the exhaust velocity u and the masses of the empty rocket m_0 and of the launch amount of fuel M.

The rocket equation was deduced, making use of the conservation of total momentum for a system: the increase dm (negative) in the mass of fuel and the increase dv in the rocket's velocity change the start moment into $(M + m + dm)(v + dv)$ and give the moment $-dm(v + u)$.

Conservation of the total momentum then gives $(M + m)v = (M + m + dm)(v + dv) - dm(v + u)$, which means that $dv = u\left(\frac{dm}{M+m}\right)$.

The students created a worksheet in Excel and used this equation to calculate actual values of velocities and masses, respectively, in a number of cases.

Taking the same equation as a starting point, the teacher then deduced the corresponding differential equation $dv = u\left(\frac{d(M+m)}{M+m}\right)$ and demonstrated its solution.

In chemistry, the following four issues were treated:

(1) Generation of energy.
(2) Temperatures of combustions.
(3) Liquid and solid fuels.
(4) Different types of rockets.

In physics, Newton's laws, circular orbits and gravitation, conservation of momentum, and ballistic motion.

General study preparation, History of rocket technology, satellites and global positioning system (GPS).[1]

5.4 Students' expected outcome

The students were requested to write individual chapters to a common report and to give a 5-min presentation of the theme of their chapter.

[1] The 'general study preparation'-subject was involved for instance in the first bullet of one of the themes formulated by the teachers:

- Give an account of the historical development of the rockets
- Give an account of the deduction of the rocket equation and of how the final velocity depends on the masses of fuel and empty rocket. Explain and give grounds for each step in the derivation
- Choose yourself one kind of (historical) rocket, for which you can find data. Calculate the final velocity. (Remember to state all data and source references.)
- Discuss which conditions influence the actual final velocity of the rocket, its height and cruising range.

The project seemed to provide conditions for students to improve their understandings and competences on two levels:

The students had the opportunity to achieve technical knowledge about rockets, i.e. to see physical, chemical and mathematical concepts and their interrelationships demonstrated in an authentic context. They could investigate different quantities of rockets and their movements by experiments and calculations on their own, and they could learn and reflect critically, on aspects of historical developments and selected civic uses of rockets.

5.5 The teachers' planning meeting

At the first planning meeting, the 2005 reform's formal regulations were tried out in practice for the first time. Interpretations, therefore, in a number of situations, were left to the individual teachers. The team of teachers in our case did the planning together and made common decisions concerning the project's content, evaluation, products and time schedule. At the school, a number of rooms were available for the teachers, for individual work and for teamwork. On 28 March 2007, the team had booked a room for a meeting for discussing the 6th lesson, which we observed. Since one of them was unavailable, but had sent some proposals by email in advance, the two teachers decided to meet and consult the third partner afterwards.

The following paragraph demonstrates the variety of issues, discussed and decided on at the meeting:

- The teachers had to plan the students' final written reports and their oral presentations. The formal regulations stated that:
  - Each student must be evaluated individually on the basis of a written report.
  - It is up to the teacher to decide the theme of each student's report
- The teachers formulated detailed requests to the written reports and, in parallel they set up criteria for assessment of the products.
- They also planned for an oral presentation; this is in accordance with Danish school culture where oral examination is used at all levels from primary school and up. It was common practice to organise students' oral presentations with an 'external' audience, i.e. students who had no previous knowledge about the theme making the presentation more authentic, according to the teachers. In this case the students from a corresponding class were invited as audience; as part of the course in study skills the audience students were requested to take written notes, which would be evaluated by teachers responsible for that course. The audience students had access to the written rocket reports electronically in advance and they were encouraged to ask questions and start discussions after each of the oral presentations.
- The teachers decided to divide the group of 17 students into group A and group B.
- The teachers decided to formulate 8 themes, and give each theme to both groups. The audience class was also divided into two groups, to make it possible to have the oral presentations in two parallel sessions during two or maybe three lessons.
- Finally, the teachers had to organise and plan their own work; find out how many working hours they were credited and decide when to carry out their work. The team was supposed to do this by itself—there was no chief manager to do it. As specified in the official regulations, they would be credited with an extra 50% working hours to guide, evaluate and give feedback to the written reports.

Next, the teachers discussed themes with relation to the four subjects involved. They aimed to combine more than one of the subjects physics, mathematics, and chemistry in each of the themes:

Teacher2: Can we have the impulse from physics combined with themes in mathematics?
Teacher1: No, because two different approaches are used.
Teacher2: Yes in mathematics the impulse is applied, eh?
Teacher1: no…, no,

They decided on four themes mostly within physics (named P_1, P_2, P_3, P_4); two themes mostly within mathematics (M_1, M_2) and two themes mostly within chemistry (C_1 and C_2)

Teacher2: Now let us put them in some kind of natural succession.

Since P2 was rather fundamental, the teachers choose it as the starter. Because of its historical aspect, M1 was next. P3 and P4 were investigations, M2 concerned with numerical solutions, and the next, C1 and C2, were realisations. P1 became the last one, because it was a broader theme than the others.

In our interpretation, it was evident that the teamwork improved the flow and the quality of the teachers' working process. These kinds of decisions belong to level K2 of professional development, according to E. Dale's taxonomy[2] for teachers' professionalism (Dale 1998, p. 169).

[2] In Dale's taxonomy, level K1 of professionalism is characterised by rationality in and beyond the teaching performance, level K2 includes (amongst others) reflections upon and evaluation of curriculum and teamwork with colleagues on organisation of courses, whereas level K3 encompass theory development based on own experiences.

5.6 Student part: the classroom case study

Data for our analysis consisted of film recordings and field notes from about eight lessons, teaching materials, and notes from informal talk with the teachers and the students' written reports, students' power point presentations and students' written homework tasks. In the following we present analysis of two selected episodes from the data: one episode took place during a lesson in chemistry and the other occurred while the students did group work in the physics classroom. The episodes were selected to illuminate how the students perceive school mathematics and relate it to physics, to chemistry and to their everyday experiences.

5.6.1 Episode 1

This episode took place in the project's first chemistry lesson. Under the teacher's introduction of the chemistry part of the project to the students, he had stated that most likely, the students would not notice any distinction between the chemistry parts of the project and the parts relating to general study preparation. He stressed that there would be no intended distinction between these parts.

The teacher went through a note on rocket engines, presuming that the students had read it in advance to prepare for the lesson.

The concept of specific momentum was introduced and explained in the note (*Raketmotorer* page 2, our translation):

"The specific momentum, which is in fact not momentum, is defined by "kg pressure power per kg consumed fuel per second". That is, if a given fuel may deliver a 200 kg pressure power by consumption of 1 kg fuel each second, then $I = \frac{200\,\text{kg}}{1\,\text{kg/s}} = 200\,\text{s}$.

Notice, that the unit of specific momentum is the second. Notice also that the specific momentum gets the same value and the same unit if it is defined by "pounds pressure power per pound consumed fuel per second".

It is easy to show that $I = \frac{u}{g}$, where u is the exhaust velocity and g is the gravitation.

After a short discussion about types of chemical reactions that give a huge amount of energy very fast, the teacher continued, referring to the note:

Teacher: Then we have a short chapter on specific momentum—have you worked with this together with G (the physics teacher)?
Student: We have talked about the concept of momentum
Teacher: Have you also talked about the concept of specific momentum? (Confirmative mumbling)
Student: And in mathematics, too
Teacher: Do we have to do it again, then?
Student1: I would like you to explain this again
Student2: Will you please do it again?
Teacher: Let's do that

It was clear that the students accepted 'specific momentum' as a part of the class' shared knowledge, even if this physic's concept had only been introduced in a mathematics lesson and now, it was repeated in chemistry, not in physics.

A few minutes later, the teacher had explained that the higher the velocity of the exhaust gasses, the higher the velocity of the rocket. He referred to the note's table of graphs of pressure, density, temperature and speed as functions of the position in the rocket nozzle. One of the students interrupted the teacher:

Student1: T (name of the teacher)? What is this: You equals u divided by g, what is it?
Teacher: I is the specific momentum, it is a property of the fuel, isn't it?
Student1: But is it a formula, or what? That is, I have never seen it before.
Teacher: Then you see it now!
Student2: We have seen it before in a different way
Student1: Yes, we have seen it before in a different way
Student3: It is the same

Apparently, student 1 used the term 'a formula' in the meaning of 'shared knowledge in the class, known from previously taught lessons. That is, in our interpretation, that Student1 wanted to know whether the expression $I = u/g$ was supposed to be well known. The students 2 and 3 agreed that it was part of the shared knowledge but gave student1 an excuse for not recognising it ('we have seen it in a different way').

In parallel, the question 'what is it?' in our interpretation revealed that student1 also felt unsure of the meaning of the concept symbolised by I. The teacher followed up with a question, apparently with the intention to find out whether the students understood the expression and the concept of specific momentum or not:

Teacher: In what form have you seen it?
Student: We know I equals… p times v, wasn't it so?
Student: I equals p times vm
Teacher: The u is the same as…
Student (interrupts): or u times v
Teacher: …is the same as momentum divided by mass, isn't it?
Student1: But is it a formula for momentum, or what?
Teacher: It is a formula for what is called the specific momentum of the fuel. It is related with the exhaust velocity so you can say that these things—it is easy to see, based on the conservation of momentum, that the

higher speed the better, isn't it? That is, the higher specific momentum the better.

In our interpretation, the problem is that the concept of *specific momentum* is distinct from the concept of *momentum*. The first one was rather new to the students, whereas the latter, supposedly, was part of their shared knowledge. The students might not be aware of the distinction.

The episode demonstrates two levels of questions:

One level concerns the pool of shared knowledge. One of the students doubts that the concept specific momentum is included in this pool and asks a question that lead to negotiations about which formulas and expressions the teacher and the students can use, when they want to refer to the pull. The student's use of the term 'formula' reveals in our interpretation her picture of the class' shared knowledge as a collection of formal expressions and formulas, rather than being clusters of concepts and their relations, etc. It is impossible, though, to say whether the student's question refers only to the structure of the formal expression, that is, to the mathematics content of it, or not.

The other level concerns the meaning of the concepts, referred to by the symbols. At this level, the two different meanings of the term 'momentum' confuse the student, although this double use is clearly mentioned in the teaching materials. The confusion, in our interpretation, demonstrates that at least one of the concepts, maybe both, is only adapted on the superficial level of terms or symbols. It also reveals that this student neither use the units (school physics) or the structure of the expression (school mathematics) to distinguish between the two.

5.6.2 *Episode 2*

The second episode reports on one group's work with a task, which formed the basis for one of the individual presentations. The episode serves to demonstrate how the students chose appropriate mathematical tools when faced with a problem.

The teacher presented the problem: what height does the rocket reach? One of the students referred to a ballistic experiment, previously done in physics lessons with the class. The previous experiment concerned a ball so the teacher argued that they calculated the height of its path based on the mass of the ball. Therefore, they could not use the same method to solve this new problem. Next, the teacher and one of the students in common referred to another experiment on height, which involved a tall house in the neighbourhood. The teacher made a sketch on the blackboard (Fig. 1).

One student asked whether it would be appropriate in the actual case to use trigonometry, and the teacher answered yes. The teacher then briefly introduced the use of laser to measure the height with the use of similar triangles.

In the following lesson, the group work started. A group of four students were requested to design an experiment, which could serve to determine the height of the rocket's path. The students made a drawing on the computer, resembling the teacher's sketch (Fig. 2).

Based on their drawings, the students discussed the design of the experiment. They neither referred to the previous experiments with the ball nor to the one with the big house. In our interpretation, they didn't because the teacher already had discussed these two strategies in plenum and rejected them. Neither did they refer to the textbook nor to any mathematics lessons. They asked the teacher if it would be all right to use similar triangles, and the teacher answered yes:

Teacher: You are allowed to use Pythagoras and similar triangles—even if the math teacher loves cosines and sinus, you are not obliged to do the same!
Student: But how can we determine this distance…we just use similar triangles.

Apparently, the teacher meant that the students' were not obliged to use trigonometry if they did not find it appropriate. The short excerpt indicates how the students managed to compare the actual problem with previous experiences and to consider different strategies before they chose an appropriate one to solve the problem. The students, seemingly, did not feel obliged to use their most advanced school mathematics (in the form of trigonometry). The teacher deliberately encouraged them to be critical towards their math teacher's motives for working with trigonometry. To understand the excerpt's exchange of words, one must imagine the atmosphere in the

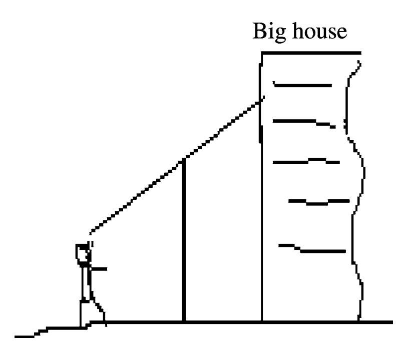

Fig. 1 The teacher's sketch

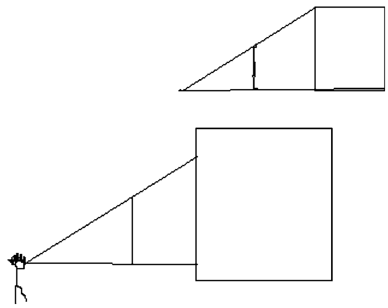

Fig. 2 The students' sketch

classroom and the relation between the teacher and the students: the level of activity was high and the students' commitment to the problem solving obvious. The teacher gave his remark about the math teacher as a joke, using the math and science teachers' prevailing, informal jargon. The joke should encourage a rule-breaking, anti-authoritarian attitude by the students, suitable for the spirit of experimental projects. It was evident from the students' reaction that they understood and accepted this message.

In our interpretation, the students were encouraged to feel personal commitment to the problem solving and, consequently, they did not care about reading any 'hidden message' in the task's text. So, they did not try to guess what strategy they were supposed to use nor did they automatically use the most advanced mathematics they had learned (trigonometry) or the mathematics they had learned most recently.

We take this as an indication of the problem's authenticity as the students perceived it. Next, we conclude that these students strongly connect mathematics with everyday use of common sense and problem solving, independent of the structure of curriculum.

5.7 Teacher part: the teachers' visions

In episode 1, the teacher told the students that they would experience little distinction, if any, between the chemistry parts of the project and the general study preparation parts. The teacher referred to the lessons in mathematics and in physics, when he asked the students if they were familiar with the relation between velocity of the exhaustion and velocity of the rocket, and with the relations between pressure, density, temperature, speed and position of the rocket, they requested he revise these topics. In that way he demonstrated that the borders between subjects should not induce borders between the activities, carried out during the single subjects' lessons. He also demonstrated competence to explain the physics' issues. This reveals, in our interpretation, that the team's common planning had also encompassed the detailed content of each part of the project, not only the framework and division of content. Further, we found it interesting to notice the teacher's answer to the student's question: "what is this: I equals u divided by g, what is it?" The teacher answered that I is the specific momentum, a property of the fuel. The answer connected the symbol with reality in the form of fuel, which has certain characteristics and, thereby, made a shortcut to pass around the formal expression and its structure. For him, apparently, the symbols had a concrete physical interpretation with or without mathematics.

In episode 2, the teacher demonstrated different possibilities for solving the task. He also demonstrated that the students had a real choice and thereby revealed a view on mathematics in this context as a tool for real problem solving rather than for demonstration of cleverness in school classes.

In our interpretation, it was clear that the team had a shared vision of multi-disciplinarity as a means for enhanced learning both in physics, chemistry and chemistry. The new subject, general study preparation, apparently, was not a main focus of the project.

6 Conclusion and perspectives

The team of teachers in this study demonstrated an example of good multi-disciplinary teaching practice as far as they managed to

- Demonstrate relations between the three subjects mathematics, chemistry and physics
- Demonstrate differences between the same three
- Engage the students in projects which, on the one hand had clear bonds to one of the three subjects and, on the other hand drew on the other subjects and the relations between them
- Organise and run this project with its very complex structure and fulfil its formal requests

The three teachers' professional experiences and competence, besides the team spirit, helped along this example of good practice. The teacher trainee, obviously, was included in the team as a peer fellow.

The results of the project, as they appeared in the classroom, were promising. We saw signs that indicated

- Students' awareness of the possibilities to transfer concepts and results from between subjects
- Students' consciousness about benefits, traps and misunderstandings caused by such transfer
- Students' reflections upon the relations between the project's subjects

The students seemed to build relations between the subjects in parallel with their formation of concepts and new skills belonging to the single subject. In our interpretation, the students and the teachers in common managed, at an introductory level, to explicate such relations and reflect upon them.

The two episodes point to a revised role for mathematics, which we could trace in our study, namely the role of an authentic tool for problem solving. In the episodes, the students showed no experience of doing mathematics as a goal in itself. We see this as a success, because doing mathematics as a goal in itself is severely criticised in Danish school culture. On the other hand, the students seemed to acknowledge the value of mathematics as a tool for understanding the physical phenomena, modelled in the rocket project. The revised role of mathematics, therefore, has the potential to embed students' mathematical competence into a broad and reflected view of math and science. It is important to notice the need for activities to balance the impact of more technical applications, during the major, non multi-disciplinary part of the teaching.

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