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The Importance of Timing Attitudes in Consumption-Based Asset Pricing Models*

Martin M. Andreasen[†] Kasper Jørgensen[‡]

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Abstract

A new utility kernel for Epstein-Zin-Weil preferences is proposed to disentangle the intertemporal elasticity of substitution (IES), the relative risk aversion (RRA), and the timing attitude. These new preferences resolve two puzzles in the long-run risk model, where consumption growth is too strongly correlated with the price-dividend ratio and the risk-free rate. The proposed preferences also enable a New Keynesian model to match equity and bond premia with a low RRA of 5. Importantly, the mechanism enabling Epstein-Zin-Weil preferences to explain asset prices in these models is *not* to separate the IES from RRA, but to introduce a strong timing attitude.

Keywords: Bond premium puzzle, Equity premium puzzle, Early resolution of uncertainty, Long-run risk.

JEL: E44, G12.

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[†]Aarhus University, CREATES, and the Danish Finance Institute. Fuglesangs Allé 4, 8210 Aarhus V, Denmark, email: mandreasen@econ.au.dk, telephone +45 87165982.

[‡]Board of Governors of the Federal Reserve System, Division of Monetary Affairs, Washington, DC, United States, email: kasper.joergensen@frb.gov, telephone +1 202-245-4233. Corresponding author.

1 Introduction

Following the seminal work of Epstein and Zin (1989) and Weil (1990), a large number of consumption-based models use so-called Epstein-Zin-Weil preferences to explain asset prices. An important property of these preferences is to disentangle the relative risk aversion (RRA) and the intertemporal elasticity of substitution (IES) which otherwise are inversely related when using expected utility. It is also well-known that the separation of the IES and RRA in Epstein-Zin-Weil preferences is achieved by imposing a timing attitude on the household, which either prefers early or late resolution of uncertainty. However, experimental evidence suggests that the timing attitude has an independent effect on decision making beyond what is implied by RRA, and that the timing attitude is unrelated to RRA (see for instance Chew and Ho (1994) and van Winden et al. (2011)). This raises the question; do Epstein-Zin-Weil preferences perform well because they separate the IES from RRA or because they imply a timing attitude?

We address this question in the present paper and explore whether a more flexible specification of the timing attitude helps to explain asset prices. The analysis is carried out by augmenting the power utility kernel adopted in Epstein and Zin (1989) and Weil (1990) with a constant u_0 to account for other aspects than market consumption C_t when modeling the household's contemporaneous utility level. The benefit of this extension of the utility kernel $u(C_t)$ is to obtain greater flexibility in setting the ratios $u_{CC}(C_t)C_t/u_C(C_t)$ and $u_C(C_t)C_t/u(C_t)$ compared to the traditional specification of Epstein-Zin-Weil preferences, where one parameter determines both ratios. Much attention in the literature has been devoted to $u_{CC}(C_t)C_t/u_C(C_t)$, because it controls the IES. The ratio $u_C(C_t)C_t/u(C_t)$, on the other hand, is often ignored but is the main focus of the present paper, because it determines how the household's timing attitude affects RRA. Thus, adding a constant to the utility kernel allows us to disentangle the IES, the RRA, *and* the timing attitude.

We start by studying the asset pricing implications of our new utility kernel in the long-run risk model of Bansal and Yaron (2004). Using an analytical second-order perturbation

approximation, we first show that the household’s timing attitude has a separate effect on asset prices beyond the IES and RRA, which is consistent with the experimental evidence cited above. Estimation results for the long-run risk model confirm the finding in Beeler and Campbell (2012) that consumption growth in the standard version of this model is too highly correlated with the price-dividend ratio due to the strong reliance on long-run risk. We further show that this property of the model also makes the contemporaneous correlation between consumption growth and the risk-free rate too high. Thus, these findings question the empirical support for the required degree of long-run risk in the model of Bansal and Yaron (2004). An important empirical finding in the present paper is to show that our utility kernel resolves these puzzles by reducing the reliance on long-run risk and instead allows the household to display strong preferences for early resolution of uncertainty. The ability of our extended model to match means, standard deviations, and auto-correlations is nearly identical to the standard version of the long-run risk model, suggesting that our extension is identified from contemporaneous correlations, which the literature mostly ignores when taking the long-run risk model to the data. Another important finding is that the satisfying performance of the long-run risk model is hardly affected by lowering RRA from 10 to 5 once u_0 is included in the utility kernel. In contrast, the fit of the standard long-run risk model deteriorates with a RRA of 5. Our results also show that the timing premium of Epstein et al. (2014) is very high for the long-run risk model (even with our extension), as it easily implies that the household is willing to give up 70% of total lifetime consumption to have all uncertainty resolved in the following period.

We also study the asset pricing implications of our new utility kernel in a New Keynesian dynamic stochastic general equilibrium (DSGE) model. Our estimates reveal that the proposed utility kernel in this setting resolves the puzzlingly high RRA required in many DSGE models to explain asset prices. More precisely, the model matches the equity premium and the bond premium (i.e. the mean and variability of the 10-year nominal term premium) with a low RRA of 5. The mechanism explaining this substantial improvement of the New

Keynesian model is similar to the one offered in the long-run risk model, namely that our new utility kernel allows strong preferences for early resolution of uncertainty to coincide with low RRA. We also find that changing RRA has a very small effect on the model's ability to match the data when using our new utility kernel. As in the long-run risk model, this suggests that it is not the high RRA in the traditional formulation of Epstein-Zin-Weil preferences that helps to match asset prices, but instead the strong timing attitude that is induced by high RRA. The New Keynesian model implies low timing premia in the order of 5% to 10% due to the endogenous labor supply, consumption habits, and a low IES. Our extension preserves this property of the New Keynesian model and hence matches asset prices with a low RRA *and* a low timing premium.

Conducting a number of counterfactual experiments, we study the asset pricing implications of the timing attitude and long-run risk in the two considered models. To examine the effects of the timing attitude, we omit Epstein-Zin-Weil preferences in both models such that RRA is tightly linked to the IES. This modification generates a small reduction in RRA for the two models, but both models are now unable to explain asset prices. A second counterfactual re-introduces strong preferences for early resolution of uncertainty but omits long-run risk. Here, we also find that the two models cannot match asset prices, although the IES, the RRA, and the timing attitude are identical to their estimated values in both models. Thus, our analysis suggests that the mechanism enabling Epstein-Zin-Weil preferences to explain asset prices in the two considered models, is *not* to separate the IES from RRA, but to introduce strong preferences for early resolution of uncertainty to amplify effects of long-run risk.

The remainder of this paper is organized as follows. Section 2 introduces our new utility kernel within the long-run risk model. Section 3 estimates this extension of the long-run risk model and studies its empirical performance. Section 4 considers a New Keynesian model with the proposed utility kernel and explores its empirical performance. Section 5 concludes, while technical details are provided in Appendix A to D and an online appendix.

2 A Long-Run Risk Model

The representative household is introduced in Section 2.1, and the exogenous processes for consumption and dividends are specified in Section 2.2. The new utility kernel is presented in Section 2.3, and the asset pricing properties of the proposed utility kernel are explored analytically in Section 2.4.

2.1 The Representative Household

Consider a household with recursive preferences as in Epstein and Zin (1989) and Weil (1990). Using the formulation in Rudebusch and Swanson (2012), the value function V_t is given by

$$V_t = u_t + \beta (\mathbb{E}_t[V_{t+1}^{1-\alpha}])^{1/(1-\alpha)} \quad (1)$$

for $u_t > 0$ and $\alpha \in \mathbb{R} \setminus \{1\}$, where $\mathbb{E}_t[\cdot]$ is the conditional expectation given information in period t .¹ Here, $\beta \in (0, 1)$ and $u_t \equiv u(C_t)$ denotes the utility kernel as a function of consumption C_t in the goods market of the economy.

An important property of (1) is to endow the household with preferences for resolution of uncertainty. To describe this property we use the monotonically increasing transformation $U_t \equiv V_t^{1-\alpha}/(1-\alpha)$ to express (1) as $U_t = f(u_t, \mathbb{E}_t[U_{t+1}])$, where $f(u_t, y_t) \equiv \frac{1}{1-\alpha} \left[u_t + \beta ((1-\alpha)y_t)^{\frac{1}{1-\alpha}} \right]^{1-\alpha}$ and $y_t \equiv \mathbb{E}_t[U_{t+1}]$. Kreps and Porteus (1978) show that the household displays preferences for early (late) resolution of uncertainty if $f(\cdot, \cdot)$ is convex (concave) in y_t . It is then easy to verify that α determines the sign of $f_{y_t y_t}$, implying preferences for early (late) resolution of uncertainty if $\alpha > 0$ ($\alpha < 0$).² Thus, the nature of the household's timing attitude is controlled by α , whereas the household is indifferent to the temporal resolution of uncertainty when $\alpha = 0$ and (1) simplifies to expected utility.

¹The standard formulation of these preferences provided in Epstein and Zin (1989), i.e. $\hat{V}_t = \left[\hat{u}_t^\rho + \beta (\mathbb{E}_t[\hat{V}_{t+1}^{\hat{\alpha}}])^{\frac{\rho}{\hat{\alpha}}} \right]^{\frac{1}{\rho}}$, is obtained by letting $V_t = \hat{V}_t^\rho$, $u_t = \hat{u}_t^\rho$, and $\hat{\alpha} = \rho(1-\alpha)$. When $u_t < 0$, we define $V_t = u_t - \beta \mathbb{E}_t[(-V_{t+1})^{1-\alpha}]^{\frac{1}{1-\alpha}}$ as in Rudebusch and Swanson (2012).

²The opposite sign restrictions apply when $u_t < 0$.

An intuitive way to measure the strenght of the household's preferences for the temporal resolution of uncertainty is to compute the timing premium Π_t of Epstein et al. (2014), which is the fraction of lifetime consumption that the household is willing to give up to have all uncertainty resolved in the following period. Epstein et al. (2014) show that Π_t depends positively on α , and we confirm this relationship below in Section 3 and 4. The timing premium also depends positively on the amount of consumption risk in Epstein et al. (2014), and it may therefore be useful to think of α as controlling the 'price of timing risk', whereas the law of motion for consumption controls the 'quantity of timing risk'. The timing premium is generally not available in closed form, and we will therefore rely on the positive correlation between the timing attitude (as captured by $|\alpha|$) and Π_t when studying the analytical properties of the proposed preferences.³

The household has access to a complete market for state contingent claims A_{t+1} . Resources are spent on C_t and A_{t+1} , giving the budget restriction $C_t + \mathbb{E}_t [M_{t,t+1} A_{t+1}] = A_t$, where $M_{t,t+1}$ denotes the real stochastic discount factor.

2.2 Consumption and Dividends

The process for market consumption C_t is specified to be compatible with production economies displaying balanced growth. Hence, we let $C_t \equiv Z_t \times \tilde{C}_t$, where $Z_t > 0$ is the balanced growth path of technology, or simply the productivity level. The variable $\tilde{C}_t$ introduces cyclical consumption risk, which in production economies originates from demand-related shocks, monetary policy shocks, or short-lived supply shocks.

³Alternatively, the strenght of the timing attitude may be quantified based on the curvature of $f(\cdot, \cdot)$. One unit-free measure is $E_t [V_{t+1}^{1-\alpha}] f_{y_t y_t} / f_{y_t} = \alpha(1 - \beta \mu_z^{1-\frac{1}{\psi}})$ for $u_t > 0$ and $E_t [(-V_{t+1})^{1-\alpha}] f_{y_t y_t} / f_{y_t} = -\alpha(1 - \beta \mu_z^{1-\frac{1}{\psi}})$ for $u_t < 0$ when evaluated in the steady state, where $\beta \mu_z^{1-\frac{1}{\psi}}$ is the effective discount factor along the balanced growth path.

Inspired by the work of Bansal and Yaron (2004), we let

$$\begin{aligned}
\log Z_{t+1} &= \log Z_t + \log \mu_z + x_t + \sigma_z \sigma_t \varepsilon_{z,t+1} \\
x_{t+1} &= \rho_x x_t + \sigma_x \sigma_t \varepsilon_{x,t+1} \\
\sigma_{t+1}^2 &= 1 - \rho_\sigma + \rho_\sigma \sigma_t^2 + \sigma_\sigma \varepsilon_{\sigma,t+1}
\end{aligned} \tag{2}$$

where σ_t^2 introduces stochastic volatility. Here, $\varepsilon_{i,t+1} \sim \mathcal{NID}(0, 1)$ for $i \in (z, x, \sigma)$ with $|\rho_x| < 1$ and $|\rho_\sigma| < 1$.⁴ Thus, x_t introduces persistent changes in the growth rate of Z_t and captures long-run productivity risk. The innovation $\varepsilon_{z,t}$ does not generate persistence in the growth rate of Z_t and is referred to as short-run productivity risk. As in Bansal et al. (2010), we let $\log \tilde{C}_{t+1} = \rho_{\tilde{c}} \log \tilde{C}_t + \sigma_{\tilde{c}} \sigma_t \varepsilon_{\tilde{c},t+1}$, where $\varepsilon_{\tilde{c},t} \sim \mathcal{NID}(0, 1)$ and $|\rho_{\tilde{c}}| < 1$.

The process for dividends D_t is given by $\Delta d_{t+1} = \log \mu_d + \phi_x x_t + \phi_{\tilde{c}} \tilde{c}_t + \sigma_d \sigma_t \varepsilon_{d,t+1}$, where $d_{t+1} \equiv \log D_{t+1}$, $\tilde{c}_t \equiv \log \tilde{C}_t$, and $\varepsilon_{d,t} \sim \mathcal{NID}(0, 1)$. Here, ϕ_x and $\phi_{\tilde{c}}$ capture firm leverage in relation to long-run and cyclical risk, respectively, as in Bansal et al. (2010). For completeness, all innovations are assumed to be mutually uncorrelated at all leads and lags.

2.3 The Utility Kernel

To motivate our new utility kernel for disentangling the IES, the RRA, *and* the timing attitude, it is useful to start with the general expression for RRA. Recall, that RRA measures the amount that the household is willing to pay to avoid a risky gamble over wealth. With recursive preferences as formulated in (1), RRA in the steady state (ss) is given by (see Swanson (2018))

$$\text{RRA} = - \left. \frac{u_{CC}(C_t) C_t}{u_C(C_t)} \right|_{ss} + \alpha \left. \frac{u_C(C_t) C_t}{u(C_t)} \right|_{ss}. \tag{3}$$

Hence, RRA depends on the timing attitude through α and the two ratios $u_{CC}(C_t) C_t / u_C(C_t)$ and $u_C(C_t) C_t / u(C_t)$. The first term in (3) is the familiar expression for the inverse of the IES, where the IES measures the percentage change in market consumption growth for a one

⁴Although (2) does not enforce $\sigma_t^2 \geq 0$, we nevertheless maintain this specification for comparison with Bansal and Yaron (2004) and Bansal et al. (2010).

percent change in the real interest rate when abstracting from uncertainty. The second term in (3) is controlled by the timing attitude through α and the ratio $u_C(C_t)C_t/u(C_t)$. The presence of the ratio $u_C(C_t)C_t/u(C_t)$ in this second term is rarely mentioned, but this ratio plays a key role for RRA because it determines how the household's timing attitude affects risk aversion. That is, for a given IES and a given timing attitude, the ratio $u_C(C_t)C_t/u(C_t)$ determines the RRA. This property of $u_C(C_t)C_t/u(C_t)$ appears to have been largely overlooked in the literature, because much focus has been devoted to the power utility kernel $\frac{1}{1-1/\psi}C_t^{1-1/\psi}$, where ψ determines both $u_{CC}(C_t)C_t/u_C(C_t)$ and $u_C(C_t)C_t/u(C_t)$.

This observation suggests that the IES, the RRA, and the timing attitude may be disentangled by considering a utility kernel, where $u_{CC}(C_t)C_t/u_C(C_t)$ and $u_C(C_t)C_t/u(C_t)$ can be determined separately. A simple way to achieve this separation is to let

$$u(C_t) = u_0 Z_t^{1-1/\psi} + \frac{1}{1-1/\psi} C_t^{1-1/\psi}, \quad (4)$$

where the constant $u_0 \in \mathbb{R}$ augments the standard power kernel. To avoid that this constant diminishes relative to the utility from market consumption C_t as the economy grows, we scale u_0 by $Z_t^{1-1/\psi}$ to ensure a balanced growth path in the model.⁵ In this extended utility kernel, the constant u_0 determines $u_C(C_t)C_t/u(C_t)$, whereas the ratio $u_{CC}(C_t)C_t/u_C(C_t)$ and the IES are controlled by ψ as in the conventional power kernel.

The presence of u_0 in (4) may be motivated by accounting for other aspects than market consumption when modeling household utility. We provide two examples. First, the household may enjoy utility from government spending G_t on roads, public parks, law and order, etc. When these spendings grow with the size of the economy, i.e. $G_t = g_{ss}Z_t$ where $g_{ss} \in \mathbb{R}_+$, and the utility from G_t is separable from C_t , then conditions for balanced growth imply a utility kernel of the form $\frac{g_{ss}}{1-1/\psi}Z_t^{1-1/\psi} + \frac{1}{1-1/\psi}C_t^{1-1/\psi}$ as captured by (4). Second,

⁵The kernel in (4) is obviously not the only way to separately determine $u_{CC}(C_t)C_t/u_C(C_t)$ and $u_C(C_t)C_t/u(C_t)$. A previous version of this paper studied a utility kernel that modifies the standard power utility kernel by changing $u_C(\cdot)$ and $u_{CC}(\cdot)$ as opposed to changing the level of $u(\cdot)$ as in (4). However, this alternative specification is slightly more complicated than (4), and we therefore prefer the specification in (4), which we are grateful to the associate editor, Eric Swanson, for proposing.

the household may also consume home-produced goods $C_{h,t}$ that are made using the technology $L_{ss}Z_t$, where L_{ss} denotes a fixed supply of labor. When utility from home-produced goods is separable from C_t , conditions for balanced growth dictates a utility kernel of the form $\frac{L_{ss}^{1-1/\psi}}{1-1/\psi}Z_t^{1-1/\psi} + \frac{1}{1-1/\psi}C_t^{1-1/\psi}$, which also has the structure captured by (4). Thus, by including u_0 in (4) we capture utility from non-market consumption goods and this allows us to disentangle the IES and RRA from the timing attitude.⁶

It is straightforward to show that RRA with (4) is given by

$$\text{RRA} = \frac{1}{\psi} + \alpha \frac{1 - \frac{1}{\psi}}{1 + u_0 \left(1 - \frac{1}{\psi}\right)}. \quad (5)$$

Thus, a high value of u_0 reduces RRA, and vice versa. To understand the intuition behind this effect, consider the case where u_0 is high, such that $u_C(C_t)C_t/u(C_t)$ is low, and hence variation in market consumption C_t only has a small effect on the overall utility level across the business cycle. This implies that the value function attains a high and stable level even when faced with a risky gamble, and the household is therefore only willing to pay a small amount to avoid this gamble, i.e. it has a low RRA. Thus, by varying u_0 , we can separately set RRA, for a given IES and timing attitude.

2.4 Understanding Asset Prices

To explain how the IES, the RRA, and the timing attitude affect asset prices, we follow Bansal and Yaron (2004) and consider a simplified version of the long-run risk model without stochastic volatility, i.e. $\sigma_\sigma = 0$. The presence of u_0 in (4) implies that we cannot obtain the household's wealth in closed form and hence eliminate the value function from the stochastic discount factor as in Epstein and Zin (1989). We are therefore unable to obtain an analytical expression for asset prices by the log-normal method as in Bansal and

⁶A constant in the utility kernel is also included in models with mortality to get positive utility when $\psi < 1$ and hence make life preferable to zero utility from being dead (see Becker et al. (2005), Hall and Jones (2007), Jones (2016) among others). Different from these papers, we include the constant u_0 to amplify the level of $u(C_t)$ instead of altering its sign.

Yaron (2004). Instead, we use the perturbation method to derive an analytical second-order approximation to the model around its balanced growth path. This is done by first using invariant transformations to obtain an equivalent representation of the model with only stationary variables, as provided in (13) of Appendix B. For instance, (1) and the stochastic discount factor are here expressed in terms of the scaled value function $\tilde{V}_t \equiv V_t/Z_t^{1-\frac{1}{\psi}}$. The second-order approximation is then computed around the steady state of this transformed model. We only highlight the solution for the scaled value function $\tilde{v}_t \equiv \log \tilde{V}_t$, the mean of the risk-free rate $r_t^f \equiv \log R_t^f$, and the mean of equity return $r_t^m \equiv \log R_t^m$ in excess of the risk-free rate, with the remaining policy functions provided in Appendix A.

Proposition 1 *The second-order approximation to $\tilde{v}_t$ around its steady state is $\tilde{v}_t = \tilde{v}_{ss} +$*

$$\begin{aligned} & \tilde{v}_{\tilde{c}}\tilde{c}_t + \tilde{v}_x x_t + \frac{1}{2}\tilde{v}_{\tilde{c}\tilde{c}}\tilde{c}_t^2 + \frac{1}{2}\tilde{v}_{xx}x_t^2 + \tilde{v}_{\tilde{c}x}\tilde{c}_t x_t + \frac{1}{2}\tilde{v}_{\sigma\sigma} \text{ with } \kappa_0 \equiv \beta\mu_z^{1-\frac{1}{\psi}} \text{ and} \\ & \tilde{v}_{ss} = \log \left(u_0 + \frac{1}{1-\frac{1}{\psi}} \right) - \log(1 - \kappa_0) \quad \tilde{v}_{\tilde{c}} = \frac{1-\kappa_0}{1-\kappa_0\rho_{\tilde{c}}} \frac{1-\frac{1}{\psi}}{1+u_0(1-\frac{1}{\psi})} \\ & \tilde{v}_x = \frac{\kappa_0}{1-\kappa_0\rho_x} \left(1 - \frac{1}{\psi} \right) \quad \tilde{v}_{\tilde{c}\tilde{c}} = \frac{1-\kappa_0}{1-\kappa_0\rho_{\tilde{c}}^2} \frac{(1-\frac{1}{\psi})^2}{1+u_0(1-\frac{1}{\psi})} - \left[\frac{1-\frac{1}{\psi}}{1+u_0(1-\frac{1}{\psi})} \frac{1-\kappa_0}{1-\kappa_0\rho_{\tilde{c}}} \right]^2 \\ & \tilde{v}_{xx} = \frac{\kappa_0}{1-\kappa_0\rho_x^2} \frac{1-\kappa_0}{(1-\kappa_0\rho_x)^2} \left(1 - \frac{1}{\psi} \right)^2 \quad \tilde{v}_{\tilde{c}x} = \frac{(1-\frac{1}{\psi})^2}{1+u_0(1-\frac{1}{\psi})} \frac{1-\kappa_0}{1-\kappa_0\rho_{\tilde{c}}} \left[\frac{\rho_{\tilde{c}}\kappa_0}{1-\kappa_0\rho_{\tilde{c}}\rho_x} - \frac{\kappa_0}{1-\kappa_0\rho_x} \right] \\ & \tilde{v}_{\sigma\sigma} = \frac{\kappa_0}{1-\kappa_0} \left[\tilde{v}_{\tilde{c}\tilde{c}}\sigma_{\tilde{c}}^2 + (1-\alpha)\tilde{v}_{\tilde{c}}^2\sigma_{\tilde{c}}^2 + \tilde{v}_{xx}\sigma_x^2 + (1-\alpha)\tilde{v}_x^2\sigma_x^2 + (1-\alpha)\left(1 - \frac{1}{\psi}\right)^2\sigma_z^2 \right] \end{aligned}$$

The steady state of the scaled value function $\tilde{v}_{ss}$ is obviously increasing in u_0 , whereas the loadings $\tilde{v}_{\tilde{c}}$ and $\tilde{v}_{\tilde{c}x}$ are decreasing in u_0 . That is, a higher value of u_0 raises the level of the value function and makes it less responsive, as argued above. The lower value of $\tilde{v}_{\tilde{c}}$ is further seen to reduce the contribution from cyclical consumption in the risk correction $\tilde{v}_{\sigma\sigma}$. A key determinant for the size of $\tilde{v}_{\sigma\sigma}$ is α and hence the timing attitude, which has a negative impact on $\tilde{v}_{\sigma\sigma}$ through cyclical, short-, and long-run risk, because $\alpha > 1$ for plausible levels of RRA with $\tilde{v}_{ss} > 0$.

Proposition 2 *The unconditional mean of the risk-free rate $\mathbb{E}[r_t^f]$ and the ex ante equity premium $\mathbb{E}[r_{t+1}^m - r_t^f]$ in a second-order approximation around the balanced growth path are*

$$\mathbb{E}[r_t^f] = r_{ss} - \frac{1}{2}\alpha\tilde{v}_x^2\sigma_x^2 - \frac{1}{2}\left[1 + (\alpha - 1)\left(1 - \frac{1}{\psi^2}\right)\right]\sigma_z^2 - \frac{1}{2}\left(\frac{1}{\psi^2} + \frac{1}{\psi}2\alpha\tilde{v}_{\tilde{c}} + \alpha v_{\tilde{c}}^2\right)\sigma_{\tilde{c}}^2$$

and

$$\mathbb{E} \left[r_{t+1}^m - r_t^f \right] = \alpha \kappa_1 \tilde{v}_x \frac{\phi_x - \frac{1}{\psi}}{1 - \kappa_1 \rho_x} \sigma_x^2 + \left(\alpha \tilde{v}_{\tilde{c}} + \frac{1}{\psi} \right) \frac{\phi_{\tilde{c}} + (1 - \rho_{\tilde{c}}) \frac{1}{\psi}}{1 - \kappa_1 \rho_{\tilde{c}}} \kappa_1 \sigma_{\tilde{c}}^2.$$

Proposition 2 shows that the mean risk-free rate is given by its steady state level $r_{ss} = -\log \beta + \frac{1}{\psi} \log \mu_z$ and uncertainty corrections for each of the shocks affecting consumption. The first term $-\frac{1}{2} \alpha \tilde{v}_x^2 \sigma_x^2$ corrects for long-run risk and is negative and increasing in the timing attitude through α . The second uncertainty correction in $\mathbb{E} \left[r_t^f \right]$ relates to short-run risk and is also negative if $\psi > 1$ and $\alpha > 1$. The final term in $\mathbb{E} \left[r_t^f \right]$ corrects for cyclical risk and is also negative and becomes larger (in absolute terms) when ψ falls and α increases. The effect of u_0 enters in the uncertainty correction for $\tilde{c}_t$ through $\tilde{v}_{\tilde{c}}$, where a low value of u_0 gives a high RRA and a high $\tilde{v}_{\tilde{c}}$ that results in a large uncertainty correction from $\tilde{c}_t$.

The equity premium depends positively on long-run risk if $\phi_x > \frac{1}{\psi}$ and $\psi > 1$, where the latter requirement is needed to ensure that $\tilde{v}_x > 0$. We also note that this uncertainty correction is increasing in i) the persistence of x_t as determined by ρ_x , ii) α , and iii) firm leverage ϕ_x . The second term in $\mathbb{E} \left[r_{t+1}^m - r_t^f \right]$ is also positive and corrects for cyclical risk. The size of this term increases in i) the persistence of $\tilde{c}_t$ as determined by $\rho_{\tilde{c}}$, ii) α , iii) firm leverage $\phi_{\tilde{c}}$, and iv) the loading $\tilde{v}_{\tilde{c}}$. The latter implies that a lower value of u_0 (to increase RRA and $\tilde{v}_{\tilde{c}}$) also increases the contribution of cyclical risk in the equity premium.

To summarize our insights from these analytical expressions, recall that existing models tend to generate too high risk-free rates and too low equity premia. Given identical returns for bonds and equity in the steady state, we thus require a negative uncertainty correction in $\mathbb{E} \left[r_t^f \right]$ and a positive uncertainty correction in $\mathbb{E} \left[r_{t+1}^m - r_t^f \right]$ to resolve the risk-free rate and equity premium puzzles. The long-run risk model does exactly so for a high timing attitude (through a large α) and a high RRA provided $\psi > 1$. That is, the IES must be larger than one to ensure that the uncertainty corrections for the risk-free rate and the equity premium have the correct sign. The proposed utility kernel also shows that the timing attitude via α has a separate effect on asset prices beyond the IES and RRA.

3 Estimation Results: The Long-Run Risk Model

This section studies the ability of the long-run risk model to explain key features of the postwar U.S. economy. We first describe the model solution and estimation methodology in Section 3.1. The estimation results for the standard long-run risk model are provided in Section 3.2 as a natural benchmark. Section 3.3 considers our extension of the long-run risk model, while Section 3.4 studies the performance of the model on moments that are not included in the estimation. We finally consider a number of counterfactuals in Section 3.5.

3.1 Model Solution and Estimation Methodology

Pohl et al. (forthcoming) show that the widely used log-normal method to approximate the solution to the long-run risk model may not always be sufficiently accurate. Our extension allows α to take on even larger values than traditionally considered, and this may generate even stronger nonlinearities in the long-run risk model than reported in Pohl et al. (forthcoming). We address this challenge by using a second-order projection solution, where we exploit properties of quadratic systems with Gaussian innovations to analytically carry out the required integration. Avoiding numerical integration allows us to greatly reduce the executing time of this projection solution to a few seconds, which makes the approximation sufficiently fast to be used inside an estimation routine. Appendix B provides the details on this approximation, which constitutes a new numerical contribution to the literature. We also show in Appendix C that this second-order projection solution is more accurate than the widely used log-normal method, and that it generally performs as well as a highly accurate fifth-order projection solution.

The estimation is carried out on quarterly data, as this data frequency strikes a good balance between getting a reasonably long sample and providing reliable measures of consumption and dividend growth. Consistent with the common calibration procedure for the long-run risk model, we let one period in the model correspond to one month and time-aggregate the theoretical moments to a quarterly frequency. When simulating model mo-

ments, Bansal and Yaron (2004) enforce the non-negativity of σ_t^2 by replacing negative draws with a small positive number. We follow their procedure and set this small number to σ_σ^2 .

Our quarterly data set is from 1947Q1 to 2014Q4, where we use the same five variables as in Bansal and Yaron (2004): i) the log-transformed price dividend ratio pd_t , ii) the real risk-free rate r_t^f , iii) the market return r_t^m , iv) consumption growth Δc_t , and v) dividend growth Δd_t . All variables are stored in this order in $\mathbf{data}_t$ with dimension 5×1 . We explore whether the model can match the means, variances, contemporaneous covariances, and persistence in these five variables, as well as the ability of pd_t to forecast excess market return $ex_t \equiv r_t^m - r_t^f$ and the inability of pd_t to forecast dividend growth. To ease the estimation, the values of μ_z and μ_d are calibrated to match the sample mean of consumption growth and dividends, respectively. Hence, for the estimation we let

$$\mathbf{q}_t \equiv \begin{bmatrix} \widetilde{\mathbf{data}_t}' & vech(\mathbf{data}_t \mathbf{data}_t')' & diag(\mathbf{data}_t \mathbf{data}_t')' & (ex_t - \overline{ex}) \times pd_{t-1} & \Delta d_t \times pd_{t-1} \end{bmatrix},$$

where $\widetilde{\mathbf{data}_t}$ contains the first three elements of $\mathbf{data}_t$, $diag(\cdot)$ denotes the diagonal elements of a matrix, and $\overline{ex}_t$ is the sample average of ex_t . The model is estimated by simulated method of moments (SMM), where the model-implied moments $\frac{1}{S} \sum_{s=1}^S \mathbf{q}_s$ are computed by simulation using $S = 250,000$ monthly observations. We adopt the conventional two-step implementation of SMM and use a diagonal weighting matrix in a preliminary first step, where moments related to consumption and dividend growth have a relatively high weight to ensure that the model does not match asset prices at the expense of a distorted fit to macro fundamentals. Based on these estimates, we then obtain our final estimates using the optimal weighting matrix computed by the Newey-West estimator with 15 lags.

A preliminary analysis reveals that σ_σ is badly identified. Given that the long-run risk model requires high persistence in σ_t^2 , we occasionally find that large estimates of σ_σ generate a fairly low probability of σ_t^2 being non-negative (e.g., $\Pr(\sigma_t^2 \geq 0) \approx 60\%$), making (2) a poor approximation for the evolution of σ_t^2 . Therefore, we impose an upper bound of 0.999 on ρ_σ as in Bansal et al. (2012) and set the value of σ_σ to 0.05. This value of σ_σ ensures that

285 $\Pr(\sigma_t^2 \geq 0)$ is at least 83% with $\rho_\sigma \leq 0.999$.⁷

286 3.2 The Benchmark Model

287 As a natural benchmark, we first consider the standard long-run risk model by letting $u_0 = 0$
 288 in (4). For comparability with nearly all calibrations of this model, we let the IES = 1.5
 289 and RRA = 10 by setting α appropriately using (5). The estimates in the second column
 290 of Table 1 show that x_t generates a small but very persistent component in consumption
 291 growth with $\hat{\sigma}_x = 1.16 \times 10^{-4}$ and $\hat{\rho}_x = 0.990$. As in the calibration of Bansal et al. (2012),
 292 σ_t^2 displays high persistence with $\hat{\rho}_\sigma = 0.9983$. Cyclical consumption risk is mean-reverting
 293 with $\hat{\rho}_c = 0.975$ and fairly volatile with $\hat{\sigma}_c = 0.0027$. We also note that the constraint on
 294 the effective discount factor $\beta\mu_z^{1-1/\psi} < 1$ is binding, because a high value of β is needed to
 295 generate a low risk-free rate.

296 Table 1 also reports the timing premium Π_t of Epstein et al. (2014). At the steady state,
 297 we find $\Pi_{ss} = 70\%$, meaning that the household is willing to give up 70% of its lifetime
 298 consumption to know all future realizations of consumption in the following period. This
 299 level of the timing premium is much higher than the reported 31% for the long-run risk
 300 model in Epstein et al. (2014), but lower than 77% as implied by the calibrated version of
 301 the long-run risk model in Bansal et al. (2012).⁸

302 Column three in Table 2 verifies the common finding in the literature that the standard
 303 long-run risk model with an IES = 1.5 and RRA = 10 is able to explain several asset pricing
 304 moments. In particular, the model provides a very satisfying fit to the means and standard
 305 deviations of the price-dividend ratio and market return. However, the risk-free rate has an
 306 elevated mean (1.96% vs. 0.83%) and displays insufficient variability with a standard devia-

⁷For comparison, Bansal et al. (2012) let $\sigma_\sigma = 0.0378$, and our calibration is thus very similar to their preferred value of σ_σ .

⁸The difference in the timing premium reported in Epstein et al. (2014) and the implied value from the calibration in Bansal et al. (2012) is mainly explained by the considered values of β and ρ_σ . Epstein et al. (2014) use $\rho_\sigma = 0.987$ and $\beta = 0.9980$, but increasing ρ_σ to 0.999 as in Bansal et al. (2012) raises Π_{ss} from 31% to 50%. If we also increase β to 0.9989 as in Bansal et al. (2012), then $\Pi_{ss} = 82\%$ and hence close to the 77% in Bansal et al. (2012). Slightly different values of σ_z and σ_x in Bansal et al. (2012) and Epstein et al. (2014) account for the remaining difference.

tion of 0.75% compared to 2.22% in the data. Table 2 also shows that our estimated version of the long-run risk model matches the standard deviation and persistence in consumption and dividend growth, although the auto-correlation for dividend growth is somewhat higher than in the data (0.52 vs. 0.40). It is, however, within the 95% confidence interval $[0.27, 0.52]$, which is derived from the reported standard error for each of the sample moments in Table 2, shown in parenthesis and computed using a block bootstrap. To evaluate the importance of cyclical consumption risk for these moments, we briefly impose $\sigma_{\tilde{c}} = 0$. Unreported results show that all means are largely unaffected, but that this simplification reduces the standard deviations in r_t^m (12.05) and Δc_t (1.34) as well as increases $\text{corr}(\Delta c_t, \Delta c_{t-1})$ to 0.67. Thus, cyclical consumption risk has only a modest effect in the benchmark model, and this may explain why $\tilde{c}_t$ occasionally is ignored in the long-run risk model.

The last part of Table 2 shows the contemporaneous correlations. We find that consumption growth is too highly correlated with the price-dividend ratio (0.37 vs. 0.03). This is similar to the finding reported in Beeler and Campbell (2012). We also find that consumption growth is too strongly correlated with the risk-free rate (0.47 vs. 0.16). Conventional two-sided t-tests further show that the differences in $\text{corr}(pd_t, \Delta c_t)$ and $\text{corr}(pd_t, r_t)$ have t-statistics of 4.26 and 3.84, respectively.⁹

To understand why consumption growth is too highly correlated with pd_t and r_t^f , recall that the standard long-run risk model relies on the power utility kernel with an IES = 1.5 and RRA = 10. Equation (5) then implies a relatively low timing attitude with $\alpha = 28$. To explain the market return, the model therefore requires high persistence in x_t to amplify the long-run risk channel (see Section 2.4). But, such a high level of persistence in x_t makes consumption growth too highly correlated with the price-dividend ratio and the risk-free

⁹Using the log-normal method and the calibration in Bansal and Yaron (2004), the long-run risk model implies $\text{corr}(pd_t, \Delta c_t) = 0.547$ and $\text{corr}(r_t^f, \Delta c_t) = 0.581$. The corresponding empirical moments on annual data are 0.061 and 0.356, respectively. The modified calibration in Bansal et al. (2012) with less long-run risk gives $\text{corr}(pd_t, \Delta c_t) = 0.368$ and $\text{corr}(r_t^f, \Delta c_t) = 0.473$. Thus, the elevated correlations for $\text{corr}(pd_t, \Delta c_t)$ and $\text{corr}(r_t^f, \Delta c_t)$ also appear in calibrated versions of the long-run risk model using annual data.

330 rate. To realize this, consider the analytical approximation in Section 2.4 which implies

$$cov(\Delta c_t, pd_t) = \frac{\phi - \frac{1}{\psi}}{1 - \kappa_1 \rho_x} \rho_x \frac{\sigma_x^2}{1 - \rho_x^2} + \frac{(1 - \rho_{\bar{c}})^2}{1 - \kappa_1 \rho_{\bar{c}}} \frac{1}{\psi} \frac{\sigma_{\bar{c}}^2}{1 - \rho_{\bar{c}}^2} \quad (6)$$

331 and

$$cov(\Delta c_t, r_t^f) = \frac{1}{\psi} \left[\rho_x \frac{\sigma_x^2}{1 - \rho_x^2} - (1 - \rho_{\bar{c}})^2 \frac{\sigma_{\bar{c}}^2}{1 - \rho_{\bar{c}}^2} \right]. \quad (7)$$

332 Both covariances are increasing in ρ_x for the parameter values in Table 1. Hence, an un-
 333 desirable effect of the high persistence in x_t is to amplify the comovement of consumption
 334 growth with pd_t and r_t^f .

335 The tight link between the timing attitude and the degree of long-run risk is seen clearly
 336 when estimating the model with RRA = 5, as shown in the first column of Table 1. This
 337 lower level of RRA weakens the effect from the timing attitude, as α falls from 28 to 13. To
 338 match asset prices, we therefore find an increase in the degree of long-run risk compared to
 339 the benchmark specification with RRA = 10, as σ_x increases from 1.16×10^{-4} to 1.57×10^{-4}
 340 and ρ_x increases from 0.990 to 0.993. The second column in Table 2 shows that this increase
 341 in long-run risk produces too much auto-correlation in consumption growth (0.72 vs. 0.31)
 342 and further amplifies $corr(\Delta c_t, pd_t)$ and $corr(\Delta c_t, r_t^f)$.

343 3.3 The Extended Model

344 We next introduce u_0 in the utility kernel and re-estimate the long-run risk model when
 345 conditioning on the familiar values of an IES = 1.5 and RRA = 10. Column seven in Table
 346 1 shows that we find $\hat{u}_0 = 9.87$ with a standard error of 0.90, meaning that $\hat{u}_0$ is statistically
 347 different from zero at all conventional significance levels. This implies that market consump-
 348 tion accounts for $\omega_{ss}^C \equiv 1 / (u_0 (1 - 1/\psi) + 1) = 23\%$ of utility along the balanced growth
 349 path, meaning that non-market consumption is an important part of household utility. With
 350 $\hat{u}_0 = 9.87$, the key ratio $u_C(C_t) C_t / u(C_t)|_{ss}$ is much lower than in the benchmark version of
 351 the model (0.078 vs. 0.333), and this allows for a higher timing attitude with α increasing

from 28 to 120 while keeping RRA at 10. Less long-run risk is therefore needed to match asset prices and this explains the fall in $\hat{\rho}_x$ from 0.990 to 0.968. As a result, $\text{corr}(pd_t, \Delta c_t)$ falls from 0.37 to 0.10 and $\text{corr}(r_t^f, \Delta c_t)$ falls from 0.47 to 0.26, implying that both moments are no longer significantly different from their empirical moments. We also see improvements in the ability of the model to match $\text{corr}(pd_t, r_t^f)$, $\text{corr}(r_t^f, \Delta d_t)$, $\text{corr}(\Delta c_t, \Delta d_t)$, and the mean of r_t^f . On the other hand, the fit to $\text{corr}(r_t^m, \Delta c_t)$, $\text{corr}(r_t^m - r_t^f, pd_{t-1})$, $\text{corr}(\Delta d_t, pd_{t-1})$, and the standard deviations of pd_t and r_t^f worsen slightly when including u_0 . To study the importance of cyclical consumption risk, unreported results show that letting $\sigma_{\tilde{c}} = 0$ reduces the mean of pd_t to 2.79 and increases the mean of r_t^m to 8.58%. Even larger effects are observed for some of the standard deviations, which fall to 7.22% for r_t^m and to 1.14% for Δc_t . Thus, cyclical consumption plays a more important role in this extended version of the long-run risk model than in the benchmark specification in Section 3.2.

To evaluate the overall goodness of fit for the long-run risk model, Table 2 also reports the value of the objective function Q^{step2} in step 2 of our SMM estimation and the related p-value for the J-test for model misspecification. The benchmark model and our extension are not rejected by the data, but we note that the J-test has low power given our short sample ($T = 271$). The values of Q^{step2} are unfortunately not comparable across models, because they are computed for model-specific weighting matrices. To facilitate model comparison, we therefore introduce the following measure for goodness of fit $Q^{\text{scaled}} = \sum_{i=1}^n ((m_i^{\text{data}} - m_i^{\text{model}}) / (1 + m_i^{\text{data}}))^2$, where m_i^{data} and m_i^{model} refer to the scaled moments in the data and the model, respectively, as reported in Table 2.¹⁰ Although the moments in Q^{scaled} are weighted differently than in the estimation, Q^{scaled} may nevertheless serve as a natural summary statistic for model comparison from an economic perspective. We find that the benchmark model implies $Q^{\text{scaled}} = 2.26$, but allowing for u_0 in the utility kernel gives $Q^{\text{scaled}} = 1.54$. This corresponds to a 32% improvement in model fit from disentangling the timing attitude from the IES and RRA.

¹⁰The difference $m_i^{\text{data}} - m_i^{\text{model}}$ in Q^{scaled} is standardized by $1 + m_i^{\text{data}}$, as opposed to just m_i^{data} , to ensure that moments close to zero do not get very large weights.

A natural way to extend the timing premium of Epstein et al. (2014) to the utility kernel in (4) is to define Π_t implicitly as

$$V_t = \left(u_0 Z_t^{1-1/\psi} + \frac{1}{1-\frac{1}{\psi}} C_t^{1-\frac{1}{\psi}} \right) (1 - \Pi_t)^{1-\frac{1}{\psi}} \quad (8)$$

$$+ \beta \left(\mathbb{E}_t \left[\left(\sum_{i=1}^{\infty} \beta^{i-1} \left(u_0 Z_{t+i}^{1-1/\psi} + \frac{C_{t+i}^{1-\frac{1}{\psi}}}{1-\frac{1}{\psi}} \right) (1 - \Pi_t)^{1-\frac{1}{\psi}} \right)^{1-\alpha} \right] \right)^{1/(1-\alpha)}.$$

That is, we combine the utility from market and non-market consumption when computing Π_t . This implies that Π_t measures the fraction of total lifetime consumption that the household is willing to pay to have all uncertainty resolved in the following period. Clearly, the definition in (8) of Π_t reduces to the one in Epstein et al. (2014) when $u_0 = 0$.¹¹ Table 1 shows that Π_{ss} increases from 70% to 86% when introducing u_0 in the utility kernel when $RRA = 10$ and $IES = 1.5$. That is, the pronounced increase in α from 28 to 120 more than outweighs the effects from less long-run risk and leads to an even higher timing premium.

The remaining columns in Table 1 and 2 explore the robustness of these findings to lowering the IES to 1.1, increasing the IES to 2, and reducing RRA to 5. We emphasize two results. First, lowering RRA from 10 to 5 hardly affects the model's ability to match asset prices once u_0 is included in the utility kernel. For instance, we find $Q^{scaled} = 1.54$ when the IES = 1.5 for both levels of RRA. In contrast, when using the traditional utility kernel with a RRA of 5, the model's ability to match the data deteriorates as Q^{scaled} increases from 2.26 to 3.35. Second, the effects of changing the IES are generally also small, mainly because of large changes in the estimate of u_0 . As a result, the largest effect of changing the IES appears for the fraction of utility from market consumption, which is between 23% and 44% for RRA = 10 and between 11% and 17% for RRA = 5. Thus, the satisfying ability of the long-run

¹¹ Another possibility is to define the timing premium purely in relation to the amount of market consumption C_t that the household is willing to give up to have all uncertainty about market and non-market consumption resolved in the following period. Such an alternative definition would clearly give a higher timing premium than implied by (8), which therefore constitutes a lower bound on the household's timing premium. However, this alternative definition does not ensure that the timing premium is between 0% and 100%, and we therefore consider (8) as the most natural extension of the definition in Epstein et al. (2014), which also guarantees a timing premium between 0% and 100%.

risk model to match asset prices extends to the case of a lower IES of 1.1 and a lower RRA of 5, once u_0 is included in the utility kernel. However, our extended utility kernel does not alleviate the problem of seemingly implausible high levels of the timing premium, which remains very high (i.e. above 70%) for all considered specifications of the IES and RRA.

3.4 Additional Model Implications

In addition to the moments used in the estimation, the long-run risk model is also frequently evaluated based on its ability to reproduce several stylized relationships for the U.S. economy. Figure 1 shows that past and future consumption growth in the standard long-run risk model are too highly correlated with the price-dividend ratio compared to empirical evidence. A similar finding is reported in Beeler and Campbell (2012) for two calibrated versions of this model. In contrast, our extension of the long-run risk model implies that past and future consumption growth display the same low correlations with the price-dividend ratio as in the data. Figure 1 considers the case where the IES = 1.5 and RRA = 5 in our extension of the long-run risk model, but the results are robust to using any of the other specifications for the IES and RRA in Table 1. Thus, disentangling the timing attitude from the IES and RRA is also supported by these stylized regressions, because a higher timing attitude reduces the amount of long-run risk and hence the degree of predictability in Δc_t .

The last two charts in Figure 1 explore the relationship between consumption volatility and the price-dividend ratio. We find that our extension of the long-run risk model preserves the good performance of the benchmark model and implies that i) a high price-dividend ratio predicts future low volatility and ii) high uncertainty forecasts a low price-dividend ratio.

3.5 The Key Mechanisms

We next illustrate some of the key mechanisms in the model. Here, we use the estimates in column four of Table 1 with an IES of 1.5 and a RRA of 5.

The first experiment we consider is to gradually increase u_0 to its estimated value. Table

3 shows that a higher value of u_0 generates a substantial increase in α to ensure a constant RRA. This in turn has desirable effects on the level of asset prices because a higher value of α reduces $\mathbb{E}[pd_t]$ as well as $\mathbb{E}[r_t^f]$ and increases $\mathbb{E}[r_t^m]$. To understand these effects of increasing α and hence the timing attitude for a *given* level of RRA, recall that the household is indifferent to the timing resolution of uncertainty when $\alpha = 0$. Now suppose we increase α to make the household prefer early resolution of uncertainty, but without changing the RRA. This modification increases the variability of the value function and hence increases the precautionary motive. The one-period risk-free bond therefore becomes more attractive, and this reduces the risk-free rate as shown in Proposition 2. On the other hand, uncertain future dividends from equity become less attractive for higher values of α due to the presence of long-run risk. A household with strong preferences for early resolution of uncertainty therefore requires a larger compensation for holding equity compared to the case of $\alpha = 0$ and this explains the increase in $\mathbb{E}[r_t^m]$ for higher values of α .

The second experiment we consider is to omit long-run risk by letting $\sigma_x = 0$. The fourth column in Table 3 shows that this modification has profound implications, as the model now generates a too high level for the price-dividend ratio (14.04 vs. 3.50 in the data) and the risk-free rate (2.24% vs. 0.83% in the data), whereas the average market return is too low (2.38% vs. 6.92% in the data). Omitting long-run risk also has a large effect on the timing premium, which falls from 86% to 18%. Thus, disentangling the timing attitude and hence α from the IES and RRA does not alleviate the reliance on long-run risk in the model.

Our third experiment imposes $\sigma_\sigma = 0$ to evaluate the importance of stochastic volatility. The fifth column in Table 3 shows that the mean of the price-dividend ratio increases to 4.46 and the mean market return falls to 3.66%. We also find that the timing premium decreases from 86% to 33%. This shows that stochastic volatility may have a much larger impact on the timing premium in the long-run risk model than suggested by the results in Epstein et al. (2014). Thus, stochastic volatility remains an important feature of the long-run risk model, even when the timing attitude is set independently of the IES and RRA.

The fourth experiment explores whether the high subjective discount factor β may help to explain the high timing premium in the long-run risk model. We address this question in the sixth column of Table 3 by reducing β from its estimated value of 0.9991 to 0.9980 as considered in Epstein et al. (2014). This small change in β reduces the timing premium from 86% to 34%, which is in the neighborhood of the 31% reported in Epstein et al. (2014). However, $\beta = 0.9980$ gives a too high mean for the risk-free rate (3.21% vs. 0.83% in the data), and hence makes the model unable to resolve the risk-free rate puzzle. This result explains why our estimation prefers a high β , although it implies a high timing premium.

Our final experiment studies the effect of the timing attitude by letting $\alpha = 0$, implying that the household is indifferent between early and late resolution of uncertainty. The seventh column of Table 3 shows that this modification only lowers the RRA from 5 to 0.67, but it nevertheless has a profound impact on the model despite the presence of long-run risk. That is, the model is simply unable to match asset prices without strong preferences for early resolution of uncertainty.

4 A New Keynesian Model

To provide further support for our extended Epstein-Zin-Weil preferences, we next show that they also help explain asset prices in an otherwise standard New Keynesian model. The processes for consumption and dividends are here determined within the model, whereas they are assumed to be exogenously given in the long-run risk model. We proceed by presenting the New Keynesian model in Section 4.1, the adopted estimation routine in Section 4.2, and the estimation results in Section 4.3. We finally examine the key mechanisms in our extended New Keynesian model in Section 4.4.

4.1 Model Description

4.1.1 Household

The household is similar to the one considered in Section 2 except for a variable labor supply L_t . To match the persistence in the growth rate of market consumption, we follow much of the New Keynesian tradition and allow for external consumption habits of the form bC_{t-1} . These modifications are included in the new utility kernel by letting

$$u(C_t, L_t) = u_0 Z_t^{1-1/\psi} + \frac{(C_t - bC_{t-1})^{1-1/\psi}}{1 - 1/\psi} + \varphi_0 Z_t^{1-1/\psi} \frac{(1 - L_t)^{1-\frac{1}{\varphi}}}{1 - \frac{1}{\varphi}} \quad (9)$$

with $\varphi_0 > 0$ and $\varphi \in \mathbb{R} \setminus \{1\}$, which reduces to the specification in Rudebusch and Swanson (2012) when $u_0 = b = 0$. The constant u_0 does not affect the IES at the steady state $\psi \left(1 - \frac{b}{\mu_{Z,ss}}\right)$, where consumption habits reduce the IES compared to the value implied by ψ . The expression for RRA is slightly more involved than the one provided in (5) due to consumption habits and the variable labor supply, where the latter gives the household an additional margin to absorb shocks. For the Epstein-Zin-Weil preferences in (1), Swanson (2018) shows that RRA in the steady state is given by $\text{RRA} = \frac{1}{\text{IES}} \left(1 + \frac{W_t}{Z_t} \Lambda_t\right)^{-1} \Big|_{ss} + \alpha \frac{u_C(C_t, L_t) C_t}{u(C_t, L_t)} \Big|_{ss}$, where $\Lambda_t \equiv -\frac{u_L(C_t, L_t) u_{CC}(C_t, L_t)}{u_C(C_t, L_t) u_{LL}(C_t, L_t)}$ accounts for the labor margin. When inserting for the utility kernel in (9) we get

$$\begin{aligned} \text{RRA} = & \frac{1}{\text{IES}} \frac{1}{\left(1 + \frac{1}{1-b\mu_{Z,ss}^{-1}} \frac{\varphi}{\psi} \frac{\tilde{W}_{ss}(1-L_{ss})}{\tilde{C}_{ss}}\right)} \\ & + \left[\frac{\alpha \left(1 - \frac{1}{\psi}\right)}{1 - b\mu_{Z,ss}^{-1}} \right] \frac{1}{\left(1 - \frac{1}{\psi}\right) u_0 \left(\tilde{C}_{ss} - b\mu_{Z,ss}^{-1} \tilde{C}_{ss}\right)^{\frac{1}{\psi}-1} + 1 + \frac{1}{1-b\mu_{Z,ss}^{-1}} \frac{1-\frac{1}{\varphi}}{1-\frac{1}{\varphi}} \frac{(1-L_{ss})\tilde{W}_{ss}}{\tilde{C}_{ss}}}. \end{aligned} \quad (10)$$

Here, $\tilde{C}_{ss}$ and $\tilde{W}_{ss}$ refer to the steady state of market consumption and the real wage in the normalized economy without trending variables, and $\mu_{Z,ss}$ denotes the deterministic trend in consumption and productivity specified below in (11). Equation (10) shows that u_0 also with

habits and a variable labor supply controls RRA through the ratio $u_C(C_t, L_t) C_t / u(C_t, L_t)$.

The real budget constraint for the household is given by $\mathbb{E}_t \left[\frac{M_{t,t+1}}{\pi_{t+1}} X_{t+1} \right] + C_t = \frac{X_t}{\pi_t} + W_t L_t + D_t$, where $M_{t,t+1}$ is the real stochastic discount factor, X_t is nominal state-contingent claims, π_t denotes gross inflation, W_t is the real wage, and D_t is real dividend payments from firms.

4.1.2 Firms

Final output Y_t is produced by a perfectly competitive representative firm, which combines differentiated intermediate goods $Y_t(i)$ using $Y_t = \left(\int_0^1 Y_t(i)^{\frac{\eta-1}{\eta}} di \right)^{\frac{\eta}{\eta-1}}$ with $\eta > 1$. This implies that the demand for the i th good is $Y_t(i) = \left(\frac{P_t(i)}{P_t} \right)^{-\eta} Y_t$, where $P_t \equiv \left(\int_0^1 P_t(i)^{1-\eta} di \right)^{\frac{1}{1-\eta}}$ denotes the aggregate price level and $P_t(i)$ is the price of the i th good.

Intermediate firms produce the differentiated goods using $Y_t(i) = Z_t A_t K_{ss}^\theta L_t(i)^{1-\theta}$, where K_{ss} and $L_t(i)$ denote capital and labor services at the i th firm, respectively. Productivity shocks are allowed to have the traditional stationary component A_t , but also a non-stationary component Z_t to generate long-run risk in the model. For the stationary shocks, we let $\log A_{t+1} = \rho_A \log A_t + \sigma_A \varepsilon_{A,t+1}$, where $|\rho_A| < 1$, $\sigma_A > 0$, and $\varepsilon_{A,t+1} \sim \mathcal{NID}(0, 1)$. Similarly for the non-stationary shocks, we introduce $\mu_{Z,t+1} = Z_{t+1}/Z$ and let

$$\log \left(\frac{\mu_{Z,t+1}}{\mu_{Z,ss}} \right) = \rho_Z \log \left(\frac{\mu_{Z,t}}{\mu_{Z,ss}} \right) + \sigma_Z \varepsilon_{Z,t+1}, \quad (11)$$

where $|\rho_Z| < 1$, $\sigma_Z > 0$, and $\varepsilon_{Z,t+1} \sim \mathcal{NID}(0, 1)$.¹²

Intermediate firms can freely adjust their labor demand at the given market wage W_t and are therefore able to meet demand in every period. Price stickiness is introduced as in Rotemberg (1982), where $\xi \geq 0$ controls the size of firms' real cost $\frac{\xi}{2} (P_t(i) / (P_{t-1}(i) \pi_{ss}) - 1)^2 Y_t$

¹²The specification of long-run productivity risk adopted in the endowment model, i.e. (2), could also be used in the New Keynesian model, but we prefer the more parsimonious specification in (11) for comparability with the existing DSGE literature (see, for instance, Justiniano and Primiceri (2008)). This difference explains the slightly different notation used in (11) for $\mu_{Z,t}$, $\mu_{Z,ss}$, σ_Z , and $\varepsilon_{Z,t+1}$ compared to the corresponding parameters in (2).

when changing the optimal nominal price $P_t(i)$ of the good they produce.¹³ As in Rudebusch and Swanson (2012), we also assume that firms use $\delta K_{ss} Z_t$ units of output to maintain the constant capital stock.

4.1.3 The Central Bank and Aggregation

The central bank sets the one-period nominal interest rate i_t according to $i_t = i_{ss} + \beta_\pi \log\left(\frac{\pi_t}{\pi_{ss}}\right) + \beta_y \log\left(\frac{Y_t}{Z_t \tilde{Y}_{ss}}\right)$, based on a desire to close the inflation and output gap. Note that the inflation gap accounts for steady-state inflation π_{ss} , and that the output gap is expressed in deviation from the steady state level of output in the normalized economy $\tilde{Y}_{ss}$ without trending variables. Summing across all firms, the resource constraint becomes $C_t + Z_t \delta K_{ss} = \left(1 - \frac{1}{2} \xi (\pi_t / \pi_{ss} - 1)^2\right) Y_t$.

4.1.4 Equity and Bond Prices

Equity is defined as a claim on aggregate dividends from firms, i.e. $D_t = Y_t - W_t L_t - \frac{1}{2} \xi (\pi_t / \pi_{ss} - 1)^2 Y_t - Z_t \delta K_{ss}$, and its real price P_t^m is therefore $1 = \mathbb{E}_t [M_{t,t+1} R_{t+1}^m]$ where $R_{t+1}^m = (D_{t+1} + P_{t+1}^m) / P_t^m$.

The price in period t of a default-free zero-coupon bond $B_t^{(n)}$ maturing in n periods with a face value of one dollar is $B_t^{(n)} = \mathbb{E}_t \left[\frac{M_{t,t+1}}{\pi_{t+1}} B_{t+1}^{(n-1)} \right]$ for $n = 1, \dots, N$ with $B_t^{(0)} = 1$. Its yield to maturity is $i_t^{(n)} = -\frac{1}{n} \log B_t^{(n)}$. Following Rudebusch and Swanson (2012), we define term premia as $\Psi_t^{(n)} = i_t^{(n)} - \tilde{i}_t^{(n)}$, where $\tilde{i}_t^{(n)}$ is the yield to maturity on a zero-coupon bond $\tilde{B}_t^{(n)}$ under risk-neutral valuation, i.e. $\tilde{B}_t^{(n)} = e^{-i_t} \mathbb{E}_t \left[\tilde{B}_{t+1}^{(n-1)} \right]$ with $\tilde{B}_t^{(0)} = 1$.

¹³Specifying nominal rigidities by Calvo pricing as in Rudebusch and Swanson (2012) gives largely similar results to those reported below. The considered specification is chosen because the solution to the New Keynesian model with Rotemberg pricing is approximated more accurately by the perturbation method than with Calvo pricing. The reason seems to be that Calvo (unlike Rotemberg) pricing induces a price dispersion index as an extra state variable that makes the New Keynesian model very nonlinear in certain areas of the state space (see Andreasen and Kronborg (2018)). Appendix D shows that the third-order perturbation solution used to estimate the considered New Keynesian model is fairly accurate and generally delivers the same level of precision as a fifth-order perturbation approximation.

4.2 Model Solution and Estimation Methodology

We solve the model by a third-order perturbation solution. The model is estimated by generalized methods of moments (GMM) using unconditional first and second moments computed as in Andreasen et al. (2018). The selected series describing the macro economy and the bond market are given by Δc_t , π_t , i_t , $i_t^{(40)}$, $\Psi_t^{(40)}$, and $\log L_t$, where one period in the model corresponds to one quarter. The 10-year nominal interest rate and its term premium (obtained from Adrian et al. (2013)) are available from 1961Q3, leaving us with quarterly data from 1961Q3 to 2014Q4. We include all means, variances, and first-order auto-covariances of these six variables for the estimation, in addition to five contemporaneous covariances related to the correlations reported at the end of Table 5. To examine whether our New Keynesian model is able to match the equity premium, we also include the mean of the net market return $r_t^m = \log R_t^m$ in the set of moments. Finally, the GMM estimation is implemented using the conventional two-step procedure for moment-based estimators as outlined in Section 3.1.

We estimate all structural parameters in the model except for a few badly identified parameters. That is, we let $\delta = 0.025$ and $\theta = 1/3$ as typically considered for the U.S. economy. We also let $\eta = 6$ to get an average markup of 20% and impose $\varphi = 0.25$ to match a Frisch labor supply elasticity in the neighborhood of 0.5. The ratio of capital to output in the steady state is set to 2.5 as in Rudebusch and Swanson (2012). We follow Andreasen (2012) and set ξ based on a linearized version of the model to match a Calvo parameter of $\alpha_p = 0.75$, giving an average duration for prices of four quarters.¹⁴ Finally, the estimates of the subjective discount factor for all considered specifications of the New Keynesian model hit the upper bound for this parameter and we therefore simply let $\beta = 0.9995$.

¹⁴The mapping is $\xi = \frac{(1-\theta+\eta\theta)(\eta-1)\alpha_p}{(1-\alpha_p)(1-\theta)\left(1-\alpha_p\beta\mu_{Z,ss}^{1-\frac{1}{\psi}}\right)}$ as derived in the online appendix.

4.3 Estimation Results

4.3.1 A Standard Power Utility Kernel

We first consider the standard implementation of Epstein-Zin-Weil preferences with $u_0 = 0$ and condition the estimation of the New Keynesian model on different values of RRA. Table 4 shows that we get fairly standard estimates when $\text{RRA} = 5$. That is, we find strong habits ($\hat{b} = 0.70$), very persistent technology shocks ($\hat{\rho}_A = 0.99$ and $\hat{\rho}_Z = 0.31$), and a central bank that assigns more weight to stabilizing inflation than output ($\hat{\beta}_\pi = 1.47$ and $\hat{\beta}_y = 0.02$). Table 5 shows that the model does well in matching the mean and variability of inflation, the short rate, the 10-year interest rate, and the 10-year term premium. The model-implied level of the market return is 3.75% and reasonably close to the empirical value of 5.53%, when accounting for its large standard error of 2.01% computed by a block bootstrap. However, the model also generates too much variability in consumption growth (2.49% vs. 1.80%) and labor supply (2.80% vs. 1.62%), predicts too strong autocorrelation in consumption growth (0.70 vs. 0.53), and is unable to match the negative correlation between consumption growth and inflation (0.14 vs. -0.18). Table 4 and 5 also show that increasing RRA to 10 does not substantially affect the estimates and performance of the New Keynesian model. Thus, these results just reproduce the finding in Rudebusch and Swanson (2008) that the standard New Keynesian model with low RRA struggles to match key asset pricing moments without distorting the fit to the macro economy.

We next increase RRA to 60, although such an extreme level of risk aversion is hard to justify based on micro-evidence. Table 5 shows that the New Keynesian model now reproduces all means without generating too much variability in the macro economy, except for a slightly elevated standard deviation in labor supply (2.45% vs. 1.62%). Thus, a high RRA of 60 implies that the model delivers a better overall fit to the data with $Q^{\text{scaled}} = 0.33$ compared to $Q^{\text{scaled}} = 0.66$ when $\text{RRA} = 5$.

To compute the timing premium in our New Keynesian model we must extend the definition in Epstein et al. (2014) to account for an endogenous labor supply. The labor margin

gives the household an extra dimension to absorb shocks and this affects its willingness to pay for getting uncertainty resolved. The problem is thus very similar to the one considered in Swanson (2018) for extending expressions of RRA to account for a variable labor supply, and we therefore follow his approach and use the equilibrium condition for the consumption-leisure trade-off. This implies that the value function can be expressed in consumption units as

$$\begin{aligned}
V_t = & Z_t^{1-\frac{1}{\psi}} u_0 + \frac{1}{1-\frac{1}{\psi}} (C_t - bC_{t-1})^{1-\frac{1}{\psi}} \\
& + Z_t^{1-\frac{1}{\psi}} \frac{\varphi_0^\varphi}{1-\frac{1}{\varphi}} \frac{Z_t^{(1-\frac{1}{\psi})(\varphi-1)}}{W_t^{(\varphi-1)}} (C_t - bC_{t-1})^{\frac{1}{\psi}(\varphi-1)} + \beta \left(\mathbb{E}_t [V_{t+1}^{1-\alpha}] \right)^{\frac{1}{1-\alpha}},
\end{aligned} \tag{12}$$

and the timing premium is then computed using the same procedure as in (8). Table 4 shows that the timing premium at the steady state Π_{ss} is 0.1% with $\text{RRA} = 5$, 4% with $\text{RRA} = 10$, and only 10% with $\text{RRA} = 60$. Note also that this increase in Π_{ss} coincides with higher levels of the timing attitude, as the absolute value of α increases for higher RRA. Importantly, the timing premium in the New Keynesian model is substantially lower than in the long-run risk model, even when considering an extreme RRA of 60.

To explore whether the labor margin helps to account for the low timing premium in the New Keynesian model, we next condition on the reported estimates in Table 4 with $\text{RRA} = 60$ and change the Frisch labor supply elasticity $\varphi (1/L_t - 1)$ by considering different values of φ . It is a priori not obvious how the timing premium should be affected by changing the variability of the labor supply. As argued by Swanson (2018) in the context of RRA, a higher labor supply elasticity allows the household to better self-insure against bad productivity shocks to reduce the variability in consumption. This effect should therefore reduce the timing premium for higher values of φ . But, a more volatile labor supply also makes the household's value function more uncertain through the direct effect of leisure in the utility kernel, and this effect should therefore increase the timing premium for higher values of φ . Panel A in Table 6 shows that the second effect dominates, as the timing premium is 10% for

$\varphi = 0.25$, 35% for $\varphi = 0.50$, and 93% for $\varphi = 0.75$. These computations are conditioned on a RRA of 60 by appropriately changing α , which increases substantially in absolute terms for higher values of φ . Panel B of Table 6 adopts another approach by conditioning on $\alpha = -36$ and instead let RRA vary as we change the value of φ . When using this alternative benchmark, we find a much more modest increase in the timing premium when increasing φ , showing that the main effect of the labor margin operates through the timing attitude as controlled by α . Two other features of the New Keynesian model that also may have a sizable impact on the timing premium are consumption habits and the low estimate of ψ . Both features help to generate a low IES, which reduces the timing premium as shown in Epstein et al. (2014). Panel C in Table 6 shows that low consumption habits and higher values of ψ increase the timing premium. For instance, we find that the timing premium is 43% with $b = 0$ and $\psi = 0.75$.

Thus, the labor margin, consumption habits, and a low estimate of ψ help to generate a low timing premium in the New Keynesian model.

4.3.2 The Extended Utility Kernel

We next let u_0 be a free parameter and estimate the New Keynesian model when conditioning on a RRA of 5. The fourth column in Table 4 shows that $\hat{u}_0 = -943$ with a standard error of 261.¹⁵ Hence, we clearly reject the null hypothesis of $u_0 = 0$ (t-statistic = -3.62) and therefore the standard utility kernel. This means that accounting for other aspects than market consumption and leisure when modeling household utility also helps the New Keynesian model to explain postwar U.S. data. The estimate of u_0 is clearly larger (in absolute terms) than any of the estimates of u_0 in the long-run risk model, but such a direct comparison is not particularly useful because of the structural differences between the two models. For instance, the New Keynesian model implies $\tilde{C}_{ss} = 0.80$, includes habits, and gives a substantial utility contribution from leisure (as $\varphi_0 = 42.81$ to match $\hat{L}_{ss}$). In

¹⁵The utility contributions from consumption and leisure in (12) are negative given our estimates, and we therefore let $u_0 < 0$ to get a negative value function as in the standard version of the New Keynesian model.

contrast, the long-run risk model has $\tilde{C}_{ss} = 1$ and abstracts from both habits and leisure. Instead, it is more informative to study the value of $u_C(C_t, L_t) C_t / u(C_t, L_t)|_{ss}$, because both models determine u_0 from this ratio to attain a given level of RRA. Table 4 shows that our large estimate of u_0 gives a fairly low value of $u_C(C_t, L_t) C_t / u(C_t, L_t)|_{ss} = -0.10$, which is remarkably close to the corresponding ratio in the long run risk model, which is 0.08 with $IES = 1.5$ and $RRA = 10$.

We generally find small effects on most of the structural parameters from including u_0 . The main exceptions are smaller consumption habits ($b = 0.48$), a reduction in the amount of long-run productivity risk (ρ_Z and σ_Z fall), and more risk related to stationary productivity shocks (ρ_A and σ_A increase). We also find a large increase in the timing attitude, as α increases from -1.3 to -27.7 when $RRA = 5$. However, this increase does not generate a substantially higher timing premium, which remains low at 6% with $RRA = 5$. A possible downside of including u_0 is that the fraction of utility from market consumption and leisure along the balanced growth path $\omega_{ss}^{C,L}$ is somewhat low at 6% when $RRA = 5$.

Table 5 shows that including u_0 in the New Keynesian model enables the model to match all means and standard deviations, except for the labor supply that displays the same degree of variability as in the standard New Keynesian model with $RRA = 60$. Subject to this qualification, the New Keynesian model now explains the equity premium (5.01% vs. 5.53% in the data) with a low $RRA = 5$ and a low timing premium. For the 10-year nominal term premium, the model also matches its mean (1.67% vs. 1.66% in the data) and its standard deviation (1.00% vs. 1.17% in the data), implying that we also explain the bond premium puzzle with low RRA and low timing premium. The auto- and contemporaneous correlations are also well matched, and the proposed extension of the New Keynesian model therefore has better overall fit with $Q^{scaled} = 0.24$ compared to $Q^{scaled} = 0.33$ for the standard New Keynesian model with $RRA = 60$.

Accordingly, the extended utility kernel in (9) makes the timing attitude a useful behavioral parameter that helps to lower RRA. This finding is thus in sharp contrast to the

traditional formulation of Epstein-Zin-Weil preferences, where Epstein et al. (2014) view the timing attitude and the related timing premium as an unavoidable cost for separating the IES and RRA.

The final two columns of Table 4 and 5 study the effects of higher RRA when allowing for an unrestricted timing attitude. We find that higher RRA does *not* improve the performance of the New Keynesian model. Actually, its performance worsens slightly with Q^{scaled} increasing from 0.24 to 0.29 when changing RRA from 5 to 60. This suggests that it is not the high RRA in the traditional formulation of Epstein-Zin-Weil preferences that helps the New Keynesian model match asset prices, but instead the high timing attitude that is induced by the high RRA.

4.4 The Key Mechanisms

We next run three experiments to explore some of the key mechanisms in the New Keynesian model with the extended utility kernel in (9). The first experiment considered in Table 7 illustrates the implications of gradually increasing u_0 . As for the long-run risk model, a numerically larger value of u_0 lowers $u_C(C_t, L_t) C_t / u(C_t, L_t)$ and allows for a strong timing attitude through a high α without affecting RRA. The large value of α then amplifies the existing risk corrections and enables the model to explain asset prices with low RRA.

Our second experiment abstracts from long-run productivity risk by letting $\sigma_Z = 0$. The fourth column in Table 7 shows that this modification has very large effects as the model now is unable to explain mainly the level of π_t , i_t , $i_t^{(40)}$, and $\Psi_t^{(40)}$. Thus, long-run risk is also an essential feature of the New Keynesian model even when including u_0 .

Our final experiment omits Epstein-Zin-Weil preferences by letting $\alpha = 0$ to make the household indifferent between early and late resolution of uncertainty. Although this modification only has a small effect on RRA (reducing it from 5 to 2.2) it nevertheless has a profound impact on the model, which largely displays the same properties as when omitting long-run productivity risk. That is, the New Keynesian model is unable to explain

asset prices without Epstein-Zin-Weil preferences, and hence strong preferences for early resolution of uncertainty.

Thus, we confirm the result from the long-run risk model, namely that the main effect of Epstein-Zin-Weil preferences with our extended utility kernel is *not* to separate the IES from RRA but instead to introduce strong preferences for early resolution of uncertainty. This finding also helps to clarify why consumption habits may struggle to match asset prices in DSGE models, although they allow for additional flexibility in setting the IES and RRA (see Rudebusch and Swanson (2008)). The explanation seems to be that consumption habits do not introduce preferences for early resolution of uncertainty, which we find are essential to explain asset prices in a standard New Keynesian model.¹⁶

5 Conclusion

The present paper highlights the importance of the timing attitude for consumption-based asset pricing. To isolate the effects of the timing attitude, we propose a slightly more general formulation of Epstein-Zin-Weil preferences than considered previously to disentangle the timing attitude from the IES and RRA. We show that this extension helps to explain several asset pricing puzzles in both endowment and production economies. In particular, we resolve two puzzles in the long-run risk model where consumption growth is too highly correlated with the price-dividend ratio and the risk-free rate. We also resolve the need for high RRA in DSGE models by enabling an otherwise standard New Keynesian model to match the equity premium and the bond premium with a low RRA of 5. Our analysis also reveals that Epstein-Zin-Weil preferences help to explain asset prices, *not* because they separate the IES from RRA, but because they introduce strong preferences for early resolution of uncertainty in the presence of long-run risk.

¹⁶See also Dew-Becker and Giglio (2016) who analyze the differences between Epstein-Zin-Weil preferences and consumption habits in the frequency domain.

A The Long-Run Risk Model: A Perturbation Approximation under Homoscedasticity

Proposition A.1 *The second-order approximation to $\tilde{e}v_t \equiv \log \mathbb{E}_t \left[e^{(1-\alpha)(\tilde{v}_{t+1} + (1-\frac{1}{\psi}) \log \mu_{z,t+1})} \right]$ with $\mu_{z,t} \equiv Z_t/Z_{t-1}$ around the balanced growth path is given by*

$$\tilde{e}v_t = \tilde{e}v_{ss} + \tilde{e}v_{\tilde{c}}\tilde{c}_t + \tilde{e}v_x x_t + \frac{1}{2}\tilde{e}v_{\tilde{c}\tilde{c}}\tilde{c}_t^2 + \frac{1}{2}\tilde{e}v_{xx}x_t^2 + \tilde{e}v_{\tilde{c}x}\tilde{c}_t x_t + \frac{1}{2}\tilde{e}v_{\sigma\sigma},$$

where

$$\begin{aligned} \tilde{e}v_{ss} &= (1-\alpha) \left(\log \left(\left| u_0 + \frac{1}{1-\frac{1}{\psi}} \right| \right) - \log(1-\kappa_0) + \left(1-\frac{1}{\psi}\right) \log \mu_z \right) \\ \tilde{e}v_{\tilde{c}} &= (1-\alpha) \rho_{\tilde{c}} \frac{1-\kappa_0}{1-\kappa_0 \rho_{\tilde{c}}} \frac{1-\frac{1}{\psi}}{1+u_0(1-\frac{1}{\psi})} \\ \tilde{e}v_x &= \frac{1-\alpha}{1-\kappa_0 \rho_x} \left(1-\frac{1}{\psi}\right) \\ \tilde{e}v_{\tilde{c}\tilde{c}} &= (1-\alpha) \rho_{\tilde{c}}^2 \frac{1-\kappa_0}{1-\kappa_0 \rho_{\tilde{c}}^2} \frac{(1-\frac{1}{\psi})^2}{1+u_0(1-\frac{1}{\psi})} - (1-\alpha) \rho_{\tilde{c}}^2 \left[\frac{1-\frac{1}{\psi}}{1+u_0(1-\frac{1}{\psi})} \frac{1-\kappa_0}{1-\kappa_0 \rho_{\tilde{c}}} \right]^2 \\ \tilde{e}v_{xx} &= (1-\alpha) \rho_x^2 \frac{\kappa_0}{1-\kappa_0 \rho_x^2} \frac{1-\kappa_0}{(1-\kappa_0 \rho_x)^2} \left(1-\frac{1}{\psi}\right)^2 \\ \tilde{e}v_{\tilde{c}x} &= (1-\alpha) \rho_x \rho_{\tilde{c}} \tilde{v}_{x\tilde{c}} \\ \tilde{e}v_{\sigma\sigma} &= \frac{1-\alpha}{1-\kappa_0} \left[\tilde{v}_{\tilde{c}\tilde{c}} \sigma_{\tilde{c}}^2 + (1-\alpha) \tilde{v}_{\tilde{c}}^2 \sigma_{\tilde{c}}^2 + \tilde{v}_{xx} \sigma_x^2 + (1-\alpha) \tilde{v}_x^2 \sigma_x^2 + (1-\alpha) \left(1-\frac{1}{\psi}\right)^2 \sigma_z^2 \right] \end{aligned}$$

Proposition A.2 *The second-order approximation to the risk-free rate r_t^f and the expected equity return $r_t^{m,e}$ around the balanced growth path are given by*

$$\begin{aligned} r_t^f &= r_{ss} + r_{\tilde{c}}\tilde{c}_t + r_x x_t + \frac{1}{2}r_{\sigma\sigma}^f \\ r_t^{m,e} &= r_{ss} + r_{\tilde{c}}\tilde{c}_t + r_x x_t + \frac{1}{2}r_{\sigma\sigma}^{m,e} \end{aligned}$$

where

$$\begin{aligned} r_{ss} &= -\log \beta + \frac{1}{\psi} \log \mu_z \\ r_{\tilde{c}} &= -(1-\rho_{\tilde{c}}) \frac{1}{\psi} \\ r_x &= \frac{1}{\psi} \\ r_{\sigma\sigma}^f &= -\alpha \tilde{v}_x^2 \sigma_x^2 - \left[1 - (1-\alpha) \left(1-\frac{1}{\psi}\right) \left(1+\frac{1}{\psi}\right) \right] \sigma_z^2 - \left(\frac{1}{\psi^2} + \frac{1}{\psi} 2\alpha \tilde{v}_{\tilde{c}} + \alpha \tilde{v}_{\tilde{c}}^2 \right) \sigma_{\tilde{c}}^2 \\ r_{\sigma\sigma}^{m,e} &= -(1-\kappa_1) p d_{\sigma\sigma} + \kappa_1 (p d_{\tilde{c}\tilde{c}} + p d_{\tilde{c}}^2) \sigma_{\tilde{c}}^2 + \kappa_1 (p d_{xx} + p d_x^2) \sigma_x^2 + \sigma_d^2 \end{aligned}$$

with $\kappa_1 \equiv \frac{e^{p d_{ss}}}{1+e^{p d_{ss}}}$.

Proposition A.3 *The second-order approximation to the log-transformed price-dividend ratio pd_t around the balanced growth path is given by*

$$pd_t = pd_{ss} + pd_{\tilde{c}}\tilde{c}_t + pd_x x_t + \frac{1}{2}pd_{\tilde{c}\tilde{c}}\tilde{c}_t^2 + \frac{1}{2}pd_{xx}x_t^2 + pd_{\tilde{c}x}\tilde{c}_t x_t + \frac{1}{2}pd_{\sigma\sigma},$$

where

$$\begin{aligned} pd_{ss} &= \log \frac{\kappa_1}{1-\kappa_1} \\ pd_{\tilde{c}} &= \frac{\phi_{\tilde{c}} + (1-\rho_{\tilde{c}})\frac{1}{\psi}}{1-\kappa_1\rho_{\tilde{c}}} \\ pd_x &= \frac{\phi_x - \frac{1}{\psi}}{1-\kappa_1\rho_x} \\ pd_{\tilde{c}\tilde{c}} &= -pd_{\tilde{c}}^2 + \frac{2\kappa_1\rho_{\tilde{c}}(1-\rho_{\tilde{c}})\frac{1}{\psi} + \phi_{\tilde{c}} + \phi_{\tilde{c}}\kappa_1\rho_{\tilde{c}}}{1-\kappa_1\rho_{\tilde{c}}}pd_{\tilde{c}} - \frac{(1-\rho_{\tilde{c}}^2)\frac{1}{\psi^2} - \phi_{\tilde{c}}(1-\rho_{\tilde{c}})\frac{1}{\psi} - 2(1-\rho_{\tilde{c}})\frac{1}{\psi^2}}{1-\kappa_1\rho_{\tilde{c}}^2} \\ pd_{xx} &= -pd_x^2 + \frac{(\phi_x - \frac{1}{\psi})^2}{1-\kappa_1\rho_x^2} + 2\kappa_1\rho_x \frac{\phi_x - \frac{1}{\psi}}{1-\kappa_1\rho_x^2}pd_x \\ pd_{\tilde{c}x} &= -pd_{\tilde{c}}pd_x + \frac{\kappa_1\rho_{\tilde{c}}pd_{\tilde{c}}(\phi_x - \frac{1}{\psi})}{1-\kappa_1\rho_{\tilde{c}}\rho_x} + \frac{(\frac{1}{\psi}(1-\rho_{\tilde{c}}) + \phi_{\tilde{c}})(\phi_x - \frac{1}{\psi} + \rho_x\kappa_1pd_x)}{1-\kappa_1\rho_{\tilde{c}}\rho_x} \\ pd_{\sigma\sigma} &= \frac{\sigma_d^2}{1-\kappa_1} + \frac{\sigma_z^2}{1-\kappa_1} \left[\alpha + (1-\alpha)\frac{1}{\psi^2} \right] \\ &\quad + \frac{\sigma_{\tilde{c}}^2}{1-\kappa_1} \left[\alpha\tilde{v}_{\tilde{c}}^2 - 2\alpha\kappa_1pd_{\tilde{c}}\tilde{v}_{\tilde{c}} + \kappa_1pd_{\tilde{c}\tilde{c}} + \kappa_1pd_{\tilde{c}}^2 + 2\alpha\frac{1}{\psi}\tilde{v}_{\tilde{c}} - 2\kappa_1pd_{\tilde{c}}\frac{1}{\psi} + \frac{1}{\psi^2} \right] \\ &\quad + \frac{\sigma_x^2}{1-\kappa_1} \left[\alpha\tilde{v}_x^2 - 2\alpha\kappa_1pd_x\tilde{v}_x + \kappa_1pd_{xx} + \kappa_1pd_x^2 \right] \end{aligned}$$

708 with $\kappa_1 \equiv \frac{e^{pd_{ss}}}{1+e^{pd_{ss}}}$.

709 B The Long-Run Risk Model: An Efficient Second- 710 Order Projection Approximation

711 The long-run risk model may be summarized by the following four equilibrium equations:

$$\begin{aligned} \tilde{V}_t &= u_0 + \frac{1}{1-\frac{1}{\psi}}\tilde{C}_t^{1-\frac{1}{\psi}} + \beta\widetilde{EV}_t^{\frac{1}{1-\alpha}} \\ \widetilde{EV}_t &= \mathbb{E}_t \left[\tilde{V}_{t+1}^{1-\alpha} \mu_{z,t+1}^{(1-\frac{1}{\psi})(1-\alpha)} \right] \\ 1 &= \mathbb{E}_t \left[\beta \left(\frac{(\widetilde{EV}_t)^{\frac{1}{1-\alpha}}}{\tilde{V}_{t+1}} \mu_{z,t+1}^{-(1-\frac{1}{\psi})} \right)^\alpha \left(\frac{\tilde{C}_{t+1}}{\tilde{C}_t} \right)^{-\frac{1}{\psi}} \mu_{z,t+1}^{-\frac{1}{\psi}} R_t^f \right] \\ (P/D)_t &= \mathbb{E}_t \left[\beta \left(\frac{(\widetilde{EV}_t)^{\frac{1}{1-\alpha}}}{\tilde{V}_{t+1}} \mu_{z,t+1}^{-(1-\frac{1}{\psi})} \right)^\alpha \left(\frac{\tilde{C}_{t+1}}{\tilde{C}_t} \right)^{-\frac{1}{\psi}} \mu_{z,t+1}^{-\frac{1}{\psi}} ((P/D)_{t+1} \mu_{d,t+1} + \mu_{d,t+1}) \right] \end{aligned} \tag{13}$$

as the market return is given by $R_t^m = ((P/D)_t + 1) \frac{1}{(P/D)_{t-1}} \mu_{d,t}$. Here, $\tilde{V}_t \equiv V_t/Z_t^{1-\frac{1}{\psi}}$, and $\widetilde{EV}_t \equiv \mathbb{E}_t [V_{t+1}^{1-\alpha}] / Z_t^{(1-\frac{1}{\psi})(1-\alpha)}$. We consider a second-order log-approximation to the four

control variables in the model, i.e. $\tilde{v}_t = g_0^{\tilde{v}} + \mathbf{g}_s^{\tilde{v}} \mathbf{s}_t + \frac{1}{2} \mathbf{s}'_t \mathbf{g}_{ss}^{\tilde{v}} \mathbf{s}_t$, $\tilde{e}v_t = g_0^{\tilde{e}v} + \mathbf{g}_s^{\tilde{e}v} \mathbf{s}_t + \frac{1}{2} \mathbf{s}'_t \mathbf{g}_{ss}^{\tilde{e}v} \mathbf{s}_t$, $r_t = g_0^r + \mathbf{g}_s^r \mathbf{s}_t + \frac{1}{2} \mathbf{s}'_t \mathbf{g}_{ss}^r \mathbf{s}_t$, and $pd_t = g_0^{pd} + \mathbf{g}_s^{pd} \mathbf{s}_t + \frac{1}{2} \mathbf{s}'_t \mathbf{g}_{ss}^{pd} \mathbf{s}_t$, where $\tilde{v}_t \equiv \log \tilde{V}_t$, $\tilde{e}v_t \equiv \log \tilde{E}V_t$, $r_t \equiv \log R_t^f$, and $pd_t \equiv \log (P/D)_t$. The law of motion for the states is known and given by

$$\underbrace{\begin{bmatrix} \tilde{c}_{t+1} \\ x_{t+1} \\ \sigma_{t+1}^2 \end{bmatrix}}_{\mathbf{s}_{t+1}} = \underbrace{\begin{bmatrix} 0 \\ 0 \\ 1 - \rho_\sigma \end{bmatrix}}_{\mathbf{h}_0} + \underbrace{\begin{bmatrix} \rho_{\tilde{c}} & 0 & 0 \\ 0 & \rho_x & 0 \\ 0 & 0 & \rho_\sigma \end{bmatrix}}_{\mathbf{h}_s} \underbrace{\begin{bmatrix} \tilde{c}_t \\ x_t \\ \sigma_t^2 \end{bmatrix}}_{\mathbf{s}_t} + \underbrace{\begin{bmatrix} \sigma_{\tilde{c}} \sigma_t^+ & 0 & 0 \\ 0 & \sigma_x \sigma_t^+ & 0 \\ 0 & 0 & \sigma_\sigma \end{bmatrix}}_{\boldsymbol{\eta}_t} \underbrace{\begin{bmatrix} \varepsilon_{\tilde{c},t+1} \\ \varepsilon_{x,t+1} \\ \varepsilon_{\sigma,t+1} \end{bmatrix}}_{\boldsymbol{\varepsilon}_{t+1}}$$

712 $\Downarrow$

$$\mathbf{s}_{t+1} = \mathbf{h}_0 + \mathbf{h}_s \mathbf{s}_t + \boldsymbol{\eta}_t \boldsymbol{\varepsilon}_{t+1}, \quad (14)$$

713 where $\mathbf{s}_t$ is a matrix of size $n_s \times 1$ and $\sigma_t^+ \equiv \sqrt{\max(\sigma_t^2, 0)}$. Below, we use the notation
714 $\begin{bmatrix} g_s^{\tilde{v}}(1, \tilde{c}) & g_s^{\tilde{v}}(1, x) & g_s^{\tilde{v}}(1, \sigma^2) \end{bmatrix}$ to index the elements in $\mathbf{g}_s^{\tilde{v}}$ and similar for $\mathbf{g}_s^{\tilde{e}v}$, $\mathbf{g}_s^r$, and $\mathbf{g}_s^{pd}$.
715 Also, $\mathbf{g}_{ss}^{\tilde{v}}(\tilde{c}, \tilde{c})$ denotes the element on the first row and first column of the matrix $\mathbf{g}_{ss}^{\tilde{v}}$, and
716 so forth. To derive the approximation, we exploit the following result which we prove in the
717 online appendix:

718 **Proposition B.1** *Let $a \in \mathbb{R}$, $\mathbf{b}$ denote an $1 \times n_s$ matrix, and $\mathbf{C}$ denote a symmetric $n_s \times n_s$*
719 *matrix. Given (14), we then have that*

$$\begin{aligned} & \mathbb{E}_t \left[\exp \{ a + \mathbf{b} \mathbf{s}_{t+1} + \mathbf{s}'_{t+1} \mathbf{C} \mathbf{s}_{t+1} \} \right] \\ &= \exp \{ a + \mathbf{b} \mathbf{h}_0 + \mathbf{h}'_0 \mathbf{C} \mathbf{h}_0 + (2 \mathbf{h}'_0 \mathbf{C} \mathbf{h}_s + \mathbf{b} \mathbf{h}_s) \mathbf{s}_t + \mathbf{s}'_t \mathbf{h}'_s \mathbf{C} \mathbf{h}_s \mathbf{s}_t \} \\ & \quad \times \exp \left\{ \frac{1}{2} (\mathbf{b} \boldsymbol{\eta}_t + 2 \mathbf{h}'_0 \mathbf{C} \boldsymbol{\eta}_t + 2 \mathbf{s}'_t \mathbf{h}'_s \mathbf{C} \boldsymbol{\eta}_t) (\mathbf{I} - 2 \boldsymbol{\eta}'_t \mathbf{C} \boldsymbol{\eta}_t)^{-1} (\mathbf{b} \boldsymbol{\eta}_t + 2 \mathbf{h}'_0 \mathbf{C} \boldsymbol{\eta}_t + 2 \mathbf{s}'_t \mathbf{h}'_s \mathbf{C} \boldsymbol{\eta}_t)' \right\} \\ & \quad \times |(\mathbf{I} - 2 \boldsymbol{\eta}'_t \mathbf{C} \boldsymbol{\eta}_t)|^{-\frac{1}{2}} \end{aligned}$$

The projection approximation can be implemented sequentially by first obtaining $\tilde{v}_t$ and then $\tilde{e}v_t$, afterwhich r_t and pd_t are easily computed using the expressions for $\tilde{v}_t$ and $\tilde{e}v_t$. To conserve space, we only show how to solve for $\tilde{v}_t$, as the remaining three controls variables are obtained in a similar way. We first note that the expression for the scaled value function reads

$$\tilde{V}_t = u_0 + \frac{1}{1 - \frac{1}{\psi}} \tilde{C}_t^{1 - \frac{1}{\psi}} + \beta \mathbb{E}_t \left[\tilde{V}_{t+1}^{1-\alpha} \mu_{z,t+1}^{(1 - \frac{1}{\psi})(1-\alpha)} \right]^{\frac{1}{1-\alpha}}$$

720 $\Downarrow$

$$\begin{aligned} \exp \{ \tilde{v}(\mathbf{s}_t) \} &= u_0 + \frac{1}{1 - \frac{1}{\psi}} \exp \left\{ \left(1 - \frac{1}{\psi} \right) \tilde{c}_t \right\} \\ & \quad + \beta \mathbb{E}_t \left[\exp \{ (1 - \alpha) \tilde{v}(\mathbf{s}_{t+1}) \} \exp \left\{ \left(1 - \frac{1}{\psi} \right) (1 - \alpha) \log \mu_{z,t+1} \right\} \right]^{\frac{1}{1-\alpha}}, \end{aligned}$$

where $\tilde{v}(\mathbf{s}_t) \equiv \log \tilde{V}(\mathbf{s}_t)$. Due to the independence of the shocks, it is possible to integrate out $\varepsilon_{z,t+1}$ manually as we have

$$\begin{aligned} \exp\{\tilde{v}(\mathbf{s}_t)\} &= u_0 + \frac{1}{1 - \frac{1}{\psi}} \exp\left\{\left(1 - \frac{1}{\psi}\right) \tilde{c}_t\right\} \\ &\quad + \beta \mathbb{E}_t \left[\exp\{\tilde{v}(\mathbf{s}_{t+1})\}^{(1-\alpha)} \right] \exp\left\{\left(1 - \frac{1}{\psi}\right) (\log \mu_z + x_t)\right\}^{1-\alpha} \\ &\quad \times \exp\left\{\frac{1}{2} \left(1 - \frac{1}{\psi}\right)^2 (1-\alpha)^2 \sigma_z^2 (\sigma_t^+)^2\right\}^{\frac{1}{1-\alpha}}, \end{aligned}$$

$\Updownarrow$

$$\begin{aligned} \exp\{\tilde{v}(\mathbf{s}_t)\} &= u_0 + \frac{1}{1 - \frac{1}{\psi}} \exp\left\{\left(1 - \frac{1}{\psi}\right) \tilde{c}_t\right\} \\ &\quad + \beta \mathbb{E}_t \left[\exp\left\{\tilde{v}(\mathbf{s}_{t+1}) + \left(1 - \frac{1}{\psi}\right) (\log \mu_z + x_t) + \frac{1}{2} \left(1 - \frac{1}{\psi}\right)^2 (1-\alpha)^2 \sigma_z^2 (\sigma_t^+)^2\right\}^{1-\alpha} \right]^{\frac{1}{1-\alpha}}. \end{aligned}$$

To avoid numerical overflow of $\exp\{\cdot\}^{(1-\alpha)}$, given the large values of $\tilde{v}(\mathbf{s}_{t+1})$ and α , we scale this term by $\tilde{V}_t$. That is,

$$\begin{aligned} \exp\{\tilde{v}(\mathbf{s}_t)\} &= u_0 + \frac{1}{1 - \frac{1}{\psi}} \exp\left\{\left(1 - \frac{1}{\psi}\right) \tilde{c}_t\right\} \\ &\quad + \tilde{V}_t \beta \mathbb{E}_t \left[\left(\frac{\exp\{\tilde{v}(\mathbf{s}_{t+1}) + (1 - \frac{1}{\psi})(\log \mu_z + x_t) + \frac{1}{2}(1 - \frac{1}{\psi})^2(1-\alpha)\sigma_z^2(\sigma_t^+)^2\}}{\tilde{V}_t} \right)^{1-\alpha} \right]^{\frac{1}{1-\alpha}}. \end{aligned}$$

Focusing on the last term we have

$$\beta \mathbb{E}_t \left[\left(\exp\left\{-\tilde{v}_t + \tilde{v}_{t+1} + \left(1 - \frac{1}{\psi}\right) (\log \mu_z + x_t) + \frac{1}{2} \left(1 - \frac{1}{\psi}\right)^2 (1-\alpha)^2 \sigma_z^2 (\sigma_t^+)^2\right\} \right)^{1-\alpha} \right]^{\frac{1}{1-\alpha}}$$

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$$\begin{aligned} &= \beta \mathbb{E}_t [\exp\{-\tilde{v}(\mathbf{s}_t) (1-\alpha) + (g_0^{\tilde{v}} + \mathbf{g}_s^{\tilde{v}} \mathbf{s}_{t+1} + \frac{1}{2} \mathbf{s}_{t+1}' \mathbf{g}_{ss}^{\tilde{v}} \mathbf{s}_{t+1}) (1-\alpha) \\ &\quad + \left(1 - \frac{1}{\psi}\right) (1-\alpha) (\log \mu_z + x_t) + \frac{1}{2} \left(1 - \frac{1}{\psi}\right)^2 (1-\alpha)^2 \sigma_z^2 (\sigma_t^+)^2\}]^{\frac{1}{1-\alpha}} \end{aligned}$$

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$$\begin{aligned} &= \beta \exp\left\{\left(1 - \frac{1}{\psi}\right) (\log \mu_z + x_t) + \frac{1}{2} \left(1 - \frac{1}{\psi}\right)^2 (1-\alpha)^2 \sigma_z^2 (\sigma_t^+)^2\right\} \\ &\quad \times \mathbb{E}_t \left[\exp\left\{\left((1-\alpha) (g_0^{\tilde{v}} - \tilde{v}(\mathbf{s}_t)) + (1-\alpha) \mathbf{g}_s^{\tilde{v}} \mathbf{s}_{t+1} + \mathbf{s}_{t+1}' \frac{(1-\alpha) \mathbf{g}_{ss}^{\tilde{v}}}{2} \mathbf{s}_{t+1}\right)\right\} \right]^{1/(1-\alpha)} \end{aligned}$$

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To apply Proposition B.1, let $a \equiv (1-\alpha) (g_0^{\tilde{v}} - \tilde{v}(\mathbf{s}_t))$, $\mathbf{b} \equiv (1-\alpha) \mathbf{g}_s^{\tilde{v}}$, and $\mathbf{C} \equiv \frac{(1-\alpha) \mathbf{g}_{ss}^{\tilde{v}}}{2}$.

This implies

$$\begin{aligned} &\mathbb{E}_t \left[\exp\left\{\left((1-\alpha) (g_0^{\tilde{v}} - \tilde{v}(\mathbf{s}_t)) + (1-\alpha) \mathbf{g}_s^{\tilde{v}} \mathbf{s}_{t+1} + \mathbf{s}_{t+1}' \frac{(1-\alpha) \mathbf{g}_{ss}^{\tilde{v}}}{2} \mathbf{s}_{t+1}\right)\right\} \right] \\ &= \exp\left\{(1-\alpha) \left(g_0^{\tilde{v}} - \tilde{v}(\mathbf{s}_t) + \mathbf{g}_s^{\tilde{v}} \mathbf{h}_0 + \mathbf{h}_0' \frac{\mathbf{g}_{ss}^{\tilde{v}}}{2} \mathbf{h}_0 + (\mathbf{h}_0' \mathbf{g}_{ss}^{\tilde{v}} \mathbf{h}_s + \mathbf{g}_s^{\tilde{v}} \mathbf{h}_s) \mathbf{s}_t + \mathbf{s}_t' \mathbf{h}_s' \frac{\mathbf{g}_{ss}^{\tilde{v}}}{2} \mathbf{h}_s \mathbf{s}_t\right)\right\} \\ &\quad \times \exp\left\{\frac{1}{2} (1-\alpha)^2 (\mathbf{g}_s^{\tilde{v}} \boldsymbol{\eta}_t + \mathbf{h}_0' \mathbf{g}_{ss}^{\tilde{v}} \boldsymbol{\eta}_t + \mathbf{s}_t' \mathbf{h}_s' \mathbf{g}_{ss}^{\tilde{v}} \boldsymbol{\eta}_t) (\mathbf{I} - \boldsymbol{\eta}_t' (1-\alpha) \mathbf{g}_{ss}^{\tilde{v}} \boldsymbol{\eta}_t)^{-1} \right. \\ &\quad \times (\mathbf{g}_s^{\tilde{v}} \boldsymbol{\eta}_t + \mathbf{h}_0' \mathbf{g}_{ss}^{\tilde{v}} \boldsymbol{\eta}_t + \mathbf{s}_t' \mathbf{h}_s' \mathbf{g}_{ss}^{\tilde{v}} \boldsymbol{\eta}_t)'\} \\ &\quad \times |(\mathbf{I} - \boldsymbol{\eta}_t' (1-\alpha) \mathbf{g}_{ss}^{\tilde{v}} \boldsymbol{\eta}_t)|^{-\frac{1}{2}}. \end{aligned}$$

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Hence, the Euler residuals for the log-transformed value function $R^{\tilde{v}}(\mathbf{s}_t)$ reads

$$\begin{aligned}
R^{\tilde{v}}(\mathbf{s}_t) = & -\exp\left\{g_0^{\tilde{v}} + \mathbf{g}_s^{\tilde{v}}\mathbf{s}_t + \frac{1}{2}\mathbf{s}_t'\mathbf{g}_{ss}^{\tilde{v}}\mathbf{s}_t\right\} + u_0 + \frac{1}{1-\frac{1}{\psi}}\exp\left\{\left(1 - \frac{1}{\psi}\right)\tilde{c}_t\right\} \\
& + \beta V_t \exp\left\{\left(1 - \frac{1}{\psi}\right)(\log \mu_z + x_t) + \frac{1}{2}\left(1 - \frac{1}{\psi}\right)^2(1 - \alpha)\sigma_z^2(\sigma_t^+)^2\right\} \\
& \times \exp\left\{\left(g_0^{\tilde{v}} - \tilde{v}(\mathbf{s}_t)\right) + \mathbf{g}_s^{\tilde{v}}\mathbf{h}_0 + \mathbf{h}_0'\frac{\mathbf{g}_{ss}^{\tilde{v}}}{2}\mathbf{h}_0 + \left(\mathbf{h}_0'\mathbf{g}_{ss}^{\tilde{v}}\mathbf{h}_s + \mathbf{g}_s^{\tilde{v}}\mathbf{h}_s\right)\mathbf{s}_t + \mathbf{s}_t'\mathbf{h}_s'\frac{\mathbf{g}_{ss}^{\tilde{v}}}{2}\mathbf{h}_s\mathbf{s}_t\right\} \\
& \times \exp\left\{\frac{(1-\alpha)}{2}\left(\mathbf{g}_s^{\tilde{v}}\boldsymbol{\eta}_t + \mathbf{h}_0'\mathbf{g}_{ss}^{\tilde{v}}\boldsymbol{\eta}_t + \mathbf{s}_t'\mathbf{h}_s'\mathbf{g}_{ss}^{\tilde{v}}\boldsymbol{\eta}_t\right)\right. \\
& \quad \times \left(\mathbf{I} - \boldsymbol{\eta}_t'(1 - \alpha)\mathbf{g}_{ss}^{\tilde{v}}\boldsymbol{\eta}_t\right)^{-1}\left(\mathbf{g}_s^{\tilde{v}}\boldsymbol{\eta}_t + \mathbf{h}_0'\mathbf{g}_{ss}^{\tilde{v}}\boldsymbol{\eta}_t + \mathbf{s}_t'\mathbf{h}_s'\mathbf{g}_{ss}^{\tilde{v}}\boldsymbol{\eta}_t\right)'\left. \right\} \\
& \times \left|\left(\mathbf{I} - \boldsymbol{\eta}_t'(1 - \alpha)\mathbf{g}_{ss}^{\tilde{v}}\boldsymbol{\eta}_t\right)\right|^{-\frac{1}{2(1-\alpha)}}.
\end{aligned}$$

We then determine $g_0^{\tilde{v}}$, $\mathbf{g}_s^{\tilde{v}}$, and $\mathbf{g}_{ss}^{\tilde{v}}$ as follows:

- Construct a multi-dimensional grid for the states based on the Cartesian set $\mathcal{S}_s \equiv \mathcal{S}_{\tilde{c}} \times \mathcal{S}_x \times \mathcal{S}_{\sigma_t^2}$.
- Generate N_s points $\{\mathbf{s}_t^i\}_{i=1}^{N_s}$ from the set $\mathcal{S}_s$.
- Determine $g_0^{\tilde{v}}$, $\mathbf{g}_s^{\tilde{v}}$, and $\mathbf{g}_{ss}^{\tilde{v}}$ by solving the nonlinear least squares problem, $(g_0^{\tilde{v}}, \mathbf{g}_s^{\tilde{v}}, \mathbf{g}_{ss}^{\tilde{v}}) = \arg \min \sum_{i=1}^{N_s} (R^{\tilde{v}}(\mathbf{s}_t^i))^2$.

The grid for the state variables $\mathcal{S}_s$ is constructed using 10 points uniformly distributed along each dimension, implying $N_s = 1,000$. The upper and lower bounds along each dimension is determined following a simulation of the states to cover the maximum and minimum levels. We evaluate $R^{\tilde{v}}(\mathbf{s}_t)$ across all N_s points simultaneously by using a vectorized implementation in **MATLAB**, where the symbolic toolbox is used to analytically compute the matrix products, matrix inversions, and determinants in the expression for $R^{\tilde{v}}(\mathbf{s}_t)$.

C The Long-Run Risk Model: Accuracy of Solution

This section evaluates the accuracy of the adopted second-order projection approximation for each of the eight estimated versions of the long-run risk model in Table 1. The performance of this approximation is benchmarked to the widely used log-normal method, a first-order projection solution, and a highly accurate fifth-order projection solution. As in Pohl et al. (forthcoming), we focus on means and standard deviations for pd_t , r_t^f , and r_t^m , because these moments are most sensitive to the adopted approximation method. The results are summarized in Table C.1, where we highlight the following results. First, the log-normal method generally underpredicts $\mathbb{E}[pd_t]$, generates too high values of $\mathbb{E}[r_t^m]$, and overpredicts the variability in pd_t . Hence, we reproduce the key findings of Pohl et al. (forthcoming) on our estimated models. Second, a first-order projection solution generally implies that these errors go in the opposite direction, as it overpredicts $\mathbb{E}[pd_t]$ and underpredicts $\mathbb{E}[r_t^m]$. Third, the proposed second-order projection solution displays no systematic biases and produces moments that are nearly identical to those from the fifth-order projection solution. The main exception is for the extended model with IES = 1.1 and RRA = 10, where we see somewhat larger deviations.

Table C.1: The Long-Run Risk Model: Accuracy of Moments

This table reports unconditional moments for the eight estimated versions of the long-run risk model in Table 1 when using the log-normal method as well as a first-, second-, and fifth-order projection solution with log-transformed variables. The projection approximations are computed by minimizing the squared Euler-equation errors on a grid of 1,000 points, with 10 points uniformly distributed along each dimension between its maximum and minimum level in a simulated sample of 250,000 observations. The fifth-order projection solution is computed using complete Chebyshev polynomials. The log-normal method is implemented using a first-order projection approximation of the value function and the traditional log-linear approximation of the price-dividend ratio at the unconditional mean of the price-dividend level, which is obtained by iterating on the approximated loadings.

	IES	RRA = 5						RRA = 10					
		Means			Stds			Means			Stds		
		pd_t	r_t^f	r_t^m	pd_t	r_t^f	r_t^m	pd_t	r_t^f	r_t^m	pd_t	r_t^f	r_t^m
Benchmark Model:	1.5												
Log-normal method		3.27	1.83	6.18	0.52	1.15	15.87	3.12	1.95	6.81	0.42	0.75	15.83
1st order		3.76	1.83	4.87	0.38	1.15	13.61	3.58	1.95	5.32	0.32	0.75	14.13
2nd order		3.49	1.84	5.70	0.42	1.14	14.10	3.30	1.96	6.32	0.34	0.75	14.48
5th order		3.49	1.84	5.72	0.44	1.14	14.47	3.31	1.96	6.28	0.36	0.75	14.89
Extended Model:	1.1												
Log-normal method		3.12	2.16	6.87	0.28	0.70	15.30	2.74	1.56	8.86	0.28	0.59	14.35
1st order		3.80	2.16	4.72	0.26	0.70	15.39	3.59	1.56	5.23	0.27	0.59	15.38
2nd order		3.28	2.16	6.32	0.26	0.70	14.83	3.29	1.68	6.22	0.26	0.59	14.77
5th order		3.27	2.16	6.36	0.27	0.70	15.08	4.26	1.67	3.83	0.29	0.59	16.99
Extended Model:	1.5												
Log-normal method		3.05	1.63	7.14	0.32	0.50	15.20	3.05	1.64	7.12	0.32	0.50	15.21
1st order		3.81	1.63	4.68	0.28	0.50	15.69	3.81	1.64	4.68	0.28	0.50	15.69
2nd order		3.29	1.69	6.26	0.28	0.50	14.80	3.29	1.71	6.28	0.29	0.50	14.78
5th order		3.28	1.69	6.33	0.29	0.49	15.00	3.28	1.71	6.33	0.29	0.50	15.00
Extended Model:	2.0												
Log-normal method		3.06	1.52	7.08	0.36	0.47	15.42	3.06	1.58	7.11	0.35	0.48	15.50
1st order		3.66	1.52	5.06	0.29	0.47	15.16	3.64	1.58	5.10	0.28	0.48	14.89
2nd order		3.28	1.59	6.30	0.30	0.45	14.73	3.28	1.63	6.33	0.30	0.47	14.69
5th order		3.29	1.59	6.30	0.31	0.45	15.00	3.28	1.63	6.32	0.31	0.47	15.02

D The New Keynesian Model: Accuracy of Solution

We evaluate the accuracy of the adopted third-order perturbation approximation by computing unit-free Euler-equation errors on a grid of 1,000 points. The accuracy of this solution is benchmarked to a standard first-order approximation and a fifth-order approximation using the codes of Levintal (2017). Table D.1 reports the root mean squared Euler-equation errors (RMSEs) for the six estimated versions of the New Keynesian model in Table 4. We

generally find that a third-order approximation improves the accuracy of the linearized solution, both for the Euler-equations relating to the macro part of the model and for the 40 Euler-equations describing bond prices. This improvement is particularly evident for bond prices. Increasing the approximation order from three to five provides only a small improvement to the macro part of the model when RRA equals 10 and 60, while accuracy actually deteriorates slightly for RRA = 5. We find even smaller effects on bond prices of going from third to fifth order, where accuracy only improves for the benchmark model with RRA = 60 and the extended model with RRA = 10. Thus, these results indicate that a third-order approximation is fairly accurate and that little would be gained by considering a fifth-order solution. In addition, going to fifth order is computationally much more demanding than the adopted third-order approximation and would not make a formal estimation of the New Keynesian model feasible.

Table D.1: The New Keynesian Model: Euler-Equation Errors

This table reports the root mean squared unit-free Euler-equation errors (RMSEs) on a grid of 1,000 points for a first-, third-, and fifth-order perturbation approximation. The grid is constructed by considering 10 points uniformly between $-2 \times \sigma_{x,i}$ and $2 \times \sigma_{x,i}$ for each state dimension, where $\sigma_{x,i}$ denotes the standard deviation of the i 'th state in a log-linearized solution. Conditional expectations in the Euler-equations are evaluated by Gauss-Hermite quadratures using 7 points. The considered model parameters are those reported in Table 4. The RMSEs to the 12 equations describing the model without bond prices are summarized under the label 'Macro Part', while the RMSEs for the 40 equations describing all bond prices are summarized under the label 'Bond Prices'. The label 'Total' refers to the RMSEs for the entire model.

	Benchmark Model			Extended Model		
	(1) RRA=5	(2) RRA=10	(3) RRA=60	(4) RRA=5	(5) RRA = 10	(6) RRA = 60
Macro Part:						
1st order	0.0277	0.0275	0.0719	0.2820	0.4262	0.2939
3rd order	0.1279	0.0565	0.0214	0.1407	0.0579	0.0196
5th order	0.1461	0.0474	0.0179	0.1559	0.0508	0.0150
Bonds Prices:						
1st order	0.0471	0.0466	0.1226	0.4831	0.7284	0.5006
3rd order	0.0016	0.0014	0.0048	0.0014	0.0016	0.0021
5th order	0.0064	0.0020	0.0033	0.0056	0.0014	0.0038
Total:						
1st order	0.0422	0.0418	0.1098	0.4326	0.6523	0.4485
3rd order	0.0597	0.0264	0.0108	0.0658	0.0271	0.0093
5th order	0.0684	0.0222	0.0088	0.0730	0.0238	0.0077

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Figure 1: Properties of Consumption Growth and Volatility

The model-implied moments are computed given the estimated parameters in Table 1 using a simulated sample path of 1,000,000 observations. The conditional volatility $\hat{\sigma}_t$ is estimated by $|\hat{u}_t|$, where $\hat{u}_t$ is the residual from the OLS regression $\Delta c_t = \alpha + \sum_{j=1}^5 \beta(j) \Delta c_{t-j} + u_t$. All the 95 percent confidence bands are computed using a block bootstrap applied jointly to the regressant and the regressor with a block length of $2 \times j$ lags.

Table 1: The Long-Run Risk Model: The Structural Parameters

Estimation results using data from 1947Q1 to 2014Q4 and a second-order projection approximation. The model has a monthly time frequency with model-implied moments time-aggregated to a quarterly time frequency based on a simulated sample of 250,000 monthly observations. The estimates are from the second step in SMM with the optimal weighting matrix estimated by the Newey-West estimator using 15 lags. Standard errors are reported in parenthesis, except when an estimate is on the boundary and its standard error is not available (n.a.). The values of μ_z and μ_d are calibrated to match the sample moments of consumption and dividend growth, respectively, implying $\mu_z = 1.0016$ and $\mu_d = 1.0020$. The value of σ_σ is set to 0.05. The fraction of utility from market consumption along the balanced growth is $\omega_{ss}^C \equiv 1/(u_0(1 - 1/\psi) + 1)$. The timing premium at the steady state Π_{ss} is defined as in (8) and computed using a second-order projection of the value function. The utility level when uncertainty is resolved in the following period is here computed by simulation, using anti-thetic sampling with 10,000 draws and 15,000 terms to approximate the lifetime utility stream.

	Benchmark Model		Extended Model					
	(1) RRA = 5 IES = 1.5	(2) RRA = 10 IES = 1.5	(3) IES = 1.1	(4) RRA = 5 IES = 1.5	(5) IES = 2.0	(6) IES = 1.1	(7) RRA = 10 IES = 1.5	(8) IES = 2.0
u_0	—	—	71.37 (3.36)	24.72 (3.09)	9.91 (0.64)	33.22 (4.12)	9.87 (0.90)	2.56 (0.30)
β	0.9991 (n.a.)	0.9991 (n.a.)	0.9995 (n.a.)	0.9991 (n.a.)	0.9988 (n.a.)	0.9995 (n.a.)	0.9991 (n.a.)	0.9988 (n.a.)
$\rho_{\bar{c}}$	0.7577 (0.3681)	0.9748 (0.0209)	0.9810 (0.0075)	0.9831 (0.0027)	0.9828 (0.0086)	0.9805 (0.0048)	0.9832 (0.0071)	0.9809 (0.0104)
ρ_x	0.9926 (0.0024)	0.9899 (0.0041)	0.9822 (0.0254)	0.9684 (0.0003)	0.9774 (0.0100)	0.9928 (0.0017)	0.9675 (0.0003)	0.9849 (0.0157)
ρ_σ	0.9986 (0.0011)	0.9983 (0.0025)	0.9974 (0.0081)	0.9990 (n.a.)	0.9990 (n.a.)	0.9990 (n.a.)	0.9990 (n.a.)	0.9986 (0.0047)
ϕ_x	3.2053 (0.2223)	4.3843 (0.0621)	3.5511 (3.5511)	4.595 (0.2558)	4.3246 (1.0974)	3.3772 (2.4230)	4.5664 (0.7024)	4.0767 (0.8778)
$\phi_{\bar{c}}$	2.4172 (0.0751)	0.2396 (0.1219)	0.2745 (0.0620)	0.2737 (0.0028)	0.2716 (0.0976)	0.3263 (0.0537)	0.2630 (0.0839)	0.2763 (0.0786)
$\sigma_{\bar{c}}$	0.00001 (n.a.)	0.0027 (0.0008)	0.0030 (0.0006)	0.0027 (0.0003)	0.0027 (0.0005)	0.0026 (0.0003)	0.0027 (0.0004)	0.0028 (0.0008)
σ_z	0.0020 (0.0003)	0.0014 (0.0012)	0.0013 (0.0011)	0.0016 (0.0004)	0.0016 (0.0006)	0.0020 (0.0002)	0.0016 (0.0003)	0.0015 (0.0010)
σ_d	0.0125 (0.0004)	0.0116 (0.0010)	0.0116 (0.0009)	0.0107 (0.0001)	0.0106 (0.0008)	0.0108 (0.0007)	0.0107 (0.0011)	0.0108 (0.0018)
σ_x	1.57×10^{-4} (2.34×10^{-5})	1.16×10^{-4} (2.30×10^{-5})	1.03×10^{-4} (6.56×10^{-5})	1.20×10^{-4} (0.70×10^{-5})	1.20×10^{-4} (4.17×10^{-5})	0.46×10^{-4} (4.56×10^{-5})	1.23×10^{-4} (1.35×10^{-5})	1.02×10^{-4} (5.04×10^{-5})
Memo								
$\Pr(\sigma_t^2 \geq 0)$	86.9%	89.2%	93.1%	82.6%	82.6%	82.6%	82.6%	86.9%
$\frac{u'(C_t)C_t}{u(C_t)} \Big _{ss}$	0.333	0.333	0.012	0.036	0.084	0.023	0.078	0.219
ω_{ss}^C	100%	100%	13%	11%	17%	25%	23%	44%
Π_{ss}	72%	70%	93%	86%	75%	99%	86%	73%
α	13.00	28.00	336.96	120.10	53.59	402.04	120.08	43.36

Table 2: The Long-Run Risk Model: Fit of Moments

The model has a monthly time frequency with model-implied moments time-aggregated to a quarterly time frequency. All means and standard deviations are expressed in annualized percent, except for the price-dividend ratio. That is, the relevant moments are multiplied by 400, except for the standard deviation of the market return which is multiplied by 200. All model-implied moments in columns (2) to (9) are from the unconditional distribution computed using a simulated sample of 250,000 monthly observations, whereas the empirical data moments in column (1) are the empirical sample moments. In column (1), figures in parenthesis refer to the standard error of the empirical moment, computed based on a block bootstrap using 5,000 draws and a block length of 32 quarters.

	Data (1)	Benchmark Model (2) RRA = 5 IES = 1.5 (3) RRA = 10 IES = 1.5		(4) IES = 1.1	(5) RRA = 5 IES = 1.5	(6) IES = 2.0	(7) IES = 1.1	(8) RRA = 10 IES = 1.5	(9) IES = 2.0
Means									
pd_t	3.495 (0.122)	3.491	3.297	3.277	3.290	3.284	3.294	3.286	3.278
r_t^f	0.831 (0.547)	1.839	1.959	2.162	1.693	1.591	1.676	1.706	1.628
r_t^m	6.919 (1.879)	5.703	6.320	6.318	6.262	6.300	6.215	6.276	6.330
Δc_t	1.905 (0.244)	1.894	1.902	1.905	1.897	1.896	1.894	1.897	1.900
Δd_t	2.391 (0.975)	2.354	2.377	2.398	2.363	2.357	2.358	2.363	2.370
Stdts									
pd_t	0.421 (0.068)	0.419	0.342	0.262	0.284	0.302	0.263	0.285	0.297
r_t^f	2.224 (0.397)	1.142	0.750	0.698	0.495	0.451	0.588	0.496	0.466
r_t^m	16.45 (1.138)	14.10	14.48	14.83	14.80	14.73	14.77	14.78	14.694
Δc_t	2.035 (0.172)	2.054	2.062	2.033	2.012	2.022	2.076	2.013	2.034
Δd_t	9.391 (1.531)	9.222	9.045	8.995	8.807	8.808	8.785	8.801	8.779
Persistence									
$corr(pd_t, pd_{t-1})$	0.982 (0.056)	0.985	0.976	0.957	0.963	0.968	0.957	0.964	0.967
$corr\left(r_t^f, r_{t-1}^f\right)$	0.866 (0.035)	0.987	0.978	0.964	0.951	0.966	0.981	0.949	0.975
$corr\left(r_t^m, r_{t-1}^m\right)$	0.084 (0.048)	0.017	0.012	0.003	0.003	0.006	0.000	0.003	0.006
$corr\left(\Delta c_t, \Delta c_{t-1}\right)$	0.306 (0.118)	0.718	0.378	0.269	0.257	0.286	0.240	0.258	0.289
$corr\left(\Delta d_t, \Delta d_{t-1}\right)$	0.396 (0.063)	0.467	0.523	0.529	0.552	0.555	0.544	0.552	0.553

Table 2: Long-Run Risk Model: Fit of Moments (continued)

	Data (1)	Benchmark Model		Extended Model					
		(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
		RRA = 5 IES = 1.5	RRA = 10 IES = 1.5	IES = 1.1	IES = 1.5	IES = 2	IES = 1.1	IES = 1.5	IES = 2
Correlations	$corr\left(pd_t, r_t^f\right)$	0.035 (0.212)	0.668	-0.084	0.040	0.303	-0.052	0.033	0.367
	$corr\left(pd_t, r_t^m\right)$	0.058 (0.062)	0.212	0.284	0.256	0.236	0.288	0.255	0.243
	$corr\left(pd_t, \Delta c_t\right)$	0.025 (0.080)	0.366	0.139	0.107	0.148	0.118	0.106	0.187
	$corr\left(pd_t, \Delta d_t\right)$	-0.017 (0.095)	0.535	0.635	0.605	0.573	0.663	0.604	0.586
	$corr\left(r_t^f, r_t^m\right)$	0.023 (0.044)	0.083	-0.021	-0.006	0.013	-0.072	-0.005	0.004
	$corr\left(r_t^f, \Delta c_t\right)$	0.161 (0.080)	0.468	0.305	0.253	0.289	0.200	0.256	0.230
	$corr\left(r_t^f, \Delta d_t\right)$	-0.168 (0.093)	0.336	-0.035	0.009	0.088	-0.163	0.011	0.072
	$corr\left(r_t^m, \Delta c_t\right)$	0.233 (0.054)	0.395	0.623	0.592	0.556	0.558	0.597	0.554
	$corr\left(r_t^m, \Delta d_t\right)$	0.104 (0.050)	0.294	0.296	0.290	0.289	0.292	0.289	0.289
	$corr\left(\Delta c_t, \Delta d_t\right)$	0.062 (0.0496)	0.236	0.069	0.075	0.107	0.028	0.076	0.107
Goodness of fit	$corr\left(r_t^m-r_t^f, pd_{t-1}\right)$	-0.134 (0.048)	-0.014	0.006	-0.002	-0.009	0.011	-0.002	-0.007
	$corr\left(\Delta d_t, pd_{t-1}\right)$	-0.0163 (0.104)	0.498	0.586	0.562	0.533	0.616	0.560	0.545
	Q^{step2}	-	0.0621	0.0624	0.0591	0.0592	0.0616	0.0592	0.0593
	J-test: P-value	-	10.93%	20.20%	24.78%	24.59%	21.24%	24.55	24.49%
Q^{scaled}	-	3.35	2.26	1.89	1.54	1.53	1.62	1.54	1.61

Table 3: The Long-Run Risk Model: Analyzing the Extended Model

The model has a monthly time frequency with model-implied moments time-aggregated to a quarterly time frequency. All means and standard deviations are expressed in annualized percent by multiplying by 400, except for the standard deviation of the market return that is multiplied by 200. The moments are from the unconditional distribution computed using a simulated sample of 250,000 monthly observations. Unless stated otherwise, all parameters attain the estimated values from column (4) in Table 1, meaning that the IES = 1.5 and RRA = 5.

	(1) $u_0 = 10$	(2) $u_0 = 20$	(3) $u_0 = \hat{u}_0$	(4) $\sigma_x = 0$	(5) $\sigma_\sigma = 0$	(6) $\beta = 0.998$	(7) $\alpha = 0$
Means							
pd_t	66.31	15.71	3.29	14.04	4.46	3.16	8.50
r_t^f	2.18	1.86	1.69	2.24	2.17	3.21	2.39
r_t^m	2.36	2.36	6.26	2.38	3.60	6.80	2.40
Std s							
pd_t	0.31	0.31	0.28	0.30	0.27	0.28	0.82
r_t^f	0.47	0.48	0.50	0.24	0.43	0.50	0.45
r_t^m	17.99	18.02	14.80	17.02	16.18	14.55	22.22
Memo							
RRA	5	5	5	5	5	5	0.67
Π_{ss}	46%	79%	86%	18%	33%	34%	0%
α	56.33	99.67	120.10	120.10	120.10	120.10	0

Table 4: The New Keynesian Model: The Structural Parameters

Estimation results using data from 1961Q3 to 2014Q4 based on a third-order perturbation approximation with model-implied moments computed as in Andreasen et al. (2018). The reported estimates are from the second step of GMM with the optimal weighting matrix estimated by the Newey-West estimator using 15 lags. The estimates of β are for all specifications on the boundary 0.9995 and therefore not reported below. The fraction of utility from market consumption and leisure along the balanced growth is denoted by $\omega_{ss}^{C,L}$. The timing premium at the steady state (Π_{ss}) is computed based on (12) and a third-order perturbation approximation, where the utility level when uncertainty is resolved in the followingt period is computed by simulation, using anti-thetic sampling with 5,000 draws and 10,000 terms to approximate the lifetime utility stream.

	Benchmark Model			Extended Model		
	(1) RRA=5	(2) RRA=10	(3) RRA=60	(4) RRA=5	(5) RRA = 10	(6) RRA = 60
u_0	-	-	-	-943.3 (260.7)	-300.7 (71.40)	-24.74 (6.61)
ψ	0.103 (0.011)	0.209 (0.022)	0.398 (0.144)	0.179 (0.039)	0.304 (0.047)	0.516 (0.057)
b	0.695 (0.015)	0.759 (0.019)	0.790 (0.026)	0.478 (0.026)	0.558 (0.031)	0.578 (0.049)
β_π	1.473 (0.057)	1.433 (0.074)	1.468 (0.239)	1.431 (0.053)	1.382 (0.058)	1.328 (0.057)
β_y	0.021 (0.004)	0.029 (0.005)	0.057 (0.025)	0.020 (0.004)	0.023 (0.011)	0.056 (0.019)
$\mu_{Z,ss}$	1.003 (0.0002)	1.004 (0.0003)	1.005 (0.0004)	1.005 (0.0004)	1.005 (0.0003)	1.005 (0.0004)
π_{ss}	1.067 (0.006)	1.046 (0.003)	1.031 (0.003)	1.069 (0.012)	1.046 (0.005)	1.029 (0.002)
L_{ss}	0.338 (0.001)	0.337 (0.001)	0.337 (0.001)	0.338 (0.002)	0.337 (0.001)	0.338 (0.002)
ρ_A	0.991 (0.001)	0.989 (0.0011)	0.987 (0.001)	0.993 (0.001)	0.990 (0.001)	0.982 (0.001)
ρ_Z	0.306 (0.067)	0.519 (0.072)	0.792 (0.077)	0.208 (0.129)	0.368 (0.168)	0.462 (0.515)
σ_A	0.018 (0.001)	0.014 (0.001)	0.012 (0.002)	0.021 (0.002)	0.018 (0.001)	0.010 (0.001)
σ_Z	0.010 (0.001)	0.007 (0.001)	0.002 (0.001)	0.005 (0.002)	0.003 (0.001)	0.002 (0.002)
Memo						
IES	0.032	0.051	0.085	0.094	0.135	0.219
$\frac{u_C C}{u} \Big _{ss}$	-1.92	-1.82	-1.59	-0.10	-0.08	-0.21
$\omega_{ss}^{C,L}$	100%	100%	100%	6%	5%	20%
Π_{ss}	0.1%	4%	10%	6%	11%	17%
α	-1.28	-4.17	-36.28	-27.67	-105.4	-277.3

Table 5: The New Keynesian Model: Fit of Moments

All variables are expressed in annualized terms in percent, except for L_t where the mean of $\log(L_t)$ is unscaled and $\text{std}(L_t)$ is expressed in percent. All model-implied moments in columns (2) to (7) are from the unconditional distribution, whereas the empirical data moments in column (1) are given by the sample moments. In column (1), figures in parenthesis refer to the standard error of the empirical moment, computed based on a block bootstrap using 5,000 draws and a block length of 32 quarters.

	(1)	Benchmark Model			Extended Model		
	Data	(2) RRA=5	(3) RRA=10	(4) RRA=60	(5) RRA=5	(6) RRA=10	(7) RRA=60
Means							
Δc_t	1.975 (0.276)	1.170	1.497	2.063	1.970	1.987	2.052
π_t	3.890 (0.793)	3.906	3.789	3.672	3.824	3.782	3.476
i_t	4.999 (0.994)	5.104	5.104	5.161	5.191	5.168	5.121
$i_t^{(40)}$	6.497 (0.904)	6.500	6.510	6.552	6.524	6.520	6.517
$\Psi_t^{(40)}$	1.663 (0.355)	1.723	1.775	1.770	1.674	1.755	1.775
$\log L_t$	-1.081 (0.004)	-1.081	-1.080	-1.080	-1.080	-1.080	-1.080
r_t^m	5.527 (2.012)	3.750	3.913	4.137	5.009	4.414	3.633
Stds							
Δc_t	1.802 (0.122)	2.488	2.259	1.442	2.139	1.875	1.398
π_t	2.716 (0.612)	2.445	2.601	2.899	2.301	2.534	2.994
i_t	3.173 (0.579)	2.944	2.944	2.935	2.378	2.648	2.842
$i_t^{(40)}$	2.621 (0.532)	2.636	2.618	2.589	2.370	2.541	2.572
$\Psi_t^{(40)}$	1.165 (0.170)	0.970	0.864	0.870	1.004	0.895	0.872
$\log L_t$	1.619 (0.163)	2.791	2.697	2.449	2.495	2.510	2.081
Persistence							
$\text{corr}(\Delta c_t, \Delta c_{t-1})$	0.529 (0.083)	0.696	0.757	0.763	0.470	0.539	0.527
$\text{corr}(\pi_t, \pi_{t-1})$	0.953 (0.056)	0.954	0.958	0.960	0.978	0.972	0.972
$\text{corr}(i_t, i_{t-1})$	0.949 (0.031)	0.927	0.926	0.910	0.956	0.952	0.955
$\text{corr}(i_t^{(40)}, i_{t-1}^{(40)})$	0.976 (0.031)	0.989	0.987	0.985	0.989	0.987	0.980
$\text{corr}(\Psi_t^{(40)}, \Psi_{t-1}^{(40)})$	0.937 (0.032)	0.991	0.988	0.986	0.993	0.990	0.982
$\text{corr}(\log L_t, \log L_{t-1})$	0.932 (0.476)	0.727	0.767	0.802	0.876	0.868	0.871

Table 5: The New Keynesian Model: Fit of Moments (continued)

	(1)	Benchmark Model			Extended Model		
	Data	(2) RRA=5	(3) RRA=10	(4) RRA=60	(5) RRA=5	(6) RRA=10	(7) RRA=60
Correlations							
$corr(\Delta c_t, \pi_t)$	-0.184 (0.150)	0.143	0.017	-0.166	-0.101	-0.180	-0.184
$corr(\Delta c_t, i_t)$	0.021 (0.199)	0.178	0.020	-0.240	-0.109	-0.204	-0.254
$corr(\pi_t, i_t)$	0.703 (0.074)	0.963	0.969	0.958	0.921	0.970	0.976
$corr(i_t, i_t^{(40)})$	0.900 (0.048)	0.846	0.854	0.878	0.914	0.939	0.961
$corr(i_t^{(40)}, \Psi_t^{(40)})$	0.757 (0.148)	0.885	0.958	0.989	0.817	0.921	0.976
Goodness of fit							
Q^{Step2}	-	0.062	0.062	0.060	0.050	0.059	0.061
J-test: P-value	-	0.428	0.437	0.470	0.566	0.401	0.374
Q^{scaled}	-	0.663	0.438	0.327	0.241	0.267	0.293

Table 6: The New Keynesian Model: Analysis of Timing Premium

In Panel A, the timing premium is computed for different values of φ and a $\text{RRA} = 60$, while the remaining parameters are as reported in column (3) of Table 4. In Panel B, the timing premium is computed for different values of φ and with a constant timing attitude given $\alpha = -36.28$, while the remaining parameters are as reported in column (3) of Table 4. In Panel C, the timing premium is computed for different values of ψ and b , while all the remaining parameters are as reported in column (3) of Table 4. The timing premium is computed based on (12) and a third-order perturbation approximation, while the utility level when uncertainty is resolved in the following period is computed by simulation, using anti-thetic sampling with 5,000 draws and 10,000 terms to approximate the lifetime utility stream.

Panel A: $\text{RRA} = 60$		$\varphi = 0.10$	$\varphi = 0.25$	$\varphi = 0.50$	$\varphi = 0.75$
Π_{ss}		0.0%	10%	35%	93%
α		-16.92	-36.28	-94.04	-267.16
$\text{std}(\Delta c_t)$		2.79	1.44	1.25	1.600
$\text{std}(\log L_t)$		7.80	2.45	3.84	14.19
ϕ_0		2.43	28.58	64.98	85.45
Panel B: $\alpha = -36.28$		$\varphi = 0.10$	$\varphi = 0.25$	$\varphi = 0.50$	$\varphi = 0.75$
Π_{ss}		0.0%	10%	11%	20%
RRA		123.73	60	23.90	8.88
$\text{std}(\Delta c_t)$		2.30	1.44	1.40	1.38
$\text{std}(\log L_t)$		7.97	2.45	2.72	3.37
ϕ_0		2.43	28.58	64.98	85.45
Panel C:		$\psi = 0.25$	$\psi = \hat{\psi}$	$\psi = 0.5$	$\psi = 0.75$
$b = 0$		15%	22%	27%	43%
$b = 0.25$		13%	18%	21%	32%
$b = 0.5$		11%	14%	15%	21%
$b = \hat{b}$		9%	10%	9%	9%

Table 7: The New Keynesian Model: Analyzing the Key Mechanisms

All moments are computed using a third-order perturbation approximation and represented as in Table 5. Unless stated otherwise, all parameters attain the estimated values from column (4) in Table 4.

	(1) $u_0 = 0$	(2) $u_0 = -450$	(3) $u_0 = \hat{u}_0$	(4) $\sigma_Z = 0$	(5) $\alpha = 0$
Means					
Δc_t	1.968	1.968	1.968	1.968	1.968
π_t	20.956	12.558	3.824	22.205	24.064
i_t	29.732	17.701	5.191	31.521	34.185
$i_t^{(40)}$	30.956	18.978	6.524	32.732	34.772
$\Psi_t^{(40)}$	1.564	1.617	1.674	1.552	0.927
$\log L_t$	-1.074	-1.077	-1.080	-1.074	-1.072
r_t^m	10.268	7.662	5.009	10.576	10.862
Stds					
Δc_t	3.156	2.612	2.139	2.825	3.358
π_t	3.399	2.548	2.301	2.733	4.476
i_t	4.231	2.855	2.378	3.379	5.843
$i_t^{(40)}$	4.043	2.866	2.370	3.316	5.135
$\Psi_t^{(40)}$	0.839	0.970	1.004	0.947	0.470
$\log L_t$	12.472	6.562	2.495	13.315	14.772
Memo					
RRA	5	5	5	5	2.2
Π_{ss}	1%	3%	6%	0.0%	0.0%
α	-1.677	-14.08	-27.67	-27.67	0