The Yield Spread and Bond Return Predictability in Expansions and Recessions*

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Abstract

This paper shows that expected excess bond returns display a positive correlation with the slope of the yield curve (i.e. yield spread) in expansions but a negative correlation in recessions. We use a macro-finance term structure model with different market prices of risk in expansions and recessions to show that a more accommodating monetary policy in recessions is a key driver behind this switch in return predictability.

Keywords: Market price of risk, Monetary policy, State-dependent bond return predictability, Taylor-rule.

JEL: E43, E44, G12.

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1 Introduction

It is well-known that bond returns in excess of the short rate are predictable from the yield spread, which measures the slope of the yield curve (see Fama and French (1989) and Campbell and Shiller (1991)). This paper provides new evidence on bond risk premia by conditioning this classic return regression on the state of the business cycle. When the economy is expanding, we find the familiar positive relation between expected excess bond returns and the yield spread. However, in recessions, we find a large increase in the intercept of this regression, to capture the countercyclical nature of bond risk premia, and a significantly negative relation between expected excess bond returns and the yield spread. As a result, investors demand compensation for carrying more slope risk in expansions but pay for this exposure in recessions. This new result on bond return predictability is present in the whole ten-year maturity spectrum, and it is robust to the choice of recession indicator, to various measures of the yield spread, to the inclusion of macro predictors, and to different choices of the sample period. The result also survives the recent test of Bauer and Hamilton (2018), meaning that the small-sample distortions they highlight cannot explain our finding.

We present a macro-finance term structure model to explain this new empirical finding. The short rate is here given by a simple Taylor-rule with time-varying policy weights on inflation and the output gap as in Ang, Boivin, Dong and Loo-Kung (2011). This implies that all bond yields are determined by inflation $\pi_t$, the output gap $\hat{y}_t$, and the two weights, $\omega_{\pi,t}$ and $\omega_{\hat{y},t}$, assigned to these macro variables in the Taylor-rule. We then introduce regime-switching in the market prices of risk attached to the macro variables and the policy weights to accommodate the possibility that the dynamics of the macro economy and the behavior of the Federal Reserve may change across the business cycle. The regimes we consider are episodes of economic expansions and recessions, where a real-time predictor of the business cycle is used to identify regimes. The model is estimated on monthly U.S. data from 1961:6 to 2016:12 using the sequential regression approach of Andreasen and Christensen (2015). We find that simulated bond yields from this model reproduce the empirical intercepts and slope coefficients from standard return regressions and, in addition, match the switch in the intercepts and slope coefficients during recessions. The estimates of our macro-finance model show that it is mainly the dynamics of the policy weights that appear to change across the business cycle. This observation also follows from the estimated samples paths for the pricing factors in the model, where especially the weight assigned to inflation $\omega_{\pi,t}$ in the Taylor-rule often drops...
substantially when the U.S. enters a recession.

The monetary policy decisions by the Federal Reserve in the proposed model are captured by the policy weights assigned to inflation $\omega_{\pi,t}$ and the output gap $\omega_{g,t}$ in the Taylor-rule. This implies that we can examine the importance of monetary policy for the switch in bond return predictability by studying how these policy weights affect bond yields. The first exercise we consider is to omit regime-switching in the policy weight related to inflation in the Taylor-rule, while the dynamics of all the other pricing factors in the model are allowed to switch between expansions and recessions. The results show that this restriction substantially reduces the level of bond returns during recessions such that the model-implied returns are too low compared to those observed in the data. The restriction also implies that the model no longer generates a sufficiently negative relation between expected excess bond returns and the yield spread during recessions. We then repeat this exercise for the policy weight to the output gap and find qualitatively the same results. This shows that the different policy responses by the Federal Reserve in expansions and recessions are essential for generating a switch in bond return predictability across the business cycle.

The second exercise we consider is to study the impulse response functions following a shock to either inflation, the output gap, or one of the two policy weights. The results show that only a shock to the policy weight on inflation $\omega_{\pi,t}$ in the Taylor-rule is able to change the comovement between expected excess bond returns and the yield spread across the business cycle as required to generate a switch in bond return predictability. In expansions, we find that a more soft monetary policy with respect to stabilizing inflation (i.e. a negative shock to $\omega_{\pi,t}$) increases expected excess bond returns and generates a steepening of the yield curve. This means that bond investors require compensation for a more accommodating monetary policy in expansions, and this helps to generate the desired positive comovement between expected excess bond returns and the yield spread in expansions. In recessions, on the other hand, a negative shock to $\omega_{\pi,t}$ reduces expected excess bond returns but is also accompanied by a steepening of the yield curve, because short rates fall by more than long-term yields. Thus, a shock to $\omega_{\pi,t}$ generates the desired negative comovement between expected excess bond returns and the yield spread in recessions, as required to match our new empirical finding. Given that $\omega_{\pi,t}$ typically falls during recessions, a key implication of the proposed model is that the observed switch in bond return predictability is closely related to the ability of the Federal Reserve to remove some of the risks attached to recessions, as investors are willing to pay for accommodating monetary policy in recessions but require compensation for this policy in expansions.
The rest of the paper is organized as follows. Section 2 presents the results from the standard return regression and its extension with regime-dependent loadings. We describe the considered macro-finance term structure model and how it is estimated in Section 3, while Section 4 explores the ability of this model to explain the switch in bond return predictability. Section 5 concludes. Appendix A contains data descriptions and some econometric details related to our analysis, while all remaining details are provided in an Online Appendix.

2 Bond Return Predictability Across the Business Cycle

We present our new empirical finding for bond returns in Section 2.1 and discuss its main economic implication in Section 2.2. Various robustness checks are provided in Section 2.3.

2.1 Predictability from the Yield Spread

To motivate the analysis, consider the standard return regression

\[ r_{x_{t+1,k}} = \alpha_k + \beta_k s_{t,k} + \varepsilon_{t+1,k}, \]

where the excess bond return from time \( t \) to time \( t + 1 \) on a \( k \)-period zero-coupon bond \( r_{x_{t+1,k}} \) is regressed on a constant and the slope of the yield curve \( s_{t,k} \) at time \( t \). As in Gargano, Pettenuzzo and Timmermann (2019), we use monthly returns although previous studies mostly consider annual returns. Monthly returns are nonoverlapping and therefore less persistent than monthly observations of annual returns, making standard inference more reliable. Another advantage of monthly returns highlighted in Gargano, Pettenuzzo and Timmermann (2019) is that any short-lived dynamics across the business cycle are easier to identify than with annual returns. Similar to the existing literature, we measure the yield spread as the difference between a long yield and the one-year interest rate, i.e. \( s_{t,k} = y_{t,k} - y_{t,12} \), where \( y_{t,k} \) refers to the \( k \)-month yield at time \( t \). Panel A in Table 1 shows the results from these regressions using monthly Gürkaynak, Sack and Wright (2007) bond yields from 1961:6 to 2016:12. We find the familiar results that \( \alpha_k \) is insignificant across all maturities and \( \beta_k \) is positive and slightly increasing with maturity. In the two- to five-year maturity spectrum, bond returns are not significantly predictable from the yield spread, but at longer maturities the predictability becomes significant at the 5\% level. We also note that the degree of explained variation when measured by the \( R^2 \) is less than 2\% for all maturities.
Now consider what happens when we allow the intercept and the slope coefficient in (1) to switch between expansions (EXP) and recessions (REC). That is, we consider the modified return regression

\[ rx_{t+1,k} = \alpha_k^{\text{EXP}} + \alpha_k^{\Delta} 1_{\{z_t < c\}} + \left( \beta_k^{\text{EXP}} + \beta_k^{\Delta} 1_{\{z_t < c\}} \right) s_{t,k} + \varepsilon_{t+1,k}. \]  

(2)

Here, \( 1_{\{z_t < c\}} \) is an indicator function with a value of one for recessions when \( z_t < c \) and zero otherwise. The variable \( z_t \) refers to the Purchasing Managers’ Index (PMI), where the threshold \( c = 44.5 \) from Berge and Jordà (2011) is used to identify recessions. The PMI is a widely watched indicator of business cycle activity and has the advantage of being available in real time without publication lags. Furthermore, Christiansen, Eriksen and Møller (2014) demonstrate that the PMI is the single best recession indicator among a large panel of economic variables. As shown below, our results are robust to using other recession indicators, including the NBER recession dates that are subject to publication lags and therefore not our preferred recession indicator. The regression coefficient for the yield spread is \( \beta_k^{\text{EXP}} \) in expansions and \( \beta_k^{\text{REC}} \equiv \beta_k^{\text{EXP}} + \beta_k^{\Delta} \) in recessions, meaning that \( \beta_k^{\Delta} \) captures the change in the regression slope when entering recessions. A similar interpretation applies for the intercept.

Panel B in Table 1 shows that the estimates of \( \alpha_k^{\Delta} \) are positive and significantly different from zero at the 5% level across all maturities. This holds both when computing standard errors as in Newey and West (1987) and by a block bootstrap. The estimates of \( \alpha_k^{\Delta} \) increase gradually from 1.27 for the two-year maturity to 3.05 at the ten-year maturity, showing that the overall level of excess bond returns increases strongly in recessions. The estimates of \( \beta_k^{\Delta} \) are negative and also significantly different from zero at the 5% level for all maturities, both when using Newey-West and bootstrapped standard errors. This implies that the regression slope \( \beta_k^{\text{REC}} \) becomes negative for all maturities, as shown in the last column of Table 1. That is, a higher yield spread predicts significantly higher excess bond returns in expansions but lower returns in recessions. Accounting for this negative effect of the yield spread is also seen to increase all slope estimates in expansions \( \beta_k^{\text{EXP}} \) compared to the standard return regression in (1). As a result, we now reject the Expectations Hypothesis of no predictability, i.e. \( \beta_k^{\text{EXP}} = 0 \) and \( \beta_k^{\text{REC}} = 0 \), across all maturities.

The overall effect of these significant changes in the intercepts and slope coefficients between expansions and recessions is a dramatic increase in the \( R^2 \) for all maturities. The largest improvement is at the two-year maturity, where the \( R^2 \) increases from 0.9% to 14.6%. The boost in explanatory power is also evident at medium- and long-term yields, although the improvement
declines gradually with maturity. For instance, the $R^2$ increases from 1.3% to 9.6% at the five-year maturity, and from 1.7% to 5.8% at the ten-year maturity. Thus, the modified return regression in (2) explains a much larger proportion of the variation in $r_{x_{t+1,k}}$ compared to (1), making this specification more informative about the dynamics of bond returns.

### 2.2 Economic Implication

The conventional understanding of bond risk premia, as measured by expected excess bond returns, is that a higher yield spread is associated with higher bond risk premia because $\beta_k > 0$. That is, the steepness of the yield curve serves as a risk factor for which bond investors require compensation. However, the modified return regression in (2) shows that this effect only holds in expansions. In recessions, we conversely have that a higher yield spread is associated with lower bond risk premia because $\beta_k^{REC} < 0$. That is, the steepness of the yield curve appears to operate as what may be called a 'hedging' factor, because investors are willing to buy bonds at a high price (i.e. low yield) although the yield spread is fairly low or even negative.

The previous section showed that this switch in the risks attached to the yield spread is statistically significant, but it also makes the predicted bond risk premia more plausible from an economic perspective. To realize this, consider the fitted values for expected excess bond returns in (1) and (2), which we plot at the top in Figure 1 for the ten-year maturity. Given that the yield spread is either close to zero or negative just before a recession, the standard return regression implies that bond risk premia are either close to zero or even negative at the start of a recession. During the course of a recession, we normally see a steepening of the yield curve as the Federal Reserve reduces its policy rate. The standard return regression therefore predicts high bond risk premia at the end of a recession, but also in the years following a recession because the yield spread tends to peak after a recession.

The modified return regression predicts a radically different evolution in bond risk premia during recessions, as the switch in the intercept $\alpha_k^{\Delta}$ generates a large increase in excess bond returns when the real-time recession indicator $z_t$ gets below its threshold $c$. This may correspond to the start of an NBER recession as in 1980, but it is more often seen in the middle of an NBER recession due to the ex post nature of these NBER recession dates.\(^1\) The yield spread increases towards the end of

\(^1\)An obvious example is the 2007-2009 recession that according to the NBER started in December 2007, although most economists at that time probably believed that the recession started in the fall of 2008, as also predicted by the FMI.
a recession, and this leads to a gradual reduction in bond risk premia with $\beta_{REC}^k < 0$. That is, our modified return regression predicts that bond risk premia peak at the start or middle of an NBER recession. In contrast, the standard return regression implies that bond risk premia peak at the end or after an NBER recession.

Thus, the key economic implication from the modified return regression in (2) is that the yield spread only serves as a risk factor in expansions but operates as a hedging factor in recessions. A prediction that is consistent with the stylized notion that bond risk premia are counter-cyclical and spike in recessions. We elaborate on the mechanisms that may explain this result in Section 3.

2.3 Robustness and Additional Analysis

This subsection shows that the significant switch in the intercept and the slope coefficient for the yield spread in (2) is robust across a wide range of alternative specifications of this regression. We only report the key results below and defer further details to the Online Appendix.

2.3.1 Speed of Transition

A relevant question is whether the switch in bond returns between recessions and expansions is best modeled by the binary specification in (2) or by some smooth function. To address this question, consider a version of our modified return regression that applies the logistic smooth transition regression (LSTR) of Teräsvirta (1994), i.e.

$$rx_{t+1,k} = \alpha_k^{EXP} + \beta_k^{EXP} s_{t,k} + \left[\alpha_k^\Delta + \beta_k^\Delta s_{t,k}\right] \left(1 - G(z_t, \gamma, c)\right) + \varepsilon_{t+1,k},$$

where $G(z_t, \gamma, c) = \left(1 + \exp\{-\gamma(z_t - c)\}\right)^{-1}$, $z_t$ is the PMI, and $c = 44.5$. The parameter $\gamma > 0$ determines the speed of transition between the two regimes, where moderately low values of $\gamma$ generate a gradual transition, while (3) converges to (2) with an instant switch for $\gamma \rightarrow \infty$. It is well-known that estimating $\gamma$ is difficult when its true value is large, and we therefore make two adjustments following Schleer (2015). First, the value of $\gamma$ generally depends on the scale of $s_{t,k}$, and we therefore divide $\gamma$ by the standard deviation of $s_{t,k}$. Second, the reparameterization $\gamma = \exp(\delta)$ is used to make the search for reasonable starting values easier, as an equidistant grid in $\delta$ generates a grid for $\gamma$ that is dense for low values of $\gamma$ and coarser for higher values of $\gamma$. The estimation of (3) is jointly across the 33 maturities \{24, 27, ..., 117, 120\} using monthly excess
returns. We find that \( \hat{\delta} = 6.6 \), which corresponds to an instantaneous switch as in (2). The very high estimate of \( \delta \) may reflect the difficulty of estimating the transition speed, and we therefore gradually reduce \( \delta \) to 3.0, 2.0, and 1.5 to investigate the effects of a more smooth transition. The results show that the change in the slope coefficients \( \beta_k^\Delta \) is negative for all maturities and for all considered degrees of smoothness. For maturities up to six years, \( \beta_k^\Delta \) is significant at the 5% level across all levels of \( \delta \). At longer maturities, the values of \( \beta_k^\Delta \) remain significant at the 5% level with \( \delta = 3.0 \) and at the 10% level with \( \delta = 2.0 \), while \( \beta_k^\Delta \) becomes insignificant at the nine- and ten-year maturity with \( \delta = 1.5 \). The implications of considering a more smooth transition is illustrated at the bottom of Figure 1, which shows that the fitted values from the LSTR model with \( \delta = 2.0 \) are less spiky around turning points than those implied by the binary specification in (2) but that the two regressions otherwise carry the same information.

Accordingly, the LSTR model confirms our finding from above that there is an instantaneous switch in bond return predictability between expansions and recessions. Given this finding, we continue to apply the parsimonious binary specification in (2), although it generates occasionally spikes around turning points.

### 2.3.2 Recession Indicators

A key feature of the modified return regression is the choice of recession indicator. However, the estimates of \( \alpha_k^\Delta \) and \( \beta_k^\Delta \) as well as their \( t \)-statistics in Table 1 are robust to estimating (2) when \( z_t \) is the business cycle index of Arouba, Diebold and Scotti (2009), the recession probabilities of Chauvet and Piger (2008), the Chicago Fed National Activity Index (CFNAI), or the NBER recession dates. The results are even somewhat stronger when using the recession probabilities of Chauvet and Piger (2008) compared to our benchmark specification with the PMI (for instance \( \beta_{120} \) changes from \(-1.61 \) to \(-1.75 \)). On the other hand, we unexpectedly find slightly weaker results when using the wider NBER recessions. Despite these minor differences, the estimates of \( \alpha_k^\Delta \) and \( \beta_k^\Delta \) remain significant across all maturities, showing that the specific choice of recession indicator is not essential for our new finding.

### 2.3.3 Controlling for Macro Predictors

The proposed regression in (2) captures a switch in the level of bond returns during recessions and an interaction effect between the yield spread and the state of the economy. But the work by Cooper
and Priestley (2009), Ludvigson and Ng (2009), Joslin, Priebsch and Singleton (2014), Cieslak and Povala (2015), among others, show that macro variables on their own help to forecast bond returns. To explore whether the switch in the intercepts and slope coefficients in (2) remains significant when controlling for the individual effect from macro variables, we augment the regression in (2) with a macro predictor $q_t$ and consider

$$r x_{t+1,k} = \alpha_k^\text{EXP} + \alpha_k^\Delta 1_{\{z_t<c\}} + \left( \beta_k^\text{EXP} + \beta_k^\Delta 1_{\{z_t<c\}} \right) s_{t,k} + \gamma_k q_t + \varepsilon_{t+1,k}.$$  

The results show that the estimates of $\alpha_k^\Delta$ and $\beta_k^\Delta$ reported in Table 1 as well as their $t$-statistics are basically unaffected by conditioning on i) a measure of current real economic conditions (GRO) computed as the three-month moving average of the CFNAI as in Joslin, Priebsch and Singleton (2014), ii) the expected rate of inflation over the next year obtained from the University of Michigan survey (INF), iii) the factor of Ludvigson and Ng (2010) (LN factor), or iv) the Cieslak and Povala (2015) factor that controls for trend inflation. This shows that the pattern in bond returns reported in Table 1 is not captured by any of these well-established macro economic predictors.

2.3.4 Switching in Macro Predictors

Another possibility is that the regression coefficients for macro predictors on their own switch across the business cycle. We address this question by replacing the yield spread in (2) by a macro predictor $q_t$ and consider

$$r x_{t+1,k} = \alpha_k^\text{EXP} + \alpha_k^\Delta 1_{\{z_t<c\}} + \left( \beta_k^\text{EXP} + \beta_k^\Delta 1_{\{z_t<c\}} \right) q_t + \varepsilon_{t+1,k}.$$  

The results show that the predictive coefficients of GRO change across all maturities. On the other hand, INF only changes at the ten-year maturity, while the LN factor and the Cieslak and Povala (2015) factor do not appear to switch. The final possibility we consider is just to use the first principal component in Ludvigson and Ng (2009), which explains a sizable proportion of the variability in their macro variables. We find that this LN1 factor generates a significant switch in the slope coefficients across all maturities. Thus, the evidence from macro predictors partly confirms our finding that the dynamics of bond returns appear to switch between expansions and recessions. However, the $R^2$s for these macro predictors are generally somewhat lower than for the yield spread in Table 1.
2.3.5 Other Yield-Based Predictors

We next replace the yield spread in (2) by the forward spread (Fama (1976)), the relative interest rate of Campbell (1991), or the tent-shaped linear combination of forward rates by Cochrane and Piazzesi (2005) to explore whether these alternative yield-based predictors also imply a switch in bond returns. The results clearly show that all three predictors generate a significant switch in both the intercepts and the regression slopes between expansions and recessions. We also find that the $R^2$s for these regressions are very high and comparable to those obtained for the yield spread in Section 2.1. Thus, the significant switch in bond returns does not only arise when using the yield spread.

2.3.6 The Considered Measure of the Yield Spread

Another important feature of (2) is the short rate used to compute the yield spread. Our benchmark specification uses the one-year bond yield from Gürkaynak, Sack and Wright (2007) to ensure consistency with longer term yields and to avoid well-known frictions in very short-term Treasury yields. For instance, Duffee (1996) shows that Treasury bills with less than three months to maturity display idiosyncratic variation due to market segmentation, and Longstaфф (2000) highlights the institutional demand for Treasury bills that make them special and exposed to liquidity risk. We therefore briefly explore the robustness of the results in Table 1 to using other measures of the yield spread. We first observe that the results are robust to using the second principal component from a panel of Gürkaynak, Sack and Wright (2007) bond yields as a measure of the yield spread. The results are also robust to using either the certificate of deposit rate or the Euro-Dollar rate as in Duffee (1996) (both at the three-month horizon) to compute the yield spread. The last alternative we consider is to construct the yield spread from the three-month Treasury bill to illustrate how frictions in the short rate may partly dilute our results. For this noisy measure of the yield spread, the positive change in the intercepts $\alpha_k^A$ remains significant, but the change in the slope coefficients $\beta_k^A$ is no longer significant. On the other hand, the slope estimates in expansions $\beta_k^{EXP}$ are not materially affected by using this noisy measure of the yield spread. Hence, some care is needed when constructing the yield spread to avoid market frictions in short rates during recessions, which easily may prevent researchers from locating the recession effect in bond returns documented in

\footnote{Gürkaynak, Sack and Wright (2007) therefore exclude all Treasury bills as well as notes and bonds with less than three months to maturity when estimating their zero-coupon bond yields. Accordingly, the estimated curves in Gürkaynak, Sack and Wright (2007) should only be used to construct yields with more than three months to maturity.}
Sections 2.1 and 4.

2.3.7 Sample Period

It is also interesting to explore how sensitive the findings in Table 1 are to omitting specific recessions. We address this question by re-estimating the modified return regression in (2) when omitting recessions from either i) 1970-1979, ii) 1980-1989, iii) 1990-1999, or iii) 2000-2009. We find that our results are remarkably robust to omitting recessions in each of these decades, and we find that the change in $\beta_k^A$ are even stronger when omitting the great recession from 2007-2009. The only exception relates to yields at the seven- and ten-year maturity for which the switch in (2) is not statistically significant when we exclude recessions from the 1980s.

To increase the number of observations where the economy is in recession, we next extend the sample back to 1926 by using the 20-year government bond return series from Ibbotson and the corresponding yield spread. The PMI is not available back to 1926 and we therefore implement (2) with NBER recessions. We consider the full sample from 1926:1 to 2016:12 and two subsamples before and after 1961:6, implying that the early period from 1926:1 to 1961:5 has no overlap with the sample analyzed so far. All three samples confirm our findings from above, by showing a significant positive switch in the intercept and a significant negative switch in the slope coefficient when entering recessions. Thus, the systematic difference in bond returns between expansions and recessions extend beyond the sample period considered so far and goes all the way back to 1926.

2.3.8 The Bauer-Hamilton Test

It is widely acknowledged that the first three principal components $\text{pca}_t$ explain virtually all of the variation in the yield curve, and it is therefore tempting to argue that they include all relevant information when forecasting bond returns. This prediction is termed the spanning hypothesis, because it implies that all information is "spanned" by $\text{pca}_t$. Despite its appealing intuition, the spanning hypothesis is often rejected empirically as other factors than $\text{pca}_t$ help to forecast bond returns (see Cochrane and Piazzesi (2005), Ludvigson and Ng (2009), Joslin, Priebsch and Singleton (2014), Cieslak and Povala (2015), among others). Our finding in Section 2.1 is related to this literature because we argue that a separate intercept in recessions $1_{(z_t<c)}$ and the yield spread in recessions $1_{(z_t<c)}s_{t,k}$ serve as two additional new forecasting factors. However, Bauer and Hamilton (2018) have recently challenged the regression evidence for these additional forecasting
factors by showing that their standard errors are biased downward, making it more likely that one rejects the spanning hypothesis than implied by the nominal significance level.

To relate our forecasting factors to the general framework of Bauer and Hamilton (2018), consider the regression

$$ r_{x_{t+1,k}} = \alpha_k + \beta_k' \text{pca}_t + \alpha^\Delta_k 1_{\{z_t < c\}} + \beta^\Delta_k 1_{\{z_t < c\}} s_{t,k} + \varepsilon_{t+1,k}. $$

(4)

The first three principal components in pca represent level, slope, and curvature of the yield curve and (4) is therefore very similar to the modified return regression in (2), except for the inclusion of the level and curvature factor. As carefully explained in Bauer and Hamilton (2018), the standard errors of $\alpha^\Delta_k$ and $\beta^\Delta_k$ are biased downwards if both pca and the new forecasting factors $1_{\{z_t < c\}}$ and $1_{\{z_t < c\}} s_{t,k}$ are highly autocorrelated, because the construction of pca makes it correlated with the error term.

We evaluate the impact of this bias by using the recommended bootstrap in Bauer and Hamilton (2018), where excess bond returns are simulated under the spanning hypothesis, i.e. with $\alpha^\Delta_k = \beta^\Delta_k = 0$. The results show that the empirical $t$-statistics for $\alpha^\Delta_k$ and $\beta^\Delta_k$ are numerically too large to be explained by sampling variability in $\alpha^\Delta_k$ and $\beta^\Delta_k$ under the spanning hypothesis. Thus, the small-sample distortions highlighted in Bauer and Hamilton (2018) cannot explain the new finding for bond returns documented in Section 2.1. A further inspection of our new forecasting factors $1_{\{z_t < c\}}$ and $1_{\{z_t < c\}} s_{t,k}$ reveals that they display a relatively low degree of persistence, as their first four autocorrelation coefficients are $\{0.83, 0.68, 0.57, 0.47\}$ for $1_{\{z_t < c\}}$ and $\{0.65, 0.43, 0.39, 0.33\}$ for $1_{\{z_t < c\}} s_{t,k}$. Thus, we largely avoid the small-sample distortion identified in Bauer and Hamilton (2018) because the proposed forecasting factors are not very persistent.

### 2.3.9 Forecast Horizon

We have so far used a short monthly horizon ($h = 1$) for excess bond returns to reduce the number of forecasts from an expansion into a recession, and vice versa. However, the switch in the intercepts and the slope coefficients in (2) remains significant for three-month returns ($h = 3$). When consid-

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3 Unreported results reveal that the level and curvature factors are generally insignificant in (4), but we nevertheless include these factors to follow the testing procedure in Bauer and Hamilton (2018).

4 Bond yields are here bootstrapped using a Gaussian VAR(1)-model for pca, and their related loading matrix, in addition to additive Gaussian measurement errors. The bootstrap distributions of our new forecasting factors are obtained from an independent AR(1)-model for $z_t$ (i.e. PMI). Further details are provided in the Online Appendix.
ering semi-annual returns \((h = 6)\), we more frequently forecast across regimes and it is therefore not too surprising that the recession effect in bond returns weakens with only \(\beta_k^A\) being significant. For annual returns \((h = 12)\), there hardly exist any observations for which we forecast from a recession and into a recession, and we therefore find that both \(\alpha_k^A\) and \(\beta_k^A\) are insignificant – although the estimates of \(\alpha_k^A\) and \(\beta_k^A\) have the same sign as with monthly returns. Thus, a relatively short forecast horizon of one month or one quarter is required to locate the difference in bond returns between expansions and recessions documented above. Given that most research on bond return predictability uses annual returns, this may explain why the result in Section 2.1 has not previously been noticed in the literature.

### 2.3.10 Economic Gains

Sarno, Schneider and Wagner (2016) show that high forecast accuracy of bond returns does not necessarily imply that one can outperform the Expectations Hypothesis out-of-sample when measured in terms of economic value. To analyze whether our modified return regression in (2) can generate economic value, we consider a mean-variance investor who chooses a risky bond weight of

\[
\omega_{t,k} = \frac{1}{\gamma} \hat{R}X_{t+1,k} / \hat{\sigma}^2_{t+1,k}
\]

in period \(t\). Here, \(\hat{R}X_{t+1,k}\) is the forecasted excess bond return from the considered model, \(\gamma\) is the coefficient of relative risk aversion, and \(\hat{\sigma}^2_{t+1,k}\) is the variance computed using a ten-year rolling window or the exponentially weighted moving average (EWMA) estimator. The portfolio weights are restricted to be between 0 and 1.5 as in Campbell and Thompson (2008). This gives rise to a portfolio return of

\[
R_{p,t+1,k} = \omega_{t,k}R_X_{t+1,k} + R_{f,t+1},
\]

where \(R_{f,t+1}\) is the risk-free return. The realized average utility is

\[
U_{t+1}(R_{p,t+1,k}) = \frac{1}{T - T_0} \sum_{t=T_0+1}^T \left( R_{p,t+1,k} - \frac{1}{2} \gamma \left( R_{p,t+1,k} - \tilde{\mu}_{p,k} \right)^2 \right), \tag{5}
\]

where \(T_0\) is the size of the estimation window used to form the first portfolio weights and \(\tilde{\mu}_{p,k}\) is the average portfolio return. We consider a standard value of risk aversion with \(\gamma = 5\) and set the investment horizon to one month. The economic gain from a given forecasting model is evaluated using certainty equivalent returns \((CER)\), where we compute the average utility from the no-predictability benchmark \((CER_{0,k})\) and from a given forecasting model \((CER_{k})\). We then report \(\Delta CER_k = CER_k - CER_{0,k}\) at a given maturity, where \(\Delta CER_k\) may be interpreted as the fee a mean-variance investor would be willing to pay to get access to the forecasting model. As in Gargano, Pettenuzzo and Timmermann (2019), we use a Diebold-Mariano test to evaluate whether
\( \Delta CER_k \) is different from 0.

Table 2 reports the utility gains when using the yield spread to predict excess bond returns as in (1) and when modifying this regression to allow for regime-switching as in (2). The change in utility relative to the no-predictability benchmark is positive for both predictability regressions across all maturities, but we generally obtain higher utility gains when allowing for regime-switching (0.85% to 2.30%) than without (0.14% to 1.33%). However, even the utility gains from the modified return regression are only significant at the 10% level for short and long maturities and insignificant at medium maturities. Thus, our results appear consistent with Sarno, Schneider and Wagner (2016), showing that it is difficult to translate evidence of time-variation in expected excess bond returns into economic gains for an investor.

### 2.3.11 Time-Variation in the Degree of Return Predictability

The switch in bond return predictability documented in Section 2.1 relates to a similar finding for stock return predictability in Henkel, Martin and Nardari (2011). However, they show that stock return predictability is absent in expansions but sizable in recessions, whereas we find bond return predictability in both expansions and recessions. For bond returns, Gargano, Pettenuzzo and Timmermann (2019) report high degrees of predictability in both regimes, but that the degree of predictability is stronger in NBER recessions than in expansions. We relate our result to these findings in Figure 2, which shows the regime-dependent \( R^2 \)s for (2) when computed separately for expansions and NBER recessions as in Gargano, Pettenuzzo and Timmermann (2019). The figure shows that the \( R^2 \)s are higher during recessions than in expansions for the two-year to five-year maturity spectrum, which is consistent with Gargano, Pettenuzzo and Timmermann (2019) that exclusively focus on these maturities. But Figure 2 also shows that this pattern is reversed for maturities beyond five years, where we find more predictability in expansions than in recessions.

### 3 A Macro-Finance Model with Regime-Switching

This section presents a macro-finance term structure model (MTSM) to provide an economic explanation for the switch in bond return predictability documented above. We start by motivating the considered model in Section 3.1, before formally presenting the model in Section 3.2 and the adopted estimation routine in Section 3.3. The proposed model is finally related to the existing
3.1 Motivation

It is useful to consider Figure 3 to motivate the model we present below. The top row of this figure shows the two variables in our modified return regression in (2), that is, the average level of monthly excess returns and the yield spread at the ten-year maturity in the first eight months after the start of a recession defined using the PMI. The bottom row of the figure shows the average changes in the three-month and ten-year bond yields in a recession. The unmarked red lines in these charts document the empirical moments in the U.S. from 1961:6 to 2016:12, while the remaining lines represent model-implied moments that we discuss in Section 4. We first note that excess bond returns attain their highest level at the beginning of a recession (the second month) and fall afterwards. This temporary increase in bond risk premia may reflect an increase in risk aversion. But the Federal Reserve may also re-evaluate the risks related to the U.S. economy at the start of a recession and consider a radical reduction in the policy rate. Such a temporary switch in monetary policy seems consistent with the substantial fall of about 2% in the three-month yield during a recession as reported in Figure 3. This change in monetary policy also appears to be in line with the expectations among investors, as the ten-year bond yield falls at the start of a recession, although by less than short-term yields, and we therefore see a steepening of the yield curve. The key observation from Figure 3 is that the yield spread during recessions may signal a very accommodating monetary policy, because the large reduction in the short rate increases the yield spread, which from the second month of a recession coincides with falling excess bond returns. This implies that the yield spread becomes a hedging factor (i.e. a factor that market participants would like to get exposure to), because investors are calling for very accommodating monetary policy in recessions.

The building blocks we use to evaluate whether this narrative may explain our new finding are as follows. The policy rate is specified by a Taylor-rule with time-varying policy weights on inflation and the output gap to allow the Federal Reserve to set monetary policy differently in expansions and recessions. To capture a temporary switch in investors’ risk compensation for holding bonds in recessions, we allow the market prices of risk to switch between expansions and recessions.\footnote{Another possibility would be to consider a Taylor-rule, where the policy weights are constant within regimes but switch between regimes. Results provided in the Online Appendix suggest that this somewhat simpler model is able to explain our new finding as well.}
3.2 Model Description

We consider a setting where the policy rate \( r_t \) is determined by the inflation rate \( \pi_t \) and the output gap \( \hat{y}_t \) as in Taylor (1993). To accommodate variation in the responses of the Federal Reserve to the macroenvironment, we follow Ang, Boivin, Dong and Loo-Kung (2011) and let the weights on inflation and the output gap in this Taylor-rule vary over time. That is,

\[
    r_t = \alpha + \omega_{\pi,t} \pi_t + \omega_{\hat{y},t} \hat{y}_t, \tag{6}
\]

where \( \alpha \) is a constant and the weights \((\omega_{\pi,t}, \omega_{\hat{y},t})\) are required to be non-negative in accordance with the New Keynesian theory. We collect the observed macro variables in \( x_{1,t} = \begin{bmatrix} \pi_t & \hat{y}_t \end{bmatrix}' \) and the unobserved policy weights in \( x_{2,t} = \begin{bmatrix} \omega_{\pi,t} & \omega_{\hat{y},t} \end{bmatrix}' \). The dynamics of the pricing factors \( x_t = \begin{bmatrix} x_{1,t}' & x_{2,t}' \end{bmatrix}' \) under the risk-neutral measure \( Q \) are given by

\[
    x_{t+1} = \mu^Q + \Phi^Q x_t + \Sigma^Q x_{x,t+1}, \tag{7}
\]

where \( e_{x,t+1}^Q \sim \mathcal{NID}(0, I) \). In the absence of arbitrage, Ang, Boivin, Dong and Loo-Kung (2011) show that the short rate specification in (6) and the Gaussian dynamics in (7) imply that bond yields \( y_k \) at maturity \( k \) are quadratic in \( x_t \), i.e.

\[
    y_k(x_t) = A_k + B_k' x_t + C_k x_t, \tag{8}
\]

for \( k = 1, 2, \ldots, K \).

We now deviate from the MTSM in Ang, Boivin, Dong and Loo-Kung (2011) by allowing the market prices of risk to be piece-wise affine in \( x_t \), with loadings depending on whether the economy is in expansion (regime 1) or recession (regime 2). That is, we let

\[
    \lambda_t = 1_{\{z_t \geq c\}} \Sigma^{-1} x_t \left( \lambda_0^{(1)} + \lambda_x^{(1)} x_t \right) + 1_{\{z_t < c\}} \Sigma^{-1} x_t \left( \lambda_0^{(2)} + \lambda_x^{(2)} x_t \right), \tag{9}
\]

but assume otherwise the standard expression for the stochastic discount factor, i.e. \( M_{t,t+1} = \exp \left\{ -r_t - \frac{1}{2} \lambda_t' \lambda_t - (e^x_{x,t+1})' \lambda_t \right\} \). The regimes are identified as in Section 2, meaning that \( z_t \) refers to the PMI and recessions correspond to episodes when \( z_t \) is below the threshold \( c = 44.5 \).\(^7\)

\(^7\)Another possibility is to consider a more smooth transition between regimes as discussed in Section 2.3.1.
Given that the factor dynamics remain conditionally Gaussian, a simple change of measure gives the dynamics under the physical measure $P$

$$
    x_{t+1} = \mu^Q + 1\{z_t \geq c\} \lambda^{(1)}_0 + 1\{z_t < c\} \lambda^{(2)}_0 \\
    + \left( \Phi^Q + 1\{z_t \geq c\} \lambda^{(1)}_x + 1\{z_t < c\} \lambda^{(2)}_x \right) x_t + \Sigma x \varepsilon_{x,t+1}.
$$

Thus, our new specification in (9) accommodates the possibility that the level and the persistence of macro-fundamental ($\pi_t, \dot{\gamma}_t$) as well as the policy responses by the Federal Reserve ($\omega_t, \omega^\gamma_t$) may differ across recessions and expansions. In contrast, when there is no regime-switching in the intercepts ($\lambda^{(1)}_0 = \lambda^{(2)}_0$) and the factor loadings ($\lambda^{(1)}_x = \lambda^{(2)}_x$), then (9) reduces to the specification in Duffee (2002) and we recover the model of Ang, Boivin, Dong and Loo-Kung (2011).

The model is closed by letting the $P$ dynamics of $z$ evolve as

$$
    z_{t+1} = \gamma_0 + \gamma_z z_t + \gamma_x x_t + \Sigma z \varepsilon_{z,t+1},
$$

where $\varepsilon_{z,t+1} \sim N(0, 1)$ and independent of $\varepsilon_{x,t+1}^P$. That is, $z_t$ may depend on its own lag if $\gamma_z \neq 0$ and lagged values of the pricing factors if $\gamma_x \neq 0$. The latter implies that the dynamics of the yield curve affect economic activity and hence introduce a feedback effect from financial markets to the real economy.\(^8\)

### 3.3 Model Estimation

It is well-known that term structure models may be challenging to estimate, mainly because the parameters describing the market prices of risk are hard to identify with very persistent pricing factors. One may therefore encounter problems with local optima or find it challenging to obtain (converged) draws from the posterior distribution within a Bayesian setting. Compared to the benchmark MTSM in Ang, Boivin, Dong and Loo-Kung (2011), our extension with regime-switching has potentially twice the number of parameters for the market prices of risk, suggesting that this model may be quite demanding to estimate. We overcome this limitation by drawing on recent innovations in estimation methods for term structure models, by extending the sequential regression (SR) approach of Andreasen and Christensen (2015) to models with regime-switching as in (10).

\(^8\)We do not allow $z_t$ to enter in the policy rate in (6) and the $Q$ dynamics in (7) for simplicity, because the effect of economic activity is here captured by the presence of the output gap $\dot{\gamma}_t$. It is easy to relax this assumption and also allow $z_t$ to directly affect bond yields by letting $z_t$ enter in (6) and/or (7).
This implies that all parameters in the market prices of risk are obtained instantaneously within the SR approach, meaning that our extension of the MTSM in Ang, Boivin, Dong and Loo-Kung (2011) comes at no additional computational costs.

We next describe the SR approach when adapted to the proposed MTSM with regime-switching in the market prices of risk. Here, we only present the three steps in the SR approach and refer to Andreasen and Christensen (2015) for technical details and how to obtain standard errors.

**Step 1:** The first step of the SR approach estimates the unobserved policy weights in $x_{2,t}$ and the "$Q$ parameters" that determine the cross-sectional relationship between the pricing factors and the bond yields in (8). The $Q$ parameters are denoted by $\theta_1 \equiv \left[ \theta_{11}^\prime \theta_{12}^\prime \right]^\prime$, where $\theta_{11} \equiv \left[ \alpha \ (\mu^Q)^\prime \ vec(\Phi^Q)^\prime \right]^\prime$ and $\theta_{12} \equiv vec(\Sigma_x)$. We select $n_y$ maturities $\{m_j\}_{j=1}^{n_y}$ along the yield curve and account for measurement errors $\{v_{t,m_j}\}_{j=1}^{n_y}$ in these yields. When $n_y$ is large relative to the number of unobserved pricing factors, the SR approach exploits the fact that $x_{2,t}$ can be estimated reliably by a sequence of cross-section regressions that minimize the distance between the observed bond yields in the data $y_{t,m_j}^{data}$ and the model-implied yields $y_{m_j}(x_t; \theta_1)$. That is, for a given value of $\theta_1$, we compute

$$\hat{x}_{2,t}(\theta_1) = \arg\min_{x_{2,t} \in \mathbb{R}^2} \frac{1}{n_y} \sum_{j=1}^{n_y} \left( y_{t,m_j}^{data} - y_{m_j}(x_t; \theta_1) \right)^2$$

for $t = 1, 2, ..., T$. The $Q$ parameters are then estimated by minimizing the pooled squared residuals from all these cross-section regressions, i.e.

$$\theta_1^{step1} = \arg\min_{\theta_1 \in \Theta_1} \frac{1}{T n_y} \sum_{t=1}^{T} \sum_{j=1}^{n_y} \left( y_{t,m_j}^{data} - y_{m_j}(\hat{x}_t(\theta_1); \theta_1) \right)^2,$$

where $\hat{x}_t(\theta_1) \equiv \left[ x_{1,t} \ x_{2,t}(\theta_1) \right]^\prime$ and $\Theta_1$ denotes the feasible domain of $\theta_1$.

**Step 2:** The second step estimates the "$P$ parameters" that determine the physical time series dynamics of $x_t$ and $z_t$. To describe the procedure, let

$$\theta_2^x \equiv \left[ \left( \lambda_0^{(1)} \right)^\prime \left( \lambda_0^{(2)} \right)^\prime \ vec(\lambda_x^{(1)})^\prime \ vec(\lambda_x^{(2)})^\prime \ vech(\Sigma_x)^\prime \right]^\prime$$

The nonlinear regressions in (12) are solved efficiently using the Levenberg-Marquardt optimizer with $\hat{x}_{2,t}(\theta_1)$ serving as good starting values for $t = 2, 3, ..., T$. The sign constraint on $x_{2,t}$ is imposed by optimizing (12) with respect to $\hat{x}_{2,t} \equiv \log x_{2,t}$, implying that $x_{2,t} \equiv \exp \{\hat{x}_{2,t}\}$. 

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and \( \theta^2 \equiv \begin{bmatrix} \gamma_0 & \gamma_z & \gamma_x' & \Sigma_{zz} \end{bmatrix}' \) contain all the parameters governing the \( \mathbb{P} \) dynamics of \( x_t \) and \( z_t \), respectively. Replacing the unobserved values of \( x_{2,t} \) in (10) by \( \hat{x}_{2,t} \left( \hat{\theta}^{1\text{step1}} \right) \) from the first step, we estimate \( \theta^2 \) by extending the SR approach of Andreasen and Christensen (2015) to \( \mathbb{P} \) dynamics with regime-switching. That is, we run an adjusted regression based on (10) that accounts for estimation uncertainty in \( \hat{x}_{2,t} \left( \hat{\theta}^{1\text{step1}} \right) \), as described in Appendix A.2. The elements in \( \theta^2 \) are estimated in a similar fashion based on (11) using the regression provided in Andreasen and Christensen (2015). Importantly, both \( \hat{x}_{2,t} \left( \hat{\theta}^{2\text{step2}} \right) \) and \( \hat{z}_{2,t} \left( \hat{\theta}^{2\text{step2}} \right) \) are given in closed form, meaning that all \( \mathbb{P} \) parameters are obtained instantaneously.

To ensure stationarity of model-implied bond yields, we require \( x_t \) to be stationary under the \( \mathbb{P} \) measure. This condition holds if the loading matrices on \( x_t \) in (10) are stable in recessions and in expansions, i.e. if all eigenvalues of \( \Phi^Q + \lambda^{(i)}_x \) for \( i = \{1, 2\} \) are inside the unit circle. If one of these conditions is not satisfied, then we downscale \( \Phi^Q + \lambda^{(i)}_x \) by \( \delta_i \) for \( i = \{1, 2\} \) using the data-driven procedure of Andreasen and Meldrum (2014), which is described in Appendix A.3.

**Step 3**: The conditional covariance matrix \( \Sigma_x \) is estimated in both the first and second step. Unreported results show that \( \hat{\Sigma}_{x}^{\text{step1}} \) is estimated very inaccurately compared to \( \hat{\Sigma}_{x}^{\text{step2}} \), which therefore is our preferred estimate.\(^{10}\) Based on this more efficient estimate of \( \Sigma_x \), we then condition on the value of \( \hat{\Sigma}_{x}^{\text{step2}} \) and re-estimate \( \theta_{11} \), i.e.

\[
\hat{\theta}^{3\text{step3}}_{11} = \arg \min_{\theta_{11} \in \Theta_{11}} \frac{1}{T_{ny}} \sum_{j=1}^{n_y} \sum_{t=1}^{T_{y,j}} \left( \hat{y}^{data}_{yt, mj} - \hat{y}_{yt, mj} \left( \hat{x}_{t} \left( \theta_{11} \hat{\Sigma}_{x}^{\text{step2}} \right) ; \theta_{11} \hat{\Sigma}_{x}^{\text{step2}} \right) \right)^2.
\]  

(13)

The improved estimates of \( \left\{ \hat{x}_{2,t} \left( \hat{\theta}^{3\text{step3}}_{11}, \hat{\Sigma}^{\text{step2}} \right) \right\}_{t=1}^{T} \) from (13) are finally used to update the estimates of \( \left\{ \lambda^{(i)}_0, \text{vec} \left( \lambda^{(i)}_x \right) \right\}_{i=1}^{2} \) and \( \theta^2 \) by re-running the two regressions from step 2.\(^{11}\)

### 3.4 Comparing to Other Models with Regime-Switching

When formulating term structure models with regime-switching there is an inherent trade-off between the richness of a given model and the computational complexity related to bond pricing and estimation. We have therefore chosen to consider the most parsimonious model capable of addressing the new evidence on bond return predictability documented in Section 2.1. This implies that we

\(^{10}\)Andreasen and Christensen (2015) report a similar finding for a Gaussian quadratic term structure model with two unobserved pricing factors.

\(^{11}\)Note that we do not update \( \hat{\Sigma}_{x}^{\text{step2}} \) to ensure that the same estimate of \( \Sigma_x \) is used for bond pricing and for describing the factor dynamics under the \( \mathbb{P} \) measure.
omit regime-switching in $\Sigma_x$ as in Ang, Bekaert and Wei (2008) or variation in $\Sigma_x$ within regimes as in Bansal and Zhou (2002). We also restrict the flexibility of the model by identifying expansions and recessions directly from a macroeconomic variable (the PMI). In contrast, most term structure models with regime-switching use bond yields to identify regimes, although their estimated values often are closely related to the business cycle as in Bansal and Zhou (2002) and Dai, Singleton and Yang (2007). Similar to these two papers, the market prices of risk in the proposed model are allowed to change freely across regimes, whereas Ang, Bekaert and Wei (2008) consider a somewhat more restricted formulation. As in Dai, Singleton and Yang (2007), we also accommodate time-varying transition probabilities between regimes under the $\mathbb{P}$ measure when $\gamma_x \neq 0$ or $\gamma_x \neq 0$, whereas these probabilities are constant in Bansal and Zhou (2002) and Ang, Bekaert and Wei (2008).

The specification of the policy rate in (6) also links our model to the literature estimating Taylor-rules (see Clarida, Gali and Gertler (2000), Coibion and Gorodnichenko (2012) among others). Taylor-rules are a useful benchmark, but actual monetary policy often deviates from these rules. For instance, the Federal Reserve accounts for the effectively zero lower bound, and it may change its preferences between stabilizing $\pi_t$ and $\hat{y}_t$ as time passes or respond to other variables as argued by Rudebusch (2006). Examples of such omitted variables could be the stock market index, house prices, credit conditions, or the volatility of long-term yields as in Stein and Sunderam (2018). We account for these deviations from the standard Taylor-rule by allowing the policy weights to vary over time, which is a parsimonious way to get a flexible specification for monetary policy.\footnote{Unreported results show no evidence of interest rate smoothing if added to the Taylor rule in (6). This finding is consistent with the results in Rudebusch (2002) and Rudebusch (2006), who also use a panel of bond yields to reject Taylor-rules with interest rate smoothing.}

The proposed model is also related to Rudebusch and Wu (2007), who document instability in the regressions of Campbell and Shiller (1991) during the mid-1980 and suggest that this change may be explained by a one-off switch in the market prices of risk within a Gaussian affine term structure model. Different from their work, our empirical finding in Section 2.1 reveals a recurrent switch in excess bond returns between expansions and recessions, but similar to Rudebusch and Wu (2007) we also allow for nonlinear variation in the market prices of risk as a possible explanation.
4 Empirical Findings

This section estimates the proposed MTSM with regime-switching, denoted RS-MTSM. For the analysis, we use the two preferred macro variables by Bauer and Rudebusch (2017) that explain the largest proportion of the variation in the U.S. policy rate with constant policy weights. Hence, inflation is represented by the year-over-year growth rate in the consumer price index excluding food and energy prices (core CPI), and the output gap is measured by the unemployment gap (UGAP), which is the difference between the unemployment rate and its estimated natural rate from the Congressional Budget Office. The series for the PMI is re-centered around the threshold value 44.5, and bond yields are measured using the same data set by Gürkaynak, Sack and Wright (2007) as applied in Section 2 with maturities of 6, 9, 12, ..., 120 months.

We proceed by presenting the estimated risk-neutral parameters in Section 4.1, while the model fit and the extracted policy weights are discussed in Section 4.2. The estimated market prices of risk are provided in Section 4.3, and the implications for bond return predictability are studied in Section 4.4. Section 4.5 examines the dynamics of bond yields during recessions in greater detail, while Section 4.6 explores the impact of monetary policy for generating a switch in bond return predictability. Estimates of term premia are finally provided in Section 4.7.

4.1 Estimated Risk-Neutral Parameters

The top panel in Table 3 shows the estimated risk-neutral parameters for the RS-MTSM, where elements in $\Phi^Q$ with ” − ” reflect the same zero-restrictions as imposed in Ang, Boivin, Dong and Loo-Kung (2011). We find that all factors display the usual high persistence under $Q$, where the standard requirement of stationarity is imposed and restricts the autoregressive term of $\hat{y}_t$ on $\hat{y}_{t-1}$ to be below one. Cross-correlation among the pricing factors is captured through $\Phi^Q$ and $\Sigma_x$, where most off-diagonal elements are significant. For comparison, we also note that the corresponding estimates for the benchmark MTSM without regime-switching at the bottom of Table 3 are very similar to those obtained for the RS-MTSM.

\footnote{To ease the interpretation, we scale UGAP by minus one, implying that $\hat{y}_t = -\text{UGAP}$.}
4.2 The Goodness of Fit and the Policy Weights

Table 4 reveals that the four pricing factors provide a very tight fit to bond yields, as the model errors $v_{t,mj}$ have means close to zero and standard deviations around 10 basis points, except at the 6-month maturity where the standard deviation is 21.8 basis points.

The two observed macro variables are shown at the top of Figure 4, and the estimated policy weights are displayed at the bottom for an annualized short rate, implying that $\omega_{\pi,t}$ and $\omega_{\hat{y},t}$ are scaled by 12. The weight assigned to inflation in the Taylor-rule starts around two in the early 1960s, but falls below one during the 1969-1970 recession. Its value increases in the early 1970s but decreases during the 1973-1975 recession to well below one, where it remains until March 1978. A unique stable equilibrium in the standard New Keynesian model requires that $\omega_{\pi,t}$ exceeds one (the so-called Taylor-principle), which we find is violated during much of the 1970s, as also emphasized in Clarida, Gali and Gertler (2000), Lubik and Schorfheide (2004), Boivin (2006) among others. The weight to inflation increases in the late 1970s and throughout much of the 1980s, except for the two recessions in the early 1980s. During the 1990s, the Federal Reserve generally assigns more weight to inflation which reaches an all time high at the end of the 1990s. The very accommodating monetary policy related to the recession in 2001 and the recession in 2007-2009 generates a sharp drop in $\omega_{\pi,t}$ and takes it well below one at the end of the sample. The variation in the weight to the output gap $\omega_{\hat{y},t}$ in the Taylor-rule is clearly less volatile compared to $\omega_{\pi,t}$, although there are notable changes in $\omega_{\hat{y},t}$ around recessions. These results are very similar to those reported in Ang, Boivin, Dong and Loo-Kung (2011), who also find that $\omega_{\pi,t}$ is more volatile than $\omega_{\hat{y},t}$ and that both policy weights display large movements around recessions.\footnote{One important exception is that Ang, Boivin, Dong and Loo-Kung (2011) find a much lower level of $\omega_{\pi,t}$ at the end of the 1990s than reported in Figure 4. The Online Appendix shows that this difference arises because Ang, Boivin, Dong and Loo-Kung (2011) use the GDP gap (i.e. GDP in deviation from potential GDP) to construct $\hat{y}_t$, which is slightly positive at the end of the 1990s, whereas our measure of $\hat{y}_t$ based on the unemployment gap is negative in the late 1990s. We use the unemployment gap to measure $\hat{y}_t$ because it gives the best in-sample fit with an overall root mean squared error for bond yields of 7.37 basis points, whereas it is 8.41 basis points when using the GDP gap.}

Thus, the estimates of $\omega_{\pi,t}$ and $\omega_{\hat{y},t}$ suggest that the Federal Reserve acts more aggressively during recessions than in expansions. In particular, the evolution in $\omega_{\pi,t}$ is noticeable, as it falls in all recessions except the one from 1990 to 1991. This implies that the average change in $\omega_{\pi,t}$ during an NBER recession is $-0.63$. This more pro-active policy by the Federal Reserve in recessions is also confirmed by a simple inspection of the Federal Funds Target Rate, which is not used for the model estimation. We find that 70% of the monthly changes in the target rate during recessions
are above 25 basis points, while the corresponding figure for expansions is only 31%. This implies that the average monthly change in the target rate during recessions is about 60 basis points but only about 34 basis points during expansions.

4.3 The Market Prices of Risk

The top panel of Table 5 reports the market prices of risk in the RS-MTSM. Unreported results show that $\lambda_0$ does not switch between expansions and recessions, and we therefore focus on a version of the model where only $\lambda_x$ is allowed to switch. The estimates show that risks related to inflation, the output gap, and the policy weight on inflation $\omega_{\pi,t}$ are priced unconditionally, as the first two elements in $\hat{\lambda}_0$ are significant at the 5% level and its third element at the 10% level.

For the estimates of $\lambda_x^{(1)}$ and $\lambda_x^{(2)}$, a Wald test robust to heteroskedasticity and autocorrelation in the regression residuals in (10) clearly rejects the null hypothesis $\lambda_x^{(1)} = \lambda_x^{(2)}$ of no regime-switching ($p$-value of 0.000). To understand what accounts for this significant switch and how risk changes across the business cycle, consider the partition

$$\lambda_x^{(i)} = \begin{bmatrix} \lambda_{x,11}^{(i)} & \lambda_{x,12}^{(i)} \\ \lambda_{x,21}^{(i)} & \lambda_{x,22}^{(i)} \end{bmatrix} \text{ for } i = \{1, 2\},$$

where each submatrix of $\lambda_x^{(i)}$ has dimensions $2 \times 2$. Recall that $x_t = [\pi_t \; \bar{y}_t \; \omega_{\pi,t} \; \omega_{\bar{y},t}]'$, implying that the first submatrices $\lambda_{x,11}^{(1)}$ and $\lambda_{x,11}^{(2)}$ reveal how past macro fundamentals $(\pi_{t-1}, \bar{y}_{t-1})$ affect current macro fundamentals in expansions and recessions, respectively. We generally find small differences between $\hat{\lambda}_{x,11}^{(1)}$ and $\hat{\lambda}_{x,11}^{(2)}$, except that past inflation only affects current inflation risk significantly in expansions but not in recessions. Testing whether $\hat{\lambda}_{x,11}^{(1)} = \hat{\lambda}_{x,11}^{(2)}$ using a robust Wald test, we reject the null hypothesis of no regime-switching in the upper left block of $\lambda_x^{(i)}$ at the 5% level ($p$-value of 0.043). In contrast, we cannot reject the null hypothesis that $\hat{\lambda}_{x,12}^{(1)} = \hat{\lambda}_{x,12}^{(2)}$ using a robust Wald test ($p$-value of 0.556), showing that past policy weights $(\omega_{\pi,t-1}, \omega_{\bar{y},t-1})$ affect current macro fundamentals in the same way during expansions and recessions. Note also that the estimates in $\hat{\lambda}_{x,12}$ are very similar to those obtained in the benchmark MTSM without regime-switching, as provided at the bottom of Table 5.

Turning to the lower left block of $\lambda_x^{(i)}$, we see that the current policy weight on inflation reacts differently to past macro fundamentals across the business cycle. In particular, a higher output
gap leads to a higher policy weight on inflation in expansions with \( \hat{\lambda}_x^{(1)}(\omega_\pi, \hat{y}) = 0.0726 \), whereas a higher output gap in recessions reduces the policy weight on inflation with \( \hat{\lambda}_x^{(2)}(\omega_\pi, \hat{y}) = -0.2720 \). Testing whether \( \lambda_{x,21}^{(1)} = \lambda_{x,21}^{(2)} \) using a robust Wald test, we reject the null hypothesis of no regime-switching in the lower left block of \( \lambda_x \) at a 10% level (\( p \)-value of 0.060). The lower right block of \( \lambda_x^{(i)} \) shows that the level of risk related to the time-varying policy weights changes significantly with past policy weights in recessions but not in expansions. For the weight to inflation, we find that \( \hat{\lambda}_x^{(2)}(\omega_\pi, \omega_\pi) = -0.2069 \) and \( \hat{\lambda}_x^{(2)}(\omega_\pi, \omega_\hat{y}) = 0.6657 \) are significant at the 5% level, while for the weight on the output gap we find that \( \hat{\lambda}_x^{(2)}(\omega_\hat{y}, \omega_\pi) = -0.0117 \) is significant at the 10% level. As a result, a robust Wald test clearly rejects the null hypothesis \( \hat{\lambda}_x^{(1)} = \hat{\lambda}_x^{(2)} \) with a \( p \)-value of 0.003. Combining the insights from the estimates of \( \lambda_{x,21}^{(i)} \) and \( \lambda_{x,22}^{(i)} \), we mainly find that the risks attached to the policy weights are priced during recessions. This ‘recession effect’ is not present in the benchmark MTSM without regime-switching, and this explains why risks attached to the two policy weights in this model are not priced significantly when conditioning on variation in the four pricing factors, as shown at the bottom of Table 5.

We draw two conclusions from this decomposition of \( \lambda_x^{(1)} \) and \( \lambda_x^{(2)} \). First, there is only weak evidence for a switch in the risks attached to macro fundamentals across expansions and recessions. A result that seems consistent with the common finding that inflation and the output gap only respond very slowly to shocks. Second, there is strong evidence for regime-switching in the risks related to monetary policy as captured by the policy weights \( \omega_{\pi,t} \) and \( \omega_{\hat{y},t} \) in the Taylor-rule.

### 4.4 Bond Return Predictability

An obvious question is whether the proposed model is able to explain the loadings in the standard predictability regressions in (1) and the regime-dependent loadings in (2). We therefore simulate a sample of 100,000 observations using the RS-MTSM to obtain the model-implied regression loadings.\(^{15}\) The left charts in Figure 5 reveal that this model matches the standard regression loadings in (1). The top right part of Figure 5 shows that the RS-MTSM also generates the desired low values of \( \alpha_k^{EXP} \) in expansions and that \( \alpha_k^{\Delta} \) increases to capture the higher level of excess bond returns in recessions. Even more encouraging are the results in the bottom right part of Figure 5, showing that the RS-MTSM generates positive slope coefficients \( \beta_k^{EXP} \) in expansions and large

---

\(^{15}\)The \( \mathbb{P} \) dynamics in the RS-MTSM do not enable us to ensure that the policy weights are non-negative in simulations. However, we find that only 3.49% and 0.00% of the draws for \( \omega_{\pi,t} \) and \( \omega_{\hat{y},t} \), respectively, are negative when simulating the model, implying that omitting these non-negativity constraints is not essential for our results.
negative changes in these slope coefficients $\beta_k^\Delta$ during recessions. Most of the values for $\beta_k^{\text{EXP}}$ and $\beta_k^\Delta$ are inside the 95% confidence bands for the sample moments, while the remaining slope coefficients lie just outside these bands. In contrast, the benchmark MTSM without regime-switching is only able to match the standard regression loadings in (1), but not the switch in these loadings during recessions, as shown in Figure 6.

Thus, the proposed regime-switching model goes a long way in explaining the switch in bond return predictability by allowing bond investors to reprice risk when the U.S. economy enters recessions.

4.5 Bond Yields During Recessions

Let us return to Figure 3 and explore how well the RS-MTSM matches the stylized features of bond yields in recessions that we used to motivate the model. We first observe that the RS-MTSM reproduces the same large increase in the ten-year excess bond returns at the start of a recession as seen in the data. The model is also able to match the large fall in the three-month bond yield and the somewhat smaller fall in the ten-year yield, although the drop in this long yield is more persistent than observed in the data. This implies that the model generates an increasing yield spread, which from the second month of a recession coincides with falling excess bond returns to generate the desired switch in bond return predictability. In contrast, the benchmark MTSM does not produce any notable change in excess bond returns during recessions. This model also generates a too small reduction in the three-month bond yield, whereas the ten-year yield even increases slightly. Section 4.1 showed that the risk-neutral parameters in the RS-MTSM and the benchmark MTSM are nearly identical and hence cannot explain the different properties of the two models. Thus, it must be the repricing of risk that allows the RS-MTSM to generate large changes in bond yields during recessions by altering the $\mathbb{P}$ dynamics of the pricing factors.

Given this observation, it seems natural to study the persistence of bond yields in the RS-MTSM to examine whether it is consistent with the data. We therefore consider the regression

$$ y_{t+1,k} = \delta_k + (\rho_k^{\text{EXP}} + \rho_k^\Delta 1_{(z_t<\kappa)}) y_{t,k} + \varepsilon_{t+1,k}, $$

where $\rho_k^{\text{EXP}}$ measures the persistence in expansions and $\rho_k^\Delta$ captures the change in persistence during recessions. Figure 7 shows that U.S. bond yields broadly display the same degree of persis-
tence in expansions with \( \rho_k^{\text{EXP}} \approx 1 \), and that yields are significantly less persistent in recessions. We find that \( \rho_k^{\Delta} \) attains the most negative value at the one-year maturity with \( \rho_{12}^{\Delta} = -0.08 \) and increases gradually to \( \rho_{120}^{\Delta} = -0.04 \) at the ten-year maturity. Figure 7 shows that the RS-MTSM is also consistent with these properties of U.S. bond yields, as the model implies that \( \rho_k^{\text{EXP}} \approx 1 \) and that short-term yields are more mean-reverting during recessions than long yields. In contrast, the benchmark MTSM is unable to generate any asymmetry in the persistence of bond yields across the business cycle as \( \rho_k^{\Delta} \approx 0 \).

Thus, the switch in the market prices of risk that we estimate in the RS-MTSM appears consistent with several stylized features of U.S. bond yields, although none of the moments summarized in Figure 3 and 7 were included in the model estimation.

### 4.6 Monetary Policy and the Switch in Bond Return Predictability

The previous analysis has shown that the ability of the RS-MTSM to produce a switch in bond return predictability is closely linked to regime-switching in \( \lambda_x \). Thus, one way to examine the importance of monetary policy for this result is to gradually "turn off" regime-switching in the rows of \( \lambda_x \) related to the policy weights \( \omega_{\pi,t} \) and \( \omega_{g,t} \). The top charts in Figure 8 show the impact on \( \alpha_k^{\Delta} \) and \( \beta_k^{\Delta} \) when we omit regime-switching in the third row of \( \lambda_x \) but allow regime-switching in the other rows. That is, we momentarily assume that the risks attached to the time-varying weight on inflation \( \omega_{\pi,t} \) in the Taylor-rule are not repriced during recessions. Figure 8 shows that this restriction reduces the level of excess bond returns in recessions, as the values of \( \alpha_k^{\Delta} \) are now outside the 95% confidence bands for maturities at or below five years. The slope coefficients \( \beta_k^{\Delta} \) are also strongly affected, as the values of \( \beta_k^{\Delta} \) are now almost entirely above the 95% confidence bands.\(^{16} \) Thus, allowing for different dynamics of \( \omega_{\pi,t} \) in expansions and recessions through a repricing of risk is essential for the proposed model to explain the higher level of excess bond returns in recessions and the switch in bond return predictability.

The bottom of Figure 8 carries out the same analysis for the time-varying weight on the output gap \( \omega_{g,t} \) in the Taylor-rule. That is, we omit regime-switching in the fourth row of \( \lambda_x \) but allow regime-switching in the other rows. The charts show that this restriction eliminates the increasing pattern in \( \alpha_k^{\Delta} \) and substantially reduces the level of excess bond returns in recessions. As a result,

\(^{16} \)The regression loadings \( \alpha_k^{\text{EXP}} \) and \( \beta_k^{\text{EXP}} \) in expansions are not materially affected when restricting the degree of regime switching in \( \lambda_x \) and therefore not displayed in Figure 8.
the values of $\alpha^\Delta_k$ are now outside the 95% confidence bands for all maturities. The slope coefficients $\beta^\Delta_k$ are also well outside the confidence bands and even turn positive for maturities beyond six years. Accordingly, allowing for different dynamics of $\omega_{\delta,t}$ in expansions and recessions through a repricing of risk is also required to explain the high level of excess bond returns in recessions and to explain the switch in bond return predictability.

Another way to examine the importance of monetary policy for the switch in bond return predictability is to compute impulse response functions (IRFs) for a shock to the policy weights $\omega_{\pi,t}$ and $\omega_{\delta,t}$. Unreported results show that only a shock to $\omega_{\pi,t}$ is able to change the comovement between expected excess bond returns $E_t [rx_{t+1,k}]$ and the yield spread $y_{t,k} - y_{t,12}$ across expansions and recessions as required to match the regime-dependent regression loadings in (2). Also shocks to the two macro fundamentals $\pi_t$ or $\delta_t$ are unable to change the comovement between $E_t [rx_{t+1,k}]$ and $y_{t,k} - y_{t,12}$, implying that shocks to $\omega_{\pi,t}$ are essential for generating a switch in return predictability. We therefore focus exclusively on a shock to $\omega_{\pi,t}$ in Figure 9, where we report the IRFs for expected excess returns on the ten-year bond yield at the top and the ten-year yield spread at the bottom in expansions and recessions. The shock hits in the first period and we report the IRFs for the following 60 months (i.e. 5 years). In expansions, a positive shock to $\omega_{\pi,t}$ decreases $E_t [rx_{t+1,k}]$ by almost 0.5% and reduces the slope of the yield curve $y_{t,k} - y_{t,12}$ by 0.25% as short rates increase more than long-term bond yields.\footnote{Note that Figure 9 plots monthly returns, whereas Ang, Boivin, Dong and Loo-Kung (2011) annualize returns when reporting IRFs.} This means that bond investors lower their required risk compensation in expansions when monetary policy focus more on stabilizing inflation. A shock to $\omega_{\pi,t}$ therefore helps to generate the desired positive comovement between $E_t [rx_{t+1,k}]$ and $y_{t,k} - y_{t,12}$ in expansions as implied by (2), where the slope of the yield curve is a risk factor.

In recessions, on the other hand, a positive shock to $\omega_{\pi,t}$ increases $E_t [rx_{t+1,k}]$ by about 2% and reduces the slope of the yield curve as in expansions. This implies that a shock to $\omega_{\pi,t}$ generates the desired negative comovement between $E_t [rx_{t+1,k}]$ and $y_{t,k} - y_{t,12}$ in recessions, as implied by (2). Given that $\omega_{\pi,t}$ typically falls during recessions (as seen from Figure 4), it follows that this change in monetary policy helps to decrease excess returns although it also steepens the yield spread.

Thus, a key implication of the RS-MTSM is that the observed switch in bond return predictability is closely related to monetary policy and the market’s perception of the risks related to the Federal Reserve’s policy weights. During recessions, investors are eager to see that the Federal Reserve reduces its policy rate by focusing less on inflation (i.e. by lowering $\omega_{\pi,t}$).
a reduction in bond risk premia following such actions by the Federal Reserve, although it increases the slope of the yield curve. This effectively means that the slope of the yield curve becomes a hedging factor in recessions, as monetary policy is able to remove some of the risks attached to recessions. The situation is completely opposite in expansions, as the same policy (i.e. an interest cut due to a softer look at inflation) increases bond risk premia, and we therefore find that the slope of the yield curve is a traditional risk factor. These state-dependent effects of monetary policy are not present in the benchmark MTSM, which therefore is unable to generate a switch in bond return predictability.

4.7 Term Premia

The presence of regime-switching in the market prices of risk has profound implications for term premia, defined as $y_{t,k} = \frac{1}{k} \sum_{i=0}^{k-1} \mathbb{E}_t [r_{t+i}]$. Figure 10 plots term premia in the RS-MTSM and the benchmark model at the two- and ten-year maturity, with NBER recessions indicated by the shaded regions. The two models provide very similar dynamics for term premia in expansions, although term premia in the RS-MTSM generally have a slightly lower level compared to the benchmark MTSM until the late 1980s. However, during recessions, we generally find that term premia in the RS-MTSM increase more than in the benchmark model. The prime examples are the two recessions in the early 1980s. This effect arises because the model with regime-switching assigns an increasing fraction of the forecast distribution for future short rates to a recession, where the switch in $\lambda_x$ implies that all interest rates are substantially lower than in expansions. For instance, the mean of the model-implied 6-month bond yield is 5.29% in expansions and 0.67% in recessions, which closely matches the corresponding moments in the data of 4.45% and 0.61%, respectively. This in turn generates a lower predicted path of future short rates and hence higher term premia in the RS-MTSM compared to the benchmark model. This 'recession amplifier' in term premia gradually disappears towards the end of a recession and explains why term premia in the RS-MTSM are more counter-cyclical than in the benchmark model.

5 Conclusion

This paper identifies a new result for the relation between expected excess bond returns and the yield spread by conditioning the standard return regression on the state of the business cycle.
When the economy is expanding, we find the familiar positive relation between expected excess bond returns and the yield spread. However, when the economy is contracting, we observe a large increase in the intercept to capture the counter-cyclical nature of bond risk premia and a negative relation between expected excess bond returns and the yield spread. To explain this new empirical finding, the MTSM of Ang, Boivin, Dong and Loo-Kung (2011) is extended with regime-switching in the market prices of risk. We show that this model reproduces the empirical intercepts and slope coefficients from standard return regressions and, in addition, matches evidence from the modified return regressions conditioning on the business cycle. The model's ability to explain the switch in return predictability is closely linked to the actions of the Federal Reserve, as we find significant changes in the conduct of monetary policy between expansions and recessions. In particular, the Federal Reserve appears to be less concerned with closing the inflation gap in recessions compared to expansions, and this more accommodating monetary policy helps to reduce the required risk compensation among bond investors in recessions. This effect of monetary policy is not present in the benchmark MTSM, which therefore provides less accurate estimates of bond risk premia compared to our regime-switching model.
A Appendix

A.1 Data

We use the Gürkaynak, Sack and Wright (2007) data set from 1961:6 to 2016:12, where yields beyond the seven-year maturity spectrum from 1961:6 to 1971:8 are calculated by extrapolation of the estimated curves in Gürkaynak, Sack and Wright (2007). These zero-coupon yields are denoted by $y_{t,k}$, which refer to the $k$-month yield in period $t$. Monthly continuously compounded excess returns are given by $r_{xt+1,k} \equiv -\frac{k-1}{12}y_{t+1,k-1} + \frac{k}{12}y_{t,k} - \frac{1}{12}y_{t,1}$. The short rate $y_{t,1}$ is not available from Gürkaynak, Sack and Wright (2007), and we therefore follow Gargano, Pettenuzzo and Timmermann (2019) and use the one-month Treasury bill from the Center for Research in Security Prices (CRSP).

A.2 Step 2 of the SR Approach: Regime-Switching in the Time Series Regression

This subsection describes how to estimate (10) by ordinary least squares when accounting for measurement errors in the estimated pricing factors. For notational convenience, we express (10) as

$$x_{t+1} = 1_{\{z_{t+1} \geq 0\}}h_0^{(1)} + (1 - 1_{\{z_{t+1} \geq 0\}})h_0^{(2)} + h_x^{(1)}x_t^{(1)} + h_x^{(2)}x_t^{(2)} + w_{t+1},$$

where $h_0^{(i)} = \eta^{(i)} + \lambda_0^{(i)}$, $h_x^{(i)} = \phi^{(i)} + \lambda_x^{(i)}$, $x_t^{(1)} \equiv 1_{\{z_t \geq 0\}}x_t$, $x_t^{(2)} \equiv (1 - 1_{\{z_t \geq 0\}})x_t$, and $w_{t+1} = \Sigma x^t_{i=1}$ for $i = \{1, 2\}$. It clearly suffices to present the case where all elements in $x_t$ are estimated, as any observed factors can be accommodated by setting the correction for measurement errors at specific elements in $x_t$ equal to zero. Using the same procedure as in Andreasen and Christensen (2015), we estimate $\theta^2$ based on

$$
\begin{bmatrix}
E [\hat{w}_{t+1} 1_{\{z_{t+1} \geq 0\}}] \\
E [\hat{w}_{t+1} (1 - 1_{\{z_{t+1} \geq 0\}})] \\
E [\hat{w}_{t+1} (\hat{x}_t^{(1)})'] \\
E [\hat{w}_{t+1} (\hat{x}_t^{(2)})'] \\
Var (\hat{w}_{t+1})
\end{bmatrix}
\begin{bmatrix}
0 \\
0 \\
Cov (u_{t+1}^{(1)}, u_t^{(1)}) - h_x^{(1)} \text{Var} (u_t^{(1)}) \\
Cov (u_{t+1}^{(2)}, u_t^{(2)}) - h_x^{(2)} \text{Var} (u_t^{(2)}) \\
\text{Var} (w_{t+1}) + \Omega_{t+1}
\end{bmatrix},
$$

where

$$\Omega_{t+1} \equiv \text{Var} (u_{t+1}) + h_x^{(1)} \text{Var} (u_t^{(1)}) (h_x^{(1)})' + h_x^{(2)} \text{Var} (u_t^{(2)}) (h_x^{(2)})'$$

$$-Cov (u_{t+1}^{(1)}, u_t^{(1)}) (h_x^{(1)})' - h_x^{(1)} \text{Cov} (u_t^{(1)}, u_{t+1}^{(1)})$$

$$-Cov (u_{t+1}^{(2)}, u_t^{(2)}) (h_x^{(2)})' - h_x^{(2)} \text{Cov} (u_t^{(2)}, u_{t+1}^{(2)}).$$

Here, $u_t$ refers to the estimation uncertainty in the estimated pricing factors, i.e. $\hat{x}_t = x_t + u_t$, where $u_t^{(1)} = 1_{\{z_t \geq 0\}}u_t$ and $u_t^{(2)} = (1 - 1_{\{z_t \geq 0\}})u_t$. As in Andreasen and Christensen (2015), all required moments of $u_t$ follow from the first step of the SR approach. The solution to the first four
moments conditions in (14) is given in closed form by

$$
\begin{bmatrix}
\hat{h}_0^{(1)} & \hat{h}_0^{(2)} & \hat{h}_x^{(1)} & \hat{h}_x^{(2)}
\end{bmatrix} = \left(\sum_{t=1}^{T-1} \hat{x}_{t+1}a'_t - \sum_{t=1}^{T-1} \hat{A}_{t+1} \right) \left(\sum_{t=1}^{T-1} a_t a'_t - \sum_{t=1}^{T-1} \bar{V}ar (u_{a,t}) \right)^{-1},
$$

where

$$a_t \equiv \begin{bmatrix} 1_{[z \geq c]} \\ 1 - 1_{[z \geq c]} \\ \hat{x}_t^{(1)} \\ \hat{x}_t^{(2)} \end{bmatrix}, \quad u_{a,t} \equiv \begin{bmatrix} 0 \\ 0 \\ u_t^{(1)} \\ u_t^{(2)} \end{bmatrix},$$

and \(\hat{A}_{t+1} = \begin{bmatrix} 0 & 0 & \bar{Cov} (u_{t+1}, u_t^{(1)}) & \bar{Cov} (u_{t+1}, u_t^{(2)}) \end{bmatrix}.\) The solution to the last moment condition in (14) is also given in closed form and implies the following estimator

$$\bar{V}ar (w_{t+1}) = \frac{1}{T-1 - 2(n_x + 1)} \sum_{t=1}^{T-1} \bar{w}_{t+1} \bar{w}'_{t+1} - \frac{1}{T-1} \sum_{t=1}^{T-1} \bar{\Omega}_{t+1}, \quad (15)$$

where

$$\bar{\Omega}_{t+1} = \bar{V}ar (u_{t+1}) + \hat{h}_x^{(1)} \bar{V}ar (u_t^{(1)}) \left(\hat{h}_x^{(1)}\right)' + \hat{h}_x^{(2)} \bar{V}ar (u_t^{(2)}) \left(\hat{h}_x^{(2)}\right)' - \bar{Cov} (u_{t+1}, u_t^{(1)}) \left(\hat{h}_x^{(1)}\right)' - \hat{h}_x^{(1)} \bar{Cov} (u_t^{(1)}, u_{t+1})$$

$$- \bar{Cov} (u_{t+1}, u_t^{(2)}) \left(\hat{h}_x^{(2)}\right)' - \hat{h}_x^{(2)} \bar{Cov} (u_t^{(2)}, u_{t+1}),$$

with \(\bar{w}_{t+1} = \hat{x}_{t+1} - 1_{[z \geq c]} \hat{h}_0^{(1)} - (1 - 1_{[z \geq c]}) \hat{h}_0^{(2)} - \hat{h}_x^{(1)} \hat{x}_t^{(1)} - \hat{h}_x^{(2)} \hat{x}_t^{(2)}.\) Note that we adopt a standard degree of freedom correction to the first term in (15) because we estimate \(2(n_x + 1)\) unknown parameters per equation in the model. The asymptotic distribution of \(\theta_2^x\) for \(T \to \infty\) follows from Hansen (1982) when applied on the moment conditions in (14). The estimated loadings in the market prices of risk are then given by \(\hat{\lambda}_0^{(i)} = \hat{h}_0^{(i)} - \hat{\mu}_x^{(i)}\) and \(\hat{\lambda}_x^{(i)} = \hat{h}_x^{(i)} - \hat{\Phi}_x^{(i)}\) for \(i = \{1, 2\}.\) Finally, the standard errors for \(\hat{\lambda}_0^{(i)}\) and \(\hat{\lambda}_x^{(i)}\) are identical to those for \(\hat{h}_0^{(i)}\) and \(\hat{h}_x^{(i)}\), respectively. That is, we omit uncertainty about \(\hat{\mu}_x^{(i)}\) and \(\hat{\Phi}_x^{(i)}\), because these estimates use \(Tn_y\) observations and therefore tend faster to infinity than \(\hat{\theta}_2^x\) when also \(n_y \to \infty\), as noted in Andreasen and Christensen (2015).

### A.3 Step 2 of the SR Approach: Inducing Stationarity

If the stability condition for \(x_t\) is not satisfied, then \(\Phi_x^{(i)} + \lambda_x^{(i)}\) is downscaled by \(\delta_i\) for \(i = \{1, 2\}\) if the eigenvalues of \(\Phi_x^{(i)} + \lambda_x^{(i)}\) are greater than or equal to one. The values of \(\delta_1\) and \(\delta_2\) are determined as in Andreasen and Meldrum (2014), i.e. by

$$
(\delta_1, \delta_2) = \arg \min_{\{\delta_{\text{lower}} \leq \delta_1 < 1\}^2} \sum_{i=1}^{n_y} \left( \frac{\sigma_{i,\text{model}}^2 (\delta_1, \delta_2) - \sigma_{i,\text{sample}}^2}{\sigma_{i,\text{sample}}^2} \right)^2.
$$
We follow Andreasen and Meldrum (2014) and estimate the unconditional variance of the \(i\)th pricing factor in the sample from \(\{\hat{x}_{i,t}\}_{t=1}^{T}\) using

\[
\hat{\sigma}_{i,\text{sample}}^2 = \frac{1}{T-1} \sum_{t=1}^{T} \left(\hat{x}_{i,t} - \hat{E}[\hat{x}_i]\right)^2 - \frac{1}{T} \sum_{t=1}^{T} \hat{\text{Var}}(u_{i,t}),
\]

where \(\hat{E}[\hat{x}_{i,t}] = 1/T \sum_{t=1}^{T} \hat{x}_{i,t}\) and \(\hat{\text{Var}}(u_{i,t})\) refers to the estimated variance of \(\hat{x}_{i,t}\). The value of the unconditional variance of \(x_{i,t}\) in the model \(\sigma_{i,\text{model}}^2(\delta_1, \delta_2)\) is computed by simulation using

\[
x_{t+1} = \mu^Q + 1_{[z_t \geq c]} \lambda_0^{(1)} + (1 - 1_{[z_t \geq c]}) \lambda_0^{(2)} + \left(\delta_1 1_{[z_t \geq c]} \left\{\Phi^Q + \lambda_x^{(1)}\right\} + \delta_2 (1 - 1_{[z_t \geq c]}) \left\{\Phi^Q + \lambda_x^{(2)}\right\}\right) x_t + \Sigma_x e_{x,t+1}^F
\]

and (11). In relation to the estimates in Table 5, we find that \(\delta_1 = 1\) and \(\delta_2 = 0.9734\) for the RS-MTSM.
References


Table 1: Return Regressions

Panel A shows results for the standard return regression \( r_{t+1,k} = \alpha_k + \beta_k s_{t,k} + \varepsilon_{t+1,k} \), where \( s_{k,t} = y_{t,k} - y_{t,12} \). Panel B shows the results for the modified return regression \( r_{t+1,k} = \alpha_k^{\text{EXP}} + \alpha_k^\Delta 1_{(z_t < c)} + \beta_k^{\text{EXP}} s_{k,t} + \beta_k^\Delta 1_{(z_t < c)} s_{t,k} + \varepsilon_{t+1,k} \), where the Purchasing Managers’ Index is used to identify recessions. All intercepts are multiplied by 100. The last two columns report \( \alpha_k^{\text{REC}} = \alpha_k^{\text{EXP}} + \alpha_k^\Delta \) and \( \beta_k^{\text{REC}} = \beta_k^{\text{EXP}} + \beta_k^\Delta \). In parentheses are \( t \)-statistics obtained from Newey-West standard errors computed with 2 lags and in brackets are bootstrap \( t \)-statistics computed using bootstrapped standard errors. The bootstrap with 10,000 draws is carried out by resampling the regressand and the regressors jointly in blocks of random size having an average block size of 2, subject to at least 20 recession observations in each bootstrap sample. Significance at the 10 and 5 percent level using Newey-West standard errors is denoted by * and **, respectively. The estimation is carried out on monthly data from 1961:6 to 2016:12 using Gürkaynak-Sack-Wright bond yields.

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<th>Panel B: Switching</th>
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<td>60</td>
<td>0.00</td>
<td>0.24*</td>
</tr>
<tr>
<td></td>
<td>(0.02)</td>
<td>(1.86)</td>
</tr>
<tr>
<td></td>
<td>[0.02]</td>
<td>[1.89]</td>
</tr>
<tr>
<td>72</td>
<td>-0.03</td>
<td>0.26**</td>
</tr>
<tr>
<td></td>
<td>(-0.20)</td>
<td>(2.12)</td>
</tr>
<tr>
<td></td>
<td>[-0.21]</td>
<td>[2.15]</td>
</tr>
<tr>
<td>84</td>
<td>-0.06</td>
<td>0.28**</td>
</tr>
<tr>
<td></td>
<td>(-0.40)</td>
<td>(2.34)</td>
</tr>
<tr>
<td></td>
<td>[-0.41]</td>
<td>[2.36]</td>
</tr>
<tr>
<td>96</td>
<td>-0.10</td>
<td>0.30**</td>
</tr>
<tr>
<td></td>
<td>(-0.57)</td>
<td>(2.50)</td>
</tr>
<tr>
<td></td>
<td>[-0.58]</td>
<td>[2.51]</td>
</tr>
<tr>
<td>108</td>
<td>-0.13</td>
<td>0.32**</td>
</tr>
<tr>
<td></td>
<td>(-0.71)</td>
<td>(2.62)</td>
</tr>
<tr>
<td></td>
<td>[-0.73]</td>
<td>[2.62]</td>
</tr>
<tr>
<td>120</td>
<td>-0.17</td>
<td>0.34**</td>
</tr>
<tr>
<td></td>
<td>(-0.83)</td>
<td>(2.71)</td>
</tr>
<tr>
<td></td>
<td>[-0.86]</td>
<td>[2.69]</td>
</tr>
</tbody>
</table>
Table 2: Utility Gains
The table reports utility gains computed as the annualized difference between the utility obtained from trading based on predictive regressions and the utility obtained from trading on the predictions of the historical average. Utility and the yield spread are computed using discrete compounding. A mean-variance investor with a relative risk aversion of 5 is considered. Excess bond returns are forecasted using the yield spread with and without switching in the predictive coefficients. The variance is estimated using either a ten-year rolling window or the exponentially weighted moving average (EWMA) estimator with a decay factor of 0.97. In parentheses are $t$-statistics obtained from Newey-West standard errors computed with 2 lags. Two-sided significance at the 10 and 5 percent level using Newey-West standard errors is denoted by * and **, respectively. The estimation is carried out on monthly data from 1961:6 to 2016:12 using Gürkaynak-Sack-Wright bond yields. An initial estimation window of 15 years is used to generate the firsts forecasts.

<table>
<thead>
<tr>
<th>$k$</th>
<th>Panel A: No switching</th>
<th>Panel B: Switching</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Rolling variance</td>
<td>EMWA variance</td>
<td>Rolling variance</td>
</tr>
<tr>
<td>24</td>
<td>0.32 0.33</td>
<td>(1.56) (1.42)</td>
<td>0.87* 0.85*</td>
</tr>
<tr>
<td>36</td>
<td>0.19 0.14</td>
<td>(0.81) (0.58)</td>
<td>1.06* 1.00*</td>
</tr>
<tr>
<td>48</td>
<td>0.26 0.25</td>
<td>(0.71) (0.73)</td>
<td>1.06 1.01</td>
</tr>
<tr>
<td>60</td>
<td>0.36 0.36</td>
<td>(0.74) (0.74)</td>
<td>1.19 1.11</td>
</tr>
<tr>
<td>72</td>
<td>0.72 0.60</td>
<td>(1.29) (1.04)</td>
<td>1.59* 1.39</td>
</tr>
<tr>
<td>84</td>
<td>0.98 0.88</td>
<td>(1.49) (1.30)</td>
<td>1.81* 1.75*</td>
</tr>
<tr>
<td>96</td>
<td>1.17 1.09</td>
<td>(1.55) (1.38)</td>
<td>1.93* 2.00*</td>
</tr>
<tr>
<td>108</td>
<td>1.29 1.23</td>
<td>(1.52) (1.38)</td>
<td>2.05* 2.17*</td>
</tr>
<tr>
<td>120</td>
<td>1.30 1.33</td>
<td>(1.40) (1.34)</td>
<td>2.25* 2.30*</td>
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</tbody>
</table>
Table 3: Estimation Results of the MTSMs: The Risk-Neutral Parameters

This table reports the estimation results for the risk-neutral parameters in the MTSM with regime-switching and the benchmark MTSM. Elements in $\Phi^Q$ with " - " reflect zero-restrictions following Ang, Boivin, Dong and Loo-Kung (2011). Asymptotic standard errors are expressed as $t$-statistics and reported in parenthesis. The standard errors for $\mu^Q$ and $\Phi^Q$ are robust to measurement errors $v_{t,m}$ displaying heteroskedasticity in the time series dimension, cross-sectional correlation, and autocorrelation. We use $w_D = 5$ and $w_T = 10$ in the provided estimator of Andreasen and Christensen (2015). The asymptotic standard errors for $\Sigma_x$ are computed based on the standard errors described in Appendix A.2. No standard error and $t$-statistic are available for $\Phi^Q(2, 2)$ which is on its boundary. Significance at the 10 and 5 percent level is denoted by * and **, respectively, except for the diagonal elements of $\Sigma_x$ due to their sign-restriction.

### Regime-Switching MTSM

<table>
<thead>
<tr>
<th>$\mu^Q$</th>
<th>$\phi^Q$</th>
<th>$\Sigma_x$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\pi_t$</td>
<td>0.002** (7.38)</td>
<td>$\pi_t$</td>
</tr>
<tr>
<td>$\hat{y}_t$</td>
<td>-0.0152** (-11.07)</td>
<td>$\hat{y}_t$</td>
</tr>
<tr>
<td>$\omega_{\pi,t}$</td>
<td>0.0041** (7.89)</td>
<td>$\omega_{\pi,t}$</td>
</tr>
<tr>
<td>$\omega_{\hat{y},t}$</td>
<td>0.0007** (42871.14)</td>
<td>$\omega_{\hat{y},t}$</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>-0.0001 (-0.18)</td>
<td>$\Sigma_x$</td>
</tr>
</tbody>
</table>

### Benchmark MTSM

<table>
<thead>
<tr>
<th>$\mu^Q$</th>
<th>$\phi^Q$</th>
<th>$\Sigma_x$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\pi_t$</td>
<td>0.0020** (9.58)</td>
<td>$\pi_t$</td>
</tr>
<tr>
<td>$\hat{y}_t$</td>
<td>-0.0152** (-11.48)</td>
<td>$\hat{y}_t$</td>
</tr>
<tr>
<td>$\omega_{\pi,t}$</td>
<td>0.0041** (8.93)</td>
<td>$\omega_{\pi,t}$</td>
</tr>
<tr>
<td>$\omega_{\hat{y},t}$</td>
<td>0.0007** (18705.18)</td>
<td>$\omega_{\hat{y},t}$</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>-0.0001 (-0.18)</td>
<td>$\Sigma_x$</td>
</tr>
</tbody>
</table>
Table 4: Goodness of Fit for the MTSMs
This table reports the means and the standard deviations of the measurement errors $v_{t,m}$ at selected maturities when evaluated at $\{\hat{x}_t\}_{t=1}^T$. The errors are expressed in annualized basis points.

<table>
<thead>
<tr>
<th>Maturities in years</th>
<th>Regime-Switching MTSM</th>
<th>Benchmark MTSM</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Means</td>
<td>Standard deviations</td>
</tr>
<tr>
<td>0.5</td>
<td>-0.5</td>
<td>21.8</td>
</tr>
<tr>
<td>1</td>
<td>0.1</td>
<td>8.9</td>
</tr>
<tr>
<td>2</td>
<td>0.3</td>
<td>5.7</td>
</tr>
<tr>
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<td>0.0</td>
<td>7.7</td>
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<tr>
<td>4</td>
<td>-0.2</td>
<td>7.7</td>
</tr>
<tr>
<td>5</td>
<td>-0.2</td>
<td>6.2</td>
</tr>
<tr>
<td>6</td>
<td>0.0</td>
<td>3.8</td>
</tr>
<tr>
<td>7</td>
<td>0.2</td>
<td>1.9</td>
</tr>
<tr>
<td>8</td>
<td>0.2</td>
<td>3.8</td>
</tr>
<tr>
<td>9</td>
<td>0.0</td>
<td>7.0</td>
</tr>
<tr>
<td>10</td>
<td>-0.4</td>
<td>10.3</td>
</tr>
</tbody>
</table>
Table 5: Estimation Results of the MTSMs: Parameters for the Physical Dynamics

This table reports the estimation results for the parameters describing the physical dynamics in the MTSM with regime-switching and the benchmark MTSM. Asymptotic standard errors are expressed as t-statistics and reported in parenthesis. The standard errors for $\lambda$ and $\lambda_x$ in the benchmark MTSM are computed as in Andreasen and Christensen (2015), whereas the standard errors of $\lambda$, $\lambda_x^{(1)}$, and $\lambda_x^{(2)}$ in the MTSM with regime-switching are computed as described in Appendix A.2. For both models, standard errors related to the parameters for the Purchasing Managers' Index (PMI) are obtained as in Andreasen and Christensen (2015). Significance at the 10 and 5 percent level is denoted by * and **, respectively, except for $\Sigma_{zz}$ due to its sign-restriction.

| Regime-Switching MTSM | | Recessions: $\lambda_x^{(2)}$ | |
|---|---|---|---|---|---|---|---|---|
| | $\lambda_0$ | Expansions: $\lambda_x^{(1)}$ | | | | | | |
| $\pi_t$ | $-0.0010^{**}$ (12.37) | $0.0611^{**}$ (11.58) | $-0.0003^{**}$ (5.33) | $0.1101^{**}$ (5.23) | $-0.0525^{**}$ (2.39) | $0.0240$ (1.42) | $-0.0085$ (1.42) | $0.0109$ (1.42) | $-0.0693^{**}$ (2.06) |
| $\tilde{y}_t$ | $0.0154^{**}$ (31.54) | $-0.2481^{**}$ (18.04) | $-0.0196^{**}$ (18.04) | $-0.0010$ (18.04) | $-0.3650^{**}$ (21.80) | $-0.2535^{**}$ (28.34) | $-0.0201$ (13.71) | $0.0092$ (13.71) | $-0.4047^{**}$ (14.24) |
| $\omega_{\pi,t}$ | $-0.0005^{*}$ (1.91) | $0.0524^{*}$ (1.91) | $0.0725$ (1.91) | $-0.0044$ (1.91) | $0.1401$ (1.91) | $0.0986$ (1.91) | $-0.2720^{**}$ (1.91) | $-0.2067^{**}$ (1.91) | $0.6651^{**}$ (1.91) |
| $\omega_{\tilde{y},t}$ | $-0.0004$ (0.96) | $0.0040$ (0.96) | $0.0003$ (0.96) | $-0.0001$ (0.96) | $0.0110$ (0.96) | $0.0069$ (0.96) | $-0.0241$ (0.96) | $-0.0117^{*}$ (0.96) | $0.0259$ (0.96) |

<table>
<thead>
<tr>
<th></th>
<th>$\gamma_0$</th>
<th>$\gamma_x$</th>
<th>$\gamma_z$</th>
<th>$\Sigma_{zz}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$z_t$</td>
<td>$0.0008$ (0.15)</td>
<td>$-0.0830$ (1.39)</td>
<td>$0.1916^{**}$ (2.16)</td>
<td>$-0.0651^{**}$ (2.75)</td>
</tr>
</tbody>
</table>

Benchmark MTSM

<table>
<thead>
<tr>
<th>$\lambda_0$</th>
<th>$\lambda_x$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\pi_t$</td>
<td>$-0.0009^{**}$ (2.12)</td>
</tr>
<tr>
<td>$\tilde{y}_t$</td>
<td>$0.0153^{**}$ (32.96)</td>
</tr>
<tr>
<td>$\omega_{\pi,t}$</td>
<td>$-0.0050^{*}$ (1.73)</td>
</tr>
<tr>
<td>$\omega_{\tilde{y},t}$</td>
<td>$-0.0004$ (1.16)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$\gamma_0$</th>
<th>$\gamma_x$</th>
<th>$\gamma_z$</th>
<th>$\Sigma_{zz}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$z_t$</td>
<td>$0.0008$ (0.15)</td>
<td>$-0.0819$ (1.37)</td>
<td>$0.1917^{**}$ (2.16)</td>
</tr>
</tbody>
</table>
Figure 1: Return Regressions: Fitted Values
This figure shows the predicted values for monthly excess bond returns in percent at the ten-year yield using the standard return regression in (1), the modified return regression in (2) at the top, and the logistic smooth transition regression in (3) with \( \delta = 2.0 \) at the bottom. The shaded areas denote the NBER recession dates.
Figure 2: Predictive Power in Recessions and Expansions

For the modified return regression in (2), this figure shows the $R^2$ values by maturity when computed separately for expansions and NBER recessions as in Gargano, Pettenuzzo and Timmermann (2019). Let $\hat{\varepsilon}_{t+1,k}$ denote the estimated residual from (2) and let $\hat{u}_{t+1,k}$ denote the estimated residual from a constant-only model. Also, let the indicator function $1_{t}^{NBER}$ be 1 if period $t$ is an NBER recession and zero otherwise. Then $R^2_{EXP} = 1 - \frac{\sum_{t=1}^{T-1} \hat{\varepsilon}_{t+1,k}^2 (1 - 1_{t+1}^{NBER})}{\sum_{t=1}^{T-1} \hat{u}_{t+1,k}^2 (1 - 1_{t+1}^{NBER})}$ and $R^2_{REC} = 1 - \frac{\sum_{t=1}^{T-1} \hat{\varepsilon}_{t+1,k}^2 1_{t+1}^{NBER}}{\sum_{t=1}^{T-1} \hat{u}_{t+1,k}^2 1_{t+1}^{NBER}}$.
Figure 3: Excess Returns and Bond Yields During Recessions

The top row of this figure reports the average level of monthly excess returns and the yield spread at the ten-year maturity in the first eight months after the start of a recession defined using the PMI. The bottom row of the figure reports the average changes in the three-month and ten-year bond yields in the first eight months after the start of a recession defined using the PMI. The data moments are computed using monthly Gürkaynak-Sack-Wright bond yields from 1961:6 to 2016:12, and the shaded areas denote the 95 percent confidence intervals for these empirical moments obtained using Newey-West standard errors with 8 lags. The corresponding model-implied moments are for the MTSM with regime-switching (RS-MTSM) and the benchmark MTSM without regime-switching.

---

Data -- RS-MTSM -- Benchmark MTSM

---

Average Level of 10-Year Excess Returns in Recessions

Average Level of 10-Year Term Spread in Recessions

Average Change in 3-Month Bond Yields in Recessions

Average Change in 10-Year Bond Yields in Recessions
Figure 4: The MTSM with Regime-Switching: The Pricing Factors
The top row displays inflation and the output gap, while the bottom row reports the estimated policy weights on these macro variables in the Taylor-rule. The policy weights are displayed for an annualized version of the short rate, implying that the weights are scaled by 12. The (very tight) 95 percent confidence bands for the policy weights are denoted by gray lines and computed without accounting for the sampling variability in $\hat{\theta}_1$. Shaded bars denote NBER recessions.
Figure 5: The MTSM with Regime-Switching: Return Predictability

This figure shows the standard regression loadings in $r_{x,t+1,k} = \alpha_k + \beta_k s_{t,k} + \varepsilon_{t+1,k}$ to the left and the regime-dependent loadings in $r_{x,t+1,k} = \alpha_k^{EXP} + \alpha_k^1(z_t<\gamma) + (\beta_k^{EXP} + \beta_k^1(z_t<\gamma)) s_{t,k} + \varepsilon_{t+1,k}$ to the right. All model-implied regression loadings are computed using a simulated sample path of 100,000 observations for the MTSM with regime-switching at the estimates in Table 3 and 5. The corresponding empirical moments are computed using monthly Gürkaynak-Sack-Wright bond yields from 1961:6 to 2016:12, and the shaded areas denote the 95 percent confidence intervals for these empirical moments obtained using Newey-West standard errors with 2 lags.
Figure 6: The Benchmark MTSM: Return Predictability
This figure shows the standard regression loadings in $r x_{t+1,k} = \alpha_k + \beta_k s_{t,k} + \varepsilon_{t+1,k}$ to the left and the regime-dependent loadings in $r x_{t+1,k} = \alpha_k^{\text{EXP}} + \alpha_k^{\Delta} 1_{\{z_t < c\}} + \left( \beta_k^{\text{EXP}} + \beta_k^{\Delta} 1_{\{z_t < c\}} \right) s_{t,k} + \varepsilon_{t+1,k}$ to the right.
All model-implied regression loadings are computed using a simulated sample path of 100,000 observations for the benchmark MTSM at the estimates in Table 3 and 5. The corresponding empirical moments are computed using monthly Gürkaynak-Sack-Wright bond yields from 1961:6 to 2016:12, and the shaded areas denote the 95 percent confidence intervals for these empirical moments obtained using Newey-West standard errors with 2 lags.
Figure 7: Persistence of Bond Yields in Expansions and Recessions

This figure reports the slope coefficients in $y_{t+1,k} = \delta_k + (\rho_k^{\text{EXP}} + \rho_k^2 1_{\{z_t < c\}}) y_{t,k} + \epsilon_{t+1,k}$ for the empirical sample, the MTSM with regime-switching (RS-MTSM), and the benchmark MTSM. The empirical moments are obtained from monthly Gürkaynak-Sack-Wright bond yields from 1961:6 to 2016:12 and shaded areas denote the 95 percent confidence intervals obtained using Newey-West standard errors with 12 lags. All model-implied moments are computed using a simulated sample of 100,000 observations from the RS-MTSM and the benchmark MTSM at the estimates in Table 3 and 5.

Persistence of yields in expansions: $\rho_k^{\text{EXP}}$

Change in persistence of yields in recessions: $\rho_k^c$
Figure 8: Time-varying Policy Weights and the Switch in Return Predictability
For the MTSM with regime-switching, the charts in the top row report $\alpha_k$ and $\beta_k$ in (2) for the full model and a reduced version of this model without a switch in the third row of $\lambda_x$ related to $\omega_{\pi,t}$. The charts in the bottom row report $\alpha_k$ and $\beta_k$ in (2) for the full model with regime-switching and a reduced version of this model without a switch in the fourth row of $\lambda_x$ related to $\omega_{\tilde{y},t}$. All model-implied regression loadings are computed using a simulated sample path of 100,000 observations. The corresponding empirical moments are computed using monthly Gürkaynak-Sack-Wright bond yields from 1961:6 to 2016:12, and the shaded areas denote the 95 percent confidence intervals for these empirical moments obtained using Newey-West standard errors with 2 lags.
Figure 9: Impulse Response Functions: A Higher Policy Weight on Inflation

This figure reports the impulse response functions (IRFs) in the MTSM with regime-switching for the monthly expected excess return on $y_{t,120}$ in percent at the top and the annualized yield spread $y_{t,120} - y_{t,12}$ in percent at the bottom in expansions and recessions following a positive one standard deviation shock to the policy weight on inflation, i.e. $\omega_{\pi,t}$. The x-axis denotes months with the shock hitting the economy in period one. The IRFs are computed as in Koop, Pesaran and Potter (1996) using simulations with 5,000 draws when the initial values of $x_t$ and $z_t$ are at their mean in expansions and recessions, respectively.
Figure 10: Estimated Term Premia

This figure shows the two- and ten-year term premia from the MTSM with regime-switching (RS-MTSM) and the benchmark MTSM using the estimates in Table 3 and 5. For the RS-MTSM, conditional short-rate expectations in a given period are computed by Monte Carlo integration using 10,000 draws. The term premia estimates are reported in annualized percent, and shaded bars denote NBER recessions.