Abstract

Why is it that a firm facing no financing costs today might deviate from a financially unconstrained firm’s investment policy? In a simple two-period model, we derive an investment threshold that is U-shaped in the firm’s cash holdings. We show analytically the relevant trade-offs leading to the U-shape: the firm balances financing costs for present and future investment, respectively. Our main argument is that financing costs today are more important than the risk of future financing costs. Therefore, the minimum point of the U-shape separates regions of cash holdings where investment today leads or does not lead to financing costs. The novel empirically testable implication is that sensitivities of investment to cash holdings are positive for low-cash firms with financing costs today, while these sensitivities are negative for cash-rich firms facing only the risk of future financing costs.

JEL Classification: G31

Keywords: investment timing, cash holdings, financing constraints
1. Introduction

What is the effect of financing constraints on the timing of corporate investment? And why is it that apparently unconstrained firms do not invest like truly unconstrained firms? More precisely, why can the investment of a cash-rich firm exhibit a negative sensitivity to its cash holdings?

In their seminal paper, Boyle and Guthrie (2003) extend the real-option framework of McDonald and Siegel (1986) by introducing financing constraints. They show that the risk of being constrained in the future might cause a firm to invest early. Thus, it might forgo some of the value of waiting from which an unconstrained firm would benefit.

Boyle and Guthrie (2003) model a firm facing only a financing capacity constraint, but no financing costs. In their model, there is endogenous acceleration of investment as cash becomes scarcer. However, the policy switch towards more delay occurs when the capacity constraint becomes binding, rather than being the result of an endogenous decision.

This issue is addressed by Hirth and Uhrig-Homburg (2010). They extend the framework of Boyle and Guthrie (2003) by introducing financing costs. Consequently, they show that the trade-off between present and future financing costs results in an endogenous U-shape of the investment threshold in cash holdings: cash-rich firms accelerate investment in constraints to avoid future costs. In contrast, low-cash firms delay investment in constraints to avoid present costs.

While the policy switch is endogenous in Hirth and Uhrig-Homburg (2010), it remains unclear how firms on either side of the switch point differ from each other. The results of Boyle and Guthrie (2003) and Hirth and Uhrig-Homburg (2010) are derived in an infinite-horizon, continuous-time framework. This precludes an analytical solution.

Therefore, a number of questions remain open: Do the mentioned results hold in general? For example, can they alternatively be obtained in a simple two-period model? What is the relevant trade-off leading to the U-shape? What are the characteristics of firms that exhibit acceleration and delay of investment in constraints, respectively? How is the switch point between these two fundamentally different policies determined?

In our paper, we aim to answer these questions by extending the two-period framework introduced by Lyandres (2007). His model is only defined for firms that face financing costs for investment today. For these firms, investment thresholds are always decreasing in cash holdings. This is in
contrast to the results derived in Boyle and Guthrie (2003) and Hirth and Uhrig-Homburg (2010). So it raises the question whether the non-monotonic effect of financing constraints on investment timing can only be found for a longer, maybe infinite, time horizon and disappears in a two-period framework. Therefore, we see the model by Lyandres (2007) as an ideal starting point to investigate whether the difference in modeling affects the results. We show that the discrepancy between the modeling worlds is resolved once we extend the definition space of the Lyandres (2007) model. More precisely, we additionally examine firms that have enough internal funds for investment today, but face the risk of financing costs in the future. This allows us to derive an investment policy very similar to that predicted by the models of Boyle and Guthrie (2003) and Hirth and Uhrig-Homburg (2010), i.e., a U-shape of investment thresholds in cash holdings.

Comparing our approach with the latter two models, we can analyze the resulting investment behavior more thoroughly, and we are able to explain the trade-offs at work. In particular, we identify financing costs for present vs. future investment as important factors for the investment timing decision. We find that the switch point is determined by whether a firm faces financing costs today. If so, then these costs are more important than the possible future costs, other things being equal. In this case, investment thresholds are decreasing in cash holdings. In contrast, if a firm has enough cash holdings to allow investment today without requiring external funds, then investment thresholds are increasing in cash holdings.

Linking investment timing to investment volume, our findings might be seen as an explanation why cash-rich firms have negative sensitivities of investment to cash holdings and cash flow, while the corresponding sensitivities of low-cash firms are positive. In this sense, the dynamic view on investment decisions that we put forward provides an alternative explanation for the puzzling empirical finding of negative investment-cash flow sensitivities, as discovered in recent empirical work by Cleary et al. (2007) and Hovakimian (2009).\footnote{Note that earlier studies such as Kaplan and Zingales (1997) have already pointed out the fact that investment-cash flow sensitivities do not have to be increasing in financing constraints. However, they still take for granted that these sensitivities should be positive, i.e., in the first order, more cash leads to more investment. Only very recently, also the idea of sensitivities possibly being negative has entered the discussion.} Complementing our emphasis on the dynamic properties of investment, Almeida et al. (2009) show how future financing constraints
can influence today’s investment policy. Still, consensus has not yet been reached about how to explain the empirical evidence of negative sensitivities. With our paper, we aim to contribute to a better understanding of this phenomenon.

Our paper is organized as follows: Section 2 presents the model and derives expressions for the expected values of investing now and postponing investment, respectively. Moreover, it introduces different regions of cash holdings with significantly different investment behavior. The resulting investment threshold is presented in Section 3. We state its properties analytically and discuss them. Then, we use a numerical example for further illustration. Section 4 highlights the empirical implications of our model and relates them to the existing literature. Section 5 concludes.

2. Model

Now we introduce a model to illustrate the trade-off between investment today and postponing the investment decision. We will show how the trade-off depends on characteristics of the firm, most importantly its level of initial cash holdings.

2.1. Setup

We consider a firm in a world with risk-neutral decision makers and a zero risk-free interest rate. There are three dates $t = 0, 1, 2$. In $t = 0$, the firm has initial cash holdings $C$, and there will be a cash flow $X$ from assets in place in $t = 1$. $X$ is uniformly distributed on $[x - \delta, x + \delta]$, where $x - \delta < 0$. Besides yielding the cash flow $X$, the assets in place do not have any remaining value.

Moreover, the firm owns an investment opportunity that can be pursued once, either in $t = 0$ or in $t = 1$. Investment requires an expense of $I$, and it gives a payoff $I + g_0$ in $t = 1$ if invested in $t = 0$, and $I + g_1$ if invested in $t = 1$, respectively.\footnote{Lyandres (2007) defines $\Delta g$ such that $g_0 = g_1 - \Delta g$ and expresses the model in terms of $g_1$ and $\Delta g$. We use the original notation of Lyandres (2005), i.e., $g_0$ and $g_1$, which simplifies some of the expressions and interpretations.}

Whenever the current cash holdings are not sufficient to fund investment,
the firm has to raise external financing, which we model as equity issuance.\textsuperscript{3} There is a market friction due to asymmetric information: given a true firm value $V$, the external investors assume a value of $\frac{V}{1+\alpha}$ with $\alpha \geq 0$. As a motivation, imagine that the firm under consideration is a “good” firm that is pooled with “bad” firms and therefore suffers a discount in the external investors’ valuation. For $\alpha = 0$, we are back in a frictionless world, and there are no additional costs for external financing.

The purpose of our model is to answer the following question: for a given $g_1$, what is the critical $g_0^*$ to make the firm indifferent between investment in $t = 0$ and in $t = 1$, i.e., to make its expected firm values equal for either decision?

In a frictionless world with zero risk-free interest rate, the firm is indifferent between investment and waiting for $g_0^* = g_1$. For $g_0 \neq g_1$, it prefers the investment with the higher payoff regardless of the timing. With market frictions, the critical $g_0^*$ can deviate from $g_1$. We interpret $g_0^* < g_1$ as early investment: the firm requires a lower return in $t = 0$ than in $t = 1$ to reach the same expected firm value. For $g_0^* < g_0 < g_1$, the firm is better off investing in $t = 0$, although the investment payoff is lower than when investing in $t = 1$. Conversely, $g_0^* > g_1$ can be interpreted as investment delay: the firm requires a higher return in $t = 0$ than in $t = 1$. For $g_1 < g_0 < g_0^*$, the firm postpones investment to $t = 1$, although it could get a higher payoff by investing in $t = 0$.

In the following, we derive the expected equity values for investing in $t = 0$ and postponing investment to $t = 1$, respectively. We first present the case $C < I$: when investing in $t = 0$, the firm has to raise a positive amount $I - C$ from external investors and faces external financing costs. This case corresponds to Lyandres (2007).\textsuperscript{4} Afterwards, we extend the analysis to the case $C \geq I$, i.e., there are enough liquid funds in the firm to finance investment today.

\textsuperscript{3}Lyandres (2007, A.2) shows that in the present modeling framework, the value for the old shareholders and thus the investment decision would be the same if external financing was raised by debt issuance.

\textsuperscript{4}The analysis is more extensive in his working paper version, Lyandres (2005). But exactly as in the published version, the case $C \geq I$ is not considered there.
2.2. Financing costs given investment today

2.2.1. Investment today

For investment in \( t = 0 \) and insufficient cash holdings \( (C < I) \), the financing amount \( I - C \) is raised as new equity. The resulting asset value in \( t = 1 \) is \( X + I + g_0 \). In the case of a negative value in \( t = 1 \) (\( X + I + g_0 < 0 \)), the old and new equity holders decide jointly to abandon the firm. To simplify the analysis, we restrict the parameter values such that there are some abandonment states, i.e., at least for the lowest possible cash flow \( X = x - \delta \) we have \( x - \delta + I + g_0 < 0 \).\(^5\) The firm value from a \( t = 0 \) perspective can thus be expressed as:

\[
V_I = \mathbb{E}[\max\{X + I + g_0, 0\}] = \int_{-I-g_0}^{x+\delta} \frac{X + I + g_0}{2\delta} dX.
\]

In return for their contribution \( I - C \), the new equity holders receive a share of the firm. Due to the market friction, the new equity holders assume a firm value of \( \frac{V_I}{1+\alpha} \) instead of the true value \( V_I \). Thus, the share demanded by the new equity holders is \( w_I = \frac{(1+\alpha)(I-C)}{V_I} \). Conversely, the value retained by the old equity holders in \( t = 0 \) is

\[
E^o_I = (1-w_I)V_I = V_I-(1+\alpha)(I-C) = \int_{-I-g_0}^{x+\delta} \frac{X + I + g_0}{2\delta} dX -(1+\alpha)(I-C).
\]

The superscript \( o \) refers to the \( C < I \) case. To simplify the analysis, we assume that even for purely external financing \( (C = 0) \), the investment in \( t = 0 \) has a positive net present value, i.e.,

\[
I + g_0 - (1+\alpha)I > 0.
\]

Therefore, it is never optimal to liquidate voluntarily in \( t = 0 \) without awaiting the realization of \( X \).

2.2.2. Postpone investment decision

If the investment decision is postponed to \( t = 1 \), we have to distinguish two cases in \( t = 1 \), dependent on the realization of \( X \):

\(^5\)Otherwise, the lower integral bound would be \( \max\{-I - g_0, x - \delta\} \).
a) $X < I - C$, i.e., external financing needed in $t = 1$.

To invest an amount of $I$, the new equity holders have to contribute $I - C - X$. The asset value in $t = 2$ and the firm value in $t = 1$ after investment are $V_W = I + g_1$ each. Again, the external investors assume only a firm value of $\frac{V_W}{1+\alpha}$ instead of the true value $V_W$ due to the market friction. Thus, the share demanded by the new equity holders is $w_W = \frac{(1+\alpha)(I - C - X)}{V_W}$. The original owners will abandon the firm instead of issuing new equity, if the new equity holders demand more than 100% of the firm, i.e., if

$$w_W = \frac{(1+\alpha)(I - C - X)}{V_W} > 1 \iff (1+\alpha)(I - C - X) > I + g_1 \iff X < I - C - \frac{I + g_1}{1+\alpha}.$$

To simplify the analysis, we restrict again the parameter values such that there are some abandonment states. Here this means that even for $C > I$, we require the following in case of the lowest possible cash flow $X = x - \delta$:

$$x - \delta < -\frac{I + g_1}{1+\alpha}. \quad (2)$$

Otherwise, for $w_W \leq 1$, investment takes place. The value for the old equity holders in $t = 1$ is

$$\max\{(1 - w_W)V_W, 0\} = \max\{V_W - (1+\alpha)(I - C - X), 0\} = \max\{I + g_1 - (1 + \alpha)(I - C - X), 0\}.$$

Consequently, the value component for the old equity holders from a $t = 0$ perspective, conditional on $X < I - C$, is

$$\int_{I-C-\frac{I+g_1}{1+\alpha}}^{I-C} \frac{I + g_1 - (1 + \alpha)(I - C - X)}{2\delta} dX. \quad (3)$$

Lyandres (2007, p. 964) requires even $x - \delta < -(I + g_1)$, which cannot be justified as required to ensure a chance of abandonment. One could even introduce a tighter restriction than ours, given that $C$ is a known value.

The $\max\{\cdot\}$ function in the following expression also captures the cases in which $w_W > 1$ and thus abandonment takes place.
b) $X \geq I - C$, i.e., no external financing needed in $t = 1$.

Now the old equity holders can remain the sole owners of the firm. The asset value in $t = 2$ and thus the firm value in $t = 1$ are

$$I + g_1 - (I - C - X) = C + X + g_1.$$  

From a $t = 0$ perspective, the firm value component conditional on $X \geq I - C$ is

$$\int_{I-C}^{x+\delta} C + X + g_1 \frac{2\delta}{dX}. \tag{4}$$  

For the whole range of possible realizations of $X$, we add the two components (3) and (4) and get as the total value for the old equity holders in $t = 0$:

$$E_{t=0}^{WO} = \mathbb{E}[\max\{I + g_1 - (I - C - X) - \alpha \cdot \max\{I - C - X, 0\}, 0\}] \tag{5}$$  

$$= \int_{I-C}^{I} \frac{I + g_1 - (1 + \alpha)(I - C - X)}{2\delta} dX + \int_{I-C}^{x+\delta} C + X + g_1 dX.$$  

As for investment in $t = 0$, we restrict the parameter values to simplify the analysis such that it is never optimal to liquidate voluntarily in $t = 1$ and retain $C + X$ without investment.

### 2.3. No financing costs given investment today

#### 2.3.1. Investment today

For investment in $t = 0$ and sufficient cash holdings ($C \geq I$), no external financing is needed. Thus, the old equity holders can remain the sole owners of the firm. Their firm value is

$$E_I = \mathbb{E}[\max\{I + g_0 + X - (I - C), 0\}]$$  

$$= \mathbb{E}[\max\{X + C + g_0, 0\}] = \int_{\max\{-C-g_0,x-\delta\}}^{x+\delta} \frac{X + C + g_0}{2\delta} dX. \tag{6}$$  

Comparing with (1), we notice that a max\{$\cdot$\} function has entered the lower integral bound. For the initial level of cash holdings $C$ being high enough, there is no more possible abandonment. Then the integration in (6) begins at $x - \delta$ and runs over the whole range of possible $X$ values.
2.3.2. Postpone investment decision

If the investment decision is postponed to \( t = 1 \), there is a risk of financing costs and possible abandonment of the firm. The outcome depends on the initial cash holdings relative to the realization of the (possibly negative) cash flow \( X \). Whenever there is a positive financing gap \( I - C - X > 0 \), there are financing costs. Likewise, whenever there would be a negative value for the old equity holders, i.e., the new equity holders would demand more than 100% of the firm, the firm is abandoned. Therefore, the value for the old equity holders from a \( t = 0 \) perspective is very similar to (5), namely

\[
E_W = \mathbb{E}[^{\text{max}\{I + g_1 - (I - C - X) - \alpha \cdot \text{max}\{I - C - X, 0\}, 0\}}] \\
= \int_{\text{max}\{I - C - \frac{I + g_1}{1 + \alpha} - x - \delta\}}^{\text{max}\{I - C, x - \delta\}} \frac{I + g_1 - (1 + \alpha)(I - C - X)}{2\delta} \, dX \\
+ \int_{\text{max}\{I - C, x - \delta\}}^{x + \delta} \frac{C + X + g_1}{2\delta} \, dX. \tag{7}
\]

However, there are additional \( \text{max}\{\cdot\} \) functions in the integral bounds, reflecting whether there is risk for financing costs and possible abandonment, respectively. For sufficiently large values of \( C \), the integrals reduce to simpler expressions. These different cases are explained in the next section.

2.4. Different regions of initial cash holdings

First of all, we define as Region \( o \) the case that \( C < I \), i.e., there are financing costs for investment in \( t = 0 \). This case has been introduced in Section 2.2.

Then, for the case \( C \geq I \), there are no financing costs in \( t = 0 \). For this case, introduced in Section 2.3, we identify four different regions, labeled I, II, III, and IV, respectively, in order of increasing initial cash holdings:

1. \( I \leq C < -g_0 - (x - \delta) \)

In Region I, there is still the risk that after investment in \( t = 0 \) (without financing costs), the cash flow realization \( X \) in \( t = 1 \) is low enough to trigger abandonment of the firm. More precisely, the firm is abandoned if \( x - \delta \leq X < -C - g_0 \). Therefore, (6) becomes

\[
E_I = \int_{-C - g_0}^{x + \delta} \frac{X + C + g_0}{2\delta} \, dX.
\]
Similarly, there is possible abandonment if investment is postponed to \( t = 1 \). Both in Regions I and II, we have that \( I - C - \frac{I + g_1}{1 + \alpha} > x - \delta \). Therefore, the lower bound of the first integral in (7) becomes \( I - C - \frac{I + g_1}{1 + \alpha} \):

\[
E_W^I = \int_{I - C - \frac{I + g_1}{1 + \alpha}}^{I - C} \frac{I + g_1 - (1 + \alpha)(I - C - X)}{2\delta} dX \\
+ \int_{I - C}^{x + \delta} \frac{C + X + g_1}{2\delta} dX.
\] (8)

In all Regions I, II, and III, we have that \( I - C > x - \delta \), which explains that the upper bound of the first integral and the lower bound of the second integral in (7) become \( I - C \).

II: \( -g_0 - (x - \delta) \leq C < I - \frac{I + g_1}{1 + \alpha} - (x - \delta) \)
For \( C \) being above the level of \( -g_0 - (x - \delta) \), there is a positive value remaining even in case of the lowest possible cash flow \( x - \delta \). Therefore there are neither financing costs nor possible abandonment, given that investment takes place in \( t = 0 \). That means that (6) becomes

\[
E_I^{II} = x + C + g_0.
\]

Still, there is possible abandonment in case investment is postponed to \( t = 1 \), and (7) is the same as in Region I:

\[
E_W^{II} = E_W^I.
\]

III: \( I - \frac{I + g_1}{1 + \alpha} - (x - \delta) \leq C < I - (x - \delta) \)
For even higher values of \( C \), there is no possible abandonment if investment is postponed to \( t = 1 \). Formally, the first integral in (7) starts from \( x - \delta \), while the upper bound of the first and the lower bound of the second integral are still \( I - C \):

\[
E_W^{III} = \int_{x - \delta}^{I - C} \frac{I + g_1 - (1 + \alpha)(I - C - X)}{2\delta} dX + \int_{I - C}^{x + \delta} \frac{C + X + g_1}{2\delta} dX.
\] (9)

Here we impose an additional restriction on the parameters, namely that

\[
g_1 < g_0(1 + \alpha) + \alpha I \iff \delta - x - g_0 < \delta - x + \frac{\alpha I - g_1}{1 + \alpha}.
\]
This ensures that Region III actually starts at a higher C level than Region II.
The value for immediate investment remains the same as in Region II:

\[ E_{III} = E_{II} \]

IV: \( I - (x - \delta) \leq C \)

In the highest region of C values, there are neither financing costs nor possible abandonment, both for investment in \( t = 0 \) and when investment is postponed to \( t = 1 \).

Formally, the first integral in (7) then disappears, and the second integral in (7) runs over the whole range of possible \( X \) values. The resulting firm value is

\[ E_{IV} = C + x + g_1. \]

Still, the value for immediate investment remains the same as in Region II:

\[ E_{IV} = E_{II} \]

The different regions of initial cash holdings C and their corresponding properties are summarized in Table 1.

[Table 1]

3. Results

3.1. Investment threshold for \( t = 0 \)

The investment threshold for \( t = 0 \) is defined as the critical \( g_0^* \) for a given \( g_1 \), which makes the firm indifferent between investment in \( t = 0 \) and in \( t = 1 \). At the threshold, the expected firm values are thus equal for either decision.

For \( C < I \) (Region o), the investment threshold is obtained by equating (1) and (5), i.e., \( E_I = E_W \), and then solving for \( g_0 \). For \( C \geq I \), we equate (6) and (7), i.e., \( E_I = E_W \), and solve again for \( g_0 \).

The notation with \( \overset{\dagger}{=} \) indicates what is to be shown. The solutions for \( g_0^* \) are given in Appendix A.
3.2. Slope and curvature of investment threshold

The following two propositions state the slope and curvature of the investment threshold for the different regions of initial cash holdings.

**Proposition 1.** In Region $o$, the investment threshold $g_0^*$ is decreasing in the initial level of cash holdings $C$, given that $C + x - \delta - 2\alpha\delta + g_1 < 0$. In Regions I, II, and III, the investment threshold $g_0^*$ is increasing in the initial level of cash holdings $C$.
Proof: See Appendix B.

**Proposition 2.** In Region $o$, the investment threshold $g_0^*$ is moderately convex for most parameter values. In terms of $\delta$, it is convex unless $\delta$ is close to zero or very large. In Regions I and III, the investment threshold $g_0^*$ is concave in the initial level of cash holdings $C$. In Region II, the investment threshold is convex in $C$.
Proof: See Appendix C.

In Region IV, the firm is unconstrained, so $g_0^* = g_1$ is constant in the initial level of cash holdings $C$, and thus there is no curvature.

3.3. Discussion

Now we discuss the relevant trade-offs leading to the proposed U-shape and curvature of the investment threshold. In general, the U-shape results from the varying importance of financing costs and possible abandonment for investment in $t = 0$ and $t = 1$, respectively. It is explained in the direction from high to low cash holdings, which allows the most intuitive discussion.

Region IV is the easiest case: the firm is unconstrained, as there are neither financing costs nor possible abandonment, both for investment in $t = 0$ and when investment is postponed to $t = 1$. Therefore, $g_0^* = g_1$ is constant; in both periods, the firm requires the same net present value for investment.

In contrast, when cash holdings are low enough to enter Region III, then $E_W$ is influenced by financing costs, while $E_I$ is not. More precisely, a decrease in cash holdings $C$ by one unit means that the integration area is moved by one unit from the second integral to the first integral in (9). At the same time, the amount of financing costs in the first integral is also negatively related and linear in $C$. Together, this leads to a quadratic decrease of $E_W$, while $E_I$ is still linear in $C$. This explains the concavity of the investment curve in Region III.
When moving from Region III into Region II, an additional effect comes into play: now postponing investment to \( t = 1 \) introduces possible abandonment. Still, the value of immediate investment is the same as in Region III. A first guess could be that early investment becomes even more favorable, so the negative relation between investment and \( C \) should become more pronounced. However, compare the definitions of \( E_W \) in Regions III and II, (9) and (8), respectively. The only difference is the lower bound of the first integral. Given that the equity holders have decided to postpone the investment, the option to abandon the firm is actually valuable to them. Assuming for a moment that (9) did still hold, then the equity holders would have to cover the negative firm value in the low \( X \) states. Therefore, (9) would yield a lower value. This difference is sufficient to explain that \( E_W \) is convex in Region II. Hence, the investment curve is also convex in Region III (\( E_I \) is still linear in \( C \)).

In Region I, we see that the possibility of abandonment is present no matter whether investment happens immediately or is postponed.

Finally, in Region 0, the firm faces financing costs even for investment in \( t = 0 \). These costs become more severe as \( C \) approaches zero. Therefore, waiting is increasingly encouraged. By postponing investment to \( t = 1 \), the firm now has the advantage that it can avoid financing costs due to a favorable cash flow realization in \( t = 1 \). In this sense, Region 0 illustrates our main argument that financing costs today are more important than the risk of future financing costs.

3.4. Numerical example

We illustrate the results of the model by a numerical example. The parameter values are given in Table 2. To ensure comparability, the values for \( I, g_1, x, \) and \( \delta \) are the same as in Lyandres (2005), whereas he considers \( \alpha \) varying between 0 and 1, possibly correlated with \( C \).

Fig. 1 shows the U-shape of the optimal investment threshold, as theoretically derived earlier in this section. It further illustrates the importance of the different regions of initial cash holdings: they have a significant influence on the slope and convexity of the curve, which is evident from the different trade-offs at work in the regions. Especially, the difference between regions with negative and positive slopes becomes apparent: In Region 0 with financing costs for immediate investment, the investment threshold shows the
traditional behavior, i.e., it is decreasing in the level of cash holdings. In contrast, the opposite holds for all the other regions.

4. Empirical implications and relation to existing evidence

4.1. Empirical implications

Our results lead to the following three empirical implications: first, low-cash firms facing financing costs today are more reluctant to invest if they have less cash. This might correspond to positive sensitivities of investment to cash holdings and cash flow. Second, cash-rich firms facing no financing costs today invest in less favorable projects (i.e., forgo their real option to wait) if they have less cash. This might correspond to negative sensitivities of investment to cash holdings and cash flow. Third, the policy switch between the first and the second implication is the result of an endogenous decision. It is driven by whether there are financing costs today.

The first implication follows the “conventional view” that investment is decreasing in constraints. So the noteworthy part is that it only holds for firms facing financing costs today. The second implication emphasizes the dynamic view of financing constraints. In static now-or-never investment models, for example Almeida and Campello (2007), Cleary et al. (2007), and Flor and Hirth (2010), a firm that has enough cash to allow first-best investment today is defined as unconstrained. In contrast to these studies, and similar to Boyle and Guthrie (2003) and Hirth and Uhrig-Homburg (2010), we emphasize that such a firm might forgo some value of waiting due to future constraints. In this sense, investing more than an unconstrained firm today can also be a sign of being constrained. In the third implication, we point out the difference from the two related investment-timing models: in Boyle and Guthrie (2003), the policy switch occurs when a financing capacity constraint becomes binding, i.e., when the firm is forced to postpone the investment. In Hirth and Uhrig-Homburg (2010) and our model, however, all investment behavior is the result of endogenous trade-off decisions. Unlike Hirth and Uhrig-Homburg (2010), we put up a very strong prediction about the properties of the switch point and thus the distinction between the two policies: we predict a one-to-one relation between financing costs today and positive sensitivity on the one hand, and no financing costs today and negative sensitivity on the other hand.
4.2. Relation to existing evidence

In the following, we present the existing empirical evidence and discuss whether it supports our implications. In cases where it does not, we give possible reasons for the discrepancies.

As pointed out earlier, the model of Lyandres (2007) examines only what we call “Region o”. Thus, he predicts in his working paper version exactly what we state for this region, namely that investment is always increasing in cash holdings. Also, the empirical results in Lyandres (2005) support this finding. However, he provides only a test for the whole sample of firms. From our extension of the model, we would expect that once the firms are grouped according to their cash holdings, there should be a negative coefficient in the regression of investment on cash holdings for the group of firms with the highest level of cash holdings.

The empirical evidence provided by Hovakimian (2009) supports our predictions to some extent: firms with negative investment cash-flow sensitivities exhibit a significantly higher level of internal liquidity, but a significantly lower cash flow than firms with positive sensitivities. Internal liquidity is a stock measure and highly related to the cash holdings in our model. Thus, the evidence supports our predictions of negative sensitivities for cash-rich firms and positive sensitivities for low-cash firms. Still, it is puzzling that the relation is the opposite if cash flow is considered instead of internal liquidity. Moreover, Hovakimian (2009) finds that cash-flow insensitive firms have the lowest amount of internal liquidity, whereas our model predicts that the insensitive firms should be those with the highest cash holdings. A possible explanation, also presented by Hovakimian (2009, p. 165) and others, is that high levels of internal liquidity can be interpreted in two opposite ways: either, as we argue, firms are not constrained and can follow the first-best investment policy, being insensitive to cash-flow shocks; or, firms have limited access to external capital, and therefore holding high cash reserves is actually a sign of being constrained.

Cleary et al. (2007) and Flor and Hirth (2010) propose that investment is U-shaped in liquid funds. This can be seen as the opposite to the U-shape of investment thresholds proposed in our work. On the empirical side,

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8Internal liquidity, or financial slack, is defined in Hovakimian (2009, p. 165) as the sum of cash and marketable securities, 0.7 times accounts receivable, 0.5 times inventories, less the accounts payable, divided by net fixed assets.
Cleary et al. (2007) confirm their theoretically derived U-shape. So they provide support for non-monotonicity in general, but not for our specific shape of the investment threshold. The discrepancy to our predictions can be explained by the different aspects that are the center of attention: the model setup in Cleary et al. (2007) and Flor and Hirth (2010) is a static now-or-never investment framework. However, they emphasize the nature of the firm’s financing costs, which they derive endogenously. In contrast, our model, as well as Hirth and Uhrig-Homburg (2010), considers financing costs as an exogenous friction. Our main focus is on the effect of investment timing flexibility and its dependence on financing constraints. We conclude that a test of our model needs to account for the emphasis on investment timing.

In between the static and dynamic modeling approaches, Almeida et al. (2009) derive empirical implications from a model in which there is an interaction between future financing constraints and today’s investment policy, as in our framework. However, the difference is that they do not consider the timing of one specific project that can be launched in different periods. Rather, they make predictions about the characteristics of different investment projects at specific points in time.

Whited (2006) provides a theoretical model and also a corresponding empirical approach for directly testing investment timing. She focuses on lumpy investment for constrained firms and the timing of large investment projects, rather than incremental investment. This is related to the timing of a fixed investment expense, as in our model, as well as in Lyandres (2007), Boyle and Guthrie (2003), and Hirth and Uhrig-Homburg (2010). Both her model and the empirical part suggest that there is always more delay between the lumpy investment events for constrained firms than for unconstrained firms. In this sense, her results are similar to our predictions: we propose the “conventional view” of a reluctance to invest that is increasing in constraints for firms with few liquid funds. On the other hand, we propose accelerated investment in constraints for the less constrained firms with substantial liquid funds.

5. Conclusion

In this paper, we analyze the investment timing of a financially constrained firm in a two-period setup. As a main result, we show that the optimal investment threshold is U-shaped in liquid funds.
We provide the following explanation for the U-shape: there is a trade-off between financing costs for present and future investment, respectively. We identify different regimes that are determined by the firm’s level of cash holdings. In each regime, a different subset of the mentioned influence factors is in force, which leads to the distinct shape of the investment curve.

Our work has important empirical implications: we predict that low-cash firms facing financing costs today are more reluctant to invest if they have less cash. In contrast, cash-rich firms facing no financing costs today invest in less favorable projects if they have less cash. These two policies might correspond to positive and negative sensitivities of investment to cash holdings and cash flow for low-cash firms and cash-rich firms, respectively. We point out that the switch between the two policies is the result of an endogenous decision. It is driven by whether there are financing costs today. If so, we argue that, other things being equal, financing costs today are more important than the risk of future financing costs.

Many researchers have analyzed investment-cash flow sensitivities empirically. It has been widely accepted that these sensitivities might be decreasing in constraints. However, only a few recent studies show that investment-cash flow sensitivities might even be negative. The predictions of our model with respect to positive and negative sensitivities complement the existing empirical evidence. Still, it remains an open empirical question which types of firms and environments actually exhibit negative investment-cash flow sensitivities.

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Table 1: Regions of initial cash holdings.
The table summarizes the different regions of initial cash holdings $C$, and it shows for each region whether each of the following properties applies: FC0 (Financing costs for investment in $t = 0$), FC1 (Risk of financing costs if investment is postponed to $t = 1$), A0 (Possible abandonment for investment in $t = 0$), and A1 (Possible abandonment if investment is postponed to $t = 1$). The letters indicate whether the properties apply (Y) or do not apply (N).

<table>
<thead>
<tr>
<th>Region</th>
<th>Bounds for initial cash holdings $C$</th>
<th>FC0</th>
<th>A0</th>
<th>A1</th>
<th>FC1</th>
</tr>
</thead>
<tbody>
<tr>
<td>$o$</td>
<td>$C &lt; I$; Lyandres (2007)</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
</tr>
<tr>
<td>I</td>
<td>$I &lt; C &lt; -g_0 - (x - \delta)$</td>
<td>N</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
</tr>
<tr>
<td>II</td>
<td>$-g_0 - (x - \delta) \leq C &lt; I - \frac{I+g_1}{1+\alpha} - (x - \delta)$</td>
<td>N</td>
<td>N</td>
<td>Y</td>
<td>Y</td>
</tr>
<tr>
<td>III</td>
<td>$I - \frac{I+g_1}{1+\alpha} - (x - \delta) \leq C &lt; I - (x - \delta)$</td>
<td>N</td>
<td>N</td>
<td>N</td>
<td>Y</td>
</tr>
<tr>
<td>IV</td>
<td>$I - (x - \delta) \leq C$</td>
<td>N</td>
<td>N</td>
<td>N</td>
<td>N</td>
</tr>
</tbody>
</table>

Table 2: Parameter values used in numerical example.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Project investment cost</td>
<td>$I = 10$</td>
</tr>
<tr>
<td>Net present value of $t = 1$ investment</td>
<td>$g_1 = 60$</td>
</tr>
<tr>
<td>Expected $t = 1$ cash flow</td>
<td>$x = 5$</td>
</tr>
<tr>
<td>Cash-flow volatility</td>
<td>$\delta = 80$</td>
</tr>
<tr>
<td>Market friction</td>
<td>$\alpha = 0.5$</td>
</tr>
</tbody>
</table>
Figure 1: Investment threshold for $t = 0$. The figure shows the investment threshold $g_0^*$ for $t = 0$ as a function of the firm’s initial cash holdings $C$. Also shown are the different $C$ regions (horizontal lines): from left to right, these are Region $o$, and then Regions I, II, III, IV. Parameter values are given in Table 2.
Appendix A. Investment threshold for $t = 0$

- Region $\alpha$:

$$g_{0,\alpha}^* = -\frac{1}{1+\alpha}(x + x\alpha + \delta + \alpha\delta + I + \alpha I - ((1 + \alpha)(x^2(1 + \alpha) + (1 + \alpha)C^2 + \delta^2 + \alpha\delta^2 - 2(1 + \alpha)C(\delta + 2\alpha\delta - g_1) + 2\delta g_1 + 2\alpha\delta g_1 + g_1^2 + 2x(1 + \alpha)(C + \delta + g_1) + 4\delta I + 8\alpha\delta I + 4\alpha^2\delta I - 2\alpha g_1 I - \alpha I^2)^{1/2}).$$

- Region I:

$$g_{0,I}^* = -\frac{1}{1+\alpha}(x + x\alpha + C + \alpha C + \delta + \alpha\delta - ((1 + \alpha)((1 + \alpha)(x + C + \delta)^2 + 2x(1 + \alpha)g_1 + 2(1 + \alpha)Cg_1 + 2\delta g_1 + 2\alpha\delta g_1 + g_1^2 - 2\alpha g_1 I - \alpha I^2)^{1/2}).$$

- Region II:

$$g_{0,II}^* = \frac{1}{4(1+\alpha)\delta}(x^2(1 + \alpha) + (1 + \alpha)C^2 + \delta^2 + \alpha\delta^2 - 2(1 + \alpha)C(\delta - g_1) + 2\delta g_1 + 2\alpha\delta g_1 + g_1^2 + 2x(1 + \alpha)(C - \delta + g_1) - 2\alpha g_1 I - \alpha I^2).$$

- Region III:

$$g_{0,III}^* = -\frac{1}{4\delta}(x^2\alpha - 4\delta g_1 - 2x\alpha(-C + \delta + I) + \alpha(-C + \delta + I)^2).$$

- Region IV:

$$g_{0,IV}^* = g_1.$$
Appendix B. Slope of investment threshold

Proof of Proposition 1:

The notation with $^1<$ and $^1>$ indicates what is to be shown.

- **Region $o$:**

  \[ \frac{\partial g_{0}^{*,o}}{\partial C} = \frac{(1 + \alpha)(C + x - \delta - 2\alpha\delta + g_{1})}{\sqrt{A}} > 0, \quad (B.1) \]

  with

  \[
  A = (1 + \alpha)(x^2(1 + \alpha) + (1 + \alpha)C^2 + (1 + \alpha)\delta^2 - 2(1 + \alpha)C(\delta + 2\alpha\delta - g_{1}) + g_{1}^2 + 2x(1 + \alpha)(C + \delta + g_{1}) - 2\alpha g_{1}I - \alpha I^2 + 2(1 + \alpha)\delta(g_{1} + 2(1 + \alpha)I)). \quad (B.2)
  \]

  Sufficient condition for $\frac{\partial g_{0}^{*,o}}{\partial C} < 0$ is that the second bracket in the numerator of (B.1) is negative, i.e,

  \[ C + x - \delta - 2\alpha\delta + g_{1} < 0. \]

  This holds for most parameter values; however, for some constellations, the condition is not satisfied. The restriction (2) makes it possible to transform the condition to

  \[ C + x - \delta - 2\alpha\delta + g_{1} < C - \frac{I + g_{1}}{1 + \alpha} - 2\alpha\delta + g_{1} < 0 \]

  \[ \Leftrightarrow (1 + \alpha)C - I + \alpha(g_{1} - 2\delta) < 0. \]

  With $C < I$, we have $\alpha(I + g_{1} - 2\delta) < 0$. So, as $\alpha \geq 0$, a sufficient condition is that $I + g_{1} < 2\delta$.

- **Region $I$:**

  \[ \frac{\partial g_{0}^{*,I}}{\partial C} = \frac{(1 + \alpha)(x + C + \delta + g_{1})}{\sqrt{B}} - 1 > 0 \]

  with

  \[
  B = (1 + \alpha)((1 + \alpha)(x + C + \delta)^2 + 2x(1 + \alpha)g_{1} + 2\delta g_{1} + 2\alpha\delta g_{1} + g_{1}^2 - 2\alpha g_{1}I - \alpha I^2). \quad (B.3)
  \]
To be shown:
\[
\frac{(1 + \alpha)(x + C + \delta + g_1)}{\sqrt{B}} > 1 \iff \frac{(1 + \alpha)^2(x + C + \delta + g_1)^2}{B} > 1
\]
\[
\iff \alpha(1 + \alpha)(g_1 + I)^2 > 0,
\]
which holds as \(\alpha, g_1, I > 0\).

- **Region II:**

  \[
  \frac{\partial g^*,_{II}}{\partial C} = \frac{x - \delta + C + g_1}{2\delta} > 0.
  \]

  In Region II, \(-g_0 - (x - \delta) \leq C \iff x - \delta \geq -g_0 - C\).

  Therefore, the numerator, \(x - \delta + C + g_1 > -g_0 - C + C + g_1 = g_1 - g_0\).

  So, as long as the resulting \(g^*_{II} < g_1\), we have \(\frac{\partial g^*_{II}}{\partial C} > 0\) (as \(\delta > 0\)).

- **Region III:**

  \[
  \frac{\partial g^*_{III}}{\partial C} = \frac{\alpha(I - C - (x - \delta))}{2\delta} > 0.
  \]

  In Region III, \(C < I - (x - \delta) \iff I - C > x - \delta\). So the bracket in the numerator is positive, and thus \(\frac{\partial g^*_{III}}{\partial C} > 0\) (as \(\alpha, \delta > 0\)).

### Appendix C. Curvature of investment threshold

**Proof of Proposition 2:**

- **Region o:**

  \[
  \frac{\partial^2 g^*_{o}}{\partial C^2} = -\frac{F}{A^{3/2}},
  \]

  with \(A\) as defined in (B.2) and

  \[
  F = (1 + \alpha)^2(-4x(1 + \alpha)^2\delta + 4\alpha^3\delta^2 + 4\alpha^2\delta(2\delta - g_1 - I) - 4\delta(g_1 + I) + \alpha(4\delta^2 - 8\delta(g_1 + I) + (g_1 + I)^2)).
  \]

  \(\frac{\partial^2 g^*_{o}}{\partial C^2}\) is positive whenever \(\delta\) is between the two roots

  \[
  \delta^*_{1/2} = \frac{G \pm \sqrt{G^2 - 4(4\alpha + 8\alpha^2 + 4\alpha^3)(\alpha g_1^2 + 2\alpha g_1 I + \alpha I^2)}}{2(4\alpha + 8\alpha^2 + 4\alpha^3)},
  \]

  with

  \[
  G = 4x + 8x\alpha + 4x\alpha^2 + 4g_1 + 8\alpha g_1 + 4\alpha^2 g_1 + 4I + 8\alpha I + 4\alpha^2 I.
  \]
• Region I:
\[
\frac{\partial^2 g_0^{*,I}}{\partial C^2} = -\frac{\alpha (g_1 + I)^2 \sqrt{B}}{H^2} < 0,
\]
with $B$ as defined in (B.3) and
\[
H = x^2 (1 + \alpha) + (1 + \alpha) C^2 + (1 + \alpha) \delta^2 + 2 (1 + \alpha) \delta g_1 + g_1^2 + 2(1 + \alpha) C (\delta + g_1) + 2 x (1 + \alpha) (C + \delta + g_1) - 2 \alpha g_1 I - \alpha I^2,
\]
which holds as $\alpha, g_1, I > 0$.

• Region II:
\[
\frac{\partial^2 g_0^{*,II}}{\partial C^2} = \frac{1}{2\delta} > 0,
\]
which holds as $\delta > 0$.

• Region III:
\[
\frac{\partial^2 g_0^{*,III}}{\partial C^2} = -\frac{\alpha}{2\delta} < 0,
\]
which holds as $\alpha, \delta > 0$. 

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