Longevity studies based on Kernel Hazard Estimation

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Abstract

We propose a nonparametric smoothing method to visualize the evolution of mortality rates. This is used to compare the mortality experiences of Denmark and Spain. Our comparison takes into account the dependence of mortality on chronological time and age. A two-dimensional surface indicating transition intensity is estimated and a number of other interesting estimators can be derived from this surface: i.e. ratios of the structural development of mortality, estimation of multiplicative structures using marginal integration (e.g., Linton and Nielsen, 1995), and smoothed trends for fixed ages. The multiplicative structure effectively separates the chronological time component and the age component.

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1 Introduction

A lot of actuarial work has traditionally been concerned with estimation and forecasting techniques for mortality curves. A review of graduating principles can be found in Benjamin and Pollard (1980). Gwilt (1956) started a series of international analyses on mortality trends. The works by Anderson and Whitehead (1960), Giles and Wilkie (1973) and Wilkie (1976) followed. The evolution over time of graduated curves and their projection is still popular for the purpose of extrapolation. One usual procedure is based on the projection of parameters as in Cramér and Wold (1935) and Buus (1960), another strategy is to analyze the evolution of mortality rates between two time periods: i.e. ratios of mortality rates. Other works are based on the assumption that the fitted curve has some kind of dependence on calendar time. Benjamin and Soliman (1995) provide a good account of practical methods for mortality projection. Forfar and Smith (1988) deal with the changing shape of English life tables.

Actuarial practice has shown that it is difficult to compare mortality trends and levels, at the same time, when studying mortality experiences. Most papers focus on either the evolution of mortality through time or on the comparison of levels. One exception is the work done by Macdonald, Cairns, Gwilt, and Miller (1998), where levels and trends are summarized simultaneously and results are used to compare mortality trends for several countries. Another contribution is the study of Renshaw, Haberman and Hatzopoulos (1996) which also modelled simultaneously the age and

time variation of mortality rates for UK insured persons.

When comparing the evolution and mortality patterns of two countries, usually, different distributional assumptions and mortality laws have been applied. Thereby, we need to compare raw data in both dimensions without any previous assumptions and with a method capable to handle efficiently scarce occurrences in some ages. Renshaw and Haberman (1997) used a method that was also designed for comparisons where data may be sparse e.g. for select mortality at low durations.

If we look at the mortality experience of a given population, we can plot a surface showing raw mortality rates. One dimension corresponds to age, the other to calendar time. In Figure 1, we can see three-dimensional graphs for two different countries: Denmark and Spain. Raw mortality rates for women have been plotted in the logarithm scale, over nineteen years and from zero to ninety years of age. The main difference between both countries is the population size. In 1996, Spain had approximately 39.3 million inhabitants while the population of Denmark totalled 5.25 million (Data from Eurostat, 1997, Demographic Statistics). Therefore, a previous smooth is required for the Danish mortality experience before any examination, although it is also convenient for the Spanish population.

Figure 1. Raw mortality rates. Danish and Spanish women, 1975-1993. In logarithm scale

The comparison of the two surfaces in Figure 1 implies some difficulty because, in particular, the Danish curve is not sufficiently smooth, in the sense of Forfar, McCutcheon and Wilkie (1988, page 14). Besides, comparing the shape of two surfaces

is not an easy task.

Haberman and Renshaw (1998) discussed some available methods for mortality comparison. Two mortality experiences may be compared before fitting any curve. In fact, this is the approach suggested by Forfar et al. (1988) and Renshaw and Haberman (1997). Another possible approach is to fit the same curve to both mortality experiences separately and then compare the fitted mortalities, as proposed by Olbricht and Miller (1998). In any case, distributional assumptions on the behaviour of the occurrences and the exposures are made before fitting. One major drawback of these proposals rises when applied to small populations (or small portfolios). Some previous smoothing is required to target the underlying pattern of mortality. Furthermore, these methods can be applied when comparing patterns through time or across countries but not simultaneously.

It is difficult to write a general model framework that is compatible to all the contributions on the study of mortality trends available at present (see Macdonald, 1997), because authors use different assumptions although they have the same objective, namely, to analyze the evolution of mortality over time. Recently, actuaries have become aware of the importance of counting process for survival models in the statistical analysis of mortality. For a complete presentation see the work by Macdonald (1996a, 1996b and 1996c). We have chosen this general framework where problems, such as censoring, find a natural presentation and have a tractable approximation.

We combine the basic ideas of counting processes where chronological time is our time scale, whereas age is treated as a covariate process. Our model assumes that the transition intensity (or hazard rate) combines the effect of calendar time (for example, exogenous such as technological evolution factors of the populations under study) and other individual covariates (in particular age). The approach is non-parametric, so that our results do not depend on distributional assumptions. Conclusions derived from this work show the interest of separating these two components: the effect of time and also the effect of each individual age.

We will shortly review the recent study of Macdonald et al. (1998). They use the UN mortality database to calculate the Comparative Mortality Factor (CMF). The fitted mortality for Sweden in 1990 is used as the baseline country. If the CMF for any given country and year is greater than one, it means that this country has a higher mortality than the Swedish experience in 1990. Figure 2 shows the evolution of the CMF for men and women in some countries of the European Union, from 1970 to 1990. For Germany, only data corresponding to West Germany is used. A reduction in the dispersion can be seen at the end of the period. CMF for Spanish men has a different evolution, since it has a minimum in 1985, but moves up to the average in 1990.

Figure 2. EU men and women Comparative Mortality Factor trends, 1970-1990

(Country labels can be found in Appendix 1)

When looking at the evolution and at the levels of the CMF for men, countries can be classified into two groups, according to their level. Denmark has a stable

behaviour through these years and has only a higher mortality factor at the end of the period. Figure 3 shows these two groups of countries.

Figure 3. Groups of EU countries according to their Comparative Mortality Factor evolution (1970-1990)

The evolution for women's mortality is similar to the one observed for men. CMF's dispersion for women always stays above the one observed for men; nevertheless, there is a reduction of dispersion throughout the considered period. Although in all countries men's mortality is higher than women's at all ages, women's CMF has been higher than men's until 1980. Since then, the average CMF for women has become closer to one.

We observe that Danish and Spanish mortalities exemplify two extreme situations. Besides, when comparing two countries, the CMF cannot ascertain the presence of significant changes in some particular age ranges. Therefore, we will propose the kernel hazard estimation approach to determine the location and nature of mortality changes in these countries. The use of kernel smoothing methods for graduating mortality rates or intensities has a number of actuarial antecedents (see, for example, some developments by Ramlau-Hansen, 1983, Copas and Haberman, 1983 and Gavin, Haberman and Verrall, 1993).

Using counting processes as a general framework, this paper is focused on the comparison of the force of mortality for two experiences, while taking simultaneously into account two dimensions: age and time (mortality trends). We introduce the use of modern kernel smoothing methods to alleviate the usual inconveniences. Our objective is first to analyze the development of Danish and Spanish population mor-

talities and find if differences exist. Secondly, we would like to determine where those differences are more prominent and, finally, to identify the way the differences have developed over time; all of these, with minimum assumptions requirements.

In section 2 the model is described in the counting processes framework. Afterwards, in section 3, the local constant estimator of the hazard rate is presented. Section 4 explains the multiplicative submodel and the marginal integration estimation method, followed by a review of the theoretical properties of the estimators in section 5. Section 6 shows the application of the estimator to Danish and Spanish mortality, by gender and a comparative analysis is presented. Finally, several conclusions are drawn and further research is pointed out.

2 The model

In order to analyze the problem of longevity, one natural way to proceed is to consider mortality as a function of chronological time and age. Therefore, in this section, we will describe the mortality model in the counting processes framework as a two-dimensional function.

Let t be chronological time and $X(t)$ be the age process. We observe n individuals $i = 1, \dots, n$. Now, let N_i count observed failures for the i^{th} individual in the time interval I_2 . We assume that $\mathbf{N} = (N_1, \dots, N_n)$ is an n -dimensional counting process with respect to an increasing, right continuous, complete filtration $\mathcal{F}_t$, $t \in I_2$, i.e. one

that obeys *les conditions habituelles*, (see Andersen et al., 1993, page 60). We model the random intensity process $\lambda = (\lambda_1, \dots, \lambda_n)$ of $\mathbf{N}$ as depending on marker values,

$$\lambda_i(t) = \alpha\{X_i(t), t\}Y_i(t),$$

but do not restrict the functional form of $\alpha(\bullet)$. Here, Y_i is a predictable process taking values in $\{0, 1\}$, indicating (by the value 1) when the i^{th} individual is under risk, while $X_i(t)$ is the predictable one-dimensional age process. In our study the age process $X_i(t)$ takes values in $[0, 90]$ and chronological time belongs to the interval $I_2 = [1975, 1993]$.

3 Local constant estimation of α

Now, we are going to introduce the local constant estimation methodology. Thereby, let the kernel K be a one-dimensional kernel and $b = (b_1, b_2)$ a two-dimensional bandwidth-vector. The estimator suggested by Fusaro, Nielsen and Scheike (1993) and Nielsen and Linton (1995) is:

$$\hat{\alpha}(x, t) = \frac{O_{x,t}}{E_{x,t}} \tag{1}$$

where

$$O_{x,t} = \sum_{i=1}^n \int_{1975}^{1993} K_{b_1}\{x - X_i(s)\} K_{b_2}(t - s) dN_i(s)$$

and

$$E_{x,t} = \sum_{i=1}^n \int_{1975}^{1993} K_{b_1} \{x - X_i(s)\} K_{b_2} (t - s) Y_i(s) ds$$

are, respectively, the smoothed occurrence and exposure. Nielsen (1998) interprets the above estimator as a local constant kernel smoother. The local constant kernel estimator hereby corresponds to the well known occurrence-exposure ratio. Let the kernel $K(x)$ equal $I(|x| < 1)$, then $O_{x,t}$ corresponds to the observed number of deaths and $E_{x,t}$ to the exposure time, both observed in the chronological time interval $[t - b_1, t + b_1]$ and in the age interval $[x - b_2, x + b_2]$. The effect of the choice of the kernel function and the bandwidths has extensively been discussed in the literature (see, Gavin, Haberman and Verrall, 1994).

4 The multiplicative submodel and the marginal integration estimation method

In this section, we will restrict our attention to the two-dimensional multiplicative submodel of the more completed model previously described. Then, the multiplicative submodel is:

$$\alpha(x, t) = \alpha_1(x) \alpha_2(t).$$

The model can be interpreted as a continuous mortality ratio and does not make any other functional assumption than a multiplicative interaction between the two components.

Next, we consider the following contrasts:

$$\tilde{\alpha}_1(x) = \int_{1975}^{1993} \alpha(x, t) dt, \quad (2)$$

$$\tilde{\alpha}_2(t) = \int_0^{90} \alpha(x, t) dx. \quad (3)$$

Then, $\tilde{\alpha}_1(x) = \alpha_1(x)c_1$ and $\tilde{\alpha}_2(t) = \alpha_2(t)c_2$, where $c_1 = \int_{1975}^{1993} \alpha_2(t) dt$ and $c_2 = \int_0^{90} \alpha_1(x) dx$. Therefore, up to identifiability, $\tilde{\alpha}_1(x)$ and $\tilde{\alpha}_2(t)$ are the univariate components of interest in the multiplicative structure. Note furthermore that, in this case, $c = \int_{1975}^{1993} \int_0^{90} \alpha(x, t) dx dt$ equals $c_1 c_2$.

The contrasts (2) and (3) are estimated by

$$\hat{\alpha}_1(x) = \int_{1975}^{1993} \hat{\alpha}(x, t) dt, \quad (4)$$

$$\hat{\alpha}_2(t) = \int_0^{90} \hat{\alpha}(x, t) dx. \quad (5)$$

Whereas the constant is estimated as

$$\hat{c} = \int_{1975}^{1993} \int_0^{90} \hat{\alpha}(x, t) dx dt. \quad (6)$$

Note that an alternative interpretation of the estimators is that, up to additive constants, $\hat{\alpha}_1(x)$ and $\hat{\alpha}_2(t)$ minimize the L_2 distance

$$\arg \min_{\alpha_1, \alpha_2} \int_{1975}^{1993} \int_0^{90} [\hat{\alpha}(x, t) - \alpha_1(x)\alpha_2(t)/c]^2 dx dt. \quad (7)$$

5 Pointwise theoretical properties of the estimators

In this section, we will review the theoretical properties of the estimators we are going to use. Properties have been derived from the n -dimensional ones presented in Nielsen and Linton (1995) and Linton and Nielsen (1999). We will state the theorems for the two-dimensional case considered in this paper.

Note that the rate of convergence of the complete two-dimensional estimator is slower than the rate of convergence of the marginal estimators. This is another advantage of applying marginal integration, particularly in situations with scarce data.

Let's define now

$$\alpha^*(x, t) = \frac{\sum_{i=1}^n \int_{1975}^{1993} K_{b_1} \{x - X_i(s)\} K_{b_2} (t - s) d\Lambda_i(s)}{\sum_{i=1}^n \int_{1975}^{1993} K_{b_1} \{x - X_i(s)\} K_{b_2} (t - s) Y_i(s) ds}.$$

with $\Lambda_i(t) = \int_0^t \lambda_i(s) ds$,

Contrasts (2) and (3) are estimated by

$$\alpha_1^*(x) = \int_{1975}^{1993} \alpha^*(x, t) dt, \quad (8)$$

$$\alpha_2^*(t) = \int_0^{90} \alpha^*(x, t) dx. \quad (9)$$

Let $F_s(x) = \Pr(X_i(s) \leq x | Y_i(s) = 1)$ be the conditional distribution function of the covariate process at time s and $f_s(x)$ the corresponding density with respect to the one-dimensional Lebesgue measure. We will assume that $E\{Y_i(s)\} = y(s)$, where $y(\bullet)$

is continuous and assume that the stochastic processes $(N_1, X_1, Y_1), \dots, (N_n, X_n, Y_n)$ are iid for the n individuals and $\mathcal{F}_t = \sigma(\mathbf{N}(s), \mathbf{X}(s), \mathbf{Y}(s); s \leq t)$, where $\mathbf{Y} = (Y_1, Y_2, \dots, Y_n)$ and $\mathbf{X} = (X_1, X_2, \dots, X_n)$. The pointwise asymptotic distribution is analyzed by separating the errors into the variable part and the stable part. Once properly normalized, the former converges in distribution and the latter converges in probability. Theorem 1 was derived in Nielsen and Linton (1995) and Theorem 2 was derived in Linton and Nielsen (1999).

THEOREM 1. [Pointwise Convergence of the two-dimensional local constant estimator] *Let $\varphi(x, t) = f_t(x)y(t)$, and assume that $\varphi(x, t) > 0$ and that α is twice and φ is once continuously differentiable at just (x, t) . We also assume, that the kernel K has bounded support, is symmetric about zero, and is continuous. Define the kernel moments: $\kappa_1 = \int_R v^2 K(v)dv$ and $\kappa_2 = \int_R K(v)^2 dv$. Finally, suppose that for $j = 1, 2$, $b_j = \gamma_j n^{-1/4}$, where b_j is fixed and known. Then*

$$(1a) \quad \gamma_1^{1/2} \gamma_2^{1/2} n^{2/5} \{\hat{\alpha}(x, t) - \alpha^*(x, t)\} \Rightarrow N[0, \kappa_2^2 \alpha(x, t) / \varphi(x, t)]$$

$$(1b) \quad \alpha^*(x, t) - \alpha(x, t) = \kappa_1 \gamma_1^2 n^{-2/5} \left\{ \frac{\partial \alpha}{\partial x}(x, t) \frac{\partial \varphi}{\partial x}(x, t) / \varphi(x, t) + \frac{\partial^2 \alpha}{\partial x^2}(x, t) / 2 \right\} +$$

$$\kappa_1 \gamma_2^2 n^{-2/5} \left\{ \frac{\partial \alpha}{\partial t}(x, t) \frac{\partial \varphi}{\partial t}(x, t) / \varphi(x, t) + \frac{\partial^2 \alpha}{\partial t^2}(x, t) / 2 \right\} + o_P(n^{-2/5})$$

THEOREM 2. [Pointwise Convergence for the marginally integrated local constant estimator] *Assume that $\varphi(x, t)$ is positive and bounded away from zero on*

$I_1 \times I_2 \subset [0, 90] \times [1975, 1990]$. We assume that α is twice and φ is once continuously differentiable on $I_1 \times I_2$ and that for $j = 1, 2$, $b_j = \gamma_j n^{-1/5}$.

(2a) Then the asymptotic pointwise properties of the chronological time marginal are

$$n^{2/5} \gamma_1^{1/2} (\hat{\alpha}_2 - \alpha_2^*)(t) \Rightarrow N[0, v_1(t)],$$

$$(\hat{\alpha}_2 - \alpha_2^*)(t) = \gamma_1^2 m_1(t) + o_P(n^{-2/5})$$

with

$$m_1(t) = \kappa_1^2/2 \int_0^{90} \left\{ \frac{\partial \alpha}{\partial t}(x, t) \frac{\partial \varphi}{\partial t}(x, t) / \varphi(x, t) + \frac{\partial^2 \alpha}{\partial t^2}(x, t) \right\} dx,$$

and

$$v_1(t) = \kappa_2 \int_0^{90} \alpha(x, t) / \varphi(x, t) dx.$$

(2b) Then the asymptotic pointwise properties of the age marginal are

$$n^{2/5} \gamma_2^{1/2} (\hat{\alpha}_1 - \alpha_1^*)(x) \Rightarrow N[0, v_2(x)],$$

$$(\hat{\alpha}_1 - \alpha_1^*)(x) = \gamma_2^2 m_2(x) + o_P(n^{-2/5})$$

with

$$m_2(x) = \kappa_1^2/2 \int_{1975}^{1993} \left\{ \frac{\partial \alpha}{\partial t}(x, t) \frac{\partial \varphi}{\partial t}(x, t) / \varphi(x, t) + \frac{\partial^2 \alpha}{\partial t^2}(x, t) \right\} dt,$$

and

$$v_2(x) = \kappa_2^2 \int_{1975}^{1993} \alpha(x, t) / \varphi(x, t) dt.$$

(2c) *The estimator of the constant is of a lower order of magnitude, namely $\hat{c} - c = O_P(n^{-1/2})$.*

6 Results

In this section, we will show the results obtained when applying the model to compare Danish and Spanish mortality experiences by gender. As stated previously, the analyzed period has been [1975,1993] and the age range [0,90]. As a matter of fact, our principal interest is a descriptive analysis; therefore the estimates will be presented graphically. Firstly, the two-dimensional hazard rate estimates for each country and gender will be studied. Afterwards, marginally integrated local constant estimates for each component, age and chronological time, will be examined and compared.

The two-dimensional surfaces estimating the force of mortality through all ages and years are plotted in Figure 4 and 5, in logarithm scale. Figure 4 shows the estimates for Danish men and women while Figure 5 presents the estimates for Spanish men and women. When compared to Figure 1, the kernel estimates of Figures 4 and 5 show the way smoothing eliminates the peaks appearing in the original data. The bandwidths b_1 and b_2 used to obtain the estimates are equal to 5 and 3, respectively.

Figure 4. Two-dimensional estimate of logarithm of mortality rate for Danish men
and women (1970,1990)

Figure 5. Two-dimensional estimate of logarithm of mortality rate for Spanish men and women (1970,1990)

Figure 4 and 5 show differences in pattern and evolution. For instance, Spanish women estimates have no "accident hump", while the rest did. Our two-dimensional graphical analysis shows, however, that mortality has not developed in a multiplicative fashion.

Therefore, we could conclude that differences exist. Moreover, we would like to determine where those differences arise from. For this purpose, we can apply the usual comparison procedures to the estimates, such as comparative ratios or plotted mortality trends for several given ages. But traditional comparative tools will require further analyses to settle where differences are and how they have developed. This could be effectively alleviate with the use of marginally integrated local constant estimators of the multiplicative components.

A comparison of the age multiplicative component, or first marginal curve, reflects similarities in mortality patterns by sex and country in a compact way, for the whole analyzed period.

Figure 6. Age marginal curve for Spain and Denmark by gender

In Figure 6 we have plotted the age marginal curve for Spain and Denmark by gender in logarithm scale. The age marginal curve is not a mortality hazard rate corresponding to one particular year, but an average mortality rate over the analyzed period. Thus, by comparing the different age marginal curves, we obtain the average differences in mortality patterns. Women have a lower mortality rate throughout the whole age range, although for childhood ages, from 0 to 6, values are similar. In teen and younger adult ages, Spain shows a higher mortality rate. Nevertheless, from adult to earlier retirement ages, Spain presents lower mortality rates.

These differences can be better explored by means of the excess mortality ra-

tio, calculated by dividing the Spanish age marginal component by the Danish one. This ratio, by gender, is represented in Figure 7. The maximum difference between Spanish and Danish women is attained at age 55, reflecting Spanish mortality to be approximately 40% lower than the Danish.

Figure 7. Excess mortality ratio. Spain versus Denmark

Figure 8 summarizes the evolution of mortality rates throughout the period, i.e. 1975-1993. In this sense, we can consider the marginally integrated estimator of the second component (i.e. chronological time) to be a mortality index (a summary measure of mortality for the whole age range), such as life expectancy at birth or the Comparative Mortality Factor; but weighting all ages equally.

Figure 8. Second component trends by country and gender

At the beginning of the period, in 1975, Denmark presented lower values than Spain for both genders. Although the second marginal curve has shown decreasing trend for both countries, the one for Spain has been more dramatic. Thus by 1993 Spanish men and women attained a lower calendar year marginally integrated estimate.

As a representation of the traditional comparative tools, the E75 ratio for 1990, as in Felipe, Guillén and Artís (1998), has been calculated for each country and gender. The ratios are obtained by dividing the estimates for each age and year by

the estimates for each corresponding age in 1975. Values greater than one mean that mortality has increased when comparing 1975 versus 1990, and vice versa.

Figure 9. E75 ratio for 1990 estimates. Spain and Denmark by gender

From Figure 9 it is noticeable that for Spanish men and women and for Danish men mortality has raised for the working active ages (40%, 5% and 13% respectively) and decreased for pensioners (18%, 30% and 8% respectively). This is, of course, a very dangerous development for the insurance industry and has to be investigated further. For Danish women development has been somewhat different: mortality has increased by 7%, but in the age range from 57 to 73.

Figure 10. Danish men estimated trends by age

It is also interesting to examine mortality trends by age. These are obtained by plotting the two-dimensional mortality estimates for a given age. In Figure 10, Danish men mortality trends are shown for working and retirement ages, by gender. Following previous analyses, decreasing mortality rates are observed except for age 40. At older ages, a recovering of the mortality rates occurs in the final years of the period.

7 Conclusions

The methodology we have used in the paper could be classified as an exploratory data analysis technique, especially designed to diagnose the two-dimensional (i.e., age and calendar time) behaviour of a mortality experience prior to the graduation of any mortality law. We have shown the usefulness of this methodology when applied to scarce data and when comparing mortalities in two dimensions simultaneously, because the multiplicative structure effectively separates the chronological time component and the age component and this is useful for a quick overview of mortality development.

Future theoretical developments will allow the calculation of confidence intervals for first and second marginal curves. This will imply a more accurate comparison between mortalities. Another refinement that should be studied in the future is the inclusion of interactions between time and age effects.

Although several boundary problems have been detected, the improvement reached

by applying local linear estimator instead of a local constant estimator must be further investigated. The local linear estimator is expected to behave better at the boundaries as shown by Nielsen (1998).

A natural development for this method would be to apply backfitting procedures to estimate the multiplicative structure of the hazard rate (see, for example, Hastie and Tibshirani, 1991). The adaptative methods used by Gavin, Haberman and Verrall (1995) also contain a backfitting approach. Linton (1998) shows that backfitting is expected to be more efficient than marginal integration and Linton and Nielsen (1999) introduce the backfitting estimator for hazard rates.

Appendix

The labels used in Figure 2 correspond to the following countries:

Label	Country
B	Belgium
DK	Denmark
D	Germany
E	Spain
F	France
IE	Ireland
I	Italy
NL	The Netherlands
P	Portugal
FIN	Finland
S	Sweden
UK	United Kingdom
A	Austria

References

- Andersen, P. K., Borgan, O., Gill, D.R. and Keiding, N. (1993). Statistical models based on counting processes. Springer Verlag, New York.
- Anderson, J.L. and Whitehead, D. (1960). The trend of mortality rates in seventeen countries in the last twenty years. Transactions of the 16th International Congress of Actuaries, 2, 379-400.
- Benjamin, B. and Pollard, J.H. (1980). The analysis of mortality and other actuarial statistics. Heinemann, London.
- Benjamin, B. and Soliman, A.S. (1995). Mortality on the move. City University Print Unit. London.
- Buus, H. (1960). Investigations of mortality variations. Transactions of the 25th International Congress of Actuaries, 2, 364-378.
- Copas, J.B. and Haberman, S. (1983). Non parametric graduation using Kernel Methods. Journal of the Institute of Actuaries, 110, 135-156.
- Cramer, H. and Wold, H. (1935). Mortality Variations in Sweden. A study in graduation and forecasting. Skandinavisk Aktuarietidskrift, 161-241.
- Felipe, A., Guillen, M. and Artis, M. (1998). Recent mortality trends in

the Spanish population". To appear in Transactions of the 26th International Congress of Actuaries, 9.

Forfar, D.O., McCutcheon, J.J. and Wilkie, A.D. (1988). On graduation by mathematical formula. Journal of the Institute of Actuaries, 115, 381-402.

Forfar, D.O. and Smith, D.M. (1988). The changing shape of English Life Tables. Transactions of the Faculty of Actuaries, 40, 98-133.

Fusaro, R., Nielsen, J.P. and Scheike, T. (1993). Marker dependent hazard estimation. An application to AIDS. Statistics in Medicine, 12, 843-865.

Gavin, J.B., Haberman, S. and Verrall, R.J. (1993). Moving Weighted Average Graduation Using Kernel Estimation. Insurance: Mathematics and Economics, 12, 113-126.

Gavin, J.B., Haberman, S. and Verrall, R.J. (1994). On the Choice of Bandwidth for Kernel Graduation. Journal of the Institute of Actuaries, 121, 119-134.

Gavin, J.B., Haberman, S. and Verrall, R.J. (1995). Graduation by Kernel and Adaptive Kernel Methods with Boundary Corrections. Transactions of the Society of Actuaries, 47, 173-209.

Giles, P. and Wilkie, A.D. (1973). Recent mortality trends: some international

comparisons (with discussion). Transactions of the Faculty of Actuaries, 33, 375-514.

Gwilt, R.L. (1956). Mortality in the past hundred years (with discussion) Transactions of the Faculty of Actuaries, 24, 40-176.

Haberman, S. and Renshaw, A.E. (1998). On the comparison of two mortality experiences. Transactions of the 26th International Congress of Actuaries, 6, 217-294.

Hastie, T.J. and Tibshirani, R.J. (1991). Generalized additive models. Chapman & Hall, London.

Linton, O.B. and Nielsen, J.P. (1995). A kernel method of estimating structured nonparametric regression based on marginal integration. Biometrika, 82, 93-101.

Linton, O.B. and Nielsen, J.P. (1999). Estimating multiplicative and additive marker dependent hazard functions by backfitting with the assistance of marginal integration. In progress.

Macdonald, A.S. (1996a). An actuarial survey of statistical models for decrement and transition data: I: multiple states, Poisson and binomial models. British Actuarial Journal, 2, 129-156.

Macdonald, A.S. (1996b). An actuarial survey of statistical models for decrement and transition data: II: competing risks, non parametric and regression models. British Actuarial Journal, 2, 429-448.

Macdonald, A.S. (1996c). An actuarial survey of statistical models for decrement and transition data: III: counting process models. British Actuarial Journal, 2, 703-726.

Macdonald, A.S. -Editor- (1997). The second actuarial study of mortality in Europe. Groupe Consultatif des Associations d'Actuaires des Communautés Europeennes.

Macdonald, A.S., Cairns, A.J.G., Gwilt, P.L. and Miller, K.A. (1998). An International comparison of recent trends in population mortality. British Actuarial Journal, 4, I, 3-141.

Nielsen, J.P. (1998). Marker dependent kernel hazard estimation from local linear estimation. Scandinavian Actuarial Journal, 2, 113-124.

Nielsen, J.P. and O.B. Linton. (1995). Kernel estimation in a marker dependent hazard model. Annals of Statistics, 23 1735-1748.

Olbricht, W. and Miller, K. (1998). Portfolio Monitoring: The Actuary as 'Controller'. Transactions of the 26th International Congress of Actuaries, 6, 381-402.

Ramlau-Hansen, H. (1983). The Choice of a Kernel Function in the Graduation of Counting Process Intensities. Scandinavian Actuarial Journal, 165-182.

Renshaw, A.E. and Haberman, S. (1997). Dual modelling and Select mortality. Insurance: Mathematics and Economics, 19, 105-116.

Renshaw, A.E., Haberman, S. and Hatzopoulos, P. (1996). Recent Mortality Trends in Male Assured Lives". British Actuarial Journal, 2, 449-477.

Wilkie, A.D. (1976). An International comparison of recent trends in population mortality. Transactions of the 20th International Congress of Actuaries, 2, 761-781.