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Specific taxation, asymmetric costs, and endogenous quality*

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Abstract

This paper shows how a specific tax - in contrast to an ad valorem tax - alters industry structure and firm-level performance in a monopolistic competition framework, where firms chose product quality endogenously and differ exogenously in productivity (i.e. marginal production efficiency). Industry equilibrium mechanisms and selection based on productivity play a significant role: A specific tax shifts market shares and profits toward firms with costs and prices above the industry average at the expense of low-cost firms. This reallocation of market shares releases a novel scale effect such that low-cost firms may quality downgrade, while high-cost firms always quality upgrade. There exists a parameter subspace, where this combines to a decrease in average quality for the industry. In comparison: An ad valorem tax only reduces the number of firms/varieties in the industry due to demand absorption, but affects neither firm-level performance nor industry structure.

JEL-codes: D43, D61, H23, H25, L11, L13

Keywords: specific tax, ad valorem tax, industry structure, monopolistic competition, heterogeneous firms, asymmetric costs, endogenous quality, political economy.

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1 Introduction

Taxes and the comparison of specific and ad valorem taxes in particular continue to be a central issue in formal public finance. Taxes interact with the economy not just through the provision of government revenue, but through their effects on the choices of individuals and business, i.e. tax policy and the actual design of tax tools interact with and affect markets. In this paper, we ask how a specific tax - as opposed to an ad valorem tax - affects quality choices, prices and profits across heterogeneous firms in an imperfect competitive market. Moreover, we ask how specific taxes, in a setting where firms choose quality endogenously, shape industry structure and industry aggregates in the long run.

The Alchian-Allen conjecture proposes that a specific shipping cost, i.e. a constant per unit cost, reduces the relative price of high-quality (and thus higher priced) products and therefore shifts demand and market shares toward higher quality. Related Barzel (1976) argued that a specific tax encourages firms to substitute quantity with quality in order to escape the tax, i.e. to quality upgrade. The present paper examines these seminal mechanisms in an augmented Dixit-Stiglitz monopolistic competition model, which accounts for industry equilibrium effects. In particular our formal model includes the empirical relevant phenomena of within-industry productivity heterogeneity (i.e. asymmetric costs) across firms (Syverson, 2011), fixed cost of production that increases in the quality of the product (Shaked and Sutton, 1983), and intra-industry reallocations including endogenous entry and exit of firms (Foster et al., 2001; Melitz, 2003). The inclusion of these empirical relevant phenomena into a theoretical framework allows us to identify novel effects that a specific tax - opposed to an ad valorem tax - has on quality and performance not only at the firm level but also at the aggregated industry level.

The key difference that arises from the inclusion of firm heterogeneity in marginal production efficiency (productivity) is the following: When firms differ in marginal costs, a specific tax - but not an ad valorem tax - distorts relative marginal costs. This, in turn, distorts relative prices as well as the relative qualities and affects firm-level performance measures under a specific tax. Most importantly, the heterogeneous responses across firms with different marginal costs add up and distort the overall industry structure. We find that the Alchian and Allen conjecture applies. A specific tax shifts market shares and profits from low-price firms (firms with prices below the quantity-weighted industry average) towards high-price firms. In fact, a specific tax reduces profits of low-price firms and increases profits of high-price firms. This finding also has political economy implications for firms’ preferences over taxation and tax instruments (specific vs. ad valorem) and provides an argument for two-way interdependence between tax design and industry structure.

Specific and/or ad valorem taxes are imposed for a wide range of products and sometimes simultaneously; including for example tobacco, gasoline and alcohol (see also Delipalla and Keen (2006) for a discussion). Moreover, and central for the current formal approach, empirical work has been able to identify effects from tax tool design on product quality. Sobel and Garret (1997) show for the US cigarette market that specific taxes have heterogeneous impacts across products of different qualities and prices. First, they show that generic (cheap) cigarettes were introduced in the market when the real value of the specific tax was at its minimum. Secondly, they show that higher specific taxes increase the market share of premium (expensive) cigarettes relative to (cheap) generic cigarettes, while an ad valorem tax has no impact. More recently, Ljunge
(2011) makes similar observations for the US wine market. The present model is exactly able to accommodate and replicate these and several other patterns.

Our theoretical finding is a combination of two effects. First, quality is affected by specific taxation through the well-known Barzel (1976) effect. In the presence of specific taxes all firms aim to increase quality and prices (and reduce quantity, i.e. the characteristic that is taxed) since this reduces the effective tax rate on revenue. Second, our paper arrives at a novel scale effect of specific taxes that relates to the Alchian and Allen mechanism of market share reallocation towards expensive products. Such a reallocation implies that the specific tax reduces the scale of low-marginal cost/low-price firms. Given a fixed cost of producing quality, such a scale reduction encourages quality downgrading for low-price firms. In contrast, for high-price firms the opposite will be true. In sum, the two effects result in the following: A specific tax unambiguously leads to quality upgrading for high-price firms, but has an ambiguous impact on the quality choice of low-price firms and thus may - for certain parameter subspaces - have an ambiguous effect for the industry as a whole. In contrast, for an ad valorem tax we find no such ambiguity. In fact, the ad valorem tax only affects the number of firms/varieties in the industry via demand absorption, but does not affect firm-level performance or the industry structure. Interestingly, despite the ambiguity that a specific tax may have for the aggregate quality level, it always raises the minimum quality level found in the industry. Endogenous selection of firms ensures that the lowest quality level able to survive in the industry is raised in response to an increase in a specific tax.

Firm heterogeneity is fundamental for the surprising and novel ambiguity that we identify for the effect that specific taxes may have on quality at the industry level. In particular, also in our framework the conventional prediction that a specific tax increases quality applies when we model firms as homogenous. At a more general level, these findings show that it is important to account for cost/productivity heterogeneity and industry dynamics (selection) also when one is only concerned with the aggregate/average effects from taxes on quality (as well as on prices, quantities, market shares and profits across the heterogeneous firms).

The novel scale effect and its interaction with taxation arises from the costs that firms face when they produce quality. Quality upgrading entails both higher fixed cost and higher marginal costs. The positive link between quality and costs is well-known in the industrial organization literature, see e.g. Shaked and Sutton (1983). It captures the fact that conceiving, designing, and producing a product that consumers are willing to pay a higher price for entails not only higher production costs but also higher fixed expenses associated with for example activities such as R&D, advertising, or quality control. Conditional on a given quality level, a high-productivity (that is lower marginal cost) firm has lower prices and therefore operates on a larger scale. A larger scale per se encourages quality upgrading (since a larger scale and thus larger operating profits will more easily cover the fixed costs associated with quality upgrading). Accordingly, the endogenous firm-level quality choice increases in firm-level productivity (efficiency). However, since marginal costs also increase with higher quality, the resulting relation between productivity (and thus profits) and prices becomes ambiguous and is determined by a parameter constraint.

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1 Related, Hummels and Skiba (2004) confirm the ’Washington apples effect’ proposed by Alchian and Allen using data on trade and shipping costs.

2 In Appendix C we provide an analysis of the case with homogenous firms.
including the sensitivity of demand and fixed costs to quality.\(^3\) It follows that parameter constraints determine whether it is high-profit or low-profit firms that benefit from a specific tax as opposed to an ad valorem tax.

The present paper contributes to several discussions in the literature. An important strand of literature comparing specific and ad valorem taxes under various forms of imperfect competition emerged after Suits and Musgrave (1953) formalized the Wicksell (1896) conjecture that ad valorem taxes may have favorable efficiency properties relative to specific taxes in monopoly markets.\(^4\) Until recently this literature extensively assumed identical costs across firms.\(^5\) However, in the words of Syverson (2011): 'Economists have shown that large and persistent differences in productivity levels across businesses are ubiquitous'. Schröder and Sørensen (2010) show that the welfare dominance of the ad valorem tax strengthens with cost asymmetries in a monopolistic competition model with CES preferences. Kaygusuz (2013) compares the two taxes conditional on aggregate output in a monopolistic competition framework with linear-quadratic demand, firm-specific mark-ups, and asymmetric costs. The present paper extends the works of Schröder and Sørensen (2010) and Kaygusuz (2013) by including endogenous quality choice.\(^6\) Few papers have included quality in the formal analysis of taxes. Recently, Wang et al. (2018) consider an oligopolistic framework with quadratic utility, an exogenous number of firms, and exogenous heterogeneity in quality and costs. They show that a specific tax may welfare dominate an ad valorem tax for equal impact on total output.\(^7\) Rojas and Shi (2011) show by simulation that the effect of a specific tax on the relative price, relative quantity, and relative advertising across quality levels is ambiguous in an oligopoly setting with an exogenous number of multi-product firms competing in prices and advertisement (quality is fixed). Even for competitive markets the equivalence of specific and ad valorem taxation vanishes when quality is endogenous: Liu (2003) shows that specific taxation results in higher quality than ad valorem taxation under three stylized preference structures.\(^8\) Delipalla and Keen (2006) derive the optimal mix of specific and ad valorem taxation, and Obara and Tsugawa (2017) compare the tax instruments when consumers decide on quality as well as quantity. The present paper contributes to the public economics literature by exploring the effects of specific (and ad valorem) taxation on industry structure and firm-level variables in a tractable imperfect competition setting with asymmetric costs, endogenous quality choice, and endogenous entry/exit.

In terms of modelling choices, the present paper formalizes the examination of specific and ad valorem taxes in a framework of monopolistic competition. This set-up provides analytical tractability and transparency and allows us to include novel key mechanisms running through endogenous entry/exit of firms and firm heterogeneity. Besides tractability and transparency, the

\(^{3}\)A similar ambiguity arises in Kugler and Verhoogen (2012) using different functional forms for the production function.

\(^{4}\)See e.g. the survey by Keen (1998) as well as Anderson et al. (2001a, 2001b).

\(^{5}\)Papers considering asymmetric costs across firms include Anderson et al. (2001a), Wang and Zhao (2009), Schröder and Sørensen (2010), Lapan and Hennessy (2011), Kaygusuz (2013), and Wang et al. (2018). See also Kotsogiannis and Serfes (2014) for a tax tool comparison when there is uncertainty regarding firms’ costs.

\(^{6}\)See Schröder (2004), Dröge and Schröder (2009) for comparisons of specific and ad valorem taxation under monopolistic competition with homogenous firms and exogenous quality in general equilibrium.

\(^{7}\)Also, Collie (2019) conducts - inter alia - a comparison of the two tax tools in an oligopoly framework (i.e. with large firms) and is able to include general equilibrium effects but abstracts from quality.

\(^{8}\)See also Bohanon and Van Cott (1991) and Kay and Keen (1991) for the importance of preference structures.
The monopolistic competition model with CES preferences has at least two further advantages. First, it is extensively applied in the macroeconomics literature and in the international economics literature\(^9\) and the mechanisms explored in the present paper should be straightforward to incorporate into these strands of literature. Second, since the industry structure conditional on aggregate expenditures is first-best in the absence of taxation, see Dhingra and Morrow (2019), one does not need to consider whether other policies could/should be used to improve efficiency when assessing welfare implications of taxation. Still, other models of imperfect competition are available. In particular the oligopoly type models mentioned above allow for strategic interaction effects which are absent in the present paper. However, Thisse and Ushchev (2018) show that monopolistic competition arises as the limiting case of oligopolistic competition when the number of firms approaches infinity and strategic interaction vanishes.\(^{10}\)

The present paper also relates to other fields of economics. There exists a substantial literature on endogenous quality choice of firms, see e.g. Podhorsky (2000) for a model where quality choices of firms relate to environmental damage\(^{11}\) or Fan et al. (2015) and Ludema and Yu (2016) for international trade applications. Moreover, international economics that studies effects of specific costs arising from shipping costs, specific tariffs, and shadow values of quotas (Hummels and Skiba, 2004; Dürceylan, 2012; Sørensen, 2014; Irarrazabal et al., 2015) and industrial organization that studies the effects of changes in prices of common input factors (Gaigné and Mener, 2014). The present paper complements these studies by allowing for endogenous quality choice in a heterogeneous firms monopolistic competition setting. While the paper considers taxation, all findings apply for any multiplicative (ad valorem) and additive (specific) costs per unit of output that is common across firms and is invariant to firm-level efficiency and quality. Such additive costs could, for example, be certain costs associated with distribution or the price of a common intermediate input or a raw material. Accordingly, the applications of our theoretical results are potentially more far-reaching than the mere tax incidence case highlighted here.

The remainder of the paper is organized as follows. Section 2 describes the monopolistic competition model with heterogeneous firms and endogenous quality. Section 3 analyzes how the specific tax affects firms and industry structure. Section 4 provides a discussion of results. Section 5 concludes. Appendices A to E provide technical details, derivations, special cases, extensions, and proofs.

\(^9\)See e.g. Melitz (2003) in the international economics literature and Bilbiie et al. (2012) in the macro literature.

\(^{10}\)Thisse and Ushchev (2018) also show that monopolistic competition for certain functional forms preserves several of the properties of oligopolistic competition. Future research may be able to consolidate these approaches further. Recently, the literature on oligopolistic competition has been able to improve in terms of tractability by exploiting aggregative game approaches, see Nocke and Schutz (2018) for a framework with an exogenous number of heterogeneous multi-product firms and Anderson et al. (2020) for a framework with single-product heterogeneous firms and endogenous entry. See also the above mentioned Collie (2019).

\(^{11}\)Apart from the interest in quality choices, Podhorsky (2000) also applies a monopolistic competition framework and compares environmental certification with a unit tax imposed on environmental damage.
2 Monopolistic competition with endogenous quality

In this section we extend the seminal Dixit-Stiglitz monopolistic competition model. Firms choose quality endogenously and are heterogeneous due to underlying exogenous differences in production efficiency (productivity). Tax revenue is assumed to be used outside the industry of interest, e.g., for public employment, thus our formulation ignores feedback effects running through the public budget. The analysis is restricted to steady state equilibria and thus describes long-run effects.

Households
Preferences of the representative household over the endogenous set of goods/varieties within the industry of interest ($\Omega$) are given by

$$U = \left[ \int_{\omega \in \Omega} [q(\omega) c(\omega)]^{\frac{1}{\sigma-1}} d\omega \right]^{\frac{\sigma}{\sigma-1}},$$

(1)

where $c(\omega)$ is consumption of variety $\omega$, $q(\omega)$ is quality of variety $\omega$, and $\sigma > 1$ is the elasticity of substitution between goods. The multiplicative interaction of quantity and quality implies that the elasticity of substitution between quantity and quality equals unity and does not vary with taxation. Moreover, households care about quality-adjusted prices, $p(\omega) / q(\omega)$, where $p(\omega)$ is price of the variety $\omega$. This quality specification is known as the lightbulb specification, see Saving (1982). Aggregate expenditures $E$ is treated as exogenous\(^\text{12}\) and aggregate demand for variety $\omega$, $c(\omega)$, reads

$$p(\omega) c(\omega) = \left( \frac{p(\omega)}{q(\omega)} \right)^{1-\sigma} E P^{\sigma-1}$$

(2)

for all $\omega \in \Omega$, where $P$ is the price index of the industry defined as $P = \left( \int_{\omega \in \Omega} \left( \frac{p(\omega)}{q(\omega)} \right)^{1-\sigma} d\omega \right)^{-\frac{1}{\sigma-1}}$.

Firms’ costs and taxes
Monopolistic firms produce with a single (possible composite) input factor, the price of which we normalize to unity. Each firm produces a unique variety and firms and varieties become synonymous. In order to enter the industry, firms face sunk costs (also paid in units of the input factor) for developing a new variety, $F_E > 0$. Each new variety is randomly associated with a firm-specific marginal production efficiency ($\varphi(\omega)$) drawn from a known distribution $G(\varphi)$ with continuous support ($\varphi_{\text{min}}, \varphi_{\text{max}}$) and no mass points.\(^\text{13}\) This may be interpreted as randomness in the R&D process of developing a new variety. We can ease notation: Due to symmetry in preferences and since cost structures - conditional on exogenous marginal efficiency $\varphi$ - are the same across varieties we suppress $\omega$ in the following. Put differently, marginal production efficiency, $\varphi$, is a sufficient statistic for firm heterogeneity. The marginal costs of producing one

\(^{12}\)If tax revenue is used for public employment a Cobb-Douglas upper-tier utility function is consistent with constant/exogenous aggregate expenditure. In fact, due to the constant elasticity of substitution (CES) preferences the aggregate expenditure level has no impact on firm-level outcomes or industry structure, but matters for the number of firms only. Hence, the loss of generality is minimal and vanishes for equal yield comparisons of ad valorem and specific taxes.

\(^{13}\)Assume that $G(\varphi)$ is differentiable and let $g(\varphi) \equiv G'(\varphi)$. 
unit of the product with quality $q$ for a firm with efficiency $\varphi$ read $mc(\varphi, q) = q^\gamma / \varphi$, where $\gamma \geq 0$ captures the sensitivity of marginal costs to the endogenously chosen and firm-specific level of quality. Also the fixed costs of production increase in quality and are given by $F(q) = F_0 + f q^\alpha$, where $F_0 > 0$, $f > 0$, and $\alpha > 0$, captures the sensitivity of fixed costs to quality.\footnote{Fan et al. (2015) analyze the link from lower tariffs on imported intermediate goods to quality choice of exports using a partial equilibrium model in which quality has similar impacts on demand, variable costs, and fixed costs as in the present model. Also Ludema and Yu (2016), in their analysis of tariff pass-through employ a quality upgrading structure with fixed and variable costs. See also Shaked and Sutton (1983).}

Firms are subject to a specific ($t \geq 0$) and an ad valorem ($\tau \geq 0$) tax, and thus marginal costs become $mc(\varphi, q, t) = q^\gamma / \varphi + t$. Crucial for the subsequent analysis the cost share of the specific tax decreases with marginal costs of production, i.e. the specific tax has a relatively larger impact on costs in low-cost firms.

### Profit maximization and firms’ quality choice

Due to a constant elasticity of demand the profit-maximizing price is a constant markup on marginal costs inflated by the ad valorem tax, i.e. $p(\varphi, q, t) = \frac{\sigma}{\sigma - 1} (1 + \tau) mc(\varphi, q, t)$. Reduced form profits become

$$\pi(\varphi, q, \Lambda, t) = \Lambda \left(\frac{\gamma}{\varphi + t}\right)^{1-\sigma} q^{\sigma - 1} - F_0 - f q^\alpha,$$

where $\Lambda \equiv \frac{1}{\sigma - 1} \left(\frac{\sigma}{\sigma - 1}\right)^{-\sigma} EP^{\sigma - 1}$ is a demand shifter that is common across firms and perceived as exogenous for the individual firm albeit endogenously determined in industry equilibrium. Inspection of (3) gives some first insights on the comparison of specific and ad valorem taxes in the present framework. The ad valorem tax, $\tau$, operates through its effect on the demand shifter, $\Lambda$, while the specific tax, $t$, interferes with firms’ decisions depending on their drawn marginal production efficiency. Hence, we can conjecture that an ad valorem tax will not interfere with heterogeneity and selection.\footnote{This result mirrors findings in Schröder and Sorensen (2010) presenting a monopolistic competition setting with heterogeneous firms but without product quality.}

Firms choose their quality level such as to balance the gain from higher demand against the higher fixed and variable costs of producing quality. Optimal quality satisfies the first-order condition

$$\frac{\partial \pi(\varphi, q, \Lambda, t)}{\partial q} = \Lambda \left(\frac{\gamma}{\varphi + t}\right)^{1-\sigma} q^{\sigma - 1} (\sigma - 1) \frac{1}{q} \left(\frac{1 - \gamma}{\varphi + t}\right) - \alpha f q^{\alpha - 1} = 0. \quad (4)$$

For the first-order condition to hold for $q > 0$ and $t = 0$, it is required that $\gamma < 1$.\footnote{For the first-order condition to hold for $q > 0$ and $t = 0$, it is required that $\gamma < 1$. Hence, we restrict attention to the parameter subspace where $\gamma \in [0, 1)$. Conditional on quality, firms with higher efficiency, $\varphi$, have lower prices and therefore operate on a larger scale. Accordingly, these firms gain more from an increase in quality and therefore choose higher quality levels,}
Because endogenous quality increases in efficiency, it is generally undetermined whether firms with higher efficiency (\( \varphi \)) and thus higher profits have higher or lower marginal costs/prices.\(^{17} \) From the first-order condition (4) - after imposing the second-order condition - it follows that

\[
\text{sign} \left[ \frac{dmc}{d\varphi} \frac{\varphi}{mc} \right] = \text{sign} \left[ (\sigma - 1) - \alpha \right].
\]

Hence, more efficient firms with higher profits will - as a result of their chosen product quality level - have higher marginal costs/prices if \((\sigma - 1) > \alpha\) (while the opposite is true when \(\sigma - 1 < \alpha\)). Why is this so? The \((\sigma - 1) > \alpha\) condition states that the sensitivity of demand to quality (conditional on price) exceeds the sensitivity of fixed costs of production to quality and implies a strong motive to quality upgrade for large (high-efficiency) firms. In this case the positive monotone relationship between efficiency and quality implies a qualitatively similar relation between quality and marginal costs/prices, i.e. if \((\sigma - 1) > \alpha\), firms with higher quality have higher marginal costs/prices and vice versa.

**Lemma 1** Quality and profits increase in efficiency and there is a positive (negative) association between efficiency and marginal costs/prices iff \((\sigma - 1)\) is larger (smaller) than \(\alpha\).

**Proof.** See text. Note that the second-order condition requires that \(\alpha - (\sigma - 1) + \left( \frac{\sigma q^\gamma / \varphi}{q^\gamma / \varphi + t} - \frac{(1-\gamma)q^\gamma / \varphi}{(1-\gamma)q^\gamma / \varphi + t} \right) \gamma > 0\), i.e. there is a lower bound for \(\alpha\) for which the equilibrium is well-behaved.

For later reference we label the two theoretically possible cases that emerge from the above Lemma.

**Definition 1** We refer to the case of \((\sigma - 1) > \alpha\) as the ‘quality case’.

**Definition 2** We refer to the case of \(\alpha > (\sigma - 1)\) as the ‘efficiency case’.

In the quality case the endogenous response of quality to efficiency is so strong that higher production efficiency, \(\varphi\), is associated with higher marginal costs as a result of quality upgrading.\(^{19} \) Conversely, in the efficiency case the initial ranking of firms according to production efficiency matches the resulting ranking in terms of marginal costs, i.e. firms with high production efficiency have low marginal costs. Since profits increase in marginal production efficiency, it follows that more profitable firms have higher prices in the quality case and lower prices in the efficiency case. In other words, the quality case resembles the common perception that high-quality goods will also be high-priced goods, while the opposite is true in the efficiency case. Obviously, the

\(^{17}\)Formally this follows from the second-order condition \(\frac{\partial^2 \pi(\varphi, \varphi, \Lambda, t)}{\partial \varphi^2} \bigg|_{\varphi = \varphi^*} > 0\) and that

\[
\frac{\partial^2 \pi(\varphi, \varphi, \Lambda, t)}{\partial \varphi^2} \bigg|_{\varphi = \varphi^*} = \alpha f \gamma^{\alpha - 1} \frac{\varphi^{\gamma - 1}}{\varphi^{\gamma + t}} - (1-\gamma)q^\gamma / \varphi^{\gamma + t} > 0.
\]

\(^{18}\)A similar ambiguity arises in Kugler and Verhoogen (2012) using different functional forms for the production function.

\(^{19}\)In the special case where quality has no impact on marginal costs, i.e. \(\gamma = 0\), the second-order condition requires \(\alpha > (\sigma - 1)\) and therefore rules out the quality case. For \(\gamma = 0\) the upward pressure on marginal costs and thus prices from higher quality disappears.

8
actual relation between quality and price will differ for different sectors and is ultimately an empirical question. Notably, in a model of tariff pass-through Ludema and Yu (2016) - albeit featuring variable markups - also arrive at two possible cases for the relationship between quality and price, which in their model depends on the scope for quality differentiation in products.\textsuperscript{20} Their empirical investigation, based on US transaction level exports, suggests that both cases are present in the data. Moreover, Jäkel and Sørensen (2020) show for Danish manufacturing that (on average) exported and thus better performing varieties have higher quality\textsuperscript{21} and lower price than their non-exported counterparts.

\textbf{Completing the model}

The model is completed with the following conditions.\textsuperscript{22} The zero profit condition determines the lowest production efficiency consistent with non-negative profits and reads

$$\pi (\varphi^*, q^*, \Lambda, t) = \Lambda ((q^*)^\gamma / \varphi^* + t)^{1-\sigma} (q^*)^{\sigma-1} - F_0 - f (q^*) = 0,$$

where $q^* \equiv q (\varphi^*)$, and $\varphi^*$ is the efficiency threshold, such that firms with efficiency draws $\varphi < \varphi^*$ exit the industry due to negative profits, while firms with $\varphi \geq \varphi^*$ stay in the industry, produce, and earn non-negative profits.\textsuperscript{23}

Free entry and exit ensure that firms enter the industry (i.e. participate in drawing efficiencies at sunk entry cost $F_E$) until the expected net present value of the stream of profits equals the entry costs. The intertemporal discount rate is assumed to be zero. Net present values of firms are kept finite and entry in equilibrium is ensured by assuming that firms in each period after entry die with constant and exogenous probability $\delta > 0$.\textsuperscript{24} The free entry condition reads

$$\int_{\varphi^*}^{\varphi_{\text{max}}} \pi (\varphi, q (\varphi), \Lambda, t) dG (\varphi) = \delta F_E$$

Finally, mass $M$ of firms/varieties ensures that aggregate revenues equal aggregate expenditures, i.e.

$$E = M \int_{\varphi^*}^{\varphi_{\text{max}}} r (\varphi, q (\varphi), \Lambda, t) \mu (\varphi) \, d\varphi$$

where $r (\varphi, q (\varphi), \Lambda, t) \equiv p (\varphi, q (\varphi), t) c (\varphi, q (\varphi), \Lambda, t)$ denotes revenue and $\mu (\varphi) \equiv g (\varphi) / [1 - G (\varphi^*)]$ denotes the density of efficiencies among producing firms.

The first-order condition for quality (4), the zero-profit condition (5), the free entry condition (6), and the market clearing condition (7) pin down the endogenous variables, i.e. \{q (\varphi), M, \Lambda, \varphi^*\}.

\textsuperscript{20}To be precise, Ludema and Yu (2016) employ a quadratic utility function instead of the CES specification applied in our model. Accordingly, in their model price is not just a simple constant markup over marginal cost.

\textsuperscript{21}Quality is in Jäkel and Sørensen (2020) inferred through demand residuals applying the Khandelwal et al. (2013) approach.

\textsuperscript{22}We use here a modeling strategy within the Dixit-Stiglitz specification that resembles the steps employed in Melitz (2003) and the literature that followed. See also Dhingra and Morrow (2019) for further discussion.

\textsuperscript{23}Appendix D considers the case where $\varphi^* < \varphi_{\text{min}}$ and the selection channel is absent.

\textsuperscript{24}By setting $\delta = 1$, we would obtain a static model with the same qualitative properties as those of the steady states of the dynamic model presented here.
3 Effects of taxes

This section explores the impact of specific and ad valorem taxes, on firm-level and industry-level variables. The presence of a specific tax implies that there is no closed-form solution. Nevertheless, a wide range of results for the tax comparison of ad valorem and specific taxes can be derived. Most importantly, an ad valorem tax - as conjectured above - operates only by the absorption of purchasing power through the demand shifter \( \Lambda \). We can state:

**Proposition 1** The ad-valorem tax, \( \tau \), does not affect firm-level quantity, quality, and net-of-tax price choices; it has no effect on firm-level profits, selection and thus industry structure, but reduces the number of firms in the industry through demand absorption.

**Proof.** See Appendix A for derivation. ■

It follows from Proposition 1 that we can focus our remaining analysis on the presentation of results for the specific tax case.

**Firm-level profits**

From Section 2 we know, that an increase in the specific tax increases marginal costs, which in turn increases the price charged by firms and reduces profits. Since the increase in marginal costs is relatively larger for low-cost firms and given constant elasticity of demand, the low-cost firms are affected disproportionately. This partial equilibrium effect is captured by the second term in Equation (8). In addition to this direct effect, there is an industry equilibrium effect. The changes in the prices of competitors as well as the change in the equilibrium number of firms that are viable active on the market do increase the price index and thus the demand shifter. In other words, the residual demand curves shift outwards as a result of an increase in the specific tax. The demand shifter adjusts until the free entry condition (6) is met, i.e. until expected net present value of profits prior to entry equals the entry costs. This industry equilibrium effect is captured by the first term in Equation (8). While the direct effect hurts low-cost firms disproportionately, the positive industry equilibrium effect works proportional to producers’ surplus. In sum, low-cost firms are relatively worse off than high-cost firms. Furthermore, in absolute terms low-cost firms are worse off while high-cost firms gain from an increase in the specific tax (Recall, that which firms are actually high-cost and low-cost, respectively, depends on the endogenous quality choice). Formally,

\[
\frac{d\Lambda}{dt} = \Lambda (mc(\varphi, q, t))^{1-\sigma} q^{\sigma-1} \left( \frac{d\Lambda}{dt} - \frac{(\sigma - 1)}{mc(\varphi, q, t)} \frac{\partial mc(\varphi, q, t)}{\partial t} \right) \quad (8)
\]

where \( \bar{p}(\varphi^*, t) (\bar{mc}(\varphi^*, t)) \) is the quantity-weighted average price (marginal costs including the unit tax) across firms.

**Proposition 2** Firm-level profits increase (decrease) with the specific tax for firms with price/marginal costs above (below) the quantity weighted industry average.

**Proof.** Follows directly from Equation (8). See Appendix A for derivation. ■
Importantly, it is always the case that some firms are better off and some are worse off after a change in the specific tax. Contrast this to a change in the ad valorem tax. While a specific tax has distributional effects across the heterogeneous firms, an ad-valorem tax has no affect on firm-level profits once industry equilibrium effects are taken into account. Moreover, the distributional consequences of a specific tax have political economy implications: high- and low-cost firms have different preferences over the economic design of tax schemes, with high-cost (low-cost) firms preferring a specific (an ad-valorem) tax. Hence, the tax instruments affect the price distribution of an industry and the price distribution of an industry affects the lobbying potential effort expended in favor of a given tax instrument.

The finding \( \frac{d}{dt} = c(p - \bar{p})/\bar{p} \) from (8) holds under a much wider set of assumptions than the ones applied in the above analysis. The ingredients required for our result are the envelope theorem (changes in quality and prices/output have second-order effects on profits) and the binding free entry condition that determines the shift in the residual demand curves. In fact, the finding extends to any utility function of the type \( U = h(R_u(q^*c^*))d\omega) \), where \( h(\cdot) \) is strictly increasing and continuous differentiable and \( u(\cdot) \) is strictly increasing, concave, and thrice continuous differentiable.25 Similarly, the functional forms of how fixed and variable costs are related to quality are not important for this finding (as long as the problem of the firm is well-defined).

**Selection and average production efficiency**

Profit changes affect selection based on profits/productivity and thereby average production efficiency across firm. Formally, we have that26

\[
\frac{d\varphi^*}{dt} \frac{1}{\varphi^*} = \frac{1}{mc(\varphi^*)} \left( 1 - \frac{mc(\varphi^*, t)}{p(\varphi^*, t)} \right) = \frac{1}{mc(\varphi^*)} \left( 1 - \frac{p(\varphi^*, t)}{\bar{p}(\varphi^*, t)} \right)
\]

An increase in the specific tax benefits high-price firms and hurts low-price firms, cf. Proposition 2. Hence, in the quality case (defined in Section 2), where low-profit firms are those with lower prices (costs), i.e. \( \bar{p}(\varphi^*, t) > p(\varphi^*, t) \), a tax increase hurts the low-profit firms and squeezes the least profitable firms out of the market. This results in a higher efficiency threshold. Conversely, in the efficiency case where low-profit firms are those with higher prices (costs), i.e. \( \bar{p}(\varphi^*, t) < p(\varphi^*, t) \), an increase in the specific tax benefits low-profit firms and allows firms with efficiency below the initial efficiency threshold to make non-negative profits.

**Proposition 3** The efficiency threshold, \( \varphi^* \), increases (decreases) in the specific tax in the quality (efficiency) case.

**Proof.** See equation (9) noting that \( \bar{p}(\varphi^*, t) > p(\varphi^*, t) \) in the quality case and \( \bar{p}(\varphi^*, t) < p(\varphi^*, t) \) in the efficiency case.

The effect on average marginal production efficiency (productivity) defined as \( \tilde{\varphi} = \int_{\phi^*}^{\phi_{\text{max}}} \frac{dG(\phi)}{\phi - \phi^*} \) follows directly, as average productivity is affected through the extensive margin only, i.e.

\[
\frac{d}{dt} = (\tilde{\varphi} - \varphi^*) \frac{d\tilde{\varphi}}{dt} \quad \text{where} \quad \tilde{\varphi} > \varphi^*.
\]

25 We show this in Appendix E.
26 See Appendix A for derivation.
Corollary 1 Average production efficiency increases (decreases) in the specific tax in the quality (efficiency) case.

Proof. Follows directly from the text and Proposition 3.

This is an important and hitherto overlooked channel by which tax tool design affects the economy. In the quality case, where the most efficient firms engage so substantially in quality upgrading that high-quality products have high prices, an increase in the specific tax raises the average production efficiency in the industry. In contrast, in the efficiency case, where the firms with high production efficiency have low marginal costs and prices, also after investing in quality upgrading, an increase in the specific tax shrinks the average production efficiency in the industry.

Firm-level quality and average quality

Apart from effects on average production efficiency our model also arrives at novel results for the interaction of specific and ad valorem taxes with quality. From the first-order condition for optimal quality, see Equation (4), it follows that the effect of an increase in the specific tax on firm-level quality reads

$$ \frac{d q}{dt} \Psi(q) = \frac{d \Lambda}{dt} \frac{1}{q} - (\sigma - 1) \frac{\partial mc(\varphi, q, t)}{mc(\varphi, q, t)} - \frac{\sigma(\frac{\partial mc(\varphi, q, t)}{mc(\varphi, q, t)} - \frac{\partial mc(\varphi, q, t)}{mc(\varphi, q, t)} \frac{q}{mc(\varphi, q, t)})}{1 - \frac{\partial mc(\varphi, q, t)}{mc(\varphi, q, t)}} $$

where $\Psi(q) = -\alpha f^q = \left(\frac{\partial^2 \sigma(q, \varphi, \Lambda, t)}{mq} \right)^{-1} > 0$ cf. the second-order condition. The first term, $\frac{d \Lambda}{dt} \frac{1}{q} > 0$, is an industry equilibrium effect and captures the increase in the demand shifter due to an increase in the specific tax, cf. above. The larger demand shifter encourages quality upgrading as firms thereby spread the increase in fixed costs from quality upgrading over more units. The remaining terms capture the partial equilibrium effect. On the one hand, a higher specific tax increases marginal costs, which reduces the scale of the firm and thereby discourages quality upgrading; this is the term $- (\sigma - 1) \frac{\partial mc(\varphi, q, t)}{mc(\varphi, q, t)}$. On the other hand, a higher tax reduces the importance of quality in marginal costs - the cost share of the specific tax increases - and this encourages quality upgrading; this is the term $\frac{\sigma(\frac{\partial mc(\varphi, q, t)}{mc(\varphi, q, t)} - \frac{\partial mc(\varphi, q, t)}{mc(\varphi, q, t)} \frac{q}{mc(\varphi, q, t)})}{1 - \frac{\partial mc(\varphi, q, t)}{mc(\varphi, q, t)}} > 0$, which is the mechanism introduced by Barzel (1976). Importantly, the partial equilibrium effect is in general ambiguous.

Inserting the industry equilibrium effect gives us

$$ \frac{d q}{dt} \Psi(q) = (\sigma - 1) \left( \frac{1}{mc(\varphi^*, t)} - \frac{1}{mc(\varphi, t)} \right) + \left( \frac{1}{mc(\varphi, t)} - \gamma mc(\varphi, 0) - \frac{1}{mc(\varphi, t)} \right) $$

The first term is the novel scale effect - namely, a combined effect of higher marginal costs and a higher demand shifter - and is positive (negative) for high-cost firms that gain (lose) market shares from an increase in the specific tax. The second term is the Barzel (1976) effect, which for $\gamma > 0$ encourages quality upgrading since the effect from quality on marginal costs is reduced. The combined effect is positive for high-cost firms and ambiguous for low-cost firms.
We now turn to the minimum quality level that survives in the industry, i.e. the quality of the 
firm that is exactly indifferent between exiting the industry or staying and producing. It follows 
from the zero-profit condition, Equation (5), and the first-order condition for quality evaluated 
for the marginal firm, Equation (4), that

$$\frac{dq^*}{dt} \frac{1}{q^*} = \left(1 + \frac{dq^*}{dt} \frac{t}{\varphi}\right) \gamma mc(\varphi^*) \left(\frac{F_0}{F_0 + f(q^*)} + \frac{\gamma mc(\varphi^*) t}{mc(\varphi^*) (1 - \gamma) t + mc(\varphi^*, t)}\right)^{-1} > 0$$  

(11)

for $\gamma > 0$ since $1 + \frac{dq^*}{dt} \frac{t}{\varphi} = \frac{mc(\varphi^*, t)}{mc(\varphi^*)} \frac{mc(\varphi^*)}{mc(\varphi^*)} > 0$. Hence, when taking the endogenous selection 
of firms into account, the minimum quality supplied in the industry unambiguously increases 
in the specific tax. The reason being that the potential quality downgrading by the marginal 
firm (with the lowest quality) in the quality case, i.e. when low-profit firms are low-cost firms, 
is dominated by the tougher selection, which squeezes low-quality firms out of the market.

**Proposition 4** Firms with above 'average’ marginal cost/price upgrade quality following an in-
crease in the specific tax, while it is ambiguous whether firms with below 'average’ marginal 
cost/price upgrade or downgrade quality. Further, the minimum quality supplied in the industry 
always increases in the specific tax.

**Proof.** Follows directly from Equation (10) and from Equation (11).

Turning to average quality, $\bar{q} = \int_{\varphi^*}^{\varphi^{max}} q(\varphi) \frac{dG(\varphi)}{1 - G(\varphi^*)}$, we have that

$$\frac{d\bar{q}}{dt} = (\bar{q} - q^*) \frac{g(\varphi^*) d\varphi^*}{1 - G(\varphi^*)} + \int_{\varphi^*}^{\varphi^{max}} \frac{dq(\varphi)}{dt} \frac{dG(\varphi)}{1 - G(\varphi^*)},$$  

(12)

where $\bar{q} > q^*$ and $dq(\varphi)/dt$ follows from Equation (10). The first term, the extensive margin, 
captures the effect running through selection. The extensive margin effect is positive in the 
quality case as low-quality firms are squeezed out of the market and oppositely negative in the 
efficiency case where selection softens. The intensive margin effect is in general ambiguous, cf. 
above, and so is the total effect. Hence, it is in general ambiguous how average quality in the 
industry responds to a change in the specific tax. If considering output weighted average quality, 
the Alchian and Allen effect will have an additional effect as weights are shifted toward high-price 
varieties (which can be of higher or lower quality than low-price varieties).

The following numerical analysis highlights the finding that average quality may fall due to 
a specific tax but not due to an ad valorem tax in the efficiency case.$^{27}$ Figure 1 plots two 
measures of average quality (output weighted average and a simple across firms average) for 
a specific tax $t = 0.05$, expressed as a ratio of the same quality average under an ad valorem tax 
(namely without a specific tax: $t = 0$). This ratio is given for $\gamma$ from 0 to 0.7. If this ratio is 
below one, quality is falling in the presence of a specific tax.

$^{27}$The numerical example uses the following parameter values: The efficiency distribution is assumed to be 
Pareto distributed with shape parameter 3 and location parameter 1, i.e. $\varphi_{min} = 1$ and $\varphi_{max} = \infty$. Further, 
$\sigma = 1.8$, $\alpha = 1.5$, $\delta = 0.15$, $F_G = 3$, $f = 1$ and $F_0 = 1$. Note, that the numerical exercise aims not to be an 
empirically calibrated version of the model, but simply illustrates that a specific tax may reduce average quality 
for certain parameter values.
We see that the ratio reacts to the sensitivity of marginal costs to quality, $\gamma$. Recall, for $\gamma = 0$, the Barzel (1976) effect on quality vanishes. Figure 1 displays that the specific tax leads to quality downgrading (on average) for low values of $\gamma$, i.e. when the Barzel (1976) effect is weak. This numerical analysis accentuates the ambiguity of the effect of the specific tax on average quality that can occur in the efficiency case. We can state:

**Corollary 2** There exists a parameter subspace for the efficiency case such that an increase in the specific tax rate reduces (output weighted) average quality of the industry.

**Proof.** Numerical analysis. 

4 Discussion

It is instructive to explore three special cases that emphasize the different mechanisms present in the above framework and relate several of the above results to existing literature and possible future research.

First, consider the special case where the cost of higher quality is only in terms of higher fixed costs, but has no impact on marginal costs, i.e. $\gamma = 0$. In this case quality may be reinterpreted as pure marketing/branding, see e.g. Rojas and Shi (2011). We note immediately, that for $\gamma = 0$ only the efficiency case is well-behaved, i.e. $\alpha > \sigma - 1$, as firms otherwise choose infinitely high
quality. Hence, the specific tax increases profits of low-profit firms with low efficiencies and reduces profits of high-profit firms with high efficiencies (Proposition 2). Importantly, the endogenous selection implies that a higher specific tax reduces average efficiency in the sector (Proposition 3). The effect on firm-level quality simplifies to \( \frac{d\Psi(q)}{dq} = (\sigma - 1) \left( \frac{1}{mc'(q,t)} - \frac{1}{mc(q,t)} \right) \), i.e. the Barzel (1976) effect vanishes and only the novel scale effect remains, and the ambiguity in Proposition 4 is resolved. Firms with below (above) average marginal costs downgrade (upgrade) quality. For the effect on average quality the extensive effect is negative as the efficiency case applies.

Second, consider the special case of homogenous firms, which is a frequently applied formal set-up in the literature comparing specific and ad valorem taxes when product quality matters (see for example Liu, 2003; Delipalla and Keen, 2006). In this case the standard Barzel (1976) finding holds (for \( \gamma > 0 \)) and the novel scale effect found in our framework vanishes. Firms will always upgrade quality in response to a specific tax in order to avoid taxation. However, the higher fixed costs associated with higher quality combined with the demand-absorbing effect of the tax reduce the number of firms supported by the market.

Third, consider the special case of heterogeneous firms with homogenous and exogenous quality across firms (resembling Schröder and Sørensen, 2010). Again the industry is in the efficiency case and the specific tax benefits low-efficiency (and low-profit) firms and hurts high-efficiency (high-profit) firms. Moreover, it follows that the specific tax - compared to an ad valorem tax - reduces average firm size (revenue), average efficiency, and as a matter of fact the Herfindahl index of industry concentration.

**Welfare ranking and optimal tax structure**

While the focus of the present paper is on firm-level implications (including distributional aspects and quality responses) as well as on aggregate industry outcomes, the welfare effects of tax policy tools can also be addressed. The classic exercise of welfare ranking of ad valorem and specific taxation in the above framework with heterogeneous firms, monopolistic competition, and endogenous quality turns out to repeat the common finding of ad valorem taxes being superior. To see why, consider the following: It is well-known from Dhingra and Morrow (2019) that the combination of CES preferences and monopolistic competition implies that the non-distorted market equilibrium is first-best. Appendix B confirms that this finding extends to the present framework with endogenous quality choice. Accordingly, the optimal tax structure (ignoring political economy and distributional issues) which minimizes the price level, \( P \), conditional on a given tax revenue, has a specific tax of zero and relies purely on ad valorem taxation.

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28. Equation (4) shows that the derivative of profits with respect to quality is strictly increasing in quality for the quality case \( (\alpha < \sigma - 1) \) when \( \gamma = 0 \). The second-order-condition is violated in this case and the solution to Equation (4) is a local minimum.

29. See Appendix C for technical details of the homogenous firms case.

30. The CES preferences imply uniform markups, which in turn - for the non-distorted market equilibrium - ensure alignment between relative prices and relative marginal costs. Given constant elasticity of demand this ensures optimal relative output across producing firms. Moreover, the CES preferences warrant that the non-distorted market equilibrium has the optimal trade-off between number of varieties (due to love of variety) and economies of scale in production (due to fixed costs). Similarly, the non-distorted market equilibrium features the efficient selection of firms into production.
Thus, throughout the various cases studied in the above framework the ad valorem tax welfare dominates the specific tax.

**Testable implications for future research**
While the present paper has focused on a formal analysis of a long standing question in public economics, it is instructive briefly to highlight the testable implications from our model for future empirical research. First, the elasticity of quality with respect to a specific tax depends on the initial price of the variety. Quality measures to test this hypothesis may not, unlike Champagne due to expert rankings (Crozet et al. 2012), be observable for most products. However, a promising avenue for future research might be the Khandelwal et al. (2013) approach, which infers quality differences from demand residuals, i.e. from sales conditional of consumer price. Second, the endogenous quality response at the core of the present model implies that pass-through of the specific tax on price depends on the initial prices of the variety. Empirical applications and tests may apply variation in specific taxation, but could also build on variation in non-tax marginal costs that are additive and independent of efficiency and quality. This could be changes in prices of raw materials, costs of distribution, taxes on packing materials etc. Third, future research could attempt to explore the tax preferences of smaller and larger firms, i.e. if larger firms actually prefer certain tax designs such as specific taxes or ad valorem sales tax. Lobbying data, for example from US tax bills, could potentially allow researchers to identify variation across firm size.31 Last but not least empirical research within international economics, where trade barriers involve ad valorem and specific tariffs, might be informed by our formal considerations.

5 Conclusion
The present paper complements existing literature on the effects of taxation by exploring the effects of specific (and ad valorem) taxation in a monopolistic competition framework with heterogeneous (i.e. asymmetric) firms and endogenous quality choice. The seminal mechanisms of the Alchian and Allen conjecture and Barzel (1976) are present, but the combination of asymmetric costs across firms and fixed costs that depend on the chosen level of quality introduces a novel link from specific taxation to quality that operates through a scale effect. A key implication is that specific taxation may reduce quality across a sub-set of firms and for the industry on average.

Specific taxation and ad valorem taxation have qualitatively different effects across firms as well as for the industry aggregate. While ad valorem taxation only affects the number of products (firms), it is shown that specific taxation distorts firm-level outcomes and the industry equilibrium. From a political economy perspective it is important that specific taxation always benefits some firms and hurts others depending on their location in the price distribution. Hence, the shape of the initial price distribution affects lobbying for taxation and/or tax tool design (ad valorem versus specific) and as shown in the paper the use of specific taxation affects the price

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31 This approach may not be as far fetched as it appears on first sight. Recently, Blanga-Gubbay et al. (2020) have been able to use this type of data to explore the role of firms in the political economy of trade agreements as suggested by theoretical frameworks such as Cole et al. (2018).
distribution. Accordingly, there could be a two-way interaction between product taxation and the price distribution.

Future research should explore the mechanisms highlighted in the present paper under alternative theoretical assumptions, departing from the monopolistic competition framework with free entry and constant mark-ups presented here. One direction is to include variable mark-ups in a monopolistic competition setting, see e.g. Dhingra and Morrow (2019). Another option is to include strategic interaction as is present in oligopoly frameworks. Although complicated, recent advances in the oligopoly literature exploit the aggregative game approach to enhance tractability and capture entry, see e.g. Nocke and Schutz (2018) and Anderson et al. (2020). Hence, it may be realistic that future research analytically can explore the effects of specific taxation in the case of oligopoly with heterogeneous firms, free entry, and endogenous quality.
References


Kaygusuz, E. D., 2013. 'The Efficiency Comparison of Taxes under Monopolistic Competition with Heterogenous Firms and Variable Markups'. mimeo.


Appendices:

A Proofs of Propositions

This appendix contains proofs of Propositions 1 to 3.

**Proof of Proposition 1 (Effects of an ad valorem tax)**

From the main text we have

\[
\text{FOC} : \quad \Lambda \left( \frac{q^\gamma}{\varphi} + t \right)^{1-\sigma} q^{\sigma-1} (\sigma - 1) \frac{q}{\varphi} \frac{1}{q} (\gamma - 1) + t - \alpha f q^{\alpha-1} = 0
\]

\[
\text{ZPC} : \quad \Lambda \left( \frac{(q^*)^\gamma}{\varphi^*} + t \right)^{1-\sigma} (q^*)^{\sigma-1} - F_0 - f (q^*)^{\alpha} = 0
\]

\[
\text{FE} : \quad \int_{q^*}^{q_{\text{max}}} \left( \Lambda \left( \frac{q^\gamma}{\varphi} + t \right)^{1-\sigma} q^{\sigma-1} - F_0 - f q^{\alpha} \right) dG(\varphi) = \delta F_E
\]

which pin down \( \Lambda, \varphi^* \) and \( q(\varphi) \). Recall that the demand shifter, \( \Lambda \), is common across firms and exogenous for the individual firm albeit endogenously determined in industry equilibrium. It follows directly, that the ad valorem tax, \( \tau \), does not affect the equilibrium values of \( \Lambda, \varphi^* \) and \( q(\varphi) \). Hence, the ad valorem tax has no effect on selection (\( \varphi^* \)) and quantities. From equation (3), it follows that profits, \( \pi(\varphi, q, \Lambda, t) = \Lambda (q^\gamma/\varphi + t)^{1-\sigma} q^{\sigma-1} - F_0 - f q^{\alpha} \), are not affected by the ad valorem tax. Quantities reads \( c(\varphi) = \Lambda (\sigma - 1) \left( \frac{(q(\varphi))^\gamma}{\varphi} + t \right)^{1-\sigma} \) and are thus invariant to \( \tau \). Prices are \( p(\varphi, q, t, \tau) = \frac{\sigma}{\sigma-1} (1 + \tau) \left( \frac{(q(\varphi))^\gamma}{\varphi} + t \right) \) and net-of-tax prices become \( \frac{p(\varphi, q, t, \tau)}{1 + \tau} = \frac{\sigma}{\sigma-1} \left( \frac{(q(\varphi))^\gamma}{\varphi} + t \right) \), where the latter do not depend on \( \tau \). Finally, the number of firms follows from

\[
E = M \int_{q^*}^{q_{\text{max}}} r(\varphi, q(\varphi), \Lambda, t) \mu(\varphi) d\varphi \quad \text{and} \quad M = \frac{E}{1 + \tau} \left( \frac{1 - G(\varphi)}{\sigma} \right) \int_{q^*}^{q_{\text{max}}} \Lambda \left( \frac{(q(\varphi))^\gamma}{\varphi} + t \right)^{1-\sigma} (q(\varphi))^{\sigma-1} dG(\varphi)
\]

Accordingly, it follows that \( \frac{dM}{d\tau} = -\frac{M}{1+\tau} < 0 \).

**Proof of Proposition 2 (Profits)**

Start by taking the derivative of the profit equation \( \pi(\varphi, q, \Lambda, t) = \Lambda \left( \frac{q^\gamma}{\varphi} + t \right)^{1-\sigma} q^{\sigma-1} - F_0 - f q^{\alpha} \)

\[
\frac{d\pi(\varphi, q, \Lambda, t)}{dt} = \frac{\partial \pi(\varphi, q, \Lambda, t)}{\partial t} \frac{dt}{dt} + \frac{\partial \pi(\varphi, q, \Lambda, t)}{\partial q} \frac{dq}{dt} + \frac{\partial \pi(\varphi, q, \Lambda, t)}{\partial \Lambda} \frac{d\Lambda}{dt}
\]

where \( \frac{\partial \pi(\varphi, q, \Lambda, t)}{\partial q} = 0 \) as quality is chosen to maximize profits, \( \frac{\partial \pi(\varphi, q, \Lambda, t)}{\partial t} = \Lambda \left( \frac{q^\gamma}{\varphi} + t \right)^{1-\sigma} q^{\sigma-1} (1 - \sigma) \frac{1}{\varphi + t} \),

and \( \frac{\partial \pi(\varphi, q, \Lambda, t)}{\partial \Lambda} = \left( \frac{q^\gamma}{\varphi} + t \right)^{1-\sigma} q^{\sigma-1} \). The free entry condition reads \( \int_{q^*}^{q_{\text{max}}} \pi(\varphi, q, \Lambda, t) dG(\varphi) = \)
\[ \delta F_E \] and from the derivative - after exploiting that \( \frac{\partial \pi(\varphi, q, \Lambda, t)}{\partial q} = 0 \) and \( \pi(\varphi^*, q^*, \Lambda, t) = 0 \) - we get

\[
\frac{d \Lambda}{dt} = \frac{\int_{\varphi^*}^{\varphi \max} \frac{\partial \pi(\varphi, q, \Lambda, t)}{\partial \varphi} dG(\varphi)}{\int_{\varphi^*}^{\varphi \max} \frac{\partial \pi(\varphi, q, \Lambda, t)}{\partial \Lambda} dG(\varphi)}
\]

Insert \( \frac{d \Lambda}{dt}, \frac{\partial \pi(\varphi, q, \Lambda, t)}{\partial \varphi}, \) and \( \frac{\partial \pi(\varphi, q, \Lambda, t)}{\partial \Lambda} \) into \( \frac{d \pi(\varphi, q, \Lambda, t)}{dt} \) to obtain

\[
\frac{d \pi(\varphi, q, \Lambda, t)}{dt} = \frac{\partial \pi(\varphi, q, \Lambda, t)}{\partial t} - \frac{\int_{\varphi^*}^{\varphi \max} \frac{\partial \pi(\varphi, q, \Lambda, t)}{\partial \varphi} dG(\varphi) \frac{\partial \pi(\varphi, q, \Lambda, t)}{\partial \Lambda}}{\int_{\varphi^*}^{\varphi \max} \frac{\partial \pi(\varphi, q, \Lambda, t)}{\partial \Lambda} dG(\varphi)}
\]

\[
= \Lambda (1 - \sigma) \left[ \frac{1}{\varphi^* + t} - \frac{\int_{\varphi^*}^{\varphi \max} \left( \frac{\varphi^*}{\varphi^* + t} \right)^{1-\sigma} \frac{1}{\varphi^* + t} dG(\varphi)}{\int_{\varphi^*}^{\varphi \max} \frac{1}{\varphi^* + t} dG(\varphi)} \right] \left( \frac{\varphi^*}{\varphi^* + t} \right)^{1-\sigma} q^{\sigma-1}
\]

\[
= \frac{r(\varphi, \Lambda)}{\sigma (1 + \tau)} (1 - \sigma) \left[ \frac{1}{\varphi^* + t} - \frac{\int_{\varphi^*}^{\varphi \max} r(\varphi, \Lambda) \frac{1}{\varphi^* + t} dG(\varphi)}{\int_{\varphi^*}^{\varphi \max} r(\varphi, \Lambda) dG(\varphi)} \right]
\]

\[
= c(\varphi) \left[ -1 + \frac{mc(\varphi, q, t)}{mc(\varphi, q, t) \int_{\varphi^*}^{\varphi \max} c(\varphi) dG(\varphi)} \right]
\]

\[
= c(\varphi) \left[ -1 + \frac{mc(\varphi, q, t)}{mc(\varphi, q, t) \int_{\varphi^*}^{\varphi \max} c(\varphi) dG(\varphi)} \right] = c(\varphi) \left[ -1 + \frac{p(\varphi, q, t)}{\bar{p}} \right]
\]

where \( \bar{m} = \int_{\varphi^*}^{\varphi \max} mc(\varphi, q, t) \int_{\varphi^*}^{\varphi \max} c(\varphi) dG(\varphi) \) and \( \bar{p} = \int_{\varphi^*}^{\varphi \max} p(\varphi, q, t) \int_{\varphi^*}^{\varphi \max} c(\varphi) dG(\varphi) \).

**Proof of Proposition 3 (Exit threshold)**

From the main text we have that

\[
\text{FOC} : \Lambda \left( \frac{\varphi^*}{\varphi^* + t} \right)^{1-\sigma} q^{\sigma-1} (\sigma - 1) \frac{\varphi^*}{\varphi^* + t} (1 - \gamma) + t - \alpha f q^{\sigma-1} = 0
\]

\[
\text{ZPC} : \Lambda \left( \frac{q^*}{\varphi^*} + t \right)^{1-\sigma} (q^*)^{\sigma-1} - F_0 - f (q^*)^\sigma = 0
\]

\[
\text{FE} : \int_{\varphi^*}^{\varphi \max} \left( \Lambda \left( \frac{\varphi^*}{\varphi^* + t} \right)^{1-\sigma} q^{\sigma-1} - F_0 - f q^\sigma \right) dG(\varphi) = \delta F_E
\]

\[
\text{FOC + ZPC} : (\sigma - 1) \frac{(\varphi^* + t) (1 - \gamma) + t}{(\varphi^* + t)^2} = \frac{\alpha f (q^*)^\sigma}{F_0 + f (q^*)^\sigma}
\]
Comparative statics reveal

\[
FE: \int_{\varphi^*}^{\varphi^{\max}} \left( \Lambda \left( \frac{q^\gamma}{\varphi} + t \right)^{1-\sigma} q^{\sigma-1} \left( \frac{d\Lambda}{\Lambda} - \frac{(\sigma - 1) dt}{\varphi^\gamma} \right) \right) dG(\varphi) = 0
\]

\[
ZPC: \frac{d\Lambda}{\Lambda} + (\sigma - 1) \left( \frac{d\varphi^*}{\varphi} - \frac{q^\gamma}{\varphi^\gamma + t} \right) = \frac{f(q^*)^{\alpha} dq^*}{F_0 + f(q^*)^{\alpha} q^*}
\]

Insert \((\sigma - 1) \frac{(q^*)^\gamma (1-\gamma) + t}{q^\gamma + t}\) into the derivative of the ZPC to obtain \(\frac{d\Lambda}{\Lambda} = \frac{(\sigma - 1) dt}{q^{\gamma} + t}\)

\[
\frac{d\varphi^*}{\varphi^*} = \frac{(q^*)^\gamma + t}{(q^*)^\gamma + t} \left[ \frac{1}{\varphi} - \int_{\varphi^*}^{\varphi^{\max}} \frac{r(\varphi, q, \Lambda, t) \frac{1}{\varphi^\gamma + t}}{dG(\varphi)} dG(\varphi) \right]
\]

\[
= \frac{1}{mc(\varphi^*; q^*, t)} \left[ 1 - \int_{\varphi^*}^{\varphi^{\max}} \frac{mc(\varphi, q, t)}{c(\varphi, q, \Lambda, t)} \frac{c(\varphi, q, \Lambda, t)}{c(\varphi, q, \Lambda, t) dG(\varphi)} dG(\varphi) \right]
\]

\[
= \frac{1}{mc(\varphi^*)} \left[ 1 - \frac{p(\varphi^*, t)}{\tilde{p}(\varphi^*, t)} \right]
\]

where it has been used that \(r(\varphi, q, \Lambda, t) = p(\varphi^*, q, t) c(\varphi, q, \Lambda, t) = \sigma (1 + \tau) \Lambda \left( \frac{q^\gamma}{\varphi} + t \right)^{1-\sigma} q^{\sigma-1}\)

and \(mc(\varphi^*, t) = \int_{\varphi^*}^{\varphi^{\max}} mc(\varphi, q, t) \frac{c(\varphi, q, \Lambda, t)}{c(\varphi, q, \Lambda, t) dG(\varphi)} dG(\varphi)\) is output-weighted average marginal costs (including the unit tax) across firms and similarly for \(\tilde{p}(\varphi^*, t)\).

**B Optimal tax structure**

In this appendix we link household preferences over tax schemes to their degree of love-of-variety.

Tax revenue reads

\[
T = tM \int_{\varphi^*}^{\varphi^{\max}} c(\varphi) \frac{g(\varphi)}{1 - G(\varphi^*)} d\varphi + E \frac{\tau}{1 + \tau}
\]

and it follows that

\[
\frac{d\tau}{dt} \bigg|_{t=0} = -M \int_{\varphi^*}^{\varphi^{\max}} c(\varphi) \frac{g(\varphi)}{1 - G(\varphi^*)} d\varphi \frac{1}{E \frac{1}{1 + \tau} \frac{1}{1 + \tau}} = -\int_{\varphi^*}^{\varphi^{\max}} r(\varphi, q, \Lambda, t) \frac{c(\varphi) g(\varphi)}{1 - G(\varphi^*)} d\varphi \frac{1}{E \frac{1}{1 + \tau} \frac{1}{1 + \tau}}
\]
From the free entry condition we have that

$$\Lambda = \frac{\delta F_E + \int_{\varphi}^{\varphi_{\text{max}}} (F_0 - f q^m) \, dG(\varphi)}{\int_{\varphi}^{\varphi_{\text{max}}} \left( \frac{q^m}{\varphi} + t \right)^{1-\sigma} q^{\sigma-1} dG(\varphi)}$$

and using the definition of $\Lambda$ we get that

$$P = \frac{\sigma}{\sigma - 1} (1 + \tau)^{1-\sigma} \left( \frac{\sigma}{E} \right)^{1-\sigma} \left( \frac{\delta F_E + \int_{\varphi}^{\varphi_{\text{max}}} (F_0 - f q^m) \, dG(\varphi)}{\int_{\varphi}^{\varphi_{\text{max}}} \left( \frac{q^m}{\varphi} + t \right)^{1-\sigma} q^{\sigma-1} dG(\varphi)} \right)^{1-\sigma} = \frac{\sigma}{\sigma - 1} (1 + \tau)^{1-\sigma} \left( \frac{\sigma}{E} \right)^{1-\sigma} (\Lambda)^{1-\sigma}$$

and it follows that

$$\frac{dP}{dt} \bigg|_{t=0} = \frac{\sigma}{\sigma - 1} \frac{1}{1 + \tau} \frac{d\tau}{dt} \bigg|_{t=0} + \frac{1}{\sigma - 1} \frac{d\Lambda}{dt} \bigg|_{t=0}$$

where

$$\frac{d\Lambda}{dt} \bigg|_{t=0} = \frac{(\sigma - 1)}{(\varphi_{\text{max}})} - (\sigma - 1) \frac{d\varphi}{dt} \frac{1}{\varphi} = \frac{\int_{\varphi}^{\varphi_{\text{max}}} mc(\varphi, q) \left( \frac{\sigma - 1}{\int_{\varphi}^{\varphi_{\text{max}}} c(\varphi, q, \Lambda, t) \, dG(\varphi)} \right)}{\int_{\varphi}^{\varphi_{\text{max}}} \left( \frac{\sigma - 1}{\int_{\varphi}^{\varphi_{\text{max}}} c(\varphi, q, \Lambda, t) \, dG(\varphi)} \right) \, dG(\varphi)}$$

It follows that

$$\frac{dP}{dt} \bigg|_{t=0} = - (1 + \tau) M \int_{\varphi}^{\varphi_{\text{max}}} c(\varphi, q, \Lambda, t) \left( \frac{q^m}{\varphi} \right) \sigma \frac{1}{1 - q(\varphi)} d\varphi - \frac{1}{\sigma - 1} \frac{d\Lambda}{dt} \bigg|_{t=0}$$

= 0

when using that $E = M \int_{\varphi}^{\varphi_{\text{max}}} r(\varphi, q, \Lambda, t) \frac{dG(\varphi)}{1 - q(\varphi)}$, \( r(\varphi, q, \Lambda, t) = p(\varphi, q, t) c(\varphi, q, \Lambda, t) \) and \( p(\varphi, q, t) = \frac{\sigma}{\sigma - 1} (1 + \tau) mc(\varphi, q, t) \).

### C Homogenous firms

Here we consider the case of homogenous firms. The first-order condition with respect to quality is unchanged

$$\frac{\partial \pi(\varphi, q, \Lambda, t)}{\partial q} = \Lambda \left( \frac{q^m}{\varphi} + t \right)^{-\sigma} q^{\sigma-1} (\sigma - 1) \frac{1}{q} \left( \frac{q^m}{\varphi} (1 - \gamma) + t \right) - \alpha f q^{\alpha-1} = 0$$

$$\frac{\partial^2 \pi(\varphi, q, \Lambda, t)}{\partial q^2} = \Lambda \left( \frac{q^m}{\varphi} + t \right)^{-\sigma} q^{\sigma-1} (\sigma - 1) \frac{1}{q} \left( \frac{q^m}{\varphi} (1 - \gamma) + t \right) \frac{1}{q} \left( -\sigma \frac{q^m}{\varphi} + t + \sigma - 2 + \frac{q^m}{\varphi} (1 - \gamma) + t \right)$$

$$- \alpha f q^{\alpha-1} (\alpha - 1) \frac{1}{q}$$

$$= - \alpha f q^{\alpha-2} \left( \alpha - (\sigma - 1) + \sigma \frac{q^m}{\varphi} + t - \frac{q^m}{\varphi} (1 - \gamma) \right)$$

$$- \alpha f q^{\alpha-2} \left( \sigma \frac{q^m}{\varphi} + t - \frac{q^m}{\varphi} (1 - \gamma) + t \right)$$
and the free entry condition reads

\[ \pi (\varphi, q, \Lambda, t) = \Lambda \left( \frac{q^\gamma}{\varphi} + t \right)^{1-\sigma} q^{\sigma-1} - F_0 - f q^\alpha = \delta F_E. \]

The derivatives of the first-order condition for quality and the free entry condition read

\[
\begin{align*}
\frac{\partial}{\partial q} \frac{\partial \pi (\varphi, q, \Lambda, t)}{\partial q} dq + \frac{\partial}{\partial \Lambda} \frac{\partial \pi (\varphi, q, \Lambda, t)}{\partial \Lambda} d\Lambda + \frac{\partial}{\partial t} \frac{\partial \pi (\varphi, q, \Lambda, t)}{\partial t} dt &= 0 \\
\frac{\partial \pi (\varphi, q, \Lambda, t)}{\partial \Lambda} d\Lambda + \frac{\partial \pi (\varphi, q, \Lambda, t)}{\partial t} dt &= 0
\end{align*}
\]

implying that

\[
\frac{dq}{dt} = \left( \frac{\partial \pi (\varphi, q, \Lambda, t)}{\partial q} \right)^{-1} \left( \frac{\partial}{\partial \Lambda} \frac{\partial \pi (\varphi, q, \Lambda, t)}{\partial \Lambda} - \frac{\partial}{\partial q} \frac{\partial \pi (\varphi, q, \Lambda, t)}{\partial q} \right)
\]

\[ = \alpha q^{\sigma-1} \left( \frac{\partial \pi (\varphi, q, \Lambda, t)}{\partial q} \right)^{-1} \left( \frac{1}{\varphi} + t - \frac{1}{\varphi^2} (1 - \gamma) + t \right) > 0 \text{ for } \gamma > 0
\]

as \( \frac{\partial \pi (\varphi, q, \Lambda, t)}{\partial q} < 0 \) cf. the second-order condition. Hence, a higher unit tax implies quality upgrading. Accordingly, marginal costs of production increases in the unit tax.

Turning to the mass of varieties supported in equilibrium we have that

\[ M = \frac{E}{\tau} = \frac{E}{\sigma (1 + \tau)} \left( \frac{q^\gamma}{\varphi} + t \right)^{1-\sigma} \Lambda = \frac{1}{1 + \tau} \frac{E}{\sigma \delta F_E + F_0 + f q^\alpha}
\]

It follows directly, that

\[
\frac{\partial M}{\partial t} = - \frac{f q^\alpha}{\delta F_E + F_0 + f q^\alpha} \frac{dq}{dt} = - \frac{q^\gamma}{\varphi} (1 - \gamma) + t \frac{dq}{dt} q < 0
\]

\[
\frac{\partial M}{\partial \tau} = - \frac{1}{1 + \tau} < 0
\]

Tax revenue reads

\[
T = Mt_c + E \frac{\tau}{1 + \tau} = \frac{E t_c + E}{\frac{\tau}{1 + \tau}} = \left( \frac{\tau}{\varphi} + \frac{\tau}{1 + \tau} \right) E = \left( \frac{t}{\varphi} + \frac{\tau}{1 + \tau} \right) E
\]

\[ = \left( \frac{\varphi - \tau}{\sigma - \tau} (1 + \tau) \left( \frac{q^\gamma}{\varphi} + t \right) + \frac{\tau}{1 + \tau} \right) E
\]

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It follows that
\[
\frac{dt}{dt} = -\frac{q^\tau}{\varphi + t} \frac{\sigma}{\sigma - 1} \left( \frac{\varphi^\gamma}{\varphi + t} - t \right).
\]

A tax reform increasing \( t \) and reducing \( \tau \) to keep revenue fixed implies the following impact on the mass of varieties
\[
\frac{dM}{dt} \bigg|_{t} = \frac{\partial M}{\partial t} \bigg|_{t} + \frac{\partial M}{\partial \tau} \bigg|_{t} \frac{d\tau}{dt}
\]
and evaluated at \( t = 0 \) we get
\[
\frac{dM}{dt} \bigg|_{t=0} = \left( 1 - \frac{\gamma \sigma}{\alpha - (\sigma - 1)(1 - \gamma)} \right) \frac{\sigma - 1}{\sigma} \frac{1}{\varphi} < 0.
\]

We have that \( \frac{dM}{dt} \bigg|_{t=0} > 0 \) for \( \gamma < \gamma = \alpha - (\sigma - 1) \) and \( \frac{dM}{dt} \bigg|_{t=0} < 0 \) for \( \gamma > \alpha - (\sigma - 1) \).

Intuitively, a larger \( \gamma \) magnifies quality upgrading and quality upgrading increases fixed costs and thus the break-even revenue.

Turning to the price index we have that
\[
P = \left( \Lambda \frac{(\sigma - 1)}{E} \right)^{\frac{1}{\sigma - 1}} \left( \frac{\sigma}{\sigma - 1} (1 + \tau) \right)^{\frac{\sigma}{\sigma - 1}} = \left( \frac{\delta F_E + F_0 + f q^\alpha (q - 1) dF_E}{\varphi + t) (\varphi + t) + (q - 1) \frac{q^\gamma}{\varphi + t} + t} \right)^{\frac{1}{\sigma - 1}} \left( \frac{\sigma}{\sigma - 1} (1 + \tau) \right)^{\frac{\sigma}{\sigma - 1}}
\]

It follows that
\[
\frac{\partial P}{\partial t} \bigg|_{t} = \frac{1}{\sigma - 1} + \tau > 0
\]
and a revenue tax reform implies that
\[
\frac{dP}{dt} \bigg|_{t} = \frac{\partial P}{\partial t} \bigg|_{t} + \frac{\partial P}{\partial \tau} \bigg|_{t} \frac{d\tau}{dt} \bigg|_{t}
\]
and evaluated at \( t = 0 \) we get
\[
\frac{dP}{dt} \bigg|_{t=0} = \frac{1}{\varphi} + t \frac{\sigma}{\sigma - 1} \left( \frac{q^\gamma}{\varphi + t} - t \right).
\]
Hence, \( t = 0 \) and \( \tau = \frac{q_f}{q_q} > 0 \) is the tax schedule that minimizes the price index for a given tax revenue constraint \( (T) \).

## D Heterogeneous firms - no selection

Here we examine the case were \( \varphi^* < \varphi_{\text{min}} \) such that the selection channel is absent.

The first-order condition with respect to quality reads

\[
\frac{\partial \pi (\varphi, q, \Lambda, t)}{\partial q} = \Lambda \left( \frac{q^\gamma}{\varphi} + t \right)^{1-\sigma} q^{\sigma-1} (\sigma - 1) \frac{q^\gamma}{q} \frac{(1-\gamma) + t}{q+}\frac{q^\gamma}{\varphi} - \alpha f q^{\sigma-1} = 0
\]

and the free entry condition reads

\[
\int_{\varphi_{\text{min}}}^{\varphi_{\text{max}}} \pi (\varphi, q, \Lambda, t) dG (\varphi) = \int_{\varphi_{\text{min}}}^{\varphi_{\text{max}}} \left( \Lambda \left( \frac{q^\gamma}{\varphi} + t \right)^{1-\sigma} q^{\sigma-1} - F_0 - f q^\gamma \right) dG (\varphi) = \delta F_E
\]

and the mass of firms reads

\[
M = \frac{E}{\int \varphi_{\text{max}}} r(\varphi, q, \Lambda, t) dG (\varphi) = \frac{E}{\Lambda \int \varphi_{\text{max}}} \sigma (1+\tau) \left( \frac{mc(\varphi, q, t)}{q} \right)^{1-\sigma} dG (\varphi)
\]

Static comparative analysis reveals that

\[
\frac{\partial \pi (\varphi, q, \Lambda, t)}{\partial \varphi} dq + \frac{\partial \pi (\varphi, q, \Lambda, t)}{\partial \Lambda} d\Lambda + \frac{\partial \pi (\varphi, q, \Lambda, t)}{\partial t} dt = 0
\]

\[
\int_{\varphi_{\text{min}}}^{\varphi_{\text{max}}} \frac{\partial \pi (\varphi, q, \Lambda, t)}{\partial \varphi} dG (\varphi) d\Lambda + \int_{\varphi_{\text{min}}}^{\varphi_{\text{max}}} \frac{\partial \pi (\varphi, q, \Lambda, t)}{\partial \Lambda} dG (\varphi) dt = 0
\]

\[
dM = \frac{dM}{M} - \frac{d\Lambda}{\Lambda} - \frac{dt}{1+\tau} = \int_{\varphi_{\text{min}}}^{\varphi_{\text{max}}} \frac{\varphi^\gamma + t}{q} \frac{1-\sigma}{(\sigma - 1)} \left( \frac{\varphi^\gamma}{q} \right)^{1-\sigma} (\sigma - 1) \frac{q^\gamma}{q} \frac{(1-\gamma) + t}{q+}\frac{q^\gamma}{\varphi} dG (\varphi)
\]

It follows that

\[
\frac{d\Lambda}{dt} = -\frac{\int \varphi_{\text{max}}} \frac{\partial \pi (\varphi, q, \Lambda, t)}{\partial \varphi} dG (\varphi) - \Lambda \left( \frac{q^\gamma}{\varphi} + t \right)^{-\sigma} q^{\sigma-1} dG (\varphi) = \Lambda (\sigma - 1) \left( \frac{q^\gamma}{\varphi} + t \right)^{-\sigma} q^{\sigma-1} dG (\varphi) > 0
\]

\[
\frac{dq}{dt} = q \left( \frac{\varphi^\gamma + t}{q} \frac{1-\sigma}{(\sigma - 1)} \left( \frac{\varphi^\gamma}{q} \right)^{1-\sigma} q^{\sigma-1} dG (\varphi) - \frac{1}{\varphi +t} \right) + \left( \frac{1}{\varphi +t} \frac{1}{\varphi +t} \right)
\]

\[
\alpha - (\sigma - 1) + \frac{\varphi^\gamma}{\varphi^\gamma + t} - \frac{\varphi^\gamma}{\varphi^\gamma + t} (1-\gamma) + t
\]

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Turning to average quality we have

$$
\bar{q} = \int_{\hat{q}_{\text{min}}}^{\hat{q}_{\text{max}}} q dG(\varphi)
$$

$$
\frac{d\bar{q}}{dt} = \int_{\hat{q}_{\text{min}}}^{\hat{q}_{\text{max}}} \frac{d\hat{q}}{dt} dG(\varphi)
$$

$$
= \int_{\hat{q}_{\text{min}}}^{\hat{q}_{\text{max}}} q \left( \frac{\hat{q}_{\text{max}}}{\hat{q}_{\text{min}}} \right)^{1-\gamma} q^{\gamma-1} dG(\varphi) - \frac{1}{\alpha} \frac{\hat{q}_{\text{max}}}{\hat{q}_{\text{min}}} + \frac{1}{\alpha} \frac{dG(\varphi)}{dt} \right)
$$

and evaluated at a zero tax rate we find

$$
\frac{d\bar{q}}{dt} = \left( \frac{\hat{q}_{\text{max}}}{\hat{q}_{\text{min}}} \right)^{1-\gamma} q^{\gamma-1} dG(\varphi) - \frac{1}{\alpha} \frac{\hat{q}_{\text{max}}}{\hat{q}_{\text{min}}} + \frac{1}{\alpha} \frac{dG(\varphi)}{dt} \right)
$$

Now impose the often applied Pareto distribution. Assume that $G(\varphi) = 1 - \left( \frac{\varphi}{\varphi_{\text{min}}} \right)^{-k}$ for $\varphi \geq \varphi_{\text{min}}$ and $\varphi_{\text{max}} = \infty$. We then have that

$$
\frac{d\bar{q}}{dt} \bigg|_{t=0} = \left( \frac{\hat{q}_{\text{max}}}{\hat{q}_{\text{min}}} \right)^{1-\gamma} \left( \frac{\hat{q}_{\text{max}}}{\hat{q}_{\text{min}}} \right)^{\gamma-1} dG(\varphi) - \frac{1}{\alpha} \frac{\hat{q}_{\text{max}}}{\hat{q}_{\text{min}}} + \frac{1}{\alpha} \frac{dG(\varphi)}{dt} \right)
$$

$$
= \left( \frac{\hat{q}_{\text{max}}}{\hat{q}_{\text{min}}} \right)^{1-\gamma} \left( \frac{\hat{q}_{\text{max}}}{\hat{q}_{\text{min}}} \right)^{\gamma-1} dG(\varphi) - \frac{1}{\alpha} \frac{\hat{q}_{\text{max}}}{\hat{q}_{\text{min}}} + \frac{1}{\alpha} \frac{dG(\varphi)}{dt} \right)
$$

Now we show that possibility of the average quality falling in the specific tax rate. We set $\gamma = 0,$
i.e. we shut down the Barzel (1976) mechanism and find that

\[
\frac{dq}{dt}\bigg|_{t=0,\gamma=0} = \frac{(\sigma - 1)^2}{(\alpha - (\sigma - 1))^3 kk - k} \frac{(\alpha - \sigma) \alpha + (\sigma - 1)}{\alpha - (\sigma - 1)}
\]

\[
\frac{dq}{dt}\bigg|_{t=0,\gamma=0,\alpha=(\sigma-1)+x} = \frac{(\sigma - 1)^2}{(\sigma - 2 + x) (x)^2} \frac{(\sigma - 1)kk - k}{x} + \frac{\sigma - 1}{x} \frac{\sigma - (\sigma - 1) + \sigma x}{x} < 0 \iff \sigma < 2 - x
\]

E General functional forms

Let \( U = h \left( \sum u (q_i, c_i) \right) \) and it follows that \( p_i = \frac{h'(\cdot)}{\lambda} u' (q_i, c_i) q_i \), where \( \lambda \) is the Lagrange multiplier, i.e. the shadow value of income. Profit of the firm reads

\[
\pi_i = \left( p_i - mc_i (q_i, \varphi_i) - t \right) c_i - F (q_i)
\]

\[
= \left( \frac{h'(\cdot)}{\lambda} u' (q_i, c_i) q_i - mc_i (q_i, \varphi_i) - t \right) c_i - F (q_i)
\]

and profit maximization requires

\[
\frac{\partial \pi_i}{\partial q_i} = \frac{h'(\cdot)}{\lambda} (u' (q_i, c_i) + u'' (q_i, c_i) q_i) c_i - \frac{\partial mc_i (q_i, \varphi_i)}{\partial q_i} c_i - F' (q_i) = 0
\]

\[
\frac{\partial \pi_i}{\partial c_i} = \frac{h'(\cdot)}{\lambda} (u' (q_i, c_i) + u'' (q_i, c_i) q_i) q_i - (mc_i (q_i, \varphi_i) + t) = 0
\]

and the second-order conditions

\[
\frac{\partial^2 \pi_i}{\partial q_i \partial c_i} = \frac{h'(\cdot)}{\lambda} (2u'' (q_i, c_i) + u''' (q_i, c_i) q_i) q_i^2 < 0
\]

\[
\frac{\partial^2 \pi_i}{\partial q_i \partial q_i} = \frac{h'(\cdot)}{\lambda} (2u'' (q_i, c_i) + u''' (q_i, c_i) q_i) c_i^2 - \frac{\partial mc_i (q_i, \varphi_i)}{\partial q_i} c_i - F'' (q_i) < 0
\]

\[
\frac{\partial^2 \pi_i}{\partial q_i \partial c_i} = \frac{h'(\cdot)}{\lambda} (u' (q_i, c_i) + u'' (q_i, c_i) q_i) c_i - \frac{h'(\cdot)}{\lambda} (2u'' (q_i, c_i) + u''' (q_i, c_i) q_i) q_i c_i - \frac{\partial mc_i (q_i, \varphi_i)}{\partial q_i}
\]

\[
0 < \frac{\partial^2 \pi_i}{\partial c_i \partial c_i} = \frac{\partial \pi_i}{\partial q_i} \frac{\partial \pi_i}{\partial c_i} - \frac{\partial \pi_i}{\partial q_i} \frac{\partial \pi_i}{\partial c_i}
\]

From the first-order-condition wrt output the price becomes

\[
p_i = \frac{mc_i (q_i, \varphi_i) + t}{1 + \frac{u'' (q_i, c_i) q_i}{u' (q_i, c_i)}} = \xi (q_i, c_i) (mc_i (q_i, \varphi_i) + t)
\]
where \( \xi (q, c_i) \equiv \left( 1 + \frac{u''(q, c_i) q_i}{u'(q, c_i)} \right)^{-1} \geq 1 \) is the mark-up. The free entry condition reads

\[
\int \left[ \left( \frac{h'(\cdot)}{\lambda} u'(q, c_i) q_i - mc_i(q_i, \varphi_i) - t \right) c_i - F(q_i) \right] dG(\cdot) = \delta F_E
\]

and using the envelope theorem it follows that

\[
\frac{d h'(\cdot)}{dt} = \int u'(q, c_i) q_i c_i dG(\cdot)
\]

Applying the envelope theorem again it follows that

\[
\frac{d\pi_i}{dt} = \frac{\partial \pi_i}{\partial q_i} \frac{dq_i}{dt} + \frac{\partial^2 \pi_i}{\partial q_i \partial c_i} dc_i + (u'(q, c_i) + u''(q, c_i) q_i c_i) c_i \frac{d h'(\cdot)}{\lambda} = 0
\]

\[
\frac{d\pi_i}{dc_i} = \frac{\partial^2 \pi_i}{\partial c_i \partial q_i} dc_i + \frac{\partial^2 \pi_i}{\partial c_i \partial c_i} dq_i - dt + (u'(q, c_i) + u''(q, c_i) q_i c_i) q_i \frac{d h'(\cdot)}{\lambda} = 0
\]

It follows that

\[
dc_i = -\frac{(u'(q, c_i) + u''(q, c_i) q_i c_i) c_i}{\frac{\partial^2 \pi_i}{\partial q_i \partial c_i}} \frac{d h'(\cdot)}{\lambda} - \frac{\partial^2 \pi_i}{\partial q_i ^2} \frac{dq_i}{dt}
\]

For quality we have that

\[
\left( \frac{\partial^2 \pi_i}{\partial c_i \partial q_i} - \frac{\partial^2 \pi_i}{\partial q_i ^2} \right) \frac{dq_i}{dt} = (u'(q, c_i) + u''(q, c_i) q_i c_i) \left( q_i - \frac{\partial^2 \pi_i}{\partial c_i \partial c_i} \frac{c_i}{\frac{\partial^2 \pi_i}{\partial q_i \partial c_i}} \right) \frac{d h'(\cdot)}{\lambda} - 1
\]

\[
= (u'(q, c_i) + u''(q, c_i) q_i c_i) \left( q_i - \frac{\partial^2 \pi_i}{\partial c_i \partial c_i} \frac{c_i}{\frac{\partial^2 \pi_i}{\partial q_i \partial c_i}} \right) \frac{\int u'(q, c_i) q_i c_i dG(\cdot)}{u'(q, c_i) q_i c_i dG(\cdot)} - 1
\]

\[
= \left( 1 - \frac{\frac{\partial^2 \pi_i}{\partial c_i \partial c_i}}{\frac{\partial^2 \pi_i}{\partial q_i \partial c_i}} \frac{c_i}{q_i} \right) \left( 1 + \frac{u''(q, c_i) q_i c_i}{u'(q, c_i)} \right) \frac{p_i}{\hat{p}} - 1
\]

\[
= \left( 1 - \frac{\frac{\partial^2 \pi_i}{\partial c_i \partial c_i}}{\frac{\partial^2 \pi_i}{\partial q_i \partial c_i}} \frac{c_i}{q_i} \right) MR_i \frac{p_i}{\hat{p}} - 1
\]

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The effect of the tax on quality comes through several channels. First, the higher tax increases marginal costs including the tax and output is therefore reduced to equate marginal revenue with marginal costs. Lower output reduces quality as the fixed costs of quality upgrading can be spread across fewer units. Second, the reduction in the competitive pressure in the industry due to the higher tax increases marginal revenue (price) and this stimulates quality upgrading. The proportional increase in marginal revenue is more valuable to firms with a high marginal revenue (price), i.e. these firms are more likely to upgrade quality than firms with a low marginal revenue.

As is common in imperfect competition models without explicit functional forms, the comparative statics depends on higher-order derivatives (second and third order) of the sub-utility function $u(\cdot)$ as properties of the marginal revenue function become key.