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Product line design with vertical and horizontal consumer heterogeneity: the effect of distribution channel structure on the optimal quality and customization levels

Parisa Bagheri Tookanlou
Department of Engineering, Operations Management,
Aarhus University, Aarhus, Denmark

Hartanto Wijaya Wong
Department of Economics and Business Economics, Aarhus University,
Aarhus, Denmark

Abstract

Purpose – The purpose of this study is to analyze the problem of optimal product line design in marketing channels where consumers are heterogeneous in both horizontal and vertical dimensions.

Design/methodology/approach – This paper develops a model to evaluate when it is preferable for a firm to extend the product line in a vertical or horizontal direction. Consumers are modeled as being vertically heterogeneous with respect to their valuation of quality and horizontally heterogeneous with respect to their preference on the esthetic component of the product. These model characteristics allow us to consider a broader set of product line extension strategies. By considering both a vertically integrated channel and a decentralized channel, this study investigates how channel structure influences optimal product line design. The problem with supplemental numerical analyses is mathematically analyzed.

Findings – The analysis shows that a horizontal product line extension strategy that offers the customized product can be used as an alternative to a vertical product line extension strategy. If the fixed cost is not too high, offering the customized product with low quality may be preferred to the quality-based segmentation strategy. Furthermore, the analysis shows that the channel structure is influential as the preference for the horizontal product line extension strategy is more pronounced in the decentralized channel than in the centralized channel.

Research limitations/implications – The analysis presented in this paper is limited by the consideration of full market coverage. Further research is needed to see how the results can be generalized to the case with partial market coverage.

Practical implications – The analysis suggests that a firm may consider product customization as part of its product line strategy. Information regarding market characteristics and channel structure is important when deciding on the optimal product line design.

Originality/value – The model reflects a more realistic marketing strategy and channel structure than previous studies that typically consider product line extension in only one direction and focus on the centralized distribution channel. Combining the standard product line extension and customization strategies also represents an important contribution to the literature. These extensions produce interesting new results and insights into a firm’s optimal product line design strategy.

Keywords: Distribution channel, Customization, Horizontal differentiation, Product line design, Vertical differentiation
1. Introduction

It can be argued that one of the most important decisions within product line design is the length of product line, which is defined as the number of product offerings in a product line. The increasing number of consumer goods and services offered in recent years shows that product line extension has always been an integral part of marketing managers’ strategies. Offering a larger product variety allows firms to increase both demand and market share (Kotler and Keller 2016; Draganska and Jain 2005; Caldieraro et al. 2015).

As consumers may have heterogeneous preferences in both vertical and horizontal dimensions of a product, product line extensions can take place either vertically or horizontally or both. When considering consumer heterogeneity in product quality valuation, firms often implement vertical product differentiation or quality-based segmentation, by offering a product line comprised of multiple products with different quality levels. The notion of quality here is represented by a set of product attributes, on which consumers agree in their preference ordering, i.e., every consumer prefers higher values on the attributes to lower values (or vice versa), ceteris paribus (Moorthy 1984; Moorthy and Png, 1992). Apparel companies such as Land’s End and Levi Strauss offer a variety of jeans that are differentiated by the quality of the fabric (organic or conventional; pure cotton or a synthetic mix), the treatment of denim (washed or unwashed), and the quality of construction (high stitch count or low stitch count). In a similar vein, BMW targets different segments by offering a wide range of products ranging from the BMW Series 3 to the BMW Series 7. While increasing demand and market share serve as the primary incentive of vertical product line extension, firms must carefully pay attention to the downside of this extension strategy, which is mainly associated with the potential cannibalization that could lead to reduced profitability.

Alternatively, firms may choose to differentiate their product lines horizontally when they focus on consumer heterogeneity in the taste or aesthetic attributes of the product. Unlike the vertical attributes, the merits and demerits of the taste or aesthetic attributes may be evaluated differently by consumers. With horizontal product differentiation, firms offer products that have the same quality level but different aesthetic attributes such as taste, color, or flavor. For example, Coca Cola offers Diet Coke and Decaffeinated Coke alongside the standard Coke without differentiating the price. Garment manufacturing firms usually offer products in different colors and sizes, but all come with the same quality and price.

In recent years, we witness an increasing number of firms offering customized products to their consumers as part of their horizontal extension strategies. The two examples above (Land’s End and Levi Strauss) offers customized shirts, jeans, and jackets by allowing consumers to choose their preferred styles, colors, inseam, etc. There are abundant examples of firms offering customized t-shirts where consumers have the possibility to choose the most preferred color and/or pictures (e.g. Spreadshirt and Rush order Tees). Sisal Rugs Direct (http://www.sisalrugsdirect.com) allows consumers to choose the shape, colors, and pattern of their rugs. The motivation for firms to offer such customized products is that besides they may increase both their consumer base and revenues, they may also achieve higher margins as consumers may be willing to pay a higher price for products that fit their preference better (Franke et al. 2010). However, this motivation does not come without
costs, as some investments are necessary to enhance flexibility in the production facility to accommodate customization.

There has also been a growing list of firms that introduce customized products in addition to their existing product line offering standard products as their attempt to deal with consumer heterogeneity in both the vertical and horizontal dimensions. Normann Copenhagen is a Danish design company who allows its customers to customize a wide range of furniture in the tone and texture of furniture fabric. This company offers consumers the ability to configure the furniture with specific quality and aesthetic attributes. A number of luxury brands in the fashion industry such as Hermès and Burberry offer personalization alongside the standard offerings.

A number of empirical studies have documented successful cases in relation to product line extension strategies. The introductions of Honda’s Acura, Toyota’s Lexus and Nissan’s Infiniti in the luxury car segment are examples of a successful vertical product line extension in the automotive industry (Shah 2018). Johnnie Walker’s strategy to offer Red Label, Gold Label Reserve and Blue Label is another example of vertical product extension (Pontes 2018). Timbuk 2’s customized bags, mymuesli’s muesli and Build-a-bear’s customized toys represent some examples of a successful product line extension involving customization. However, despite the many advantages of the product line extension, a study by Nielsen (2015) reports a significantly high failure rate. Of over 60,000 new SKUs introduced in Europe between 2011 and 2013, 55% made it to 26 weeks, and only 24% lived to reach a full year. In a similar vein, there are many companies and startups failing in implementing customization, as documented in Walcher and Piller (2013).

These developments have motivated a number of scholars to examine many factors that may affect the effectiveness of a product line extension strategy (see e.g. Kim et al. 2000, Pontes et al. 2017 and Pontes 2018 for research on vertical extensions, and Liu and Cui 2010, for research on horizontal extensions). However, despite the growing interest in the factors affecting vertical or horizontal extension’s success, there have been no studies examining the preference of a horizontal extension (involving customization) as opposed to a vertical extension. This leaves an interesting but less understood question: which extension direction would be more desirable: a vertical product line extension or a horizontal product line extension (with customization), and what are the affecting factors? Understanding the preference of an extension direction over the other would contribute to increasing the likelihood of a product line extension’s success.

Firms implementing quality-based segmentation may want to gain additional benefits by adding customized offerings to their existing product line, i.e., by combining both vertical and horizontal extension strategies. An important question that marketing managers are facing is: which segment should they target the customized product to, i.e. should the customization be offered to the high-quality or low-quality product? Finding the right answer to this question would contribute to enhancing the effectiveness of the customization strategy.

To complicate the problem further, the strategic decisions stated above may also be influenced by channel structure. The firms in the above examples sell their product lines using various channel structures. Timbuk2 offers customized bags online and sells more standard bags through resellers. Many car manufacturers rely heavily on a decentralized dealer network. Hence, it is important for the
manufacturing firms to consider channel structure when designing and extending a product line. When a manufacturing firm sells directly to consumers i.e. uses a centralized distribution channel, it has the full control over the ultimate targeting of the products to the different consumer segments through its pricing and quality level decisions. In a decentralized channel, however, the manufacturing firm depends on intermediate parties, e.g. retailers or dealers, who may choose a different market coverage strategy. For example, in the case where the manufacturer intends to cover the whole market, the retailer may prefer to cover the market only partially. This potential misalignment of the manufacturer’s and the retailer’s strategies may create channel inefficiency that results in a reduction of channel profitability. A dual channel structure is also adopted in some of the examples above, especially when the manufacturer sells the customized product. That is, the standard product is sold through the retailer, and the customized product is sold online directly by the manufacturer. Despite the extensive marketing literature on various distribution channel issues, there has been no research that has analyzed the interaction between a firm’s product line extension decision and distribution channel structure.

The above discussions reflect some of the managerial questions that are worth addressing to enhance the understanding of how different marketing and operations related factors may play a role in the effectiveness of a product line extension strategy. Our paper seeks to shed some lights on the preference of product line extension strategies in the market and how this preference is influenced by the distribution channel structure. More specifically, we provide answers to the following questions:

1. When is the horizontal product line extension preferable to the vertical product line extension?

2. In the customization strategy, which segment should the customized product be targeted to?

3. How are the answers to the above questions influenced by the distribution channel structure, and which operations and marketing factors are influential?

Our analysis of these research questions advances a number of important messages suggested by previous studies in the following ways:

- When the potential cannibalization effect is too severe in a vertical product differentiation, firms may find a horizontal extension with customization as a more attractive strategy as long as the cost associated with flexibility is not too costly.
- Offering customization in addition to the standard vertical product differentiation may improve profitability. No specific segment targeting strategy dominates, i.e., there are certain conditions under which the customized product should be targeted to the high-valuation or low-valuation segment.
- Channel decentralization has an impact on the optimal extension strategy. Moreover, offering customization in a decentralized channel may lead to a reduction in quality distortion.

Methodologically, we follow the lead of the previous studies on product line design in the economic and marketing literature by developing stylized models that allow us to perform a set of “logical experiments” to explore the research questions (Moorthy 1993; Chung and Lee 2014). These models
have been proved to be useful for generating strategic insights about how firms should operate. In the case where a decentralized or dual channel is used, we adopt the game theoretic approach to capture the strategic interaction between the manufacturer and the retailer. The settings we consider in this paper are noticeably more complex than those considered in the previous studies for the following two reasons. Firstly, we relax one of the restrictive assumptions commonly found in previous models that only consider either vertical or horizontal differentiation. As we need to evaluate the preference of a vertical or horizontal extension, our model needs to consider consumer heterogeneity in both dimensions. This model extension also allows us to evaluate the strategy offering a customized product to the existing product line with vertical product differentiation. Secondly, in addition to the bilateral model used in most previous studies in representing a decentralized channel structure, we consider a dual channel for the product line with customization, which mimics the real world practice of most companies with customized offerings. In this dual channel, we model the competition between the retailer selling the standard product and the online channel owned by the manufacturer selling the customized product. This allows us to provide richer insights into the role of channel structure in the optimal product line extension.

Despite all the additional complexities in our model, we limit the analysis to a monopolistic setting in the sense that there is no competition between manufacturing firms. The choice of this setting is conventional in the literature studying channel structure in product line design (see e.g. Villas-Boas 1998, Liu and Cui 2010, and Jerath et al. 2018), and allows us to isolate the impact of operations and marketing factors on the relative performances of the different product line extension strategies without any interference from the market competition. Our model should be seen as a building block for developing an extended model considering market competition.

The remainder of this paper is structured as follows. Section 2 surveys the related literature. In Section 3, we present two comparison scenarios under a centralized channel. In the first comparison scenario, we focus on contrasting the horizontal and vertical extension strategies. We continue with the second comparison scenario where the focus is on examining the effects of offering a customized product to the existing quality-based segmentation strategy. In Section 4, we analyze the two comparison scenarios under a decentralized channel. We conclude the paper in Section 5.

2. Literature Survey

Product line design is a topic widely studied in the literature. Previous work has been concerned with product line strategies and their effects on revenues and costs, both empirically and analytically. Among the empirical papers, the literature has been concerned with trying to understand the effects of product line breadth at the market level (see e.g. Kekre and Srinivasan 1990, Bayus and Putsis 1999, and Moreno and Terwiesch 2017). While these empirical studies are useful for the accumulation of evidence that support or challenge the established theories, the generalization of their results is also limited due to the narrow scope of products and/or variables examined.

More closely related to our work are papers that explore various aspects related to product line design through the use and development of analytical models. We classify these papers into four streams as
discussed in the following. We summarize the main differences of these papers in Table 1. The first relevant stream considers product line design with vertical product differentiation. Mussa and Rosen (1978) and Moorthy (1984) are among the first introducing quality differentiation into the marketing literature. The classical result from their studies is that due to potential cannibalization, only consumers with the highest valuation for quality receive their optimal quality level, whereas consumers in the other segments receive quality levels that are lower than their optimal quality levels. Subsequent related papers explore several other aspects related to product line design with quality differentiation, such as the impact of production technology on the optimal quality and price decisions (Netessine and Taylor 2007), the choice of product attributes (Lauga and Ofek 2011), the trade-off between component commonality and product differentiation (Kim and Chhajed 2000, Desai et al. 2001, and Heese and Swaminathan 2006). While this stream of research provides a solid understanding of the advantages and disadvantages of vertical product differentiation, its application is limited as firms are also often interested in the horizontal product differentiation.

By contrast, the second stream of research considers product line design with horizontal product differentiation. This stream of research is not concerned with product quality and therefore, the reservation price of consumers is assumed constant. A common feature, as we also use in this paper, is that consumers are modeled to be uniformly distributed over Hotelling’s line that is characterized by a closed interval of product space [0, 1] (Hotelling, 1929). Each point on the line represents the consumer’s preference (or location) for the aesthetic attribute(s) of the product. De Groote (1994) studies the role of flexibility in accommodating product line extension in the horizontal direction. Several papers in this stream particularly study the comparison between mass production offering a limited set of standard products and mass customization offering unlimited and customized products (see e.g. Gaur and Honhon 2006, Jiang et al. 2006, Alptekinoğlu and Corbett 2010, and Wong and Eyers 2011). Several authors, e.g. Syam and Kumar (2006), Alptekinoğlu and Corbett (2008), Mendelson and Parlaktürk (2008a and 2008b), and Xia and Rajagopalan (2009) examine the competition between the standard and customized products in a duopoly market. Takagoshi and Matsubayashi (2013) present a model considering a two-attribute space where one attribute indicates a characteristic in relation to the function of the product and the other attribute indicates a taste or flavour of the product. They define customization as a continuous extension of their product line from the core product only along the function attribute, and study a competition between two firms. Some authors, e.g. Dewan et al. (2003) and Xiao et al. (2014) model the product variety in a circular spatial market instead of the traditional Hotelling model. Similar to the first stream, the main limitation of this second stream is mainly due to the single dimension of consumer heterogeneity considered. As the models in this stream do not pay attention to quality and market segmentation, their applications to product line extension decisions are restricted. In this paper, we try to deal with the main limitations of these two research streams by explicitly considering consumer heterogeneity in both the vertical and horizontal dimensions. This increases the model complexity, but is necessary for the purpose of comparing the preference of vertical or horizontal extension, which is central in our paper.

There are indeed a few papers studying product line design that considers vertical and horizontal consumer heterogeneity, which we classify in the third stream. Desai (2001) considers a market where consumers are two-dimensionally heterogeneous. They focus on the effect of cannibalization on
product line design and do not pay attention to product line extension. Lacourbe et al. (2009) study the optimal product portfolio positioning in a market where consumers are two-dimensionally heterogeneous. Shi et al. (2013) consider a market where consumers can be horizontally or vertically differentiated, but they do not look into the quality-based segmentation problem. Although these papers lend support to the analysis presented in our paper, our paper is different in the two following respects. Firstly, despite the similarity in considering two-dimensional consumer heterogeneity, our paper aims at examining when a vertical extension is preferable to a horizontal extension, and vice versa, a topic which does not represent their main interest. Secondly, they consider horizontal product differentiation on Hotelling’s line but does not relate them to the customization level as we consider in our model. With the growing interest in customization, focusing on the horizontal extension with customization options is certainly of relevance.

The final stream of research considers product line design that takes into account the choice of distribution channel structure. A number of papers (Villas-Boas 1998, Hua et al. 2011, Chung and Lee 2014, and Wong and Lesmono 2019) considers product line design with vertical differentiation in a distribution channel that consists of a manufacturer selling a product line through a retailer. By contrast, the following papers study product line design in a distribution channel where products are differentiated horizontally. Liu and Cui (2010) focus on the optimal number of products in a product line. Li et al. (2015) consider competition between the manufacturer’s online customization channel and the conventional retailer. Similar to their model, our modeling framework also includes a dual channel in the case where both the standard and customized products are offered. While the analysis and modeling framework developed in these papers provide a basis for our analysis, it is not adequate for answering the research questions considered in this paper. This is particularly true when one would like to examine the effect of channel structure on the preference of vertical or horizontal extension. No previous studies have examined this issue. Moreover, working with consumer heterogeneity in two dimensions allows us to obtain a more comprehensive understanding of quality distortion prevalent in a decentralized channel. Interestingly, we are able to identify customization distortion in addition to quality distortion in a dual channel setting. Through our extended model, it is also possible to capture what effects a customization strategy may have on the extent of quality distortion.

To sum up, our paper contributes to advancing the existing literature by considering more comprehensive product line extension strategies and the effect of channel structure. Our literature review suggests that no previous studies have tried to analyze a product line design problem where both quality and customization level represent important design decisions. Our analysis on differentiating product line design in a centralized channel as opposed to a dual channel while considering two dimensional consumer heterogeneity at the same time represents an important novelty of this paper.
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<th>Customization</th>
<th>Main decision variables</th>
<th>Product line extension strategy</th>
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3. Comparison of Product Line Extension Strategies in a Centralized Channel

We consider two scenarios for the comparison of product line extension strategies, as depicted in Figure 1. In the first scenario, we focus on comparing the direction of the extension (vertical vs. horizontal). Using the baseline strategy that offers one standard product as a benchmark, we evaluate the preference of the vertical product line extension over the horizontal product line extension. In the second scenario, we consider a manufacturer already implementing quality-based segmentation that extends its product line horizontally by adding a customized product. Our focus here is on the comparison of the quality of this customized product (high vs. low). In this section, we present and discuss the results for the centralized channel, wherein the manufacturer is the only player involved. This can also be seen as equivalent to the setting where the manufacturer sells its product directly to consumers without any intermediaries. In Section 4, we continue the comparisons for the decentralized channel.

<table>
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<tr>
<td>Dewan et al. (2003)</td>
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<td>Desai (2001)</td>
<td>Product quality and price</td>
</tr>
<tr>
<td>Lacourse et al. (2009)</td>
<td>Size of the product portfolio, product quality, product feature, and price</td>
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<tr>
<td>Xiao et al. (2014)</td>
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<td>Shi et al. (2013)</td>
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<td>Chung and Lee (2014)</td>
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<td>Hua et al. (2011)</td>
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<td>Wong and Lesmono (2019)</td>
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<td>Li et al. (2015)</td>
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<td>This paper</td>
<td>Price, price, and customization level</td>
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*Investigating the effect of channel structure on optimal product line design problem
3.1 Model Notation and Assumptions

We consider a market where consumers are two-dimensionally heterogeneous, and hence, extend most previous studies that consider consumer heterogeneity a single dimension. To capture consumer heterogeneity in the vertical dimension, we assume that consumers belong to either the high-valuation segment or to the low-valuation segment, indexed by $i=\{L, H\}$ (see Table 2 for the notation of the model). Consumers in segment $i$ have valuation $\theta_i q$ for the product, where $\theta_i$ represents consumers’ quality valuation in segment $i$, and $q$ represents the quality level of the product. Here the quality variable, $q$, may represent a summary measure of all more-is-better attributes of the product. For example, for a bag or luggage, the quality variable $q$ can be a composite of the material composition and size.
Consumers in the high-valuation segment have a higher valuation per unit of quality than consumers in the low-valuation segment. That is,  \( \theta_H > \theta_L > 0 \). In the case where two products of different quality are offered, the quality level of the product targeted at segment \( i = \{L, H\} \) is denoted by \( q_i \). The size of each segment is denoted by \( \alpha_i, i = \{L, H\} \). Without loss of generality, we assume that the total market size of both segments is normalized to be one, i.e., \( \alpha_L + \alpha_H = 1 \).

Table 2: Notation of the model

<table>
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<td>( p_{ij} )</td>
<td>Retail price of product ( j, (j = S, C) ) targeted at segment ( i, i = {L, H} );</td>
</tr>
<tr>
<td>( x_i )</td>
<td>Customization level of the customized product targeted at segment ( i, i = {L, H} );</td>
</tr>
<tr>
<td>( q_i )</td>
<td>Quality of the product targeted at segment ( i, i = {L, H} );</td>
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<td>( w_i )</td>
<td>Wholesale price of the standard product targeted at segment ( i, i = {L, H} );</td>
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<td>( t_i )</td>
<td>Unit mismatch cost for consumers in segment ( i, i = {L, H} );</td>
</tr>
<tr>
<td>( \theta_i )</td>
<td>Consumer’s quality valuation in segment ( i, i = {L, H} );</td>
</tr>
<tr>
<td>( y_i )</td>
<td>Consumer’s location along the Hotelling’s line in segment ( i, i = {L, H} );</td>
</tr>
<tr>
<td>( \alpha_i )</td>
<td>Size of segment ( i, i = {L, H} );</td>
</tr>
<tr>
<td>( D_{ij} )</td>
<td>Portion of consumers in segment ( i, i = {L, H} ) buying product ( j, (j = S, C) );</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Production parameters</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>( a )</td>
<td>Coefficient of the production cost;</td>
</tr>
<tr>
<td>( b )</td>
<td>Coefficient of the fixed investment cost or flexibility cost to accommodate customization;</td>
</tr>
<tr>
<td>( c )</td>
<td>Additional marginal cost for the customized product;</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Others</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>( \pi_{SC,c}^{(k)} )</td>
<td>Total profit in the centralized channel for Strategy ( k );</td>
</tr>
<tr>
<td>( \pi_{SC,d}^{(k)} )</td>
<td>Total profit in the decentralized channel for Strategy ( k );</td>
</tr>
<tr>
<td>( \pi_{R}^{(k)} )</td>
<td>Retailer’s profit in the decentralized channel for Strategy ( k );</td>
</tr>
<tr>
<td>( \pi_{M}^{(k)} )</td>
<td>Manufacturer’s profit in the decentralized channel for Strategy ( k ).</td>
</tr>
</tbody>
</table>

To model the horizontal consumer heterogeneity, we assume that consumers in each of the two segments are uniformly located on Hotelling’s line that is characterized by a closed interval of product space \([0,1]\). The horizontal distribution of consumers represents taste differences among consumers. For example, the taste attribute for a bag or luggage can be represented by its color. Each consumer in segment \( i = \{L, H\} \) is represented by his/her aesthetic preference of the product, which is denoted as \( y_i \in [0,1] \).

Following Li et al. (2015), when the manufacturer decides to offer a customized product, the customization level of the product is represented by \( x \in (0,1) \), which measures the fraction of aesthetic attributes of the product that the manufacturer chooses to customize. A standard product is represented by the left-end point, i.e., \( x = 0 \), whereas a fully customized product is represented at the right-end point, i.e., \( x = 1 \). Suppose a standard product is sold to consumers with price \( p \) and quality \( q \). The utility derived by a consumer in segment \( i \) located at \( y_i \in [0,1] \) is \( u_i = \theta_i q - t_i y_i - p \). The linearity in the consumer utility function considered in this paper is consistent with the spirit of the
multi-attribute utility models widely employed in the marketing literature (see e.g. Kotler and Keller 2012). A similar approach is also used in conjoint analysis, which is one of the widely used marketing research techniques in real-world applications (Cattin and Wittink 1982).

A customized product with customization level \( x \) can satisfy the preference for consumers in segment \( i \) located at \( y_i \in [0, x] \). However, consumers in segment \( i \) located at \( y_i \in (x, 1] \) incurs a mismatch cost \( t_i \) per unit deviation between their preference and that of the purchased product. We assume that consumers in the high-valuation segment are more sensitive to this deviation than consumers in the low-valuation segment, i.e. \( t_H > t_L > 0 \). This assumption, which is also used in Desai (2001), reflects the situation where consumers in the high valuation segment also have stronger taste preferences.

In the product line extension strategies we consider, the manufacturer may offer a customized product with low quality or high quality alongside the standard product. For notational convenience, we define \( x_i, i = \{L, H\} \), as the customization level targeted at segment \( i \). Following the standard assumption in the product line design literature (see e.g. Moorthy 1984; and Desai 2001), the manufacturer’s unit production cost is a quadratic function of product quality, \( q \), and equal to \( aq^2 \). This assumption can be justified in many real settings where the cost of providing a unit of quality increases at an increasing rate as the level of quality increases, which can be due to increased production complexity that requires higher costs of materials and man-hours.

To capture the fact that offering a higher customization level will imply higher production complexity and therefore require higher flexibility, we follow Li et al. (2015) and assume that the manufacturer incurs a fixed flexibility cost, \( bx^2 \), when a customized product with customization level \( x \) is offered in the product line. For example, a customized-rug manufacturer needs to invest in a new production system or reconfigure the existing one that can produce the rugs with different patterns and/or color. The quadratic form of flexibility cost reflects the situation where it becomes increasingly difficult for a manufacturer to extend its customization ability, which may require a larger amount of machine and labor resources.

We consider situations where the unit production cost is primarily driven by product quality, i.e., there is no difference in the unit production cost between the standard product and the customized product with the same quality level. However, as the fulfillment of the customized product may require additional order processing, we assume that there is an additional marginal cost \( c \) for the customized product. This additional cost may represent an aggregate measure of costs for processing online customer orders and for shipping finished customized products to customers.

Suppose a customized product is sold to consumers with price \( p \), quality \( q \), and customization level \( x \). The utility derived by a consumer in segment \( i \) located at \( y_i \in [0, x] \) is \( u_i = \theta_i q - p \). The utility derived by a consumer in segment \( i \) located at \( y_i \in (x, 1] \) is \( u_i = \theta_i q - t_i(y_i - x) - p \). A consumer will make a purchase as long as her utility is non-negative. We use index \( j = S, C \), to differentiate the standard product(s) from the customized product.

### 3.2 The First Comparison Scenario – horizontal vs. vertical product line extension

We present the following three product line strategies:
Strategy 1: offering one standard product (with low quality) targeted at both segments;
Strategy 2: offering one standard product and one customized product with the same quality;
Strategy 3: offering two standard products with two different quality levels, each targeted at each of the two segments.

Baseline strategy (Strategy 1 - one standard product)
As described in the standard literature on product line design (see e.g. Moorthy and Png 1992), the manufacturer basically offers a low-quality standard product targeted at the two segments with quality level $q_L$ and price $p_{LS}$. Consequently, we use index $L$ in this strategy. The manufacturer thus solves the following optimization problem:

$$\max_{p_{LS}, q_L} \pi_{SC,c}^{(1)} = (p_{LS} - aq_L^2)(\alpha_L D_{LS} + \alpha_H D_{HS})$$

In (1), the term in the first parentheses on the right hand side denotes the manufacturer’s marginal profit from selling the standard product with low-quality level. The term in the second parentheses represents the total demand of the standard product in both the high-valuation and low-valuation segments. Note that we use $SC$ in the subscript of the profit since we are particularly interested in the channel or supply chain profit in both the centralized and decentralized channels. Figure 2 below depicts the consumers’ utility in the two segments.

![Figure 2: Consumer’s utility and demand in Strategy 1](image)

Demands in the low-valuation and high-valuation segments can be expressed as $\alpha_L D_{LS}$ where $D_{LS} = \min \left( \frac{\theta_L q_L - p_{LS}}{t_L}, 1 \right)$, and $\alpha_H D_{HS}$ where $D_{HS} = \min \left( \frac{\theta_H q_L - p_{LS}}{t_H}, 1 \right)$, respectively. Note that in this paper, we focus on the situation where the whole market is covered for all product line strategies, i.e., $D_{LS} = \frac{\theta_L q_L - p_{LS}}{t_L} = 1$ and $D_{HS} = \frac{\theta_H q_L - p_{LS}}{t_H} = 1$. With a full market coverage, the manufacturer sets the price such that $\frac{\theta_L q_L - p_{LS}}{t_L} = 1 \iff p_{LS} = \theta_L q_L - t_L$. The problem in (1) is solved by substituting $p_{LS}$ with $\theta_L q_L - t_L$ and by setting $D_{LS} = D_{HS} = 1$. The reader is referred to Table 3 for the solutions to the all strategies in the first comparison scenario to Appendix A for the complete derivation of the solutions.
Table 3: The optimal solutions for all strategies under the centralized channel in the first comparison scenarios

<table>
<thead>
<tr>
<th>Strategies</th>
<th>Optimal solutions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Strategy 1</td>
<td>$q^*_L = \frac{\theta_L}{2a}$</td>
</tr>
<tr>
<td>Strategy 2</td>
<td>$q^*_L = \frac{\theta_L}{2a}$</td>
</tr>
<tr>
<td></td>
<td>$x^*_L = \frac{t_L}{2b}$</td>
</tr>
<tr>
<td>Strategy 3</td>
<td>$q^*_L = \frac{\theta_L}{2a} - \frac{\alpha_H(\theta_H - \theta_L)}{2a \alpha_L}$, where $\frac{\theta_L}{\theta_H} &gt; \alpha_H$</td>
</tr>
<tr>
<td></td>
<td>$q^*_H = \frac{\theta_H}{2a}$</td>
</tr>
</tbody>
</table>

Horizontal extension strategy (Strategy 2 - one standard product and one customized product)

Under this strategy, the manufacturer extends the product line in the horizontal direction. The product line now consists of the standard and customized products of the same quality, and both products are offered to the two segments. The price of the customized product is denoted as $p_{LC}$. Introducing the customized product alongside the standard product will certainly create competition between the two products, i.e. demands from the two segments will now be distributed over the two products. This will give the manufacturer a possibility to raise the price of the standard product and gain additional revenues from a portion of consumers who will be more interested in buying the customized product offered at a price that is higher than the price of the standard product. In other words, offering a customized product will give the manufacturer the possibility to sell the standard product at a higher price than the price in Strategy 1 while the full market is still covered. In the derivation of the solution to this strategy, we show that the manufacturer will be better off by increasing the price of the standard product so that it is higher than the price in the baseline strategy (see Appendix A).

The manufacturer solves the following optimization problem:

$$
\max_{p_{LS}, q_L, x_L, p_{LC}} \pi^{(2)}_{SC} = (p_{LS} - a q_L^2)(\alpha_L D_{LS} + \alpha_H D_{HS}) + (p_{LC} - a q_L^2 - c)(\alpha_L D_{LC} + \alpha_H D_{HC}) - b x_L^2 \tag{2}
$$

In (2), the first term denotes the manufacturer’s profit from selling the standard product with low quality to both segments. The second term represents the profit from selling the customized product with low-quality level to both segments, and the last term is the fixed investment cost that the manufacturer incurs for offering the customization. With full market coverage, the manufacturer needs to determine the customization level, $x_L > 0$, in such a way that the consumer’s utility in the low-valuation segment located at $y_L = 1$ is equal to zero when buying the customized product. Since the whole market is covered, the manufacturer sets the price for the customized product equal to $p_{LC} = \theta_L q_L - (1 - x_L) t_L$. We denote $y_L^0 = \frac{p_{LC} - p_{LS}}{t_L}$ as the location of the consumer who is indifferent between buying the standard product and buying the customized product in the low valuation segment. The demands from the low-valuation segment can be expressed as $D_{LS} = y_L^0 = \frac{p_{LC} - p_{LS}}{t_L}$ for the standard product, and $D_{LC} = 1 - y_L^0$ for the customized product. Similarly, the demands from the
The high-valuation segment can be expressed as $D_{HS} = \frac{p_{LS} - p_{LS}}{t_H}$ and $D_{HC} = 1 - \frac{p_{LS}}{t_H}$. The problem in (2) reduces to determining the optimal $q_L$, $p_{LS}$ and $x_L$.

**Vertical extension strategy (Strategy 3 - two standard products)**

Under this strategy, the manufacturer offers two standard products, each targeted at each segment. In other words, this strategy represents a product line extension in a vertical direction. The two quality levels are $q_L$ and $q_H$ ($q_H > q_L$), and the corresponding prices are $p_{LS}$ and $p_{HS}$. The manufacturer’s problem can be written as follow:

$$\max \pi_{SC,c}^{(3)} = (p_{LS} - aq_L^2)\alpha_L D_{LS} + (p_{HS} - aq_H^2)\alpha_H D_{HS},$$

Subject to

$$\theta_L q_L - p_{LS} \geq \theta_L q_H - p_{HS} \quad (3.1)$$

$$\theta_H q_H - p_{HS} \geq \theta_H q_L - p_{LS} \quad (3.2)$$

In (3), the first term represents the profit from selling the standard product with low quality level to the low-valuation segment, and the second term denotes the profit from selling the standard product with high quality level to the high-valuation segment. A consumer may find a product designed for another segment more attractive than the product designed for her own segment, and so the firm faces a cannibalization problem. To prevent this cannibalization, the two self-selection constraints (3.1) and (3.2) must be included. Following the standard product line design model, the optimal solution is obtained by making (3.2) binding, which gives $p_{HS} = p_{LS} + \theta_H (q_H - q_L)$. It is optimal for the manufacturer to set the price of the high-quality product such that consumers in the high-valuation segment are indifferent between buying the high-quality product and buying the low-quality product. This price is lower than the maximum price that the manufacturer could have charged to cover the whole market in the high-valuation segment ($\theta_H q_H - t_H$). As in Strategy 1, the price of the low-quality product is $p_{LS} = \theta_L q_L - t_L$, and $D_{LS} = D_{HS} = 1$.

**Discussion**

We can now make a comparison between the two extension strategies, and examine if the horizontal product line extension strategy can be used as an alternative to the vertical product line extension strategy, and if so, which conditions are necessary. It can be shown that the difference in profit between the two extension strategies can be negative or positive, which suggests that none of the two extension strategies dominate.

There may be several ways to represent the condition for the preference of one extension strategy over the other. We choose to focus on the condition based on the flexibility cost $b$ and summarize the main finding in Proposition 1.
Proposition 1: In a centralized channel, the horizontal product line extension strategy with customization is preferred to the vertical product line extension strategy if \( b < b_{\text{max}} \), where

\[
b_{\text{max}} = \frac{t_L^2}{4c - c^2(t_L + t_H) + \frac{\alpha_H(t_H - \alpha_L)^2}{\alpha_L}}
\]

(Proofs for all propositions are provided in Appendix B).

The optimal results show that the optimal quality level offered in the baseline strategy is not affected when the manufacturer extends the product line horizontally by offering the customized product. This quality level also represents the efficient quality level for the low-valuation segment obtained in the market with perfect information in which there is no risk of cannibalization (Moorthy and Png 1992, and Netessine and Taylor 2007). When the manufacturer extends the product line horizontally through the offering of the customized product, it can generate higher revenues from consumers who purchase the product that matches their preference. A major factor that affects how this extension strategy performs is the cost of providing flexibility to accommodate customization. In Proposition 1, we show that when this cost is not too costly, the manufacturer may consider the horizontal extension strategy as a better option than the vertical extension strategy.

Under the vertical extension strategy, while the high-valuation segment gets the efficient quality level, the low-valuation segment now gets the quality that is lower than the efficient quality level. This downward quality distortion occurs due to the existence of potential cannibalization of the high-quality product by the low-quality product. The vertical extension strategy is appealing for the manufacturer as it may generate higher profits by serving consumers in each segment with a product targeted to them. However, although the vertical extension always improves the baseline profit, the manufacturer must be careful with the presence of cannibalization. As the size of the high-valuation segment increases, the cannibalization problem becomes more severe.

The car industry is an example where we have seen an increasing number of classes offered in a specific product line, highlighting the fact that the focus has been on extending quality differentiation. The applications of customizing the taste attributes are still quite limited, even though few brands such as Mini have started (Williams 2019). The lack of customization applications in this industry may be inhibited by costly investments to accommodate customization whilst most consumers still put more weight on the quality rather than taste attributes.

Common intuition might suggest that vertical extension would be necessary to keep up with competition. However, our result in Proposition 1 indicates that extending the product line horizontally by offering customization may turn out to be a more effective strategy. In some other industries like apparel and clothing, we witness an increasing number of firms emphasizing customization in their product line strategies. With the help of 3D scanning technologies, firms such as MTailor and Redthread offer their consumers individualized products tailored to fit their bodies. Traditionally, the technologies have been expensive and difficult to roll out at scale. However, as reported by McKinsey (Gandhi et al. 2013), the technologies have become more mature. Companies like Styku have sold a large number of 3D measurement terminals with total hardware costs under $3000. Those examples give affirmation to our finding on the important role played by the flexibility cost in making the horizontal extension with customization as the preferred strategy.
This finding complements the existing results in the literature that predominantly focuses solely on either the vertical extension (quality-based segmentation) or the horizontal extension. This study can inform marketing managers about the importance of considering the two dimensions when they plan to extend the existing product line. Before making a decision regarding the number of products and the corresponding quality levels (in a vertical extension) or regarding the number of products and the level of customization (in a horizontal extension), this study shows the importance of questioning whether the extension should be made horizontally or vertically. Focusing solely on one extension direction may lead to a sub-optimal product line design. Moreover, we provide a qualitative guideline as to when the horizontal or vertical extension is preferred. These, in our belief, would increase the likelihood of a product line extension’s success.

3.3 The Second Comparison Scenario – low quality vs high quality in customization

The focus in the second comparison is on the effect of adding a customized product to the existing product line that consists of two standard products of different quality. We intend to assess the role played by the horizontal product line extension offering the customized product when firms already apply a quality-based segmentation. We compare two strategies that differ in whether the customized product is targeted to the high or low valuation segment. In this scenario, we use the standard product line consisting of two products (Strategy 3) as the baseline strategy.

Customization with low quality (Strategy 4a - two standard products and one customized product with low quality)

In this extension strategy, the manufacturer offers a customized product targeted to the low-valuation segment in addition to the existing two standard products. In order to make quality-based segmentation meaningful, the manufacturer needs to ensure that consumers in the high-valuation segment prefer buying the high quality product rather than buying the other two products targeted at the low-valuation segment. The manufacturer solves the following problem:

\[
\max_{\pi_{SC}^{(4a)}, x_L, p_{LS}, q_{LS}, q_H, p_{HS}} \pi_{SC}^{(4a)} = (p_{LS} - a q_L^2) \alpha_L D_{LS} + (p_{LC} - a q_L^2 - c) \alpha_L D_{LC} + (p_{HS} - a q_H^2) \alpha_H D_{HS} - bx_L^2
\]

Subject to:

\[\theta_L q_L - p_{LS} \geq \theta_L q_H - p_{HS}\]  
\[\theta_H q_H - p_{HS} \geq \theta_H q_L - p_{LS}\]  
\[\theta_H q_H - p_{HS} \geq \theta_H q_L + t_H x_L - p_{LC}\]

In the problem defined in (4), the first and second terms represent the manufacturer’s profit from selling the standard and customized products, respectively, to the low-valuation segment. The third term is the profit from selling the standard product with high quality to the high-valuation segment, and the last term denotes the fixed investment cost for offering the customization. There are three self-selection constraints in the above problem formulation. The first two constraints are necessary to prevent cannibalization between the two standard products. The third self-selection constraint is
required to prevent consumers in the high-valuation segment from buying the customized product. Constraint (4.3) will give the first upper bound on \( p_{HS} \), \( \bar{p}_{HS1} = p_{LC} + \theta_H(q_H - q_L) - t_H x_L \), and constraint (4.2) will give the second upper bound \( \bar{p}_{HS2} = p_{LS} + \theta_H(q_H - q_L) \). Since \( \bar{p}_{HS1} < \bar{p}_{HS2} \), we make constraint (4.3) binding. As in the horizontal extension in the first comparison scenario, the price of the customized product is \( p_{LC} = \theta_L q_L - (1 - x_L) t_L \). With full market coverage, we have \( D_{LS} = \frac{p_{LC} - p_{LS}}{t_L} \), \( D_{LC} = 1 - \frac{p_{LC} - p_{LS}}{t_L} \), and \( D_{HS} = 1 \). In Table 4, we present the optimal solutions to the two strategies (see Appendix A for the complete derivation of the solutions).

Table 4: The optimal solutions for all strategies under the centralized channel in the second comparison scenario

<table>
<thead>
<tr>
<th>Strategies</th>
<th>Optimal solutions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Strategy 4a</td>
<td>( q_L^* = \frac{\theta_L}{2a} - \frac{a_H(\theta_H - \theta_L)}{2a a_L} ), where ( \frac{\theta_L}{\theta_H} &gt; \alpha_H ) ( q_H^* = \frac{\theta_H}{2a} )</td>
</tr>
<tr>
<td></td>
<td>( x_L^* = \frac{t_L}{2b} - \frac{a_H \theta_H}{2b} ), where ( \frac{t_L}{t_H} &gt; \alpha_H )</td>
</tr>
<tr>
<td>Strategy 4b</td>
<td>( q_H^* = \frac{\theta_H}{2a} - \frac{a_H(\theta_H - \theta_L)}{2a a_L} ), where ( \frac{\theta_L}{\theta_H} &gt; \alpha_H ) ( x_H^* = \frac{a_H(c + t_H t_H)}{2(b + a_H t_H)} )</td>
</tr>
</tbody>
</table>

**Customization with high quality (Strategy 4b - two standard products and one customized product with high quality)**

In this strategy, the customized product is offered with high quality and targeted to the high-valuation segment. The customization level and price of the customized product are denoted by \( x_H \) and \( p_{HC} \), respectively. The manufacturer’s problem can be written as:

\[
\max_{q_{LS} \in \{H, L\}, p_{HS}, p_{HC}} \pi^{(4b)}_{M,C} = (p_{LS} - a q_L^2) \alpha_L D_{LS} + (p_{HS} - a q_H^2) \alpha_H D_{HS} + (p_{HC} - a q_H^2 - c) D_{HC} - b x_H^2
\]  

Subject to:

\[
\theta_L q_L - p_{LS} \geq \theta_L q_H - p_{HS} \tag{5.1}
\]
\[
\theta_H q_H - p_{HS} \geq \theta_H q_L - p_{LS} \tag{5.2}
\]
\[
\theta_L q_L - p_{LS} \geq \theta_L q_H + t_L x_H - p_{HC} \tag{5.3}
\]

In this strategy, the manufacturer needs to maintain the price of the standard (high-quality) product based on the self-selection constraint (5.2) to prevent the consumers in the high-valuation segment from switching to the low-quality product. This is in contrast to the counter-part strategy (Strategy 4a: customization with low quality) where it is possible for the manufacturer to increase the price of the standard (low-quality) product,
By making constraint (5.2) binding, the price of the high-quality product is \( p_{HS} = p_{LS} + \theta_H (q_H - q_L) \), while the price of the low quality product is \( p_{LS} = \theta_L q_L - t_L \). When the manufacturer sets the price for the customized product equal to \( p_{HC} = p_{HS} + t_H x_H \), the consumers in the high-valuation segment located at \( y_H \in [0, x_H] \) will buy the standard product, whereas the consumers located at \( y_H \in [x_H, 1] \) will be indifferent between buying the standard product and the customized one, i.e., \( D_{HS} = x_H \) and \( D_{HC} = 1 - x_H \). It can be shown that with the above price for the customized product, constraint (5.3) will always be satisfied (see Appendix C).

**Discussion**

The results show that offering the customized product to the low-valuation segment has no effect on the optimal quality of the existing standard products. We also note that the optimal customization level in Strategy 4a is lower than the customization level offered in Strategy 2, which shows the existence of distortion in both the quality and customization levels. The additional distortion in the customization level occurs because of the manufacturer’s action to prevent cannibalization of the high-quality standard product by the customized product. The extent of distortion in customization level is increasing in the difference of the mismatch costs of consumers in both segments.

A notable advantage of offering a customization option for the low-valuation segment (moving from Strategy 3 to Strategy 4a) can be drawn from what we have observed in the horizontal extension strategy in the first comparison scenario. That is, the manufacturer has the possibility to increase the profit from the low-valuation segment by introducing the customized product, and this possibility can be justified when the fixed flexibility cost is not too high. However, there is also a detrimental effect due to the reduction in the price of the high-quality product to prevent cannibalization by the customized product. Thus, there is no guarantee that the manufacturer is better off by offering the customized product to the existing quality-based segmentation strategy. In the following proposition, we define the threshold value of the flexibility cost.

**Proposition 2:** In a centralized channel, offering a customized product with low quality to the existing quality-based segmentation strategy will increase the profit if \( b < \frac{\alpha_H t_L (\alpha_H t_H - 2t_L) + t^2_L}{\alpha_L (4t_L - c)} \).

When we compare Strategy 3 and Strategy 4b, we find no difference in the profit obtained from the low-quality product offered to the low-valuation segment. Hence, the difference comes only from the profit generated in the high-valuation segment. Since the prices of the standard products are not affected, the difference is then solely due to the profit generated from a fraction of consumers in the high-valuation segment who switch from buying the standard product in the baseline strategy to buying the customized product in Strategy 4b. As in Strategy 4a, the preference for Strategy 4b over Strategy 3 cannot be guaranteed, and depends on factors such as the flexibility cost associated with the customized product.

**Proposition 3:** In a centralized channel, offering a customized product with high quality to the existing quality-based segmentation strategy will increase the profit if \( b < \frac{\alpha_H (c - t_H)^2}{4c} \).
In Figure 3, we illustrate how the profits in Strategy 4a and Strategy 4b are influenced by the fixed flexibility cost using a numerical example with the following parameter values: $\alpha_H = 0.2, \alpha_L = 0.8, \theta_H = 5.0, \theta_L = 4.0, t_H = 1.5, t_L = 0.8, c = 0.2, a = 1$. The profit in the baseline strategy is also included for comparison purposes. In this numerical example, we show how the flexibility cost affects the preference of each of the three strategies. In the range with low flexibility cost values, there is an incentive to offer a customized product with low quality. As the flexibility cost increases, the preference shifts to offering a customized product with high quality, and this lasts until a point where the flexibility cost is too costly such that extending the product line is no longer appealing.

Examples of firms that offer customization for their higher-end segments can be found in the luxury-goods industry. As today’s luxury-goods consumers are heterogeneous, it is essential to target customization options to the right segment. Bain & Company identifies seven main segments of luxury-goods consumers based on their attitudes and spending (D’Arpizio 2014). Personalization or pure customization, in particular, has gained increased significance to luxury-goods consumers who are increasingly demanding in their requests and put emphasis on uniqueness and exclusivity through high quality services. One of the most iconic brands, Hermès, has introduced Hermès Le Sur-Mesure, a concept where consumers can place requests for extraordinary made-to-measure objects and interiors (e.g. furniture, gloves, and saddles) tailored to their individual taste and needs resulting in one-off bespoke designs. With such bespoke offerings, made exclusively in its Paris-based workshop, the flexibility cost must be quite high and the firm certainly targets consumers in the high-end segment who are willing to pay premium for such highly personalized products. Another example is Burberry that launched its Burberry Bespoke offering fragrances which can be personalized to match individual needs.

However, it is important to note that when the flexibility cost is sufficiently low, our finding points to the attractiveness of customization for the low-end products, as showcased by the following examples. Timbuk2 sells customized messenger bags and backpacks, but only for a limited set of their standard or classic styles. The higher-end styles are sold without customization. Similarly,
custom watches offered by Swatch or Fossil are mainly targeted to their lower end segments as consumers can only customize the more standard designs. The more deluxe designs targeted at the high-end segment are offered without customization.

The above results offer new insights into the effectiveness of a customization strategy in product line extension. Most studies in the existing literature on customization (see e.g. Li et al 2015) considers only horizontal consumer heterogeneity, and hence, they fail to consider further potential cannibalization that arises between the high-quality standard product and the customized product, which appears to be essential in this paper. In a similar vein, our study also complements the well-established literature focusing only on vertical differentiation (see e.g. Moorthy and Png; Netessine and Taylor 2007) by demonstrating the possible occurrence of customization distortion aside from the well-known quality distortion. This study could inform marketing managers about the potential of pursuing the customization strategy to their existing product line as an attempt to deal with consumer heterogeneity in both the vertical and horizontal dimensions. More importantly, we show the importance of choosing the right segment to target in offering the customized product.

4. Comparison of Product Line Extension Strategies in a Decentralized Channel

We next present the models and results of the two comparison scenarios in a decentralized channel. In a decentralized channel, the manufacturer sells the standard product(s) through an independent retailer. In the case where a customized product is also offered, the customized product is sold through a direct (online) channel owned by the manufacturer. We adopt a game theoretic approach to capture the strategic interaction between the manufacturer and the retailer. The general sequence of decisions is as follows:

i) The manufacturer decides on the quality level(s) of the standard product(s) and the corresponding wholesale prices charged to the retailer. In the case where a customized product is offered, the manufacturer also decides on the customization level and the price of the customized product.

ii) Given the quality level(s) and wholesale price(s) offered by the manufacturer, the retailer decides on the retail price(s) of the standard product(s). We consider the scenario in which the manufacturer is powerful so that she can make a take-it-or-leave-it offer to the retailer. This means that the retailer will carry the standard product(s) offered by the manufacturer. This scenario is also used in e.g. Liu and Cui (2010) and Wong and Lesmono (2019).

4.1 The First Comparison Scenario – horizontal vs. vertical product line extension

Baseline strategy (Strategy 1 - one standard product)

For given \( q_L \) and \( w_L \) offered by the manufacturer, the retailer solves the following problem:

\[
\max_{p_{LS}} \pi_R^{(1)} = (p_{LS} - w_L)(\alpha_L D_{LS} + \alpha_H D_{HS})
\]
As in the centralized channel, with a full market coverage, the price set by the retailer is determined by the equation $D_{LS} = \frac{\theta_L q_L - p_{LS}}{t_L} = 1$, which gives $p_{LS} = \theta_L q_L - t_L$. It can be shown that the retailer’s optimal pricing decision when the demands are represented as $\frac{\theta_L q_L - p_{LS}}{t_L}$ and $\frac{\theta_H q_L - p_{LS}}{t_H}$ is $p_{LS}^* = \frac{w_L}{2} + \gamma_1 q_L$, where $\gamma_1 = \frac{\theta_L + \theta_H}{t_L + t_H}$. Using backward induction, as the manufacturer knows the optimal response of the retailer, it sets the wholesale price equal to $w_L = (2\theta_L - \gamma_1) q_L - 2t_L$. The fact that the wholesale price set by the manufacturer is lower than the retail price shows the existence of double marginalization in the decentralized channel. The manufacturer sets the wholesale price lower than the retail price to ensure that the retailer is willing to cover the whole market as intended by the manufacturer. The manufacturer solves the following problem:

$$\max_{q_L} \pi^{(1)}_{M,d} = ((2\theta_L - \gamma_1) q_L - 2t_L - a q_L^2) (\alpha_L D_{LS} + \alpha_H D_{HS})$$  \hspace{1cm} (7)$$

The optimal solutions for the two strategies in the decentralized channel are presented in Table 5.

<table>
<thead>
<tr>
<th>Strategies</th>
<th>Optimal solutions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Strategy 1</td>
<td>$q_L^* = \frac{2\theta_L - \gamma}{2a}$, where $\gamma = \frac{\alpha_L \theta_L + \alpha_H \theta_H}{t_L + t_H}$, $2\theta_L &gt; \gamma$.</td>
</tr>
<tr>
<td>Strategy 2</td>
<td>$q_L^* = \frac{\theta_L}{2a}$, $x_L^* = \frac{t_L}{2b}$</td>
</tr>
<tr>
<td>Strategy 3</td>
<td>$q_L^* = \frac{\theta_L - a_H (\theta_H - \theta_L)}{a \alpha_L}$, $q_H^* = \frac{\theta_H}{2a}$</td>
</tr>
</tbody>
</table>

**Horizontal extension strategy (Strategy 2 - one standard product and one customized product)**

In this strategy, the retailer must determine the retail price of the standard product, given its quality and wholesale price, and the price of the customized product sold directly by the manufacturer. As in the centralized channel, $D_{LS} = \frac{p_{LC} - p_{LS}}{t_L}$ and $D_{HS} = \frac{p_{LC} - p_{LS}}{t_H}$. The retailer’s problem is:

$$\max_{p_{LS}} \pi^{(2)}_R = (p_{LS} - w_L) \left( \alpha_L \frac{p_{LC} - p_{LS}}{t_L} + \alpha_H \frac{p_{LC} - p_{LS}}{t_H} \right)$$  \hspace{1cm} (8)$$

It can be shown that the retailer’s optimal pricing decision is $p_{LS}^* = \frac{w_L + p_{LC}}{2}$. The manufacturer solves:
By substituting $p_{LC} = \theta_L q_L - (1 - x_L)t_L$, and $p_{LS}^* = \frac{w_L + p_{LC}}{2}$, the above problem reduces to determining the optimal $q_L, x_L$, and $w_L$.

**Vertical extension strategy (Strategy 3 - two standard products)**

Under this strategy, the retailer solves

$$
\max_{\pi_{M,d}} \pi_{M,d}^{(2)} = (w_L - aq_L^2)\left(\frac{p_{LC} - p_{LS}}{t_L} + \alpha_H \frac{p_{LC} - p_{LS}}{t_H}\right) + (p_{LC} - aq_L^2 - c)\left(\alpha_L \left(1 - \frac{p_{LC} - p_{LS}}{t_L}\right)\right) - bx_L^2
$$

(9)

In the case of full market coverage, the retail price of the low-quality product is set to $p_{LS} = \theta_L q_L - t_L$. To prevent cannibalization, the retail price of the high-quality product is set to $p_{HS} = p_{LS} + \theta_H (q_H - q_L)$. It can be shown that the retailer’s optimal pricing decision when the demands are represented as $\theta_L q_L - p_{LS}$ and $\theta_L q_H - p_{HS}$ is $p_{LS}^* = \frac{\theta_L q_L + w_L}{2}$ and $p_{HS}^* = \frac{\theta_H q_H + w_H}{2}$. Using backward induction, the manufacturer needs to set the wholesale prices equal to $w_L = 2p_{LS} - \theta_L q_L$ and $w_H = 2p_{HS} - \theta_H q_H$. These two equations on the wholesale price will motivate the retailer to cover the whole market.
retailer to increase the retail price of the standard product and sell it to a fraction of the market. This finding is interesting because it runs counter to the established belief that quality distortion becomes more severe in the decentralized channel (see e.g. Villas-Boas 1998; Wong and Lesmono 2019).

Companies like Zara and Gap in the apparel industry that design and sell their own products, i.e., they are operating in a centralized channel, may benefit from the absence of potential quality distortion. In contrast, The North Face and Columbia that predominately distributes its product indirectly through retail channels cannot avoid the quality distortion. However, our study suggests that these companies can reduce the extent of quality distortion by offering customized products in their product lines.

The optimal results in Strategy 3 show that the quality of the low-end product is lower than the quality level in Strategy 2, and also lower compared to the quality level in the centralized channel under the same strategy. In line with the standard results in product line design literature, the optimal quality of the high-end product is not affected and the high-valuation segment always get the efficient quality level.

We next examine the preference for Strategy 2 over Strategy 3, or vice versa. The following proposition summarizes the condition for the preference for Strategy 2 over Strategy 3.

**Proposition 4**: In a decentralized channel, the horizontal product line extension strategy with customization is preferred to the vertical product line extension strategy if

\[ b < b_{2,3}^{\text{Decentralized}} = \frac{t_L^2}{4c - 2c^2 \left( \frac{\theta_H}{t_L}, \frac{\theta_H}{t_H} \right) - \frac{\theta_H(\theta_H - 2\theta_L)}{a} + \frac{2\theta_L \theta_H(\theta_H - \theta_L)}{a}}. \]

Similar to the centralized channel case, in Proposition 4, we show how the preference of the horizontal product line extension is affected by the flexibility cost to accommodate customization. By comparing the threshold values of the flexibility cost in the centralized and decentralized channels, it can be shown that in a certain region, \( b_{2,3}^{\text{Decentralized}} > b_{2,3}^{\text{Centralized}} \). This means that there is a more stringent requirement regarding the flexibility cost in the centralized channel than in the decentralized channel. This also shows that channel structure is influential as the manufacturer has a stronger motivation to opt for the horizontal extension in the decentralized channel.

We provide Figure 4 below to illustrate how the threshold values of the flexibility cost \( b \) in the two channel structures are affected by \( c \) and \( \alpha_L \). In this numerical example, the parameter values are: \( b = 0.5, \theta_H = 5, \theta_L = 4, t_H = 1.0, t_L = 0.8, a = 1 \).

Figure 4 shows that both threshold values increase in the size of the low valuation segment, but the rate of increase for the centralized channel is higher. The preference of the horizontal extension over the vertical extension in the decentralized channel seems to be strong when the size of the low-valuation segment is small, and gets weaker as the size of the low-valuation segment increases. The figure also shows the effect of the additional marginal cost of selling the customized product, \( c \). With a higher \( c \) value (\( c = 0.8 \) as opposed to \( c = 0.2 \)), the difference in the threshold values of the two channel structures seems to be less significant.
While this result is in line with some previous studies e.g. Li et al. (2015) and Xiao et al (2014), our result provides further information about how the customization strategy performs compared to the vertical extension strategy in a decentralized channel. Such information is impossible to obtain in the setting that only considers consumer heterogeneity in the horizontal dimension.

Our study complements the existing literature by showing the importance of considering two factors, namely direction (vertical vs horizontal) and channel structure (centralized vs decentralized) when choosing the appropriate line extension strategy. Our model provides a better understanding of the interdependence of these two factors. While there are several motivations for manufacturing firms to sell their products through retailers (e.g. shortening lead times and relying on retailers reputation), our study demonstrates some downsides of this distribution structure when viewed from a product line design perspective.

![Figure 4: Impact of \(c\) and \(\alpha_L\) on threshold values of \(b\) in the centralized and decentralized channels](image)

4.2 The Second Comparison Scenario – low quality vs high quality in customization

We first derive the optimal results for Strategy 4a and Strategy 4b under a decentralized channel.

**Customization with low quality (Strategy 4a - two standard products and one customized product with low quality)**

Based on the demands derived in the centralized channel, in this strategy, the retailer solves

\[
\max_{p_{LS},p_{HS}} \pi_R^{(4a)} = (p_{LS} - w_L)\alpha_L(D_{LS}) + (p_{HS} - w_H)\alpha_H(D_{HS})
\]

Subject to:

\[
\theta_L q_L - p_{LS} \geq \theta_L q_H - p_{HS} \tag{12.1}
\]

\[
\theta_H q_H - p_{HS} \geq \theta_H q_L - p_{LS} \tag{12.2}
\]

\[
\theta_H q_H - p_{HS} \geq \theta_L q_L + \tau_H x_L - p_{LS} \tag{12.3}
\]
The retailer’s optimal prices are \( p_{LS}^* = \frac{w_L + p_{LC}}{2} \) and \( p_{HS}^* = \frac{\theta_H q_H + w_H}{2} \) (see Appendix D). As in the centralized channel, we have \( p_{LC} = \theta_L q_L - (1 - x_L) t_L \), and \( p_{HS} = p_{LC} + \theta_H (q_H - q_L) - t_H x_L \). 

\[ D_{LS} = \frac{p_{LC} - p_{LS}}{t_L} \] 

and \( D_{HS} = 1 \). The manufacturer’s objective function can be written as:

\[
\max \pi^{(Aa)}_M = (w_L - a q_L^2) \alpha_L D_{LS} + (w_H - a q_H^2) \alpha_H D_{HS} + (p_{LC} - a q_L^2 - c) \alpha_L D_{LC} - b x_L^2 
\]

By substituting \( w_H \) and \( p_{LC} \), the above problem reduces to determining the optimal \( q_L, q_H, w_L \) and \( x_L \).

**Customization with high quality (Strategy 4b - two standard products and one customized product with high quality)**

In this strategy, the retailer solves:

\[
\max \pi^{(4b)}_R = (p_{LS} - w_L) \alpha_L D_{LS} + (p_{HS} - w_H) \alpha_H D_{HS},
\]

Subject to:

\[
\begin{align*}
\theta_L q_L - p_{LS} &\geq \theta_L q_H - p_{HS} \quad (14.1) \\
\theta_H q_H - p_{HS} &\geq \theta_H q_L - p_{LS} \quad (14.2) \\
\theta_L q_L - t_L x_H - p_{LS} &\geq \theta_L q_H - p_{HC} \quad (14.3)
\end{align*}
\]

\( D_{LS} = 1 \) and \( D_{HS} = \frac{p_{HC} - p_{HS}}{t_H} \). The retailer’s optimal responses to the manufacturer’s offer are \( p_{LS}^* = \frac{\theta_L q_L + w_L}{2} \) and \( p_{HS}^* = \frac{\theta_H q_H + w_H}{2} \). Like in the centralized channel, \( p_{LS} = \theta_L q_L - t_L \), and by making (14.2) binding, we have \( p_{HS} = p_{LS} + \theta_H (q_H - q_L) \). The manufacturer’s problem is:

\[
\max \pi_M^{(4b)} = (w_L - a q_L^2) \alpha_L D_{LS} + (w_H - a q_H^2) \alpha_H D_{HS} + (p_{HC} - a q_H^2 - c) \alpha_H D_{HC} - b x_H^2
\]

The manufacturer sets the price for the customized product equal to \( p_{HC} = p_{HS} + t_H x_H \). Knowing the retailer’s optimal responses, the manufacturer sets the wholesale prices equal to \( w_H = 2p_{HS} - p_{HC} \) and \( w_L = 2p_{LS} - \theta_L q_L \) (see Appendix D). By substitutions, \( D_{HS} = x_H \) and \( D_{HC} = 1 - x_H \), and \( p_{HS} \) and \( p_{LS} \) the manufacturer’s problem reduces to determining the optimal \( q_L, q_H, x_H \). The solutions for the two strategies are presented in Table 6.
Table 6: The optimal solutions for all strategies under decentralized channel in the second comparison scenario

<table>
<thead>
<tr>
<th>Strategies</th>
<th>Optimal solutions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Strategy 4a</td>
<td>( q_L^* = \frac{\theta_l}{2a} - \frac{a_H(\theta_H-\theta_l)}{2a_L} ), where ( \frac{\theta_l}{\theta_H} &gt; \frac{2a_H}{1+\alpha_H} )</td>
</tr>
<tr>
<td>Strategy 4b</td>
<td>( q_H^* = \frac{\theta_H}{2a} )</td>
</tr>
<tr>
<td></td>
<td>( x_L^* = \frac{t_L}{2b} - \frac{a_H(2\alpha_H-3t_L)}{2a_H} ), where ( (1+3a_H)t_L &gt; t_H )</td>
</tr>
</tbody>
</table>

Discussion

Like in the centralized channel, offering the customized product with low quality in Strategy 4a in the decentralized channel does not affect the optimal quality of the existing standard products in baseline strategy. However, it is interesting to see that offering the customized product with high quality in Strategy 4b in the decentralized channel increases the optimal quality level of the low-quality product compared to the quality level in the baseline strategy. Recall that the quality levels are not affected in the centralized channel. In fact, the optimal quality level of the low-quality product in Strategy 4b in the decentralized channel is the same as the optimal quality level in the centralized channel. As we also observe in the first comparison scenario, offering a customized product can alleviate the quality distortion effect in the decentralized channel.

Like in the centralized channel, offering the customized product to the existing quality-based segmentation strategy provides the opportunity to increase the channel profit. In the following propositions, we derive the preference for Strategy 4a or Strategy 4b over Strategy 3 that depends on the flexibility cost.

**Proposition 5:** In the decentralized channel, offering customized product with low quality to the existing quality-based segmentation strategy will increase the channel profit when

\[
b < \frac{2a_H\alpha_l(t_H-2t_L)-a_L t_L^2}{4(1+3a_H)^2+4a_Ht_L-\frac{a_H \theta_H^2}{4a_L}+a_H \theta_H \theta_L-\frac{a_H \theta_H \theta_L}{4a_L}-(a_L+1)\frac{\theta_L}{2a}}\]

**Proposition 6:** In the decentralized channel, offering the customized product with high quality to the existing quality-based segmentation strategy will increase the channel profit when

\[
b < \frac{a_H^2(c+t_H)(a_L(a_L(c^2+t_H^2)-4a_H\theta_H(\theta_H-\theta_L)^2)+2a_H^2t_H(\theta_H-\theta_L)^2+a_H^2\theta_H^2(c^2+t_H^2)-6a_H^2a_Lt_Hc}{2a_H(4a_aLc-\alpha_H(\theta_H-\theta_L)^2)} - \alpha_H t_H.
\]

We use the numerical example \((\alpha_H = 0.1, \alpha_L = 0.9, \theta_H = 8, \theta_L = 5.5, t_H = 1, t_L = 0.6, c = 0.3, a = 1)\) to illustrate how the profits for the three strategies would change by increasing the flexibility cost \(b\). As we can see in Figure 5, similar to the centralized channel case, the total profit channel for strategies 4a and 4b are decreasing in the flexibility cost and Strategy 4a is more sensitive to the flexibility cost than Strategy 4b.
Our finding that offering a customized product to the existing product line consisting two standard products may alleviate quality distortion in a decentralized channel provides a new insight into the attractiveness of the customization strategy. This is particularly true when the customized product is targeted at the high-valuation segment. For this reason, and the fact that offering a customized product with low quality is quite sensitive to the change in flexibility cost, manufacturing some firms operating in a decentralized channel would find it more attractive to extend their existing product line by adding the customization option for the high-valuation segment. Most examples featured in the previous section that offer customization to the high-end segment are from the luxury-goods industry. It is common for those firms to design and sell their own products through their centralized channels. Based on our finding, firms operating in a decentralized channel such as Land’s End and Levi Strauss, may also consider offering customization targeted at their high-end segment as a way to reduce quality distortion in their standard products. However, this strategy would particularly be justifiable if the customization cost is high enough for which only consumers in the high-end segment are prepared to pay.

5. Conclusions

5.1 Main Contributions

Frequently, marketing managers must make decisions on product line extension in order to enhance their market base and revenues. Although previous studies on product line design provide a solid understanding of the advantages and disadvantages of extending the product line length, they have been conducted under the assumption of consumer heterogeneity in either a vertical or a horizontal dimension. Consequently, a direct application of their findings to the real settings where consumer heterogeneity exists in both the vertical and horizontal dimensions would be limited. An important question marketing managers need to ask in relation to product line extension decisions is if the extension should be performed vertically or horizontally. To answer this question, there is a need to extend the current models by explicitly considering consumer heterogeneity in both dimensions, and...
this is the primary motivation of our study. This extension has resulted in interesting findings that are new to the literature, and helpful for enhancing the understanding of how the operations and marketing related factors have an influence on the effectiveness of a product-line extension strategy.

We have focused on two scenarios for the comparison of extension strategies. In the first scenario, we use a single product strategy as the baseline strategy and examine the preference for the horizontal extension strategy that offers the customized product of the same quality to the vertical extension strategy that offers two products of different quality. In the second scenario, we use the quality-based segmentation strategy as the baseline strategy, and examine the effect of adding the customized product to the existing product line. Furthermore, we examine if the channel structure affects the results in those two comparison scenarios.

Some of the most important results are summarized as follows. When the single-product strategy is used as the baseline strategy, adding a new product with a higher quality, i.e., applying the vertical product line extension strategy, always increases the profit. The effect of adding the customized product, i.e., applying the horizontal extension strategy, depends on several factors. One notable factor is the fixed flexibility cost for accommodating the customization. The horizontal extension strategy may increase the profit if this cost is not too high. This finding highlights the importance of considering the choice of extending the product line vertically or horizontally beyond the decision of product line length. The results in the second comparison scenario show that offering customization in addition to the standard products may increase the channel profit. In line with the results in the first scenario, the fixed cost for accommodating customization plays an important role. A very interesting result is that offering the customized product targeted at the high-valuation segment may alleviate the quality distortion effect, and hence, motivates the manufacturer to increase the quality level of the low-quality product.

Our extended analysis on the effect of channel structure shows interesting results. For the first comparison scenario, even though the result regarding the possible preference for the horizontal or vertical extension strategy is also observed in a decentralized channel, we show that the channel structure is influential. That is, the preference for the horizontal extension strategy is stronger in the decentralized channel than in the centralized channel. In the decentralized channel, the vertical product line extension strategy suffers from channel efficiency loss because of the potential misalignment of the manufacturer’s and the retailer’s coverage strategies. This finding suggests that when manufacturing firms rely on retailers to sell their products, they should have a stronger incentive to offer the customized product to their existing product line. Our study also demonstrates the effect of channel structure on the decision of whether the customized product should be targeted at the low or high valuation segment. We show that offering a customized product with low quality is quite sensitive to the change in flexibility cost, and this is more so in a decentralized channel than in a centralized channel. Consequently, there seems to be a stronger incentive for manufacturing firms operating in a decentralized channel to offer the customization option to the high-valuation segment.

5.2 Managerial implications

With a significantly high failure rate of product line extension (Nielsen 2015), there is a huge business potential in improving product line extension decisions. The results of our study provide marketing
managers with a better understanding of the inter-related factors that influence the effectiveness of product line extension strategies. In particular, our modelling framework that facilitates an examination of consumer heterogeneity in both the vertical and horizontal dimensions should help inspire practitioners to consider in which direction they should make a product line extension before they decide on the length of the product line.

Our results point to the interesting interdependence between a firm’s distribution channel structure and line extension strategy. When a manufacturer operates in a decentralized channel, i.e. sells its products through retailers, the success of the manufacturer’s line extension strategy critically depends on the extent of the misalignment between the manufacturer and retailers’ market targeting strategies. Interestingly, we have shown that offering customized products to the standard products may alleviate the quality distortion effect that is prevalent in a decentralized channel. This finding also provides a new insight into the value of moving away from a traditional distribution channel by adopting, e.g. a dual channel. Finally, a managerial message that encompasses the specific results regarding the customization strategy is that introducing customization in a product line could indeed be beneficial. However, to enhance the effectiveness of this strategy, there is a need for a more proactive approach to integrate the marketing and operations decisions such that enhanced manufacturing flexibility can support the right targeting of the customized product.

5.3 Future Research

The present study has a number of limitations and we consequently suggest several topics for future research. First, our study only considers a full market coverage, and hence, further research is needed in order to see how the results presented in this paper can be generalized to the case with a partial market coverage. Additional results for the case with partial market coverage will give more complete information on the benefits of offering the customized product in the product line. Considering the case with partial market coverage, however, would require an extensive numerical study since the analytical results are hard to obtain in most of the product line strategies studied in this paper. Second, like most of the existing studies in the product line design literature, our model setup does not capture the possible presence of demand uncertainty. In many realistic settings, demand uncertainty cannot be neglected and may have an influence on the optimal product line decisions. In the presence of demand uncertainty, the manufacturer must also determine an optimal production or inventory level in addition to the product quality and customization levels, and there might be interdependence between them. In the case of a decentralized channel, there is an even a more interesting and complex problem because for instance the retailer’s order quantity decision may influence the pricing decision, and vice versa. We further foresee a coordination issue between the manufacturer and the retailer that must be investigated in order to maximize the channel profit. Finally, as the main insights in this paper are generated from stylized models, it would be useful to conduct empirical studies that may reveal, for example, the heterogeneity of consumers’ valuation for quality and their preference for the aesthetic components of the product. For this purpose, it may be necessary to apply marketing research techniques such as conjoint analysis to validate the functional form and parameter values. It would also be interesting to characterize and quantify investments for enhancing flexibility, which we show is crucial in determining the preferred strategy. There would be a need to assess the scope of flexibility that is relevant for specific real operations and marketing environments. For the
strategies offering customization through an online channel, it is essential to look beyond manufacturing flexibility as the fulfilment of customer orders does not only rely on manufacturing but also on order processing and execution (pre-production) and distribution and transportation (post-production). Such empirical studies will be very valuable when considering the development of decision tools for product line managers. Furthermore, conducting multiple case studies and survey research to map the market segmentation of customized products would also represent an important supplement to this paper.

Appendix A

In this part, we provide the derivatives for determining the optimal solutions in all the strategies, for both the centralized and decentralized channels.

**Strategy 1- Centralized channel:**

\[
\frac{\partial \pi^{(1)}_{SC,c}}{\partial q_L^2} = -2a < 0 \text{ implies that } \pi^{(1)}_{SC,c} \text{ is concave in } q_L. \text{ Therefore, solving } \frac{\partial \pi^{(1)}_{SC,c}}{\partial q_L} = \theta_L - 2aq_L = 0 \text{ gives us the optimal quality level of the standard product.}
\]

**Strategy 2- Centralized channel:**

Before providing derivatives for Strategy 2, we show how demand for the standard product will be affected by introducing a customized product. Suppose the manufacturer offers both a standard product and a customized one in the market. The manufacturer’s profit function is as follows:

\[
\pi^{(2)}_{SC} = (p_s - aq^2)\alpha D_s + (p_c - aq^2 - c)\alpha D_c - bx^2
\]  

(A1)

where \( D_s = \frac{p_c - p_s}{t} \) and \( D_c = 1 - \frac{p_c - p_s}{t} \).

Using transformation \( p_s = p_c - D_s t \), the resulting manufacturer’s profit is written as:

\[
\pi^{(2)*}_{SC} = (p_c - D_s t - aq^2)\alpha D_s + (p_c - aq^2 - c)\alpha (1 - D_s) - bx^2
\]  

(A2)

Since \( \frac{\partial^2 \pi^{(2)*}_{SC}}{\partial D_s^2} = -2aT < 0 \), \( \pi^{(2)*}_{SC} \) is concave in \( D_s \). From the first-order condition, the stationary point of \( \pi^{(2)*}_{SC} \) is \( D_s^* = -\frac{p_c - aq^2 - c}{T} < 0 \). Since \( D_s \) should be positive, the feasible solution for \( D_s \) is represented on interval \((0, \frac{2c}{t})\). \( \pi^{(2)*}_{SC} \) is a decreasing function on this interval. Therefore, by decreasing \( D_s, \pi^{(2)*}_{SC} \) will increase. Under the full market coverage, \( p_c \) will be set equal to \( \theta q - (1 - x) T \), and an increase in \( p_s \) leads to a decrease in \( D_s \) which consequently increases \( \pi^{(2)*}_{SC} \). Thus, the manufacturer is better off by decreasing the demand for standard product. Figure A-1 shows the demand for the standard product with and without accommodating a customized product.
Following Wu et al. (2012), since \[ \frac{\partial \pi_{SC,c}^{(2)}}{\partial q_L} = -\frac{2\theta_L^2(a_L t_H + a_H t_L) + 2a_H t_L}{t_H t_L} < 0, \] \[ \frac{\partial \pi_{SC,c}^{(2)}}{\partial q_H} = -\frac{2\theta_H^2(a_L t_H + a_H t_L) + 2a_L t_L}{t_H t_L} < 0, \] \[ \frac{\partial^2 \pi_{SC,c}^{(2,1)}}{\partial p_{LS} \partial x_L} = -\frac{4ab}{\theta_L^2} > 0. \] Thus, from the first-order conditions of \( \pi_{SC,c}^{(2,1)} \) the optimal result can be found. Figure A-2 below is provided to illustrate the consumers’ utility in the two segments.

Strategy 3 - Centralized channel:

The conditions \( \frac{\partial^2 \pi_{SC,c}^{(3)}}{\partial q_L^2} = -2a_L < 0, \) \( \frac{\partial^2 \pi_{SC,c}^{(3)}}{\partial q_H^2} = -2a_H < 0, \) and \( \left( \frac{\partial^2 \pi_{SC,c}^{(3)}}{\partial q_L^2} \right) \left( \frac{\partial^2 \pi_{SC,c}^{(3)}}{\partial q_H^2} \right) - \left( \frac{\partial^2 \pi_{SC,c}^{(3)}}{\partial q_L \partial q_H} \right)^2 = 4a^2a_L a_H > 0 \) denote that \( \pi_{SC,c}^{(3)} \) is jointly concave in \( q_L \) and \( q_H. \) Thus, the stationary point obtained from the first-order conditions \( \frac{\partial \pi_{SC,c}^{(3)}}{\partial q_L} = \alpha_L (\theta_L - 2aq_L) - \alpha_H (\theta_H - \theta_L) = 0 \) and \( \frac{\partial \pi_{SC,c}^{(3)}}{\partial q_H} = \alpha_H (\theta_H - 2aq_H) = 0 \) give us the optimal result. Figure A-3 illustrates the consumers’ utility in Strategy 3.
Strategy 4a Centralized channel:

In this strategy, $p_{LS}$, $q_L$, $q_H$, $x_L$ are decision variables. The second partial derivatives of the profit function w.r.t. the decision variables are as follows:

$$\frac{\partial^2 \pi_{SC,c}^{(4a)}}{\partial p_{LS}^2} = -\frac{2\alpha_L}{t_L} < 0, \quad \frac{\partial^2 \pi_{SC,c}^{(4a)}}{\partial q_L^2} = -\frac{2\alpha_L (\theta_L^H + \alpha t_L)}{t_L} < 0,$$

$$\frac{\partial^2 \pi_{SC,c}^{(4a)}}{\partial q_H^2} = -2a\alpha_H < 0, \quad \frac{\partial^2 \pi_{SC,c}^{(4a)}}{\partial x_L^2} = -2b - 2\alpha_L t_L < 0.$$  

Also, the determinant of the Hessian matrix is

$$\frac{16a^2b^2\alpha_H^2}{t_L} > 0.$$  

Therefore, the stationary point that comes from the first-order conditions is the optimal solution. Figure A-4 below shows the consumers’ utility in Strategy 4a.

Strategy 4b Centralized channel:

The manufacturer’s profit function $\pi_{SC,c}^{(4b)}$ is not concave in $q_L$, $q_H$, and $x_L$. However, $\frac{\partial^2 \pi_{SC,c}^{(4b)}}{\partial q_H^2} = -2a\alpha_H < 0$ implies that, for given values of $q_L$ and $x_L$, the manufacturer’s profit function is concave in $q_H$. Thus, we first solve for $q_H$ for given $q_L$ and $x_L$, then substitute the optimal $q_H$ into the profit function. The resulting expected profit function $\pi_{M,c}^{(4b,1)}$ is jointly concave in $q_L$ and $x_L$, because we
have \( \frac{\partial^2 \pi_{MC}^{(4b,1)}}{\partial q_L^2} = -2a < 0 \), \( \frac{\partial^2 \pi_{MC}^{(4b,1)}}{\partial x_L^2} = -2b - 2a t_H < 0 \), and \( \left( \frac{\partial^2 \pi_{SC,e}^{(4b,1)}}{\partial q_L^2} \right) \left( \frac{\partial^2 \pi_{SC,e}^{(4b,1)}}{\partial x_L^2} \right) - \left( \frac{\partial^2 \pi_{SC,e}^{(4b,1)}}{\partial q_L \partial x_L} \right)^2 = 4a > 0 \). From solving the first-order conditions of \( \pi_{MC}^{(4b,1)} \), the optimal solutions for \( q_L \) and \( x_L \) will be derived.

**Strategy 1 - Decentralized channel:**

Since \( \frac{\partial \pi_{MC}^{(1)}}{\partial q_L^2} = -2a < 0 \), solving \( \frac{\partial \pi_{MC}^{(1)}}{\partial q_L} = 2\theta_L - \lambda - 2a q_L = 0 \) for \( q_L \) results in the optimal quality level. Figure A-5 shows the consumers’ utility in Strategy 4b.

![Figure A-5: Consumer’s utility and demand in Strategy 4b](image)

**Strategy 2 - Decentralized channel:**

Using a backward induction approach, we first maximize the retailer’s profit function. Since \( \frac{\partial \pi_{MC}^{(2)}}{\partial p_{LS}} = -2a H - 2\frac{a L}{t_L} < 0 \), the first-order condition gives us the retailer’s optimal pricing decision: \( p_{LS}^* = \frac{w_L}{2} + \frac{\theta_L q_L}{2} + \frac{t_L(x_L - 1)}{2} \) for given values of \( w_L, q_L \), and \( x_L \). By knowing the retailer’s best response on retail price for the standard product, the manufacturer’s profit function can be written as a function of \( w_L, q_L, \) and \( x_L \). The determinant of the Hessian matrix of \( \pi_{MC,d}^{(2)}(w_L, q_L, x_L) \) is not negative definite.

Therefore, we first solve for \( q_L \) because \( \frac{\partial \pi_{MC,d}^{(2)}}{\partial q_L^2} = -\left( (a_L t_H + a_H t_L) \frac{\theta_L^2}{t_H t_L} + 2a t_L t_H \right) < 0 \), i.e., the manufacturer’s profit function is concave in \( q_L \). Next, we substitute the optimal \( q_L^* \) into the original manufacturer’s profit function. The sufficient conditions of the resulting profit function \( \pi_{MC,d}^{(2,1)} \) indicate that it is jointly concave in \( w_L \) and \( q_L \).
Strategy 3- Decentralized channel:

Solving \( \frac{\partial \pi^{(3)}_{M,d}}{\partial q_L} = \alpha_L(\theta_L - 2aq_L) - 2\alpha_H(\theta_H - \theta_L) = 0 \) and \( \frac{\partial \pi^{(3)}_{M,d}}{\partial q_H} = \alpha_H(\theta_H - 2aq_H) = 0 \) for \( q_L \) and \( q_H \) respectively, gives us the optimal solutions because the sufficient conditions are \( \frac{\partial^2 \pi^{(3)}_{M,d}}{\partial q_L^2} = -2a\alpha_L < 0 \), and \( \frac{\partial^2 \pi^{(3)}_{M,d}}{\partial q_H^2} = -2a\alpha_H < 0 \); and \( \left( \frac{\partial^2 \pi^{(3)}_{M,d}}{\partial q_L^2} \right) \left( \frac{\partial^2 \pi^{(3)}_{M,d}}{\partial q_L \partial q_H} \right) - \left( \frac{\partial^2 \pi^{(3)}_{M,d}}{\partial q_L \partial q_H} \right)^2 = 4a^2 \alpha_L \alpha_H > 0 \), which implies that the profit function is jointly concave in \( q_L \) and \( q_H \).

Strategy 4a- Decentralized channel:

\( w_L, q_L, q_H, x_L \) are decision variables. The determinant of the Hessian matrix is \( \frac{16a^2 b^2 \alpha_H \alpha_L^2}{t_L} > 0 \). Also, we have \( \frac{\partial^2 \pi^{(4a)}_{M,d}}{\partial w_L^2} = -2a\alpha_L < 0 \), \( \frac{\partial^2 \pi^{(4a)}_{M,d}}{\partial q_L^2} = -2a\alpha_L(\theta_L + at_L) \frac{\partial L}{\partial q_L} < 0 \), \( \frac{\partial^2 \pi^{(4a)}_{M,d}}{\partial q_H^2} = -2a\alpha_H < 0 \), \( \frac{\partial^2 \pi^{(4a)}_{M,d}}{\partial x_L^2} = -2b - 2a\alpha_L t_L < 0 \). Therefore, the first-order conditions give us the optimal solutions.

Strategy 4b- Decentralized channel:

The determinant of the Hessian matrix of \( \pi^{(4b)}_{M,d}(q_L, q_H, x_L) \) is not negative definite. However, \( \pi^{(4b)}_{M,d} \) is concave in \( q_H \) as \( \frac{\partial \pi^{(4b)}_{M,d}}{\partial q_H} = -2a\alpha_H < 0 \). First we find the optimal solution for \( q_H \) and substitute the result back into \( \pi^{(4b)}_{M,d} \) to find \( \pi^{(4b,1)}_{M,d}(q_L, x_H) \). The derivatives \( \frac{\partial^2 \pi^{(4b,1)}_{M,d}}{\partial x_H^2} = -2b - 4\alpha_H t_H < 0 \), \( \frac{\partial^2 \pi^{(4b,1)}_{M,d}}{\partial q_L^2} = -2a\alpha_L < 0 \), and \( \left( \frac{\partial^2 \pi^{(4b,1)}_{M,d}}{\partial q_L^2} \right) \left( \frac{\partial^2 \pi^{(4b,1)}_{M,d}}{\partial x_H^2} \right) - \left( \frac{\partial^2 \pi^{(4b,1)}_{M,d}}{\partial q_L \partial x_H} \right)^2 = 4a\alpha_L (b + 2\alpha_H t_H) \) show that the resulting profit function is jointly concave in \( q_L \) and \( x_H \).

### Appendix B

In this section, we prove Propositions 1 to 4. The solution approaches applied to prove these propositions are the same. First, we create a comparison function that represents the difference between the profits of the two strategies compared. Second, we show that the comparison function is strictly increasing w.r.t. \( b \). Therefore, for all \( b \) that is less than the threshold value, the manufacturer is better-off moving from the first strategy to the second one. The threshold value of \( b \) is the single root of the comparison function.

For Propositions 5 and 6, the single root of the comparison function only indicates that there is no difference between the two strategies.

Table B-1: proofs for Propositions 1 to 4

<table>
<thead>
<tr>
<th>Proposition</th>
<th>Condition</th>
<th>Increasing function</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 ( \lambda_{c,1}(b) = \pi_{SC,1}^{(3)} - \pi_{SC,1}^{(2)} = -\frac{t_L^2}{4b} + \rho_1 )</td>
<td>( \rho_1 = \frac{a_H(\theta_{H,H} - \theta_{L})^2}{4a\alpha_L} + c - \frac{\alpha_{H}^2}{4a\alpha_L} )</td>
<td>( \frac{\partial \lambda_{c,1}(b)}{\partial b} = \frac{t_L^2}{4b} &gt; 0 )</td>
</tr>
</tbody>
</table>
### Appendix C

In this section, we prove that when the manufacturer offers a customized product with the high quality level, there is no cannibalization between the standard product with the low-quality and the customized one. Consumers in the high-valuation segment are indifferent between buying the standard product and the customized one when $p_{HC} = p_{HS} + t_H x_H$. From this, $p_{HC} - p_{HS} > t_L x_H$ ⇒ $\theta_L q_H - t_L y_L - p_{HS} > \theta_L q_H - t_L (y_L - x_H) - p_{HC}$ ⇒ $\theta_L q_H - t_L y_L - p_{LS} > \theta_L q_H - t_L (y_L - x_H) - p_{HC}$. Therefore, we can conclude that there is no cannibalization between the standard low-quality product and the customized product targeted at the high-valuation segment.

### Appendix D

In this part, we show how to set the wholesale prices in strategies 3, 4a, and 4b in the decentralized channel:

**In Strategy 3:**

The retailer’s profit function is as follows:

$$\pi^{(3)}_R = (p_{LS} - w_L)\alpha_L (\frac{\theta_L q_L - p_{LS}}{t_L}) + (p_{HS} - w_H)\alpha_H (\frac{\theta_H q_H - p_{HS}}{t_H})$$  \hspace{1cm} (D1)

Since $\frac{\partial^2 \pi^{(3)}_R}{\partial p_{LS}^2} \frac{\partial^2 \pi^{(3)}_R}{\partial p_{HS}^2} = (\frac{\partial^2 \pi^{(3)}_R}{\partial p_{LS} \partial p_{HS}})^2 = \frac{4\alpha_L \alpha_H}{t_H t_L} > 0$, the first-order conditions $\frac{\partial \pi^{(3)}_R}{\partial p_{LS}} = 0$ and $\frac{\partial \pi^{(3)}_R}{\partial p_{HS}} = 0$ give us the retailer’s optimal pricing decisions:

$$p_{LS}^* = \frac{\theta_L q_L}{2} + \frac{w_L}{2} \hspace{1cm} (D2)$$

$$p_{HS}^* = \frac{\theta_H q_H}{2} + \frac{w_H}{2} \hspace{1cm} (D3)$$

Then, the wholesale prices in Strategy 3 are $w_L = 2p_{LS} - \theta_L q_L$ and $w_H = 2p_{HS} - \theta_H q_H$. 

---

<table>
<thead>
<tr>
<th>Channel</th>
<th>λ_{d,1}</th>
<th>λ_{c,2}</th>
<th>λ_{c,2}</th>
<th>\frac{\partial \lambda_{d,1}(b)}{\partial b}</th>
<th>Increasing function</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$\lambda_{d,1}^2 = \lambda_{d,1}^{(3)<em>} - \lambda_{d,1}^{(2)</em>}$</td>
<td>$\lambda_{c,2}^3 = \lambda_{c,2}^{(3)<em>} - \lambda_{c,2}^{(4a)</em>}$</td>
<td>$\lambda_{c,2}^4 = \lambda_{c,2}^{(3)<em>} - \lambda_{c,2}^{(4b)</em>}$</td>
<td>$\frac{\partial \lambda_{d,1}(b)}{\partial b} = \frac{\varepsilon^2}{4b^2}$</td>
<td>$\lambda_{c,2}^2 &gt; 0$</td>
</tr>
<tr>
<td>$\rho_2 = \frac{a_L \theta_L^2}{4a} + \frac{a_H (\theta_H - 2\theta_L)^2}{4a} + \frac{\theta_L^2}{2a} + \frac{\theta_H^2}{2a} + c - \frac{3c}{16} \left( \frac{\alpha_H}{t_H} + \frac{\alpha_L}{t_L} \right)$</td>
<td>$\rho_3 = \alpha_L c - \frac{a_L \varepsilon}{4t_L} + \frac{\theta_L \theta_L \varepsilon}{2a} + \frac{\theta_L^2 (a_L - \varepsilon)^2}{4a^2} + \frac{\varepsilon^2}{4t_L}$</td>
<td>$\rho_4 = \frac{a_L \varepsilon}{4t_L} + \frac{\theta_4 (a_L - \varepsilon)^2}{4a^2} + \frac{\varepsilon^2}{4t_L}$</td>
<td>$\frac{\partial \lambda_{c,2}^2}{\partial b} = \frac{(\alpha_H t_H - t_L)^2}{4b^2} &gt; 0$</td>
<td>Increasing function</td>
<td></td>
</tr>
</tbody>
</table>
In Strategy 4a:

The retailer’s profit function is as follow:

\[
\pi_{R}^{(4a)} = (p_{LS} - w_{L})\alpha_{L}\left(\frac{p_{LC} - p_{LS}}{t_{L}}\right) + (p_{HS} - w_{H})\alpha_{H}\left(\frac{\theta_{H}q_{H} - p_{HS}}{t_{H}}\right)
\]  
(D4)

The first-order and sufficient conditions are determined as:

\[
\frac{\partial \pi_{R}^{(4a)}}{\partial p_{LS}} = \alpha_{L}\left(\frac{p_{LC}}{t_{L}} - \frac{2p_{LS}}{t_{L}} + \frac{w_{L}}{t_{L}}\right)
\]  
(D5)

\[
\frac{\partial \pi_{R}^{(4a)}}{\partial p_{HS}} = \alpha_{H}\left(\frac{\theta_{H}q_{H}}{t_{H}} - \frac{2p_{HS}}{t_{H}} + \frac{w_{H}}{t_{H}}\right)
\]  
(D6)

\[
\frac{\partial^{2}\pi_{R}^{(4a)}}{\partial p_{LS}^{2}} = -\frac{2\alpha_{L}}{t_{L}} < 0, \quad \frac{\partial^{2}\pi_{R}^{(4a)}}{\partial p_{p_{HS}}^{2}} = -\frac{2\alpha_{H}}{t_{H}} < 0, \quad \text{and} \quad \frac{\partial^{2}\pi_{R}^{(4a)}}{\partial p_{p_{LS}}\partial p_{p_{HS}}} - \left(\frac{\partial^{2}\pi_{R}^{(4a)}}{\partial p_{p_{HS}}^{2}}\right)^{2} = \frac{4\alpha_{L}\alpha_{H}}{t_{L}t_{H}} > 0
\]  
(D7)

Consequently, the retailer’s optimal pricing decisions are as follows:

\[
p^{*}_{LS} = \frac{p_{LC}}{2} + \frac{w_{L}}{2}
\]  
(D8)

\[
p^{*}_{HS} = \frac{\theta_{H}q_{H}}{2} + \frac{w_{H}}{2}
\]  
(D9)

Thus, \(w_{L} = 2p_{LS} - p_{LC}\) and \(w_{H} = w_{L} + \theta_{H}(q_{H} - 2q_{L}) + \theta_{L}q_{L} + (3t_{L} - 2t_{H})x_{L} - t_{L}\).

In strategy 4b:

The retailer’s profit function is as follows:

\[
\pi_{R}^{(4b)} = (p_{LS} - w_{L})\alpha_{L}\left(\frac{\theta_{L}q_{L} - p_{LS}}{t_{L}}\right) + (p_{HS} - w_{H})\alpha_{H}\left(\frac{p_{HC} - p_{HS}}{t_{H}}\right)
\]  
(D10)

From the first-order and sufficient conditions, we have:

\[
p^{*}_{LS} = \frac{\theta_{L}q_{L}}{2} + \frac{w_{L}}{2}
\]  
(D11)

\[
p^{*}_{HS} = \frac{p_{HC}}{2} + \frac{w_{H}}{2}
\]  
(D12)

Since \(p_{LS} = \theta_{L}q_{L} - t_{L}\), therefore, \(w_{L} = \theta_{L}q_{L} - 2t_{L}\) and \(w_{H} = \theta_{L}q_{L} - t_{L} + \theta_{H}(q_{H} - q_{L}) - x_{H}t_{H}\).
References


