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Realized Beta GARCH:

A Multivariate GARCH Model with Realized Measures of Volatility and CoVolatility

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Abstract

We introduce a multivariate GARCH model that utilizes and models realized measures of volatility and covolatility. The realized measures extract information contained in high-frequency data that is particularly beneficial during periods with variation in volatility and covolatility. Applying the model to market returns in conjunction with an individual asset yields a model for the conditional regression coefficient, known as the beta. We apply the model to a set of highly liquid stocks and find that conditional betas are much more variable than usually observed with rolling-window OLS regressions with daily data. In the empirical part of the paper we examine the cross-sectional as well as the time variation of the conditional beta series.

The model links the conditional and realized second moment measures in a self-contained system of equations, making it amenable to extensions and easy to estimate. A multi-factor extension of the model is briefly discussed.

Keywords: Financial Volatility; Beta; Realized GARCH; High Frequency Data.

JEL Classification: G11, G17, C58

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1 Introduction

The availability of high frequency data paved the way for relatively accurate measurements of volatility and covolatility. In this paper we propose a multivariate GARCH-type model which utilizes and models realized measures of volatility and covolatility, inspired by the realized GARCH model mentioned above. The model is based on a single-factor structure that specifies a system of equations describing the dynamics of asset returns, realized volatilities and realized correlations of the assets with the factor. A multi-factor extension is conceptually straightforward to obtain. If the factor is chosen to be the market return, our approach allows for the estimation of a model-based realized beta related to a conditional CAPM framework. The concept of realized betas is not new. Existing approaches (see Bollerslev and Zhang (2003), Andersen et al. (2006), Patton and Verardo (2009), Dovocon et al. (2010)), however, are mainly reduced-form, lacking the equation that relates the realized measure to the conditional variance. This measurement equation is important since the realized measure is only a proxy for the true conditional variance. Furthermore, it allows for the incorporation of important empirical relationships between the return and volatility, such as the leverage effect.

Whether betas are indeed time-varying or not is a controversial topic in the empirical finance literature. The studies of Ferson and Harvey (1991, 1993), Shanken (1990) specify parametric relationships between betas and proxies for the state of the economy and find support for time-varying betas. A time-varying conditional beta specification can arise from a dynamic general equilibrium production economy as shown by Gomes et al. (2003). Conditional betas have been modeled by means of conventional GARCH models by Braun et al. (1995) and Bekaert and Wu (2000), among others. Ghysels (1998) launches a critique against conditional CAPM models based on the substantial risk of model misspecification. He argues that if the beta risk is misspecified, traditional constant-beta CAPM models can lead to superior asset pricing. Support for these findings has been documented by Wang (2003). Lewellen and Nagel (2006) argue that variation in betas would have to be “implausibly large” to explain important asset-pricing anomalies. We believe that our modeling framework can prove useful in resolving some of the controversial issues discussed above.

The research devoted to high-frequency volatility measures was catalyzed by Andersen and Bollerslev (1998), who documented that the sum of squared intraday returns, known as the realized variance, provides an accurate measurement of daily volatility. The theoretical foundation of realized variance was developed in Andersen, Bollerslev, Diebold and Labys (2001) and Barndorff-Nielsen and Shephard (2002). Currently a large number of related estimators, such as realized bipower variation, realized kernels, multiscale estimators, preaveraging estimators and Markov chain estimators have been proposed to deal with issues such as jumps and market microstructure frictions (see Barndorff-Nielsen and Shephard (2004b), Barndorff-Nielsen, Hansen, Lunde and Shephard (2008), Zhang (2006), Jacod et al. (2009), Hansen and Horel (2009) and also references therein). The multivariate extensions of the concept of realized volatility is theoretically devel-

oped in Barndorff-Nielsen and Shephard (2004a). Noise- and non-synchronicity-robust estimators have been proposed by Hayashi and Yoshida (2005), Voev and Lunde (2007), Griffin and Oomen (2010), Christensen et al. (2010), Barndorff-Nielsen et al. (2010). In this paper we will rely on the multivariate kernel approach developed in Barndorff-Nielsen, Hansen, Lunde and Shephard (2010) which ensures positivity of the realized covariance measure.

While volatility is unobservable, the use of realized measures allow us to construct very precise ex-post volatility proxies. Currently, a growing body of research investigates the issue to what extent realized measures can be used to specify better models of volatility dynamics and provide more accurate volatility forecasts. Hansen and Lunde (2010) categorize the existing approaches into two broad classes: reduced-form and model-based. Reduced-form volatility forecasts are based on a time series model for the series of realized measures, while a model-based forecast rests on a parametric model for the return distribution. Model-based approaches effectively build on GARCH models in which a realized measure is included as an exogenous variable in the GARCH equation. Examples include Engle (2002), Barndorff-Nielsen and Shephard (2007), etc (see references in Hansen and Lunde (2010)).

A complete framework that jointly specifies models for returns and realized measures of volatility was first proposed by Engle and Gallo (2006), who refer to their model as the Multiplicative Error Model (MEM). Shephard and Sheppard (2010) subsequently analyzed a simplified MEM structure, which they refer to as the HEAVY model.

The realized GARCH model by Hansen et al. (2010) involves a different approach to the joint modeling of returns and realized volatility measures with the key difference being the measurement equation in the Realized GARCH model, that links the realized measure with the underlying conditional variance.

The rest of the paper is structured as follows. The theory of the model and its estimation are presented in sections 2 and 3. Section 4 contains the empirical application of the model, and Section 5 concludes.

2 A Hierarchical Realized GARCH Framework

The modeling strategy we propose combines a marginal model for market returns and the corresponding realized measure of volatility, with a conditional model for the asset-specific return, its realized volatility and the covolatility between the asset and the market (the factor).

The marginal model we use for the market-specific time series is a variant of the Realized GARCH model discussed in Hansen et al. (2010), section 6.3. This model is called the Realized EGARCH model because it shares certain features with the EGARCH model by Nelson (1991). The conditional realized EGARCH model that is used to build a multivariate model is new.

We first consider a bivariate setup, and subsequently discuss the extension to an arbitrary number of assets.

2.1 Notation and Modeling Strategy

We use $r_{0,t}$ to denote the market return, with a realized measure of volatility denoted by $x_{0,t}$. The corresponding time series for the individual asset are denoted by $r_{1,t}$ and $x_{1,t}$, respectively, and the realized correlation measure is denoted by $y_{1,t}$. The realized volatility and correlation measures are obtained using the multivariate kernel methodology of Barndorff-Nielsen et al. (2010). The natural filtration is thus given by

$$\mathcal{F}_t = \sigma(\mathcal{X}_t, \mathcal{X}_{t-1}, \dots) \quad \text{with} \quad \mathcal{X}_t = (r_{0,t}, r_{1,t}, x_{0,t}, x_{1,t}, y_{1,t})'.$$

Our modeling approach takes advantage of the following simple decomposition of the conditional density,

$$f(r_{0,t}, x_{0,t}, r_{1,t}, x_{1,t}, y_{1,t} | \mathcal{F}_{t-1}) = f(r_{0,t}, x_{0,t} | \mathcal{F}_{t-1}) f(r_{1,t}, x_{1,t}, y_{1,t} | r_{0,t}, x_{0,t}, \mathcal{F}_{t-1}).$$

We will adopt the Realized EGARCH model as our specification of the first term, $f(r_{0,t}, x_{0,t} | \mathcal{F}_{t-1})$, and utilize a new model structure for the second conditional density. It is the specification of the latter that enables us to extend the Realized GARCH framework to a multivariate setting. The specification for the second conditional density, $f(r_{1,t}, x_{1,t}, y_{1,t} | r_{0,t}, x_{0,t}, \mathcal{F}_{t-1})$, defines how the time series associated with the individual asset evolves conditional on contemporary market variables. Our specification of this conditional density has a structure that is similar to that of the univariate Realized GARCH model, but has some important adaptations for the modeling of the correlation structure. Assuming a single-factor structure is very convenient as there is no need to introduce realized correlation measures between the individual assets. The estimation proceeds by first estimating the model for the market data $(r_{0,t}, x_{0,t})$ and then estimating each conditional model for $(r_{i,t}, x_{i,t}, y_{i,t})$ separately for $i = 1, 2, \dots, n$, where n is the number of assets. We report results for thirty assets (the S&P500 exchange traded fund SPDR as the market index and 29 individual stocks from the DJIA index) in the empirical section below.

2.2 Realized EGARCH Model for Market Returns

The Realized EGARCH model for market returns and realized measures of volatility is given by the following three equations

$$r_{0,t} = \mu_0 + \sqrt{h_{0,t}} z_{0,t}, \tag{1}$$

$$\log h_{0,t} = a_0 + b_0 \log h_{0,t-1} + c_0 \log x_{0,t-1} + \tau_0(z_{0,t-1}) \tag{2}$$

$$\log x_{0,t} = \xi_0 + \varphi_0 \log h_{0,t} + \delta_0(z_{0,t}) + u_{0,t}, \tag{3}$$

where we model $z_{0,t} \sim \text{iid}N(0, 1)$, $u_{0,t} \sim \text{iid}N(0, \sigma_u^2)$. As is the case in conventional GARCH models, $h_{0,t}$, denotes a conditional variance, $h_{0,t} = \text{var}(r_{0,t} | \mathcal{F}_{t-1})$, the key difference being that the information set, \mathcal{F}_t , is richer than in the conventional framework. The normality of $u_{0,t}$ is motivated by the findings in Andersen, Bollerslev, Diebold and Labys (2001), Andersen, Bollerslev, Diebold and Ebens (2001) and Andersen

et al. (2003), who document that realized volatility is approximately log-normal. Furthermore, Andersen, Bollerslev, Diebold and Ebens (2001) find that returns standardized by realized volatility are close to normally distributed. Since the conditional variance $h_{0,t}$ incorporates the information on the realized volatility measure, we expect that $z_{0,t}$ will be normal.

The functions $\tau(z)$ and $\delta(z)$ are called leverage functions because they model aspects related to the *leverage effect*, which refers to the dependence between returns and volatility. Hansen et al. (2010) found that a simple second-order polynomial form provides a good empirical fit. We will adopt this structure in our framework, and set $\tau(z) = \tau_1 z + \tau_2(z^2 - 1)$ and $\delta(z) = \delta_1 z + \delta_2(z^2 - 1)$. This leads to a GARCH equation that is somewhat similar to an EGARCH model. The important difference is that we also utilize the realized measure x_{t-1} on the right hand side of the equation.

We refer to the first two equations, (1) and (2), as the *return equation* and the *GARCH equation*, respectively. These two equations define a GARCH-X model, similar to those that were estimated by Engle (2002), Barndorff-Nielsen and Shephard (2007), and Visser (2008). See also Chen et al. (2009) for additional variants of the GARCH-X model and some related models.

The third equation, (3) called the *measurement equation*, completes the model. Tying the realized measure, x_t , to the conditional variance, h_t , is nicely motivated by the fact that conventional GARCH models trivially imply that

$$\log(r_t - \mu)^2 = \log h_t + \log z_t^2.$$

Since the realized measure, x_t , is similar to r_t^2 (just a more accurate measure of volatility), it is natural to expect that $\log x_t \approx \log h_t + f(z_t) + \text{error}_t$. Because we may compute realized measures of volatility over a shorter period of time than the one spanned by the return (e.g., if we use only data from the trading session, which often excludes the overnight period), some flexibility in the specification may be required motivating the “intercept” ξ_0 and the “slope” φ_0 . So long as $x_{0,t}$ is roughly proportional to $h_{0,t}$, we should expect $\varphi_0 \simeq 1$, and $\xi_0 < 0$, which is always the case empirically.

Note that we do not follow the conventional GARCH notation, because we want to reserve the notation “ β ” for

$$\beta_t = \text{cov}(r_{1,t}, r_{0,t} | \mathcal{F}_{t-1}) / \text{var}(r_{0,t} | \mathcal{F}_{t-1}), \quad (4)$$

We are particularly interested in the dynamic properties and the cross-sectional variation of β_t .

2.3 Conditional Model for Individual Asset Returns, Volatility, and Covolatility

To extend the framework to a joint model for the market returns/volatility and another asset’s return/volatility and their covolatility, we shall formulate a model for the time series associated with the individual asset, conditional on contemporaneous “market” variables, i.e., a specification for $f(r_{1,t}, x_{1,t}, y_{1,t} | r_{0,t}, x_{0,t}, \mathcal{F}_{t-1})$. We utilize a further decomposition of this conditional density, specifically

$$f(r_{1,t}, x_{1,t}, y_{1,t} | r_{0,t}, x_{0,t}, \mathcal{F}_{t-1}) = f(r_{1,t} | r_{0,t}, x_{0,t}, \mathcal{F}_{t-1}) f(x_{1,t}, y_{1,t} | r_{1,t}, r_{0,t}, x_{0,t}, \mathcal{F}_{t-1}).$$

The first part, $f(r_{1,t}|r_{0,t}, x_{0,t}, \mathcal{F}_{t-1})$, is modeled with three equations. The first two have the Realized EGARCH structure as above,

$$r_{1,t} = \mu_1 + \sqrt{h_{1,t}}z_{1,t}, \quad (5)$$

$$\log h_{1,t} = a_1 + b_1 \log h_{1,t-1} + c_1 \log x_{1,t-1} + d_1 \log h_{0,t} + \tau_1(z_{1,t-1}). \quad (6)$$

The difference between the two GARCH equations, (2) for the market return and (6) for the asset return, is the presence of the term, $d_1 \log h_{0,t}$. This term relates the market conditional variance to the conditional variance of an individual asset under consideration. Note that since $h_{0,t}$ is \mathcal{F}_{t-1} -measurable, $h_{1,t}$ may still be considered as the conditional variance of $r_{1,t}$. The parameter d_1 can be interpreted as a spillover effect that measures the extend to which the market's volatility affects the volatility of the individual asset after having accounted for the asset-specific volatility dynamics.

Because we condition on the contemporaneous market data, i.e, we specify $f(r_{1,t}|r_{0,t}, x_{0,t}, \mathcal{F}_{t-1})$, we allow $z_{1,t}$ to be correlated with $z_{0,t}$ and therefore we cannot model $z_{1,t}$ as an iid sequence.

To capture the dependence between market returns and individual returns, we introduce the conditional covariance

$$\rho_t = \text{cov}(z_{0,t}, z_{1,t}|\mathcal{F}_{t-1}),$$

and it follows directly that ρ_t is the conditional correlation between $r_{0,t}$ and $r_{1,t}$.

To complete this part of the model we need to specify the dynamic properties of ρ_t , and this is where we introduce the realized correlation measure, $y_{1,t}$. For this purpose we make use of the Fisher transformation (also known as the inverse hyperbolic tangent, arctanh), $\rho \mapsto F(\rho) \equiv \frac{1}{2} \log \frac{1+\rho}{1-\rho}$, which is a one-to-one mapping from $(-1, 1)$ into \mathbb{R} . The GARCH-type equation for the Fisher transformation of ρ_t is thus given by

$$F(\rho_t) = a_{01} + b_{01}F(\rho_{t-1}) + c_{01}F(y_{1,t-1}),$$

where we have assumed that there are no leverage-type effects in the conditional correlation.

Finally, the model is completed using the following two measurement equations:

$$\log x_{1,t} = \xi_1 + \varphi_1 \log h_{1,t} + \delta_1(z_{1,t}) + u_{1,t}, \quad (7)$$

and

$$F(y_{1,t}) = \xi_{01} + \varphi_{01}F(\rho_t) + v_{1,t}. \quad (8)$$

Conditional on contemporaneous market variables, the covariance structure for the error terms in the three measurement equations (3), (7) and (8), $(u_{0,t}, u_{1,t}, v_{1,t})$, is explicitly specified as

$$\Sigma = \text{var} \begin{pmatrix} u_{0,t} \\ u_{1,t} \\ v_{1,t} \end{pmatrix} = \begin{bmatrix} \sigma_{u_0}^2 & \sigma_{u_0,u_1} & \sigma_{u_0,v_1} \\ \bullet & \sigma_{u_1}^2 & \sigma_{u_1,v_1} \\ \bullet & \bullet & \sigma_{v_1}^2 \end{bmatrix}.$$

2.4 Extensions

The model we propose can be extended to a multi-factor framework to incorporate other priced factors beyond the market factor. With m factors, the multivariate realized kernel method can be used to obtain a series of covariance matrices for the $m + 1$ vector of factors returns and the asset return. Having obtained the covariance structure of the factors (the $m \times m$ submatrix of the realized kernel matrix) a multivariate extension of the system of equations (1)-(3) can be specified to model the conditional covariance of the factors. Furthermore, the asset return variance and the m correlations with the individual factors can be modeled conditional on the factors' return and covariance structure as in the single-factor model detailed in the paper. In terms of practical implementation, the factors can be chosen among the SPDR sector indices. Again, the factor structure allows adding individual assets one by one, since the cross-asset correlations are captured through the correlation of the individual assets with the factors.

3 Estimation

To formulate the quasi likelihood function of the joint model, we adopt a Gaussian specification, where $(z_{0,t}, z_{1,t})$ are independent of $(u_{0,t}, u_{1,t}, v_{1,t})$. This assumption together with the decomposition of the joint density

$$f(r_{0,t}, x_{0,t}, r_{1,t}, x_{1,t}, y_{1,t} | \mathcal{F}_{t-1}) = f(r_{0,t}, x_{0,t} | \mathcal{F}_{t-1}) f(r_{1,t} | r_{0,t}, x_{0,t}, \mathcal{F}_{t-1}) f(x_{1,t}, y_{1,t} | r_{1,t}, r_{0,t}, x_{0,t}, \mathcal{F}_{t-1}), \quad (9)$$

implies that the log-likelihood of the model under normality is given by

$$\ell(r_0, x_0, r_1, x_1, y_1) = -\frac{1}{2} (\ell_{z_0} + \ell_{u_0} + \ell_{z_1|z_0} + \ell_{u_1,v_1|u_0}),$$

where the single components on the right hand side will be elaborated upon below. The likelihood contributions from the (marginal) model for the market return and the corresponding realized volatility measure are simply given by

$$\begin{aligned} \ell_{z_0} &= \sum_{t=1}^T \log h_{0,t} + z_{0,t}^2 \\ \ell_{u_0} &= \sum_{t=1}^T \log \sigma_{u_0}^2 + \frac{u_{0,t}^2}{\sigma_{u_0}^2}. \end{aligned}$$

The second component on the right hand side of the decomposition in Equation (9) is the density of the individual asset return conditional on the contemporaneous market variables. Since $\text{cov}(r_{1,t}, r_{0,t} | \mathcal{F}_{t-1}) = \rho_t \sqrt{h_{0,t} h_{1,t}}$, the conditional mean of $r_{1,t}$ given $r_{0,t}$ and \mathcal{F}_{t-1} is given by

$$\mu_1 + \beta_t(r_{0,t} - \mu_0) = \mu_1 + \frac{\rho_t \sqrt{h_{0,t} h_{1,t}}}{h_{0,t}}(r_{0,t} - \mu_0) = \mu_1 + \rho_t \sqrt{h_{1,t}} z_{0,t}$$

and the conditional variance is given by

$$h_{1,t} - \frac{(\rho_t \sqrt{h_{0,t} h_{1,t}})^2}{h_{0,t}} = (1 - \rho_t^2) h_{1,t}.$$

Thus we have that

$$\ell_{z_1 | z_0} = \sum_{t=1}^T \log[(1 - \rho_t^2) h_{1,t}] + \frac{(y_{1,t} - \mu_1 - \rho_t \sqrt{h_{1,t}} z_{0,t})^2}{(1 - \rho_t^2) h_{1,t}}.$$

Under multivariate normality of $(u_{0,t}, u_{1,t}, v_{1,t})$, the density of $(u_{1,t}, v_{1,t})$ given $u_{0,t}$ and \mathcal{F}_{t-1} , corresponding to the third component on the right hand side of the decomposition in Equation (9) is normal with mean

$$\begin{pmatrix} \sigma_{u_1, u_0} / \sigma_{u_0}^2 \\ \sigma_{v_1, u_0} / \sigma_{u_0}^2 \end{pmatrix} u_{0,t},$$

and variance

$$\Omega = \begin{bmatrix} \sigma_{u_1}^2 & \sigma_{u_1, v_1} \\ \bullet & \sigma_{v_1}^2 \end{bmatrix} - \begin{bmatrix} \sigma_{u_1, u_0} \\ \sigma_{v_1, u_0} \end{bmatrix} \frac{1}{\sigma_{u_0}^2} \begin{bmatrix} \sigma_{u_0, u_1} & \sigma_{u_0, v_1} \end{bmatrix}.$$

It follows that the last term in the log-likelihood is given by

$$\ell_{u_1, v_1 | u_0} = \sum_{t=1}^T \log \det \Omega + U'_{1,t} \Omega^{-1} U_{1,t},$$

where we have defined

$$U_{1,t} = \begin{pmatrix} u_{1,t} \\ v_{1,t} \end{pmatrix} - \begin{pmatrix} \sigma_{u_1, u_0} / \sigma_{u_0}^2 \\ \sigma_{v_1, u_0} / \sigma_{u_0}^2 \end{pmatrix} u_{0,t}.$$

The parameters in the model can be split into parameters in “market” model

$$\theta_0 = (\mu_0, \omega_0, a_0, b_0, c_0, \tau_{01}, \tau_{02}, \xi_0, \varphi_0, \delta_{01}, \delta_{02}, \sigma_{u_0}^2),$$

and parameters in conditional asset model

$$\theta_1 = (\mu_1, \omega_1, a_1, b_1, c_1, d_1, \tau_{11}, \tau_{12}, \xi_1, \varphi_1, \delta_{11}, \delta_{12}, \sigma_{u_1}^2, \xi_{01}, \varphi_{01}, \sigma_{v_1}^2, \sigma_{u_0 u_1}, \sigma_{u_0, u_1}, \sigma_{u_1, v_1}).$$

3.1 Simplification in Estimation

In order to reduce the number of parameters in the estimation of the model, the likelihood function can be concentrated with respect to the covariance matrix of $(u_{0,t}, u_{1,t}, v_{1,t})$. Defining $\hat{u}_{0,t}$, $\hat{u}_{1,t}$ and $\hat{v}_{1,t}$ as residuals from the corresponding measurement equations, from the properties of the Gaussian likelihood it follows that

$$\hat{\sigma}_{u_0}^2 = \frac{1}{T} \sum_{t=1}^T \hat{u}_{0,t}^2$$

and

$$\hat{\sigma}_{u_1 u_0} = \frac{1}{T} \sum_{t=1}^T \hat{u}_{1,t} \hat{u}_{0,t}, \quad \hat{\sigma}_{v_1 u_0} = \frac{1}{T} \sum_{t=1}^T \hat{v}_{1,t} \hat{u}_{0,t}, \quad \hat{\Omega} = \frac{1}{T} \sum_{t=1}^T \hat{U}_{1,t} \hat{U}'_{1,t},$$

where

$$\hat{U}_{1,t} = \begin{pmatrix} \hat{u}_{1,t} \\ \hat{v}_{1,t} \end{pmatrix} - \begin{pmatrix} \hat{\sigma}_{u_1 u_0} / \hat{\sigma}_{u_0}^2 \\ \hat{\sigma}_{v_1 u_0} / \hat{\sigma}_{u_0}^2 \end{pmatrix} \hat{u}_{0,t}.$$

So we can also express the log-likelihood function as

$$2\ell(r_0, x_0, r_1, x_1, y_1) = 2\ell(r_0, x_0) + 2\ell(r_1, x_1, y_1 | r_0, x_0),$$

where

$$\begin{aligned} -2\ell(r_0, x_0) &= \left(\sum_{t=1}^T \log h_{0,t} + z_{0,t}^2 \right) + T(\log \hat{\sigma}_{u_0}^2 + 1), \\ 2\ell(r_1, x_1, y_1 | r_0, x_0) &= \left(\sum_{t=1}^T \log \left[(1 - \rho_t^2) h_{0,t} \right] + \frac{(y_{1t} - \mu_1 - \rho_t \sqrt{h_{1t}} z_{0t})^2}{(1 - \rho_t^2) h_{1t}} \right) + T(\log \det \hat{\Omega} + 2), \end{aligned}$$

with $\hat{\sigma}_{u_0}^2$, $\hat{\Omega}$, and $\hat{U}_{1,t}$ are defined as above.

This reduces the number of parameter that the optimizer has to search over, to

$$\theta_0 = (\mu_0, \omega_0, a_0, b_0, c_0, \tau_{01}, \tau_{02}, \xi_0, \varphi_0, \delta_{01}, \delta_{02})$$

in the market model and

$$\theta_1 = (\mu_1, \omega_1, a_1, b_1, c_1, d_1, \tau_{11}, \tau_{12}, \xi_1, \varphi_1, \delta_{11}, \delta_{12}, \xi_{01}, \varphi_{01})$$

in the conditional asset model.

If one is interested only in one-step ahead modeling, specifying the dynamics of the realized measures (the measurement equations) becomes redundant and the parameters in the model are further reduced.

4 Empirical Analysis

In estimating our proposed model, we use high-frequency assets prices for thirty assets. The ticker symbols of these assets are AA, AIG, AXP, BA, BAC, C, CAT, CVX, DD, DIS, GE, GM, HD, IBM, INTC, JNJ, JPM, KO, MCD, MMM, MRK, MSFT, PFE, PG, SPY, T, UTX, VZ, WMT, and XOM. The SPY is an exchange-traded fund that holds all of the S&P 500 Index stocks and has enormous liquidity. We use the SPY as a proxy for the market. The data source is the collection of trades and quotes recorded on the NYSE, taken from the TAQ database through the Wharton Research Data Services (WRDS) system. In this paper we follow the step-by-step cleaning procedure used and detailed in Barndorff-Nielsen et al. (2009). The realized measures are computed using the multivariate kernel methodology developed in Barndorff-Nielsen et al. (2010).

The sample period runs from January 3, 2002 to the end of 2009, delivering 2008 distinct trading days. The actual number of days for each individual stock varies due to stock-specific missing data and cleaning of outlying observations.

Estimation results for the dynamic parameters of the model are contained in Table 1.

The results are quite consistent across stocks which lends support for the plausibility of the modeling framework. The realized measure loadings (c coefficients) are large and the typical GARCH effects (b coefficients) are of a smaller magnitude compared to traditional GARCH models. This finding is in line with the literature on GARCH-X models mentioned above. The parameter d_1 is significantly positive, indicating that there is a contemporaneous impact linking the market volatility to the individual asset volatilities. The estimates of the parameters in the leverage functions τ and δ are similar to the ones reported in Hansen et al. (2010) and describe an asymmetric volatility response to positive vs. negative shocks (news impact curve). The relation of the news impact curve and the leverage functions is detailed in Hansen et al. (2010). As the realized measures are based on data spanning the trading session only, as expected ξ_0 and ξ_1 are negative, while φ_0 and φ_1 are close to one.

In Figure 1 we present the realized variance of CVX and SPY against the model-implied conditional variance.¹ Clearly, the conditional variance tracks the realized series closely but exhibits smaller variation (note that the measurement equation implies that the conditional variance is equal to a deterministic function of the realized measure multiplied by a noise term). The apparent downward bias of the realized measure is due to the fact that it is computed over the trading period only and is related to the coefficients ξ_0 and ξ_1 being negative.

¹The plots for all stocks are available from the authors upon request.

Table 1: Estimates of the dynamic parameters of the Realized GARCH Beta model.

Volatility parameters													Correlation Parameters						
	h_0	μ_0	a_0	b_0	c_0	d_1	τ_{01}	τ_{02}	ξ_0	φ_0	δ_{01}	δ_{02}	ρ_1	a_{01}	b_{01}	c_{01}	ξ_{01}	φ_{01}	
SPY	1.066	0.002	0.147	0.647	0.325		-0.107	0.026	-0.455	1.008	-0.116	0.038							
	h_1	μ_1	a_1	b_1	c_1	d_1	τ_{11}	τ_{12}	ξ_1	φ_1	δ_{11}	δ_{12}							
AA	2.683	-0.007	0.319	0.593	0.284	0.061	-0.023	0.015	-0.534	1.082	-0.047	0.081		0.479	0.067	0.743	0.204	-0.204	1.087
AIG	5.725	-0.060	0.243	0.596	0.368	0.033	-0.060	0.021	-0.476	0.971	-0.011	0.072		0.146	-0.018	0.753	0.326	0.105	0.691
AXP	3.919	0.001	0.190	0.618	0.322	0.042	-0.080	0.018	-0.432	1.038	-0.044	0.070		0.842	0.021	0.698	0.379	0.085	0.650
BA	3.855	0.050	0.261	0.587	0.245	0.080	-0.045	0.013	-0.471	1.125	-0.039	0.077		0.482	0.059	0.660	0.264	-0.052	1.020
BAC	3.191	-0.018	0.188	0.539	0.435	0.026	-0.057	0.005	-0.366	0.980	-0.063	0.036		0.657	0.091	0.714	0.205	-0.298	1.224
C	4.214	-0.039	0.170	0.571	0.389	0.054	-0.055	0.039	-0.269	0.950	-0.058	0.078		0.986	-0.005	0.707	0.365	0.086	0.724
CAT	0.932	0.060	0.323	0.538	0.290	0.064	-0.028	0.008	-0.598	1.172	-0.041	0.074		0.627	0.106	0.718	0.179	-0.392	1.328
CVX	1.214	0.041	0.143	0.604	0.313	0.009	-0.040	0.030	-0.349	1.114	-0.082	0.052		0.639	0.026	0.734	0.267	-0.042	0.922
DD	2.515	-0.017	0.202	0.520	0.322	0.109	-0.042	0.003	-0.227	1.010	-0.053	0.057		0.630	0.069	0.663	0.309	-0.079	0.924
DIS	4.737	0.015	0.206	0.563	0.318	0.081	-0.039	0.007	-0.306	1.022	-0.056	0.069		0.417	0.023	0.717	0.338	0.011	0.740
GE	3.318	-0.027	0.170	0.588	0.315	0.082	-0.054	0.030	-0.316	1.012	-0.032	0.063		0.735	0.030	0.648	0.390	0.034	0.786
GM	2.564	-0.096	0.385	0.654	0.226	0.021	-0.037	0.014	-1.308	1.333	-0.020	0.070		0.570	-0.035	0.809	0.319	0.136	0.551
HD	2.365	-0.027	0.217	0.553	0.300	0.092	-0.037	0.014	-0.263	1.047	-0.038	0.068		0.641	0.009	0.689	0.342	0.098	0.741
HPQ	5.184	0.060	0.249	0.579	0.300	0.042	-0.048	0.005	-0.489	1.121	-0.049	0.076		0.619	-0.027	0.634	0.458	0.197	0.597
IBM	1.907	-0.008	0.141	0.618	0.283	0.040	-0.059	0.008	-0.352	1.086	-0.043	0.055		0.641	0.009	0.695	0.315	0.086	0.822
INTC	15.82	-0.016	0.347	0.552	0.308	0.040	-0.043	0.004	-0.700	1.166	-0.018	0.031		0.926	0.030	0.748	0.223	-0.020	0.989
JNJ	1.123	0.005	0.031	0.622	0.260	0.042	-0.054	0.023	-0.123	1.150	-0.018	0.080		0.414	0.072	0.682	0.198	-0.203	1.330
JPM	5.066	-0.021	0.197	0.580	0.356	0.076	-0.067	0.034	-0.301	0.971	-0.057	0.069		0.604	0.099	0.677	0.299	-0.129	0.879
KO	1.695	0.003	0.044	0.514	0.343	0.109	-0.023	0.010	-0.079	0.959	-0.035	0.058		0.336	0.085	0.718	0.132	-0.450	1.805
MCD	2.108	0.061	0.157	0.602	0.257	0.044	-0.026	0.007	-0.340	1.173	-0.058	0.094		0.331	0.032	0.719	0.219	-0.040	1.085
MMM	0.491	0.013	0.175	0.476	0.320	0.101	-0.047	0.004	-0.277	1.073	-0.052	0.058		0.640	-0.069	0.624	0.500	0.239	0.622
MRK	1.690	-0.011	0.377	0.459	0.256	0.113	-0.016	0.003	-0.613	1.239	-0.045	0.062		0.055	-0.024	0.707	0.361	0.120	0.715
MSFT	3.830	-0.008	0.290	0.511	0.315	0.094	-0.023	0.007	-0.505	1.098	-0.018	0.032		0.871	-0.014	0.710	0.312	0.120	0.830
PFE	3.245	-0.044	0.253	0.472	0.306	0.120	-0.002	0.023	-0.321	1.058	-0.042	0.070		0.443	0.034	0.719	0.279	-0.038	0.889
PG	1.888	0.009	0.038	0.499	0.330	0.101	-0.046	0.011	-0.088	0.988	-0.051	0.070		0.441	0.062	0.680	0.227	-0.112	1.154
T	2.651	-0.005	0.195	0.525	0.346	0.080	-0.047	0.035	-0.238	1.005	-0.044	0.088		0.186	-0.014	0.738	0.327	0.095	0.722
UTX	4.328	0.014	0.193	0.497	0.322	0.119	-0.065	0.009	-0.222	0.994	-0.033	0.077		0.464	0.019	0.623	0.432	0.106	0.696
VZ	1.097	-0.023	0.148	0.570	0.297	0.072	-0.038	0.020	-0.236	1.078	-0.041	0.066		0.459	0.039	0.725	0.247	-0.048	0.953
WMT	1.962	-0.027	0.107	0.625	0.229	0.035	-0.037	0.017	-0.295	1.276	-0.017	0.068		0.677	0.042	0.784	0.145	-0.208	1.364

The table reports point estimates of the dynamic parameters of the Realized GARCH Beta model described in Equations (1)-(8). The starting values for the conditional variance of the market, the conditional variance of the asset and the conditional correlation are treated as parameters and the reported as h_0 , h_1 and ρ_1 , respectively.

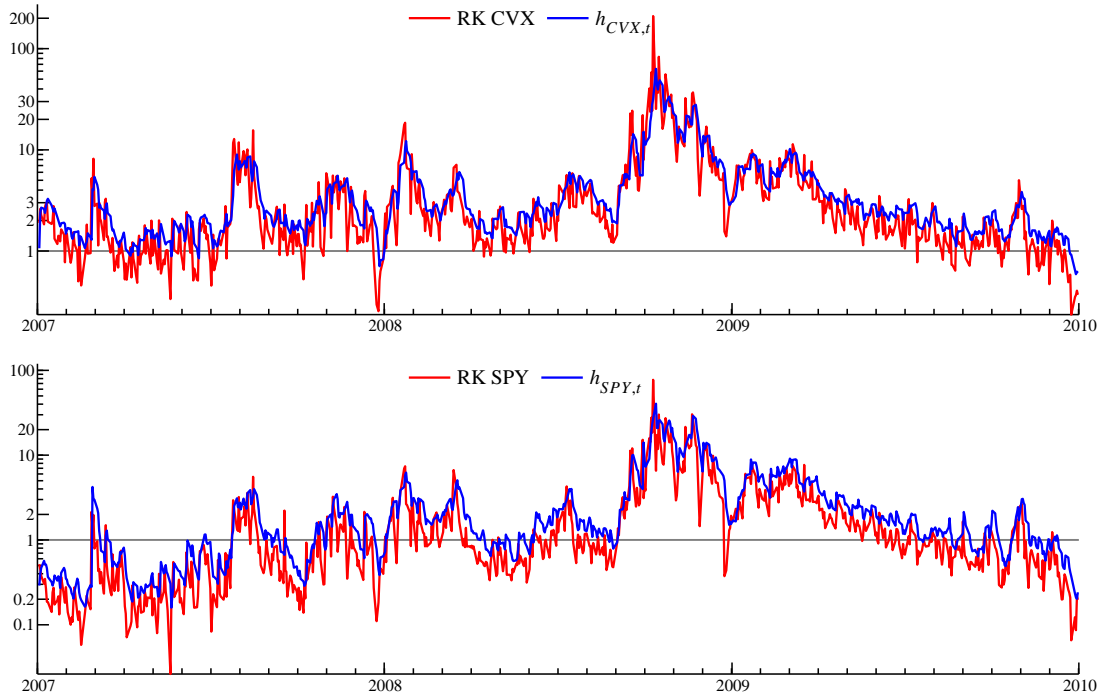


Figure 1: Realized kernel (RK) variance and conditional variance of CVX (upper panel) and SPY (lower panel) over the period 2007 – 2009.

For completeness, we also report estimates of the covariance matrix of the measurement innovations Σ in Table 2.

Table 2: Covariance matrix estimates for measurement innovations.

AA	AIG	AXP	BA
$\begin{bmatrix} 0.149 & & \\ 0.076 & 0.172 & \\ 0.026 & 0.016 & 0.028 \end{bmatrix}$	$\begin{bmatrix} 0.152 & & \\ 0.092 & 0.262 & \\ 0.028 & 0.020 & 0.029 \end{bmatrix}$	$\begin{bmatrix} 0.151 & & \\ 0.101 & 0.195 & \\ 0.030 & 0.018 & 0.030 \end{bmatrix}$	$\begin{bmatrix} 0.150 & & \\ 0.085 & 0.174 & \\ 0.024 & 0.009 & 0.029 \end{bmatrix}$
BAC	C	CAT	CVX
$\begin{bmatrix} 0.150 & & \\ 0.111 & 0.215 & \\ 0.031 & 0.027 & 0.026 \end{bmatrix}$	$\begin{bmatrix} 0.150 & & \\ 0.105 & 0.204 & \\ 0.031 & 0.026 & 0.028 \end{bmatrix}$	$\begin{bmatrix} 0.149 & & \\ 0.087 & 0.162 & \\ 0.028 & 0.017 & 0.029 \end{bmatrix}$	$\begin{bmatrix} 0.148 & & \\ 0.088 & 0.157 & \\ 0.027 & 0.021 & 0.029 \end{bmatrix}$
DD	DIS	GE	GM
$\begin{bmatrix} 0.151 & & \\ 0.097 & 0.174 & \\ 0.030 & 0.017 & 0.032 \end{bmatrix}$	$\begin{bmatrix} 0.150 & & \\ 0.085 & 0.176 & \\ 0.030 & 0.018 & 0.027 \end{bmatrix}$	$\begin{bmatrix} 0.149 & & \\ 0.110 & 0.203 & \\ 0.034 & 0.027 & 0.028 \end{bmatrix}$	$\begin{bmatrix} 0.152 & & \\ 0.073 & 0.311 & \\ 0.019 & 0.000 & 0.021 \end{bmatrix}$
HD	HPQ	IBM	INTC
$\begin{bmatrix} 0.150 & & \\ 0.093 & 0.179 & \\ 0.029 & 0.019 & 0.027 \end{bmatrix}$	$\begin{bmatrix} 0.151 & & \\ 0.088 & 0.199 & \\ 0.029 & 0.017 & 0.027 \end{bmatrix}$	$\begin{bmatrix} 0.150 & & \\ 0.096 & 0.165 & \\ 0.030 & 0.021 & 0.027 \end{bmatrix}$	$\begin{bmatrix} 0.149 & & \\ 0.094 & 0.168 & \\ 0.023 & 0.016 & 0.021 \end{bmatrix}$
JNJ	JPM	KO	MCD
$\begin{bmatrix} 0.149 & & \\ 0.079 & 0.203 & \\ 0.028 & 0.014 & 0.030 \end{bmatrix}$	$\begin{bmatrix} 0.150 & & \\ 0.110 & 0.195 & \\ 0.031 & 0.025 & 0.027 \end{bmatrix}$	$\begin{bmatrix} 0.149 & & \\ 0.084 & 0.165 & \\ 0.031 & 0.021 & 0.030 \end{bmatrix}$	$\begin{bmatrix} 0.150 & & \\ 0.075 & 0.199 & \\ 0.028 & 0.013 & 0.027 \end{bmatrix}$
MMM	MRK	MSFT	PFE
$\begin{bmatrix} 0.149 & & \\ 0.093 & 0.172 & \\ 0.029 & 0.016 & 0.031 \end{bmatrix}$	$\begin{bmatrix} 0.149 & & \\ 0.080 & 0.224 & \\ 0.027 & 0.011 & 0.026 \end{bmatrix}$	$\begin{bmatrix} 0.149 & & \\ 0.100 & 0.186 & \\ 0.028 & 0.022 & 0.025 \end{bmatrix}$	$\begin{bmatrix} 0.149 & & \\ 0.077 & 0.193 & \\ 0.028 & 0.015 & 0.028 \end{bmatrix}$
PG	T	UTX	VZ
$\begin{bmatrix} 0.150 & & \\ 0.081 & 0.172 & \\ 0.031 & 0.023 & 0.028 \end{bmatrix}$	$\begin{bmatrix} 0.150 & & \\ 0.081 & 0.204 & \\ 0.028 & 0.016 & 0.028 \end{bmatrix}$	$\begin{bmatrix} 0.149 & & \\ 0.094 & 0.171 & \\ 0.030 & 0.019 & 0.032 \end{bmatrix}$	$\begin{bmatrix} 0.149 & & \\ 0.083 & 0.192 & \\ 0.033 & 0.018 & 0.031 \end{bmatrix}$
WMT			
$\begin{bmatrix} 0.150 & & \\ 0.088 & 0.174 & \\ 0.028 & 0.018 & 0.027 \end{bmatrix}$			

The table reports point estimates of the covariance matrix of $(u_{0,t}, u_{1,t}, v_{1,t})$ for the Realized GARCH Beta model described in Equations (1)-(8).

We turn next to the model-implied betas given by

$$\hat{\beta}_t = \frac{\hat{\rho}_t \sqrt{\hat{h}_{0,t} \hat{h}_{1,t}}}{\hat{h}_{0,t}} = \frac{\hat{\rho}_t \sqrt{\hat{h}_{1,t}}}{\sqrt{\hat{h}_{0,t}}}, \quad (10)$$

where “hats” denote estimated quantities. It is important to note the difference between the model-implied betas and the raw realized betas defined as

$$\tilde{\beta}_t = \frac{y_{1,t}\sqrt{x_{1,t}x_{0,t}}}{x_{0,t}} = \frac{y_{1,t}\sqrt{x_{1,t}}}{\sqrt{x_{0,t}}}. \quad (11)$$

The model-implied betas take into account the presence of measurement error in the realized quantities as well as the dynamic linkages between realized measures and conditional moments.

The difference between $\hat{\beta}$ and $\tilde{\beta}$ can be seen as the core economic motivation of the paper. Table 3 contains descriptive statistics of $\hat{\beta}_t$ across the stocks in our analysis.

Table 3: Descriptive statistics of the model-implied beta.

	<i>Model-implied beta</i>					<i>Realized beta</i>				
	Mean	Median	Min	Max	St.dev.	Mean	Median	Min	Max	St.dev.
AA	1.413	1.372	0.645	2.411	0.270	1.208	1.171	-0.311	3.224	0.431
AIG	1.309	1.186	0.323	9.789	0.643	1.127	1.004	-0.673	12.989	0.712
AXP	1.308	1.250	0.436	3.023	0.382	1.117	1.062	0.075	3.027	0.440
BA	0.961	0.959	0.406	1.586	0.147	0.955	0.950	-0.013	2.592	0.271
BAC	1.154	0.938	0.471	4.330	0.585	1.085	0.903	0.053	6.271	0.633
C	1.322	1.180	0.522	4.090	0.514	1.237	1.120	-0.210	5.327	0.554
CAT	1.257	1.239	0.511	2.195	0.245	1.133	1.102	-0.103	2.740	0.337
CVX	0.901	0.866	0.079	1.671	0.242	0.880	0.866	-0.421	2.507	0.339
DD	1.047	1.027	0.535	1.702	0.155	1.028	1.019	0.135	2.173	0.276
DIS	1.065	1.058	0.560	1.729	0.186	0.942	0.921	-0.066	2.896	0.299
GE	1.076	1.045	0.449	2.380	0.228	1.025	0.990	-0.018	3.508	0.313
GM	1.271	1.258	0.107	3.707	0.401	1.012	0.921	-0.229	4.792	0.523
HD	1.053	1.043	0.348	2.015	0.171	1.075	1.052	-0.034	3.336	0.314
HPQ	1.076	1.057	0.499	1.917	0.205	1.048	0.996	0.247	2.558	0.320
IBM	0.863	0.862	0.436	1.277	0.118	0.882	0.871	0.212	1.873	0.204
INTC	1.425	1.416	0.511	2.806	0.310	1.394	1.333	0.219	3.284	0.413
JNJ	0.521	0.536	0.205	0.961	0.127	0.593	0.580	-0.190	1.645	0.238
JPM	1.404	1.323	0.667	3.426	0.381	1.236	1.159	0.306	4.090	0.451
KO	0.550	0.555	0.286	0.907	0.096	0.657	0.650	-0.111	1.753	0.227
MCD	0.709	0.710	0.294	1.205	0.153	0.795	0.778	-0.454	2.087	0.277
MMM	0.794	0.796	0.260	1.236	0.140	0.856	0.845	0.054	2.147	0.214
MRK	0.777	0.773	0.069	1.520	0.192	0.770	0.771	-0.597	3.638	0.277
MSFT	1.006	0.975	0.473	1.774	0.195	1.026	0.987	-0.297	2.670	0.298
PFE	0.868	0.864	0.403	2.253	0.152	0.814	0.794	-0.214	5.557	0.278
PG	0.565	0.562	0.263	0.951	0.108	0.655	0.639	-0.152	1.578	0.214
T	0.817	0.819	0.236	1.527	0.201	0.832	0.828	-0.314	2.773	0.311
UTX	0.948	0.946	0.490	1.498	0.138	0.927	0.921	-0.011	1.863	0.240
VZ	0.843	0.836	0.436	1.470	0.147	0.869	0.856	-0.364	1.935	0.286
WMT	0.725	0.742	0.344	1.319	0.148	0.844	0.834	-0.268	3.895	0.257
Overall	1.001	0.972	0.388	2.299	0.241	0.966	0.928	-0.129	3.404	0.343

The table reports descriptive statistics of $\hat{\beta}_t$ and $\tilde{\beta}_t$ for the sample period January 3, 2002 to the end of 2009. The last row of the table reports the cross-sectional (across stocks) average of each statistic.

The betas of the 29 stocks in our sample exhibit a fairly large variation, although much smaller than that of realized betas which is attributed to the smoothness of the conditional moments compared to their raw realized counterparts. Furthermore, realized betas are often negative, which is not realistic. Since the

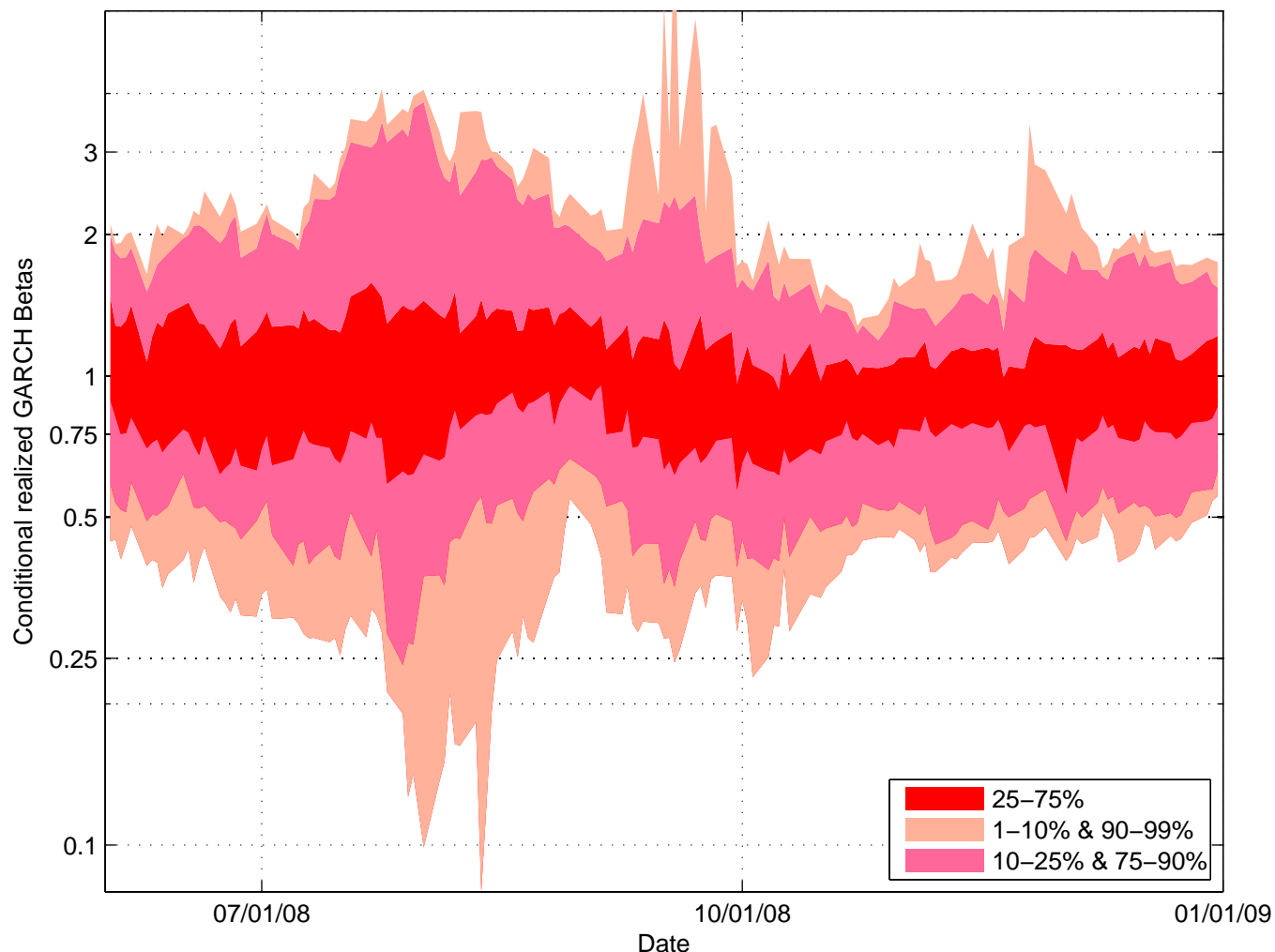


Figure 2: Quantile time series plot of conditional realized GARCH betas for the period 06.2008 – 12.2008.

capitalization of the 29 companies in our dataset relative to the S&P 500 is quite large, the average beta is practically equal to one.

In Figure 2 we present a quantile time series plot of the cross sectional variation in the conditional beta for the initial crisis period June - December 2008 around events such as the collapse of Lehman Brothers. Intriguingly, betas and correlations do not behave similarly. While there is an overwhelming evidence that correlations increased dramatically (the cross-sectional average of the correlation increased from between 50% to 75% to over 75%), betas do not show a recognizable upward trend. It is interesting to note that around August 2008, the increased volatility of the stocks in the financial sector increased the volatility of the market index substantially. For stocks in other industries (see e.g., CVX below) that were not initially affected by the turmoil and maintained normal levels of volatility, the increase of market volatility led to a sharp decrease in correlations and betas evident in the fat lower tail of the cross-sectional quantile plots in the figures above. As the crisis became more ubiquitous, correlations surged, but the distribution of betas across stocks remained fairly stable.

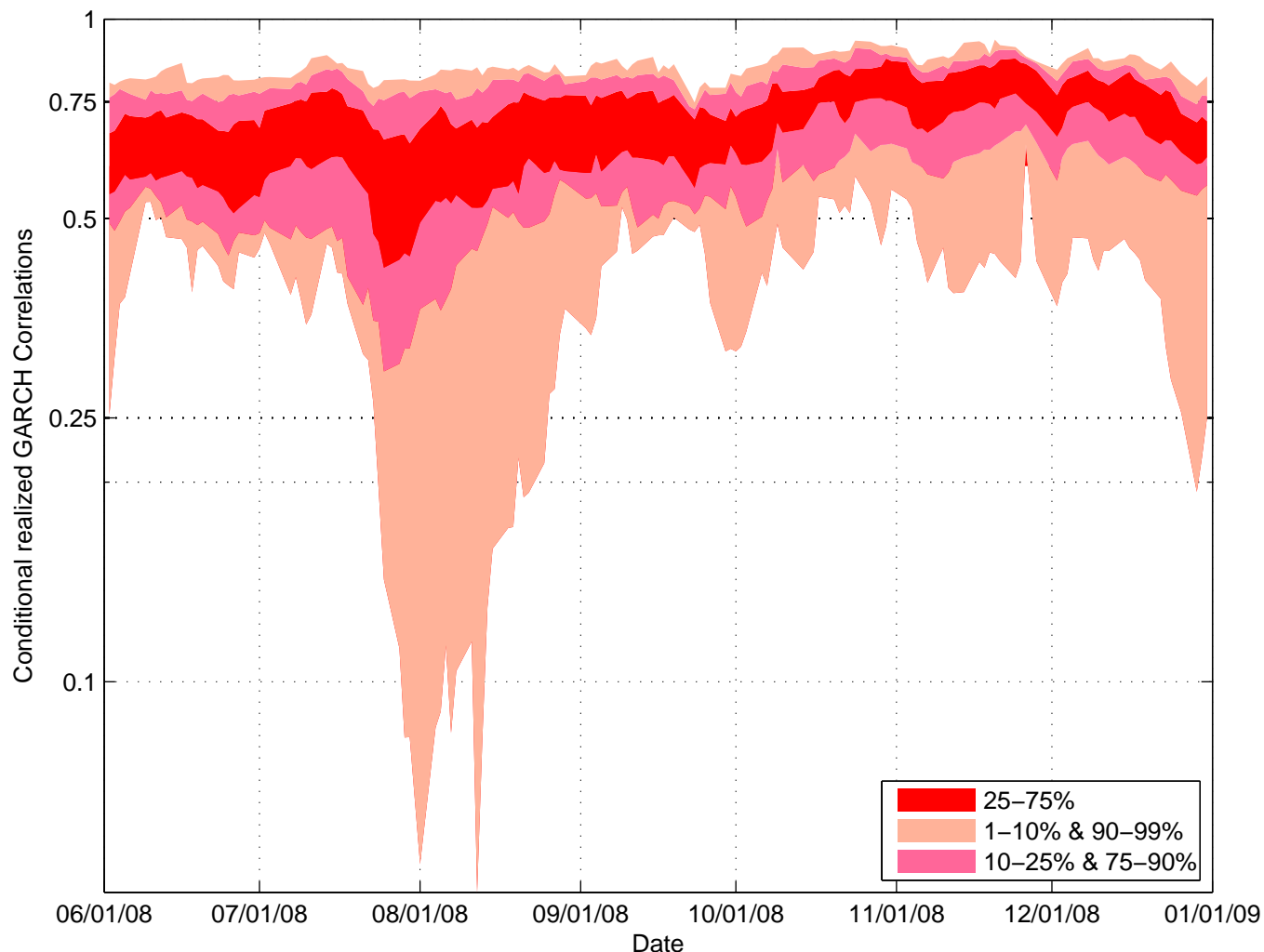


Figure 3: Quantile time series plot of conditional realized GARCH correlations for the period 06.2008 – 12.2008.

To examine the time variation of $\hat{\beta}_t$ in more detail, we continue with our previous example, and present graphic results for the realized and the conditional beta and correlation of CVX over the last three years in our sample in Figure 4.

In the example above correlation changes rapidly during the sample period, which is reflected in the systematic risk of CVX represented by its beta ranging from close to zero to over one. Variation of this magnitude would be close to impossible to obtain with standard approaches using rolling window OLS techniques based on daily returns. The question which arises is whether short-term variations in the systematic risk of a company of such magnitude are plausible and can be rationalized from an asset-pricing perspective. In light of the findings of Lewellen and Nagel (2006), large time variation in betas can help explain asset pricing “anomalies”. We plan to address this issue in future research.

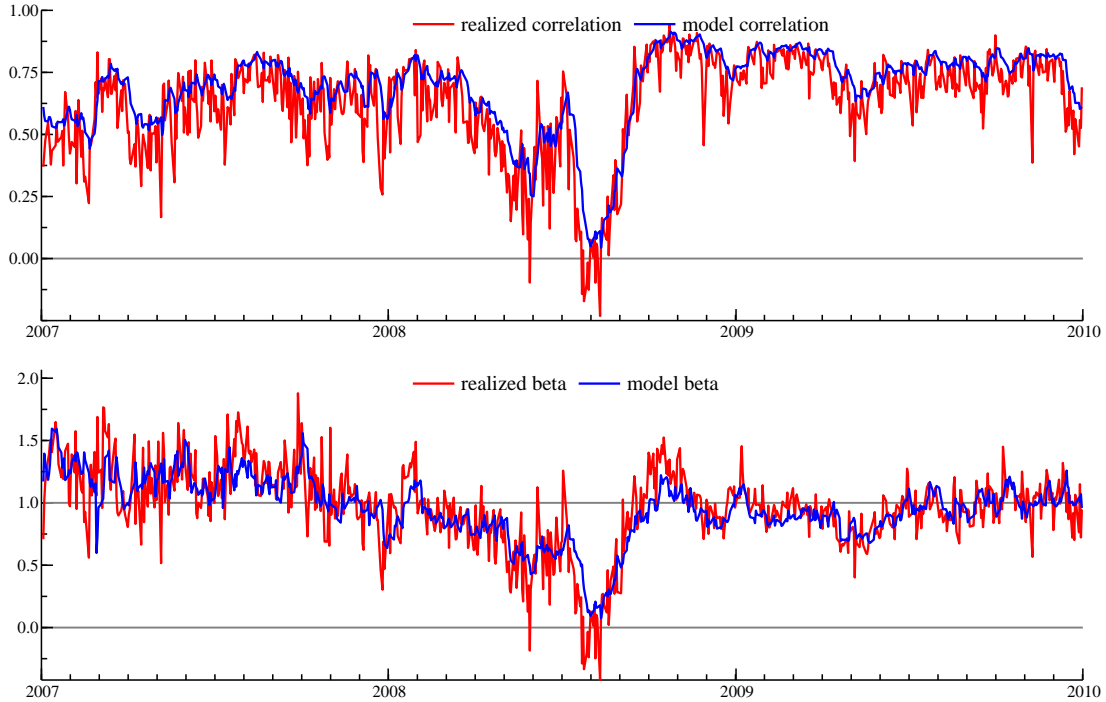


Figure 4: Realized and conditional correlation (upper panel) and beta (lower panel) of CVX over the period 2007 – 2010.

5 Conclusion

In this paper we propose a model for the joint dynamics of conditional and realized measures of volatility and correlation. The model builds on a self-contained system of equations linking realized to conditional measures and allows the construction of precise conditional betas often used as measures of systematic risk in finance. The approach is easy to apply to multiple assets and amenable to a number of extensions. We provide a detailed roadmap of the estimation procedure in the case of single-factor structure and an outlook into the multi-factor extension. The specification we employ allows for leverage effects and spillover effects between the assets' and the market volatility. In this respect the model combines the flexibility of the GARCH modeling framework with the statistical precision in volatility measurement resulting from the use of high-frequency data.

The empirical study we undertake reveals some interesting features of the cross-sectional variation of the conditional betas, as well as their time-series variation. In particular, we find that betas are far more variable than typically obtained with rolling-window OLS regressions, which has potential implications for the validity of the conditional CAPM, which is to be investigated in future work. Notably, however, our conditional betas are less variable than raw realized betas, since we filter the noise in the realized measures through a measurement equation linking the conditional to the ex-post realized moments.

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