DEPARTMENT OF ECONOMICS AND BUSINESS

Economics Working Paper

2011-13

Quantity Precommitment and Price Matching

Norovsambuu Tumennasan

Department of Economis and Business
Business and Social Sciences
Aarhus University
Bartholins Allé 10, Building 1322
DK-8000 Aarhus C - Denmark
Phone +45 8942 1610
Mail: oekonomi@econ.au.dk
Web: www.econ.au.dk

Quantity Precommitment and Price Matching

Norovsambuu Tumennasan*
The School of Economics and Management, Aarhus University
Denmark

First Version: March 2010
This Version: September 2011

Abstract

We revisit the question of whether price matching is anti-competitive in a capacity constrained duopoly setting. We show that the effect of price matching depends on capacity. Specifically, price matching has no effect when capacity is relatively *low*, but it benefits the firms when capacity is relatively *high*. Interestingly, when capacity is in an intermediate range, price matching benefits only the *small* firm but does not affect the *large* firm in any way. Therefore, one has to consider capacity seriously when evaluating if price matching is anti-competitive.

If the firms choose their capacities simultaneously before pricing decisions, then the effect of price matching is either pro-competitive or ambiguous. We show that if the cost of capacity is *high*, then price matching can only (weakly) decrease the market price. On the other hand, if the cost of capacity is *low*, then the effect of price matching on the market price is ambiguous due to the multiplicity of equilibria. Therefore, this paper challenges the widely accepted belief that price matching is an anti-competititive practice if the firms choose their capacities simultaneously before pricing decisions.

Keywords: Price matching, capacity constraint, quantity precommitment
JEL Classifications: L00, L01, L02

1 Introduction

Many businesses offer price matching, i.e., if one of the firm's competitors is selling the same product for a lesser price, then the firm will sell the product for the same price. Examples of firms using price matching range from the electronic retail giant, BestBuy to tiny pizza parlors.[1] Given its prevalence[2] in practice, price matching has attracted a considerable interest among economists.

*This paper is a chapter of my Ph.D. dissertation from Brown University. I am indebted to my advisors Roberto Serrano, Geoffroy de Clippel, and Sergio Turner for their continual advice and encouragement. I would like to thank Antonio Cabrales, Alexander Koch, John Kennes, Julia Nafziger, Per Baltzer Overgaard and participants at the Brown University and Aarhus University seminars for their many useful comments and suggestions. All remaining errors are my own.

[1] Pizza parlors often accept competitors' coupons which is one form of price matching.

[2] Moorthy and Winter (2006) point out that a Google search on "price matching" returns more than two hundred thousand hits demonstrating how widespread price matching is.

A price matching firm informs the market that its price is the lowest, hence, it seems to embrace competition. However, the price matcher warns its competitors that it will not be undersold, thus, it eliminates the rivals' incentive to undercut the price (Salop, 1986). As a result, any price which is usually reached through collusion is a market price when firms have an option to price match.[3] From this point of view, price matching is a tool for firms to enforce collusive agreements.

Another line of research argues that price matching is a form of price discrimination: Belton (1987), Png and Hirshleifer (1987) and Edlin (1997) show that by offering to match its competitors' price, a firm gives discounts to customers who are aware of the market prices but it keeps the price high to other customers. As a result, economists as well as legal scholars[4] predominantly view price matching as an anti-compettitive practice. However, the literature on price matching implicitly assumes that firms can adjust their capacities instantly. This naturally leads to the question we consider in this paper: What are the effects of price matching if the competing firms are constrained in terms of capacity? Specifically, in this paper, we study the effects of price matching in two well studied models: (1) a price-setting duopoly in which each firm has limited capacity and (2) firms select their capacity simultaneously before pricing decisions.

We adopt the setting of Kreps and Scheinkman (1983) (KS) in which firms install their capacity in the first period and name their price in the second period. As pointed out in the original paper, the KS model can be interpreted as follows: In the first period, firms produce, and then in the second period, having observed the other's production level, they engage in a Bertrand (price) competition. However, each firm cannot sell more than its first period production. The KS model covers a wide range of applications because customers, instead of calling to pre-order, usually visit a store to buy what they need. As a result, firms must hold a stock of their product before they meet customers. In addition, because firms meet their customers after producing their stock, the firms' prices are set in the second period.

Formally, this paper considers a dynamic model in which firms install their capacity in the first period and choose their price and price matching option in the second period. What we add to the KS model is that the firms can price match in the second period. We believe that the price matching decision belongs in the second stage after installing capacities because capacity cannot be instantly changed, while the pricing decisions can be modified almost instantly.

First we show that when the capacity of each firm is limited, the effects of price matching vary with the firms' capacities. Specifically, the larger the capacity, the stronger are the effects of price matching on the firms. This is intuitively plausible because for price matching to be effective, the equilibrium price (in the absence of price matching) needs to be low enough that some price beyond it simultaneously improves the firms. But when capacity is relatively small, without price matching, the equilibrium price is already very high (Kreps and Scheinkman (1983) and Osborne and Pitchik (1986)). Thus price matching does not affect the firms. At the other extreme, when capacity is relatively high, without price matching, the equilibrium price is sufficiently low, thus price matching affects both firms in a positive way. Most interestingly, when industry capacity is in an intermediate range, price matching benefits the small firm but

[3] Salop (1986) shows that the equilibrium price in the presence of price matching option ranges from the monopolistic to the Bertrand price. Doyle (1988) further points out that only the monopolistic price survives the process of iterative elimination of weakly dominated strategies.

[4] For a comprehensive literature review, see Arbatskaya et al. (2004).

not the large one. Without price matching in this case, the equilibrium strategies involve randomization (Kreps and Scheinkman (1983) and Osborne and Pitchik (1986)) because whenever the small firm offers a price exceeding the market clearing price, the large firm prefers a price slightly lower than that of the small firm. With price matching, the small firm can eliminate the large firm's incentive to underprice. Therefore, the equilibrium price increases but the firms split the equilibrium market demand so that the small firm sells its full capacity. This is the reason why price matching has disproportionate effects on the firms. Our analysis on capacity constrained duopoly suggest that capacity is a key factor to whether price matching is anti-competitive or not.

Second, we show that if firms choose their capacities simultaneously before pricing decisions, then the effect of price matching is either pro-competititive or ambiguous. We prove the following 2 results. (1) If the cost of capacity is low,[5] then some SPE prices are higher than the Cournot price – the only SPE price in the KS setting – while some are lower. (2) If the cost of capacity is high, then the SPE prices are always (weakly) lower than the Cournot price. The reason is as follows: to take advantage of price matching, firms need to have a capacity exceeding a certain threshold, but this threshold does not depend on the cost of capacity because it is sunk once the second period starts. Furthermore, when firms' capacity exceeds this threshold, there is a range of equilibrium prices as in Salop (1986), and the highest price in this range is always the Monopolistic price associated with the cost of production. But the Cournot price is lower than the Monopolistic price associated with the cost of production only if the cost of capacity is low. Therefore, price matching can only decrease the market price below the Cournot one when the cost of capacity is high. However, when the cost of capacity is low, some SPE prices exceed the Cournot price while some do not. Therefore, the effect of price matching on the market price is ambiguous if the cost of capacity is low.

We furthermore use an equilibrium refinement that requires the firms to coordinate on the best equilibrium from their perspective in the second period. In the Salop (1986) model, this refinement leads to the equilibria that result in the Monopolistic price (Doyle, 1988). However, when we apply the refinement to our model, SPE does not exist if the installation cost is low when price matching could potentially have an anti-competititive effect. Therefore, this refinement does not help in determining the effect of price matching on the market price when the installation cost is low.

A handful of papers challenge the conventional wisdom that price matching is anti-competitive. Corts (1995) checks the robustness of the anticompetitive effect of price matching. He extends the price matching policy to the price beating policy and restores the Bertrand price as the unique equilibrium.[6] The difference between the price beating and matching policies is that the former allows firms to undercut the price of others, while the latter only allows firms to tie their price to those of the others. This is the reason behind Corts's result. Also, Hviid and Shaffer (1999) introduce hassle costs, i.e., consumers have to bear certain costs to convince a price matching firm that there is a lower price in the market. In their model, a firm can steal the other's market share by underpricing because customers save the hassle costs by buying from the

[5]The formal condition requires the total Cournot quantity with the combined cost of capacity installation and production to exceed the monopolistic quantity with the production cost. However, for the ease of presentation, the introduction uses the condition that coincides with the formal condition in the case of linear demand and cost.

[6]Kaplan (2000) further extends the strategy set to include effective price strategies and restores the possibility of monopoly pricing.

price cutter, thus, restoring the Bertrand price.[7] The common thread between Corts (1995) and Hviid and Shaffer (1999) is that in both papers, the firms' incentive to undercut the others' price is restored. In this paper, we do not restore the firms' incentive to undercut the others' price, but introduce capacity as a choice variable. Moorthy and Winter (2006) introduce cost heterogeneity among firms and shows that only a low cost firm uses price matching to signal that it is low priced.

The paper is organized as follows: Section 2 lays out the model. Section 3 investigates the effect of price matching in a capacity constrained duopoly. Section 4 studies the effect of price matching in the full game. Lastly, section 5 concludes.

2 Model

Two identical firms offer the same product and the market demand for this product is $P(x)$ or $D(p) = P^{-1}(p)$ where x and p are quantity and price, respectively.

The two firms compete in two stages: In the first stage, each firm installs its capacity which is the maximal quantity that the firm can sell in the second stage. Firm i's cost of capacity $k_i \in \mathbb{R}_+$ is $c(k_i)$.

In the second period, after observing each other's capacity, each firm i chooses its *announced* price p_i and price matching option $o_i \in \{0, 1\}$ where 1 means "match" and 0 means "do not match". Without loss of generality, we normalize the cost of production in this period to 0. The buyers are informed about the firms' second period actions.[8] Consequently, by choosing different price matching options, a firm alters the actual price of its product. Specifically, if firm i *does* price match, then it sells its product for the lowest price on the market. But if firm i does not price match, then it sells its product for its announced price. We use the terminology ***effective price*** of firm i to refer to the price the firm sells its product for, i.e., $p_i^e(p_i, o_i, p_j, o_j) \equiv (1 - o_i)p_i + o_i \min\{p_i, p_j\}$. A firm can offer any effective price by properly choosing its price and price matching option, but it cannot underprice the other if the rival price matches. We use the effective prices extensively because these prices ultimately determine the sales quantity of the firms.

Now let us formulate the sales quantity of firm i which of course depends on the firms' capacities and effective prices. Let p_1^e and p_2^e be the corresponding effective prices for firms 1 and 2. Then firm i sells

$$x_i(p_i^e, p_j^e, k_i, k_j) = \begin{cases} \min\{k_i, D(p_i^e)\} & \text{if } p_i^e < p_j^e \\ \min\left\{k_i, \max\left\{D(p_j^e) - k_j, \frac{D(p_j^e)}{2}\right\}\right\} & \text{if } p_i^e = p_j^e \\ \min\{k_i, \max\{0, D(p_i^e) - k_j\}\} & \text{if } p_i^e > p_j^e. \end{cases} \tag{1}$$

The above formulation implicity assumes that the firms split the market if they announce the same price as long as each has a sufficient capacity. In addition, the efficient rationing rule is used, i.e., the consumers with a higher valuation buy from the firm with the lower effective price.

Now let us consider the full game. The strategy set of firm i is $\mathbb{R}_+ \times \{(p_i, o_i) : \mathbb{R}_+^2 \to \mathbb{R}_+ \times \{0, 1\}\}$.[9]

[7] Dugar and Sorensen (2006) take the model of Hviid and Shaffer (1999) to an experimental lab, and find a significantly different price than the Bertrand price.

[8] Perhaps through newspaper or internet advertising.

[9] When discussing the strategies, the notations $p_i(k_1, k_2)$ and $o_i(k_1, k_2)$ are the functions of the capacities chosen in period 1.

Let (s_1, s_2) denote a generic strategy profile. Then the profit function of firm i is

$$\pi_i(s_i, s_j) = p_i^e(s_i, s_j)x_i(s_i, s_j) - c(k_i)$$

where $p_i^e(s_i, s_j)$ and $x_i(s_i, s_j)$ are the effective price and sales quantity of firm i corresponding to the strategy profile (s_i, s_j). We use the notion of subgame perfect equilibrium (SPE) to analyze the full game.

As in KS, we make the following two assumptions.

Assumption 1. *$P(x)$ is strictly positive on some bounded interval $(0, \bar{x})$ on which it is twice continuously differentiable, strictly decreasing, and concave. For $x \geq \bar{x}$, $P(x) = 0$.*

Assumption 2. *The cost function is twice differentiable, increasing and convex, i.e., $c' > 0$ and $c'' \geq 0$. In addition, $c(0) = 0$. Furthermore, $c'(0) < P(0)$ - production at some level is profitable.*

Osborne and Pitchik (1986) study a model similar to KS's under the relaxed assumption that demand is decreasing but not necessarily concave. They show that there could be multiple SPEs but this complicates our goal of determining the effects of price matching in the full game.[10] To avoid this problem, we maintain assumption 1.

We now turn our attention to the standard Cournot competition with cost function b, where $b(x) = 0$ or $b(x) = c(x)$. Thanks to assumptions 1 and 2, one can easily show that the profit function $P(x+y)x - b(x)$ is concave on $[0, \bar{x} - y]$. Let $r_b(y)$ be the Cournot best response to the rival's production y, i.e., $r_b(y) = \arg\max_{0 \leq x \leq \bar{x} - y} xP(x+y) - b(x)$.

The following lemma, which is instrumental in our analysis, is from KS.

Lemma 1. *(a)For cost function b, r_b is nonincreasing in y, and r_b is continuously differentiable and strictly decreasing over the range where it is strictly positive*
(b) $r_b' \geq -1$ with strict inequality for y such that $r_b(y) > 0$
(c) If b and d are two cost functions such that $b' > d'$, then $r_b(\cdot) < r_d(\cdot)$
(d) If $x \geq r_b(x)$, then $x \geq r_b(r_b(x))$

Proof. See KS. □

As pointed out in KS, thanks to assumptions 1 and 2, there is a unique Cournot equilibrium associated with cost b, with each firm supplying x_b^c. Let $p_b^c \equiv P(2x_b^c)$ and $\pi_b^c \equiv p_b^c x_b^c - b(x_b^c)$. Also, let $x_b^m \equiv \arg\max_x P(2x)x - b(x)$, $p_b^m \equiv P(2x_b^m)$ and $\pi_b^m \equiv p_b^m x_b^m - b(x_b^m)$. Observe that if each firm supplies x_b^m when $b = 0$, then the total market supply, $2x_b^m$, equals the monopolistic quantity associated with 0 cost. Hence, we refer to x_b^m as the monopolistic quantity. Since the second period cost of production is 0, the $b(\cdot) = 0$ case is used extensively in our analysis. To simplify the notations, we omit the subscripts from the notations.

Before we move to the next section, let us review Salop (1986) and KS which this paper is based upon. Salop (1986) studies the effect of price matching in the standard Bertrand setting in which firms have

[10]However, in capacity constrained games, we believe that relaxing assumption 1 as in Osborne and Pitchik (1986) does not change the effects of price matching found in this paper.

unlimited capacity. Therefore, his model corresponds to the subgame in our model in which both firms have a sufficiently large capacity. Salop finds that all the prices ranging from 0 to p^m can be supported as a Nash equilibrium.

KS, on the other hand, models capacity but not price matching. As we mentioned earlier, our model is an extension of the KS model in which firms can price match. Consequently, to see the effects of price matching on the market price, we compare the equilibrium prices found in our model to the ones found in KS.

3 Price Matching in a Capacity Constrained Doupoly

In this section, we analyze the effect of price matching in capacity constrained games, i.e, the second stage of our full game.

Let us fix a capacity constrained game in which the firms' respective capacities are k_1 and k_2. The total capacity is $k = k_1 + k_2$ and without loss of generality, we assume that $k_1 \leq k_2$. Sometimes we refer to firm 2 (firm 1) as the large (small) firm. Since we are concentrating capacity constrained games, we simplify the notations by using p_i and o_i for $p_i(k_1, k_2)$ and $o_i(k_1, k_2)$, respectively. In this paper, we will only concentrate on pure strategies.[11] If $k_1 \geq D(0)$, our model is equivalent to the standard Salop model, hence, any price in the interval $[0, p^m]$ can be supported as an equilibrium price. Henceforth, we will concentrate on the $k_1 < D(0)$ cases.

If firm i sets its effective price to p_i^e and firm j to p_j^e, then firm i nets:

$$R_i(p_i^e, p_j^e) = \begin{cases} L_i(p_i^e) \equiv p_i^e \min\{k_i, D(p_i^e)\} & \text{if } p_i^e < p_j^e \\ E_i(p) \equiv p \min\left\{k_i, \max\left\{D(p) - k_j, \frac{D(p)}{2}\right\}\right\} & \text{if } p_i^e = p_j^e = p \\ G_i(p_i^e) \equiv p_i^e \min\{k_i, \max\{0, D(p_i^e) - k_j\}\} & \text{if } p_i^e > p_j^e. \end{cases} \tag{2}$$

Observe that each of L_i, E_i, and G_i is a continuous function and $L_i(p) \geq E_i(p) \geq G_i(p)$. In figure 1, we show the examples of the functions L_i, E_i and G_i. Generally, E_2 has a more complex shape than its counterpart E_1 because firm 2 has a higher capacity than firm 1. To see this, let us examine E_1 and E_2 for different prices. If the price is lower than $P(k)$, there is undercapacity in the market, thus each firm sells its full capacity. Consequently, $E_i(p) = pk_i$ if $p \leq P(k)$. The firms have enough capacity to meet the market demand as soon as the price reaches $P(k)$. Then half of the total market demand is allocated to each firm which the small firm cannot meet if the market price is lower than $P(2k_1)$. If this is the case, then the excess demand is allocated to firm 2. Consequently, $E_1(p) = pk_1$ and $E_2(p) = p(D(p) - k_1)$ on the interval $[P(k), P(2k_1)]$. Once the price passes $P(2k_1)$, firm 1's capacity is large enough to meet half of the market demand. As a result, for both firms, $E_1(p) = E_2(p) = \frac{pD(p)}{2}$ if $p \geq P(2k_1)$. Here, we note that E_2 is equal to G_2 for all prices $p \leq P(2k_1)$ and to $\frac{pD(p)}{2}$ for all $p \geq P(2k_1)$. This will be used repeatedly in our subsequent analysis.

Because the functions L_i, E_i and G_i play important roles in characterizing the equilibria, we investigate

[11] One can consider mixed strategies which complicates the analysis significantly without providing additional insights. The analysis that considers mixed strategies can be obtained from the author.

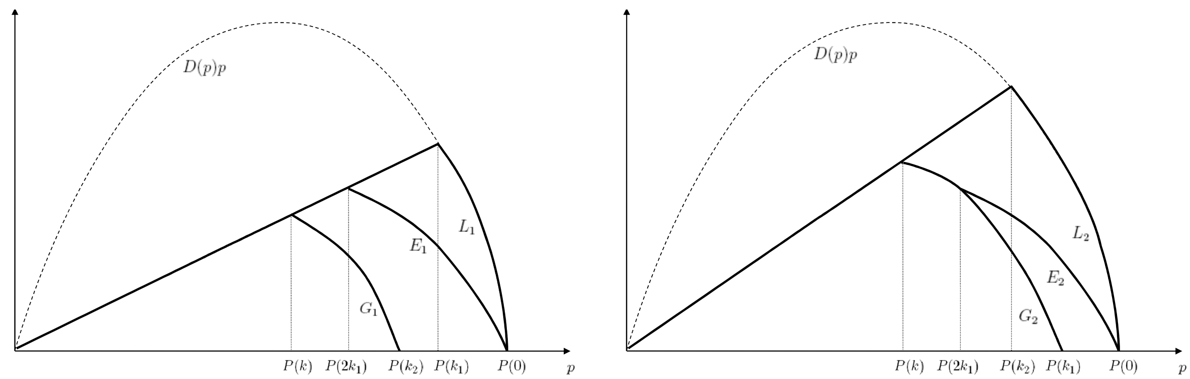


Figure 1: The Functions L_i, E_i and G_i for $i = 1, 2$

them in detail. Let $\bar{L}_i(k_1, k_2) \equiv \max_{p \geq 0} L_i(p)$ and $p_i^L(k_1, k_2) \equiv \arg\max_{p \geq 0} L_i(p)$. In a similar way, we define $\bar{E}_i(k_1, k_2)$, $p_i^E(k_1, k_2)$, $\bar{G}_i(k_1, k_2)$ and $p_i^G(k_1, k_2)$. For these notations, we often exclude the arguments when this does not cause confusion. For example, $\bar{L}_i$ is the shorthand notation for $\bar{L}_i(k_1, k_2)$.

First let us examine G_i because L_1, L_2 and E_1 turn out to be special cases of G_i. First we identify the maximizer and the maximal value of G_i depending on the capacities of the firms. Suppose firm i has unlimited capacity. Then at price p, $G_i(p) = p(D(p) - k_j)$ which can be interpreted as the revenue firm i earns by selling quantity $D(p) - k_j$ for price p. From the perspective of firm i, this is equivalent to the case in which firms i and j bring respective quantities $D(p) - k_j$ and k_j to the market, and the market price adjusts to clear the market — the quantity (Cournot) competition in which firms i and j produce $D(p) - k_j$ and k_j, respectively. Accordingly, we know that firm i maximizes $G_i(p)$ by selling $r(k_j)$ (the Cournot best response to firm j's capacity) for $P(k_j + r(k_j))$. Now let us consider the cases in which firm i is capacity constrained. If firm i has a capacity that exceeds $r(k_j)$, firm i can always sell $r(k_j)$ by setting its price to $P(k_j + r(k_j))$. As a result, for firm i, $G_i\left(P(k_j + r(k_j))\right)$ equals the maximal G_i for capacity unconstrained firm i. As a capacity *constrained* firm i cannot have a higher $\bar{G}_i(k_i, k_j)$ than a capacity *unconstrained* firm i, it must be that $\bar{G}_i(k_i, k_j) = P(k_j + r(k_j))r(k_j)$ and $p_i^G(k_i, k_j) = P(k_j + r(k_j))$ if firm i's capacity exceeds $r(k_j)$. If firm i's capacity is less that $r(k_j)$, then by setting its price to $P(k_j + r(k_j))$, firm i cannot sell $r(k_j)$ because of its capacity. Consequently, in this case, $p_i^G(k_i, k_j)$ must be $P(k)$ — the minimal price at which firm i can meet the market demand. Now that we have fully identified the maximizer of G_i, let us consider the shape of G_i which is composed of two functions: pk_i on the right of $P(k)$ and $p(D(p) - k_j)$ on the left of it. The former function is an increasing function while the latter one is a hump-shaped (inverted U) function. In addition, because G_i is continuous, it must be a hump-shaped function. We summarize these results in the following lemma.

Lemma 2. *(a) For each firm i,*

$$p_i^G = \begin{cases} P(k) & \text{if } k_i \leq r(k_j) \\ P(k_j + r(k_j)) & \text{if } k_i > r(k_j) \end{cases} \quad \text{and } \bar{G}_i = \begin{cases} P(k)k_i & \text{if } k_i \leq r(k_j) \\ P(r(k_j) + k_j)r(k_j) & \text{if } k_i > r(k_j). \end{cases}$$

In addition, G_i strictly increases on $\left[0, p_i^G\right)$ and strictly decreases on $\left(p_i^G, P(k_j)\right)$.

(b) For each firm i,

$$p_i^L = \begin{cases} P(k_i) & \text{if } k_i \le 2x^m \\ p^m & \text{if } k_i > 2x^m \end{cases} \quad \text{and } \bar{L}_i = \begin{cases} P(k_i)k_i & \text{if } k_i \le 2x^m \\ 2p^m x^m & \text{if } k_i > 2x^m. \end{cases}$$

In addition, L_i strictly increases on $[0, p_i^L)$ and strictly decreases on $\left(p_i^L, P(0)\right)$.

(c) For firm 1,

$$p_1^E = \begin{cases} P(2k_1) & \text{if } k_1 \le x^m \\ p^m & \text{if } k_1 > x^m \end{cases} \quad \text{and } \bar{E}_1 = \begin{cases} P(2k_1)k_1 & \text{if } k_1 \le x^m \\ p^m x^m & \text{if } k_1 > x^m. \end{cases}$$

In addition, E_1 strictly increases on $\left[0, p_1^E\right)$ and strictly decreases $\left(p_1^L, P(0)\right)$.

Proof. See Appendix. □

Now we turn our attention to E_2 which can have two local (maybe global) maxima because E_2 is the upper envelope of two hump-shaped functions, G_2 on the left of $P(2k_1)$ — the intersection of G_2 and $pD(p)/2$ — and $pD(p)/2$ on the right of it. If G_2 peaks on the left of $P(2k_1)$ and $pD(p)/2$ peaks on the right of it, then E_2 has two local maxima.

Now we will find the maxima of E_2 depending on the capacities of the firms. As the previous paragraph suggests, this can be done by analyzing where the peaks of $G_2(p)$ and $pD(p)/2$ are positioned relative to $P(2k_1)$. The function G_2 peaks, by definition, at p_2^G which turns out to be on the left of $P(2k_1)$ only if $k_1 < x^c$ (see the proof of lemma 3). The function $pD(p)/2$ peaks at p^m which is located on the left of $P(2k_1)$ only if $k_1 < x^m$. As $x^m < x^c$, we consider 3 cases: (I) $k_1 < x^m$ (II) $k_1 \in [x^m, x^c]$ and (III) $k_1 > x^c$. In each case, we show the corresponding E_2 in figure 2.

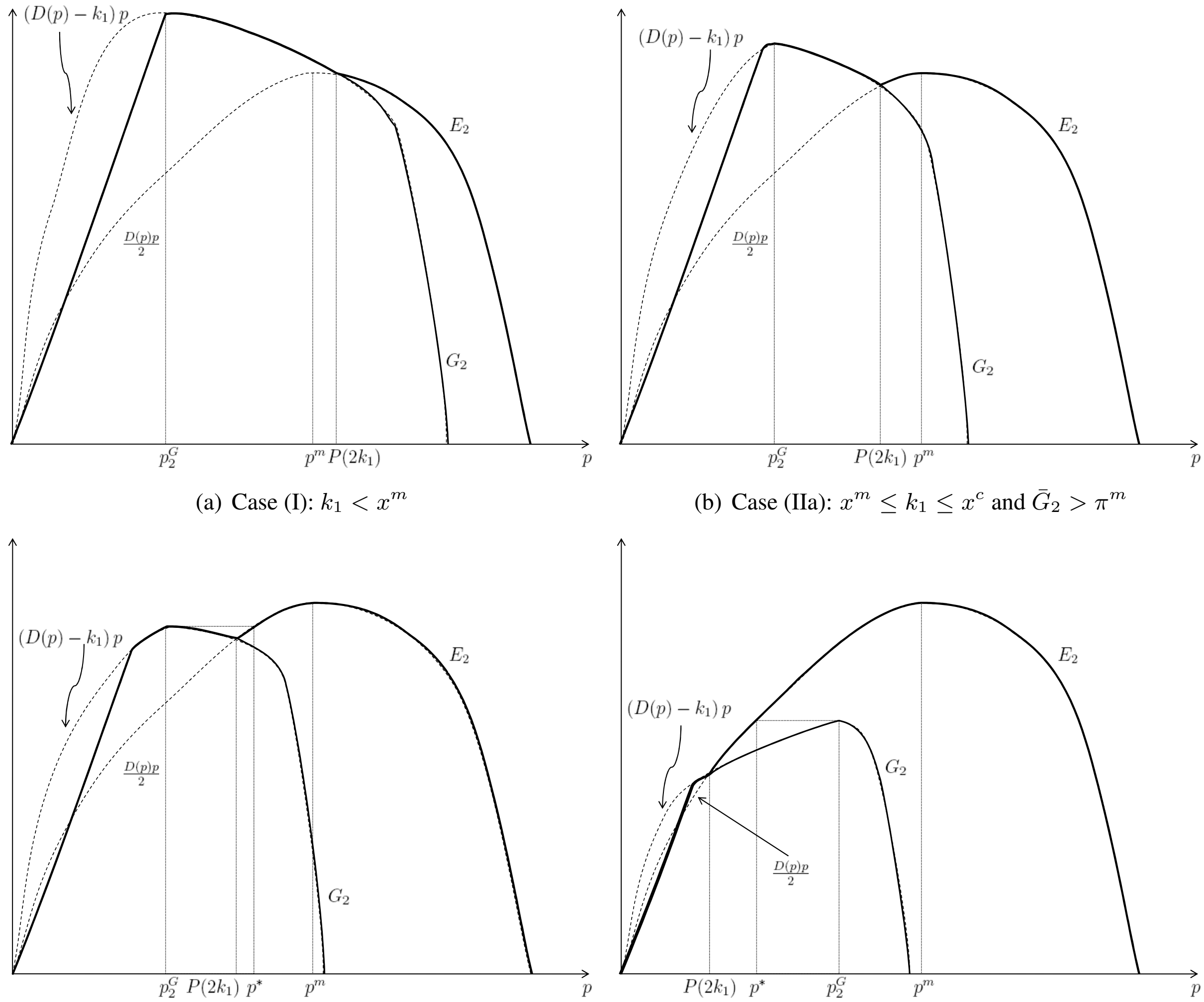
From figures 2a and 2d, one can see that the function E_2 is hump-shaped in cases (I) and (III) thanks to the fact that both G_2 and $D(p)p/2$ are peaked on one side of the intersection of the two functions. In case (II), E_2 is (usually) double-hump-shaped because G_2 and $D(p)p/2$ are peaked on different sides of the intersection of the two functions. In other words, in case (II), E_2 has two local maxima. The following lemma summarizes our findings on E_2.

Lemma 3. *For firm 2,*

$$\bar{E}_2 = \begin{cases} \bar{G}_2 & \text{if } k_1 < x^m \\ \max\left\{\bar{G}_2, \pi^m\right\} & \text{if } k_1 \in [x^m, x^c] \\ \pi^m & \text{if } k_1 > x^c \end{cases} \quad \text{and } p_2^E = \begin{cases} p_2^G & \text{if } \bar{E}_2 = \bar{G}_2 \\ p^m & \text{if } \bar{E}_2 = \pi^m \end{cases}$$

In addition, if $k_1 < x^m$ or if $k_1 > x^c$, then E_2 strictly increases on $\left[0, p_2^E\right)$ and strictly decreases on $\left(p_2^E, P(0)\right)$. If $k_1 \in [x^m, x^c]$, then E_2 strictly increases on the intervals $\left[0, p_2^G\right)$ and $[P(2k_1, p^m)$ and strictly decreases on the intervals $\left(p_2^G, P(2k_1)\right]$ and $[p^m, P(0))$.[12]

[12] Sometimes, some of these intervals are empty. For example, when $k_1 = k_2 = x^m$, $p_2^G = P(2k_1) = p^m$, and then the intervals $[P(2k_1, p^m)$ and $\left(p_2^G, P(2k_1)\right]$ are empty.



(a) Case (I): $k_1 < x^m$

(b) Case (IIa): $x^m \leq k_1 \leq x^c$ and $\bar{G}_2 > \pi^m$

(c) Case (IIb): $x^m \leq k_1 \leq x^c$ and $\bar{G}_2 \leq \pi^m$

(d) Case (III): $k_1 > x^c$

Figure 2: The Functions E_2 and G_2

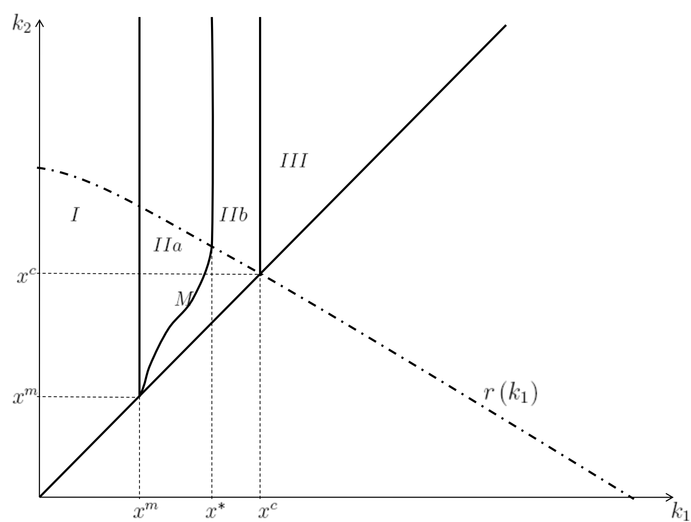


Figure 3: The function E_2 is maximized at p_2^G in regions I and IIa, and at p^m in regions IIb and III

Proof. See Appendix. □

In figure 3, we depict the capacity constrained games in which E_2 is maximized at p_2^G (or at p^m). This information turns out to be crucial in characterizing the equilibria as we will see later. We already know that E_2 is maximized at p_2^G in region I of figure 3 and at p^m in region III. Hence, we only concentrate on regions IIa and IIb which are separated by curve M. For the games on this curve, $\bar{G}_2 = \pi^m$ and $k_1 \in [x^m, x^c]$. Curve M starts at $k_2 = k_1 = x^m$ and always stays above the $45°$ line because on this line, $\bar{G}_2(k_1, k_2) = P(2k_1)k_1$ by lemma 3a (recall $k_1 \leq x^c$) and the function $P(2k_1)k_1$ reaches its peak value π^m at $k_1 = x^m$ and decreases when $k_1 > x^m$. Also, it is not hard to show that M continues northeast[13] starting $k_2 = k_1 = x^m$ until it intersects with $r(k_1)$ (at x^* in figure 3) at which point curve M turns into a vertical half line (recall that $\bar{G}_2(x^*, k_2) = \bar{G}_2(x^*, r(x^*))$ for all $k_2 > r(k_2)$ by lemma 2a). Finally, let us point out that $\bar{E}_2$ is maximized at p_2^G in the games above M (region IIa) and at p^m in the ones below M (region IIb).

In the next two lemmas, we narrow down the set of strategy profiles that can arise at equilibrium. Lemma 4 shows that the firms must offer the same effective price at any equilibrium. The argument for this result is simple: if the firms offer different effective prices, then the firm with the higher *announced* price must be not price matching. Consequently, the firm with the lower price must be earning what a monopolist (with a capacity constraint) would have earned; otherwise the firm with the lower price can profit by increasing its price slightly. But a monopolist's price is always high enough that its capacity can meet the corresponding market demand. As a result, the firm with the higher price earns 0 revenue because its price is too high that there is no excess demand beyond what the firm with the lower price can handle. Certainly, the firm with the higher price can earn a revenue higher than 0.

Lemma 4. *At equilibrium both firms must offer the same effective price, i.e.,* $p_1^e = p_2^e$.

Proof. Suppose $p_i^e < p_j^e$. This means $p_i < p_j$ and $o_j = 0$. Consequently, firm i nets $L_i(p_i)$ while firm j nets $G_j(p_j)$. It must be that $p_j > p_i^L$ because if $p_j \leq p_i^L$, firm i can increase its revenue by naming a price in the interval (p_i, p_j) as $L_i(\cdot)$ is a strictly increasing function on $\left[0, p_i^L\right)$ (lemma 2b). But for all $p \geq p_i^L$, $k_i \geq D(p)$, thus $k_i > D(p_j)$. Consequently, $G_j(p_j) = p_j \max\{0, D(p_j) - k_i\} = 0$. However, by price matching firm j nets a positive revenue, which is a contradiction. □

We use the terminology *market price* at an equilibrium to refer to the unique effective price the firms offer at this equilibrium. In addition, at any equilibrium, each firm i's revenue must be E_i at the market price corresponding to this equilibrium. Now let us determine whether there is any equilibrium in which one of the firms does not price match. If such an equilibrium exists, the market price corresponding to this equilibrium must be the announced price of the non price matching firm. But then the rival firm must have an incentive to undercut the market price as long as the rival firm does not sell its full capacity . In other words, a firm may not price match at an equilibrium only when the rival firm must sell its full capacity. But firms 1 and 2 sell their respective full capacities, only if the market price must not exceed $P(2k_1)$ and $P(k)$, respectively. Consequently, we find that if firm 1 does not price match, then the market price is $P(k)$ —

[13]One can show the continuity of M on the interval $k_1 \in [x^m, x^*]$ by using the implicit function theorem on the expression $\bar{G}_2(k_1, M(k_1)) = \pi^m$.

the lowest possible equilibrium price. On the other hand, if firm 2 does not price match, then market price cannot exceed $P(2k_1)$. We present this result in the following lemma.

Lemma 5. *If $o_1 = 0$ at an equilibrium, then the corresponding market price to this equilibrium must be $P(k)$. However, if $o_2 = 0$, then the market price cannot exceed $P(2k_1)$.*

Lemma 5 implies that by unilaterally deviating from an equilibrium, the revenue of firm 2 is either E_2 or G_2. Specifically, if firm 2 sets its effective price (weakly) below firm 1's price, then firm 2 nets E_2. This is because at equilibrium, firm 1 must either price match or set its price to $P(k)$, and in the latter case L_2 and E_2 coincide for all prices below $P(k)$ (figure 1). On the other hand, by setting its effective price above firm 1's, firm 2 nets G_2.

Finally, we are ready to characterize the equilibria. First let us consider the games in which $k_1 < x^m$ (region I in figure 3) or in which $k_1 \in [x^m, x^c]$ and $\bar{G}_2(k_1, k_2) > \pi^m$ (region IIa). In this case, the equilibrium market price must be p_2^G. Otherwise, by setting its effective price to $p_2^E = p_2^G$ (lemma 3), firm 2 nets $\bar{E}_2 = \bar{G}_2$, the maximal value of E_2. To support p_2^G as an equilibrium market price, consider the strategy profile in which both firms name p_2^G and price match. Observe that the market price corresponding to this strategy profile is p_2^G. To argue that this proposed strategy profile is an equilibrium, we only need to show that firm 1 cannot profit by deviating unilaterally. At the proposed strategy, firm 1 nets $E_1(p_2^G)$ and sells its full capacity because $p_2^G \leq P(2k_1)$ (recall $p_2^G \leq P(2k_1)$ whenever $k_1 \leq x^c$) Thus, there is no incentive to name a lower price. Naming a higher price also will not profit firm 1 because $E_1(p_2^G) \geq G_1(p_2^G)$ (see figure 1) and G_1 decreases for the prices higher than p_2^G (see the formal proof).

Next we consider the games in which $k_1 \in [x^m, x^c]$ and $\bar{G}_2(k_1, k_2) \leq \pi^m$ (region IIb) or in which $k_1 > x^c$ (region III). To determine the equilibria in these subgames, we need one more notation: p^* is the price level in $[P(2k_1), p^m]$ such that $E_2(p^*) = D(p^*)p^*/2 = \bar{G}_2$. The examples of p^* are shown in figures 2c and 2d. Sometimes we will write $p^*(k_1, k_2)$ because p^* depends on the capacities of the firms. First, note that $p^* < p_2^G$ only in the subgames with $k_1 > x^c$. Also, from figures 2c and 2d, one can see that on the interval $[0, \bar{p}]$ where $\bar{p} \in [p^*, p^m]$, E_2 is maximized at $\bar{p}$ and $E_2(\bar{p}) \geq \bar{G}_2$.

Now let us identify the set of equilibrium market prices in the games we started considering in the previous paragraph. The equilibrium market price cannot strictly exceed p^m: otherwise, firm 2 would bring down its effective price to p^m and net π^m, the highest E_2 (see figures 2c and 2d). In addition, the equilibrium market price cannot be strictly lower than p^* unless it is p_2^G (recall that $p_2^G < p^*$ only in region IIb). Otherwise, by setting its effective price to p_2^G, firm 2 can earn at least $\bar{G}_2$ which is superior to E_2 at any price lower than p^* (see figures 2c and 2d). As a result, we conclude that the equilibrium market price must be in the set $p_2^G \cup [p^*, p^m]$.

To show that any price in the interval $p_2^G \cup [p^*, p^m]$ can be an equilibrium market price, consider a strategy profile in which both firms name $\bar{p}$ in the set $[p^*, p^m]$ and price match. At this strategy profile, both firms earn $E_2(\bar{p}) = E_1(\bar{p})$. There is no incentive to deviate for firm 2: by increasing its effective price, firm 2 nets G_2 but unfortunately, even the highest value of G_2 cannot exceed firm 2's income $E_2(\bar{p})$ (see figures 2c and 2d). By decreasing its effective price, firm 2 nets E_2 but $E_2(\bar{p})$ is the maximal value of E_2 on the interval $[0, \bar{p}]$. Now we need to argue that firm 1 cannot profitably deviate. Because the proposed strategy profile is symmetric for the firms and yield the same revenue, whenever there is a profitable deviation for the small firm from the proposed strategy, the same deviation must be also profitable for the large firm. Therefore, the small firm cannot profitably deviate. Now we are left to show that $p = p_2^G \notin [p^*, p^m]$ is an equilibrium price. We know that $p_2^G < p^*$ only in games in which $k_1 \in [x^m, x^c]$ and $\bar{G}_2(k_1, k_2) \leq \pi^m$. For these games $p_2^G \leq P(2k_1)$, hence, using a similar logic used in the games in region IIa, we show that both firm naming p_2^G and price matching is an equilibrium. Consequently, the set of equilibrium market prices is $p_2^G \cup [p^*, p^m]$. We summarize these results in the following proposition.

Proposition 1.

(a) *If $k_1 < x^m$ (region I in figure 3) or if $k_1 \in [x^m, x^c]$ and $\bar{G}_2(k_1, k_2) > \pi^m$ (region IIa in figure 3), then the only equilibrium market price is p_2^G and firms 1 and 2 net $p_2^G k_1$ and $\bar{G}_2$, respectively.*

(b) *If $k_1 \in [x^m, x^c]$ and $\bar{G}_2(k_1, k_2) \leq \pi^m$ (region IIb), then the set of equilibrium market prices is $p_2^G \cup [p^*(k_1, k_2), p^m]$. If the equilibrium market price is p_2^G, then firms 1 and 2 net $p_2^G k_1$ and $\bar{G}_2$, respectively. If the market price is $p \in [p^*(k_1, k_2), p^m]$, then each firm nets $\frac{D(p)p}{2} \in \left[\bar{G}_2, \pi^m\right]$.*

(c) *If $k_1 > x^c$ (region III), then the set of equilibrium market prices is $[p^*(k_1, k_2), p^m]$. If the equilibrium market price is $p \in [p^*(k_1, k_2), p^m]$, then each firm nets $\frac{D(p)p}{2} \in \left[\bar{G}_2, \pi^m\right]$.*

Proof. See Appendix. □

Proposition 1 fully characterizes the equilibrium market prices in each capacity constrained game. Now we investigate the effects of price matching in capacity constrained games. For this we need the results found in the KS model as our model is an extension of the KS model that allows the firms to price match. The following proposition summarizes the results of KS in capacity constrained games.

Proposition 2. *(*Proposition 1 in KS*) Suppose that the firms do not have a price matching option. Then the equilibrium outcomes are given as follows:*

(a) *if $k_2 \leq r(k_1)$ (the region between the $45°$ line and the $r(k_1)$ curve in figure 3), then there is a unique pure equilibrium in which each firm i names $p_i^G = P(k)$ and nets $\bar{G}_i = P(k)k_i$.*

(b) *if $k_2 > r(k_1)$ (the region above the $45°$ line and the $r(k_1)$ curve), then there is no pure equilibrium. At any mixed equilibrium, the highest price in the support of equilibrium strategy is $p_2^G = P(k_1 + r(k_1))$. In addition, firm 2 nets $\bar{G}_2 = P(k_1 + r(k_1))r(k_1)$ while firm 1's equilibrium revenue does not exceed $\bar{G}_2$.*

With the help of propositions 1 and 2 we are ready to identify the effects of price matching in a given capacity constrained game. Once the firms have an option to price match, there are many new equilibria in certain games. Most interestingly, in the games with $k_2 > r(k_1)$, pure equilibria exists in our model but not in the KS model.[14] Let us explain why any of our equilibria is not an equilibrium in the KS model. In our setting, the equilibrium price always (strictly) exceeds $P(k)$ in the games with $k_2 > r(k_1)$. Consequently,

[14]In fact, the mixed equilibria in the KS model is no longer an equilibrium in our model because for those strategies, firm 1 must price match once it has a price matching option. This in turn alters firm 2's responses.

firm 2 does not sell its full capacity at any of our equilibria. In the KS setting, by underpricing, firm 2 improves as its market share increases significantly. Therefore, none of our equilibrium prices can be an equilibrium price in the KS model. In our model, when firm 1 price matches, by underpricing, firm 2 cannot steal the market share of firm 1. Thus, these strategy profiles can be supported as equilibria in our model.

Now let us turn our attention to how price matching affects the market price and the firms' revenues in different games. We summarize these effects in table 3, for which we use figure 3.

		(Region I or IIa)	(Region IIb)	(Region III)
	The Market Price	—	↗	
$k_2 \leq r(k_1)$	Firm 1's revenue	—	↗	n.a.
(on or below $r(k_1)$)	Firm 2's revenue	—	↗	
	The Market Price	↗	↗	ambiguous
$k_2 > r(k_1)$	Firm 1's revenue	↗	↗	↗
(above $r(k_1)$)	Firm 2's revenue	—	↗	↗

Table 1: The effect of price matching on the market price and the revenues of the firms.

We highlight the key results in table 3 below:

1. Price matching has absolutely no effect if $k_2 \leq r(k_1)$ and either $k_1 \leq x^m$ or $\bar{G}_2 > \pi^m$. In these games, there is undercapacity in the market, hence, no firm wants to "compete" with the other: each firm produces at capacity, and so cannot benefit from undercutting its rival. In addition, the market price (without price matching) is already high enough that no price higher than the market price can benefit both firms simultaneously. Therefore, price matching has no effect in these games.

2. Price matching (weakly) increases the market price if $k_1 \leq x^c$. This is because in these games the lowest equilibrium market price in our model is $p_2^G(k_1, k_2)$ but this is the highest equilibrium price in the KS model.

3. The effect of price matching on the market price is ambiguous if $k_1 > x^c$. This is because in these games the lowest equilibrium market price in our model is $p^*(k_1, k_2)$ but it is lower than $p_2^G(k_1, k_2)$ – the highest price in the KS model.

4. Price matching affects the firms positively. This effect on firm 2 can be seen easily because in the KS model firm 2 always nets $\bar{G}_2$ but it is the lowest equilibrium revenue of firm 2 in our model. To see the effect of price matching on firm 1, observe that at equilibrium, firm 1 either nets the same revenue as firm 2 or sells its full capacity at price p_2^G. In the former case, firm 1's revenue (weakly) exceeds $\bar{G}_2$, the highest revenue firm 1 can net in the KS model (proposition 2). In the latter case, firm 1 sells its full capacity for the highest possible equilibrium price in the KS model.

Our analysis of this section demonstrates that price matching is not anti-competitive if the capacities of the firms are low. However, as the firms' capacities increase, price matching becomes increasingly anti-competitive. Hence, capacity must be considered seriously when evaluating the effects of price matching in situations where capacity cannot be changed instantly.

In the next section, we analyze the full game, hence, answer the question of how price matching affects the market price if the firms choose their capacity simultaneously before pricing decisions.

4 Equilibria in the Full Game

First of all, let us fix some terminologies: to avoid confusions, the second period monopolistic price (quantity) refers to p^m (x^m), while the Cournot price (quantity) refers to p_c^c (x_c^c). The (k_1, k_2) subgame refers to the subgame in which firms 1 and 2 have capacities of k_1 and k_2, respectively.

Now we consider a class of strategy profiles that plays an important role in our subsequent analysis: firms 1 and 2 install capacities $k_1 \leq x^c$ and $k_2 > k_1$, respectively, and in each capacity constrained subgame, the equilibrium market price is the large firm's p_i^G (which is always possible thanks to proposition 1). Within this class of strategy profiles, let us look for an SPE capacity pair (k_1^*, k_2^*). If $k_2^* > r(k_1^*)$, firm 2 nets a constant revenue $\bar{G}_2 = P(k_1^* + r(k_1^*))r(k_1^*)$ (proposition 1 and lemma 2). But additional capacity is costly, and thus, $k_2^* \leq r(k_1^*)$. From proposition 1 and lemma 2, we know that when $k_1 \leq x^c$ and $k_2 \leq r(k_1)$, each firm i nets $p_2^G k_i = P(k_i + k_j)k_i$, but this revenue is the one firm i would have earned if there was no price matching available (see proposition 2). In other words, when $k_1 \leq x^c$ and $k_2 \leq r(k_1)$ we are in the KS world. From KS, we know that unless firm i's capacity is $r_c(k_j^*)$, then it has an incentive to deviate towards $r_c(k_j^*)$ (without leaving the region of subgames in which $k_1 \leq x^c$ and $k_2 \leq r(k_1)$). This means the only SPE within the class of strategy profiles we are considering is the one in which both firms install x_c^c ($< x^c$ by lemma 1c) and the market price is p_c^c. This has 2 important consequences:

1. The Cournot price p_c^c is always an SPE price because $p_1^G = p_2^G = p_c^c$ in the subgame in which both firms' capacity is x_c^c.
2. If price matching affects the market price, i.e., if there is an SPE price other than p_c^c, then both firms must install capacity which strictly exceeds x^m. To see this, observe that when one of the firms, say firm 1, installs a capacity less than x^m, the equilibrium market price always p_2^G. Now thanks to the arguments above, one obtains that if there exists any SPE in which firm 1 installs less than x^m, then at this SPE, the capacities of both firms must be x_c^c and the market price p_c^c.

Now let us consider the question of whether price matching could increase the market price above the Cournot price p_c^c. The answer hinges on whether the second period monopolistic quantity x^m exceeds the Cournot quantity x_c^c (or equivalently, on whether p^m is higher than p_c^c). The key insight for this is simple: we know that if price matching affects the market price, then both firms must install a capacity exceeding x^m. But for these subgames, the highest equilibrium market price is the second period monopolistic price p^m (see proposition 1) which exceeds the Cournot price only if $x^m < x_c^c$. This means that the condition $x^m < x_c^c$ is necessary for price matching to increase the market price. This condition also turns out to be sufficient for price matching to increase the market price beyond the Cournot price p_c^c.

Let us explain why the second period monopolistic price p^m is an SPE price if $x^m > x_c^c$. Earlier we argued that the Cournot price emerges as the market price at the SPE in which both firms install x_c^c and the market price is the large firm's p_i^G in each subgame. Now we modify this SPE profile only on its equilibrium path so that the market price is p^m (instead of p_c^c) in the (x_c^c, x_c^c) subgame. As a result, both firms' revenue increases from $p_c^c x_c^c$ to π^m on the equilibrium path, but stays the same off the equilibrium path. Now observe that the firms should not have any incentive to deviate from the modified strategy profile because they do not have any from the original one. Therefore, p^m must be an SPE. The following theorem extends this argument and shows that any price in $[p_c^c, p^m]$ is an SPE price.

Theorem 1. *There exists a pure SPE which results in a price greater than p_c^c if and only if $x^m < x_c^c$. In addition, if $x^m < x_c^c$, then any $p \in [p_c^c, p^m]$ is an SPE price.*

Proof. See Appendix. □

To prove the sufficiency part of the equilibrium, we show that each firm installing x_c^c and naming a price in the interval (p_c^c, p^m) is an SPE. Observe that at none of these SPEs, the firms do sell their capacity. We remark that there are other SPEs in which the market price is higher than p_c^c yet firms install capacities lower than x_c^c. However, as we will see later in proposition 4, firms must hold extra capacity if they want a market price higher than p_c^c. To see the intuition behind this, let us interpret the strategy profile in the proof of theorem 1 as follows: if both firms install x_c^c, then they cooperate and set the market price to $\hat{p} > p_c^c$. However, if one of the firms deviate, then the other firm retaliates by setting the market price to the lowest equilibrium price. But this price will not be low enough to deflect the other if the firms do not have any extra capacity. This is the reason why the firms carry extra capacity.

Theorem 1 demonstrates that price matching could increase the market price only if the Cournot quantity is less than the second period Monopolistic quantity. To understand this condition better, let us consider the following example:

Example 1. *Consider a market whose demand is linear, $P(x) = a - bx$. In addition, let $c(x) = cx$. Then $x^m \leqq x_c^c$ if and only if $c \leqq a/4$.*

In the above example, price matching potentially increases the market price only if the cost of installation is high. The main intuition behind this result is the following: to take advantage of price matching, the firms need to install a high capacity which is simply too expensive when the cost of installation is high. Hence, price matching does not increase the market price if the cost of installation is high.

So far we have investigated whether price matching can increase the market price. However, in order to fully understand the impact of price matching, we need to know whether price matching can decrease the market price. To answer this question, let us define the following property which plays a key role in our analysis.

Definition 1. *Property α is satisfied if there exists $\hat{x} \in (\max\{x_c^c, x^m\}, x^c)$ such that*

$$p_c^c x_c^c - c(\hat{x}) > P(\hat{x} + r_c(\hat{x}))\, r_c(\hat{x}) - c(r_c(\hat{x})). \tag{3}$$

To understand property α, suppose that the firms collude to build a capacity of $\hat{x}$ and charge the Cournot price. To enforce the collusion, if a firm deviates and installs a capacity other than $\hat{x}$, then in the resulting subgame, the firms name the large firm's p_i^G, which minimizes the deviator's equilibrium revenue. Hence, if the firms follow the agreement, then each firm's profit is given by the left hand side of inequality 3. If one deviates, then the maximal profit the deviating firm can earn is given by right hand side of inequality 3 thanks to our earlier discussion. Hence, property α is satisfied if there exists a capacity $\hat{x}$ for which the collusion described above is enforceable. In other words, when property α is satisfied, one can find an SPE which results in market price p_c^c and in which each firm installs a capacity $\hat{x} > x_c^c$. In fact, as the left hand side of inequality 3 strictly exceeds the right hand side, there should be an SPE price slightly below p_c^c.

Theorem 2. *If property α is satisfied, then there exists an SPE price $\hat{p} < p_c^c$.*

Proof. See appendix. □

Theorem 2 shows that whenever property α satisfied, there is always an SPE which results in a price lower than the Cournot price. We also remark that the firms carry an extra capacity in the strategy profile used in the proof of theorem 2. Again the firms hold extra capacity for the retaliation purposes.

Now we turn our attention to the question of when property α is satisfied. First we consider the $x_c^c > x^m$ case for which price matching could increase the market price. Then it turns out that property α is satisfied as shown in proposition 6. To prove the proposition, observe that the 2 sides of inequality 3 are the same when evaluated at x_c^c. However, when we move to the right of this quantity, the left hand side decreases at a slower rate than the right hand side. This means that there must be a capacity level $\hat{x}$ slightly higher than x_c^c for which inequality 3 is satisfied.

Lemma 6. *If $x_c^c > x^m$, then property α is satisfied.*

Proof. See Appendix □

A direct consequence of this lemma is that there are SPEs lower than the Cournot price if $x^m < x_c^c$. Therefore, when combined with theorem 1, this proposition shows that if $x^m < x_c^c$, then two types of SPEs exist: the ones that increase the market price and the ones that decrease the market price. Therefore, when the cost of installation is reasonably high, whether price matching increases the market price is completely dependent on which SPE is realized. This begs the following question: can SPEs be refined in a reasonable way to eliminate some of the SPEs. We will investigate this possibility in the next subsection.

Now let us turn our attention to the $x^m \geq x_c^c$ case, for which price matching does not increase the market price (theorem 1). Proposition 3 shows that there exists a case in which $x^m = x_c^c$ and property α is satisfied. To prove this, consider any setting in which $x^m = x_c^c$, $P'' < 0$ on $[0, \bar{x}]$ and $c'' > 0$. Then at x_c^c, not only do the two sides of inequality 3 coincide but their first order derivatives also coincide. Yet the second order derivative of the right hand side exceeds the one of the left hand side. However, for any $x^* > x_c^c$, the proof shows that one can perturb c'' so that (1) x_c^c and $r_c(x^*)$ are not affected and (2) the second derivative of the left hand side is 0 while the one of the right hand side is negative. When x^* is close enough to x_c^c, applying the Taylor theorem, one can show that there exists a quantity (1) which exceeds x^* and (2) at which inequality 3 is satisfied.

Proposition 3. *Property α is sometimes satisfied even if $x^m = x_c^c$.*

Proof. See Appendix. □

By modifying the proof of proposition 3, we can find an example satisfying property α when x^m is slightly greater than x_c^c. As a result, when $x^m \geq x_c^c$, there are SPEs that yield a market price lower than p_c^c. This and theorem 1 show that price matching could only decrease the market price if $x^m \geq x_c^c$. Therefore, if firms are starting its business from scratch, i.e., if firms install their capacity before pricing decisions, and if the cost of capacity installation is reasonably high, the belief that price matching has an anti-competitive effect seems misleading.

Finally, we now show that at any SPE that yields a price different than the Cournot price, at least one firm carries extra capacity. As we already hinted, extra capacity allows the firms to retaliate in the case that one of them deviates from the pre-coordinated capacity.

Proposition 4. *At any SPE resulting in a price different than the Cournot price, at least one firm carries extra capacity.*

Proof. Contrary to proposition 4, suppose there exists an SPE that results in a price different than the Cournot price and in which each firm sells its full capacity. Let k_1 and k_2 be the corresponding capacity that each firm builds at this SPE. Since both firms are selling their full capacity, the effective price in the (k_1, k_2) subgame must be $P(k_1 + k_2)$ and each firm must earn a profit of $P(k_1 + k_1)k_i - c(k_i)$. This means that if firm i has an incentive to deviate towards $r_c(k_j)$ unless $k_i = r_c(k_j)$. But if $k_i = r_c(k_j)$, then each firm's capacity is x_c^c, yielding the desired contradiction. □

4.1 Equilibrium Refinement

In this subsection, we investigate the question of whether the equilibrium refinements used in the standard Bertrand setting can help determine the effect of price matching on the market price.

In the standard Bertrand setting, when the price matching option is available to firms, the equilibrium market price ranges from the Bertrand to the monopolistic price. However, one can argue that the monopolistic price is most likely to arise in practice for several reasons: (1) the only strategy that survives the iterative elimination of weakly dominated strategies results in the monopolistic price (Doyle, 1988) (2) the only strong Nash[15] equilibrium results in the monopolistic price and (3) the equilibrium resulting in the monopolistic price Pareto dominates all other equilibria from the firms' perspective. According to these criteria, one can identify the equilibrium prices in each capacity constrained subgame. Specifically, one can show that when $k_1 \leq k_2$, the most likely equilibrium price is (i) p_2^G if $k_1 < x^m$ or $k_1 \in [x^m, x^c]$ and $\bar{G}_2 > \pi^m$ and (ii) p^m in all other subgames. This result lends support to the argument that price matching is an anti-competitive practice in capacity constrained games as long as the firms' capacities are sufficiently high.

Now we investigate whether there is any SPE in which the firms coordinate on a Pareto dominant equilibrium in each subgame. In other words, we assume that the firms are rational enough to realize that they will reach the best equilibrium for themselves once they fix their capacities. Hence, such an SPE is likely to arise

[15] A Nash equilibrium is a *strong Nash equilibrium* in which no coalition can cooperatively deviate in a way that benefits all of its members.

in practice if it exists. Unfortunately, when the effect of price matching is ambiguous, i.e., $x^m < x_c^c$, no SPE exists within the class of strategy profiles in which the firms coordinate on a Pareto dominant equilibrium in each subgame.

Proposition 5. *Within the class of strategy profiles in which the firms coordinate on a Pareto dominant equilibrium in each subgame, there is no SPE when $x^m < x_c^c$.*

Proof. See Appendix. □

Proposition 5 shows that no SPE exists if the refinement used to the Bertrand setting is extended to our setup. The refinements that lend support to the argument that price matching is anti-competitive in the standard Bertrand setting do not help determine the effects of price matching if the firms choose their capacities simultaneously before pricing decisions.

5 Conclusion

We have studied the effects of price matching in the setting of KS in which the firms install their capacities in the first period and set their prices in the second period.

We show that when the capacities of the firms are fixed, the effect of price matching varies with the capacities of the firms. If the total capacity is low, then price matching has no effect; if the total capacity is high, then price matching affects both firms positively. When the total capacity is in an intermediate range, price matching affects only the small firm.

If firms choose their capacities simultaneously before pricing decisions, then the effect of price matching is pro-competitive or ambiguous. We show that if the cost of installation is "low," then the availability of the price matching option to firms generates two types of SPEs: the ones that increase the market price and the ones that decrease the market price. Hence, whether price matching increases or decreases the market price depends on which SPE is reached at equilibrium. As a result, in an ex-ante stage, it is difficult to determine the effect of price matching on the market price when the cost of installation is "low." On the other hand, if the cost of installation is "high," then price matching does not increase the market price. In fact, in some settings, price matching can decrease the market price. Hence, if the cost of installation is "high," then price matching can only reduce the market price. As a result, in the setting of KS, price matching seems not to be as anti-competitive as feared.

References

Maria Arbatskaya, Morten Hviid, and Greg Shaffer. On the incidence and variety of low-price guarantees. *Journal of Law & Economics*, 47(1):307–32, 2004.

Terrence M. Belton. A model of duopoly and meeting or beating competition. *International Journal of Industrial Organization*, 5(4):399–417, 1987.

Kenneth S. Corts. On the robustness of the argument that price-matching is anti-competitive. *Economics Letters*, 47(3-4):417–421, 1995.

Christopher Doyle. Different selling strategies in bertrand oligopoly. *Economics Letters*, 28(4):387–390, 1988.

Subhasish Dugar and Todd Sorensen. Hassle costs, price-matching guarantees and price competition: An experiment. *Review of Industrial Organization*, 28(4):359–378, June 2006.

Aaron S. Edlin. Do guaranteed-low-price policies guarantee high prices, and can antitrust rise to the challenge? *Harvard Law Review*, 111(2), 1997.

Morten Hviid and Greg Shaffer. Hassle costs: The achilles' heel of price-matching guarantees. *Journal of Economics & Management Strategy*, 8(4):489–521, 1999.

Todd R. Kaplan. Effective price-matching: a comment. *International Journal of Industrial Organization*, 18(8):1291–1294, 2000.

David M. Kreps and Jose A. Scheinkman. Quantity precommitment and bertrand competition yield cournot outcomes. *Bell Journal of Economics*, 14(2):326–337, 1983.

Sridhar Moorthy and Ralph A. Winter. Price-matching guarantees. *RAND Journal of Economics*, 37(2): 449–465, 2006.

Martin J. Osborne and Carolyn Pitchik. Price competition in a capacity-constrained duopoly. *Journal of Economic Theory*, 38(2):238–260, April 1986.

I P L Png and D Hirshleifer. Price discrimination through offers to match price. *Journal of Business*, 60(3): 365–83, 1987.

Steven C. Salop. Practices that (credibly) facilitate oligoply coordination. In Joseph E. Stiglitzs and Frank G. Mathewson, editors, *New Developments in the Analysis of Market Structure*, pages 265–290. MIT Press, Cambridge, MA, 1986.

Appendix

Proof of Lemma 2. First observe that all p_i^G, p_i^E, and p_i^L must (weakly) exceed $P(k)$. This is because $G_i(p) = E_i(p) = L_i(p) = pk_i$ for all $p < P(k)$ and pk_i is a strictly increasing function.

(a). As $p_i^G \geq P(k)$, $\bar{G}_i = \max_{p \geq P(k)} p \max\{0, D(p) - k_j\}$ because $D(p) - k_j \geq k_i$ for all $p \geq P(k)$. If $k_j \geq D(0)$, then $\bar{G} = 0$, proving the lemma. If $k_j < D(0)$, then $\bar{G}_i = \max_{p \geq P(k)} (p \max\{0, D(p) - k_j\})$. It must be that $p_i^G \leq P(k_j)$. By setting $x = D(p) - k_j$, one obtains $\bar{G}_i = \max_{x \in [0,k_i]} P(x + k_j)x$. Because $P(x + k_j)x$ is concave and maximized at $r(k_j)$, we obtain the desired result:

$$\bar{G}_i(k_i,k_j) = \begin{cases} P(k)k_i & \text{if } k_i \le r(k_j) \\ P(r(k_j)+k_j)r(k_j) & \text{if } k_i > r(k_j). \end{cases}$$

To see that G_i is a hump shaped function, observe that $p(D(p)-k_j)$ is a strictly concave function maximized at $P(k_j + r(k_j))$. Also recall that G_i is a continuous function which is equal to pk_i on the interval $[0, P(k)]$ and to $p(D(p) - k_j)$ on $[P(k), P(k_j)]$. In addition, because the function pk_i is increasing and the function $p(D(p) - k_j)$ is hump-shaped, one obtains the desired result.

(b). Observe that $G_i(p) = L_i(p)$ if $k_j = 0$. In addition, $r(0) = 2x^m$ and $P(0 + r(0)) = p^m$. These results along with (a) yield (b).

(c). Observe that for $p \ge P(k)$, $D(p)-k_2 \le \frac{D(p)}{2}$ because $k_1 \le k_2$. Consequently, $E_1(p) = p\min\left\{k_i, \frac{D(p)}{2}\right\}$ for any $p \ge P(k)$. Set $d(p) = D(p)/2$ and observe that $\max pd(p) = p^m x^m$. With the new notation $E_1(p) = p\min\{k_i, d(p)\}$. Now using (b) one obtains the result in (c). □

Proof of Lemma 3. We know that

$$E_2(p) = \begin{cases} G_2(p) & \text{if } p \le P(2k_1) \\ \frac{pD(p)}{2} & \text{if } p \in [P(2k_1), P(0)]. \end{cases}$$

We will show that (i) $p_2^G(k_1,k_2)$, the maximizer of G_2 (lemma 2a), satisfies $p_2^G(k_1,k_2) \le P(2k_1)$ if and only if $k_1 \le x^c$ and (ii) The function $\frac{pD(p)}{2}$ is a hump-shaped function maximized on the left of $P(2k_1)$ if and only if $k_1 \leqq x^m$. Then (i) and (ii) prove lemma 3, because E_2 is continuous, G_2 and $\frac{pD(p)}{2}$ are hump-shaped, and $x^m < x^c$.

(i) Recall that $k_1 \leqq r(k_1)$ if and only if $k_1 \leqq x^c$. In addition, because $P(\cdot)$ is decreasing, we obtain $P(k_1 + r(k_1)) \leqq P(2k_1)$ if and only if $k_1 \leqq x^c$. Consequently, if $k_1 \leqq x^c$, then $P(k_1 + r(k_1)) \leqq P(2k_1)$ and in addition, $P(k) \le P(2k_1)$ as $k_2 \ge k_1$. These inequalities and that p_2^G is either $P(k)$ or $P(k_1 + r(k_1))$ (lemma 2a) yield that $p_2^G \le P(2k_1)$ if $k_1 \le x^c$. Now we are left to consider the $k_1 > x^c$ case. In this case, $P(k_1 + r(k_1)) > P(2k_1)$ and in addition, $p_2^G = P(k_1 + r(k_1))$ because $k_2 \ge k_1 > r(k_1)$ (lemma 2a). These imply $p_2^G > P(2k_1)$ if $k_1 > x^c$.

(ii) This is a consequence of assumption 1 and the definition of p^m.

□

Proof of Proposition 1. (a) First we show that the strategy profile in which both firms play $(p_2^G, 1)$ is an equilibrium. Recall that in this case, $p_2^E = p_2^G \le P(2k_1)$ and $\bar{E}_2 = \bar{G}_2$. Thus firm 2 nets $\bar{E}_2$. Because firm 1 is price matching, by setting its effective price to $p \ne p_2^G$, firm 2 nets either $E_2(p)$ or $G_2(p)$, but $G_2(p) \le E_2(p)$ (see figure 1) and $E_2(p) < \bar{E}_2$ (because E_2 has a unique maximizer at p_2^G). Consequently, firm 2 cannot profitably deviate. Firm 1, on the other hand, nets $E_1(p_2^G)$ at the proposed profile. If firm 1 sets its effective price to $p < p_2^G$, it earns $E_1(p)$, but $E_1(p) = pk_1$ because $p < p_2^G \le P(2k_1)$. Consequently, $E_1(p) < E_1(p_2^G)$, thus firm 1 does not deviate to $p < p_2^G$. If firm 1 sets its effective price to $p > p_2^G$, then it earns $G_1(p)$. Recall that G_1 is maximized at p_1^G and G_1 is a decreasing function on $(p_1^G, P(k_2))$ (lemma 2a). Using the definition of p_i^G and lemma 1b, $p_1^G \le p_2^G$ or equivalently, $p_2^G \in [p_1^G, P(k_2))$. Consequently, for all $p > p_2^G$, $G_1(p) < G_1(p_2^G)$ and in addition, as $G_1(p_2^G) \leq E_1(p_2^G)$, $p > p_2^G$ cannot be a profitable deviation for firm 1. Therefore, the strategy profile in which both firms play $(p_2^G, 1)$ is an equilibrium. To prove that p_2^G is the unique market price, recall that both firms offer the same effective price at equilibrium (lemma 4). If both firms set their effective price to $p \neq p_2^G$, then firm 2 nets $E_2(p)$, but as we argued earlier, $E_2(p) < \bar{E}_2$. By setting its effective price to p_2^G, firm 2 nets at least $\bar{E}_2$, thus the market price cannot be $p \neq p_2^G$. As a result, the market price is p_2^G which yields the remaining part of proposition 1a.

(b) First we show that the strategy profile in which both firms play $(p_2^G, 1)$ is an equilibrium. Recall that in this case, $p_2^G \leq P(2k_1)$ and $E_2(p_2^G) = \bar{G}_2$. Thus firm 2 nets $E_2(p_2^G) = \bar{G}_2$. If firm 2 sets its effective price to $p < p_2^G \leq P(2k_1)$, then firm 2 nets $E_2(p) = G_2(p)$. Hence, $E_2(p) < E_2(p_2^G)$. If firm 2 sets its effective price to $p > p_2^G$, it nets $G_2(p)$, but $G_2(p) < \bar{G}_2 = E_2(p_2^G)$. Consequently, firm 2 has no profitable deviation. Following the exact same steps as in (a), one can prove that firm 1 has no profitable deviation. Therefore, the strategy profile in which both firms play $(p_2^G, 1)$ is an equilibrium.

From the definition of p^* we know that $p^* \in [P(2k_1), p^m]$ and $E_2(p^*) = \bar{G}_2$. Fix any $\bar{p} \in [p^*, p^m]$. Then $E_2(\bar{p}) \geq E_2(p^*) = \bar{G}_2$ because E_2 is an increasing function on $[P(2k_1), p^m]$ (lemma 3) and $[p^*, \bar{p}] \subseteq [P(2k_1), p^m]$. Thanks to lemma 3, for all $p \in [0, P(2k_1)]$, $E_2(p) \leq E_2(p_2^G) = \bar{G}_2$ and for all $p \in [P(2k_1), \bar{p}]$, $E_2(p) \leq E_2(\bar{p})$. Combining the previous 3 inequalities, we obtain that $E_2(\bar{p}) \geq E_2(p)$ for all $p \leq \bar{p}$. Now we show that the strategy profile in which both firms play $(\bar{p}, 1)$ is an equilibrium. At the proposed strategy, both firms net $E_2(\bar{p}) = E_1(\bar{p})$. If firm 2 sets its effective price to $p < \bar{p}$, then firm 2 nets $E_2(p)$, but $E_2(p) < E_2(\bar{p})$ as we have already shown. If firm 2 sets its effective price to $p > \bar{p}$, it nets $G_2(p)$ but $G_2(p) \leq \bar{G}_2 \leq E_2(\bar{p})$. As a result firm 2 cannot profitably deviate from the proposed strategy. Observe that the firms play a symmetric strategy at the proposed equilibrium and net the same revenue. This and that firm 2 (the large firm) cannot profitably deviate from the proposed strategy imply that firm 1 (the small firm) cannot profitably deviate. Hence, the strategy profile in which both firms play $(\bar{p}, 1)$ is an equilibrium.

To prove that the market price is in the set $p_2^G \cup [p^*, p^m]$, recall that both firms offer the same effective price at equilibrium (lemma 4). If both firms set their effective price to $p > p^m$, then firm 2 nets $E_2(p)$, but $E_2(p) < \bar{E}_2 = \pi^m$ (lemma 4). By setting its effective price to p^m, firm 2 nets at least $\bar{E}_2$, thus the market price cannot be $p > p^m$. If the market price is p such that $p \neq p_2^G$ and $p < p^*$, then firm 2 nets $E_2(p)$, but $E_2(p) < \bar{G}_2 = E_2(p_2^G)$ (lemma 4). By setting its effective price to p_2^G, firm 2 nets at least $\bar{G}_2$, thus the market price cannot be $p \neq p_2^G$ and $p < p^*$). This shows that the market price must be in the set $p_2^G \cup [p^*, p^m]$ and this in turn, yields the remaining part of proposition 1b.

(c) By following the same steps used in the proof of part (b), one can prove part (c).

□

Proof of Theorem 1. Necessity. Suppose there exists an SPE resulting in a price higher than p_c^c when $x_c^c \leq x^m$. At this equilibrium, let firms 1 and 2 install capacities k_1 and k_2, respectively. Let firm i be the firm with $k_i = \min\{k_1, k_2\}$ and firm j be the one with $k_j = \max\{k_1, k_2\}$. There can be 2 possible cases: (1) $k_i \geq x^m$ and (2) $k_i < x^m$.

Case (1). In this case, the maximum equilibrium price in the (k_1, k_2) subgame is p^m (see proposition 1) but $p^m \leq p_c^c$, contradicting that the SPE price is higher than p_c^c.

Case (2). It suffices to show that $k_1 = k_2 = x_c^c$ because p_c^c is the unique equilibrium price in the (x_c^c, x_c^c) subgame thanks to proposition 1 (recall that $x_c^c \leq x^m$). As a result, we only need to show that $k_i = r_c(k_j)$ and $k_j = r_c(k_i)$.

In this case, $k_i < x^m$ and consequently, the equilibrium price is $p_j^G(k_i, k_j)$ (proposition 1). First, $k_j \leq r(k_i)$ because for all subgames with $k_j > r(k_i)$, firm j nets the constant revenue $\bar{G}_i^2 = P(k_i + r(k_i))r(k_i)$ and therefore, whenever $k_j > r(k_i)$, firm j can increase its profit by installing some capacity in the interval $(r(k_1), k_2)$ (since firm j saves its cost of capacity without losing any revenue). Because $k_j \leq r(k_i)$, $p_j^G(k_1, k_2) = P(k)$ (lemma 2) and the profits of firms i and j are $P(k)k_i - c(k_i)$ and $P(k)k_j - c(k_j)$, respectively. Recall that the function $P(k)k_j - c(k_j)$ is maximized at $r_c(k_i)$ (lemma 1). In addition, because $r_c(k_i) < r(k_i)$ (lemma 1)c), the equilibrium price in the $(k_i, r_c(k_i))$ subgame is $P(k) = P(k_i + r_c(k_i))$ and consequently, it must be $k_j = r_c(k_i)$ (otherwise; firm j will deviate to $r_c(k_i)$). Similarly, the function $P(k)k_i - c(k_i)$ is maximized at $r_c(k_j)$, but in order to to argue that whenever $k_i \neq r_c(k_j)$, firm i must deviate and install $r_c(k_j)$, we need to make sure that the equilibrium price is $P(k) = P(k_j + r_c(k_j))$ in the $(k_j, r_c(k_j))$ subgame. This is always true when $r_c(k_j) < x^m$ thanks to proposition 1. Recall that $k_i < x^m$ and $k_j = r_c(k_i)$. Consequently, $k_j > r_c(x^m)$ as $r_c(\cdot)$ is a decreasing function (lemma 1b). This and $r_c(\cdot)$ is a decreasing function yield $r_c(k_j) < r_c(r_c(x^m))$. We know that $x_c^c \leq x^m$, which means $x^m \geq r_c(x^m)$. Now using lemma 1d, we obtain that $r_c(r_c(x^m)) < x^m$. This in turn yields that $r_c(k_j) < x^m$, the result we seek.

Sufficiency. Pick any $\hat{p} \in (p_c^c, p^m]$. Consider the following strategy profile in the full game which results in $\hat{p}$:

1. In the first stage both firms choose the capacity of $\hat{x}_c^c$.

2. In each (k_1, k_2) capacity constrained subgame,
$$(p_1, o_1) = (p_2, o_2) = \begin{cases} (\hat{p}, 1) & \text{if } k_1 = k_2 = x_c^c \\ (p_j^G(k_1, k_2), 1) & \text{otherwise} \end{cases}$$
where j is the firm with $k_j \geq k_i$.

We need to show that the strategy profile above is indeed an SPE. In other words, we need to show that (a) the proposed strategy profile is an equilibrium in each subgame and (b) no firm has a profitable first stage deviation.

(a) Because $p_j^G(k_1, k_2)$ is an equilibrium price in each subgame, (a) is proved for all subgames except for the one where $k_1 = k_2 = x_c^c$. Because $x_c^c > x^m$ and $x_c^c < x^c$ (lemma 1c), in the $k_1 = k_2 = x_c^c$ subgame, the equilibrium price must be somewhere in $[p^*(x_c^c, x_c^c), p^m]$ (proposition 1). In addition, the proposed strategy profile yields $\hat{p}$ in the (x_c^c, x_c^c) subgame, and consequently, we need to show that $\hat{p} \in [p^*(x_c^c, x_c^c), p^m]$. Because $x_c^c < x^c$, it must be that $x_c^c < r(x_c^c)$. Hence, $p_2^G(x_c^c, x_c^c) = P(2x_c^c) = p_c^c$ and $\bar{G}_2 = p_c^c x_c^c$. Using the definition of $p^*(x_c^c, x_c^c)$, we obtain that $p^* = p_c^c$. Hence, $[p^*(x_c^c, x_c^c), p^m] = [p_c^c, p^m] \ni \hat{p}$. This completes the proof that the proposed strategy profile is an equilibrium in each subgame.

(b) Because the proposed strategy profile is symmetric, we only show that firm 2 has no profitable first stage deviation. First observe that at the proposed strategy, the firms earn $\frac{\hat{p}D(\hat{p})}{2} - c(x_c^c)$. Let us find the best first period unilateral deviation from firm 2 perspective. According to the proposed strategy, each deviation with $k_2 \geq r(x_c^c)$ brings a constant revenue $\bar{G}_i^2 = P(x_c^c + r(x_c^c))r(x_c^c)$ to firm 2 (lemma 2a) but the deviation $k_2 = r(x_c^c)$ requires the lowest cost of capacity among all the deviations with $k_2 \geq r(x_c^c)$. Hence, at the best deviation of firm 2, it must be that $k_2 \leq r(x_c^c)$. If firm 2 deviates to $k_2 \leq r(x_c^c)$, according to the proposed strategy, the resulting market price is $p_2^G = P(k) = P(x_c^c + k_2)$ and firm 2 earns a profit of $\bar{G}_2 - c(k_2) = k_2 P(x_c^c + k_2) - c(k_2)$. By definition, $k_2 P(x_c^c + k_2) - c(k_2)$ is maximized at $k_2 = r_c(x_c^c)$ and because $r_c(x_c^c) < r(x_c^c)$ (lemma 1c), the most profitable deviation for firm 2 is $r_c(x_c^c)$. At this deviation, firm 2's profit is $P(x_c^c + r_c(x_c^c))r_c(x_c^c) - c(r_c(x_c^c))$ but this does not exceed $\frac{\hat{p}D(\hat{p})}{2} - c(x_c^c)$ — firm 2's profit if it does not deviate. To see this observe that by definition, $P(x_c^c + r_c(x_c^c))r_c(x_c^c) = \frac{\hat{p}D(\hat{p})}{2}$ and in addition, $c\left(r_c(x_c^c)\right) \geq c(x_c^c)$ because $x_c^c \leq x_c^c$ and $r_c(x) \geq x$ for all $x \leq x_c^c$. This completes the proof.

□

Proof of Theorem 2. Fix $\hat{x} \in (\max\{x_c^c, x^m\}, x^c)$ for which

$$p_c^c x_c^c - c\left(\hat{x}\right) > P\left(\hat{x} + r_c(\hat{x})\right) r_c(\hat{x}) - c\left(r_c(\hat{x})\right).$$

One can write $p_c^c x_c^c$ as $\frac{p_c^c D(p_c^c)}{2}$. When $x_c^c \geq x^m$, choose $\hat{p} \in (P(2\hat{x}), p_c^c)$ such that

$$\frac{\hat{p}D(\hat{p})}{2} - c\left(\hat{x}\right) > P\left(\hat{x} + r_c(\hat{x})\right) r_c(\hat{x}) - c\left(r_c(\hat{x})\right). \tag{4}$$

This is feasible because the function $\frac{pD(p)}{2}$ is continuous. When $x_c^c < x^m$, choose $\hat{p} \in (P(2\hat{x}), p^m)$ so that inequality 4 is satisfied. This is feasible because $\frac{pD(p)}{2}$ is a hump shaped function maximized at p^m. Observe that in any case, $\hat{p}$ must satisfy the following conditions (in addition to inequality 4): $\hat{p} < p_c^c$, $\hat{p} \in (P(2\hat{x}), p^m)$, and $\hat{x} > \frac{D(\hat{p})}{2}$.

Consider the following strategy profile in the full game that results in $\hat{p}$:

1. In the first stage both firms install a capacity of $\hat{x}$.

2. In each (k_1, k_2) capacity constrained subgame,
   $$(p_1, o_1) = (p_2, o_2) = \begin{cases} (\hat{p}, 1) & \text{if } k_1 = k_2 = \hat{x} \\ (p_j^G(k_1, k_2), 1) & \text{otherwise} \end{cases}$$
   where j is the firm with $k_j \geq k_i$

We need to show that the strategy profile above is an SPE. In other words, we need to show that (a) the proposed strategy profile is an equilibrium in each subgame and (b) no firm has a profitable first stage deviation.

(a) Because $p_j^G(k_1, k_2)$ is an equilibrium price in each subgame, (a) is proved for all subgames except for the one where $k_1 = k_2 = \hat{x}$. Let us fix the $(\hat{x}, \hat{x})$ subgame. Observe that in this subgame, the the proposed strategy yields $\hat{p}$. As $\hat{x} < x^c$, thanks to lemma 2a, $p_2^G = P(2\hat{x})$ in the $(\hat{x}, \hat{x})$ subgame, $p_2^G = P(k) = P(2\hat{x})$ and $\bar{G}_2 = P(2\hat{x})\hat{x}$. To complete the proof we need to show that $\hat{p} \in [p^* = P(2x_1), p^m]$ (proposition 1), but this is true as $\hat{p}$ is chosen to satisfy this.
(b) Because the proposed strategy profile is symmetric, we only show that firm 2 has no profitable first stage deviation. First observe that at the proposed strategy, the firms earn $\frac{\hat{p}D(\hat{p})}{2} - c(\hat{x})$. Now following the same steps in the proof of the sufficiency part of theorem 1, by deviating, the maximal profit firm 2 can earn is $P(\hat{x} + r_c(\hat{x}))r_c(\hat{x}) - c(r_c(\hat{x}))$. But this never exceeds the profit firm 2 earns at the proposed strategy profile (inequality 4).

□

Proof of Lemma 6. As $x_c^c > x^m$, we need to find $\hat{x} \in (x_c^c, x^c)$ satisfying inequality 3. Let $g(x) = p_c^c x_c^c - c(x)$ and $f(x) = P(x + r_c(x))r_c(x) - c\left(r_c(x)\right)$. This means that we need to find $\hat{x} \in (x_c^c, x^c)$ such that $g(\hat{x}) - f(\hat{x}) > 0$. This we will accomplish by showing that there must exist a small enough $\Delta > 0$, such that $g(x_c^c + \Delta) - f(x_c^c + \Delta) > 0$. By the Taylor theorem,

$$g(x_c^c + \Delta) - f(x_c^c + \Delta) = g(x_c^c) - f(x_c^c) + \left(g'(x_c^c) - f\prime(x_c^c)\right)\Delta + o_2.$$

Because $r_c(x_c^c) = x_c^c$,

$$g(x_c^c) = p_c^c x_c^c - c(x_c^c) = P(x + r_c(x))r_c(x_c^c) - c(r_c(x_c^c)) = f(x_c^c).$$

Consequently, we only need to show $g'(x_c^c) > f'(x_c^c)$.

Clearly, $g'(x_c^c) = -c'\left(x_c^c\right)$. Applying the envelope theorem on $f(x)$ and substituting $r_c(x_c^c) = x_c^c$, we obtain that $f'(x_c^c) = P'(2x_c^c)x_c^c$. By the definition of $r_c(x)$, $P(x_c^c) + P'(2x_c^c)x_c^c - c'\left(x_c^c\right) = 0$. Accordingly,

$$f'(x_c^c) = P'(2x_c^c)x_c^c + 0 = P'(2x_c^c)x_c^c + P(x_c^c) + P'(2x_c^c)x_c^c - c'\left(x_c^c\right) = P(2x_c^c) + 2P'(2x_c^c)2x_c^c - c'\left(x_c^c\right).$$

To show that $g'(x_c^c) > f'(x_c^c)$, it suffices to show that $P(2x_c^c) + 2P'(2x_c^c)2x_c^c < 0$. But this is true because $x_c^c > x^m$ implies that

$$\left.\frac{\partial P(2x)x}{\partial x}\right|_{x = x_c^c} = 2P'(2x_c^c)x_c^c + P(2x_c^c) < 0.$$

□

Proof of Proposition 3. Suppose $P(x)$ is a concave and decreasing function. We now show that if $c(x)$ is strictly convex and satisfies assumption 2, then $c(x)$ can be perturbed so that the resulting example satisfies property α and assumption 2.

Let $f(x) = P(x + r_c(x))r_c(x) - c\left(r_c(x)\right)$ and $g(x) = p_c^c x_c^c - c(x)$. Clearly, $f(x_c^c) = g(x_c^c)$. Now consider $f'(x_c^c)$ and $g'(x_c^c)$. We know that $f'(x) = P'(x + r_c(x))r_c(x)$ and $g'(x) = -c'(x)$. Hence, $f'\left(x_c^c\right) = P'(2x_c^c)x_c^c$ and $g'\left(x_c^c\right) = -c'\left(x_c^c\right)$. Since $x_c^c = x^m$, $2P'(2x_c^c)x_c^c + P(2x_c^c) = 0$. Therefore,

$g'(x_c^c) = 2P'(x_c^c)x_c^c + P(2x_c^c) - c'(x_c^c)$. Because $x_c^c = r_c(x_c^c)$, the definition of $r_c(\cdot)$ yields $P'(2x_c^c)x_c^c + P(2x_c^c) - c'(x_c^c) = 0$. Hence, $g'(x_c^c) = P'(2x_c^c)x_c^c$. As a result, $f'(x_c^c) = g'(x_c^c)$.

On the other hand, $g''(x) = -c''(x)$ and $f''(x) = P''(x+r_c(x))(1+r_c'(x))r_c(x) + P'(r_c(x)+x)r_c'(x)$. Then $r_c'(x) = -\frac{P''(x+r_c(x))r_c(x)+P'(x+r_c(x))}{P''(x+r_c(x))r_c(x)+P'(x+r_c(x))-c''(r_c(x))}$ thanks to the implicit function theorem. Substituting $r_c'(x)$, $f''(x) = -\frac{P''(x+r_c(x))r_c(x)c''(r_c(x))+(P'(x+r_c(x)))^2}{P''(x+r_c(x))r_c(x)+P'(x+r_c(x))-c''(r_c(x))}$. Observe that when $c''(r_c(x))$ is sufficiently large, then $f''(x) < 0$.

Since $f(x_c^c) = g(x_c^c)$ and $f'(x_c^c) = g'(x_c^c)$, for any $\delta > 0$, pick $x^* > x_c^c$, such that $|f(x^*) - g(x^*)| < \delta$ and $|f'(x^*) - g'(x^*)| < \delta$. Consider $f(x^*+\Delta) - g(x^*+\Delta)$. By the Taylor theorem $f(x^*+\Delta) - g(x^*+\Delta) = f(x^*) - g(x^*) + (f'(x^*) - g'(x^*))\Delta + \frac{1}{2}(f''(x^*) - g''(x^*))\Delta^2 + o_3$.

Claim. Suppose $P(x)$ is a strictly concave and decreasing function. If $c(x)$ is strictly convex and satisfies assumption 2, then $c''(\cdot)$ can be modified as follows:

1. For any given $x^* > x_c^c$, both $r_c(x_c^c)$ and $r_c(x^*)$ stay unchanged.

2. $c''(\cdot)$ satisfies assumption 2 and $c''(r_c(x^*))$ is large enough and $c''(x^*) = 0$ so that $f''(x^*)) < 0$ but $g''(x^*) = 0$.

For now assume the claim is true. Then we can modify the cost function without upsetting x_c^c and $r_c(x^*)$ and so that $f''(x^*)) < 0$ and $g''(x^*)) = 0$. Hence, $f(x^* + \Delta) - g(x^* + \Delta) = f(x^*) - g(x^*) + (f'(x^*) - g'(x^*))\Delta + \frac{1}{2}f''(x^*)\Delta^2 + o_3$. When $\delta \to 0$, $f(x^* + \Delta) - g(x^* + \Delta) \to \frac{1}{2}f''(x^*)\Delta^2 < 0$. This proves the proposition.

Proof of the Claim. Pick any x that satisfies $P'(x+r_c(x))r_c(x) + P(x+r_c(x)) - c'(r_c(x)) = 0$. This implies that as long as $c'(r_c(x))$ is unchanged, the cost function can be modified in any way without upsetting $r_c(x)$. Therefore, whatever change is considered, $c'(x_c^c/2)$ and $c'(x^*)$ must be unchanged. Thanks to the fundamental theorem of calculus, $c'(r_c(x^*)) = c'(0) + \int_0^{r_c(x^*)} c''(t)dt$ and $c'(x_c^c) = c'(r_c(x^*)) + \int_{r_c(x^*)}^{x_c^c} c''(t)dt$. Since $c'' > 0$, to keep $c'(r_c(x^*))$ unchanged, $\int_0^{r_c(x*)} c''(t)dt$ needs to stay unchanged which is easily achieved regardless of how big $c''(r_c(x^*))$ is made. Similarly, to keep $c'(x_c^c))$ unchanged, only $\int_{r_c(x*)}^{x_c^c} c''(t)dt$ needs to stay unchanged which is again easily achieved regardless of how big $c''(r_c(x^*))$ is made. Also, observe that any change to $c''(\cdot)$ when $x > x_c^c$ will not even affect $c''(r_c(x^*))$ and $c'(x_c^c))$. Hence, modifying $c''(\cdot)$ so that $c''(x^*) = 0$ without affecting $r_c(x^*)$ or x_c^c can be achieved easily. This concludes the proof of the claim. □

Proof of Proposition 5. Suppose otherwise. Since in the second period, the firms choose the Pareto optimal price among the equilibrium prices, in any (k_i, k_j) subgame, the following conditions are satisfied:

(a) If $k_j \geq k_i \geq x^m$ and $\bar{G}_j(k_1, k_2) \leq \pi^m$, then the equilibrium market price is p^m

(b) In all other cases, the equilibrium effective price is $p_j^G(k_1, k_2)$ where j is the firm with $k_j \geq k_i$.

Let $\hat{x}_1$ and $\hat{x}_2$ be the corresponding SPE capacities for firm 1 and 2. Assume that $\hat{x}_1 \leq \hat{x}_2$ without loss of generality. We consider the following 2 cases:

1. $\hat{x}_1 \geq x^m$ and $\bar{G}_2(\hat{x}_1, \hat{x}_2) \leq \pi^m$

2. $\hat{x}_1 < x^m$ or $\hat{x}_1 \in [x^m, x^c]$ and $\bar{G}_2(\hat{x}_1, \hat{x}_2) > \pi^m$

1. First suppose $\hat{x}_2 > x^m$. Then $\hat{x}_1 > x^m$; otherwise, $\bar{G}_2$ always exceeds the monopolistic revenue. Hence, if firm 2 decreases its capacity slightly, then the market price will still be p^m, hence, its revenue stays the same. However, its cost of capacity decreases, hence firm 2 has a profitable deviation which is a contradiction. If $\hat{x}_1 = \hat{x}_2 = x^m$, then each firm sells its full capacity at a price of p^m. Then each firm has an incentive to deviate to $r_c(x^m)$ because the price in the $(x^m, r_c(x^m))$ subgame is $P(x^m + r_c(x^m))$ and $P(x^m + r_c(x^m))\, r_c(x^m) - c(r_c(x^m)) > P(2x^m)\, x^m - c(x^m)$ by the definition of $r_c(\cdot)$.

2. In this case, $\hat{x}_1 < x^c$ (otherwise, it must be $\bar{G}_2(\hat{x}_1, \hat{x}_2) \leq \pi^m$). Now using the proof of theorem 1 proof (sufficiency part), we obtain that $\hat{x}_2 = r_c(\hat{x}_1)$. In addition, because $\bar{G}_2(\hat{x}_1, r_c(\hat{x}_1)) = P(\hat{x}_1 + r_c(\hat{x}_1))r_c(\hat{x}_1) > \pi^m$ and $x^m < x_c^c$, one can show that $\hat{x}_1 < x_c^c$ and $\hat{x}_2 = r_c(\hat{x}_1) > x_c^c$ (see figure **??**). Because $\hat{x}_1 < x_c^c$, by lemma 1d, $r_c(\hat{x}_2) = r_c(r_c(\hat{x}_1)) > x_1$. This means that firm 1 has an incentive to increase slightly its capacity as the market price is still $P(k)$ in the resulting subgame.

□

Economics Working Paper

2010-19: Torben M. Andersen and Marias H. Gestsson: Longevity, Growth and Intergenerational Equity - The Deterministic Case

2011-01: Torben M. Andersen: Welfare State - The Scandinavian Model

2011-02: Torben M. Andersen: Collective risk sharing: The social safety net and employment

2011-03: Rune Vejlin: Optimal Unemployment Insurance: How Important is the Demand Side?

2011-04: Mikkel Nørlem Hermansen: Non-existence of Steady State Equilibrium in the Neoclassical Growth Model with a Longevity Trend

2011-05: John Kennes, Daniel Monte and Norovsambuu Tumennasan: The Daycare Assignment Problem

2011-06: Rene Saran and Norovsambuu Tumennasan: Whose Opinion Counts?

2011-07: Mark Strøm Kristoffersen: Liquidity Constraints and Fiscal Stabilization Policy

2011-08: Martin Paldam: The cycle of development in Africa. A story about the power of economic ideas

2011-09: Thorvardur Tjörvi Ólafsson, Ásgerdur Pétursdóttir and Karen Á. Vignisdóttir: Price setting in turbulent times. Survey evidence from Icelandic firms

2011-10: Torben M. Andersen: Social policies and activation in the Scandinavian welfare model: the case of Denmark

2011-11: Norovsambuu Tumennasan: To Err is Human: Implementation in Quantal Response Equilibria

2011-12: Norovsambuu Tumennasan: Do Contracts Help? A Team Formation Perspective

2011-13: Norovsambuu Tumennasan: Quantity Precommitment and Price Matching