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BEHAVIOUR OF PARALLEL GIRDERS STABILISED WITH U-FRAMES

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Abstract. Lateral torsional buckling is a key factor in the design of steel girders. Stability can be enhanced by cross-bracing, reducing the effective length and thus increasing the ultimate capacity. U-frames are an option often used to brace the girders, when designing through type of bridges and where overhead bracing is not practical. This paper investigates the effect of the U-frame spacing on the stability of the parallel girders. Eigenvalue buckling analysis was undertaken with four different spacings of the U-frames. Results were extracted from finite element analysis, interpreted and conclusions drawn.

Keywords: bridges, lateral torsional buckling, stability, U-frames.

1. Introduction

Economical design of steel girder or truss bridges normally requires the use of bracing to provide adequate stability both in the construction phase and in service. The prime function of the bracing is to limit undesirable out-of-plane deformations likely to occur due to lateral-torsional buckling of the girder or the out-of-plane buckling of the entire truss. The direct advantage of bracing is to reduce the effective unrestrained length of the girder, resulting in a much greater strength. This increase in strength outweighs the cost of bracing. Since the unstable compression zone is at the top in a typical simply supported span, bracing is most effective at the level of the compression flange or compression chord in truss bridges.

Several bracing systems are used, typically between pairs of girders: X bracing, K bracing and transverse girders with moment connections.

The above systems are effective if the spacing between girders is not excessive. In some cases, the width of the bridge is relatively large and in others there exist constraints on the clearance underneath the deck. In such cases, bracing at the top of the girder, or at the level of the top chord in trusses, becomes impractical and U-frames provide an effective solution. Fig. 1 shows a schematic of typical U-frame configuration with two parallel girders. The horizontal cross-beams serve two functions. One, they support the deck above and two, form the horizontal part of the U-frame. The vertical members, typically, also serve two functions. They act as stiffeners for the girder webs and form the vertical members of the U-frame. A rigid connection between the verticals such as stiffeners and the horizontal cross-beam is essential for U-frame action. The efficiency of the U-frame depends upon the flexibility of the top of the verticals of the U-frame as well as on their spacing.

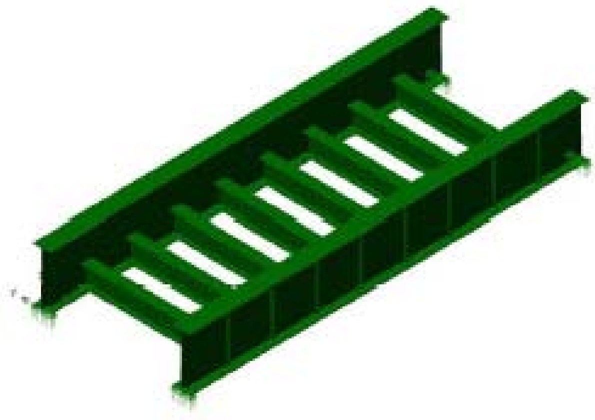

Fig. 1. Typical girders with U-frame

In the UK there are numerous half through girder bridges dating from the latter part of the 19th century that continue to provide a vital part of the transport infrastructure. Similarly, a large number of new bridges, particularly those carrying railway lines, are being designed using this configuration.

2. Previous research

There is a limited number of papers published on the subject. However, a comprehensive treatment was given in Jeffers (1990), in which also considered were some practical aspects of construction of this form of bridge. He followed this with another paper with theoretical treatment of stability of girders braced at the compression flange level.

Yuen (1992) conducted tests on scaled down models of I-girders with U-frames. He compared his results with

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BS5400:Part 3 (1982) and suggested that the radius of gyration of the whole girder section was less important than the radius of gyration of the compression flange together with a contribution from the adjoining web section. He proposed a modification to the limiting stress curve for lateral buckling of bare steel I-section girders. It is questionable whether small scale models are appropriate for forming the basis of changes in code provisions in view of the importance of imperfections.

Bradford (1998) studied inelastic buckling of I-beams with continuous elastic tension flange restraint, using the U-frame model.

More recently, Mehrkar-Asl *et al.* (2005) demonstrated the effectiveness of U-frame restraint in obtaining an optimum solution during assessment of 30 existing bridges. Similarly, Palmer and Wilkins (2006) described the efficiency in construction of 'U' type bridges particularly when the obstacle crossed consists of a busy railway line.

The concept of U-frame action has been adopted for the study of restrained distorsional buckling of continuous composite T-beam sections by Vrcelj and Bradford (2009).

3. Codified rules

BS 5400: Part 3 (1982, 2000) offers guidance on calculating the effective length to be used for girders braced with U-frames. The effective length, which is equal to the wavelength of the deflected shape, is used to determine the design strength of the girder assuming that the girder is unrestrained over its effective length. The theoretical model for calculating the effective length is a strut with regularly spaced elastic lateral restraints. The buckled shape of the girder with U-frame is sketched in Fig. 2. The formula given in the code is similar to the one given below:

$$l_e = k(EI_C L_R \delta_R)^{0.25},$$

where, l_e is the effective length of the girder, EI_C is the rigidity of the compression flange against sideways deflection, L_R is the spacing of U-frame restraints and δ_R is the is the lateral deflection which would occur in the restraint, at the level of the centroid of the flange being considered, when a unit force acts laterally to the restraint only at this point.

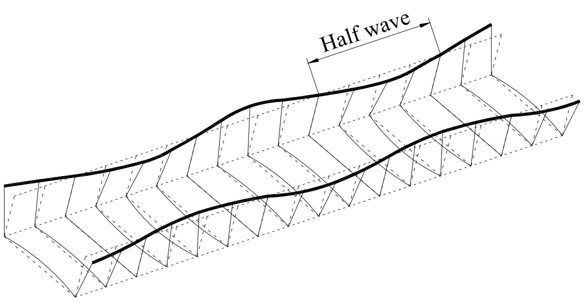

Fig. 2. Buckled shape of girders with U-frames

The constant k had been assigned a value of 2.5 in earlier edition of BS5400:Part 3, but in the 2000 edition takes into account effects of girder dimensions, variation in bending moments along the length of the girder, among others.

4. Finite Element Analysis

In order to assess the validity of the rules given in BS5400:Part 3 (1982, 2000), a parametric study was undertaken using the finite element method. The approach is to obtain the eigenvalues from the finite element model and to deduce the effective length from the corresponding eigenmodes.

4.1. U-frame models

Four different models have been analysed. All models represent the same single span half-through deck type bridge with a span length of 18 m. The width of the bridge is 6.3 m.

The main edge beams are 1.8 m deep giving a span to depth ratio of 10. The transverse beams are 0.8m deep. In all four models, the main and transverse girders have the same cross-section and properties.

The top and bottom flanges of the main beams are 550 mm wide and 30 mm thick. The web is 1740 mm deep and 20 mm thick. Fig. 3 shows the cross-section of the main girder and Fig. 4 that of the transverse beam.

The cross sectional area of the steel beam, excluding the stiffener is 67800 mm^2. The first moment of area about the major axis is 3.46×10^{10} mm^4. The first moment of area about the minor axis is 8.33×10^8 mm^4. This results in the radius of gyration in the minor axis of 111 mm.

Mild steel structural steel with a Modulus of Elasticity of 205 kN/mm^2 has been used as the constitutive material for the beams. Only the steel beams have been modelled.

As the transverse beams would behave compositely with the concrete deck slab for live loading, sectional properties taking into account composite action between both materials using a modular ratio of 6.6 have been used to input the transverse beam properties.

The four bridge deck models had different spacing of U-frames. The values of U-frame spacing used were 2000 mm, 2250 mm, 2571 mm and 3000 mm.

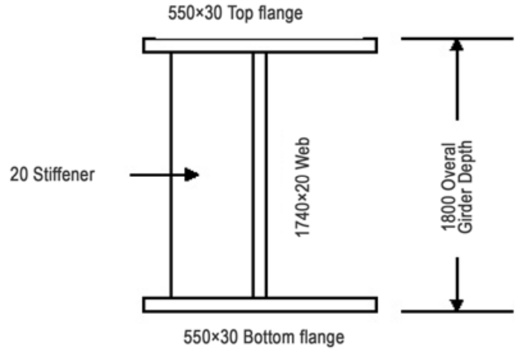

Fig. 3. Cross section of the main girders

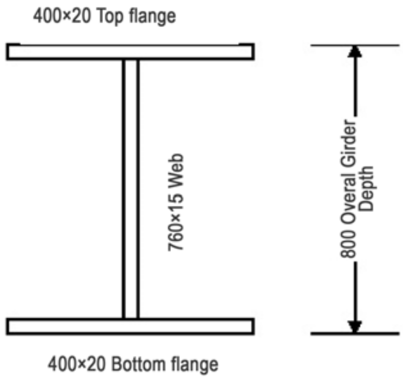

Fig. 4. Cross section of the transverse girders

This is equivalent to having 8, 7, 6 and 5 U-frames respectively.

In order to simulate practical loading on this types of bridges, the applied loading consisted of concentrated loads applied at the connections between the main edge beams and the transverse beams. Ultimate Limit State loading including permanent loads, wind loads and full HA Live Load in accordance with BS5400:Part 2 (2006) are used to calculate load factors at the bifurcation point. These loads were then converted into the concentrated loads applied at the connection with the transverse beams.

In order to compensate for the difference in the number of concentrated forces for different models, adjustment factors have been applied so that the total load applied to the structure is the same for all four models.

Distribution of the load through the structural steelwork down the depth of the neutral axis has been assumed to be uniform down the depth so as to avoid unrealistic local stress values.

Fig. 5 shows the model with U-frame spacing of 2000 mm. This is equivalent to having 8 U-frames along the length

The models consist of 3D second order thin shell elements, located at the centroids of the plates. Thick shell formulation is not considered necessary, as shear deformations normal to the plates are not significant.

Boundary conditions are provided at the location of the supports. One end of one main girder is fixed in the longitudinal direction and both ends are fixed in the lateral direction to prevent horizontal rigid body motion. Vertical non-linear contact joints are used to model any up-lift at the bearings. The vertical springs are released if up-lift occurs and at the up-lift locations there is no vertical bearing support. This analysis ensures that the models accurately represent the behaviour of the main beams and does not over-estimate the torsional restraint at the supports, which might give an unrealistic lateral torsional buckling capacity for the girder.

The vertical stiffness of the support springs is derived in accordance with commercially available information from bridge bearings manufacturers. The bearing area is accurately modelled to produce a reasonable vertical deflection that distributes the reaction over a finite area hence preventing the tendency for the girders to be supported along narrow strips

This assumption is based on the fact that the girders are supported on bearings, which would redistribute the load in such a manner. The possibility of transverse load eccentricity at the bearing stiffeners and the consequential reduction in stiffener capacity is therefore incorporated in the analysis. Results will be presented in detail for this spacing only. A summary of all results is also included.

4.2. Linear Buckling Analysis

Linear buckling analysis of the models is carried out in order to obtain the likely modes of failure. This is achieved by solving the associated eigenvalue problem. Most finite element programs offer an option to determine the eigenvalues and eigenfunctions (modes of deflections).

Sometimes the initial stress stiffness matrix may not be positive-definite, causing the eigensolution method to fail. When using this technique the load level must be adjusted to ensure that all the load factors are greater than unity. In other words, the load applied should be below

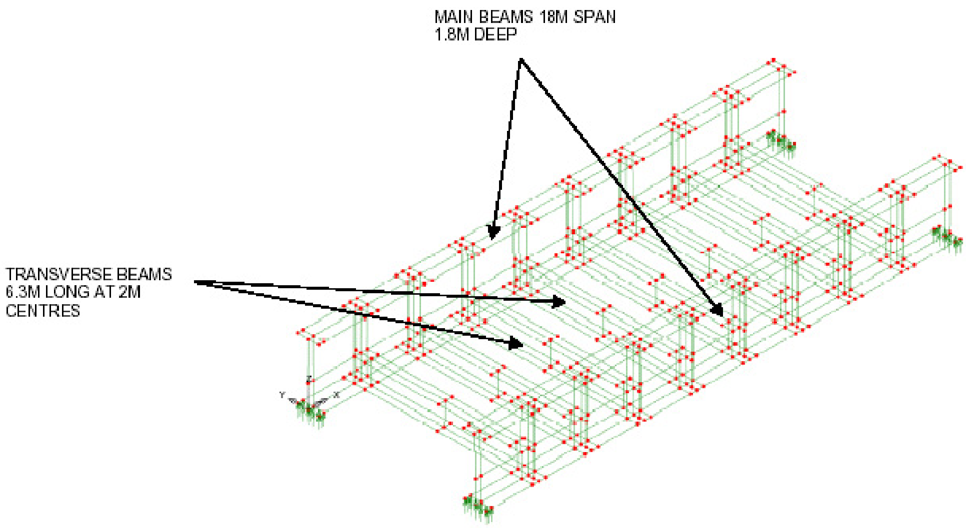

Fig. 5. FE model of girders with U-frames

the lowest expected buckling mode of the structure. An accurate load factor will however only be obtained if the specified load is close to or lower than the collapse load.

The actual buckling load for a given mode is obtained by multiplying the specified magnitude of the applied loading by the load factor obtained from the eigenvalue analysis.

Absolute displacement output is not available from any eigenvalue analysis. It is available, however in a normalised state. For buckling analyses the eigenvectors (mode shapes) are normalised to unity, where the maximum translational degree of freedom is set to one.

The mode shapes are, therefore, accurate representations of the buckling deformation but do not quantitatively define the displacements of the structure at the buckling load.

The results are presented on the basis of relative values of buckling loads and effective lengths.

5. Results

5.1. Dominant buckling mode

For the finite element model with the U-frames spaced at 2000 mm, the buckling load factor for the higher modes appear not to change significantly. For instance, the fifth mode is within 4% of the first buckling mode.

This can be explained by the fact that the thickness of the webs at the supports was explicitly increased in the numerical model in order to avoid local buckling phenomenon for plate elements the supports. This would be in line with normal practice, where extra stiffeners would be introduced at the supports for the same reason.

The deformed shape of the bridge deck for the first buckling mode is shown in Fig. 6. An inspection of the figure suggests that the first, dominant, buckling mode relates to lateral torsional buckling of the main girder.

It is noteworthy that the maximum transverse deflections and torsion of the girder occur at mid-span. This also relates to the boundary condition imposed on the girder that the ends are restrained against torsion.

For the finite element models with the U-frames spaced at 2250 mm, 2571 mm and 3000 mm, the buckling load factors for the fifth mode were within 6.3%, 9.7% and 11.3% of the relevant first buckling mode.

The mode in Fig. 7 relates to buckling of the web due to longitudinal compression. Other modes can be related to dominant half waves in panels further from the mid-span and also to whether the buckled modes are symmetrical or ant-symmetrical in relation to mid-span.

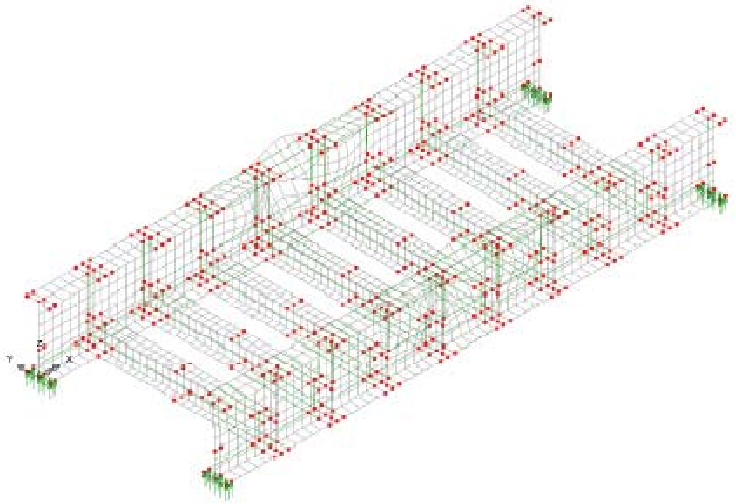

Fig. 6. First buckling mode for U-frame spacing of 2000 mm

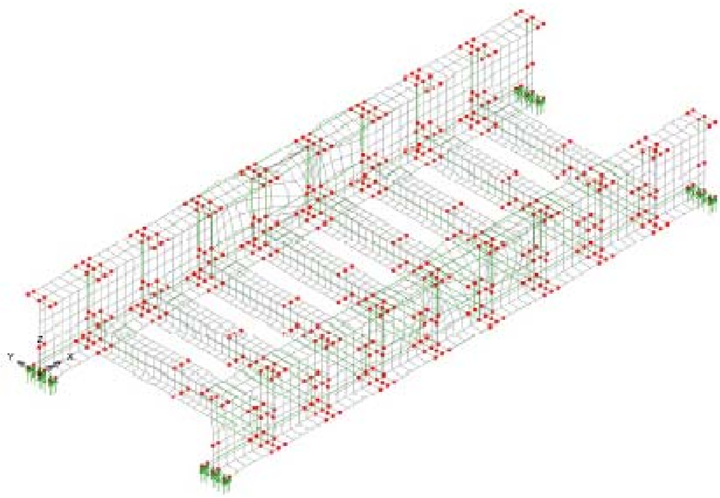

Fig. 7. Second buckling mode for U-frame spacing of 2000 mm

It should be emphasised that the sequence of these modes may well be different depending upon the relative dimensions of the constituent elements.

5.2. Relative values of buckling loads

Table 1 shows the first five buckling load factors for each model. These values appear as significantly greater than 1. This is by design, since the buckling mode analysis is based on elastic behaviour, while the actual design for strength would be based on ultimate strength. The parallel is with the relationship between Euler type buckling of columns against their ultimate strength.

Table 1. Load Factors for different U-frame spacing

Mode	2000 mm	2250 mm	2571 mm	3000 mm
1	5.01	4.34	3.99	3.55
2	5.08	4.44	4.10	3.78
3	5.63	4.97	4.63	4.29
4	5.71	5.23	4.81	4.66
5	5.89	5.48	5.27	5.13

The results are also presented in graphical form in Fig. 8. The curve of interest in that figure is the lowest curve (for Buckling Mode 1), since for any given structure the lowest buckling load is of main significance. The differences between the values of buckling loads for successive modes are not large.

It may be argued that with small differences between various buckling modes, the design may be considered as optimum, since no part of the structure is excessively overdesigned.

5.3. Relative values of effective lengths

Table 2 shows relative values of the effective length, using the value obtained for Mode 1 with 2000 mm spacing of U-frames as the normalizing value. The results are also shown graphically in Fig. 9. The curve of interest now is the top curve. It is noted that for higher modes, the effective lengths are reduced, as is to be expected. Thus, for design, it is not necessary to consider modes other than the first mode for establishing the effective length.

Table 2. Effective Lengths for different U-frame spacing, normalized for Mode 1, 2000 mm

Mode	2000 mm	2250 mm	2571 mm	3000 mm
1	1.00	1.07	1.12	1.19
2	0.99	1.06	1.10	1.15
3	0.94	1.00	1.04	1.08
4	0.93	0.98	1.02	1.04
5	0.92	0.96	0.98	0.99

It is noted that increasing the spacing from 2000 mm to 3000 mm results in an increase in the effective length by about 20%. It would be appreciated that this increase in effective length can result in a significant reduction in the strength of the girder, when using design rules based on ultimate strength.

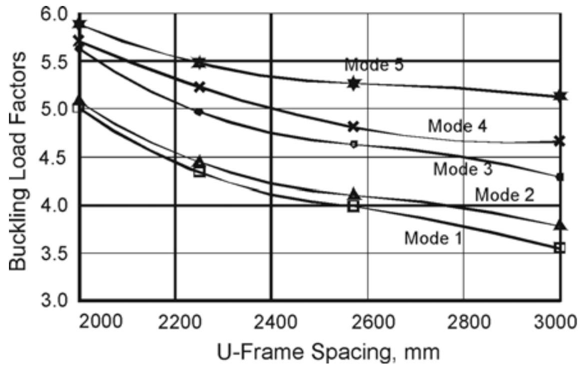

Fig. 8. Variation of Buckling Load factor with U-frame spacing

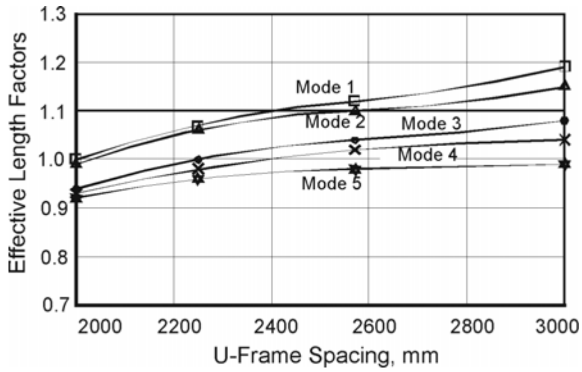

Fig. 9. Variation of Effective Length with U-frame spacing

6. Conclusions

The paper presents results from finite element eigenvalue analysis of two parallel steel plate girders stabilised with U-frame restraints. The dimensions of the girders and the loading on the assembly were obtained using BS 5400: Part 2 (2006) and Part 3 (2000). It is shown that the finite element method can be used effectively to obtain the critical buckling lengths for U-frames. A parametric study has been carried out to quantify the relative values of effective lengths for different spacing of U-frames.

The present analysis used the same sections of stiffener and transverse beam assembly as part of the U-frame, while the U-frame spacing was varied. Although the applied loading on each U-frame was adjusted to take into account the variation in the number of U-frame, in reality, the dimensions of these members would also change. Further study would include this effect.

The results presented were obtained from a linear elastic eigenvalue analysis. In order to assess the validity of the present design rules included in the British and other standards, it would be necessary to supplement this kind of study with a study of ultimate strength of girders braced with U-frames.

It is noteworthy that full-scale test results for this form of bridge construction are not available. Clearly, any modifications to the present design rules in British Standards, and eventually in Euronorms, will need to be validated against experimental results.

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U-FORMIAIS RĖMAIS STABILIZUOTŲ LYGIAGREČIŲJŲ SIJŲ ELGSENA

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S a n t r a u k a

Projektuojant plienines sijas šoninis sukamasis klupumas yra svarbiausias veiksnys. Pastovumas gali būti padidintas skersiniais ryšiais, mažinančiais veikiamąjį ilgį ir padidinančiais ribinę galią. U-formiai rėmai yra dažna priemonė sijoms išramstyti, kai projektuojami tiltai, kurių laikančiosios konstrukcijos yra virš pakloto, o viršutiniai ryšiai yra nepraktiški. Šiame straipsnyje nagrinėjamas U-formių rėmų tarpatramio poveikis lygiagrečiųjų sijų pastovumui. Tikravertis klupumo skaičiavimas buvo atliktas esant keturiems skirtingiems U-formių rėmų tarpatramiams. Aptarti rezultatai, gauti apskaičiavus baigtinius elementus, padarytos išvados.

Reikšminiai žodžiai: tiltai, šoninis sukamasis klupumas, pastovumas, U-formiai rėmai.

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