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Good Volatility, Bad Volatility, and the Cross Section of Stock Returns

Tim Bollerslev, Sophia Zhengzi Li, and Bingzhi Zhao

Abstract

Based on intraday data for a large cross section of individual stocks and newly developed econometric procedures, we decompose the realized variation for each of the stocks into separate so-called realized up and down semi-variance measures, or “good” and “bad” volatilities, associated with positive and negative high-frequency price increments, respectively. Sorting the individual stocks into portfolios based on their normalized good minus bad volatilities results in economically large and highly statistically significant differences in the subsequent portfolio returns. These differences remain significant after controlling for other firm characteristics and explanatory variables previously associated with the cross section of expected stock returns.

I. Introduction

Asset return volatility is naturally decomposed into “good” and “bad” volatility associated with positive and negative price increments, respectively. Based on high-frequency intraday data for almost 20,000 individual stocks spanning more than 20 years, along with new econometric procedures, we show that stocks with relatively high good-to-bad realized weekly volatilities earn substantially lower returns in the subsequent week than stocks with low good-to-bad volatility ratios. Our results remain robust to the inclusion of a variety of controls and systematic risk factors previously associated with the cross-sectional variation in expected returns.
stock returns, and cannot be accounted for by other high-frequency-based realized variation measures recently analyzed in the literature.

The relation between equity return and volatility has been extensively studied in the literature. Even though a number of studies have argued that variance risk is priced at the individual firm level (Adrian and Rosenberg (2008), Da and Schaumburg (2011), and Bansal, Kiku, Shaliastovich, and Yaron (2014)), with changes in variances commanding a negative risk premium, these cross-sectional relationships are generally not very strong. The strong negative relation between idiosyncratic volatility and future stock returns first documented by Ang, Hodrick, Xing, and Zhang (2006b) has also subsequently been called into question by several more recent studies (for a recent discussion of the idiosyncratic volatility puzzle and the many related empirical studies, see, e.g., Stambaugh, Yu, and Yuan (2015), Hou and Loh (2016)). Meanwhile, another strand of the literature has argued that the notion of a traditional linear return-volatility tradeoff relationship is too simplistic, and that more accurate cross-sectional return predictions may be obtained by separately considering the pricing of “upside” and “downside” volatilities (see, e.g., Ang, Chen, and Xing (2006a), Farago and Tédongap (2018)), or higher order conditional moments (see, e.g., Harvey and Siddique (2000), Dittmar (2002), and Conrad, Dittmar, and Ghysels (2013)).

Set against this background, we rely on recent advances in financial econometrics coupled with newly available high-frequency intraday data for accurately measuring “good” and “bad” volatility in a large cross section of individual stocks. In parallel to the standard realized variance measure defined by the summation of high-frequency intraday squared returns (see, e.g., Andersen, Bollerslev, Diebold, and Labys (2001)), the pertinent up and down realized semi-variance measures of Barndorff-Nielsen, Kinnebrock, and Shephard (2010) are simply defined by the summation of the squared positive and negative intraday price increments, respectively. Rather than separately considering the two semi-variance measures, we focus on their relative difference as a succinct scale-invariant measure for each of the individual stock’s good versus bad volatility. We show that this simple summary measure strongly predicts the cross-sectional variation in the future returns.

In particular, on sorting the individual stocks into portfolios based on their weekly relative good minus bad volatility measures, we document a value-weighted weekly return differential between the stocks in the lowest quintile and the stocks in the highest quintile of 29.35 basis points (bps), or approximately 15% per year. The corresponding robust $t$-statistic of 5.83 also far exceeds the more stringent hurdle rates for judging statistical significance recently advocated by Harvey, Liu, and Zhu (2016). These highly statistically significant spreads remain intact in equal-weighted portfolios, double portfolio sorts controlling for other higher order realized variation measures, as well as in firm-level Fama–MacBeth (1973) type cross-sectional predictability regressions that simultaneously control for other higher order realized variations measures and a long list of other firm characteristics previously associated with the cross section of expected stock returns (e.g. firm size, book-to-market ratio, momentum, short-term reversal, and idiosyncratic volatility).

Our results are closely related to the recent work of Amaya, Christoffersen, Jacobs, and Vasquez (ACJV) (2016), and their finding of significant spreads in
the weekly returns on portfolios comprised of stocks sorted according to a high-frequency-based realized skewness measure. However, while we confirm the key finding of ACJV that the realized skewness significantly negatively predicts the cross-sectional variation in weekly returns, this effect is completely reversed after controlling for the relative good minus bad volatility, with the realized skewness significantly positively predicting the future weekly returns. In contrast, the association between the relative good minus bad volatility remains highly statistically significant after controlling for the realized skewness with a value-weighted weekly quintile spread of 36.45 bps, which is even higher than the spread for the single-sorted quintile portfolios.

The difference between the realized up and down semi-variance measures formally either converges (for increasingly finer sampled intraday returns) to the variation due to positive minus negative price discontinuities, or the signed squared price jumps. Intuitively, since the variation associated with continuous variation, or Brownian price increments, is symmetric and (in the limit) therefore the same for the up and down semi-variance measures, their difference only formally manifests variation stemming from jumps. Meanwhile, instead of this “raw” jump measure, we rely on the relative signed jump (RSJ) variation, defined as the difference between the up and down semi-variance measures divided by the total return variation. The high-frequency-based realized skewness measure of ACJV similarly converges to a scaled version of the price jumps raised to the third power.¹ However, RSJ provides a much easier to estimate and interpret summary measure, directly motivated by the idea that stocks with different levels of good versus bad volatility, as manifest in the form of price jumps, might be priced differently in the cross section.

To better understand the determinants of the RSJ effect, we relate the performance of RSJ-sorted portfolios to other firm characteristics, including firm size, volatility and illiquidity. Larger firms, in particular, tend to have better information disclosure, while firms with more stable prices tend to have less asymmetric information. Consistent with investor overreaction, we document that both of these sets of firms are indeed associated with weaker RSJ performance. We also find that the RSJ effect is stronger among more illiquid stocks, consistent with the idea that higher transaction costs limit the forces of arbitrage (see, e.g., Shleifer and Vishny (1997)). Taken together this therefore suggests that the predictability associated with RSJ may be driven by investor overreaction and limits to arbitrage. Further along these lines, our decomposition of the profitability of a simple RSJ-based strategy following the approach of Lo and MacKinlay (1990) suggests that most of the profits arise from autocovariances in the returns and the stock’s own RSJ, as opposed to covariation with other stocks’ RSJ. This is consistent with an overreaction-based explanation.

A number of studies have previously suggested that jumps may be priced differently from continuous price moves, both at the aggregate market level (see, e.g., Eraker, Johannes, and Polson (2003), Bollerslev and Todorov (2011b)), and in the cross section (see, e.g., Yan (2011), Cremers, Halling, and Weinbaum (2015)).

¹As discussed in Feunou, Jahan-Parvar, and Tedongap (2016), the signed jump variation may alternatively be interpreted as a high-frequency measure of skewness.
In contrast to these studies, which rely on additional information from options prices to identify jumps, our empirical investigations rely exclusively on actual high-frequency prices for each of the individual stocks. The study by Jiang and Yao (2013) also utilizes actual high-frequency data together with a specific test for jumps to show that firms with more jumps have higher returns in the cross section. However, they do not differentiate between positive and negative jumps, or good and bad volatility.

Our cross-sectional pricing results are also related to Breckenfelder and Tédongap (2012) and the time-series regression results reported therein, in which the good minus bad realized semi-variance measure for the aggregate market portfolio negatively predicts future market returns. The equilibrium asset pricing model in Breckenfelder and Tédongap (2012), based on a representative investor with recursive utility and generalized disappointment aversion, also provides a possible explanation for this differential pricing of good versus bad volatility at the aggregate market level. The closely related model developed in Farago and Tédongap (2018) similarly implies the existence of a systematic risk factor explicitly related to downside aggregate market volatility. By contrast, our return predictability results rely on the firm specific good minus bad volatility measures, or the relative signed jump variation for the individual stocks.

The paper is also related to the recent work of Feunou, Jahan-Parvar, and Tedongap (2013), Feunou and Okou (2019), Feunou, Lopez Aliouchkin, Tedongap, and Xu (2017), and Feunou, Jahan-Parvar, and Okou (2018) on downside volatility. The last two studies, in particular, rely on similar ideas and techniques to those used here for decomposing both the actual realized volatility and the options implied volatility, and in turn the volatility risk premium defined as the difference between the expected future volatility and the options implied volatility, into up and downside components. From existing empirical results (see, e.g., Bollerslev, Tauchen, and Zhou (2009), Dreschler and Yaron (2011)), it appears that most of the return predictability associated with the variance risk premium is attributable to the downside portion of the premium, again underscoring the differential pricing of good versus bad volatility.

The rest of the paper is organized as follows: Section II formally defines the semi-variances and other realized measures that underlie our empirical findings. Section III discusses the high-frequency data used in the construction of the realized measures, together with the additional control variables that we also rely on. Section IV summarizes the key distributional features of the high-frequency realized measures, including the relative signed jump variation in particular. Section V presents our main empirical findings related to the cross-sectional variation in returns and firm level good minus bad volatility based on simple single-sorted portfolios, single-sorted portfolios with controls, double-sorted portfolios, including RSJ and firm characteristic sorted portfolios, as well as firm level cross-sectional regressions. Section VI provides further evidence on the RSJ-based portfolio performance and where the returns come from. Section VII concludes. More specific details about the data and the variable construction, along with more in depth

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2In a different vein, Bollerslev, Li, and Todorov (2016) have recently documented that stocks that jump more tightly together with the market tend to have higher returns on average.
econometric discussions of some of the empirical findings, are deferred to the Appendix. Additional empirical results and robustness checks are provided in the Supplementary Material.

II. Realized Variation Measures: Theory

The theoretical framework formally justifying the realized variation measures, and the up and down realized semi-variance measures in particular that underly our empirical investigations, are based on the notion of fill-in asymptotics, or increasingly finer sampled returns over fixed time-intervals. This section provides a brief summary of the relevant ideas and the actual estimators that we rely on.

To set out the notation, let $p_T$ denote the natural logarithmic price of an arbitrary asset on day $T$. The price is assumed to follow the generic jump diffusion process,

\begin{equation}
  p_T = \int_0^T \mu \, d\tau + \int_0^T \sigma \, dW_\tau + J_T,
\end{equation}

where $\mu$ and $\sigma$ denote the drift and diffusive volatility processes, respectively, $W$ is a standard Brownian motion, and $J$ is a pure jump process, and the unit time-interval corresponds to a trading day. We will assume that high-frequency intraday prices $p_t, p_{t+1}, \ldots, p_{t+1/n}$ are observed at $n+1$ equally spaced times over the trading day $[t, t+1]$. We will denote the natural logarithmic discrete-time return over the $i$th time-interval on day $t+1$ by $r_{t+i/n} = p_{t+i/n} - p_{t+(i-1)/n}$.

The daily realized variance (RV) is then simply defined by the summation of these within-day high-frequency squared returns,

\begin{equation}
  \text{RV}_t = \sum_{i=1}^{n} \left( r_{t-1+i/n} \right)^2.
\end{equation}

By well-known arguments (see, e.g., Andersen et al. (2001), Andersen, Bollerslev, Diebold, and Labys (2003)), the realized variance converges (for $n \to \infty$) to the quadratic variation comprised of the separate components due to “continuous” and “jump” price increments,

\begin{equation}
  \text{RV}_t \to \int_{t-1}^{t} \sigma^2_s \, ds + \sum_{t-1 \leq \tau \leq t} J^2_\tau,
\end{equation}

thus affording increasingly more accurate ex post measures of the true latent total daily price variation for ever finer sampled intraday returns.\(^3\)

The realized variance measure in equation (2) does not differentiate between “good” and “bad” volatility. Instead, the so-called realized up and down semi-variance measures, originally proposed by Barndorff-Nielsen et al. (2010), decompose the total realized variation into separate components associated with the

\(^3\)In practice, data limitations and market microstructure complications invariably put an upper limit on the value of $n$. As subsequently discussed, following common practice in the realized volatility literature, we rely on a 5-minute sampling scheme, or $1/n = 78$.\(^{10}\)
positive and negative high-frequency returns,

\[ \text{RV}_t^+ = \sum_{i=1}^{n} r_{t-1+i/n} 1_{r_{t-1+i/n} > 0} \], \quad \text{RV}_t^- = \sum_{i=1}^{n} r_{t-1+i/n} 1_{r_{t-1+i/n} < 0}. \]

The positive and negative realized semi-variance measures obviously add up to the total daily realized variation, \( \text{RV}_t = \text{RV}_t^+ + \text{RV}_t^- \). Moreover, it is possible to show that

\[ \text{RV}_t^+ \to \frac{1}{2} \int_{t-1}^{t} \sigma^2 \, ds + \sum_{t-1 \leq \tau \leq t} J^2_\tau 1_{(J_\tau > 0)}, \]

\[ \text{RV}_t^- \to \frac{1}{2} \int_{t-1}^{t} \sigma^2 \, ds + \sum_{t-1 \leq \tau \leq t} J^2_\tau 1_{(J_\tau < 0)}, \]

such that the separately defined positive and negative semi-variance measures converge to one-half of the integrated variance plus the sum of squared positive and negative jumps, respectively.

These limiting results imply that the difference between the semi-variances removes the variation due to the continuous component and thus only reflects the variation stemming from jumps. We will refer to this good minus bad realized volatility measure as the signed jump (SJ) variation,

\[ \text{SJ}_t = \text{RV}_t^+ - \text{RV}_t^- \to \sum_{t-1 \leq \tau \leq t} J^2_\tau 1_{(J_\tau > 0)} - J^2_\tau 1_{(J_\tau < 0)}. \]

The level of the volatility differs substantially across different stocks. As such, the signed jump variation for a particular stock may appear relatively high/low in a cross-sectional sense because the overall level of the volatility for that particular stock is relatively high/low. To help circumvent this, we normalize the signed jump variation by the total realized variation, defining the relative signed jump variation,

\[ \text{RSJ}_t = \frac{\text{SJ}_t}{\text{RV}_t}. \]

This normalization naturally removes the overall volatility level from the SJ measure, rendering RSJ a scale-invariant measure of the signed jump variation restricted to lie between \(-1\) and \(1\).

In addition to these realized variation measures based on the separately defined up and down intraday return variation, we also calculate the daily realized skewness (RSK),

\[ \text{RSK}_t = \frac{\sqrt{n} \sum_{i=1}^{n} r_{t-1+i/n}^3}{\text{RV}_t^{3/2}}, \]

and realized kurtosis (RKT),

\[ \text{RKT}_t = \frac{n \sum_{i=1}^{n} r_{t-1+i/n}^4}{\text{RV}_t^2}, \]
measures analyzed by ACJV. In parallel to the RSJ measure defined above, the
limiting values (for $n \to \infty$) of the RSK and RKT measures similarly manifest
functions of the variation attributable to jumps. In contrast to RSJ, however, which
has a clear interpretation as a measure of the relative signed jump variation, RSK
and RKT converge to scaled versions of the intraday jumps raised to the third and
fourth power, respectively, and are not directly interpretable as standard measures
of skewness and kurtosis. Furthermore, compared to the realized variation mea-

sures, the use of higher order (greater than 2) return moments in the calculation
of the realized skewness and kurtosis measures render them more susceptible to
“large” influential observations, or outliers, and thus generally more difficult to
precisely estimate.

Our main cross-sectional asset pricing analyses are conducted at the weekly
frequency. This directly mirrors ACJV. We construct the relevant weekly real-
ized variation measures by summing the corresponding daily realized variation
measures over the week. Specifically, if day $\tau$ is a Tuesday, we compute the (an-

nualized) weekly realized volatility as,

\[
RVOL_{\tau}^{{\text{WEEK}}} = \left( \frac{252}{5} \sum_{i=0}^{4} RV_{\tau-i} \right)^{1/2},
\]

while for the RSJ, RSK, or RKT measures, their weekly equivalents are defined
as,

\[
RM_{\tau}^{{\text{WEEK}}} = \frac{1}{5} \left( \sum_{i=0}^{4} RM_{\tau-i} \right),
\]

where $RM_i$ refers to the relevant daily measure. For notational convenience, we
will drop the explicit Week superscript and the time $\tau$ subscript in the sequel and
simply refer to each of the weekly measures by their respective abbreviations.

We turn next to a discussion of the data that we use in actually implementing
these measures, as well as the additional control variables that we also rely on in
our portfolio sorts and cross-sectional pricing regressions.

III. Data

Our empirical investigation relies on high-frequency intraday data obtained
from the Trade and Quote (TAQ) database. The TAQ database contains consoli-
dated intraday transactions data for all securities listed on the New York Stock
Exchange (NYSE), the American Stock Exchange (AMEX), NASDAQ and
SmallCap issues, as well as stocks traded on Arca (formerly Pacific Stock Ex-
change) and other regional exchanges. This resulted in a total of 19,896 unique
securities over the Jan. 4, 1993 to Dec. 31, 2013 sample period when matched ac-
cording to the Center for Research in Securities Prices (CRSP) unique PERMNO
numbers. Following the extant literature, we use all common stocks listed on the

\footnote{Whereas the population mean of $RV$ is invariant to the sampling frequency of the returns under-
lying the estimation, the population means of RSK and RKT depend directly on the frequency of the
data used in their estimation. The Supplementary Material also provides direct evidence that the RSJ
estimates are more robust across sampling frequencies than the estimates for RSK.}

\footnote{We also report the results for other definitions of a week in the Supplementary Material.
NYSE, NASDAQ, and AMEX exchanges, with share codes 10 or 11 and prices between $5 and $1,000. We rely on the consolidated trade files provided by TAQ to extract second-by-second prices, but only keep the observations for Monday to Friday from 9:30 AM to 4:00 PM. All-in-all, this leaves us with 1,085 calendar weeks, or around 20 million stock-week observations. Further details concerning the high-frequency data and our cleaning procedures are provided in the Appendix Section A.1.

We also rely on the CRSP database for extracting daily and monthly returns, number of shares outstanding, and daily and monthly trading volumes for each of the individual stocks. To help avoid survivorship bias, we further adjust the individual stock returns for delisting, using the delisting return provided by CRSP as the return after the last trading day. We also use the stock distribution information from CRSP for calculating the overnight returns. We rely on Kenneth R. French’s Web site (https://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html) for obtaining daily and monthly returns on the Fama–French–Carhart 4-factor (FFC4) portfolios.

Lastly, we employ a number of additional explanatory variables and firm characteristics previously associated with the cross-sectional variation in returns. These include: the CAPM beta (BETA), size (ME) and book-to-market ratio (BM) (Fama and French (1993)), medium-term price momentum (MOM) (Jegadeesh and Titman (1993)), weekly reversal (REV) (Lehmann (1990), Jegadeesh (1990)), idiosyncratic volatility (IVOL) (Ang et al. (2006b)), coskewness (CSK) (Harvey and Siddique (2000), Ang et al. (2006a)), cokurtosis (CKT) (Ang et al. (2006a)), the maximum (MAX) and minimum (MIN) daily return (Bali, Cakici, and Whitelaw (2011)), and illiquidity (ILLIQ) (Amihud (2002)). The data for most of these variables are obtained from the CRSP and Compustat databases. Our construction of each of these variables follow standard procedures, as further detailed in the Appendix Section A.2.

IV. Realized Variation Measures: Empirical Distributions

Our calculation of the realized volatility measures are based on high-frequency transactions prices. We resort to a “coarse” 5-minute sampling scheme, resulting in 390 (78 × 5) return observations for the calculation of each of the individual firms’ weekly realized measures. Our choice of a 5-minute sampling scheme aims to balance the bias induced by market microstructure effects when sampling “too finely,” and the theoretical continuous-time arguments underlying the consistency of the realized volatility measures that formally hinge on increasingly finer sample intraday returns. This particular choice also mirrors common practice in the literature (see, e.g., the survey in Hansen and Lunde (2006)).

The high frequency TAQ database only contains the raw prices without consideration of the price differences before and after any distributions.

This same choice was also adapted by ACJV in their construction of the high-frequency realized skewness and kurtosis measures, RSK and RKT. As discussed further in the Supplementary Material, we also experimented with the use of alternative subsampling estimators, high-frequency returns based on mid-quotes, and a sample of more liquid NYSE listed stocks only. Our empirical findings remain robust across all of these alternative estimators and a more restricted sample.
Figure 1 displays the kernel density estimates for the unconditional distributions of the resulting realized measures across all firms and weeks in the sample. Graph A, in particular, depicts the distribution of the new RSJ measure. The distribution is approximately symmetric around 0 with support almost exclusively between −0.5 and 0.5. This finding of approximately the same up and down semi-variance measures, or as implied by equation (5) close to identical positive and negative jump variation at the individual firm level, is in line with the previous evidence for the aggregate market portfolio and the close to symmetric high-frequency-based positive and negative jump tail distributions documented in Bollerslev and Todorov (2011a). Meanwhile, the unconditional distribution in Graph A still implies substantial variation in the RSJ measures across firms and time. From the summary statistics reported in Panel A of Table 1, the time-series mean of the cross-sectional standard deviations of RSJ equals 0.16. It follows readily from the definition in equation (6) that for a firm-week with RSJ 1 standard deviation above 0, the up semi-variance is approximately 38% larger than the down semi-variance, while for a firm-week with RSJ equal to 0.5, the up semi-variance is approximately 3 times as large as the down semi-variance. As we

### FIGURE 1
Unconditional Distributions

Graphs A–D of Figure 1 show the kernel density estimates of the unconditional distributions of the relative signed jump variation (RSJ), realized volatility (RVOL), realized skewness (RSK), and realized kurtosis (RKT) measures, respectively, averaged across all firms and weeks in the sample.

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Formally, for a firm-week RSJ 1 standard deviation away from 0, \( \sum_{i=0}^{4} \frac{RV_{i,t}^{+} - RV_{i,t}^{-}}{RV_{i,t}^{+} + RV_{i,t}^{-}} / 5 = 0.16 \). Thus, assuming that the ratios \( RV_{i,t}^{+} / RV_{i,t}^{-} \) stay approximately constant within a week, this expression implies that \( RV_{i,t}^{+} \) is 38% higher than \( RV_{i,t}^{-} \).
 subsequently document, this nontrivial variation in the RSJ measures across firms and time is associated with strong cross-sectional return predictability.

Mirroring the prior empirical evidence in ACJV, Graphs B–D in Figure 1 show the same across firm and time unconditional distributions for the RVOL, RSK, and RKT realized measures. In parallel to the distribution of RSJ in Graph A, the distribution of RSK in Graph C is approximately symmetric. In comparison with RSJ, however, the distribution of RSK is substantially more heavy-tailed and peaked around 0.9 The unconditional distributions of the realized volatility and kurtosis in Graphs B and D, respectively, are both heavily skewed to the right.

To help illuminate the temporal variation in each of the realized measures implicit in these unconditional distributions, Figure 2 plots the 10-week moving averages of the 10th, 50th, and 90th percentile for the RSJ, RVOL, RSK, and RKT measures. The plot in Graph A reveals a remarkably stable dispersion in the distributions of the RSJ measures over time. The percentiles for RSK shown in Graph C also appear quite steady through time, albeit not as stable as those for RSJ. By contrast, the percentiles of the realized volatilities RVOL shown in Graph B are clearly time-varying, with marked peaks around the time of the burst of the dot-com bubble and the financial crisis of 2007–2008. Consistent with the recent findings in Herskovic, Kelly, Lustig, and Van Nieuwerburgh (2016) and the existence of a common factor structure in idiosyncratic, or firm-specific, volatilities, there is also a distinct commonality in the temporal variation in the RVOL
percentiles. The apparent increase in the cross-sectional dispersion in the RKT measures observed in the final Graph D is in line with the evidence reported in ACJV.

Panel A in Table 1 provides the time-series averages of cross-sectional means and standard deviations for each of the 4 realized measures, as well as the other control variables that we also rely on in our analysis. Panel B of that same table reports the corresponding weekly cross-sectional correlations. RSJ and RSK are the two most highly correlated variables, with a highly significant correlation coefficient of 0.93. The new RSJ measure also correlates significantly with REV, MAX, and MIN, with correlation coefficients of 0.37, 0.24, and 0.32, respectively. The corresponding correlations for RSK are slightly lower, at 0.27, 0.19, and 0.25, respectively.10 This high pairwise correlation and similar correlations with the other control variables are not necessarily surprising, as RSJ and RSK both reflect notions of asymmetry in the intraday return distributions. At the same time, however, our empirical results discussed below suggest that the return

10 The time-series plots of the 4 realized measures sorted by REV (Figure A.1), RSJ (Figure A.2) and RSK (Figure A.3) given in the Supplementary Material further illustrate the cross-sectional dependencies inherent in these correlations.
predictability afforded by RSJ is both stronger and more robust than the return predictability of RSK.

To further clarify the relation between the realized measures and the different control variables, we also employ a series of simple portfolio sorts. At the end of each Tuesday, we sort the stocks by their past weekly realized variation measures and form 5 equal-weighted portfolios. We then calculate the time-series averages of the various firm characteristics for the stocks within each of these quintile portfolios. The results based on these RSJ, RVOL, RSK, and RKT sorts are reported in Panels A–D of Table 2, respectively. Consistent with the correlations discussed above, the portfolio sorts reveal that high RSJ firms tend to be firms with high RSK, REV, MAX, and low MIN. Further, while firms with high RVOL and high RKT tend to be small and less liquid firms, there is no obvious relation between

<table>
<thead>
<tr>
<th>Quintile</th>
<th>Panel A. Sorted by RSJ</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 (Low)</td>
<td>−0.200 0.654 −0.888</td>
</tr>
<tr>
<td>2</td>
<td>−0.061 0.599 −0.247</td>
</tr>
<tr>
<td>3</td>
<td>0.007 0.588 0.026</td>
</tr>
<tr>
<td>4</td>
<td>0.077 0.582 0.307</td>
</tr>
<tr>
<td>5 (High)</td>
<td>0.227 0.620 0.985</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Quintile</th>
<th>Panel B. Sorted by RVOL</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 (Low)</td>
<td>0.014 0.231 0.050</td>
</tr>
<tr>
<td>2</td>
<td>0.013 0.351 0.046</td>
</tr>
<tr>
<td>3</td>
<td>0.019 0.503 0.090</td>
</tr>
<tr>
<td>4</td>
<td>0.009 0.721 0.034</td>
</tr>
<tr>
<td>5 (High)</td>
<td>0.004 1.236 0.012</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Quintile</th>
<th>Panel C. Sorted by RSK</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 (Low)</td>
<td>−0.189 0.665 −0.042</td>
</tr>
<tr>
<td>2</td>
<td>−0.059 0.593 −0.255</td>
</tr>
<tr>
<td>3</td>
<td>0.007 0.578 0.026</td>
</tr>
<tr>
<td>4</td>
<td>0.075 0.576 0.313</td>
</tr>
<tr>
<td>5 (High)</td>
<td>0.215 0.631 1.041</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Quintile</th>
<th>Panel D. Sorted by RKT</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 (Low)</td>
<td>0.005 0.493 0.015</td>
</tr>
<tr>
<td>2</td>
<td>0.007 0.570 0.021</td>
</tr>
<tr>
<td>3</td>
<td>0.008 0.609 0.027</td>
</tr>
<tr>
<td>4</td>
<td>0.011 0.641 0.039</td>
</tr>
<tr>
<td>5 (High)</td>
<td>0.019 0.728 0.081</td>
</tr>
</tbody>
</table>

Further, while firms with high RVOL and high RKT tend to be small and less liquid firms, there is no obvious relation between

Table 2 reports time-series averages of equal-weighted characteristics of stocks sorted by the 4 realized measures. The sample consists of all the NYSE, AMEX, and NASDAQ listed common stocks with share codes 10 or 11 and prices between $5 and $1,000 over the 1993–2013 sample period. RSJ, RVOL, RSK, and RKT denote the relative signed jump variation, realized volatility, realized skewness, and realized kurtosis, respectively. BETA denotes the standard CAPM beta. ME denotes the logarithm of the market capitalization of the firms. BM denotes the ratio of the book value of common equity to the market value of equity. MOM is the compound gross return from day $t - 252$ through day $t - 21$. REV is the lagged 1-week return. IVOL is a measure of idiosyncratic volatility. CSK and CKT are measures of coskewness and cokurtosis, respectively. MAX and MIN represent the maximum and minimum daily raw returns over the previous week. ILLIQ refers to the natural logarithm of the average daily ratio of the absolute stock return to the dollar trading volume over the previous week. Panel A displays the results sorted by RSJ, Panel B by RVOL, Panel C by RSK, and Panel D by RKT.

---

1. The Supplementary Material provides additional cross-sectional regression-based evidence for a negative relationship between RSJ and firm leverage and credit ratings. None of these relations, are very strong, however.
RSK and firm size and illiquidity. Instead, in parallel to the results for the RSJ sorts, firms with high RSK tend to have higher REV and MAX returns and lower MIN returns.

We turn next to a discussion of how similarly constructed portfolio sorts manifest in future return differentials.

V. Good Minus Bad Variation and Future Stock Returns

We begin our empirical investigations related to return predictability by documenting highly statistically significant negative spreads in the returns on quintile portfolios comprised of stocks sorted according to their individual relative signed jump variation measure RSJ, and their realized skewness measure RSK. We then show that the spreads based on RSJ remain highly significant in double portfolio sorts designed to control for other higher order realized variation measures and control variables. In contrast, the double portfolio sorts based on RSK control for RSJ results in the completely opposite cross-sectional relation, with high/low RSK firms associated with higher/lower future returns. We further corroborate these findings in firm level cross-sectional return predictability regressions that simultaneously control for multiple higher order realized measures and other firm characteristics.

A. Single-Sorted Portfolios

At the end of each Tuesday, we sort the stocks into quintile portfolios based on their realized variation measures. We then compute value- and equal-weighted returns over the subsequent week for each of these different quintile portfolios. We also report the results for a self-financing long–short portfolio that buys stocks in the top quintile and sells stocks in the bottom quintile.

Panel A of Table 3 displays the weekly portfolio returns (in basis points) from sorting on RSJ. The column labeled “Return” for the value-weighted portfolios reveals a clear negative relation between the relative signed jump variation and the average future realized returns. The average weekly return decreases monotonically from 32.75 bps for quintile 1 (Low) to 3.40 for quintile 5 (High), yielding a High–Low spread of $-29.35$ bps, with a robust $t$-statistic of $-5.83$. The equal-weighted portfolio returns show the same decreasing pattern between RSJ and the returns over the subsequent week, with a High–Low spread of $-38.54$ bps per week.

To investigate whether these return differences result from exposure to systematic risks, we rely on the popular Fama–French–Carhart 4-factor model (Fama and French (1993), Carhart (1997)). In particular, we regress the excess returns for each of the quintile portfolios and the High–Low spread against the 4 factors for calculating the FFC4 alphas, defined as the regression intercepts. The column in Table 3 labeled “FFC4” shows a similarly strong negative relation between RSJ and the abnormal future returns measured in terms of these FFC4 alphas. The FFC4 alpha of the self-financing value-weighted RSJ strategy, in particular, equals $-28.80$ bps per week and remains highly significant with a $t$-statistic of $-5.77$, while for the equal-weighted portfolios the risk-adjusted FFC4 alpha for the High–Low spread equals $-37.89$ bps, with a $t$-statistics of $-9.95$. 

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https://www.cambridge.org/core
The results for the RVOL, RSK and RKT single-sorted portfolios reported in Panels B–D in Table 3 confirm the findings in ACJV related to the predictability of these same realized measures. Panel C in particular, shows that there is a strong negative relation between the realized skewness and next week’s stock returns. A strategy that buys stocks in the lowest realized skewness quintile and sell stocks in the highest realized skewness quintile generates an average abnormal return of $-17.11$ bps per week, with a robust $t$-statistics of $-4.11$ for the value-weighted portfolios, and $-30.75$ bps with a $t$-statistics of $-10.08$ for the equal-weighted portfolios.\(^\text{12}\) On the other hand, the $t$-statistics associated with the High–Low spreads based on RVOL and RSK in Panels B and D, respectively, are much smaller and statistically insignificant for the value-weighted portfolios. Thus, consistent with the vast existing empirical literature and the lack

\(^{12}\)The stronger RSJ and RSK effects observed for equal-weighted portfolios naturally raise the question of whether the two measures pick up a different feature for less-liquid small-cap stocks compared to more-liquid large-cap stocks. Still, recall that our current sample already is restricted to stocks listed on the NYSE, AMEX, and NASDAQ exchanges, with share codes 10 or 11 and prices between $5$ and $1,000$. Nonetheless, in an effort to help further alleviate this concern, we repeated the single-sorting analyses using only the sample of the Standard & Poor’s (S&P) 500 stocks. RSJ and RSK remain strong predictors for next-week returns within this more limited sample of stocks. In particular, when sorting by RSJ, the FFC4-adjusted High–Low spread equals $-26.23$ bps, with a $t$-statistic of $-5.22$, for the value-weighted portfolios, and $-19.16$ bps, with a $t$-statistic of $-4.77$, for the equal-weighted portfolios. We thank the referee for highlighting this issue.
of a simple risk-return tradeoff relationship, there is at best scant evidence for the realized volatility and kurtosis measures being able to predict the future returns.

To help more clearly illuminate how the long–short portfolios based on the different realized variation measures perform over time, Graphs A and B in Figure 3 depict the cumulative profits for the resulting value-weighted and equal-weighted strategies based on an initial investment of $W_0 = 1$. Specifically, let $W_\tau$ denote the cumulative profit at the end of week $\tau$. The cumulative profits depicted in the figure are then calculated as

$$W_\tau = W_{\tau-1} \times (1 + r_{\text{LONG},\tau} - r_{\text{SHORT},\tau} + r_{f,\tau}),$$

where $r_{\text{LONG},\tau}$ and $r_{\text{SHORT},\tau}$ denote the weekly returns on the long-leg and the short-leg of the portfolio, respectively, and $r_{f,\tau}$ denotes the weekly risk-free rate.

As Figure 3 shows, the RSJ-based strategy outperforms the RSK strategy by quite a wide margin. Moreover, even though the value-weighted RSK-based strategy does result in statistically significant excess return when evaluated over the full sample, most of the superior performance occurs right around the collapse of the dot-com bubble, with almost "flat" returns post 2001. By contrast, the RSJ-based long–short strategy delivers superior returns throughout the sample, both for the value-weighted and equal-weighted portfolios. As a benchmark, the figure also reports the cumulative profits based on REV. The REV-based strategies obviously result in the largest overall cumulative profits.

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Graph A of Figure 3 shows the cumulative gains for a value-weighted long–short portfolio based on the relative signed jump variation (RSJ), realized skewness (RSK), and lagged 1-week return (REV). Graph B shows the cumulative gains for an equal-weighted long–short portfolio. All of the portfolios are rebalanced and accumulated on a weekly basis, as described in the main text.

---

13 The (unreported) cumulative profits based on RKT and RVOL are both close to 0.

14 Following the approach of Tulchinsky (2015), we also calculated the portfolio turnovers for the RSJ-, RSK-, and REV-based strategies to be 63.22%, 64.12%, and 71.76%, respectively. Thus, the turnover is approximately the same for RSJ and RSK, and slightly higher for REV. Correspondingly, explicitly accounting for transaction costs, using the effective spread as a proxy (see, e.g., Hasbrouck (2009)), results in the highest net returns for the RSJ-based long–short strategy.
between the RSJ asymmetry measure and the REV return measure is only 0.37, and as such the two strategies effectively profits from different effects. Section B further delineates these separate effects.

B. Single-Sorted Portfolios with Controls

The sample correlations and the portfolio sorts reported in Tables 1 and 2, respectively, revealed a strong contemporaneous relation between the two RSJ and RSK realized measures, and somewhat weaker, albeit statistically significant, correlation with the weekly return reversal REV. To help clarify the unique informational content in each of the realized measures, we further sort the stocks by the residuals obtained from controlling for the past week returns. Specifically, at the end of each Tuesday, we run a cross-sectional regression of RSJ against REV, or RSK against REV, and sort the stocks based on the resulting regressions residuals.

The results in Panel A of Table 4 shows that these RSJ residual-based sorts still negatively predict future returns. The weekly FFC4 alpha of the High–Low spreads equals $-17.67$ bps, with a $t$-statistic of $-4.00$, for the value-weighted portfolios, and $-15.27$ bps, with a $t$-statistic of $-5.35$, for the equal-weighted portfolios. These FFC4 alphas are smaller than those produced by the simple RSJ sorts in Panel A, indicating that part of the predictability of RSJ is indeed attributable to the short-term reversal effect. Nonetheless, the results remain economically and statistically significant, implying a substantial amount of additional information in the new RSJ measure beyond that of the weekly return reversal REV.

Panel B of Table 4 repeats this same analysis in which we sort the stocks into quintile portfolios according to the residuals from the weekly cross-sectional regressions of RSK on REV. Although the RSK sorts that control for REV still result in a weekly abnormal return spread of $-10.45$ bps, with a $t$-statistic of $-4.42$, for the equal-weighted portfolios, the spread for the value-weighted portfolios is reduced to only $-4.96$ bps, with an insignificant $t$-statistic of $-1.27$. These results for the residual-based sorts are also in line with ACJV, who report much weaker $t$-statistics for the High–Low spread for the value-weighted RSK-sorted portfolios at different REV levels.

Taken together, the results in Panels A and B clearly suggest that RSJ contains more independent information beyond REV compared to RSK.

This same residual-based sorting approach also allows us to assess the inherent predictability of the RSJ and RSK measures against each other, by sorting firms into portfolios based on the residuals from the weekly cross-sectional regressions of RSJ against RSK, or RSK against RSJ. The portfolio returns and FFC4 alphas obtained from sorting based on these RSJ residuals, orthogonal to RSK, are reported in Panel C of Table 4. The value- and equal-weighted portfolios both result in highly significant abnormal return spreads. The High–Low FFC4 alpha for the value-weighted portfolios, in particular, equals $-36.11$ bps with a $t$-statistic of $-7.11$. This reveals an even stronger degree of predictability

\footnote{This approach is useful in delineating the unique predictability in RSJ and RSK over and above that of REV, as the regression residuals are orthogonal to the regressor by construction.}

\footnote{These results for the residual-based sorts are also in line with ACJV, who report much weaker $t$-statistics for the High–Low spread for the value-weighted RSK-sorted portfolios at different REV levels.}


Table 4 reports the average returns for predictive single-sorted portfolios with controls. The sample consists of all the NYSE, AMEX, and NASDAQ listed common stocks with share codes 10 or 11 and prices between $5 and $1,000 over the 1995–2013 sample period. At the end of each week, stocks are sorted into quintiles according to realized measures with controls computed from previous week high-frequency returns. Each portfolio is held for 1 week. The column labeled "Return" reports the average 1-week ahead excess returns of each portfolio. The column labeled "FFC4" reports the corresponding Fama–French–Carhart 4-factor alpha for each portfolio. The row labeled "High–Low" reports the difference in returns between portfolio 5 and portfolio 1, with Newey-West (1987) robust $t$-statistics in parentheses. RSJ, RSK, and REV denote the relative signed jump variation, realized skewness, and lagged 1-week return, respectively. In each panel, the first 2 columns report the value-weighted sorting results and the last 2 columns report the equal-weighted sorting results. Panel A displays the results sorted by RSJ residual controlling for REV, Panel B by RSK residual controlling for REV, Panel C by RSJ residual controlling for RSK, and Panel D by RSK residual controlling for RSJ.

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<th>Value FFC4</th>
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<td>(-1.27)</td>
<td>(-4.39)</td>
<td>(-4.42)</td>
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In sharp contrast, the sorts based on the RSK residuals orthogonal to RSJ fail to reproduce the same negative return predictability as the simple RSK sorts in Panel C of Table 3. Instead, the two FFC4 columns in Panel D show that the negative relation between RSK and the future returns, but not change the sign. Instead, the results suggest that RSJ and RSK share a common component that accounts for their high contemporaneous correlation and the strong negative return predictability observed in the conventional single sorts reported in Table 3. However, once the influence of this common component, which manifests most strongly in the RSJ measure, is controlled for, the effect of the realized skewness is completely reversed. Appendix Section A.3 provides a more formal econometric rational for how this change of sign may arise in the data.
C. Double-Sorted Portfolios

The results from the standard single portfolio sorts and the single sorts with controls reported in Tables 3 and 4, respectively, reveal a strong negative relation between the relative signed jump variation and the future returns. This cannot be accounted for by the previously documented weekly reversal effect, nor the realized skewness effect. By contrast, the negative predictability of the realized skewness is considerably diminished after controlling for the weekly return reversals, and completely reversed after controlling for the relative signed jump variation. Another popular approach to control for the effect of other variables associated with the cross-sectional variation in returns is to rely on double portfolio sorts. Table 5 reports the results from a series of such sequential sorts in which we alternate between the ordering of the RSJ and RSK based sorts.

Panel A of Table 5, in particular, shows the results obtained by sorting on RSJ after first sorting on RSK. At the end of each Tuesday, we first sort all of the stocks into quintile portfolios based on their individual RSK measures. Within each of these characteristic portfolios, we then sort the stocks into quintiles based on their RSJ measure for that same week, and compute the returns over the subsequent week for the resulting 25 (5 \times 5) portfolios. The row labeled “High–Low” reports the average return spread between these High and Low quintile portfolios within each RSK quintile, while the row labeled “FFC4” reports the return spreads adjusted by the 4 Fama–French–Carhart risk factors. To focus more directly on the effect of RSJ, we also compute the returns averaged across RSK quintiles as a way to produce quintile portfolios with large variations in RSJ, but small variations in RSK. These returns are reported in the last column labeled “Average.”

The resulting FFC4 alpha for the difference in the value-weighted returns on the fifth and first RSJ quintile portfolios obtained by first sorting on RSK equals \(-21.39\). This economically large spread also has a highly significant \(t\)-statistic of \(-5.16\), again underscoring that RSK cannot explain the predictability of RSJ. Interestingly, looking at the RSJ effect within each of the RSK quintiles, shows that RSJ negatively predicts the future returns within 4 of the 5 quintiles. The equal-weighted double sorts exhibit even more significant patterns, both within and across RSK quintiles.

Panel B of Table 5 shows the results from similarly constructed sequential double sorts based on RSK, in which we first sort on RSJ. The FFC4 alpha for the value-weighted portfolios given in the bottom right entry equals 11.73, with a \(t\)-statistic of 3.45, thus suggesting that RSK positively predicts future returns after controlling for RSJ. This positive predictability of RSK contradicts the strong negative predictability of RSK previously documented in ACJV and the single sorts in Panel C of Table 3. However, the results are consistent with the residual sorts reported in Panel D of Table 4 that control for the influence of RSJ.18 Interestingly, this positive predictability of RSK after first sorting on RSJ holds

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17 The Supplementary Material reports the results from additional double sorts based on these same two realized measures that first sort on other explanatory variables.

18 This again is also consistent with the idea that RSJ provides a more accurate proxy for some underlying latent factor that drives the returns, as discussed more formally in Appendix Section A.3.
Table 5 reports the average 1-week ahead returns sorted by the realized signed jump variation measure RSJ controlling for the realized skewness RSK, and vice versa. The sample consists of all the NYSE, AMEX, and NASDAQ listed common stocks with share codes 10 or 11 and prices between $5 and $1,000 over the 1993–2013 sample period. In Panel A, for each week, all stocks in the sample are first sorted into 5 quintiles on the basis of RSJ. Within each quintile, the stocks are then sorted into 5 quintiles according to RSK. For each 5 × 5 grouping, we form a value-weighted portfolio (left panel) and an equal-weighted portfolio (right panel). These 5 RSJ portfolios are then averaged across the 5 RSK portfolios to produce RSJ portfolios with large cross-portfolio variation in their RSJ but little variation in RSK. In Panel B, we reverse the order to first sort on RSK and then on RSJ. RSJ and RSK denote the relative signed jump and realized skewness.

In Panel A/B, the first 5 rows in both value-weighted and equal-weighted sorting results report time-series averages of weekly excess returns for the RSJ/RSK quintile portfolios. The row labeled “High–Low” reports the difference in the returns between portfolio 5 and portfolio 1. The row labeled “FFC4” reports the average Fama–French–Carhart 4-factor alphas. The corresponding Newey–West (1987) robust t-statistics are reported in parentheses.

### Panel A. Sorted by RSJ Controlling for RSK

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true across all quintiles. The equal-weighted double sorts show the same positive return predictability patterns associated with RSK after first sorting on RSJ.

In sum, the negative relation between the future returns and the relative signed jump variation measure remains intact after controlling for the influence of the realized skewness. On the other hand, the tendency for high/low realized skewness measuring to be negatively associated with future returns remains intact after controlling for the realized skewness.
skewness to be associated with low/high future returns completely disappears, and even reverses, after controlling for the relative signed jump variation.

D. Firm-Level Cross-Sectional Regressions

The portfolio sorts discussed above ignore potentially important firm-level information by aggregating the stocks into quintile portfolios. Also, even though the single-sorted portfolios with controls and the double-sorted portfolios both allow for the possibility that more than one explanatory variable might be linked to the cross-sectional variation in the returns, they only control for 1 variable at a time. In an effort to further corroborate and expand on these results, this section reports the results from a series of standard Fama–MacBeth (1973) cross-sectional type regressions that simultaneously control for multiple explanatory variables.

Specifically, for each of the weeks in the sample, we run the following cross-sectional regressions,

\[ r_{i,t+1} = \gamma_{0,t} + \sum_{j=1}^{K} \gamma_{j,t} Z_{j,i,t} + \epsilon_{i,t+1}, \quad i = 1,2,\ldots,N, \]

where \( r_{i,t+1} \) denotes the return for stock \( i \) over week \( t+1 \) (Tuesday-close to Tuesday-close), and the \( K \) stock-specific control variables \( Z_{j,i,t} \) are all measured at the end of week \( t \). Having estimated the slope coefficients \( \gamma_{j,t} \) for each of the weeks in the sample, we compute the time-series averages of the \( \hat{\gamma}_{j,t} \) estimates to assess whether the different controls are able to predict the future returns. Table 6 reports the resulting \( \bar{\gamma}_{j} \) averages and corresponding \( t \)-statistics for a number of different specifications.

Panel A of Table 6, in particular, focuses on simple regressions, in which we regress the returns against a single explanatory variable at a time. Consistent with the results for the single-sorted portfolios, RSJ and RSK both negatively predict the subsequent weekly returns, with highly statistically significant \( t \)-statistics of \(-8.20\) and \(-7.97\), respectively. In addition, REV stands out as the only other explanatory variable with a clearly significant \( t \)-statistic. The estimated slope coefficients for the other explanatory variables generally also have the “correct” sign, but with the exception of MAX, none of the other variables are significant at the 5% level.

Turning to Panel B of Table 6, and the multiple regressions that simultaneously control for more than 1 variable at a time, the relative signed jump variation RSJ is always highly significant. Putting the estimates into perspective, the average cross-sectional standard deviation of RSJ equals 0.16. Hence, the average slope of \(-159.73\) in Regression XIV that includes all of the variables implies that a 2-standard-deviation decrease in RSJ predicts a rise of approximately 26% in the annual returns \((2 \times 159.73 \times 0.16/10,000 \times 52 = 26\%)\). Thus, not only is the predictability afforded by RSJ highly statistically significant, it is also highly significant economically.

The realized kurtosis RKT also negatively predicts the future returns across all of the different specifications. However, the level of significance is much lower than for RSJ. By contrast, the realized skewness, consistent with the prior empirical evidence in ACJV, negatively predicts the future returns in the simple
regression in Panel A of Table 6 and Regression III in Panel B, and positively predicts the future returns in all of the regressions that include RSJ as a control. This, of course, mirrors the findings based on the double sorts discussed in Section VI.C, and further highlights the fragility of RSK as a predictor.

VI. Dissecting RSJ-Based Portfolio Strategies

The results in the previous section naturally raise the questions of where the superior performance of the RSJ-based portfolio strategies are coming from, and whether the resulting “paper portfolio” profits are somehow linked to the informational environment and costs of trading. This section provides some partial answers to these questions.
A. RSJ Profitability and Other Control Variables

We begin by examining how the performance of the RSJ-sorted portfolios vary across firm size, volatility, illiquidity and return reversal. The first 3 variables, in particular, are often used to assess how anomalies are affected by the informational environment and trading costs more generally. The mechanics behind these additional double sorts, reported in Table 7, closely mirrors those discussed in Section V.C. The only difference is that to more explicitly highlight the impact of a given control variable, we report the differences in the value-weighted return spreads between the High- and Low-level of the specific control variable, rather than the averages across these levels.

Panel A of Table 7 shows that the RSJ strategy tends to be more profitable for smaller stocks. For the value-weighted portfolios, the weekly 4-factor alpha of the RSJ strategies equals −59.16 bps for small firms compared to −26.67 bps for large firms, with t-statistics of −10.35 and −5.03, respectively. Also, the difference in the alphas between small and large firms equals 32.49 bps per week, with a highly statistically significant t-statistic of 4.76. These results are consistent with the idea that large firms tend to have better information environments and more sophisticated traders, resulting in less overreaction to extreme price movements, and thus weaker RSJ effects.

### Table 7

**RSJ Portfolios Sorted on Control Variables**

<table>
<thead>
<tr>
<th>Panel A: ILLIQ</th>
<th>Panel B: RVOL</th>
<th>Panel C: ILLIQ</th>
<th>Panel D: REV</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Value-Weighted</strong></td>
<td><strong>Low</strong></td>
<td><strong>2</strong></td>
<td><strong>3</strong></td>
</tr>
<tr>
<td><strong>(Low)</strong></td>
<td>30.01</td>
<td>29.13</td>
<td>36.67</td>
</tr>
<tr>
<td><strong>2</strong></td>
<td>20.74</td>
<td>25.44</td>
<td>22.97</td>
</tr>
<tr>
<td><strong>3</strong></td>
<td>13.84</td>
<td>16.21</td>
<td>14.68</td>
</tr>
<tr>
<td><strong>4</strong></td>
<td>8.61</td>
<td>11.54</td>
<td>7.22</td>
</tr>
<tr>
<td><strong>5</strong></td>
<td>3.86</td>
<td>3.13</td>
<td>4.32</td>
</tr>
<tr>
<td><strong>High-Low</strong></td>
<td>−26.15</td>
<td>−26.00</td>
<td>−32.35</td>
</tr>
</tbody>
</table>
| **RSJ Portfolios Sorted on Control Variables**

Table 7 reports the performance of portfolios sorted by RSJ and control variables. The sample consists of all the NYSE, AMEX, and NASDAQ listed common stocks with share codes 10 or 11 and prices between $5 and $1,000 over the 1993–2013 sample period. In each panel, for each week, all stocks in the sample are first sorted into 5 quintiles on the basis of 1 control variable. Within each quintile, the stocks are then sorted into 5 quintiles according to their RSJ. For each 5×5 grouping, we form an value-weighted portfolio. Then we consider the performance of the 25 portfolios from the intersection of the double sorts. RSJ and RVOL denote the relative signed jump and realized volatility. ME denotes the natural logarithm of the market capitalization of the firms. ILLIQ refers to the natural logarithm of the average daily ratio of the absolute stock return to the dollar trading volume over the previous week. REV denotes the lagged 1-week return. In each panel, the rows/columns labeled as 1–5 represent 5 levels of RSJ/the control variable. The row/column labeled “High-Low” reports the difference in value-weighted returns between Portfolio 5 and Portfolio 1 constructed according to RSJ/the control variable. The row labeled “FFC4” reports the Fama–French–Carhart 4-factor alphas. The corresponding Newey–West (1987) robust t-statistics are reported in parentheses.
Panel B of Table 7 examines RSJ portfolios for different volatility levels. For the value-weighted portfolios, the 4-factor alpha for the RSJ strategy equals $-26.07$ bps per week for the least volatile quantile, compared to $-70.16$ bps for the most volatile quantile. Also, the difference in the 4-factor alphas between the two groups is $-44.09$ bps per week, with a t-statistic of $-4.09$. Assuming that firms with more volatile stock prices exhibit greater informational asymmetries, arbitrage risks may thus help explain the more pronounced RSJ effects for more volatile firms.

Panel C of Table 7 relies on the Amihud illiquidity measure to assess the effect of trading costs. The results indicate a stronger RSJ effect for stocks that are more costly to trade. For the bottom quintile of stocks with the lowest illiquidity measure, the 4-factor alpha of the RSJ strategy equals $-27.61$ bps per week, with a t-statistic of $-4.76$, whereas for the top quintile of stocks with the highest illiquidity measure, the 4-factor alpha is $-61.62$ bps, with a t-statistic of $-10.03$. Moreover, the difference in the 4-factor alphas equals $-34.02$ bps per week, with a t-statistic of $-4.64$. Since illiquid stocks are more costly and risky to trade, these results indirectly support the notion that limits to arbitrage prevent traders from betting against perceived mispricing (see, e.g., Shleifer and Vishny (1997)), and why the RSJ effect is the strongest for the most difficult to trade stocks.

Panel D of Table 7 further examines the interaction between the RSJ and reversal strategies. The weekly 4-factor alpha of the RSJ strategies equals $-22.88$ bps for the firms with the lowest lagged 1-week returns, compared to $-18.95$ for the firms with highest lagged 1-week returns, with t-statistics of $-2.72$ and $-2.63$, respectively. The difference in the alphas between low and high REV firms equals $3.93$ bps per week, with an insignificant t-statistic of 0.37. The smaller FFC4 alphas for the RSJ strategies across REV quantiles indicate once again that part of the predictability of RSJ is attributable to the short-term reversal effect.

B. Autocovariances and RSJ Profits

To help further understand where the profits obtained by buying low-RSJ stocks and selling high-RSJ stocks come from, we follow Lo and MacKinlay (1990) and decompose the cross-sectional predictability from an RSJ-based strategy into 3 sources: autocorrelations in the returns, lead-lag relations among the different stocks, and cross-sectional dispersion in the unconditional mean returns.

Specifically, consider the zero-cost portfolio with weights determined by the relative magnitude of the individual stocks’ RSJ,

$$w_{i,t} = \frac{1}{c_t}(\text{RSJ}_{i,t} - \text{RSJ}_{m,t}), \tag{11}$$

where $c_t = (\sum_{i=1}^{N} |\text{RSJ}_{i,t} - \text{RSJ}_{m,t}|)/2$ denotes a scaling factor, and $\text{RSJ}_{m,t} = \sum_{i=1}^{N} \text{RSJ}_{i,t}/N$ equals the average RSJ for week $t$. The scaling factor $c_t$ ensures that the strategy is always $1$ long and $1$ short so that the magnitude of the profits are easily interpretable (see, e.g., the discussions in Lehmann (1990), Nagel (2012)). Let $\pi_{t+1}$ denote the week $t+1$ profit on the portfolio,

$$\pi_{t+1} = \sum_{i=1}^{N} w_{i,t} r_{i,t+1} = \frac{1}{c_t} \sum_{i=1}^{N} (\text{RSJ}_{i,t} - \text{RSJ}_{m,t}) r_{i,t+1}. \tag{12}$$
Then, following Lo and MacKinlay (1990), the expected profit is naturally decomposed into 3 separate terms,

\[
E(\pi_{t+1}) = \frac{N-1}{N^2} \text{tr}(\Gamma) - \frac{1}{N^2} \left[ (1' \Gamma 1 - \text{tr}(\Gamma)) \right] + \sigma_{\text{RSJ},R}^2, \tag{13}
\]

where \( \Gamma = \text{cov}(N/c_t \times \text{RSJ}_t, R_{t+1}) \) for \( \text{RSJ}_t \equiv (\text{RSJ}_{1,t}, \text{RSJ}_{2,t}, \ldots, \text{RSJ}_{N,t})' \), \( R_{t+1} \equiv (r_{1,t+1}, r_{2,t+1}, \ldots, r_{N,t+1})' \) denotes the covariance between the scaled RSJ in week \( t \) and the return in week \( t+1 \), while \( \sigma_{\text{RSJ},R}^2 \) denotes the cross-sectional covariance between the unconditionally expected RSJ and the future returns. The first term in equation (13), \( (N-1)\text{tr}(\Gamma)/N^2 \), represents the average autocovariance of the individual stocks. It would be positive/negative if low RSJ stocks became future losers/winners. The second term, \( -(1' \Gamma 1 - \text{tr}(\Gamma))/N^2 \), gives the average cross-serial covariance. It would be positive/negative if firms with high RSJ predict low/high future returns of other firms. Lastly, the third cross-sectional covariance term, \( \sigma_{\text{RSJ},R}^2 \), would be positive/negative if firms with high unconditional RSJ also have high/low unconditional returns.

Table 8 reports the unconditional sample values for the 3 separate terms based on the stocks with complete return and RSJ histories, along with the corresponding robust standard errors. Panel A gives the results for the raw returns, while Panel B gives the results for the FFC4-adjusted returns. The findings are generally consistent with an overreaction-based explanation. The total profit based on the raw returns equals \(-23.56 \text{ bps per week} \), with a \( t \)-statistic of \(-7.87 \), while the FFC4-adjusted returns produce a profit of \(-25.55 \text{ bps} \), with a \( t \)-statistic of \(-10.55 \). Most of this profitability is attributable to the autocovariance component, which generates a weekly profit of \(-30.02 \text{ bps} \), with a \( t \)-statistic of \(-4.07 \), for the raw returns, and a FFC4 alpha of \(-28.24 \text{ bps} \), with a \( t \)-statistic of \(-10.68 \). By comparison, the cross-serial component equals just \(5.87 \text{ bps} \), with a \( t \)-statistic of \(1.00 \text{ for the raw returns, and 2.24 bps for the FFC4 alpha, with a t-statistic of 1.98. The unconditional covariance components are nearly 0. In other words, the profitability arises from variation in the stock’s own RSJ, not other stocks’ RSJ, suggestive of an overreaction to large and sudden price moves.

| TABLE 8 | High–Low Portfolio Profit Decomposition |
|-----------------|-----------------|-----------------|-----------------|-----------------|
|                | Panel A. Raw Return | Panel B. FFC4   |
| Auto           | Cross           | Mean            | Total           | Auto           | Cross           | Mean            | Total           |
| \(-30.02\)     | 5.87           | 0.59            | \(-23.56\)      | \(-28.24\)     | 2.24           | 0.45            | \(-25.55\)      |
| \((-4.07\)     | (1.00)         | (1.98)          | \((-7.87\)      | \((-10.68\)     | (1.24)         | (3.78)          | \((-10.55)      |
C. RSJ and News Announcements

Do the “jumps” captured by RSJ, and the corresponding trading profits documented above, correspond to readily identifiable corporate events and/or firm-specific news? In an effort to ascertain whether they do, we follow the literature on post-earnings-announcement drifts (e.g., Bernard and Thomas (1989)), and plot the typical evolution of the RSJ measure around earnings announcements. Specifically, for each week in the sample, we compute the average and cumulative RSJ in the $[-12, 12]$ week window surrounding the announcement week conditional on positive and negative earnings surprises, respectively. The resulting averages depicted in Figure 4 reveal a clear positive “jump” in the RSJ measure 1 week prior to the announcement for positive earnings surprises, along with a negative “jump” in RSJ during the actual announcement week for negative surprises. Moreover, consistent with the extant literature and the well established pre- and post-announcement drifts (see, e.g., Fig. 1 of Bernard and Thomas (1989)), there are also clear pre- and post-announcement trends evident in the cumulative RSJ measures.

![Graph A: Average RSJ](https://doi.org/10.1017/S0022109019000097)
![Graph B: Cumulative RSJ](https://doi.org/10.1017/S0022109019000097)

**FIGURE 4**
RSJ Around Earnings Announcement

Graph A of Figure 4 shows the average weekly relative signed jump variation RSJ in a $[-12, 12]$ week window surrounding positive and negative earnings surprise, respectively. Graph B shows the cumulative RSJ.

Meanwhile, it is important to recognize that the differences in returns associated with RSJ is not merely restricted to the time around earnings announcements. Indeed, many other, both public macroeconomic and firm specific news announcements, similarly manifest in the form of firm specific jumps and in turn large (in an absolute sense) RSJs (see, e.g., Lee (2012) and the discussion therein). Related to this, Savor and Wilson (2014) have also recently argued that cross-sectional return patterns are different on news announcement days.

VII. Conclusion

We document that firms with relatively high/low “good” minus “bad” volatilities, constructed from the summation of high-frequency intraday positive and negative squared returns, respectively, are associated with low/high future returns. Sorting stocks into portfolios based on their individual relative signed jump variation results in an economically large value-weighted weekly return spread.
between the stocks in the lowest and highest quintile portfolios of 29.35 bps, or approximately 15% per year. The corresponding \( t \)-statistic of 5.83 also far exceeds the hurdle rate for judging the cross-sectional return predictability recently advocated by Harvey et al. (2016). Adjusting the returns for the influence of the Fama–French–Carhart systematic risk factors hardly changes the magnitude of this return spread, nor its statistical significance. The return spreads also remain highly statistically significant and economically large in equal-weighted portfolios, double-sorted portfolios, and Fama–MacBeth type regressions that control for other firm characteristics and explanatory variables previously associated with the cross-sectional variation in expected stock returns. By contrast, the negative predictability of the realized skewness measure recently documented by ACJV, is completely reversed after controlling for the relative “good” minus “bad” realized volatility.

Our empirical findings raise the question of why the firm specific relative signed variation so strongly predicts the future returns. Following the arguments of Brekenfelder and Tédongap (2012) and Farago and Tédongap (2018), it is possible that investors with disappointment aversion rationally price downside volatility more dearly than upside volatility. This in turn results in a priced systematic downside volatility factor. Empirically, however, it remains to be seen whether the strong predictability afforded by the individual firms’ high-frequency-based realized volatility measures can be accounted for by a common systematic priced risk factor. Also, the return predictability, albeit highly statistically significant at the weekly level, is relatively short-lived, dissipating over longer monthly horizons, casting some doubt on a purely risk-based explanation. Instead, the differential pricing of the firm specific “good” versus “bad” volatilities might reflect behavioral biases, stemming from overreaction to large sudden price declines, or negative “jumps.” Further along these lines, we find that the predictability of RSJ is stronger among small firms, firms with more volatile stock prices and more illiquid firms. This is consistent with investor overreaction to extreme price movements and limits to arbitrage. We leave it for future research to more clearly delineate the roles played by these competing explanations.

**Appendix. Data Cleaning, Explanatory Variables and RSK Sign Change**

In this Appendix, we provide the high-frequency data cleaning rules, definitions of additional explanatory variables used in the article, and explanation of the sign change of RSK after controlling for RSJ.

1. **High-Frequency Data Cleaning**

We begin by removing entries that satisfy at least one of the following criteria: A time stamp outside the exchange open window between 9:30 AM and 4:00 PM; a price less than or equal to 0; a trade size less than or equal to 0; corrected trades, that is, trades with Correction Indicator, CORR, other than 0, 1, or 2; and an abnormal sale condition, that is, trades for which the Sale Condition, COND, has a letter code other than @, *, E, F, @E, @F, *E, or *F. We then assign a single value to each variable for each second within the 9:30 AM–4:00 PM time interval. If one or multiple transactions have occurred in that second, we calculate the sum of volumes, the sum of trades, and the volume-weighted average price.
within that second. If no transaction has occurred in that second, we enter 0 for volume and trades. For the volume-weighted average price, we use the entry from the nearest previous second. Motivated by our analysis of the trading volume distribution across different exchanges over time, we purposely incorporate information from all exchanges covered by the TAQ database.

2. Additional Explanatory Variables

Our empirical investigations rely on the following explanatory variables and firm characteristics.

- **Size (ME):** Following Fama and French (1993), a firm’s size is measured by its market value of equity, that is, the product of the closing price and the number of shares outstanding (in millions of dollars). Market equity is updated daily and is used to explain returns over the subsequent week. Following common practice, we also transform the size variable by its natural logarithm to reduce skewness.

- **Book-to-market ratio (BM):** Following Fama and French (1993), the book-to-market ratio in June of year \( t \) is computed as the ratio of the book value of common equity in fiscal year \( t-1 \) to the market value of equity (size) in December of year \( t-1 \). Book common equity is defined as the book value of stockholders’ equity, plus balance sheet deferred taxes and investment tax credit (if available), minus book value of preferred stock for fiscal year \( t-1 \).

- **Momentum (MOM):** Following Jegadeesh and Titman (1993), the momentum variable at the end of day \( t \) is defined as the compound gross return from day \( t-252 \) through day \( t-21 \), skipping the short-term reversal month.

- **Reversal (REV):** Following Jegadeesh (1990), Lehmann (1990) and ACJV (2016), the short-term reversal variable is defined as the weekly return over the previous week from Tuesday to Monday.

- **Idiosyncratic volatility (IVOL):** Following Ang et al. (2006b), a firm’s idiosyncratic volatility at the end of day \( t \) is computed as the standard deviation of the residuals from the regression based on the daily returns between day \( t-20 \) and day \( t \):

\[
\begin{align*}
1 & \quad r_{i,d} - r_{f,d} = \alpha_i + \beta_i (r_{0,d} - r_{f,d}) + \gamma_i \text{SMB}_d + \phi_i \text{HML}_d + \epsilon_{i,d},
\end{align*}
\]

where \( r_{i,d} \) and \( r_{0,d} \) are the daily returns of stock \( i \) and the market portfolio on day \( d \), respectively, and \( \text{SMB}_d \) and \( \text{HML}_d \) denote the daily Fama and French (1993) size and book-to-market factors.

- **Coskewness (CSK):** Following Harvey and Siddique (2000) and Ang et al. (2006a), the coskewness of stock \( i \) at the end of day \( t \) is estimated using daily returns between day \( t-20 \) and day \( t \) as

\[
\text{CSK}_i = \frac{1}{N} \sum_d (r_{i,d} - \bar{r}_i)(r_{0,d} - \bar{r}_0)^2 \left( \frac{1}{N} \sum_d (r_{0,d} - \bar{r}_0)^2 \right)^{-1/2},
\]

where \( N \) denotes the number of trading days, \( r_{i,d} \) and \( r_{0,d} \) are the daily returns of stock \( i \) and the market portfolio on day \( d \), respectively, and \( \bar{r}_i \) and \( \bar{r}_0 \) denote the corresponding average daily returns.
• Cokurtosis (CKT): Following Ang et al. (2006a), the cokurtosis of stock \( i \) at the end of day \( t \) is estimated using the daily returns between day \( t - 20 \) and day \( t \) as

\[
\hat{\text{CKT}}_{i,t} = \frac{1}{N} \sum_d (r_{i,d} - \bar{r}_i)(r_{0,d} - \bar{r}_0)^3 \sqrt{\frac{1}{N} \sum_d (r_{i,d} - \bar{r}_i)^2 \left( \frac{1}{N} \sum_d (r_{0,d} - \bar{r}_0)^2 \right)^{3/2}}, \tag{A-3}
\]

where variables are the same as for CSK.

• Maximum daily return (MAX): Following Bali et al. (2011) and ACJV (2016), the \( \text{MAX} \) variable is defined as the largest total daily raw return observed over the previous week.

• Minimum daily return (MIN): Following Bali et al. (2011) and ACJV (2016), the \( \text{MIN} \) variable is defined as the smallest total daily raw return observed over the previous week.

• Illiquidity (ILLIQ): Following Amihud (2002), the illiquidity for stock \( i \) at the end of day \( t \) is measured as the average daily ratio of the absolute stock return to the dollar trading volume from day \( t - 4 \) through day \( t \):

\[
\text{ILLIQ}_{i,t} = \frac{1}{N} \sum_d \left( \frac{|r_{i,d}|}{\text{volume}_{i,d} \times \text{price}_{i,d}} \right), \tag{A-4}
\]

where \( \text{volume}_{i,d} \) is the daily trading volume, \( \text{price}_{i,d} \) is the daily price, and other variables are as previously defined. We further transform the illiquidity measure by its natural logarithm to reduce skewness.

3. Change of Sign of RSK

To understand how the predictability of RSK changes sign (from significantly negative to significantly positive) after controlling for RSJ, consider the following data generating process (for simplicity and without loss of generality we assume the mean return to be 0),

\[
\begin{align*}
  r_{t+1} &= a A_t + e_{t+1}, \tag{A-5} \\
  \text{RSJ}_t &= b A_t + u_t, \tag{A-6} \\
  \text{RSK}_t &= c A_t + v_t, \tag{A-7}
\end{align*}
\]

where \( r_{t+1} \) denotes the return for week \( t + 1 \), and \( A_t \) denotes a latent common factor with zero mean and unit variance. Further assume that \( a < 0 \) (since the latent variable \( A \) negatively predicts future returns), and \( b > 0 \) and \( c > 0 \) (since RSJ and RSK both provide “noisy” proxies for the latent predictor variable). Now consider the joint regression,

\[
\begin{align*}
  r_{t+1} &= \gamma_1 \text{RSJ}_t + \gamma_2 \text{RSK}_t + \epsilon_{t+1}. \tag{A-8}
\end{align*}
\]

The ordinary least squares (OLS) estimates of \( (\gamma_1, \gamma_2) \) may be succinctly expressed as,

\[
\begin{align*}
  \begin{pmatrix} \hat{\gamma}_1 \\ \hat{\gamma}_2 \end{pmatrix} &= \left( \begin{pmatrix} \sigma_{\text{RSJ}}^2 & \rho \sigma_{\text{RSJ}} \sigma_{\text{RSK}} \\ \rho \sigma_{\text{RSJ}} \sigma_{\text{RSK}} & \sigma_{\text{RSK}}^2 \end{pmatrix} \right)^{-1} \times \begin{pmatrix} \text{Cov(RSJ, } r_{t+1} \text{)} \\ \text{Cov(RSK, } r_{t+1} \text{)} \end{pmatrix} \\
  &= \frac{ab}{\sigma_{\text{RSK}}^2 \sigma_{\text{RSJ}}^2 (1 - \rho^2)} \begin{pmatrix} \sigma_{\text{RSK}} - \frac{c}{b} \rho \sigma_{\text{RSJ}} \\ \frac{c}{b} \sigma_{\text{RSJ}} - \rho \sigma_{\text{RSK}} \end{pmatrix}, \tag{A-9}
\end{align*}
\]
where all of the right-hand-side variables refer to the corresponding sample analogues. Thus, the $\hat{y}_2$ estimate will be positive if $\rho$ and $\sigma_{RSK}$ are sufficiently large. To put it differently, let $R^2_{RSJ} = b^2/\sigma^2_{RSJ}$ and $R^2_{RSK} = c^2/\sigma^2_{RSK}$ denote the $R^2$'s from the respective univariate (latent) regressions of RSJ and RSK on $A$. The $\hat{y}_2$ estimate will be positive when $R^2_{RSK}/R^2_{RSJ} < \rho^2$, or whenever RSJ provides a sufficiently more accurate proxy (as measured by the $R^2$) for the latent predictor variable than RSK.

Translating this to the actual data, equations (A-6) and (A-7) readily imply that,

\begin{equation}
\text{RSK}_t = \frac{c}{b} \text{RSJ}_t + w_t.
\end{equation}

Correspondingly, regressing the weekly RSK on RSJ results in an estimate of $\frac{c}{b} = 4.19$. Moreover, $\sigma_{RSK} - (c/b)\rho\sigma_{RSJ} = 0.77 - 4.19 \times 0.93 \times 0.16 = 0.15 > 0$ and $(c/b)\sigma_{RSJ} - \rho\sigma_{RSK} = 4.19 \times 0.16 - 0.93 \times 0.77 = -0.05 < 0$, which by the above reasoning implies that $\hat{y}_2 < 0$ and $\hat{y}_3 > 0$, consistent with the observed change of sign for RSK from negative to positive between the simple and multiple return regressions.

**Supplementary Material**

Supplementary Material for this article is available at [https://doi.org/10.1017/S00221090901900097](https://doi.org/10.1017/S00221090901900097).

**References**


