Migration Options for Skilled Labor and Optimal Investment in Human Capital

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Abstract

This paper develops a model of optimal education choice of an agent who has an option to emigrate. Using a real options framework, we analyze the time evolution of human capital in the country of origin and investigate the role of migration possibilities in the accumulation of different types of human capital. The analysis shows that the accumulation of human capital depends crucially on the level of uncertainty and the transferability of human capital across countries. Government subsidies are an important determinant of the composition of different types of human capital and can be crucial in alleviating the brain drain problem.

1 Introduction

The relation between migration flows on the one hand, and economic growth and level of human capital in developing countries on the other, has been of much interest in the labor and development economics literatures. Although the early literature recognizes the benefits associated with migration such as remittances and return migration,1 studies such as Miyagiwa (1991) and Haque and Kim (1995) conclude that migration leads to the loss of the educated and the highly skilled people. Haque and Kim (1995) find that the subsequent brain drain can result in a permanent decrease in income and growth of the source country.

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1See Bhagwati and Hamada (1974) and McCulloch and Yellen (1977) among others.
This view has been increasingly challenged in the last decade by a number of papers. These studies argue that the possibility of emigration induces optimizing agents to invest more resources in human capital. Because only a fraction of those who invest in human capital can indeed emigrate, average level of human capital and average productivity in the country of origin can increase even after emigration is netted out, a phenomenon labelled as "brain gain". In a cross-section of 127 countries, Beine, Docquier, and Rapoport (2008) find that countries with low levels of human capital and sufficiently low emigration rates are more likely to benefit from the brain gain.

Although the literature investigates the impact of a migration possibility on the formation and the level of human capital, it is relatively silent on the type of education such an option induces an agent to take. Countries that have traditionally been the destination of immigration flows are increasingly introducing laws that aim to attract skilled labor. The US Immigration Act of 1990 favors the immigration of workers with academic backgrounds or certain professional skills (Docquier and Marfouk (2006)). In 2000, Germany introduced special work visas to specialists in information, communications and technology (Lowell (2002)). Docquier and Marfouk (2006) report that the share of skilled immigration has also increased from 33% in 1990 to 37% in 2000. Acquiring globally applicable skills, therefore, is ever more important to increase one’s mobility in the international labor market. To the extent that the accumulation of globally applicable skills increases a country’s competitiveness, educational choices of the population will be one of the key determinants of future economic growth in the source country.

This paper develops a real options model of emigration that analyzes both the brain drain phenomenon and the education choices. The agent in our model holds an option to emigrate and determines the optimal time of emigration. At the same time, the agent optimizes her investment in human capital. The model distinguishes between two types of human capital. The first is the *global human capital*. Global human capital refers to skills that are highly valued in international labor markets. Education in the IT field, for instance, can potentially increase an agent’s mobility in the international labor markets. Similarly, learning English can prove to be an important prerequisite to emigration. On the other hand, *local human capital* is valued mainly in the country of origin. The study of the local law system or language can be considered as part of the local human capital.

The migration literature emphasizes the human capital development in the country of origin. However, an individual often has the opportunity to accumulate human capital in the immigration country. This is particularly true for the international students. Dreher and Poutvaara (2005) find that the level of foreign students in the US is a significant predictor of the subsequent migration decision. Therefore, it is important to investigate the implications of human capital accumulation in the destination country for the level of human capital in the home country. The model in this paper contributes to the literature by taking into account the full dynamics of the human capital accumulation before and after emigration.

In the analysis, we focus on factors such as uncertainty regarding the wage differential, the extent of labor market integration and cultural proximity between the host and the destination countries. The migration literature shows that an individual is more likely to emigrate if the the destination
country is culturally close to the country of origin. A common language, for instance, can be an important factor.\textsuperscript{3} Similarly, an economic union or a more integrated labor market across countries such as the one in Europe can facilitate emigration by increasing the transferability of local human capital. We explore the impact of the transferability of human capital on the brain drain problem.

The role of uncertainty in the migration decisions has also been increasingly recognized in the literature over the last two decades. Early models of the migration decision take a deterministic approach. In these models,\textsuperscript{4} the agent compares the expected discounted value of the gains after migration to the migration costs.\textsuperscript{5} If the benefit of migration exceeds the costs, the agent immediately emigrates. However, this is not in line with the empirical observations. Burda (1995) observes that despite the high wage differential, migration from East Germany to West Germany after the fall of the Berlin Wall proceeded at a slow rate. The positive value of delaying emigration is also documented in Locher (2002) and Anam, Chiang, and Hua (2008). Furthermore, the stylized fact that migration flows tend to occur in jumps\textsuperscript{6} suggests that the irreversibility of migration in the face of uncertainty plays an important role. In line with this literature, our paper takes a real options approach to model the migration decision. We contribute to the literature by investigating the effects of uncertainty and irreversibility associated with the option to emigrate on the accumulation of different types of human capital.

\textsuperscript{4}See, for instance, Borjas (1987).
\textsuperscript{5}Migration costs are not necessarily pecuniary costs of moving from one country to another. Loss of social networks in the country of origin, the presence or absence of an immigrant community in the destination country can significantly contribute to the overall cost of migration.
\textsuperscript{6}See Moretti (1999) and Moretto and Vergalli (2008) for further discussion.
The possibility of migration can be an important factor affecting the education policies of country of origin. In this regard, our paper is closely related to Poutvaara (2008). He studies the effects of migration on governments’ and individuals’ investment in internationally applicable and country-specific education. He demonstrates that the externalities associated with the migration possibility induces governments to underinvest in the internationally applicable skills. Poutvaara (2008) proposes a tax system to mitigate the problem. In addition to the education policies of the source country, it is also important to consider the policies of the destination country. Destination countries can subsidize the education so as to facilitate the human accumulation. This policy can help the destination country to attract more skilled migrants. In this paper, we investigate how the governments of both the source country and the destination country can influence the composition of the total human capital by providing education subsidies.

The main results of the paper are the following. The model shows that the government can alleviate the brain drain problem by subsidizing country-specific education. Such subsidies reduce the relative cost of education in local human capital and discourage emigration. Secondly, we find that the destination countries’ immigration policies are an determinant of the brain drain in the source country. While too strict or too lenient immigration policies exacerbate the brain drain phenomenon, a moderate probability of emigration results in a higher level of global human capital in the source country. Finally, the model emphasizes the importance of economic integration between the home and the destination countries to the development of global human capital in the home country. Economic integration reduces the loss of local human capital due to emigration since a higher portion of
local human capital is valued in the destination country. This encourages agents to accumulate more global human capital in the home country.

The rest of the paper is organized as follows. Section 2 introduces the model. In Section 3, we discuss the implications of our model and carry out comparative statics. Section 4 summarizes and concludes the paper.

2 The Model

Our model is based on the decision problem of an agent in the country of origin. The agent must determine both the timing of the emigration and her investment in human capital. For ease of exposition, the model is developed in the following subsections.

2.1 The Emigration Option

There are two countries, designated as the home and the destination countries. We assume that the decision to emigrate is completely irreversible. Return migration, therefore, is ruled out once the agent has emigrated from the home country to the destination country. This assumption is also in line with the experience of many European countries such as France, Germany and Belgium that have received foreign labor in the 1950s and 1960s as a remedy for their labor market shortages. Although the immigration of foreign labor was intended as a temporary solution, the workers tended to remain in their destination countries. Furthermore, immigration after 1973 has been increasingly characterized by family reunions, indicating that many immigrants did not consider returning to their home countries (Dustmann, Bentilola, and Faini (1996)).
The decision to migrate is motivated by an index of differences between the home and destination countries in the quality of life. The migration literature shows that the wage differences *per se* are not sufficient to explain the migration behavior. Vergalli (2008) draws attention to the role of an existing immigrant community in the destination country while Dreher and Poutvaara (2005) link the study opportunities abroad to the subsequent migration flows. Therefore, we let $x_h(t)$ and $x_d(t)$ denote the quality of life in the home and the destination countries. For simplicity, we normalize the quality of life in the home country and assume $x_h(t) = 1$. Define the ratio of quality of life at time $t$ as:

$$x(t) \equiv \frac{x_d(t)}{x_h(t)}$$

The ratio is assumed to follow a geometric Brownian motion:

$$dx(t) = \mu x(t) dt + \sigma x(t) dB(t)$$

where $dB(t)$ denotes the increments of a standard Brownian motion and $\mu$ and $\sigma$ are the constant drift and diffusion parameters. The agent’s time preference parameter, $r$, is taken as given with the assumption $r > \mu$.

Emigration to another country is costly. Besides the direct monetary costs of moving to another country, the individual incurs indirect costs in the form of losing established networks in the home country and transferability of existing benefits such as pension plans. The model summarizes these costs by $I$. Economically, $I$ can be considered as the opportunity cost of

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7Empirically, such an index could be proxied by the quality of life surveys of the Economist Intelligence Unit, which ranks countries based on criteria such as health, community life and job security. The OSCE reports or the life quality index developed by the Institute of Risk Research can be used as guidelines.
migration. Although the cost of migration can be incurred over time, we assume for parsimony that $I$ is incurred once when the agent decides to emigrate.

Given the above benefits and costs, the agent must choose the optimal time to emigrate to the destination country. The agent emigrates when the quality of life in the destination country is sufficiently higher than in the home country taking into account the opportunity cost of migration. Note that conventionally, emigration would take place when the expected discounted benefit from emigration exceeds the costs. However, in the context of uncertainty, the agent might be better off by postponing the emigration decision and remaining in the home country if the life quality ratio is not expected to grow. The optimal emigration time is then characterized by a stopping time, $\tau$, defined as:

$$\tau = \inf \{ t > 0 : x(t) \geq x^* \}$$

where $x^*$ is the critical value of the life quality ratio that induces the agent to emigrate and must be endogenously determined.\(^8\)

Apart from the life quality differential, a second source of uncertainty arises from the immigration policy of the destination country. The destination countries often have laws that regulate the immigration. Depending on the strictness of these regulations, the agent may not emigrate even if the life quality differential justifies the emigration decision. We denote by $q$ the exogenous probability that the agent will be allowed to immigrate to

\(^{8}\)See Dixit and Pindyck (1994), pp.103-104 for the conditions under which the optimal stopping time corresponds to a single trigger.
the destination country. The treatment of this case, however, is relegated to the appendix. In the sequel, we develop the model without this source of uncertainty.

In the next subsection, we describe the individual preferences and education choice given the above structure in the economy.

2.2 Human Capital Accumulation

Consider an agent in the home country. Besides determining the optimal time to emigrate to the destination country, the agent must also determine her investment in human capital. The amount of accumulated human capital together with the quality of life indices determine the welfare of the agent in both the home and the destination countries.

The agent can accumulate two different types of human capital. The first is the *global* human capital. As briefly discussed in Introduction, global human capital refers mainly to skills that can be applied on an international basis. A degree in the IT field can substantially increase an Indian student’s migration probability to a country such as Germany. An individual who seeks a job opportunity in the finance industry will see her job opportunities abroad broadened if she also speaks English. In other words, accumulating global human capital is similar to acquiring a liquid asset. An individual can transfer her global human capital worldwide at little or no decrease in value. Let \( g(t) \) denote the stock of global human capital at time \( t \). The agent is assumed to invest in her global human capital at rate \( u(t) \):

\[
dg(t) = u(t)dt
\]  

\[ (2) \]
where \( u(t) \in [0, \bar{u}] \) and \( u(0) = u_0 \) is given.

As opposed to the global human capital, \textit{local} human capital is specific to the home country. These skills are valued mainly in the home country and can only be partly transferred to the destination country. Getting education of the local law can be an example of local human capital. We let \( k_h(t) \) denote the level of local human capital the agent possesses at time \( t \). The law of motion of \( k_h(t) \) is given by:

\[
dk_h(t) = q_h(t)dt
\]

where \( q_h(t) \in [0, \bar{q}] \) and \( q_h(0) \) is given.

The agent can transfer only a fraction of her local human capital to the destination country. Let \( \beta \in [0, 1] \) denote the portion of local human capital transferred to the destination country. The parameter \( \beta \) can be perceived as measuring the extent of economic integration between the home and the destination countries. Migration of skilled labor need not flow from the developing to the developed countries (i.e. South-North immigration) but could also occur among the developed countries (i.e. North-North immigration). In particular, economic and political unions such as the European Union can enable individuals to transfer a larger part of their local human capital to the destination country. The transferability parameter can also relate to the cultural and historical proximity of the home and the destination countries. The more similar the countries are, the higher is the fraction of local human capital that can be transferred to the destination country.
An important contribution of this paper is to consider the full dynamics of human capital accumulation before and after emigration. We assume that after emigration, the agent can no longer accumulate human capital related to the home country. Therefore, after the emigration at $\tau$, the level of local human capital is fixed at $k_h(\tau) = \bar{k}$. However, the agent continues to accumulate global human capital and local human capital pertaining to the destination country. Accumulation of further global human capital can be seen as proxying for student flows. Accumulation of local human capital related to the destination country, on the other hand, can be an important factor in facilitating the integration to the society in the host country.\footnote{An example would be a student studying in English in a non-English speaking country and learning the local language.} Let $k_d(t)$ denote the stock of local human capital in the destination country. We assume that:

$$dk_d(t) = q_d(t)dt$$

(4)

where $q_d(t) \in [0, \bar{q}]$ and $q_d(0)$ is given.

The accumulation of both global and local human capital comes at a cost. We assume the agent incurs quadratic costs both in the home and the destination countries:

$$c_h(t) = \frac{c_1 u(t)^2}{2} + \frac{c_2 q_h(t)^2}{2}$$
$$c_d(t) = \frac{c_3 u(t)^2}{2} + \frac{c_4 q_d(t)^2}{2}$$

(5)

where $c_i \in \mathbb{R}_{++}$, $i \in \{1, 2, 3, 4\}$ are constants. The parameters $c_i$ can be seen as a policy tool of the government. For example, a subsidy that supports...
the accumulation of local human capital can achieve this by lowering the
cost of local human capital education relative to that of global human capi-
tal. Similarly, the government of the destination country can attract skilled
migrants by lowering the cost of human capital accumulation.

The next section describes the individual’s preferences and payoffs in
terms of her human capital and the quality of life in the country of resi-
dence.

2.3 Preferences and Payoffs

The agent lives for a finite period of time $T$ and is assumed to be risk
averse with a utility function $U_h(\cdot)$ in the home country and $U_d(\cdot)$ in the
destination country after emigration. The agent derives utility from the
income she generates in the country of residence. Risk aversion implies
$U_i' > 0$ and $U_i'' < 0$ for $i \in \{h, d\}$.

Let $s_h = \{g, k_h, x\}$ and $s_d = \{g, k_d, x\}$ denote the state variables in the
home and the destination countries, respectively. Similarly, define $m_h =$
$\{u, q_h\}$ and $m_d = \{u, q_d\}$ as the control variables before and after emigration,
respectively. The net payoffs to the agent after accounting for the cost of
accumulating human capital, $\pi_h(s_h, t, m_h)$ and $\pi_d(s_d, t, m_d)$, are then given
by:

$$
\begin{align*}
\pi_h(s_h, t, m_h) &= U_h(g(t) + k(t)) - c_h(t) \\
\pi_d(s_d, t, m_d) &= U_d(x(g(t) + k_d(t) + \beta \bar{k})) - c_d(t)
\end{align*}
$$

Note that we have substituted 1 for the life quality in the home country,
$x_h(t)$ and $x(t)$ for $x_d(t)$ following our discussion in Section 2.1. We are now
in a position to characterize the problem and discuss the solution in the next section.

2.4 Problem Formulation and Solution

The agent chooses her rate of investment in both types of human capital and the optimal time to emigrate to the destination country to maximize her lifetime expected utility. Let $V_h(g, k_h, x, t)$ denote the value function before the emigration decision has been made. Then the problem can be posed as:

$$V_h(s_h, t) = \max_{m_h, \tau} \mathbb{E}_0 \left\{ \min_{\tau, T} \int_0^{\tau} \pi_h(s_h, t, m_h) e^{-rt} dt 
+ e^{-r\tau}[V_d(s_d, t) - I] \right\}$$

subject to equations (1), (2) and (3). The maximization is carried out with respect to the controls $m_h = \{u, q_h\}$. The first term in the expectation captures the utility accruing to the agent until her decision to emigrate at $\tau$. After emigration, the agent continues to optimize her investment in human capital, which is captured by the value function $V_d(s_d, t)$. Note that the cost of emigration, $I$, is incurred at the emigration time, $\tau$.

To solve the problem in (7), we first analyze the agent’s optimization conditioning on the emigration. After the agent emigrates, she optimizes her investment in $g(t)$ and $k_d(t)$ and solves:

$$V_d(s_d, t) = \max_{m_d} \mathbb{E}_\tau \left\{ \int_\tau^T \pi_d(s_d, t, m_d) e^{-rt} dt \right\}$$

(8)
subject to equations (1), (2) and (4). The following proposition characterizes the optimal investment rates in the human capital.

**Proposition 1:** Investment in global and local human capital after emigration is given by:

\[
\begin{align*}
    u^*_d(t) &= \frac{1}{c_3} \frac{\partial V_d}{\partial q} \\
    q^*_d(t) &= \frac{1}{c_4} \frac{\partial V_d}{\partial k_d}
\end{align*}
\]  

(9)

Before emigration, the agent’s investment policy is characterized by:

\[
\begin{align*}
    u^*_h(t) &= \frac{1}{c_1} \frac{\partial V_h}{\partial q} \\
    q^*_h(t) &= \frac{1}{c_2} \frac{\partial V_h}{\partial k_h}
\end{align*}
\]  

(10)

**Proof:** See Appendix A

The investment rates before and after emigration are given by the ratio of the marginal product of human capital to its respective marginal cost. Note that government subsidies that would lower the costs of acquiring education increases the agent’s investment in human capital.

In the next section, we resort to numerical procedures to explore the optimal policies of the agent and the governments.

### 3 Numerical Analysis and Comparative Statics

Our aim in this section is to characterize the optimal path of levels of both the global and local human capital and analyze the impact of labor market factors as well as government policies on the agents’ endogenous decisions.
to opt for different types of education. The benchmark parameter set used in the analysis is given in Appendix C.

3.1 Education Subsidies and Brain Drain

Governments often provide education subsidies to encourage investment in human capital. Subsidies in the form of flat income tax rates or tuition support and scholarships serve to reduce the cost of education. Keane and Wolpin (1997) show, for example, that a $2000 tuition subsidy increases college graduation rate by 8.4%.

As Poutvaara (2008) observes, in an open economy, the migration opportunity enables the destination country to partially capture the benefits of human capital investment in the source country. To encourage investment in local human capital and increase the aggregate level of human capital net of the migration, the source country can augment its provision of public subsidies to the accumulation of local human capital. In our model, the effect of government subsidies is captured by the cost parameters $c_1$ and $c_2$ in equation (5).
Figure 2: Local Human Capital Subsidization and Remaining Aggregate Human Capital

Figure 1 illustrates the impact of a government subsidy of local human capital on the accumulation of both types of human capital net of the migration flows. The left panel of the figure shows that a government subsidy substantially increases the amount of local human capital accumulated. As expected, the subsidy leads to a decrease in the amount of global human capital accumulated. Note, however, that due to the migration option, the decrease in the global human capital is less than the increase in the local human capital.

It is also important to consider the effect of education subsidies on the total level of remaining human capital in the source country. This exercise would allow us to assess whether education subsidies can alleviate the brain drain phenomenon caused by the migration opportunity. Figure 2 shows
Figure 3: Volatility of Quality of Life Differential and Human Capital Accumulation

this is indeed the case. Compared to the benchmark case of no subsidies, the source country government can increase the level of aggregate human capital by lowering the cost of education in local human capital.

Government subsidies not only increase the level of aggregate human capital in the economy but also extends the period of investment. Figure 2 demonstrates that in the absence of public subsidies, agents stop accumulating human capital after 8 years. Beyond this point, the marginal benefit from acquiring additional human capital does not justify the marginal cost, indicated by the horizontal portion of the figure. By providing subsidies, the governments reduce the cost, thereby prolonging the time allocated to human capital accumulation.
3.2 Uncertainty and Human Capital Accumulation

Figure 3 explores the effect of the volatility of life quality differential, $\sigma$, between the destination and the home countries on the level of aggregate human capital in the source country. The figure graphs the ratio of the global human capital to the local human capital for various $\sigma$ values. The ratio decreases over time for any $\sigma$ specification. This is due to the age effect. As the agent postpones the migration, it becomes increasingly difficult to migrate since the agent can only look forward to a short period in which she can benefit from the life quality differential. Therefore, investing in global human capital becomes relatively unattractive and the ratio decreases. Note also that the ratio is positively related to $\sigma$. A higher $\sigma$ implies a greater probability that the life quality differential, $x_t$, hits the migration trigger point, $x^*$. This makes investment in global human capital more attractive compared to investment in local human capital.

The migration policy of the destination country creates another source of uncertainty for the agent. The destination country may, for instance, allocate quotas and place screening procedures before the migration. Therefore, the agent does not necessarily emigrate even if $x_t > x^*$. We analyze
the effect of an exogenous probability of migration, $q$, on the formation of global and local human capital in the home country in Figure 4. We can conclude from the left panel that the level of local human capital is higher when the destination country regulations are stricter. This is intuitive since the agent values local human capital more when the destination country regulations are strict even when the quality of life differential is high. On the other hand, the effect of the migration probability is nonmonotonous for the stock of global human capital, as shown in the right panel of the figure. For low values of $q$, the stock of global human capital is low since the agents opt for the education in local human capital. When $q$ is high, the agents do invest in global human capital but a substantial portion of these agents eventually emigrate. This leads to the brain drain effect. For moderate values of $q$, however, a brain gain effect dominates since a significant portion of the agents who have accumulated global human capital still remains in the source country.

3.3 Transferability and Human Capital Accumulation

Figures 5 and 6 explore the effect of varying transferability parameter, $\beta$, on the accumulation of global human capital in the source country. The parameter $\beta$ proxies for ease with which an agent can integrate into the destination country. The migration literature suggests that language and cultural proximities are important determinants of the eventual migration decision. It is also possible to think of $\beta$ as the extent to which the home and the destination countries are economically and institutionally integrated. For instance, if the two countries have similar bodies of business law, an agent in the home country can benefit from her knowledge of the local law system in the destination country.
Figures 5 and 6 show that the high transferability of local human capital can benefit the stock of global human capital in the home country. This is illustrated in Figure 5. Figure 6, on the other hand, shows a setting in which the agent invests more in her local human capital as the transferability parameter increases. When $\beta$ is high, the agent’s expected value in the destination country is higher since the loss in human capital is less. This gives the agent the incentive to invest in her global human capital in the source country. At the same time, a high $\beta$ blurs the distinction between the local and global human capital. This allows the agent to invest relatively more in the cheaper education type and still take advantage of the migration option. In Figure 5 the cost of education in the global human capital is relatively less. This induces agents to accumulate more global human capital and the level of global human capital is positively related to $\beta$. Figure 6,
on the other hand, illustrates a case in which accumulation of local human capital is cheaper than that of global human capital. Accordingly, the agents accumulate more local human capital as $\beta$ increases.

3.4 Student Mobility and Brain Drain

One crucial factor in explaining the brain drain phenomenon is the international flow of students. Countries such as the US and Canada attract foreign students from developing countries. As Dreher and Poutvaara (2005) observe, studying abroad can provide a toehold for the students and ease the subsequent migration.

An important implication of the student flows for the source country is that agents can defer the accumulation of part of their human capital until...
after they have moved to the destination country. For instance, a student may invest resources to learn English in the source country before she moves to the US for studies in the IT field. In this case, the brain drain for the source country may be exacerbated compared to a case in which education is completed in the source country.

By analyzing the full dynamics of human capital investment both in the source and the destination countries, our model captures the effect of deferring human capital accumulation until after the migration decision. Figure 7 compares the setting in which the agent continues to accumulate human capital in the destination country to that in which human capital accumulation is assumed away in the destination country. The results support the hypothesis of an exacerbated brain drain problem in the source country. When the agent has the opportunity to invest in her human capital after emigration, she invests less in the global human capital in the home country. The left panel of the figure shows, moreover, that the total amount of human capital in the source country diminishes compared to the setting where there is no human capital accumulation in the destination country.
4 Conclusion

This paper revisits the brain drain phenomenon and explores how the migration option affects agents’ education choices. Importantly, the model distinguishes between global human capital and local human capital. While the migration option favors the investment in global human capital, local human capital is important to provide subsistence in the home country.

Our model shows that the source country can experience brain gain for moderate probabilities of migration. When the migration probability is too low or too high, the agents either have no incentive to invest in global human capital or emigrate in large numbers. In these settings, the brain drain effect dominates the brain gain effect and the total stock of human capital decreases in the source country. Our results also indicate that international flow of students can also be detrimental to the amount of human capital in the home country.

Governments of the source country can follow education policies aiming at reducing the negative impact of the brain drain. In particular, public subsidies can alleviate the problem by reducing the cost of education. Providing subsidies to support the local human capital results in a higher level of total human capital in the source country.

Finally, our model emphasizes the importance of economic and legal integration to the stock of human capital. A higher degree of integration between the source and the destination countries encourages agents to accumulate more global human capital and experience a smaller decrease in value associated with the accumulated local human capital.
Appendix

Appendix A

Proof of Proposition 1: The Bellman equation for value functions $V_i(s_i, t), i \in \{h, d\}$, is given by

$$rV_i(s_i, t) = \max_{m_i} \left\{ \pi_i (s_i, t, m_i) + \frac{1}{dt} \mathbb{E}(dV_i) \right\}$$  \hspace{1cm} (11)

Expanding the value functions by Itô’s Lemma, and taking expectations, we obtain the following expressions for the value functions $dV_h(s_h, t)$ and $V_d(s_d, t)$:

$$dV_i = \frac{\partial V_i}{\partial t} + \frac{1}{2} \sigma^2 x^2 \frac{\partial^2 V_i}{\partial x^2} + \mu x \frac{\partial V_i}{\partial x} + u \frac{\partial V_i}{\partial g} + q_i \frac{\partial V_i}{\partial k_i}$$  \hspace{1cm} (12)

Substituting equation (12) into equation (11) yields:

$$0 = \max_{m_i} \left\{ \pi_i (s_i, t, m_i) + \frac{\partial V_i}{\partial t} + \frac{1}{2} \sigma^2 x^2 \frac{\partial^2 V_i}{\partial x^2} + \mu x \frac{\partial V_i}{\partial x} + u \frac{\partial V_i}{\partial g} + q_i \frac{\partial V_i}{\partial k_i} - rV_i \right\}$$  \hspace{1cm} (13)

Optimizing with respect to the controls $m_i = \{u, q_i\}$ yields the expressions in the proposition. Using (9) and (10) in the Bellman equations yields a system of nonlinear, second-order PDEs. In particular, we have the following system before and after emigration, respectively:

$$\begin{align*}
\frac{\partial V_h}{\partial t} &+ \frac{1}{2} \sigma^2 x^2 \frac{\partial^2 V_h}{\partial x^2} + \mu x \frac{\partial V_h}{\partial x} + \frac{1}{2c_1} \left( \frac{\partial V_h}{\partial g} \right)^2 + \frac{1}{2c_2} \left( \frac{\partial V_h}{\partial k_h} \right)^2 - rV_h + \pi_h = 0 \\
\frac{\partial V_d}{\partial t} &+ \frac{1}{2} \sigma^2 x^2 \frac{\partial^2 V_d}{\partial x^2} + \mu x \frac{\partial V_d}{\partial x} + \frac{1}{2c_3} \left( \frac{\partial V_d}{\partial g} \right)^2 + \frac{1}{2c_4} \left( \frac{\partial V_d}{\partial k_d} \right)^2 - rV_d + \pi_d = 0
\end{align*}$$  \hspace{1cm} (14)

Appendix B

This appendix incorporates the exogenous probability that the destination country admits an agent when the life quality differential justifies emigration. Let $N(t)$ denote a Poisson process that tracks the visa policy of the destination country:

$$dN = \begin{cases} 
1, & \text{with } qdt \\
0, & \text{with } 1 - qdt
\end{cases}$$  \hspace{1cm} (15)
We say that an agent is accepted upon application to the destination country when the Poisson process has jumped. The parameter $q$ in equation (15) captures the intensity or the probability of acceptance. We assume that the processes $N(t)$ and $x(t)$ are independent.

Corresponding to the Poisson process in (15), we denote the arrival time of a visa by $\tau_a$. Note that $\tau_a$ is exponentially distributed with the parameter $q$. We assume that the agent is accepted to the destination country if the $N(t)$ has jumped prior to the emigration decision of the agent. That is, the agent emigrates if $\tau_a < \tau$. We can now reformulate the problem posed in equation (7):

$$V_h(s_h, t) = \max_{m_h, \tau} \left\{ \min(\tau, T) \int_0^{\min(\tau, T)} \pi_h(s_h, t, m_h)e^{-rt}dt \right.$$

$$\left. + \mathbb{I}_{\{\tau_a < \tau < T\}} e^{-r\tau} [V_d(s_d, t) - I] \right\}$$

(16)

Note that the agent’s problem conditional on emigration remains as specified in (8). To solve (16) subject to equations (1), (2) and (3), we resort again to the Bellman equation in (11). By Itô’s lemma, the expression for $dV_h$ is given by:

$$dV_h = \frac{\partial V_h}{\partial t} + \frac{1}{2} \sigma^2 x^2 \frac{\partial^2 V_h}{\partial x^2} + \mu x \frac{\partial V_h}{\partial x} + u \frac{\partial V_h}{\partial g} + q_h \frac{\partial V_h}{\partial k_h}$$

$$+ q [V_d - I - V_h]$$

(17)

Substituting equation (17) into equation (11), we get:

$$0 = \max_{m_h} \left\{ \pi_h(s_i, t, m_i) + \frac{\partial V_h}{\partial t} + \frac{1}{2} \sigma^2 x^2 \frac{\partial^2 V_h}{\partial x^2} + \mu x \frac{\partial V_h}{\partial x} + u \frac{\partial V_h}{\partial g} + q_h \frac{\partial V_h}{\partial k_h} - (r + q) V_h + q(V_d - I) \right\}$$

(18)

It is important to note that the optimization in equation (18) yields the same investment policy as in Proposition 1. However, notice that the discount factor of the agent has now been augmented by the probability of acceptance to the destination country. The last term in (18) reflects the fact that the value after emigration can be captured only with intensity $q$.

Appendix C

The numerical analysis uses a CRRA utility. The benchmark case parameters are provided in the table.
Table 1: Parameter Values

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Time Horizon ((T))</td>
<td>13</td>
</tr>
<tr>
<td>Discount Rate ((r))</td>
<td>0.1</td>
</tr>
<tr>
<td>Drift ((\mu))</td>
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</tr>
<tr>
<td>Volatility ((\sigma))</td>
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</tr>
<tr>
<td>Initial Life Quality Diff ((x_0))</td>
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</tr>
<tr>
<td>Max. Local Human Capital Investment Rate ((\bar{q}))</td>
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</tr>
<tr>
<td>Max. Global Human Capital Investment Rate ((\bar{u}))</td>
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</tr>
<tr>
<td>Migration Cost ((I))</td>
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</tr>
<tr>
<td>Risk Aversion ((\gamma))</td>
<td>0.5</td>
</tr>
<tr>
<td>Migration Probability ((q))</td>
<td>0.6</td>
</tr>
</tbody>
</table>

References


