The Educational Asset Market: A Finance Perspective on Human Capital Investment[♣]

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Abstract: Like the stock market, the human capital market consists of a wide range of assets, i.e. educations. Each young individual chooses the educational asset that matches his preferred combination of risk and return in terms of future income. A unique register-based data set with exact information on type and level of education enables us to focus on the shared features between human capital and stock investments. An innovative finance-labor approach is applied to study the educational asset market. A risk-return trade-off is revealed which is not directly related to the length of education.

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1. Introduction

In his presidential address, Rosen (2002) describes how markets value diversity. He argues that markets accommodate diversity by establishing prices that make differentiated items close substitutes at the margin. Like the markets for goods, jobs, and financial assets, the market for education is characterized by diversity, though this is largely overlooked in the literature. Therefore, we introduce the concept of educational assets in this paper.

Traditionally, human capital investments have been viewed within a life-cycle framework. Early in the life cycle, individuals allocate time to human capital production, and the more time invested the higher the future earnings (Becker, 1964; Ben-Porath, 1967; Mincer, 1974). Relevant restrictions lead to the standard Mincerian earnings equation where log-earnings are regressed on years of schooling, experience, and experience squared. The rate of return to education is given as the coefficient to years of schooling. Slightly more flexible applications hereof are the earnings equation with educational level effects, degree effect, stepwise linear return, and varying return by major or varying return by age or experience. However, it is by and large neglected that using years of schooling conceals most of the diversity of educations.

In this paper, we place the focus on the features of the human capital market that are shared with the stock market. Like the stock market, the human capital market consists of a wide range of assets. Each young individual chooses the exact asset (i.e. education) that matches his or her preferred combination of risk and return in terms of future income. Thus, the type of education is as important as the level of education. Similarly, the variance of the return to schooling is as important as the return to schooling.

For decades, the finance literature has been occupied with the trade-off between risk and return of financial assets such as stocks. The Mean-Variance Model and the Capital Asset Pricing Model (CAPM) have been used extensively in this respect. In this paper, we demonstrate how this way of thinking can be successfully applied to human capital investments. Thereby, the paper provides a novel interdisciplinary approach to analyzing human capital investment decisions.

In our analysis, we take the CAPM framework as our starting point, but we do not use the CAPM as such. The CAPM is a general equilibrium portfolio selection model. Merely the expected return and the variance of the portfolio influence the investor's portfolio selection; therefore, mean-variance plots are used to identify the efficient investments (the efficient frontier). The CAPM provides a simple way to measure the performance of a portfolio of stocks taking the undertaken risk into account. The efficient frontier and the performance measure are transferred to the human capital investment problem.

Implicitly, the economics literature has been aware of the trade-off between high incomes and high risk for different educations since Smith (1776). However, the risk-return trade-off has only received little explicit attention in the labor economics literature, and as of yet, it is by no means standard to incorporate this issue in a study of returns to education.

From a theoretical point of view, Levhari and Weiss (1974) and Williams (1979) show that earnings risk induces people to invest less in education, whereas the optimal stopping model by Hogan and Walker (2001) results in the opposite conclusion. The model by Snow and Warren (1990) accommodates both possibilities, but they ask for empirical evidence on the matter.

In the empirical literature two approaches have been followed to accommodate diversity in return to human capital investments. The first approach is the random coefficient approach. Carneiro, Hansen and Heckman (2001) estimate the distributions of the return to schooling among different schooling groups whilst accounting for self-selection and attrition. Harmon, Hogan and Walker (2001) estimate a random rate of return to education and allow this to vary with all other explanatory variables. By incorporating uncertainty, both papers represent great improvements compared to the standard Mincer regression. However, both studies rely on either the rate of return to education or level effects (drop-out/high-school/some college/college graduate). Using these specifications, a large part of the dispersion of the return may stem from diversity in educational choices, which has nothing to do with earnings risk as such.

The second strand of literature estimates the risk compensation in incomes. Taking both a theoretical and an empirical stand, Weiss (1972) supplies the first study of the mean-variance trade-off. He applies the coefficient of variation (i.e. the standard deviation normalized by the mean) to correct the return to education across age and educational groups within a sample of scientists. To some degree, Hartog and Vjiverberg (2002) support that approach, because they find that including a measure of risk within an occupation-education cell is a good way to incorporate the risk-return trade-off. However, to test for the separate effect of skewness affection, McGoldrick (1995) and Hartog and Vjiverberg (2002) apply a two-step approach where relative variance and skewness are estimated in the first step and then inserted into a Mincer equation in the second step. Pereira and Martins (2002) use a different approach to the risk-return trade-off. They use cross-country

Ordinary Least Squares (OLS) returns from Mincer equations and correlate those with the spread in returns as measured by the difference in coefficients from quantile regressions. These studies assume a linear risk-return trade-off, which has the unfortunate feature that the market is assumed to provide a single price of risk. According to Rosen (1974, 2002), it does not make sense to require the Law of One Price to hold for characteristics (here: variance) of diverse items (here: education). An additional shortcoming is the lack of detailed education data, which means that occupation-education cells must be used to approximate diversity of educations.

A related strand of literature focuses on the time series variation in log-earnings paths allowing for complex error structures and more flexible specifications than usually applied. Alvarez, Browning and Ejrnæs (2001) advocate “lots of heterogeneity” in earnings processes, since they find support for a different ARMA(1,1) process for each single individual. These studies focus on error structures rather than explanatory variables to capture the variation in earnings.

Our contribution to the literature is to analyze the risk-return trade-off in a flexible world that allows for a flexible valuation of diversity, as suggested by Rosen (2002). We take the outset in the finance literature and gradually move towards a more standard labor economic analysis. We exploit similarities between human capital and stock investments to explain variation in annual income. We address the issue of the mean-variance trade-off in human capital investments, while taking into account the fact that some educational choices are typically guided by strong feelings. Furthermore, we calculate a performance measure, which ranks educations to guide individual investments.

In the empirical analysis we use a register-based data set that is unique because precise information about level and exact type of education achieved is registered for each individual. This enables new and interesting analyses, since we can go beyond years of schooling or educational level effects as measures of the human capital investment. As a consequence, we are able to investigate whether income risk varies systematically with the length of education as discussed in the theoretical papers cited above. When we account for type and level of education, as well as income variance, we are able to explain the majority of the variation in annual income.

The remaining part of the paper is organized as follows. The traditional finance approach to the risk-return relationship is introduced in Section 2, whereas Section 3 is concerned with the labor economics approach hereto. The data are presented in Section 4, the empirical findings are presented in detail in Section 5, and finally, Section 6 concludes. Various data details are deferred to an appendix.

2. A Theoretical Financial Economics Approach

In the finance literature, the trade-off between risk and return has been studied extensively. Most predominantly, this relationship has been the focus of portfolio selection models that ask which combination of financial assets is optimal with respect to risk and return. In the labor market, we are more interested in asset selection than portfolio selection, since we are interested in finding the optimal educational asset with respect to risk and return.

We apply the finance approach to investigate the risk-return trade-off on education. The efficient frontier of the Markowitz (1952) model and the Capital Asset Pricing Model of Sharpe (1964) and Lintner (1965) is a very useful devise in order to study the educational assets. We do not use the CAPM theory at face value, rather we use it as an outset for our analysis. Also, we are inspired by the CAPM performance measure, the Sharpe (1965) index, to evaluate the performance of educational assets by their standardized excess return.

In Section 2.1 we describe the main relevant features of the Markowitz model and the CAPM. Subsequently, in Section 2.2, we investigate how the analysis can be qualified to the human capital market.

2.1. The Efficient Frontier

The analysis of the trade-off between the risk and the return of (portfolios of) stocks goes back to the *mean-variance* framework of Markowitz (1952). Subsequently, Sharpe (1964) and Lintner (1965) have extended the mean-variance framework into the so-called CAPM.[1]

In the Markowitz (1952) mean-variance model, agents make their investment decisions based solely on the expected return and the variance of their portfolio: Investors prefer higher expected return ceteris paribus and equivalently prefer less risk (variance) ceteris paribus. This behavior is consistent with quadratic utility functions.

Agents maximize expected utility. In a quadratic utility function, utility is a parabola of the level of wealth. The expected utility depends positively on the expected wealth and negatively on the variance of the wealth. In other words, the same conclusions arise whether investors have quadratic utility functions or they maximize expected returns and minimize variance. The main argument against the quadratic utility function is that it shows increasing relative risk aversion (RRA) in

[1] The textbook by Elton and Gruber (1995) contains an accessible discussion of the Markowitz model and the CAPM.

wealth. It is also noticed that higher order moments such as the skewness and the kurtosis do not enter into the expected utility. In spite of these undesirable features, the quadratic utility specification has gained outspread popularity in the finance literature.

Sharpe (1964) and Lintner (1965) expand the Markowitz (1952) model by assuming that all agents agree on the statistical distribution of the asset returns (i.e. mean, variance, and covariance). These assumptions give rise to the CAPM.[2] The CAPM is a general equilibrium model because it considers all investors in all capital markets simultaneously, whereas the mean-variance framework of Markowitz (1952) is only concerned with individual investors.[3] In other words, the mean-variance framework represents the microeconomic approach to asset pricing, and the CAPM represents the macroeconomic approach to asset pricing.

Insert Figure 1

In the CAPM (and the mean-variance model), it is common to plot the mean return for each stock as a function of its standard deviation. All the feasible investment strategies (including portfolios) are contained in the *feasible set* in the mean-variance graph in Figure 1. All investors hold portfolios that are located on the *efficient frontier* and the agents agree on the efficient frontier. The efficient frontier is the envelope curve that starts in the minimum-variance point (MV) and goes northeast through the market portfolio, M. The market portfolio consists of all the stocks in the economy according to their capitalization weights. Consider the points A and B. For the same amount of risk, by choosing B the investor increases his expected return; investors prefer B to A. Equivalently, investors prefer C to A (same return less risk). The exact point on the efficient frontier chosen by the agent depends on the shape of his indifference curves.

It is usually assumed that a risk-free (zero variance) asset, F, (i.e. a bond) exists. Now, the efficient frontier is the straight line from F through M. Thus, when a risk-free asset exists, all investors will invest in the risk-free asset, the market portfolio, or a combination (portfolio) hereof.

The CAPM framework provides a simple way to evaluate the performance of stocks and portfolios of stocks. The performance measures provide a number for each stock, and therefore the stocks can be ranked according to their performance. On the one hand, these performance measures punish the

[2] Additional assumptions are necessary in order to obtain the CAPM. However, here we only list the most crucial assumptions.

[3] Liberman (1980) shows that the financial and the human capital markets can be treated separately in the CAPM framework.

undertaken risk, and on the other hand they assign positive value to higher expected returns. Here we concentrate on the Sharpe (1965) index. The Sharpe (1965) index standardizes the excess return above the risk-free return by the risk (the volatility):

$$(1) \qquad Sharpe_i = \frac{\overline{R}_i - R_F}{\sigma_i}.$$

$\overline{R}_i$ is the average return on asset *i*, R_F is the risk-free return, and σ_i is the volatility of asset *i*.

There is more to the CAPM than the efficient frontier of the mean-variance plot and the performance measure that we concentrate on here. Often, the model is reformulated in a way that the risk of the stock is measured by its beta, which is proportional to the covariance of the stock and the market portfolio. The beta is particularly useful with respect to portfolio selection because it captures the diversification aspect of holding a portfolio. However, because we do not consider portfolios on the educational asset market, we will not go further into the beta risk measure.

2.2. Qualifications to the Labor Market

On the human capital market, the assets of interest are various educations. Thus, we study the return and the risk to the educations. The return to an education is the average annual income for workers within that group, and the corresponding risk measure is the standard deviation of the annual income for workers within that educational group.

The main difference between the educational asset market and the stock market is the following limitations to the educational asset market: (i) diversification is not possible and (ii) gearing is not possible. Still, the mean-variance approach can be applied successfully to the educational asset market. A modified version of the mean-variance framework is applied as an analogy, whereas the CAPM model as such is not applied. Both the efficient frontier and the performance measure can be applied meaningfully to human capital investments. In contrast, other parts of the CAPM way of thinking such as the beta analysis cannot be transferred to the human capital market. We denote the performance measure based on the Sharpe (1965) ratio, the *standardized excess return*. The unemployment insurance benefit is applied as the risk-free asset.[4]

[4] Utility functions might exist where the worker will not choose the educational asset with the highest standardized excess return. However, we will ignore this unlikely problem in the empirical analysis.

Returning to (i), small educational portfolios can be obtained by holding an interdisciplinary education. Moreover, another kind of portfolio is obtainable if couples or extended families choose their education jointly, optimizing simultaneously. Yet, most people do not get married before undertaking an education and hardly any people live in extended families in the developed world. In the literature on the economics of marriage (Becker, 1991), a standard assumption is that the education, the earnings prospect of a potential partner, and the potential gains from specialization are primary motivations for marriage formation.[5] Therefore, marriage could be considered an educational portfolio choice.

Regarding (ii); by gearing we mean that the investment cannot be scaled arbitrarily which is the way that arbitrage opportunities are done away with in financial markets. Investing in a specific education is a binary choice variable, either you invest in a certain education or you do not. Moreover, once you hold a certain education, you are not able to sell it again.

Thus, we consider the mean-variance plot (i.e. the efficient frontier) as investment in schooling. Due to the limitations listed above the mean-variance plot is a scatter-plot where the empirical efficient frontier consists of points rather than a continuous envelope curve. When considering the efficient frontier, we ignore the risk-free asset. The mean-variance plot tells us which educations are efficient in the sense of an investment asset. In other words, if agents act as rational investors, the plot has obvious implications for educational choices. Since the public spending on education per year varies significantly across types of education, this may not be seen as a guide to policy makers about educational policy. Rather, it is a guide to individuals about what constitutes an efficient human capital investment from their point of view.

Financial economics analysis applies percentage returns (instead of $ returns) in order to make investments comparable. When assessing human capital investments by using raw annual income, we neglect correcting for the fact that different types of education represent a different amount of investment in terms of time used (foregone earnings). The analysis of the Mincer-residuals in Section 5.2 accommodates this issue.

[5] Traditionally, the relevant specialization is into market work (males) and homework (females). However, in modern families of today the relevant issue is specialization in different sorts of market work.

In classic finance, it is assumed that only pecuniary returns provide utility to the investors, e.g. the benefit from social responsible investments are not priced.[6] Equivalently, agents may choose a certain education for other reasons than investment purposes. One reason could be that some people have a vocation for a certain education, e.g. nursing.[7]

In the empirical work, we study annual income within an educational group, which reflects the combined effect of the state of the economy (business cycle), employment, occupation, sector, hours, and hourly wage outcomes. The risk inherent in the annual income includes unemployment risk as well as low-income risk due to employment in unfavorable occupations or sectors. In addition, it includes risk due to uncertainty of the individual's ability to fare well compared to others with same education, cf. Hartog and Vjiverberg (2002). Notice, that some of these risk factors are things that workers might actively choose, e.g. a person might decide to work only part-time. We implicitly assume that the individual only cares about mean and variance of annual income. However, we separate out some of the risk effects as a robustness check.

The study of return to education by Weiss (1972) is related to the mean-variance framework. He applies the coefficient of variation as a risk measure and finds that CRRA-agents maximize their utility by maximizing expected income and minimizing the coefficient of variation. Notice, this is not identical to the assumed behavior here. We find it more likely that agents care about the variance of their income rather than the relative variance.

3. A Theoretical Labor Economics Approach

Unlike in the finance literature, it is not yet standard to consider uncertainty in studies of return to human capital investments. This is true even though studies generally confirm its relevance to human capital investments. What is more, it is straightforwardly incorporated into a standard human capital model.

Section 3.1 concerns the standard human capital model, while Section 3.2 discusses how the previous literature has incorporated uncertainty herein. In Section 3.3 we introduce a new educational asset model with uncertainty.

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[6] Another category of this type is supporters' investments in sports clubs, which need not be driven by pecuniary motives either.

[7] This is the non-market benefit of education that Heckman (1976) introduces.

3.1. The Human Capital Model

In human capital theory, education is considered an investment of time plus the direct costs of schooling in exchange for enhanced future earnings; see Becker (1964) and Ben-Porath (1967). Let $U(W_0)$ be the utility of the annual income earned in case of no schooling and $U(W_S)$ be the utility on the annual income earned after S years of schooling. The discount rate is denoted δ. If earnings are time constant and the horizon is infinite, individuals are indifferent between no education and S years of schooling, if:

(2) $$U(W_0) = U(W_S)e^{-\delta S}.$$

When earnings (not utility) are maximized, we approach the standard Mincerian earnings equation. Replacing the assumption of time-constant earnings after leaving school with the assumption that a (declining) proportion of time is continuously invested in human capital (experience), we arrive at the standard earnings equation as derived by Mincer (1974):

(3) $$\ln W_i = \alpha_0 + \alpha_1 S_i + \alpha_2 X_i + \alpha_3 X_i^2 + \varepsilon_i,$$

where $\varepsilon_i \sim N(0,\sigma^2)$ and X_i denotes the years of experience and S_i the years of schooling. It is usually assumed that $\alpha_0 = \ln W_{0i} = Z_i\gamma$, where Z_i is a set of characteristics. Sometimes, a less restrictive specification is applied, where schooling is specified as a set of indicator variables each reflecting a given educational level (e.g. Psacharoupolos and Ng, 1994). Or, for studies based on NLSY, a distinction is made between the college majors (e.g. Berger, 1988; Eide, 1994; Grogger and Eide, 1995), which clearly introduce an important source of variation in educations of identical level.

In the original work by Mincer (1974), schooling is assumed exogenous even though the benchmark theoretical model treats time allocated to schooling as the control variable. Empirical studies find that the return to schooling is influenced by a negative endogeneity bias of varying magnitude. However, surveys of the literature concerning the issue of endogeneity of schooling and ability bias show that reported Instrumental Variables estimates are often more biased than OLS estimates due to the use of invalid instruments, cf. Card (1999) and Harmon, Walker and Westergaard-Nielsen (2001).

3.2. Incorporating Uncertainty in the Human Capital Model

Some attempts have been made to incorporate uncertainty in the return to schooling in the standard human capital model. All studies do so within the traditional Mincerian framework thereby relying on the return to schooling (as measured in years or levels).

The first strand of literature incorporates uncertainty by allowing returns to education to be stochastic. Carneiro et al (2001) estimate the distributions of the return to schooling among different schooling groups while correcting for self-selection and attrition. They find a high return dispersion, which is slightly lower for college graduates than for others. Harmon, Hogan and Walker (2001) estimate a random coefficient model, where the random return to education is allowed to vary with all other explanatory variables. Their main interest is whether the educational expansion in the UK has depressed returns and increased dispersion over time. This does not seem to have been the case.

The second strand of literature concerns estimation of the risk compensation in incomes. All the studies that we are aware of, assume that the Law of One Price holds for valuation of risk.[8] The noticeable study by Hartog and Vjiverberg (2002) concerns the compensation for risk aversion and skewness affection in the above-mentioned framework. They show that if the error terms are normal, structural models with, say, constant relative risk aversion (CRRA) utility functions result in risk, skewness, and the risk premium being simple functions of the estimated variance in the relevant education-occupation cell. Hence a straightforward way to correct for uncertainty of incomes in the human capital model would be to include a measure of risk within a certain education-occupation cell as an extra variable in the earnings equation (3).[9]

For non-normal errors, this result does not hold, and both variance and skewness must be estimated initially. Both McGoldrick (1995), Hartog, Plug, Serrano and Vieira (1999), and Hartog and Vjiverberg (2002) find reasonable results from reduced-form estimation confirming that incomes compensate for risk. However, the results from the estimation of the structural models are less clear, although several data sets are applied. Unless sufficient restrictions are imposed, discount rates are high and marginal utility is rising with income, see Hartog and Vjiverberg (2002).

Since detailed information about education has not been available previously, these studies assume that individuals choose a certain education-occupation cell. This is clearly a rough approximation to

[8] Rosen (2002) opposes this assumption.

[9] The combined assumptions of CRRA and normal error terms are used by Weiss (1972) for risk correction of earnings.

real life, since the individual's choice mainly concerns type and length of education. The occupational choice follows completion of education. Admittedly, the choice of education is to some degree directed towards a certain occupation, but the allocation of workers across occupations is to a large extent governed by the demand side of the economy, and not only the supply side.

Using a less conventional approach, Pereira and Martins (2002) investigate the relationship between the estimated return to education and risk in a cross-country study. The return is measured by the annual return as estimated from equation (3), and the risk is measured as the difference in returns between the 90^{th} and 10^{th} percentile estimates from quantile regressions. The study finds a positive relationship between risk and return across countries. However, since the study relies entirely on years of schooling, this risk-return link may stem from the mere fact that longer educations range from Anthropology and Philology to Computer Science, Law, and Economics. The fact that the earnings of individuals holding an MA/MSc vary a lot across subjects contributes to the finding of an increasing variation in earnings with years of education.

This critique to a lesser extent also applies to the other mentioned studies, since the variation across subjects contaminates their risk measures. In the paper by Carneiro et al (2001) this issue may explain why they find a substantial proportion of returns to be negative for each schooling level. In the papers by McGoldrick (1995), Hartog et al. (1999) and Hartog and Vjiverberg (2002), this effect contaminates the risk compensation to the extent that their 25 occupations do not pick up subject variation. However, the critique is most severe in the case of Pereira and Martins (2002), where the focus is placed on establishing a risk-return trade-off, which may be entirely explained by variation in subjects within a given length of education.

3.3. Incorporating Educational Assets in the Human Capital Model

In contrast to the above-mentioned studies, we incorporate the risk-return trade-off in a more flexible manner that does not rely entirely on length of education. Because individuals choose length of education and field of study simultaneously, an increasing earnings variation with length of education does not need to have anything to do with risk. To test this hypothesis, we would have to think of returns to schooling as related to completing a certain degree conditional on investing a number of years in education. Consequently, the S_i's should be complemented by a set of indicator variables measuring the type of education or the type of degree obtained instead.

Suppose that each individual simultaneously chooses the length of education, *S*, and the type of education, *j*. We assume that when the individual allocates a given number of years to education, she also buys a certain educational asset, A_{S_j}. Hence, annual income not only reflects how long time is spent in the educational system, but also her chosen educational asset. The return to the educational asset is assumed uncertain, whereas the return to the years of education is assumed certain. A_0 indicates the return to the asset "no education" which is assumed non-random.

Compensation for work is assumed to be

$$(4) \qquad W_{S_j} = \begin{cases} \mu_0, \text{ if } A_0 \text{ is bought} \\ \mu_{S_j} \eta_{S_j} e^{\varepsilon_{S_j}}, \text{ if } A_{S_j} \text{ is bought} \end{cases}$$

where μ indicates the certain money compensation, the component $\eta \geq 1$ indicates a vocation effect and ε is the uncertain money component of incomes.[10] Hence observed incomes are:

$$(5) \qquad W^*_{S_j} = \begin{cases} \mu_0, \text{ if } A_0 \text{ is bought} \\ \mu_{S_j} e^{\varepsilon_{S_j}}, \text{ if } A_{S_j} \text{ is bought} \end{cases}$$

In case of uncertainty about the return to the educational asset, a risk premium, Θ_{S_j}, is paid for A_{S_j} above the risk-free endowment, A_0

$$(6) \qquad U(W_0 + \Theta_{S_j}) = e^{-\delta S} EU(W_{S_j}).$$

Assume CRRA utility[11]

$$(7) \qquad U(W) = \frac{1}{1-\rho} W^{1-\rho}$$

then the left-hand side of equation (6) becomes

$$(8) \qquad U(W_0 + \Theta_{S_j}) = \frac{1}{1-\rho}(W_0 + \Theta_{S_j})^{1-\rho} = \frac{1}{1-\rho}(W_0 \pi_{S_j})^{1-\rho}$$

[10] For example, nurses are said to have a "Florence Nightingale" vocation.

[11] Quadratic utility represents the uninteresting special case of CRRA where RRA is zero.

where π_{S_j} is one plus the risk premium measured in percent, $\pi_{S_j} = 1 + \frac{\Theta_{S_j}}{\mu_0}$. The right-hand side of equation (6) becomes

$$e^{-\delta S} EU(W_{S_j}) = \frac{e^{-\delta S}}{1-\rho} (\mu_{S_j} \eta_{S_j})^{1-\rho} E\left[e^{(1-\rho)\varepsilon_{S_j}} \right]. \tag{9}$$

Now we combine the left-hand side and the right-hand side of equation (6) again. If we assume that $\varepsilon_{S_j} \sim N(0, \sigma_{S_j})$, take *ln* and divide by (*1–ρ*) equation (6) now reads:

$$\ln \mu_{S_j} = \ln \mu_0 + \ln \pi_{S_j} - \ln \eta_{S_j} + \frac{\delta S}{1-\rho} + \frac{\sigma_{S_j}^2 (1-\rho)^2}{2}. \tag{10}$$

We observe $W_{S_j}^*$:

$$\ln W_{S_j}^* = \ln \mu_0 + \ln \pi_{S_j} - \ln \eta_{S_j} + \frac{\delta S}{1-\rho} + \frac{\sigma_{S_j}^2 (1-\rho)^2}{2} + \varepsilon_{S_j}. \tag{11}$$

For CRRA utility, the Arrow-Pratt local risk premium is, cf. Pratt (1964):

$$\Theta_{S_j} = -\frac{1}{2} \sigma_{S_j}^2 \frac{U''}{U'} = \frac{\rho \sigma_{S_j}^2}{2W}. \tag{12}$$

Hence $\ln \pi_{S_j} = \ln(1 + \frac{\rho \sigma_{S_j}^2}{2\mu_0^2}) \sim \frac{\rho \sigma_{S_j}^2}{2\mu_0^2}$, which is one-half the coefficient of variation times the coefficient of relative risk aversion, ρ. The equation to be estimated is:

$$\ln W_{S_j}^* = \ln \mu_0 + \frac{\delta}{1-\rho} S - \ln \eta_{S_j} + \frac{\mu_0^2 (1-\rho)^2 + \rho}{2\mu_0^2} \sigma_{S_j}^2 + \varepsilon_{S_j}. \tag{13}$$

As introduced above, the stochastic term is assumed to be $\varepsilon_{S_j} \sim N(0, \sigma_{S_j})$. The return to education is divided by one minus the coefficient of relative risk aversion, which reflects the declining marginal utility of income. A *restrictive* as well as a *flexible* model is estimated.

The most restrictive assumption is that $\ln \eta_{S_j} = 0$. Adding individual sub-scripts and experience terms, and using conventional notation, we get the restrictive specification:

$$\ln W_{i,S_j}^* = \alpha_0 + \alpha_1 S_i + \alpha_2 X_i + \alpha_3 X_i^2 + \alpha_4 \sigma_{S_j}^2 + \varepsilon_{i,S_j}. \tag{14}$$

The $\sigma_{S_j}^2$ parameter of the error distribution is estimated simultaneously with the other parameters; otherwise this approach is similar to that of Hartog and Vjiverberg (2002) among others.

To obtain the flexible approach we add an educational group fixed effect which is to be estimated, ψ_{S_j} . The educational group fixed effect allows for both the unobserved vocation effect, η_{S_j} , and the effect of risk, $\sigma_{S_j}^2$:

(15) $$\ln W_{i,S_j}^* = \alpha_0 + \alpha_1 S_i + \alpha_2 X_i + \alpha_3 X_i^2 + \psi_{S_j} + \varepsilon_{i,S_j} .$$

The parameters to be estimated in the two models are $\alpha_0, \alpha_1, \alpha_2, \alpha_3, \alpha_4, \sigma_{S_j}$ and $\alpha_0, \alpha_1, \alpha_2, \alpha_3, \psi_{S_j}, \sigma_{S_j}$, respectively. A further investigation of the fixed effects for educational groups would reveal whether a risk-return trade-off on the educational asset prevails and which types of educations are characterized by a vocation effect. Thus, a low mean return asset, $A_{S,j}$, may be combined with a low variance or with favorable non-pecuniary job-characteristics.

Theoretically, it would be natural to treat educational choices as endogenous to the income formation. We regard this issue to be beyond the scope of this paper.[12]

4. Data on Earnings and Education

For the empirical analysis, we apply a register-based panel data set containing a representative 5%-sample of the Danish population. For the period 1987-1997, we follow the cohorts born in 1947-1957 to obtain a sample of core workers. Each year the gross income minus capital income is recorded for each individual and converted to real amounts with 1997 as the base year. We pool the observations of the real income for each of the 11 years of observation into one large data set in order to accommodate both variations across individuals and over the business cycles. It would be preferable to apply the present value of the lifetime income stream instead of the yearly income. However, this is not a viable approach due to data restrictions.

Detailed information on the highest level of education achieved is available. Therefore, we group the sample into educational groups where all individuals have identical level and type of education. We have 110 groups each consisting of at least 50 observations. Examples of types of education include (number of years of schooling in parenthesis): Appr. Shop Assistant (12), Appr. Bank

[12] Endogeneity bias is found to be small for Denmark, see Christensen and Westergaard-Nielsen (2001).

Office Clerk (12), Appr. Electrician (12), Appr. Graphic (12), Appr. Agriculture (12), Appr. Health Care (12), SCHE Armed forces (14), MCHE School Teacher (16), MCHE Social Sciences (16), MCHE Nurse (16), MCHE Transport (16), MSc Economics (18), MSc Medicine (18), MSc Pharmacy (18), PhD Social Sciences (20), PhD Engineering (20), and PhD Medicine (20).[13]

In the empirical analysis, these groups will receive special attention because they are thought to cover the wide spectrum of educations fairly well.[14] A more thorough description of all the educational groups is contained in the Appendix.

Descriptive statistics for the data are shown in Table 1. The data provide detailed information on more than 479,000 worker-years. In Table 1a we describe the entire sample, whereas Table 1b provide less detailed information for each year in the sample. The average yearly income is DKK 255,050 (USD 38,666).[15] The standard deviation is DKK 151,078 (USD 22,903). The average age of the workers in the sample is 40.1 years and they have on average 12.1 years of education and 13.1 years of work experience. The probability of being in full employment is 59%. The income distribution is skewed to the right and shows excess kurtosis.

Insert Table 1

There are certain concerns as to the choice of data. The reader might object that the data are from a fairly small European country. Thus, our results might not carry over to the much larger US labor market. Even though we are unable to draw certain conclusions about the US labor market, we think that the data provide additional information that justifies considering the Danish labor market. Firstly, the data provide detailed information about the education of the individuals in the data set. As we consider each education as an investment asset this is essential for our analysis. Secondly, the data stem from a register database, which means that the data are highly reliable. In summary, data of this kind are necessary to investigate the risk-return trade-off as explained above.

In comparison, Hartog and Vjiverberg (2002) apply US data that are either self-reported or obtained from interviews. What is more, to obtain a sufficient level of detail, they have to rely on occupation

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[13] Appr. denotes Apprenticeship education, SCHE denotes short-cycle higher education and MCHE denotes medium-cycle higher education, respectively.

[14] Basic school, high school and BA are not included because these groups include dropouts. A worker who drops out of a BA-program will enter the file as a high-school graduate. Thus, we find it safer not to study these educational assets in detail. Alternatively, dropouts should be isolated.

[15] The DKK amounts have been transferred into USD by using the average exchange rate for 1997, which is 0.1516 USD/DKK.

in addition to length of education. As far as we are concerned, the closest we get to a data set that includes similar information for the US, is the “High School and Beyond” (HS&B) statistics by the US National Center for Education Statistics, which has recorded the college major for two cohorts of high school graduates. The HS&B excludes short educations, which precludes studying the importance of the year of schooling effect once the specific education has been accounted for. Two additional drawbacks is discussed shortly, namely, the possibility of changing study subject and the Harvard effect.

For now, we settle with the rich data for Denmark. All the principles of the analysis carry over to the US. For this reason, we do not place too much emphasis on the detailed findings and we only provide just enough information about Denmark so as to understand its special features compared to the US.

The educational system in Denmark is structured roughly as follows. At age 7 children enter basic school lasting 9 years. Afterwards adolescents choose between a qualifying apprenticeship education and high-school education. Denmark has a well-developed apprenticeship system starting out with general school-based training followed by work-based specialization. In high school the main focus is on general academic skills, business skills, or technical skills. All of these educations correspond to a norm of 12 years of schooling.

Any high-school equivalent education formally qualifies for university education, which is structured as a three-year BA degree, which may be followed by a two-year MA degree, which again may be followed by a three-year PhD degree. Unlike the US, a university degree only qualifies for a higher level of study in the same field. On the one hand, forming a portfolio of educations is easier in the US than in Denmark, but on the other hand it is easier to identify people with identical education in Denmark rendering the present analysis less messy. Also, the signal of which university you graduated from is hardly relevant in the Danish labor market, i.e. there is no Harvard effect that needs to be considered. These two issues constitute the main objectives against applying the HS&B data for our analysis.

Outside the apprenticeship and university system various short-cycle higher educations (SCHE) and medium-cycle higher educations (MCHE) are available. These correspond to 14 and 16 years of education, respectively.

In Denmark, subsidies for the educational sector are large, and studies are for free. Literally all students are eligible for a Government grant that suffices for costs of living. As a consequence, time spent in the educational system is proportional to the amount invested in education in terms of foregone earnings from unskilled work. Hence, the return to education coefficient from a Mincer (1974) equation is a measure of the private return to education.

5. Empirical Findings

The empirical findings are presented in three steps. Firstly, the pure finance approach is taken and the distribution of raw income is studied. Secondly, the mixed finance-labor approach is implemented by analyzing the residuals from a Mincer regression. Thirdly, the pure labor approach is taken where the proposed human capital model is studied. In each step, we analyze the efficient frontier of the educational asset market. Merely in the second step, do we show the performance measure (standardized excess return).

5.1. Finance Approach

We investigate the risk-return trade-off for the educational assets. In Figure 2 we have plotted the mean yearly income versus its standard deviation. The 17 education groups singled out above are indicated with labels. The reader is reminded that the efficient frontier is not the envelope curve as for financial assets. Rather, the efficient frontier is given as the "optimal" observation points. For instance, doing an MCHE Transport is preferable to doing Apprenticeship Electrician, because it gives a higher return while the risk is the same.

Insert Figure 2

The efficient educations include Apprenticeship Agriculture, SCHE Armed Forces, MCHE Transport, PhD Medicine, PhD Engineering, and MCHE Social Sciences. It is interesting to notice that the efficient educations are comprised of longer as well as shorter educations. This provides evidence that the years of education is not the only factor to consider when assessing the economic consequences of undertaking a given education.[16]

There is clearly a positive relation between mean income and its standard deviation, i.e. the risk-return trade-off is present. We conduct the Weighted Least Squares (WLS) regression where the weights are the scaled number of observations in each education group. The slope coefficient is

[16] The results are robust to outliers. Taking out observations of incomes below DKK 50,000 and above either DKK 800,000 or DKK 1,000,000 does not alter the plot of the efficient frontier of the labor market.

significantly positive (0.70 with a standard deviation of 0.16) and the coefficient of determination (R^2) is extremely high and amounts to 97%.

The analysis so far has not considered the unemployment risk. For a given education, the probability of being full-time employed is not significantly related to the income risk. Thus, varying unemployment incidence is not spuriously driving our results. Likewise, differences between the private and public sectors do not drive the results either. Gender differences are present.[17] For men, the educational assets are more spread out than for women, and the plot for women is centered further southwest than for men. However, it is not our interest to pursue these differences further in the present paper.

5.2. Finance-Labor Approach

We revisit the efficient frontier, now for the residuals from a Mincerian regression: Initially, the Mincer regression in equation (3) is conducted simultaneously for all the observations in the sample. Subsequently, the residuals are grouped according to education, and the means and the variances are calculated. Thus, a combination of finance and labor economics is applied to study the risk-return trade-off on the human capital market.[18]

Insert Figure 3

Figure 3 illustrates the efficient frontier of the human capital market based on Mincer-residuals. Figure 2 does not account for the fact that the educational assets represent different amounts of time invested, whereas Figure 3 does. Figure 3 also accounts for differences in experience. The positive relationship between risk and return is less clear from the graphical presentation than before. Still, the WLS regression reveals a significantly positive slope coefficient, 0.28 (standard deviation of 0.068). The R^2 has decreased quite a lot, namely to 16%. So, even after correcting for differences in the length of education and years of experience, the positive risk-return relationship persists.

Focusing on the 17 educations singled out in Section 4, we find that the pattern is very similar to that seen in Figure 2, though small changes do occur. For instance, PhD Medicine moves down in the diagram due to correction for years of schooling and is no longer efficient. Similarly, MCHE Social Science moves from the efficient frontier to the inefficient interior. The efficient educations

[17] The results are available upon request.

[18] We only consider the observations with positive net income. Hereby we loose around 2% of the observations.

include SCHE Armed Forces, PhD Engineering, MSc Pharmacy, MSc Economics, MSc Medicine, and Apprenticeship Agriculture.[19] Again we see that risk is not necessarily closely linked to the years of schooling.

In order to further assess the risk-return trade-off between the investment opportunities (i.e. educations), it is useful to apply the one-figure performance measure: The standardized excess return, cf. Equation (1). As we are dealing with residuals, we apply a risk-free return of zero. It is not so much the standardized excess return itself, which is of interest; rather it is the ranking of educations. The standardized excess return enables us to rank educational assets with respect to risk and return simultaneously.

Insert Table 2

In Table 2 we have listed the standardized excess return for selected educational assets. We have shown the groups with the 5 largest and the 5 smallest standardized excess returns as well as the ranking of the 17 groups of special attention. The top-performing educations include mainly long educations. Still, long educations with poor performance exist, and short educations with poor performance exist. The low-performance educations are dominated by medium-cycle higher educations. The shorter Apprenticeship educations appear to fare fairly well. Again, we conclude that when investing in educational assets, the investor should find the *type* of education at least as interesting as the *length* of education.

It might be objected, that so far we have conducted our analysis without taking the worker's ability into account. We therefore make the thought-experiment that there are two kinds of workers: Workers with manual abilities and workers with academic abilities.[20] Manual workers can choose amongst the Apprenticeship educations, whereas the academic workers are restricted to the educations at the master level. Figure 4 shows these two efficient frontiers.

Insert Figure 4

The level of mean and standard deviation across the two plots are similar (notice: identical scale for both graphs). The WLS regression reveals no relation between risk and return for the manual

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[19] We have investigated the differences across time by drawing the efficient frontier for Mincer residuals using only observations from the economic upturn in 1987-1988 and the economic downturn in 1993-1994, respectively. Surprisingly, not much of the income variation within each educational asset stems from business cycle variation. However, the standard deviation (risk) decreases from 0.58 to 0.49 as measured from an average over groups. Results are available upon request.

[20] There is a whole range of different ways to divide the educational assets into ability sets.

worker's choice set (the slope coefficient is –0.16 with a p-value of 54%). For the academic worker, the relationship is borderline negative or insignificant (the slope coefficient is –1.05 with a p-value of 8%). So once the workers ability set is taken into account, workers do not appear to be compensated for risk.

Looking at Figure 4b in a little more detail, the master educations in Natural Sciences and Social Sciences tend to have high mean and low risk, whereas the Humanities are characterized by low mean and high risk. However, there are exceptions from this rule, e.g. History/Archaeology, Biology/Sports and Business Language. Thus, it is not sufficient to distinguish between majors. Some of the master educations are popular and require top GPA from high school (e.g. Medicine and Law), whereas others are less popular and therefore most often have free entry (e.g. most fields within Natural Sciences, Business Language and Economics). The diagram reveals no systematic differences between these two categories of educations.

5.3 Labor Approach

Insert Table 3

Table 3 presents the results of the estimation of earnings equations with and without educational assets.[21] The first column contains the results from estimation of the standard Mincer equation (i.e. without educational assets), whereas the last columns contain the restrictive and flexible versions of the earnings equation with educational assets.

Allowing for educational assets increases the return to education from 6% to 12.7% and 10.4% in the restrictive and flexible model, respectively. When we include educational assets, long educations with a low pay-off are no longer "allowed" to drive the return to education down. A systematic low pay-off is now attributed to the relevant educational assets. LR tests show that both the restrictive and the flexible model are preferred over the simple Mincer equation. Both reported information criteria suggest that the flexible model is preferred over the restrictive model and the simple Mincer equation.

[21] For computational reasons, the results of this section are based upon a representative fourth of the sample and a reduced number of educational assets (81 instead of 110). As a robustness check another representative fourth has been applied. This does not affect the results in any way.

The restrictive specification assumes that the education-specific risk is the only education-specific factor affecting income.[22] This specification shows a significant effect of variance on earnings; increasing the variance by one unit increases log earnings by 2.4 units. The result is illustrated in Figure 5a.

Insert Figure 5

Turning to the flexible specification, the rate of return to education is 10.4% but it is less well determined (higher standard error) than in the other model. This indicates that the educational assets hold the majority of the explanatory power, whereas the importance of the years of schooling diminishes.

The mean-variance plot based on the flexible educational assets model in Table 3 is illustrated in Figure 5b. In this figure, the mean is the coefficient of the indicator variable for each asset (plus a level adjustment), whereas the standard deviation is the value estimated from the heteroscedastic error terms. The first thing to notice is that the mean return varies by as much as 80%.[23] Each end of this interval is represented by educations of different lengths. This clearly supports the hypothesis that the number of years spent in the educational system is not as important as information about the exact education chosen. Secondly, there is a positive relationship between mean and variance: The estimated slope coefficient from WLS is significantly positive, (the slope coefficient is 1.27 with a standard error of 0.12), although a linear specification is probably not appropriate. Thirdly, it is dubious whether there is a one-to-one correspondence between mean and variance, and thus it does not suffice to specify the mean as a function of some measure of the variance (and potentially skewness).

The educational assets on the efficient frontier are characterized by efficient combinations of risk and return, whereas educational assets inside the feasible set are characterized by non-pecuniary rewards. Educations in the interior of the feasible set may attract people with a vocation for the subject or they may be characterized by being popular educations with excess demand.[24] Either of these explanations implies that they pay off less than educations of similar length. No matter which of these (or other) explanations hold true, the conclusion is that individuals who take these

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[22] This model is similar to that of Hartog and Vjiverberg (2002) and McGoldrick (1995). Estimating their model, we are able to replicate their results regarding relative variance. The existence of skewness affection depends on the exact specification. Also the conclusions of Hartog and Vjiverberg (2002) depend on the exact data set used.

[23] Disregarding the outlier Appr (Agri).

educations do it for other reasons than (pecuniary) investment purposes, and this is what we denote the vocation effect. The existence of the vocation effect invalidates analyses that are based upon a linear relationship – or any other one-to-one correspondence - between mean and variance of return.

On the efficient frontier, we only find Apprenticeship Agriculture and MSc Economics of the educations that were previously efficient. MSc Medicine moves into the interior set.[25] On the top of the return to educational assets, each year of education adds an annual return of 10.4%, which is not seen from the figure.

The analysis of returns to education has evolved progressively from a pure finance to a pure labor approach. In this stepwise analysis, we gradually allowed for a higher return to education and a return to educational assets that is more independent of length of education.[26] They represent three different approaches to studying the risk-return on educational assets. In Figure 5 the low paying master educations are identified, and they are not allowed to drive down the rate of return to schooling as they were in Figure 3.

6. Conclusion

In this paper, we investigate the human capital asset market while applying insights from the financial economics literature. We presume that individuals decide not only how long time they are willing to invest in education, but also which particular education to invest in. Moreover, individuals make their educational choice on the basis of a number of characteristics of the educations available. As is usual in the labor economics literature, we mainly focus on the economic consequences of education, i.e. pecuniary returns, though we also discuss how vocation effects might enter the analysis. The individuals decide their education based on the return and risk, and in particular they pay special attention to the trade-off between risk and return. A unique register-based data set enables us to draw spectacular conclusions. Overall, our findings support the fact that there is a trade-off between high earnings and low risk. Moreover, our findings confirm that it is highly relevant to consider educations as investment assets rather than restricting the attention to the years of schooling.

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[24] Also other non-pecuniary rewards such as flexible working conditions or part-time work are included herein.

[25] The reduction of the number of educational assets makes it impossible to identify MSc Pharmacy and PhD educations by subject.

On the one hand, we analyze the trade-off between risk and return from the finance perspective. The return to an education is measured by the average income received by those holding a given education, and the corresponding risk measure is the variance of the income of those holding the education. The modified mean-variance framework describes the human capital market. We determine the efficient educational assets and rank the educations according to their standardized excess returns. Thus, we provide agents with information to guide them in buying educational assets in accordance with their preferences.

On the other hand, we investigate the trade-off between risk and return from a more conventional labor economics perspective. The Mincerian framework is modified to account for the fact that people choose the type of education and not just the length of education. Moreover, it is extended to include the link between risk and return in such a way that it distinguishes between the various educational assets. We recover the trade-off between risk and return to education. We reject the presumption that income risk is merely explained by years of schooling. Rather, it is the type of education that matters.

The main innovative aspect of this paper is the introduction of educational assets. This new concept opens the possibilities of analyzing many basic microeconomic questions from an alternative viewpoint.

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[26] In Figure 2 he rate of return was implicitly assumed to be 0%, in Figure 3 it was estimated to 6%, whereas in Figure 5 it was estimated simultaneously to 11%.

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Appendix A. Educational groups.

Table A1: Descriptive statistics.

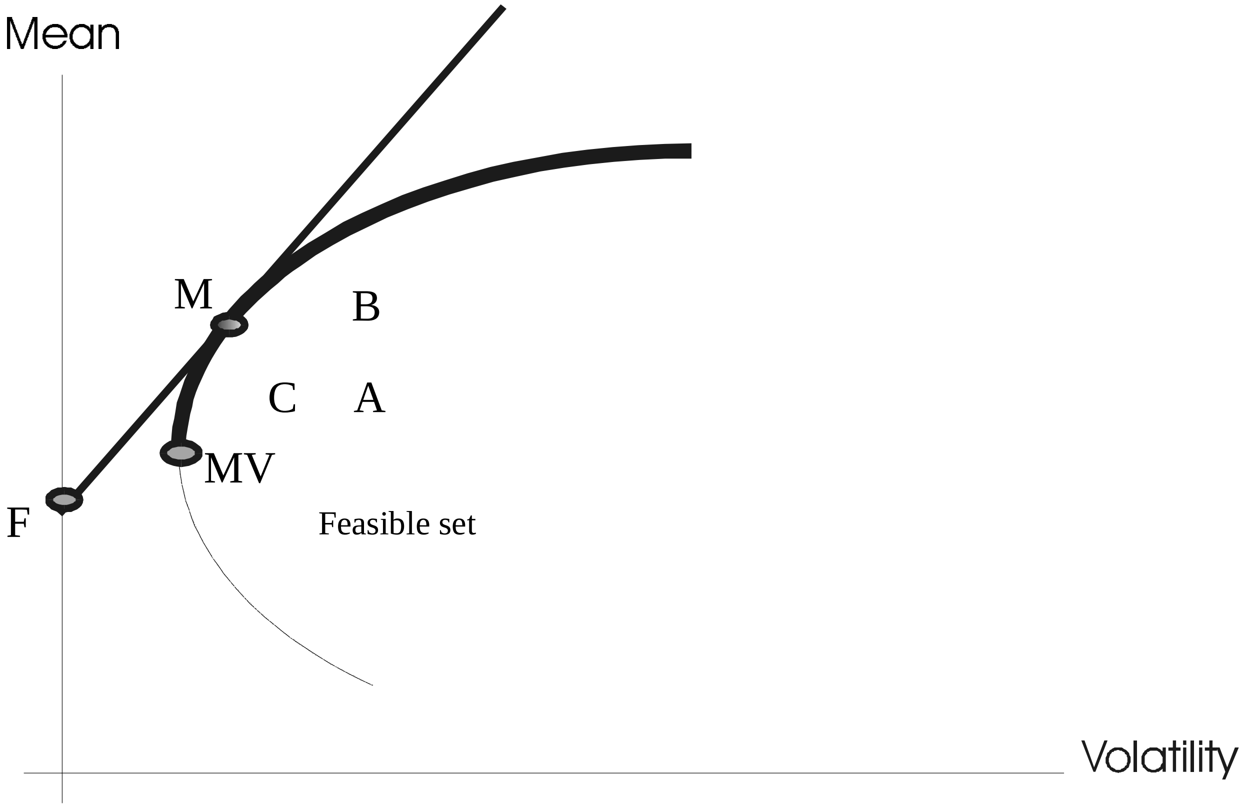
Explanation	Mean	Std. Dev.	Skewness	Kurtosis	N	Length of education
Basic School, 7 years	171377	85705	0.30	3.18	176	7
Basic School. 9 years	206659	117624	1.80	12.83	59133	9
Basic School. 9 years (Old System)	188351	127220	1.72	11.33	79888	9
Preparatory School	227433	144362	2.04	11.96	18469	10
Misc. 10 Years Education	230385	139022	2.41	13.70	2484	10
Misc. 11 Years Education	152702	100757	0.73	3.76	418	11
High School	301126	201697	1.68	7.93	18318	12
Appr. Education	177012	72057	0.61	4.12	183	12
Appr. General Business	247009	147265	2.27	13.18	32206	12
Appr. Shop Assistant	227780	125765	2.30	15.72	11581	12
Appr. Wholesale Shop Assistant	309327	153967	2.50	13.47	2631	12
Appr. Office Clerk	251910	124173	1.87	10.87	14303	12
Appr. Bank Office Clerk	304075	128102	1.93	12.15	8268	12
Appr. IT Office Clerk	358733	164414	0.68	4.28	1284	12
Appr. Builder	281118	119184	1.65	10.30	2594	12
Appr. Pavor	284507	85791	0.22	3.02	66	12
Appr. Carpenter	282922	110483	1.85	14.83	5179	12
Appr. Joiner	272171	102868	1.66	11.59	2286	12
Appr. Plumbing	285965	129587	1.71	11.27	1447	12
Appr. Painter	256137	124245	2.01	13.26	2351	12
Appr. Electrician	309469	113111	1.75	13.23	4625	12
Appr. Construction	288284	147685	2.19	13.65	6542	12
Appr. Metal	272281	115565	1.21	10.63	4341	12
Appr. Jeweler	215558	98416	0.67	2.42	91	12
Appr. Fitter	286637	110144	1.49	13.85	4799	12
Appr. Mechanics	286235	104509	1.44	12.18	9344	12
Appr. Electronics Mechanics	345817	143053	1.06	6.63	2130	12
Appr. IT Mechanics	291081	77709	0.02	4.21	466	12
Appr. Misc. Iron. Metal	293936	141933	1.89	11.82	8030	12
Appr. Graphic	330811	167441	1.55	9.42	3310	12
Appr. Photography	282268	144613	0.83	3.60	229	12
Appr. Misc. Technical	227531	108149	1.62	12.01	3920	12
Appr. Service	195308	123200	1.74	8.90	4521	12
Appr. Dairyman. Butcher	296384	132296	2.06	13.66	926	12
Appr. Baker	277890	140894	1.15	6.24	788	12
Appr. Cook. Waiter	272845	136264	1.84	11.82	1999	12
Appr. Food	235000	122847	1.67	10.77	3212	12
Appr. Agriculture	367379	192857	1.43	6.42	4162	12
Appr. Gardener	236161	105866	1.14	9.35	714	12
Appr. Forestry	227892	56038	0.53	3.02	55	12
Appr. Fishing	402781	261617	1.22	4.86	164	12
Appr. Misc. Agriculture. Fishing	308788	189782	1.97	9.30	1581	12
Appr. Transport	253603	129802	0.74	4.49	1211	12

Appr. Dental Assistant	182613	76812	0.47	6.44	2998	12
Appr. Health Care	189827	61077	0.71	10.61	13840	12
Appr. Health Care Assistant	196298	61278	1.04	12.61	2288	12
Misc. 12 Years Education	254017	134835	2.12	12.31	11636	12
SCHE Education	223563	78790	0.30	5.26	860	14
SCHE Business Language	266931	124838	2.20	15.89	2368	14
SCHE Music. Aesthetics	248934	160006	1.91	9.63	1335	14
SCHE Social Sciences	296464	115615	1.03	5.63	463	14
SCHE Laboratory Assistant	222335	110280	3.09	24.17	1386	14
SCHE Graphic	381330	203048	0.64	3.72	145	14
SCHE Misc. Technical	334309	157243	1.55	8.48	5526	14
SCHE Food	244706	97474	0.91	5.75	1595	14
SCHE Agriculture. Fishing	298807	169428	1.49	6.93	709	14
SCHE Transport	377807	162064	1.23	4.28	79	14
SCHE Health Care	217374	81219	2.02	20.57	4397	14
SCHE Police. Warder	326669	85837	0.99	9.83	2916	14
SCHE Armed Forces	341525	74987	1.86	9.20	306	14
SCHE Misc.	255070	105477	1.15	7.18	680	14
MCHE Educator	221782	79782	0.78	8.70	18869	16
MCHE School Teacher	290290	90001	1.86	20.15	16961	16
MCHE Needlework Teacher	168838	73554	0.85	6.39	469	16
MCHE Journalism	367796	137099	0.12	4.74	874	16
MCHE Business Language	256311	85047	0.75	6.30	1558	16
MCHE Music. Aesthetics	257404	159097	0.65	3.60	212	16
MCHE Social Worker	247121	87697	0.28	4.84	2711	16
MCHE Social Sciences	559842	268078	0.79	3.94	809	16
MCHE Engineering	451204	173752	0.54	5.54	5749	16
MCHE Misc. Technical	371042	181746	1.09	6.50	1243	16
MCHE Food	236083	81419	0.24	3.06	311	16
MCHE Agriculture. Fishing	358096	218358	2.65	10.30	94	16
MCHE Transport	364015	118067	0.02	6.21	1657	16
MCHE Nurse	243917	77544	1.01	9.63	10264	16
MCHE Midwife. Radiologist	242566	96510	0.65	5.89	999	16
MCHE Physiotherapist etc.	232183	95355	1.55	9.57	1864	16
BA Humanities	184099	164533	1.54	4.76	143	16
BA Natural Sciences	135772	83745	0.01	2.32	58	16
BA Social Sciences	451822	271459	1.10	4.39	1320	16
MCHE Misc.	364877	192921	1.49	6.23	673	16
MA Education	270913	113288	0.50	6.61	322	18
MA Humanities	258440	136278	0.62	5.07	1148	18
MA Theology	255588	125173	0.23	6.17	647	18
MA History. Archaeology	341566	133672	0.99	6.24	873	18
MA Letters	266681	140345	0.77	4.24	514	18
MA Business Language (LSP)	328350	122458	0.63	7.97	2773	18
MA Music. Aesthetics	241280	124758	0.54	3.93	901	18
MSc CompSci. Math. Statistics	448028	193646	1.57	8.23	509	18
MSc Physics. Astronomy. Chemistry	401581	154378	0.62	7.04	441	18
MSc Geology. Geography	370581	153407	0.59	6.98	245	18
MSc Biology. Sports	346380	167571	2.04	13.64	947	18

MSc Economics	527702	232853	0.79	4.45	1064	18
MA Law (LLM)	501002	268065	1.24	4.77	2753	18
MA Political Sciences. Sociology	398366	179134	0.87	4.75	752	18
MA Misc. Social Sciences	405181	246102	1.41	5.65	3003	18
MSc Engineering	518456	203106	0.72	5.64	1943	18
MA Architecture (MAA)	341432	167214	1.05	6.21	1733	18
MA Food	377604	160314	0.55	4.53	1026	18
MSc Medicine	542343	207713	0.73	4.64	3757	18
MSc Dentistry	449941	225944	0.99	4.13	1182	18
MSc Pharmacy	485987	205367	1.69	6.98	482	18
MSc Armed Forces	409625	144057	2.10	10.28	301	18
MSc Misc.	331897	271991	1.73	5.49	54	18
PhD Humanities	423680	263991	1.90	6.43	142	20
PhD Social Sciences	495590	214639	1.28	3.32	73	20
PhD Agriculture	339061	134023	0.92	3.42	104	20
PhD Natural Sciences	377797	143152	0.76	5.54	263	20
PhD Engineering. Technology	551715	177818	0.61	4.65	273	20
PhD Medicine	435642	134728	0.46	4.62	208	20

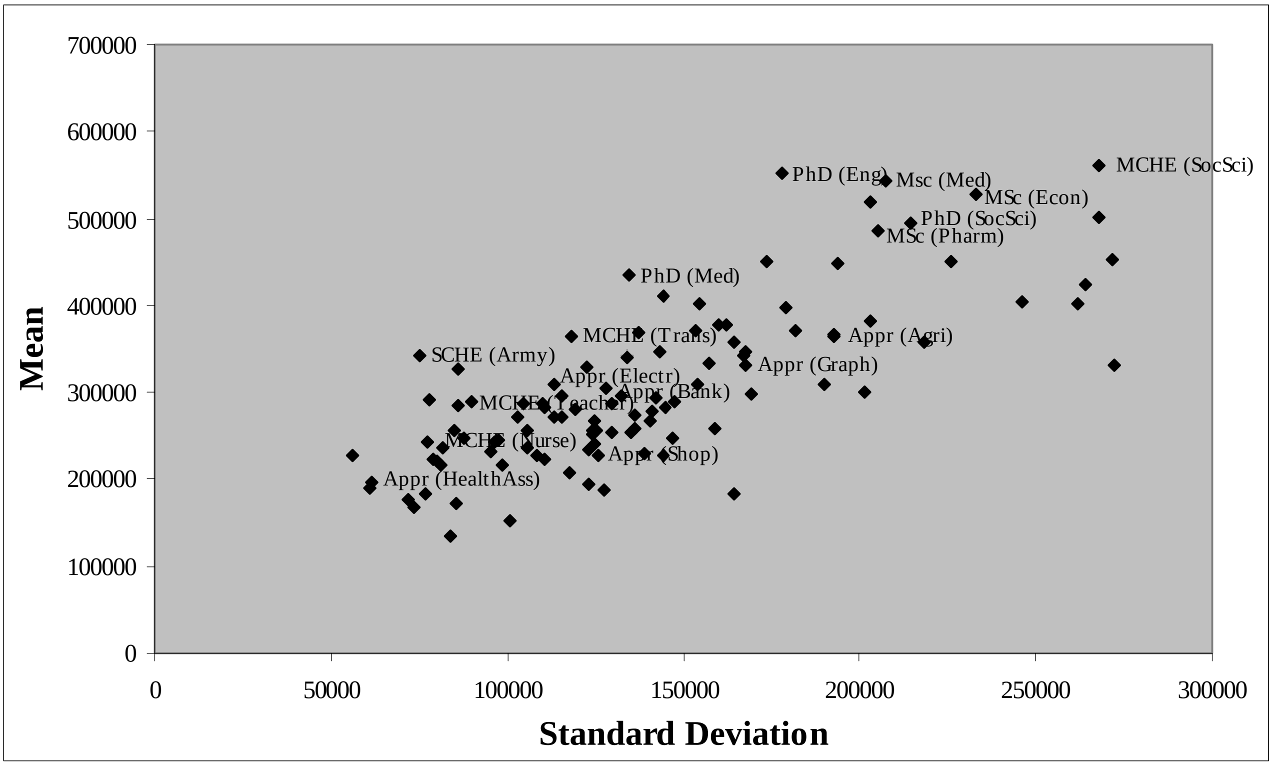
Note: The amounts are in DKK. The average exchange rate for 1997 is 0.1516 USD/DKK. Appr. is short for Apprenticeship, SCHE denotes short-cycle higher education, MCHE denotes medium-cycle higher education.

Figure 1: Efficient frontier



Note: The efficient frontier without a risk-free asset is the bold envelope curve starting at MV. The efficient frontier with a risk-free asset is the bold straight line from F through M.

Figure 2: Efficient frontier for raw income.



Note: The amounts are in DKK. The average exchange rate for 1997 is 0.1516 USD/DKK.

Figure 3: Efficient frontier for Mincer residuals.

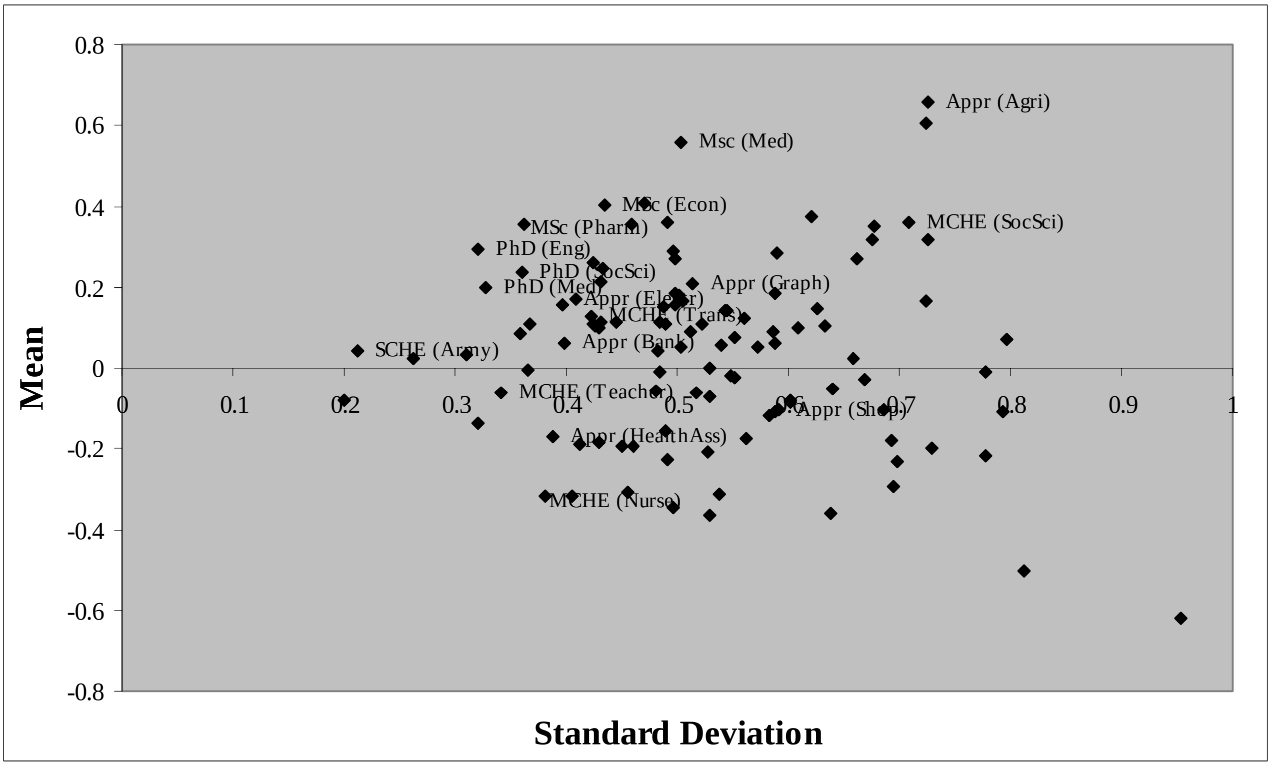


Figure 4: Efficient frontier for Mincer residuals for different abilities

Figure 4a: Manual abilities

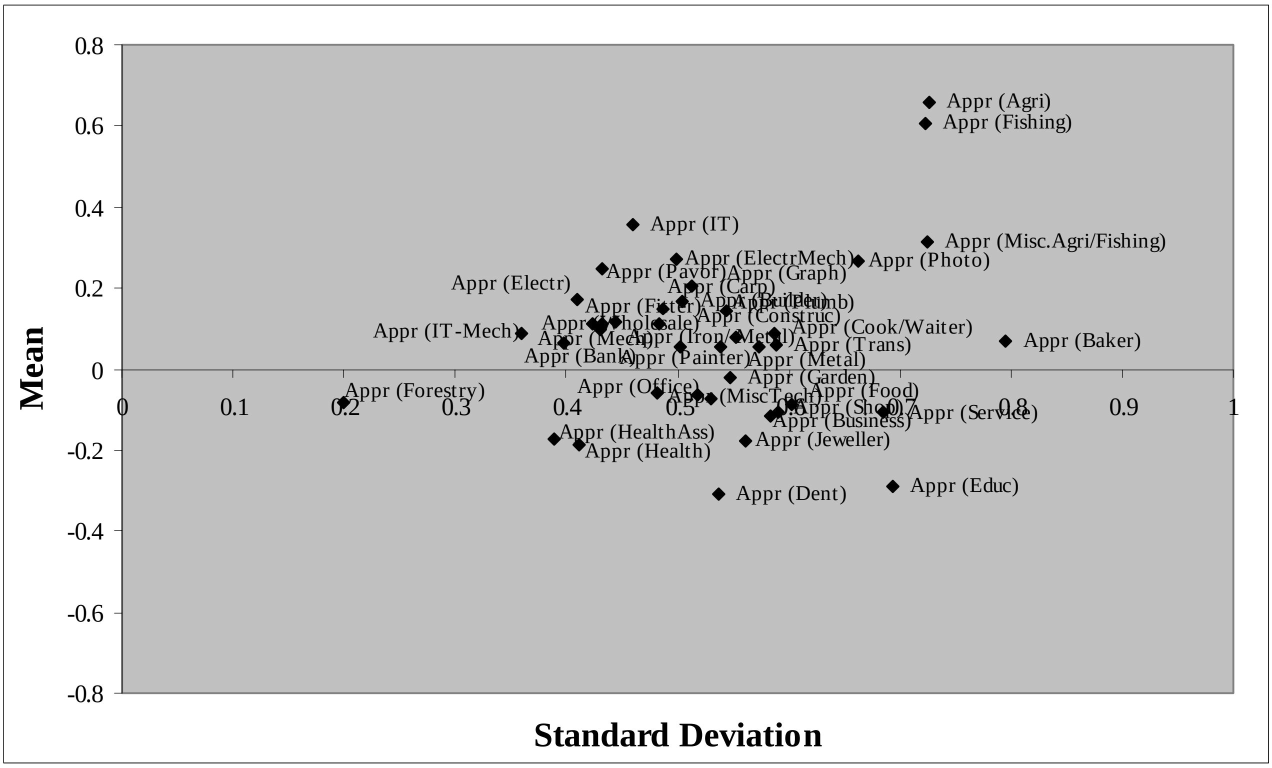


Figure 4b: Academic abilities

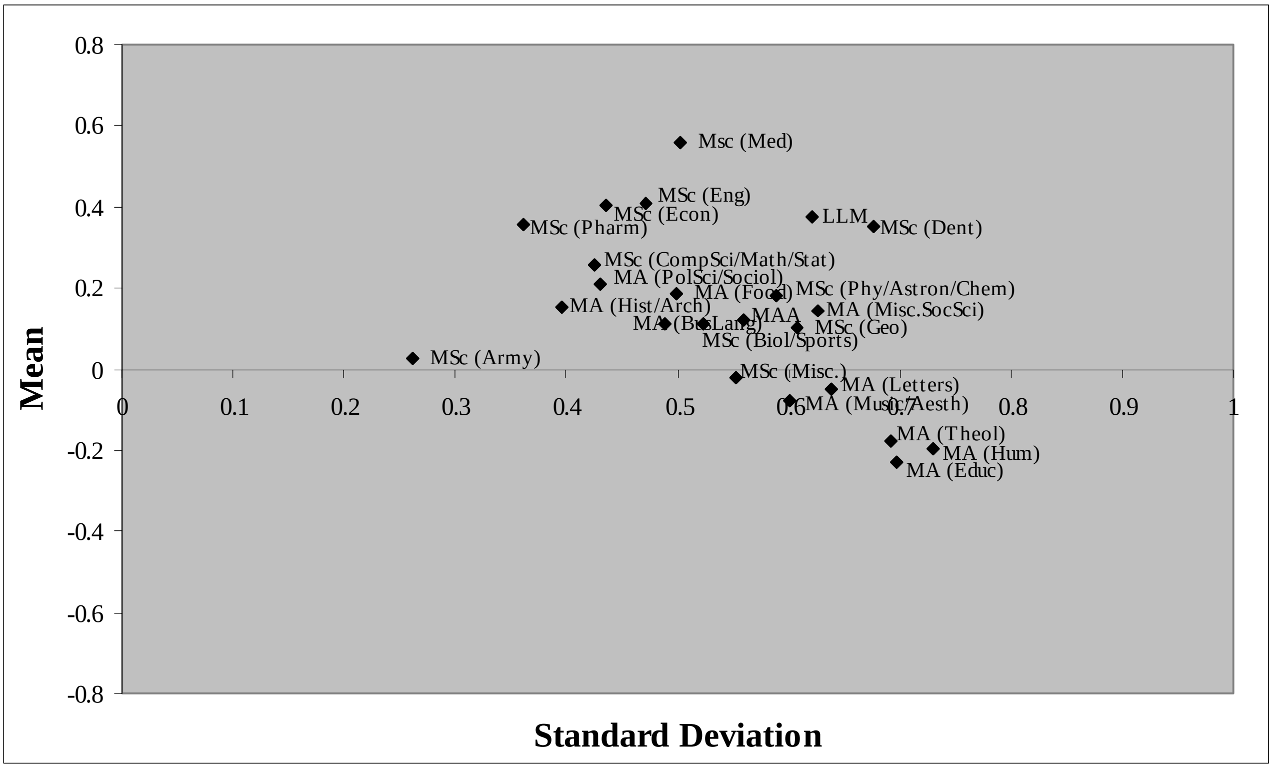


Figure 5: Efficient frontier for human capital model.
Figure 5a: Restrictive model.

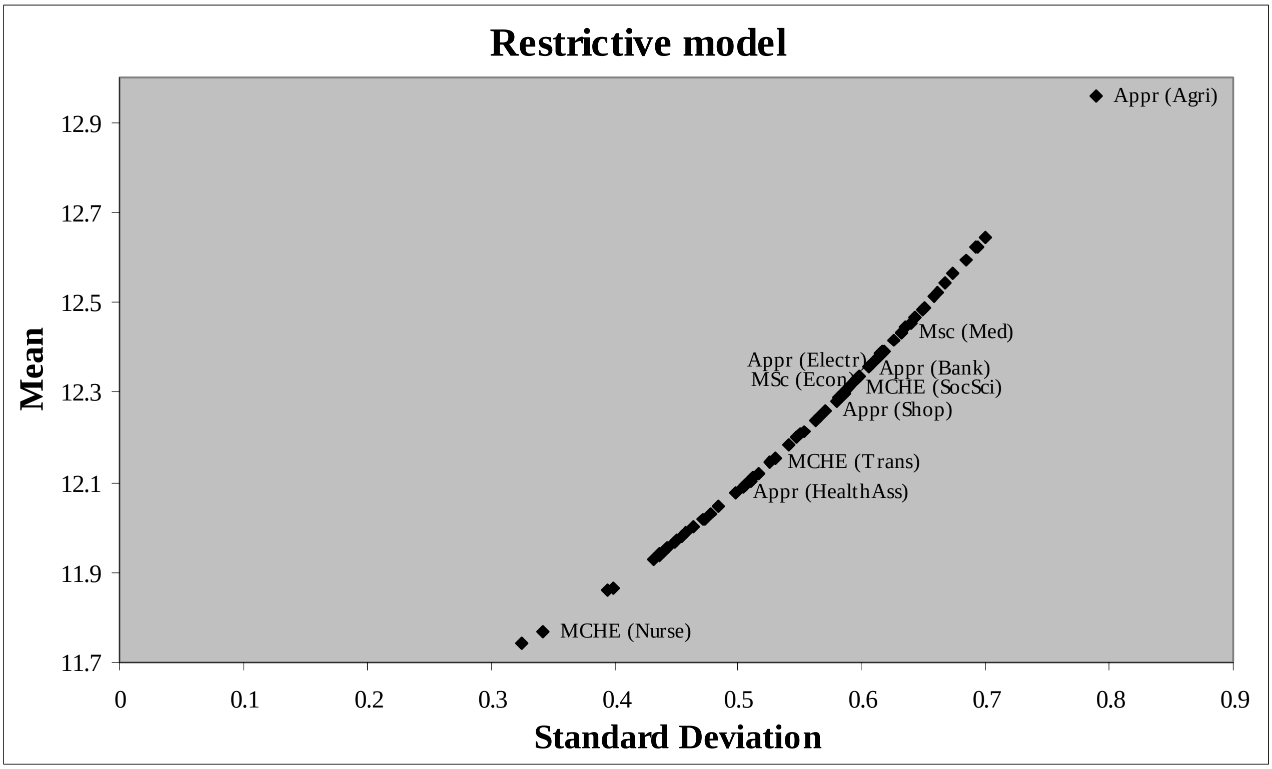
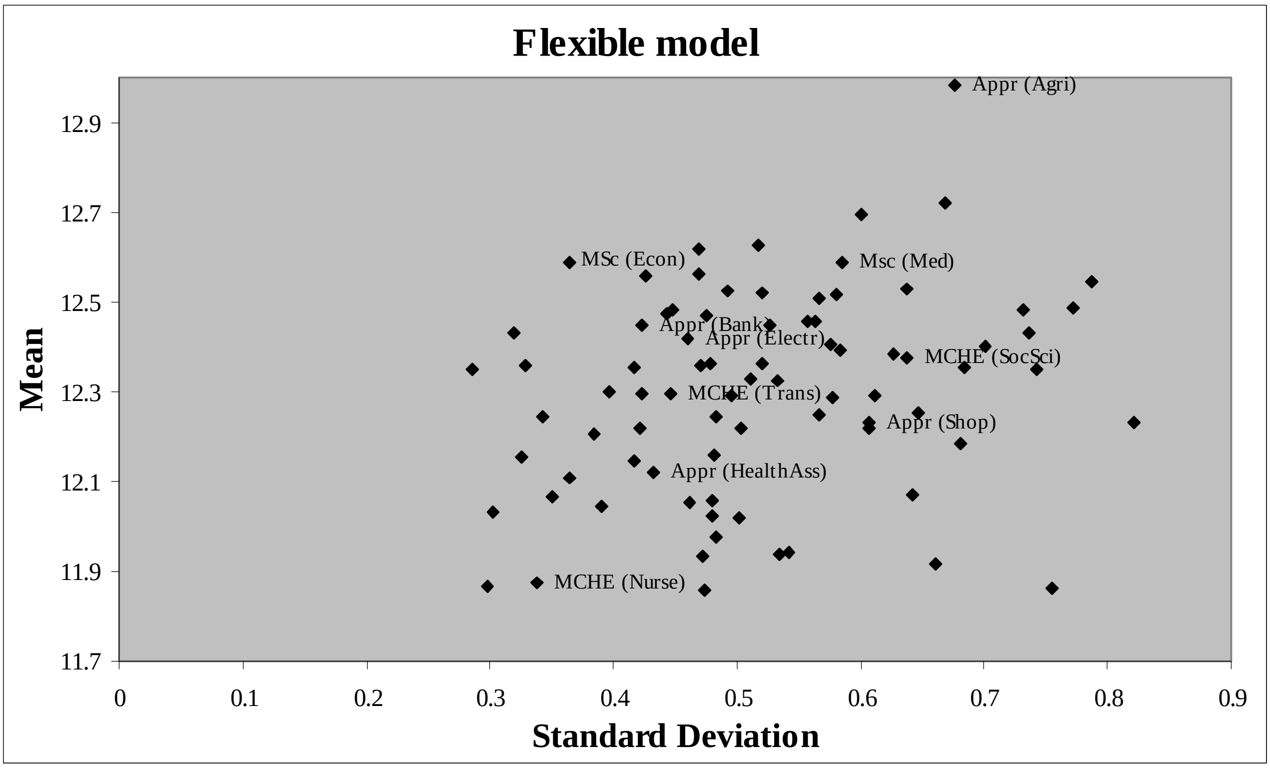


Figure 5b: Flexible model.



Note: The mean is computed for a standard person with 13 years of experience and 12 years of education. This only matters for the scaling of the y-axis. The standard deviation is allowed to be education specific and it is estimated simultaneously.

Table 1: Descriptive statistics.

Table 1a: Descriptive statistics for pooled sample.

	Mean	Std. Dev.
Annual income (DKK)	255050	151078
Age (years)	40.1	4.5
Employed hours in year	1228	675
Accumulated education	12.1	2.7
Experience	13.1	6.7
Indicator for full employment	0.59	
Number of observations	479223	
Skewness	1.88	
Kurtosis	10.74	

Table 1b: Descriptive statistics by year.

Year	Mean	Std. Dev.	Skewness	Kurtosis	N
1987	251037	151322	1.71	10.26	43610
1988	254173	152321	1.73	10.32	43505
1989	252041	151788	1.78	10.30	43433
1990	253773	154816	1.89	11.03	43427
1991	253984	155272	1.87	10.72	43643
1992	253991	154852	1.84	10.52	43632
1993	253904	155130	1.85	10.48	43625
1994	255717	144043	2.04	11.54	43592
1995	257215	148022	2.18	12.59	43671
1996	258473	144994	1.91	10.01	43634
1997	261240	148488	1.97	10.33	43451

Note: The amounts are in DKK. The average exchange rate for 1997 is 0.1516 USD/DKK.

Table 2: Standardized excess returns for Mincer residuals

Rank	Std. Excess return	Education
1	1.11	MSc Medicine
2	0.98	MSc Pharmacy
3	0.93	MSc Economics
4	0.91	PhD Engineering, Techology
5	0.91	Appr. Agriculture
10	0.66	PhD Social Sciences
12	0.60	PhD Medicine
18	0.51	MCHE Social Sciences
23	0.42	Appr. Electrician
25	0.40	Appr. Graphic
33	0.30	MCHE Transport
49	0.21	SCHE Armed Forces
53	0.16	Appr. Bank Office Clerk
82	-0.18	Appr. Shop Assistant
83	-0.18	MCHE School Teacher
99	-0.44	Appr. Health Care Assistant
106	-0.67	MCHE Educator
107	-0.69	MCHE Midwife, Radiologist
108	-0.70	SCHE Education
109	-0.78	SCHE Health Care
110	-0.83	MCHE Nurse

Table 3: Results from estimation of earnings equations.

Variables	Standard Mincer		Restrictive model		Flexible model	
Years of schooling (*S*)	0.060	(0.001)	0.127	(0.001)	0.104	(0.045)
Experience	0.086	(0.001)	0.089	(0.001)	0.082	(0.001)
Experience squared/100	-0.175	(0.004)	-0.172	(0.004)	-0.160	(0.003)
σ^2			2.357	(0.057)		
Intercept	10.809	(0.008)	9.105	(0.023)		
Indicators for educational groups	No		No		Yes	
Standard deviation of residuals, σ.	0.617	(0.001)	$\sigma=\sigma(A_{sj})$		$\sigma=\sigma(A_{sj})$	
Log likelihood	-109296		-96820		-93598	
Number of observations	116678		116678		116678	
Number of parameters	5		86		165	
AIC	218602		193812		187526	
SC	218650		194643		189121	

Note: Standard errors in parentheses. $\sigma=\sigma(A_{sj})$ indicates that sigma is allowed to differ across 81 different educational assets.