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ASSESSING MATHEMATICAL MODELLING COMPETENCY

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Abstract—Focusing on the concept of competence in mathematics education has many analytical implications. Multidimensionality is a necessary, but challenging, approach and in this chapter I address the need to work with at least three dimensions to make a valid assessment of someone's possession of competence. This is demonstrated in regard to mathematical modelling competency, before the presentation of two challenges of bringing this multidimensional approach into educational practice. The two challenges are: The conflict with simple ranking as an educational goal and the conflict with the dominant focus on technical levels in mathematics education.

1. INTRODUCTION

One of the main goals of mathematics education at all educational levels should be to assist the students' development of mathematical modelling competency. Such developments need to be assessed in a valid way, as is the case with all the main goals within educational systems framed by assessment structures, if these goals are to remain the focus of the educational endeavour. These are two not terribly controversial claims. The first is a priority in educational policy with a lot of analytical back-up in the mathematics education research literature, not least expressed in the proceedings from the ICTMA conferences (for example, Ye et al., 2003, and Lamon et al., 2003), where the sharing of this goal can be seen as the "constitutional glue" of this research community. The second claim is even less controversial as an analytical statement (the degree to which the consequence of accepting it is put into educational practice is a different issue). I do not know of any analysis arguing against it, and it is defended by many experts in the mathematics education research literature, for example, Niss (1993).

The two claims will not be defended here. They are mentioned because they jointly lead to a statement that is the premise for the arguments and conclusions put forward in this paper, namely that it is crucial to develop and implement valid assessments of the students' mathematical modelling competency as part of the development of mathematics education.

Based on this premise, the objective of the paper is to argue that a multidimensional approach is a necessary, but challenging, condition for valid assessments of someone's mathematical modelling competency. The analysis is structured as follows:

First, I propose coherent definitions of the concepts “competence” and “mathematical modelling competency” and address some of the implications of these definitions.

Then, I focus on one such implication by arguing that the proposed definition of competence suggests three dimensions to work with when assessing someone’s mathematical modelling competency.

Using such a three-dimensional approach is a challenging task, and I close the paper by offering two examples of this. One example has to do with the political wish for simple ranking systems as an educational tool, and the other example has to do with the dominance of the traditional strong focus on the technical level when assessing someone’s performance in mathematics education.

2. FROM “COMPETENCE” TO “MATHEMATICAL MODELLING COMPETENCY”

2.1 “Competence” as an Analytical Concept

Competence is defined as someone’s insightful readiness to act in response to the challenges of a given situation (Blomhøj & Jensen, 2006).

The most important characteristic of this definition is that it makes competence headed for action. As argued in Blomhøj and Jensen (2003), “action” must be interpreted broadly, as the term “readiness to act” in the definition of competence could imply a positive decision to refrain from performing a physical act, or indirectly being guided by one’s awareness of certain features in a given situation. However, no competence follows from being immensely insightful, if this insight cannot be activated in this broad interpretation of the word “action”.

Secondly, all competencies have a sphere of exertion, that is, a domain within which the competency can be brought to maturity (ibid.). This does not mean that a competency is contextually tied to the use of a specific method for solving a given task. If this was the case, the attempt to define general competencies would have no meaning. Competencies are only contextual in the sense, that they are framed by the historical, social, psychological etc. circumstances of the “given situation” mentioned in the definition of competence (cf. Wedege, 1999).

2.2 A Mathematical Competency

Mathematical competence is when the challenges mentioned in the definition of competence are mathematical.

This is a both straightforward and rather uninteresting extension of the general definition of competence. To make the analysis interesting and binding for the development of mathematics education, we need to be more specific by analysing and discussing what constitutes a mathematical challenge. This leads to a focus on *a mathematical competency* defined as someone’s insightful readiness to act in response to *a certain kind of mathematical challenge* of a given situation.

Hence, mathematical competence is analytically spanned by a set of such mathematical competencies, and it is a very interesting challenge to try to come up with a suggestion for, and exemplification of, the elements of such a set. Niss and

Jensen (to appear) represent an attempt to meet this challenge that has proven to promote a lot of good discussions and further analysis in the mathematics education community (cf. Blum et al., 2006).

2.3 Mathematical Modelling Competency

Mathematical modelling competency is – in accordance with the general definition of competence – here defined as someone’s insightful readiness to carry through all parts of a mathematical modelling process in a given situation (Blomhøj & Jensen, 2003).

Again, to avoid arguing tautologically, there is a call for being more specific by expressing what is meant by “all parts of a mathematical modelling process”. The model I use of this process, which is inspired by and not far from many of the other models presented in the literature, is depicted in Figure 1. It is analysed and presented in greater detail in Blomhøj and Jensen (2003) and Jensen (2006).

Figure 1. A visual representation of the mathematical modelling process (Blomhøj & Jensen, 2003).

3. ASSESSING (MATHEMATICAL MODELLING) COMPETENCY – A MULTIDIMENSIONAL APPROACH

The definition, proposed here, suggests three dimensions to work with when assessing someone’s possession of a competency (Niss & Jensen, to appear):

Degree of coverage, indicating which *aspects* of the competency someone can activate and the degree of *autonomy* with which this activation takes place.

Radius of action, indicating the spectrum of *contexts and situations* in which someone can activate the competency.

Technical level, indicating how *conceptually and technically advanced* the mathematics is that someone can integrate relevantly in activating the competency.

3.1 Degree of Coverage

Focusing on mathematical modelling competency, the degree of coverage addresses which part of the modelling process someone can work with and the level of the reflections involved (Blomhøj & Jensen, 2006).

One of the advantages of working with an explicitly expressed model of the mathematical modelling process in mathematics education is that it stresses the necessity of caring for this dimension: A person able to systematize an open situation in a way reflecting the wish for developing a mathematical model clearly has a higher degree of coverage than someone who can only handle pre-systematized situations. Further, a person who can enter an inner dialogue regarding the validation of a modelling process has a higher degree of coverage than someone who can only evaluate the model results and not the process leading to them.

In both these examples, someone who can carry through the various sub-processes in the mathematical modelling process, but only when prompted to do so, is more competent than someone who can not enter these processes at all, but less competent than the one who autonomously initiates the work.

3.2 Radius of Action

Focusing on mathematical modelling competency, the radius of action addresses the domain of situations in which someone can perform mathematical modelling activities (ibid.).

The need for this dimension is a consequence of the contextual nature of competencies, cf. the above discussion. Differences experienced between domains can be related to the overall mathematical approach one is “invited” to use and/or the characteristics of the extra-mathematical challenge one is trying to deal with by means of a mathematical model: Someone generally capable of modelling challenges of a geometrical nature, for example, need not be as competent when it comes to discrete mathematics or statistics, for example. Also, the fact that someone is very competent when it comes to developing and using optimization models in everyday shopping situations, for example, does not guarantee the same competence when it comes to design problems, for example.

3.3 Technical Level

Focusing on mathematical modelling competency, the technical level addresses which kind of mathematics someone can use and how flexible they are in their use of mathematics (ibid.).

This dimension represents the size and content of the “mathematical toolbox”: Someone who can model a situation by means of establishing a functional relationship is more competent than another person who can only work with one

variable “tied up” by an equation, but less competent than someone who can also consider using differential equations.

3.4 A Geometrical Model

The three dimensions almost suggest themselves to be visualised geometrically as in Figure 2. In this model the possession of a competence is represented by a volume and, consequently, progression is represented by an increasing volume.

This has two analytical consequences, with associated morals, that make them worth pointing out. Firstly, if the level on one of the axes is zero, that is, if the competence has not been developed at all along one of the dimensions, then the volume is also zero, that is, then the competence in its entirety has not been functionally developed either. Moral: We need to pay attention to all the dimensions when we attempt to support the development of a competence, for example, mathematical modelling competency, among a group of students.

Secondly, a significant increase in volume, that is, a significant progression in someone’s competence, is easily detectable, whereas a certain volume, that is, a certain level of competence, cannot be pointed out uniquely, since it can be achieved in infinitely many ways. Moral: We can use the multidimensional approach for assessment to recognize and acknowledge progression in someone’s particular mathematical competency, for example, modelling, but we cannot identify the same level of a competence across people in any simple way, and a direct ordering is impossible.

Figure 2. A visual representation of three dimensions to work with when assessing someone’s competence (Niss & Jensen, 2006 ch9).

4. CHALLENGES OF A MULTIDIMENSIONAL APPROACH

As we know from mathematics, moving from one to several dimensions when analysing something is a dramatic change of perspective. It is therefore not surprising that this is also the case when it comes to addressing the challenge of

assessing a mathematical competency: Difficulties are to be expected and need to be seen as challenges, of which I will shortly address two of the more serious ones.

4.1 The Wish for Simple Ranking Systems

The most fundamental challenge has already been touched upon in the discussion of the geometric model in the previous section: Traditionally, one of the main goals of including assessment as a fundamental part of the educational enterprise have been to summatively compare and rank the performance of different people by mapping this performance to a simple grading scale, and a multidimensional approach to assessment fundamentally conflicts with this “simple ranking goal”, *a priori*.

We have stated it this way in Niss and Jensen (to appear, ch4): “It is important to emphasise that even though we have chosen terms which imply the possibility of simple quantitative measurement, there is no such assumption in the following considerations. The only thing we are implying in this regard is that each of the dimensions allows us some kind of ordering, that is, that one version of a competence can, in relation to a specific dimension, be more or less comprehensive than another version of the same competence. Since this is only a partial ordering, it may well happen that two arbitrary versions of the same competence cannot be compared in this way.”

4.2 The Traditional Dominance of the Technical Level

The second challenge I want to address is *a posteriori*: It is my experience, both from informal discussions with many mathematics teachers and from being involved in several developmental projects, that the mathematics education community in general and the mathematics teachers in particular, have a tradition-ridden focus on the technical level when assessing someone’s mathematical performance.

This tradition can make it difficult to give space, both literally and figuratively speaking, cf. the geometric model, for, and to acknowledge, the students progression in the radius of action and, not least, the degree of coverage. This is one of the results of a long term developmental research project, that focused on developing mathematical modelling competency among a group of students in the Danish upper secondary school, suggesting one possible explanation as to why developing mathematical modelling competency is not, in general, the hub of mathematics education (Jensen, 2006).

5. SUMMING UP AND LOOKING AHEAD

Let me sum up the argument given as follows: The three dimensions in the possession and progression of a mathematical competency discussed in this paper is a vocabulary for discussing quality in mathematical performance, and hence a potentially powerful contribution to maintaining the focus of mathematics education by sharpening the lenses through which we look.

The result of a multidimensional competence-based assessment process can be statements about the performance along each of the chosen dimensions, as

exemplified in Niss and Jensen (to appear). Such an approach represents a more culture-based, more valid but also less reliable alternative to mark-schemes (Wiliam, 1994), known from the way large reports such as Masters Theses and PhD dissertations are often assessed.

To my knowledge, experiences with assessing the possession of explicitly expressed competencies this way are scarce (I am involved in one such ongoing project and the project documented in Jensen (2006) touched upon the idea), and to develop the idea further, both theoretically and experimentally, is definitely a challenge, which calls for more attention within the mathematics education research community.

Technically, it is of course possible to compress multidimensional statements, as the ones in Niss and Jensen (to appear), into one single grade (by assigning one grade to each dimension and then find a balanced average of these grades). This will, however, violate what I consider to be the core of the multidimensional approach discussed in this paper: This is an analytical attempt to respect the complexity of what we are striving for, when we try to assist development of mathematical competencies in general, and mathematical modelling competency in particular, and therefore in deliberate conflict with the assumption that such a development can be, and should be, assessed by means of a simple grading scale.

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