Accepted Manuscript

Intergenerational conflict and public sector size and structure: A rationale for debt limits?

Torben M. Andersen

PII: S0176-2680(17)30588-8

DOI: [10.1016/j.ejpoleco.2018.07.005](https://doi.org/10.1016/j.ejpoleco.2018.07.005)

Reference: POLECO 1732

To appear in: *European Journal of Political Economy*

Received Date: 13 December 2017

Revised Date: 15 June 2018

Accepted Date: 17 July 2018

Please cite this article as: Andersen, T.M., Intergenerational conflict and public sector size and structure: A rationale for debt limits?, *European Journal of Political Economy* (2018), doi: 10.1016/j.ejpoleco.2018.07.005.

This is a PDF file of an unedited manuscript that has been accepted for publication. As a service to our customers we are providing this early version of the manuscript. The manuscript will undergo copyediting, typesetting, and review of the resulting proof before it is published in its final form. Please note that during the production process errors may be discovered which could affect the content, and all legal disclaimers that apply to the journal pertain.

Intergenerational conflict and public sector size and structure: A rationale for debt limits?*

Torben M. Andersen[†]

Department of Economics and Business Economics, Aarhus University

PeRCent, CESifo, CEPR and IZA

June 2018

Abstract

How do budget rules influence the size and structure of the public sector? This question is considered in an overlapping generations political economy model. The different horizons of agents influence their voting behaviour and lead to excessive deficit financing and debt accumulation, reducing welfare of all subsequent generations. Along an equilibrium trajectory with increasing debt, the tax burden increases and public sector activities are crowded out due to the increasing costs of debt servicing. A debt ceiling may halt this process leading to less debt, more public consumption and higher welfare.

*Comments and suggestions from anonymous referees as well as from participants, and in particular the discussant Michael Burda, at the conference *Fiscal Frameworks in Europe: Background and Perspectives*, Copenhagen, June 2017, are gratefully acknowledged.

[†]Author contact: Department of Economics, Aarhus University, Fuglsangs Alle, 8210 Aarhus V, Denmark. Phone: 004587165557, e-mail: tandersen@econ.au.dk

1 Introduction

Fiscal frameworks (institutional/procedural rules, e.g., a budget law) have become widespread and are an integral part of the EU policy framework¹. An essential element is numerical fiscal targets, or intermediary fiscal policy targets. These frameworks have been developed as a response to deficit biases and debt accumulation (or failure to consolidate debt after crises) observed for a number of countries as well as a concern for fiscal sustainability in relation to changing demographics. Fiscal rules are thus serving a commitment role to provide fiscal discipline² and to increase transparency and accountability in fiscal policy to achieve better outcomes in the medium to long run.

The discussion on fiscal rules has been heated. The criticism of budget rules has both a business cycle and a structural component. The former relates to the ability to pursue a sufficiently active countercyclical policy, and thus whether such norms leave sufficient room for fiscal flexibility. An excessively rigid interpretation of budget rules has been argued to be at the root of austerity policies, causing a slow recovery after the financial crisis³. The structural issue relates to the size and structure of the public sector, in particular, whether tight budget rules restraining deficit financing or debt accumulation are part of a political agenda to attain a leaner public sector. Related, it has been claimed that tight budget rules introduce a bias in public activities away from investments, detrimental to the long-term objectives the rules are meant to serve.

Considering the empirical evidence, there is no clear-cut relation between debt levels and the size of the public sector⁴. Figure 1 gives a

¹Within the EU the well-known rules include the fiscal norms associated with the Growth and Stability Pact (strengthened in the so-called six pack), stipulating that the actual general government deficit must not exceed 3% of GDP and public debt must not exceed 60% of GDP (or at least diminish sufficiently towards the 60% threshold). The Fiscal Compact (Treaty on Stability, Coordination and Governance) imposes a lower limit on the structural deficit of 0.5% of GDP (1.0% of GDP for Member States with a debt ratio significantly below 60% of GDP), see European Commission (2017). For an analysis and discussion, see e.g. Wyplosz (2014).

²Fiscal councils play an important role in this context, see Beetsma and Debrun (2016).

³On the other hand, it has been argued that fiscal rules in the EU have been administered too laxly, and this is one key reason why many countries entered the financial crisis with high debt levels. This made many countries vulnerable to negative shocks, which eventually caused the sovereign debt crisis.

⁴While the size of the public sector (measured relative to GDP) is relatively stable for most countries, there has been some variability in the debt-to-gdp level. This makes it difficult to precisely relate public sector size and debt level. The chart uses averages as a simple metric of structural levels.

cross plot of debt and core public expenditures (total expenditures less expenditures on debt servicing), and the two are not related⁵. Hence, the simple view that the extent of the welfare state is related to debt levels is not substantiated. It is particularly noteworthy that the Nordic countries known for their extended welfare states are among the countries with the lowest debt levels.

Figure 1: Public debt and core public expenditures, OECD countries, average 1995-2015.

Note: Debt measured by Maastricht debt, and country selection determined by data availability. Core public expenditures are total expenditures net of debt servicing expenditures. Both measured as averages over the period 1995 (earliest data point) and 2015. Data source: www.oecd-ilibrary.org

High debt and debt servicing costs may crowd out other expenditure types for a given size of the public sector measured by total expenditures or tax revenue. Total expenditures less debt servicing equal the expenditures on the core activities of the public sector like education, health, pensions etc. Figure 2 shows debt servicing expenditures as a share of core expenditures as a simple metric of the budget off-setting effect; that is, for an unchanged tax burden it shows by how much core expenditures can be increased if there were no debt servicing costs. For

⁵The correlation between debt level and core expenditures across the countries is 0.3 (not significantly different from zero). The correlation between debt and total expenditures is 0.55. The higher correlation is to be expected, since higher debt leads to higher debt servicing and thus higher total expenditures.

some countries this off-set is rather large, and for countries like Italy, Greece and Hungary expenditures on debt servicing exceed those on e.g. education. This is suggestive that debt may have implications for the structure of the public sector.

Figure 2: Debt servicing expenditures in percentage of core expenditures, average 1995-2015

Note: See Figure 1.

Although there is by now a rather large literature on the institutional specificities of fiscal rules and their possible effects, see e.g. Lledó et al. (2017), the above-mentioned "structural" criticism that fiscal rules affect the size and structure of the public sector has not been much analysed. Is this criticism justified when considering explicitly both the reasons for deficit biases and how a deficit/debt restraint affects the political process? This paper analyses these issues considering why the political process causes a deficit/debt bias and explores its consequences for the size and structure of the public sector. It is considered how the political process and thus fiscal policies are affected by a budget/debt restraint.

A political economy literature has developed arguments why deficit biases may arise; for an overview see, e.g., Persson and Tabellini (2000), Calmfors and Wren-Lewis (2011) and Alesina and Passalacqua (2017). The key explanations run in terms of short horizons of voters, informational problems, political fragmentation and common-pool problems. This literature has laid the foundation for understanding the basic reasons why deficit biases arise. However, the arguments are developed in two-period models ruling out the dynamics which is at the heart of the

problem⁶. This is important since the debt dynamics may feedback into the political process causing a deficit/debt bias and therefore affect the structure of the public sector (different expenditure types and taxation). Moreover, two-period models force a repayment of debt in the second period, whereas it is well-known that a positive level of debt can be sustained in steady state.

This paper develops a dynamic political economy model with overlapping generations of voters⁷. Specifically, a standard overlapping generations model of the type used to analyse macroeconomic dynamics is merged with the so-called probabilistic voting model (see Lindbeck and Weibull (1987)). The key mechanism is the finite horizon of agents in combination with a recurrent conflict of interest between old and young (and yet unborn) voters; cf. also Musgrave (1988). The analysis considers both the mechanism causing a deficit/debt bias and its implication for the size and structure of the public sector as well as the welfare consequences. The role of a fiscal rule in the form of a binding debt ceiling (balanced budget requirement) is analysed with particular focus on how it affects the level and composition of public expenditures.

The role of finite horizons for voters is also considered in a rather large political economy literature on pensions, starting with Browning (1975); for a survey see e.g. Galasso and Profeta (2002) and Mulligan and Sala-I-Martin (2004). A key insight from this literature is that the political equilibrium may support the introduction of a PAYG pension, even though such a scheme reduces welfare in the long run. More recent work along the same line considers the entire welfare package, including education for the young and pensions for the old; see e.g. Lancia and Russo (2016) and Bishnu and Wang (2015).

The paper is organized as follows. Section 2 sets out the overlapping generations model and the political process in the case where the fiscal platform includes public consumption financed either by current taxation or debt. Section 3 extends the political decision problem to include both productive expenditures and pensions as well as distortionary taxes.

⁶These issues have also been analysed in infinite-horizon agents models, see Battaglini (2011) for a discussion and references.

⁷Röhrs (2016) presents a fully-specified OLG model where parents are altruistic towards children. Moreover, public debt is the only financial asset and is therefore needed as a savings device for young agents; i.e., debt is also associated with efficiency gains. Debt accumulation is considered in the case of both commitment and absence of commitment. Song et al. (2012) present a quantitative analysis of a "global" model with a set of small open economies and an OLG-structure similar to the one in this paper. Important in their setting is the difference in preferences across countries and how this affects international capital flows. Mueller et al. (2016) consider how the political colour of the government affects deficit/debt biases.

Section 4 offers a few concluding comments. Technical details and proofs are given in Appendices.

2 A political economy model of the public sector

Consider a standard two-period overlapping generations model where agents work and save as young and care about consumption both when young and old. Agents are assumed to be non-altruistic to bring forth the role of finite horizons among voters. The result also holds allowing for altruism⁸. The population size is assumed constant (normalized to unity)⁹. Factor prices are exogenous (small open economy). For a start, labour supply/production is assumed exogenous, but this assumption is relaxed in Section 4.

Life-time utility for a representative agent belonging to the cohort being young in t is given as

$$\Omega_t^y \equiv u(c_t^y) + v(g_t) + \delta [u(c_{t+1}^o) + v(g_{t+1})] \quad (1)$$

where $u(\cdot)$ gives the utility from private consumption (c), where a superscript y refers to the young, and o to the old. The utility function $u(\cdot)$ satisfies all standard assumptions, including $u_c > 0$, $u_{cc} < 0$. Utility from public consumption (g) is denoted by $v(\cdot)$, and it satisfies all standard conditions, including $v_g > 0$, $v_{gg} < 0$. Additive separability is assumed to simplify the exposition. Similarly, it is assumed that the young and the old have the same period utility functions specified over private and public consumption, respectively, to show how the results may derive solely from differences in time horizons across voters. These assumptions can easily be relaxed¹⁰. Disutility from work is disregarded, since labour supply is exogenous. The subjective discount factor is $\delta < 1$.

As old, utility is given as

$$\Omega_{t+1}^o \equiv u(c_{t+1}^o) + v(g_{t+1}) \quad (2)$$

⁸Altruism would imply that the voters care for the debt (negative bequest) they leave to their descendants. This increases the marginal costs of debt in the political process and therefore leads to a lower debt level. However, it does not eliminate the incentive to push financing forward in time and the excessive debt accumulation unless utility of future cohorts are fully internalized as in the dynasty model with Ricardian equivalent, see Barro (1974).

⁹Allowing for exogenous population growth would be straightforward. The relative political power of the old, introduced later, will be lower, the higher the population growth rate. Except for that, nothing changes.

¹⁰It is easily verified that a more general specification with utility to the young being $v^y(g_t)$ and for the old $v^o(g_t)$ does not change anything qualitatively. To simplify the exposition, the utility functions are assumed identical.

The individual faces the following budget constraints

$$c_t^y = y - T_t - s_t \quad (3)$$

$$c_{t+1}^o = Rs_t \quad (4)$$

where T_t is the lump-sum tax levied on the young (see discussion below), s_t savings, and $R > 1$ is the gross market return¹¹. The first-order condition to the household problem of finding the savings level maximizing (1) subject to (3) and (4) reads

$$u_c(y - T_t - s_t) = R\delta u_c(Rs_t) \quad (5)$$

which defines a savings function

$$s_t = s(R, y - T_t)$$

where $\frac{\partial s(R, y - T_t)}{\partial y} = \frac{u_{cc}(c_t^y)}{u_{cc}(c_t^y) + R^2 \delta u_{cc}(c_{t+1}^o)} \in [0, 1]$.

The public sector provides the public consumption/activity g and finances it via current taxation T or debt D . End of period debt D_t thus evolves as

$$D_t = RD_{t-1} + g_t - T_t \quad (6)$$

It is assumed that debt is always honoured; i.e., there is a commitment not to renege on debt¹².

Political process

Policies are decided by a government elected for one period according to a majority rule; i.e., there are repeated elections, and current governments cannot commit future governments. The political outcome is modelled using the so-called probabilistic voting model¹³; see Lindbeck and Weibull (1987) and Persson and Tabellini (2000). The foundation of this model is a setting with two contesting parties running for government aiming at maximizing their expected number of voters and thus

¹¹The setting may be interpreted as having a funded pension scheme; i.e., agents voluntarily save for old age at the market return R . There are no capital market imperfections, myopia or the like motivating a mandated (PAYG) pension scheme. On public pensions, see below.

¹²The issue of whether not to renege on debt in political equilibrium has been studied in a number of papers. Tabellini (1991) shows in a setting with intergenerational altruism and income heterogeneity that there are opposite interests in honouring the debt; e.g., holders of the debt and their children clearly support it to be honoured. Dixit and Londregan (1998) show that debt is honoured if debt holders are more politically powerful.

¹³For recent applications in overlapping-generations models with balanced budget settings, see e.g. Lancia and Russo (2016) and Bishnu and Wang (2015).

the probability of being elected¹⁴. The policy equilibrium is the policy package maximizing a weighted sum of the utilities of the voters with weights depending on the size of the groups and their political power¹⁵.

In the present setting, there are two groups of voters, young and old, and the equilibrium political outcome is the policy package maximizing a weighted sum of the utility of the two groups given as

$$\Omega_t = \Omega_t^u + \phi \Omega_t^o$$

where ϕ is the relative political power of the old $\phi \in [0, \infty)$. Observe that since agents are non-altruistic, the utility of future cohorts does not directly influence the political outcome. Importantly, varying ϕ traces out the set of Pareto-efficient allocations among living cohorts (those affecting the political decision). Note also that the median voter model arises as special cases in the sense that either the old (corresponds to $\phi \rightarrow \infty$) or the young ($\phi \rightarrow 0$) may be the median voter.¹⁶

Using the definition of utility for the young and old, respectively, we have that the political objective function can be written

$$\Omega_t = u(c_t^y) + v(g_t) + \delta [u(c_{t+1}^o) + v(g_{t+1})] + \phi [u(c_t^o) + v(g_t)]$$

Since D_{t-1} is predetermined, the policy choice determines public consumption (g_t) and taxation (T_t), and thereby via (6) also the outstanding debt (D_t) of the public sector. It turns out to be easier to characterize the political process in terms of g_t and D_t (T_t then follows from (6), $T_t = RD_{t-1} + g_t - D_t$). Hence, the outcome of the political process is found as the solution to the problem

$$\max_{g_t, D_t} \Omega_t$$

subject to (5) and (6). The first-order conditions to this problem read (for second order conditions, see Appendix A)

¹⁴It is essential that the mapping from the policy platform to the support of a policy/party is not one-to-one. The political parties face uncertainty in the support a given platform generates, and this makes a unique Nash equilibrium possible for the game between the two contesting parties. Note that since the parties only care about winning the election, they propose the same policies in equilibrium; see e.g. Persson and Tabellini (2000).

¹⁵The outcome may also be interpreted in the sense that the political outcome is Pareto-efficient, and by changing the relative weight of the two groups ϕ , the Pareto-efficient possibility frontier is traced out.

¹⁶Note, that if the old is deciding (the median voter) there is no concern for the debt level and the equilibrium is thus not well-defined. If the young is the median voter then debt is well-defined given the youngs perception of how debt affects the future level of public consumption.

$$g_t : u_c(y - RD_{t-1} - g_t + D_t - s_t) = [1 + \phi] v_g(g_t) \quad (7)$$

$$D_t : u_c(y - RD_{t-1} - g_t + D_t - s_t) = -\delta v_g(g_{t+1}) \frac{\partial g_{t+1}}{\partial D_t} \quad (8)$$

The first condition (7) is a variant of the standard condition (Samuelson rule) for the determination of public consumption (when lump-sum taxation is feasible). The optimal level is where the marginal cost given by the marginal utility of private consumption (here of the young, since the tax is levied on the young) equals the marginal benefit or utility of public consumption. Note that the latter is weighted by $1 + \phi$, the sum of the weights to the utility of the young and the old in the political process.

The extent of debt financing is determined by (8). A higher debt level allows a lower tax burden, which benefits the young by allowing higher consumption. Hence, the marginal benefit of debt is given by the marginal utility of consumption to the young. Higher debt is passed on to future cohorts, and may thus affect future supply of public consumption adversely, a cost which is anticipated by the current young voters. The marginal cost of debt is determined by marginal utility consequences of the lower future level of public consumption. The optimal level of debt is where the marginal gain (temptation) equals the marginal cost (punishment). Observe that (8) does not depend directly on the political power of the old. This is the natural consequence of debt financing postponing debt servicing to future generations, and thus of no direct importance for the non-altruistic old¹⁷. There is thus a clear asymmetry in the political decision process - both the young and the old have a direct interest in public consumption, but debt financing is only influenced by the young. This asymmetry is a source for deficit bias and debt accumulation, cf. below.

It can be shown (see Appendix A) that there exists a Markov equilibrium to the political-economy model of the form

$$g_t = g(D_{t-1}, \phi) \quad (9)$$

$$D_t = D(D_{t-1}, \phi) \quad (10)$$

and the tax is given by $T_t = RD_{t-1} + g_t - D_t = RD_{t-1} + D(D_{t-1}, \phi) - g(D_{t-1}, \phi)$. The equilibrium outcomes depend on the debt level inherited from the past, the state variable (note that all other variables/parameters

¹⁷Cukierman and Meltzer (1989) show in a context with intergenerational altruism that bequest-constrained voters (individuals ideally wanting negative bequest) support deficit financing.

except D_{t-1} and ϕ have been suppressed). Appendix A works out an explicit solution for the case of log-utility functions.¹⁸

The debt level evolves according to a first-order difference equation, see (10), and stability requires: $0 < \frac{\partial D_t}{\partial D_{t-1}} < 1$ (see Appendix A). A higher initial debt level implies, other things being equal, a lower initial income/wealth for the young, which lowers both private and public consumption. This gives the current young an incentive to allow debt to increase. However, this comes at the costs of lower future supply of public consumption. This constrains the increase in debt, and therefore debt increases less than one-to-one. Hence, the political equilibrium, and thus the debt level, converges to a well-defined steady state equilibrium level with a debt level $D^{ss} : D^{ss} = f(D^{ss}, \phi) > 0$. The steady state levels of the other variables can be determined similarly.

The higher the initial debt, the lower the level of public consumption¹⁹, i.e. $\frac{\partial g_t}{\partial D_{t-1}} < 0$. Therefore, starting with a low level of initial debt ($D < D^{ss}$), the equilibrium trajectory displays an increasing debt level, an increasing tax burden and a decreasing level of public consumption as illustrated in Figure 3. The political process has a deficit bias implying an increasing debt level. Debt servicing causes a higher tax burden but also crowds out public consumption. Hence, measured by the tax burden the public sector grows over time, but its structure is twisted with declining public consumption and increasing expenditures on debt servicing. To see this clearly, note that the steady-state has²⁰

$$D^{ss} = RD^{ss} + g^{ss} - T^{ss}$$

or

$$T^{ss} = g^{ss} + (R - 1)D^{ss}$$

The political process has a deficit/debt bias which, via debt servicing, "consumes" space in the public budget constraint. In the short

¹⁸This specific case implies that debt converges to the maximum debt level which can be served based on the constant income flow y and the gross rate of return R . Therefore the entire budget is eventually absorbed by debt servicing. This is clearly extreme, and does not hold for more general utility functions. Other specific assumptions including exogenous production, exogenous factor prices and absence of any form of altruism should also be kept in mind.

¹⁹It follows directly from (8) that an interior solution requires $\frac{\partial g_{t+1}}{\partial D_t} < 0$.

²⁰This is a variant of the standard budget arithmetic for debt dynamics implying that there is a steady state relation between the level of debt measured relative to GDP ($d^{ss} = \frac{D^{ss}}{Y}$) and the total budget balance measured relative GDP (Γ) given as $d^{ss} = \frac{1+\hat{y}}{\hat{y}}\Gamma^{ss}$, where $\hat{y}$ is the growth rate of GDP. Systematic deficits are thus consistent with stationary debt levels and hence in this sense sustainable. In the main text $\hat{y} = 0$, but the analysis easily generalizes to the case of exogenous output growth.

run deficit financing makes it possible to attain a higher level of public consumption, but in the long run the opposite relation holds due to the alrger burden of debt servicing consuming space in the public budget, which, other things being equal, crowds out public consumption.

Figure 3: Trajectories for debt, public consumption and taxes

Observe that if the initial debt level is high ($D > D^{ss}$), the trajectory would be the reverse, that is, a declining debt level, lower tax burden and an increasing level of public consumption. The political equilibrium can therefore be consistent with debt consolidation (e.g. after a recession increasing the debt level above the steady-state level).

The political power of the old (relative to the young) matters for both the level of public consumption and debt,

$$\frac{\partial g_t}{\partial \phi} > 0; \frac{\partial D_t}{\partial \phi} > 0$$

For a given initial debt level, a higher political power to the old leads to more public consumption and a higher debt level. The intuition follows straightforwardly from the asymmetry between the interests of the young and the old concerning public consumption and debt. Both the old and the young have a direct interest in current public consumption, cf. (7), but only the young are concerned about debt (since it influences the future level of public consumption), cf. (8). Hence, more political power to the old increases public consumption. The financing burden falls on the young, and to avoid a too sharp drop in private consumption, they allow debt to increase. It is an implication that more political power to the old (higher ϕ) leads to an increase in steady state public consumption, debt, and taxes. Note that the deficit/debt bias does not disappear in the limiting case where the political power of the old vanishes ($\phi \rightarrow 0$). The reason is that the young have a finite horizon, and therefore do not care enough about the long-run consequences of debt.

2.1 Welfare effects

How is welfare affected by the policy choices? A simple metric is to consider how decisions made by past generations affect subsequent generations. The intergenerational linkage arises via debt, and the question is thus how the life-time utility of a cohort is affected by the initial debt level "inherited" from past generations. From (1) it follows that

$$\frac{\partial \Omega_t^u}{\partial D_{t-1}} = -u_c(c_t^y) \frac{\partial T_t}{\partial D_{t-1}} + v_g(g_t) \frac{\partial g_t}{\partial D_{t-1}} + \delta v_g(g_{t+1}) \frac{\partial g_{t+1}}{\partial D_t} \frac{\partial D_t}{\partial D_{t-1}} < 0$$

The higher the initial debt level in period t (D_{t-1}), the lower the life-time utility of the period t cohort (Ω_t^u). Starting from an initial low level of debt ($D < D^{ss}$) - see Figure 1 - it follows that the equilibrium trajectory has an increasing level of debt, and hence decreasing life-time utility, i.e.

$$\Omega_{t+j+1}^u \leq \Omega_{t+j}^u \text{ for all } j \geq 0 \text{ if } D_{t-1} < D^{ss}$$

The finite horizons underlying policy choices induce an inefficiency, making cohorts successively worse off; debt accumulation is excessive. Any cohort is worse off than the previous - the political decision process creates a vicious circle. This result holds for any value of ϕ .

2.2 Budget rules

As a remedy to the excessive debt accumulation pointed out above, consider a debt ceiling $\hat{D} < D^{ss}$. It is assumed that the ceiling is fully credible and binding. Observe that when the debt ceiling is binding it corresponds to a balanced budget constraint.

In the absence of the ceiling, the political equilibrium implies a sequence of policy choices $\{g_{t+j}, D_{t+j}\}_{j=0}^{\infty}$ determined by (9) and (10) given an initial debt level $D_{t-1} < D^{ss}$. Define the time period $t+k$ by

$$D_{t+k-1} < \hat{D} \text{ and } D_{t+k} \geq \hat{D}$$

i.e. along the unconstrained policy trajectory, period $t+k$ would be the first period, where the debt level is higher than (or equal to) the debt ceiling. Clearly, this is not feasible, and hence policies from period $t+k$ onwards would be constrained by the ceiling and thus determined by

$$\max_{g_{t+k+j}, D_{t+k+j}} \Omega_{t+k+j} \text{ for } j \geq 0$$

subject to $D_{t+k+j} = \hat{D}$ for all $j > 0$. Since $D_{t+k} > \hat{D}$, initial debt will also be constrained to $\hat{D}$, and the first-order conditions to the policy

problem read

$$\begin{aligned}\hat{g}_t : u_c(y - R\hat{D} - \hat{g}_{t+k} + \hat{D} - s(y - R\hat{D} - \hat{g}_{t+k} + \hat{D})) &= [1 + \phi] v_g(g_{t+k}) \\ \hat{D} : u_c(y - R\hat{D} - \hat{g}_{t+k} + \hat{D} - s(y - R\hat{D} - \hat{g}_{t+k} + \hat{D})) &> -\delta v_g(g_{t+k+1}) \frac{\partial g_{t+k+1}}{\partial D_t}\end{aligned}$$

where $\hat{g}$ denotes the optimal level of public consumption when the debt ceiling is binding (under a balanced budget requirement). Observe that the second first-order condition states that the political process is constrained by the debt ceiling; i.e., in the absence of the ceiling, debt would be higher. Since a debt level at the ceiling level implies that debt is constant over time ($D_{t+k+j} = \hat{D}$ for all $j > 0$), it follows that

$$\hat{g}_{t+k+j} = \hat{g} \text{ for all } j \geq 0$$

i.e. when debt accumulation has ceased, public consumption remains constant across periods, and therefore the tax payment also remains constant. A consequence of the debt ceiling is that the life-time utility of future cohorts will remain unchanged. The debt accumulation is stopped, and as a consequence the decline in public consumption and the increase in the tax burden are halted. Therefore the decline in welfare of future cohorts is stopped, which gives a welfare rationale for the debt ceiling.

The preceding characterizes policies from period $t + k$ and onwards. But when will the debt ceiling start binding? In the political process, debt financing is influenced by the young perceiving that it affects their future access to public consumption negatively. However, if debt reaches the debt ceiling $\hat{D}$ in period $t + k$, there is no incentive for the young in period $t + k - 1$ to abstain from more debt financing, and hence the debt level may be reached before $t + k$. In other words, approaching the debt ceiling makes fiscal policy more loose, since there is no "reward" to a more prudent policy. To see this more precisely, recall that only the young voice an interest in the debt level, and they will follow the unconstrained trajectory as long as it delivers higher utility than by increasing debt financing immediately and striking the debt target. That is, the young are better off along the unconstrained trajectory (participation constraint) as long as

$$\begin{aligned}u(y - RD_{t+h-1} - g_{t+h} + D_{t+h} - s(y - RD_{t+h-1} - g_{t+h} + D_{t+h})) + \delta v(g_{t+h+1}) \\ > u(y - RD_{t+h-1} - g_{t+h} + \hat{D} - s(y - RD_{t+h-1} - g_{t+h} + \hat{D})) + \delta v(\hat{g}) \text{ for } h < k\end{aligned}$$

It follows that evaluated for $h = k$ we have that the condition

$$\begin{aligned}u(y - RD_{t+k-1} - g_{t+k} + D_{t+k} - s(y - RD_{t+k-1} - g_{t+k} + D_{t+k})) + \delta v(\hat{g}) \\ > u(y - RD_{t+k-1} - g_{t+k} + \hat{D} - s(y - RD_{t+k-1} - g_{t+k} + \hat{D})) + \delta v(\hat{g})\end{aligned}$$

is violated since $D_{t+k} < \hat{D}$. This shows that there is an incentive to deviate from the unconstrained trajectory before reaching period $t+k$. The period at which there is a jump in policies²¹ is in period $t+\hat{h}$, cf. Figure 4, for which

$$\begin{aligned} & u(y - RD_{t+\hat{h}-1} - g_{t+\hat{h}} + D_{t+\hat{h}} - s(y - RD_{t+\hat{h}-1} - g_{t+\hat{h}} + D_{t+\hat{h}}) + \delta v(g_{t+\hat{h}+1})) \\ &= u(y - RD_{t+\hat{h}-1} - g_{t+\hat{h}} + \hat{D} - s(y - RD_{t+\hat{h}-1} - g_{t+\hat{h}} + \hat{D})) + \delta v(\hat{g}) \end{aligned}$$

Anticipating that the debt ceiling is reached thus affects the political process and leads to more debt financing. This moves forward in time when the debt ceiling is binding.

Figure 4: Debt dynamics under a binding debt ceiling

²¹Notice that this has some similarity to a well-known result from the literature on balance-of-payment-crises; see Obstfeld (1984). A pegged exchange rate policy is threatened by a systematic monetization of public deficits, which via the increase in the money supply leads to continuous depletion of foreign reserves making the peg unsustainable in the long run. Market participants perceive the non-sustainability of the policy trajectory, and therefore the peg has to be given up before the level of foreign reserve reaches the lowest feasible level. In the present case, the discipline constraining debt financing is eroded when the economy is close to the ceiling, leading to a more loose fiscal policy bringing forward in time the moment the economy reaches the debt target.

3 Extensions

The analysis in the previous section has worked out the dynamics implied by the political equilibrium in the overlapping generations economy in the simple case to distill the key mechanisms. The following generalizes the analysis by changing the political platform to allow for productive expenditures, pensions and distortionary taxation.

3.1 Productive expenditures

The preceding considered public consumption as the main activity of the public sector. Many public sector activities (e.g. education, health, infrastructure) may have an effect on production and thus income²²²³. In a stylized way such activities can be introduced by assuming that period t income depends on period $t - 1$ human capital (h), which in turn can be affected by education (e). Specifically, assume that output depends on human capital in the following way such that income is

$$y_t = y(h_t)$$

where $\frac{\partial y_t}{\partial h_t} > 0$, and $\frac{\partial^2 y_t}{\partial h_t^2} < 0$, and human capital is determined by a decreasing returns production function

$$h_t = h(h_{t-1}, e_{t-1})$$

depending on the past level of human capital (h_{t-1}) and investments in human capital (e_{t-1}), where $(0 < \frac{\partial h_t}{\partial h_{t-1}} < 1, \frac{\partial h_t}{\partial e_{t-1}} > 0, \frac{\partial^2 h_t}{\partial e_{t-1}^2} < 0)$. The educational input affects the level of human capital with a one period lag; human capital is thus predetermined.

Define period t net private income at the disposal of the young (income minus debt servicing) as

$$\omega_t = \omega(h_t, D_{t-1}) \equiv y(h_t) - RD_{t-1}$$

which is pre-determined depending on the human capital stock and debt decided in period $t - 1$, hence the subscript $t - 1$. The political objective function can now be written (using the government budget constraint $T_t = RD_{t-1} + e_t + g_t - D_t$)

$$\Omega_t = u(\omega_t - g_t - e_t + D_t - s_t) + v(g_t) + \delta [u(Rs_t) + v(g_{t+1})] + \phi [u(Rs_{t-1}) + v(g_t)]$$

²²Via effects on either labour supply or productivity. See empirical evidence given in e.g. Gemmell et al. (2011).

²³Peletier et al. (1997) consider public investment in the Tabellini and Alesina (1990) two-period political economy model with political uncertainty/fragmentation.

The policy outcome is the choices of g_t , e_t and D_t maximizing Ω_t given that private savings are determined by

$$u_c(\omega_t - g_t - e_t + D_t - s_t) = R\delta u_c(Rs_t)$$

The first-order condition to this problem reads²⁴

$$g_t : u_c(\omega_t - g_t - e_t + D_t - s_t) = [1 + \phi] v_g(g_t) \quad (11)$$

$$e_t : u_c(\omega_t - g_t - e_t + D_t - s_t) = \delta v_g(g_{t+1}) \frac{\partial g_{t+1}}{\partial e_t} \quad (12)$$

$$D_t : u_c(\omega_t - g_t - e_t + D_t - s_t) = -\delta v_g(g_{t+1}) \frac{\partial g_{t+1}}{\partial D_t} \quad (13)$$

The "new" first-order condition (12) has the same interpretation as (7) that education expenditures should be increased up to the point where the marginal costs equal the marginal future gain in terms of higher supply of public goods.

In Appendix B it is shown that there exists a Markov equilibrium where the policy variables can be expressed in terms of the following implicit functions

$$\begin{aligned} g_t &= g(\omega_t, \phi) ; g_\omega > 0 \\ e_t &= e(\omega_t, \phi) ; e_\omega > 0 \\ D_t &= D(\omega_t, \phi) ; D_\omega < 0 \end{aligned}$$

Note that period $t-1$ decisions affect period t solely via the net private income $\omega_t = y(h_t) - RD_{t-1}$; i.e., if there in period $t-1$ is invested more in education (higher e) or borrowed less (lower D), then net income is higher in period t , and this works to increase public consumption and investment, and to reduce debt. The response of education and debt to the past debt level arises due to the link to the future supply of public consumption.

On the margin, a change in education affects net private income by its marginal product ($\frac{\partial y(h_{t+j})}{\partial h_{t+j}} \frac{\partial h_{t+j}}{\partial e_{t+j-1}}$) and a reduction in debt by the gross rate of return (R). The policy outcome ensures equality between these two, since (12) and (13) imply²⁵

²⁴Note that in equilibrium $g_{t+1} = g(\omega_{t+1}, \phi)$ hence e.g. $\frac{\partial g_{t+1}}{\partial e_t} = \frac{\partial g(\omega_{t+1}, \phi)}{\partial \omega_{t+1}} \frac{\partial \omega_{t+1}}{\partial h_{t+1}} \frac{\partial h_{t+1}}{\partial e_t}$. The simpler expression is used in the text to avoid to simplify the exposition.

²⁵Note that this is not the level of educational input maximizing steady-state welfare (Golden Rule) which is higher due to a lower implicit rate of return than the market rate in a PAYG scheme in a dynamically efficient economy, cf. Andersen and Bhattacharya (2017).

$$\frac{\partial y(h_{t+j})}{\partial h_{t+j}} \frac{\partial h_{t+j}}{\partial e_{t+j-1}} = R \text{ for all } j \geq 0 \quad (14)$$

If left unconstrained, the level of human capital and educational inputs would reach a steady state ($h_{t+k} = h^{ss}$ and $e_{t+k} = e^{ss}$ for all $k \geq 0$).

Note that the level of education is independent of both the political power of the old (ϕ) and the debt level (D_{t-1}). In the setting considered in Section 2 with public consumption only, the young have an incentive to avoid too much debt financing, since it reduces future public consumption from which they benefit as old. In this setting with education, there is a similar incentive to invest in future income to improve the future level of public consumption. Therefore, the young support such investments even if they are non-altruistic and the investment does not give them any higher income directly²⁶. On the margin, productive expenditures and debt are similar ways of influencing future political decisions, and therefore the optimal level of productive expenditures is where the marginal product equals the gross return on debt. Except for this modification, the basic results remain unchanged relative to the base model. The equilibrium trajectory for public consumption, taxation and debt are qualitatively as in Figure 1, if starting from a situation where debt is below the steady-state level ($D < D^{ss}$). Welfare is decreasing along the equilibrium trajectory, and in that sense there is excessive debt accumulation.

Consider next how the debt ceiling works in this case. Let the sequence of policy variables found in the unconstrained case are $\{g_{t+j}, e_{t+j}, D_{t+j}\}_{j=0}^{\infty}$. Define time period $t+k$ by

$$D_{t+k-1} \leq \hat{D} \text{ and } D_{t+k} > \hat{D}$$

i.e. along the unconstrained policy trajectory, period $t+k$ is the first period, where the debt level is higher than the debt ceiling. From this point in time, the debt constraint is binding. As shown in the previous section, there is an incentive to give in to the debt restraint earlier, but this is qualitatively as in Section 2. When the debt ceiling is binding,

²⁶There are also indirect effects through which the old may benefit by investing in the young or middle-aged; e.g., if it either increases the tax base and thus future pensions or increases the return to savings; see e.g. Konrad (1995) and Poutvaara (2003).

the policy variables are determined by (for $k > 0$)

$$\begin{aligned}\widehat{g}_{t+k} : u_c(\omega_{t+k} - \widehat{g}_{t+k} - \widehat{e}_{t+k} + D_{t+k} - s(\cdot)) &= [1 + \phi] v_g(\widehat{g}_{t+k}) \\ \widehat{e}_{t+k} : u_c(\omega_{t+k} - \widehat{g}_{t+k} - \widehat{e}_{t+k} + \widehat{D} - s(\cdot)) &= \delta v_g(\widehat{g}_{t+k+1}) \frac{\partial \widehat{g}_{t+k+1}}{\partial \widehat{e}_{t+k}} \\ \widehat{D}_{t+k} : u_c(\omega_{t+k} - \widehat{g}_{t+k} - \widehat{e}_{t+k} + \widehat{D} - s(\cdot)) &> R \delta v_g(\widehat{g}_{t+k+1}) \frac{\partial \widehat{g}_{t+k+1}}{\partial \omega_{t+k}}\end{aligned}$$

where $\widehat{\cdot}$ denotes the policy choice when the debt ceiling is binding, and $\omega_{t+k} = y(h_{t+k}) - R\widehat{D}$. Note that even though debt is constant, there is an intergenerational link via investments in education.

It follows from the first-order conditions given above that

$$\frac{\partial y(h_{t+k})}{\partial h_{t+k}} \frac{\partial h_{t+k}}{\partial e_{t+k-1}} > R$$

i.e. the level of investment in education is reduced $\widehat{e}_t < e_t$ when the debt ceiling is binding. In the constrained case, the level of educational investment becomes dependent on the policy power of the old (ϕ), since more power to the old increases public consumption, and this crowds out investments in education. It is shown in Appendix B that (for $k > 0$)

$$\begin{aligned}\widehat{g}_{t+k} &= \widehat{g}(\widehat{e}_{t+k-1}, \phi) ; \frac{\partial \widehat{g}_{t+k}}{\partial \widehat{e}_{t+k-1}} > 0, \frac{\partial \widehat{g}_{t+k}}{\partial \phi} > 0 \\ \widehat{e}_{t+k} &= \widehat{e}(\widehat{e}_{t+k-1}, \phi) ; 0 < \frac{\partial \widehat{e}_{t+k}}{\partial \widehat{e}_{t+k-1}}, \frac{\partial \widehat{e}_{t+k}}{\partial \phi} < 0\end{aligned}$$

Stability further requires (see Appendix B) $\frac{\partial \widehat{e}_{t+k}}{\partial \widehat{e}_{t+k-1}} < 1$. When the debt ceiling is binding, public education and consumption increases along the equilibrium trajectory due to an increasing level of human capital.

When the debt ceiling is reached, there is less incentive to invest in education, since it is competing with public consumption in the budget. This in turn reduces future income, which leads to lower public consumption and education. The debt ceiling thus has a welfare cost in terms of squeezing out productive investments. This captures one of the criticisms of budget rules, namely that they are too crude not distinguishing between "productive" and "non-productive" expenditures. Could this problem be circumvented by a Golden Rule budgeting principle, see e.g. Blanchard and Giavazzi (2004), where public investments are not included in the debt concept for which the ceiling applies. To analyse this situation, it is useful to decompose total tax payments (T_t) into two components, (T_t^g) to finance public consumption and (T_t^e) to finance education ($T_t = T_t^g + T_t^e$). The two types of expenditures and

taxes are kept separate. Debt related to consumption is denoted D_t^g and evolves according to

$$D_t^g = RD_{t-1}^g + g_t - T_t^g$$

and debt for education

$$D_t^e = RD_{t-1}^e + e_t - T_t^e$$

If $T_t^e = e_{t-1}R$, it follows that debt is $D_t^e = e_t$ (assuming initial education debt to be zero), and investment in education would be as in (14). For public consumption and the associated debt, the situation is similar to the base model, and the reasoning from there applies. Hence, applying the "Golden Rule" budgeting approach such that productive investments/expenditures are not included under the debt ceiling, would make it possible to counteract the deficit/debt bias without causing a reduction in productive expenditures/investments. The preceding has disregarded the practical problems associated with a distinction between the two types of expenditures.

3.2 Pensions

Public pensions are a prime public activity with an intergenerational link. The following adds pay-as-you-go pensions (b) to the political agenda (productive expenditures are disregarded to simplify) to analyse the consequences. With public expenditures on consumption and pensions, debt evolves according to

$$D_t = RD_{t-1} + g_t + b_t - T_t$$

The possibility of voluntary savings is maintained²⁷, but given the asymmetry in political power, there may be political support to introduce a public pension as well. The policy choice is now found as the solution to the following programme

$$\begin{aligned} \max_{g_t, b_t, D_t} \Omega_t = & u(y - RD_{t-1} - g_t - b_t + D_t - s_t) + v(g_t) + \delta [u(Rs_t + b_{t+1}) + v(g_{t+1})] \\ & + \phi [u(Rs_{t-1} + b_t) + v(g_t)] \end{aligned}$$

subject to the first-order condition for savings (s_t) given as

$$u_c(y - RD_{t-1} - g_t - b_t + D_t - s_t) = R\delta u_c(Rs_t + b_{t+1}) \quad (15)$$

²⁷The focus of this analysis is entirely the political support for pensions, and not possible efficiency effects of intervention in pension saving.

The first-order condition to the policy problem reads

$$g_t : u_c(y - RD_{t-1} - g_t - b_t + D_t - s_t) = [1 + \phi] v_g(g_t) \quad (16)$$

$$b_t : u_c(y - RD_{t-1} - g_t - b_t + D_t - s_t) = \phi u_c(Rs_{t-1} + b_t) \quad (17)$$

$$\begin{aligned} D_t : u_c(y - RD_{t-1} - g_t - b_t + D_t - s_t) \\ = -\delta v_g(g_{t+1}) \frac{\partial g_{t+1}}{\partial D_t} - \delta u_c(Rs_t + b_{t+1}) \frac{\partial b_{t+1}}{\partial D_t} \end{aligned} \quad (18)$$

The new aspect here is condition (17), saying that the optimal pension equalizes its marginal cost measured by the marginal utility of consumption to the young financing the pension to the marginal benefit given as the marginal utility of consumption to old weighted by their relative political power. It is a well-known result from the pensions literature that a public pension (PAYG) reduces long-run welfare in a representative agent model when the economy is dynamically efficient (corresponding to $R > 1$ in the absence of population growth), see e.g. Aaron (1966). However, such a pension is introduced if the political power of the old is sufficiently large²⁸ ($\phi > R\delta$); i.e., the old benefit from the pension, and the cost falls on future cohorts. This is a generalization of a well-known result; see e.g. Browning (1975).

The intergenerational political linkage now has two elements; leaving a higher debt level to future cohorts may affect both the future level of public consumption and pensions. This "cost" of debt financing may constrain the incentive to debt-finance public expenditures. To see more clearly how the debt left to the next generation (D_t) affects future pensions (b_{t+1}), we have by use of the first-order condition (15) that equation (18) can be written

$$u_c(y - RD_{t-1} - g_t - b_t + D_t - s_t) \left[1 + \frac{1}{R} \frac{\partial b_{t+1}}{\partial D_t} \right] = -\delta v_g(g_{t+1}) \frac{\partial g_{t+1}}{\partial D_t} \quad (19)$$

In the special case where the political choice is only over pensions and debt ($v_g(\cdot) = 0$: $g_{t+j} = 0$ for all j), this reduces to

$$u_c(y - RD_{t-1} - g_t - b_t + D_t - s_t) \left[1 + \frac{1}{R} \frac{\partial b_{t+1}}{\partial D_t} \right] = 0$$

which is satisfied for

$$\frac{\partial b_{t+1}}{\partial D_t} = -R > 0 \quad (20)$$

²⁸The outcome may be a corner-solution with no pension, $b_t = 0$, if $u_c(y - s_t) > \phi u_c(Rs_{t-1})$ or using the individual first-order condition $R\delta u_c(Rs_t) > \phi u_c(Rs_{t-1})$. If the initial situation has no pension, then $s_{t-1} = s_t$ and the above condition holds if $R\delta > \phi$. Hence, $\phi > R\delta$ is required for a pension to be introduced. Note that this holds for any debt level inherited from the past, see Appendix C.

i.e. an increase in debt reduces the future pension by the gross rate of return. This brings out a well-known result from the pensions literature (see e.g. Atkinson and Stiglitz (1978), Sinn (2000), de la Croix and Michel (2002)) that a PAYG pension financed by a lump-sum tax is equivalent to a debt²⁹. The equivalence follows immediately by noting that the intertemporal budget constraint for cohort t reads

$$c_t^y + \frac{c_{t+1}^o}{R} = y - RD_{t-1} - g_t - b_t + D_t + \frac{b_{t+1}}{R}$$

Hence, (20) implies that life-time income of the agents belonging to cohort t is invariant to a change in the debt level D_t , i.e.

$$\frac{\partial \left[y - RD_{t-1} - g_t - b_t + D_t + \frac{b_{t+1}}{R} \right]}{\partial D_t} = 1 + \frac{1}{R} \frac{\partial b_{t+1}}{\partial D_t} = 0$$

This is saying that if the young are contemplating raising the debt level to shift some financing burden forward in time, it would release a corresponding decline in their future pensions, leaving their life-time income and thus utility unchanged. Debt and pensions are equivalent. The interesting point here is that this equivalence holds in the political equilibrium in the special case where the political agenda considers a choice only between pensions and debt.

Returning to the general case ($v_g > 0$), it follows directly from (19) that $1 + \frac{1}{R} \frac{\partial b_{t+1}}{\partial D_t} > 0$ and hence

$$\frac{\partial b_{t+1}}{\partial D_t} > -R$$

i.e. the pension-debt equivalence result does not apply when the political decision also includes public consumption; "crowding out" is less than complete, and this gives a welfare rationale for a debt ceiling. Clearly, the mechanisms are otherwise as in the base model, cf. above.

²⁹To see the equivalence, note that the life-time income of cohort t under a balanced budget regime ($T_t = g_t + b_t$) is

$$I_t^{PAYG} = y - g_t - b_t + \frac{b_{t+1}}{R}$$

Compare this to a debt regime with no pensions where it is

$$I_t^{DEBT} = y - RD_{t-1} - g_t + D_t$$

Clearly, $I_t^{PAYG} = I_t^{DEBT}$ if pension benefits are determined by $b_{t+j} = RD_{t+j-1}$ for all j .

3.3 Distortionary taxation

To simplify, the preceding assumed lump-sum taxation, but the key arguments straightforwardly generalize to the case where taxes distort economic activity. To show this, it suffices to specify income (y) as a function of the income tax rate (τ_t), assuming that $\frac{\partial y(\tau_t)}{\partial \tau_t} < 0$, $\frac{\partial^2 y(\tau_t)}{\partial \tau_t^2} < 0$. This captures the essence of tax distortions in standard political economy models with endogenous labour supply, cf. Meltzer and Richard (1981). In this case the tax revenue is

$$T_t = T(\tau_t) = \tau_t y(\tau_t)$$

where it is assumed that $\frac{\partial T(\tau_t)}{\partial \tau_t} > 0$ (ruling out that the economy is on the downward sloping part of the Laffer curve). The public budget constraint reads

$$g_t = D_t - RD_{t-1} + T(\tau_t)$$

and the tax rate is given by

$$\tau_t = T^{-1}(RD_{t-1} + g_t - D_t)$$

which implies $\frac{\partial \tau_t}{\partial RD_{t-1} + g_t - D_t} > 0$. Disposable income can now be written

$$\begin{aligned} y(\tau_t) - T(\tau_t) &= y(T^{-1}(RD_{t-1} + g_t - D_t)) - [RD_{t-1} + g_t - D_t] \\ &= \chi(RD_{t-1} + g_t - D_t) \end{aligned}$$

where

$$\frac{\partial \chi(RD_{t-1} + g_t - D_t)}{\partial [RD_{t-1} + g_t - D_t]} = y_\tau \frac{\partial \tau_t}{\partial RD_{t-1} + g_t - D_t} - 1$$

Utility for the young is therefore

$$\begin{aligned} \Omega_t^u &= u(y(\tau_t) - T(\tau_t) - s_t) + v(g_t) + \delta [u(Rs_t) + v(g_{t+1})] \\ &= u(\chi(RD_{t-1} + g_t - D_t) - s_t) + v(g_t) + \delta [u(Rs_t) + v(g_{t+1})] \end{aligned}$$

It is readily apparent that the structure is similar to the base model, cf. above. The costs of increasing g are now higher, since it is financed by a distortionary tax reducing output and thus consumption, and this tends to reduce public activities, but otherwise the underlying mechanisms are unchanged.

4 Concluding remarks

With finite horizons (and no altruism), a deficit/debt bias arises because current voters have an incentive to shift financing forward in time on future generations. The enforcement is young agents perceiving that this in the future via debt servicing crowds out core activities of the public sector from which they benefit. Along the equilibrium trajectory (starting with low debt) debt increases, total taxes increase, and core expenditures are crowded out. As a consequence, generations are worse off along the trajectory. A debt ceiling (balanced budget) rule can curtail this tendency to excessive debt accumulation, and is thus potentially welfare improving. With a binding ceiling core expenditures are higher and the total tax burden lower. This applies for various types of public expenditures. Criticism that fiscal rules enforce a smaller public sector is not justified, rather such rules can prevent that debt crowds out core activities of the public sector.

However, for expenditures with an investment element like education, a binding debt ceiling may lead to underinvestments. This is a potential cost of a debt ceiling and raises the issue of so-called golden rule budgeting rules keeping investment expenditures outside the fiscal rules. This is, however, not without problems since in practice it is difficult to define such investment activities precisely.

The political process has been modelled using the probabilistic voting model. This model can be readily applied, and has the advantage that it traces out the Pareto-efficient possibility frontier for the political choices. A shortcoming is that it is difficult to relate to more specific institutional and political factors which are likely to play a role, and which the stylized facts shown in the introduction point to as important.

The primary aim has been to demonstrate how actually implemented fiscal rules may affect the size and structure of the public sector. Finding that simple rules are associated with welfare improvements raises questions on optimal rules. While simple rules are more easy to implement, they are likely dominated by more sophisticated rules, and it is an interesting question to analyse optimal rule design.

The analysis has purposely been very stylized to focus on the basic effects released by debt dynamics and how it may feedback into the political process. In future work it would be interesting to consider these questions in more rich environments including, e.g., a changing demographic structure.

References

Aaron, H., 1966, The social insurance paradox. *Canadian Journal of Economics*, 32, 371-4.

Alesina, A. and A. Passalacqua, 2017, The political economy of government debt, in J. Taylor and H. Uhlig (eds.) *Handbook of Macroeconomics*, Vol 2b, Chapter 34, North Holland.

Andersen, T.M. and J. Bhattacharya, 2017, The intergenerational welfare state and the rise and fall of pay-as-you-go pensions, *Economic Journal*, 127, 896–923.

Atkinson, A.B. and J.E. Stiglitz, 1980, *Lectures on Public Economics*, McGrawHill, London.

Barro, R.J., 1974, Are government bonds net wealth, *Journal of Political Economy*, 82(6), 1095–1117.

Battaglini, M., 2011, The political economy of public debt, *Annual Economic Review*, 3, 161–189.

Beetsma, R.W.M.J. and X. Debrun, 2016, Fiscal councils: Rationale and effectiveness, IMF WP/16/86.

Blanchard, O. and F. Giavazzi, 2004, Improving the SGP through a proper accounting of public investment, CEPR Discussion Paper 4220, London.

Bishnu, M. and M. Wang, 2015, The political intergenerational welfare state, *Journal of Economic Dynamics and Control*, 77, 93–110.

Browning, E.K., 1975, Why the social insurance budget is too large in a democracy, *Economic Inquiry*, 16, 373–388.

Calmfors, L. and S. Wren-Lewis, 2011, What should fiscal councils do? *Economic Policy*, 26(68), 649–695.

Cukierman, A. and A. Meltzer, 1989, A political theory of government debt and deficits in a neo-Ricardian framework, *American Economic Review*, 79, 713–32.

de la Croix, D. and P. Michel, 2002, *A Theory of Economic Growth: Dynamics and Policy in Overlapping Generations*, Cambridge University Press.

Dixit, A. and J. Londregan, 1998, Political power and the credibility of government debt, *Journal of Economic Theory*, 94, 80–105.

European Commission, 2017, *Vade Mecum on the Stability and Growth Pact - 2017 Edition*, Brussels. https://ec.europa.eu/info/sites/info/files/ip052_en_0.pdf.

Galasso, V. and P. Profeta, 2002, The political economy of social security: a survey, *European Journal of Political Economy*, 18, 1–29.

Gemmell, N., R. Kneller, and I. Sanz, 2011, The timing and persistence of fiscal policy impacts on growth, *Economic Journal*, 121, F33–F58.

Konrad, K., 1995, Social security and strategic inter-vivos transfers of social capital, *Journal of Population Economics*, 8(3), 315–326.

Lancia, F. and A. Russo, 2016, Public education and pensions in democracy: A political economy theory *Journal of the European Eco-*

conomic Association, 14(5), 1038-1073.

Lindbeck, A. and J. Weibull, 1987, Balanced budget redistribution as the outcome of political competition, *Public Choice*, 52, 273-297.

Lledó, V., S. Yoon, X. Fang, S. Mbaye, and Y. Kim, 2017, *Fiscal Rules at a Glance*, IMF, Washington.

Meltzer, A.H. and S.F. Richard, 1981, A rational theory of the size of government, *Journal of Political Economy*, 89, 914-927.

Mueller A., K. Storesletten, and F. Zilibotti, 2016, The political color of fiscal responsibility, *Journal of the European Economic Association*, 14(1), 252-302.

Mulligan, C.B. and X. Sala-i-Martin, 2004, Political and economic forces sustaining social security, *Advances in Economic Analysis and Policy*, B.E: Journals in Economic Analysis and Policy, 5, 51-74.

Musgrave, R., 1988, Public debt and intergenerational equity, in K. Arrow and M. Boskin (eds.), *The Economics of Public Debt*, Macmillan, London.

Obstfeld, M., 1984, Balance of payments crisis and devaluation, *Journal of Money, Credit and Banking*, 16, 208-217.

Peletier, D., R. Dur, and H. Swank, 1997, Voting on the budget deficit: comment, *American Economic Review*, 89, 1377-1381.

Persson T. and G. Tabellini, 2000, *Political Economics - Explaining Economic Policy*, MIT Press, Cambridge.

Poutvaara, P., 2003, Gerontocracy revisited: unilateral transfer to the young may benefit the middle-aged, *Journal of Public Economics*, 88, 161-174.

Tabellini G., 1991, The politics of intergenerational distribution, *Journal of Political Economy*, 99, 335-57.

Tabellini G. and A. Alesina, 1990, Voting on the budget deficit, *American Economic Review*, 80, 37-49.

Röhrs, S., 2016, Public debt in political equilibrium, *Macroeconomic Dynamics*, 20, 1282-1312.

Sinn, H.W., 2000, Why a funded pension system is useful and why it is not useful, *International Tax and Public Finance*, 7, 389-410.

Song, Z., K. Storesletten, and F. Zilibotti, 2012, Rotten parents and disciplined children: A politico-economic theory of public expenditure and debt, *Econometrica* 80, 2785-2803.

Wyplosz, C., 2014, Fiscal rules: Theoretical issues and historical experiences, in A. Alesina and F. Giavazzi (eds.): *Fiscal Policies after the Financial Crisis*, 495-529, University of Chicago Press and National Bureau of Economic Research.

A Public consumption and debt

Standard assumptions are made ensuring that Ω is differentiable, and the first-order conditions for the policy choices of g_t and D_t are

$$\begin{aligned}\Omega_g &= -u_c(y - RD_{t-1} - g_t + D_t - s_t) + [1 + \phi] v_g(g_t) = 0 \\ \Omega_D &= u_c(y - RD_{t-1} - g_t + D_t - s_t) + \delta v_g(g_{t+1}) \frac{\partial g_{t+1}}{\partial D_t} = 0\end{aligned}$$

and the second-order conditions ensuring a local maximum are

$$\begin{aligned}\Omega_{DD} &< 0 \\ \Omega_{DD}\Omega_{gg} - \Omega_{Dg}^2 &> 0\end{aligned}$$

Conjecture a solution of the form

$$g_{t+1} = g(D_t, \phi) \quad (21)$$

$$D_{t+1} = D(D_t, \phi) \quad (22)$$

and define

$$h(D_t, \phi) \equiv \delta v_g(g_{t+1}) \frac{\partial g_{t+1}}{\partial D_t}$$

The foc condition for the policy choices of g_t and D_t can therefore be written

$$\begin{aligned}F_g(g_t, D_t, D_{t-1}, \phi) &\equiv -u_c(y - RD_{t-1} - g_t + D_t - s_t) + [1 + \phi] v_g(g_t) = 0 \\ F_D(g_t, D_t, D_{t-1}, \phi) &\equiv u_c(y - RD_{t-1} - g_t + D_t - s_t) + h(D_t, \phi) = 0\end{aligned} \quad (23)$$

This equation system solves for g_t and D_t as functions of D_{t-1} and ϕ of the form (21) and (22). The steady-state equilibrium satisfies

$$\begin{aligned}F_g(g^{SS}, D^{SS}, D^{SS}, \phi) &= 0 \\ F_D(g^{SS}, D^{SS}, D^{SS}, \phi) &= 0\end{aligned}$$

To determine the properties of the function $g(\cdot)$ totally differentiate (23) and (24) and evaluate at the steady-state equilibrium yields

$$\begin{aligned}F_{gg}dg_t + F_{gD}dD_t + F_{gD_{t-1}}dD_{t-1} + F_{g\phi}d\phi &= 0 \\ F_{Dg}dg_t + F_{DD}dD_t + F_{DD_{t-1}}dD_{t-1} + F_{D\phi}d\phi &= 0\end{aligned}$$

or in matrix form

$$\begin{bmatrix} F_{gg} & F_{gD} \\ F_{Dg} & F_{DD} \end{bmatrix} \begin{bmatrix} dg_t \\ dD_t \end{bmatrix} = - \begin{bmatrix} F_{gD_{t-1}} & F_{g\phi} \\ F_{DD_{t-1}} & F_{D\phi} \end{bmatrix} \begin{bmatrix} dD_{t-1} \\ d\phi \end{bmatrix}$$

where

$$\begin{aligned} F_{gg} &= u_{cc}(\cdot) [1 - s_y(\cdot)] + [1 + \phi] v_{gg}(\cdot) < 0 & F_{DD} &= u_{cc}(\cdot) [1 - s_y(\cdot)] + h_d(\cdot) \\ F_{gD} &= -u_{cc}(\cdot) [1 - s_y(\cdot)] > 0 & F_{Dg} &= -u_{cc}(\cdot) [1 - s_y(\cdot)] > 0 \\ F_{gD_{t-1}} &= Ru_{cc}(\cdot) [1 - s_y(\cdot)] < 0 & F_{DD_{t-1}} &= -Ru_{cc}(\cdot) [1 - s_y(\cdot)] > 0 \\ F_{g\phi} &= v_g(\cdot) > 0 & F_{D\phi} &= 0 \end{aligned}$$

Note that (23) and (24) are the first-order conditions to the policy problem, and hence the 2nd order conditions can be written as

$$\begin{aligned} F_{DD} &< 0 \\ F_{DD}F_{gg} - F_{Dg}^2 &> 0 \end{aligned}$$

Note that they have to hold evaluated under the $g(\cdot)$ - function to have a solution of the conjectured form.

To work out the comparative statistics using Cramer's rule, we have

$$\begin{aligned} dg_t &= - \frac{\begin{vmatrix} F_{gD_{t-1}} & F_{gD} \\ F_{DD_{t-1}} & F_{DD} \end{vmatrix}}{\begin{vmatrix} F_{gg} & F_{gD} \\ F_{Dg} & F_{DD} \end{vmatrix}} dD_{t-1} = - \frac{F_{gD_{t-1}}F_{DD} - F_{gD}F_{DD_{t-1}}}{F_{DD}F_{gg} - F_{Dg}^2} \\ &= - \frac{Ru_{cc}(\cdot) [1 - s_y(\cdot)] h_D(\cdot)}{[1 + \phi] v_{gg}(\cdot) [u_{cc}(\cdot) [1 - s_y(\cdot)] + h_D(\cdot)] + u_{cc}(\cdot) [1 - s_y(\cdot)] h_D(\cdot)} \end{aligned}$$

Since it has already been established that $\frac{dg_t}{dD_{t-1}} < 0$ it implied by the above expression that $h_D(g_t, \phi) < 0$.

Similary, we have

$$\begin{aligned} dD_t &= - \frac{\begin{vmatrix} F_{gg} & F_{gD_{t-1}} \\ F_{Dg} & F_{DD_{t-1}} \end{vmatrix}}{\begin{vmatrix} F_{gg} & F_{gD} \\ F_{Dg} & F_{DD} \end{vmatrix}} dD_{t-1} = - \frac{F_{gg}F_{DD_{t-1}} - F_{gD_{t-1}}F_{Dg}}{F_{DD}F_{gg} - F_{Dg}^2} \\ &= \frac{[1 + \phi] v_{gg}(\cdot) Ru_{cc}(\cdot) [1 - s_y(\cdot)]}{[1 + \phi] v_{gg}(\cdot) [u_{cc}(\cdot) [1 - s_y(\cdot)] + h_D(\cdot)] + u_{cc}(\cdot) [1 - s_y(\cdot)] h_D(\cdot)} > 0 \end{aligned}$$

Local stability around the steady state equilibrium requires $\frac{dD_t}{dD_{t-1}} < 1$ or

$$\frac{R[1 + \phi] v_{gg}(\cdot) u_{cc}(\cdot) [1 - s_y(\cdot)]}{[1 + \phi] v_{gg}(\cdot) [u_{cc}(\cdot) [1 - s_y(\cdot)] + h_D(\cdot)] + u_{cc}(\cdot) [1 - s_y(\cdot)] h_D(\cdot)} < 1$$

or

$$[R - 1] [1 + \phi] v_{gg}(\cdot) u_{cc}(\cdot) [1 - s_y(\cdot)] < [[1 + \phi] v_{gg}(\cdot) + u_{cc}(\cdot) [1 - s_y(\cdot)]] h_D(\cdot)$$

$$[R - 1] [1 + \phi] \frac{v_{gg}(\cdot) u_{cc}(\cdot) [1 - s_y(\cdot)]}{[[1 + \phi] v_{gg}(\cdot) + u_{cc}(\cdot) [1 - s_y(\cdot)]]} > h_D(\cdot)$$

Hence, a necessary condition for a stable equilibrium is $h_D(\cdot) < 0$, which holds.

Finally, the effects of a change in the political power of the old are

$$dg_t = - \frac{\begin{vmatrix} F_{g\phi} & F_{gD} \\ F_{D\phi} & F_{DD} \end{vmatrix}}{\begin{vmatrix} F_{gg} & F_{gD} \\ F_{Dg} & F_{DD} \end{vmatrix}} d\phi = - \frac{F_{g\phi} F_{DD}}{F_{DD} F_{gg} - F_{Dg}^2} d\phi > 0$$

and

$$dD_t = - \frac{\begin{vmatrix} F_{gg} & F_{g\phi} \\ F_{Dg} & F_{D\phi} \end{vmatrix}}{\begin{vmatrix} F_{gg} & F_{gD} \\ F_{Dg} & F_{DD} \end{vmatrix}} d\phi = \frac{F_{g\phi} F_{Dg}}{F_{DD} F_{gg} - F_{Dg}^2} d\phi > 0$$

A.1 Log-case

Assume that $u(c) = \ln(c)$ and $v(c) = \ln(c)$. Savings is from (5) determined as

$$s_t = \frac{\delta}{1 + \delta} [y - RD_{t-1} - g_t + D_t]$$

Conjecture the following solutions for the level of debt and public consumption, respectively

$$D_t = A_0 + A_1 D_{t-1} \quad (25)$$

$$g_t = B_0 + B_1 D_{t-1} \quad (26)$$

Equation (23) can be written

$$\frac{1}{y - RD_{t-1} - g_t + \tilde{D}_t - s_t} = [1 + \phi] \frac{1}{g_t}$$

implying

$$\begin{aligned} g_t &= \frac{1 + \phi}{2 + \delta + \phi} [y - RD_{t-1} + D_t] \\ &= \frac{\phi}{2 + \delta + \phi} [y - RD_{t-1} + A_0 + A_1 D_{t-1}] \end{aligned}$$

which is consistent with (26) for

$$\begin{aligned} B_0 &= \frac{\phi}{2 + \delta + \phi} [y + A_0] \\ B_1 &= \frac{\phi}{2 + \delta + \phi} [A_1 - R] \end{aligned}$$

Equation (24) can be written

$$\frac{1}{y - RD_{t-1} - g_t + D_t - s_t} = -\delta \frac{B_1}{g_{t+1}}$$

or

$$B_0 + B_1 \tilde{D}_t = -\delta B_1 \frac{1}{1 + \delta} [[y - RD_{t-1} + D_t] - g_t]$$

Solving for D_t

$$D_t \left[1 + \frac{\delta}{1 + \delta} \left[\frac{1 + \delta}{2 + \delta + \phi} \right] \right] = -\frac{\delta}{1 + \delta} \left[\frac{1 + \delta}{2 + \delta + \phi} \right] [y - RD_{t-1}] - \frac{B_0}{B_1}$$

Which is consistent with (31) for

$$A_0 = \frac{-\frac{\delta}{1 + \delta} \left[\frac{1 + \delta}{2 + \delta + \phi} \right] y - \frac{B_0}{B_1}}{\left[1 + \frac{\delta}{1 + \delta} \left[\frac{1 + \delta}{2 + \delta + \phi} \right] \right]}$$

$$A_1 = \frac{\frac{\delta}{1 + \delta} \left[\frac{1 + \delta}{2 + \delta + \phi} \right] R}{\left[1 + \frac{\delta}{1 + \delta} \left[\frac{1 + \delta}{2 + \delta + \phi} \right] \right]}$$

and hence

$$A_0 = \left[\frac{2 + 2\delta + \phi - \delta R}{2 + 2\delta + \phi} \right] \frac{1}{R - 1} y$$

$$A_1 = \frac{\delta R}{2 + 2\delta + \phi} > 0$$

Stability requires $A_1 < 1$ or $\frac{\delta R}{2 + 2\delta + \phi} < 1$. Steady state (ss) debt and public consumption is

$$D^{ss} = \frac{A_0}{1 - A_1} = \frac{1}{R - 1} y$$

Note that the debt level is the maximal level which can be served with a constant income flow y at the gross rate of return R . Hence, in steady-state the entire budget is absorbed by debt servicing, and public consumption is driven to zero.

$$g^{ss} = B_0 + B_1 D^{ss} = \frac{\phi}{2 + \delta + \phi} \left[y + A_0 - \frac{1}{R - 1} y [A_1 - R] \right] = 0$$

B Productive expenditures

The first-order conditions for optimal policies read

$$g_t : u_c(\omega_t - g_t - e_t + D_t - s_t) = [1 + \phi] v_g(g_t) \quad (27)$$

$$e_t : u_c(\omega_t - g_t - e_t + D_t - s_t) = \delta v_g(g_{t+1}) \frac{\partial g_{t+1}}{\partial e_t} \quad (28)$$

$$D_t : u_c(\omega_t - g_t - e_t + D_t - s_t) = -\delta v_g(g_{t+1}) \frac{\partial g_{t+1}}{\partial D_t} \quad (29)$$

where

$$k(\omega_t) \equiv \delta v_g(g(\omega_t, \phi)) \frac{\partial g(\omega_t, \phi)}{\partial \omega_t} \frac{\partial \omega_t}{\partial e_t}$$

We thus have three equations (27)-(29) in the three endogenous variables $\{g_t, e_t, D_t\}$ with ω_t the state variable as conjectured in the text. It is implied that

$$\frac{\partial \omega_t}{\partial e_t} = -\frac{\partial \omega_t}{\partial D_t}$$

or

$$R = \frac{\partial y(h_{t+1})}{\partial h_{t+1}} \frac{\partial h_{t+1}}{\partial e_t}$$

B.1 Binding debt ceiling

If debt is at the debt ceiling ($\hat{D}$), the policy problem is

$$\text{Max}_{g_t, e_t} \Omega_t = u(\omega_t - g_t - e_t + \hat{D} - s_t) + v(g_t) + \delta [u(Rs_t) + v(g_{t+1})] + \phi [u(Rs_{t-1}) + v(g_t)]$$

where $\omega_t = y(h_t) - R\hat{D}$. The first-order conditions to this problem are

$$\begin{aligned} g : \hat{\Lambda}_g(\cdot) &\equiv -u_c(\omega - g_t - e_t + \hat{D} - s_t) + [1 + \phi] v_g(g_t) = 0 \\ e : \hat{\Lambda}_e(\cdot) &\equiv -u_c(\omega - g_t - e_t + \hat{D} - s_t) + \hat{k}(e_t) = 0 \end{aligned}$$

where

$$\hat{k}(e_t) \equiv \delta v_g(g_{t+1}) \frac{\partial g_{t+1}}{\partial e_t} > 0$$

implying $\frac{\partial g_{t+1}}{\partial e_t} > 0$.

The 2nd order conditions to the policy problem are

$$\begin{aligned} \hat{\Lambda}_{gg}(\cdot) &< 0 \\ \hat{\Lambda}_{gg}(\cdot) \hat{\Lambda}_{ee}(\cdot) - [\hat{\Lambda}_{ge}(\cdot)]^2 &> 0 \end{aligned}$$

where

$$\begin{aligned}\widehat{\Lambda}_{gg} &= u_{cc}(\cdot) [1 - s_y(\cdot)] + [1 + \phi] v_{gg}(\cdot) < 0 & \widehat{\Lambda}_{ee}(\cdot) &= u_{cc}(\cdot) [1 - s_y(\cdot)] + \widehat{k}_e(\cdot) < 0 \\ \widehat{\Lambda}_{ge}(\cdot) &= u_{cc}(\cdot) [1 - s_y(\cdot)] < 0 & \widehat{\Lambda}_{eg}(\cdot) &= u_{cc}(\cdot) [1 - s_y(\cdot)] < 0 \\ \widehat{\Lambda}_{ge_{t-1}}(\cdot) &= -u_{cc}(\cdot) y_e(\cdot) [1 - s_y(\cdot)] > 0 & \widehat{\Lambda}_{ee_{t-1}}(\cdot) &= -u_{cc}(\cdot) y_e(\cdot) [1 - s_y(\cdot)] > 0 \\ \widehat{\Lambda}_{g\phi} &= v_g(g_t) > 0 & \widehat{\Lambda}_{e\phi}(\cdot) &= 0\end{aligned}$$

Totally differentiating the first-order conditions for the policy choice yields

$$\begin{bmatrix} \widehat{\Lambda}_{gg} & \widehat{\Lambda}_{ge}(\cdot) \\ \widehat{\Lambda}_{ge}(\cdot) & \widehat{\Lambda}_{ee}(\cdot) \end{bmatrix} \begin{bmatrix} dg_t \\ de_t \end{bmatrix} = - \begin{bmatrix} \widehat{\Lambda}_{ge_{t-1}}(\cdot) & \widehat{\Lambda}_{g\phi} \\ \widehat{\Lambda}_{ee_{t-1}}(\cdot) & \widehat{\Lambda}_{e\phi}(\cdot) \end{bmatrix} \begin{bmatrix} de_{t-1} \\ d\phi \end{bmatrix}$$

Using Cramer's rule, we get

$$\begin{aligned}\frac{dg_t}{de_{t-1}} &= - \frac{\begin{vmatrix} \widehat{\Lambda}_{ge_{t-1}}(\cdot) & \widehat{\Lambda}_{ge}(\cdot) \\ \widehat{\Lambda}_{ee_{t-1}}(\cdot) & \widehat{\Lambda}_{ee}(\cdot) \end{vmatrix}}{\begin{vmatrix} \widehat{\Lambda}_{gg} & \widehat{\Lambda}_{ge}(\cdot) \\ \widehat{\Lambda}_{ge}(\cdot) & \widehat{\Lambda}_{ee}(\cdot) \end{vmatrix}} \\ &= - \frac{\widehat{\Lambda}_{ge_{t-1}}(\cdot) \widehat{\Lambda}_{ee}(\cdot) - \widehat{\Lambda}_{ge}(\cdot) \widehat{\Lambda}_{ee_{t-1}}(\cdot)}{\widehat{\Lambda}_{gg}(\cdot) \widehat{\Lambda}_{ee}(\cdot) - [\widehat{\Lambda}_{ge}(\cdot)]^2} \\ &= \frac{u_{cc}(\cdot) y_e(\cdot) \widehat{k}_e(\cdot) [1 - s_y(\cdot)]}{[u_{cc}(\cdot) [1 - s_y(\cdot)] + [1 + \phi] v_{gg}(g_t)] [u_{cc}(\cdot) [1 - s_y(\cdot)] + \widehat{k}_e(\cdot)] - [u_{cc}(\cdot) [1 - s_y(\cdot)]]^2} \\ &= \frac{u_{cc}(\cdot) y_e(\cdot) \widehat{k}_e(\cdot) [1 - s_y(\cdot)]}{[1 + \phi] v_{gg}(g_t) [1 - s_y(\cdot) u_{cc}(\cdot) + \widehat{k}_e(\cdot)] + [1 - s_y(\cdot)] u_{cc}(\cdot) \widehat{k}_e(\cdot)} > 0\end{aligned}$$

Since, $\frac{dg_t}{de_{t-1}} > 0$ this implies $\widehat{k}_e(\cdot) < 0$. Similarly,

$$\begin{aligned}\frac{de_t}{de_{t-1}} &= - \frac{\begin{vmatrix} \widehat{\Lambda}_{gg} & \widehat{\Lambda}_{ge_{t-1}}(\cdot) \\ \widehat{\Lambda}_{ge}(\cdot) & \widehat{\Lambda}_{ee_{t-1}}(\cdot) \end{vmatrix}}{\begin{vmatrix} \widehat{\Lambda}_{gg} & \widehat{\Lambda}_{ge}(\cdot) \\ \widehat{\Lambda}_{ge}(\cdot) & \widehat{\Lambda}_{ee}(\cdot) \end{vmatrix}} \\ &= - \frac{\widehat{\Lambda}_{gg} \widehat{\Lambda}_{ee_{t-1}}(\cdot) - \widehat{\Lambda}_{ge}(\cdot) \widehat{\Lambda}_{ge_{t-1}}(\cdot)}{\widehat{\Lambda}_{gg}(\cdot) \widehat{\Lambda}_{ee}(\cdot) - [\widehat{\Lambda}_{ge}(\cdot)]^2} \\ &= \frac{[1 + \phi] v_{gg}(g_t) u_{cc}(\cdot) y_e(\cdot) [1 - s_y(\cdot)]}{[1 + \phi] v_{gg}(g_t) [1 - s_y(\cdot) u_{cc}(\cdot) + \widehat{k}_e(\cdot)] + [1 - s_y(\cdot)] u_{cc}(\cdot) \widehat{k}_e(\cdot)} > 0\end{aligned}$$

Stability requires $\frac{de_t}{de_{t-1}} < 1$ or

$$[1 + \phi] v_{gg}(g_t) u_{cc}(\cdot) y_e(\cdot) [1 - s_y(\cdot)] < [1 + \phi] v_{gg}(g_t) \left[[1 - s_y(\cdot)] u_{cc}(\cdot) + \hat{k}_e(\cdot) \right] + [1 - s_y(\cdot)] u_{cc}(\cdot) \hat{k}_e(\cdot)$$

which can be written

$$[y_e(\cdot) - 1] [1 + \phi] v_{gg}(g_t) u_{cc}(\cdot) [1 - s_y(\cdot)] < [[1 + \phi] v_{gg}(g_t) + [1 - s_y(\cdot)] u_{cc}(\cdot)] \hat{k}_e(\cdot)$$

or

$$\frac{[y_e(\cdot) - 1] [1 + \phi] v_{gg}(g_t) u_{cc}(\cdot) [1 - s_y(\cdot)]}{[[1 + \phi] v_{gg}(g_t) + [1 - s_y(\cdot)] u_{cc}(\cdot)]} > \hat{k}_e(\cdot)$$

Finally, considering the relative political power we have

$$\frac{dg_t}{d\phi} = - \frac{\hat{\Lambda}_{g\phi} \hat{\Lambda}_{ge}(\cdot) - \hat{\Lambda}_{e\phi}(\cdot) \hat{\Lambda}_{ge}(\cdot)}{\hat{\Lambda}_{gg}(\cdot) \hat{\Lambda}_{ee}(\cdot) - [\hat{\Lambda}_{ge}(\cdot)]^2} > 0$$

and

$$\begin{aligned} \frac{de_t}{d\phi} &= - \frac{\begin{vmatrix} \hat{\Lambda}_{gg} & \hat{\Lambda}_{g\phi} \\ \hat{\Lambda}_{ge}(\cdot) & \hat{\Lambda}_{e\phi}(\cdot) \end{vmatrix}}{\begin{vmatrix} \hat{\Lambda}_{gg} & \hat{\Lambda}_{ge}(\cdot) \\ \hat{\Lambda}_{ge}(\cdot) & \hat{\Lambda}_{ee}(\cdot) \end{vmatrix}} \\ &= - \frac{\hat{\Lambda}_{gg} \hat{\Lambda}_{e\phi}(\cdot) - \hat{\Lambda}_{g\phi} \hat{\Lambda}_{ge}(\cdot)}{\hat{\Lambda}_{gg}(\cdot) \hat{\Lambda}_{ee}(\cdot) - [\hat{\Lambda}_{ge}(\cdot)]^2} < 0 \end{aligned}$$

C Pensions

The following considers the situation where the policy choice involves pensions and debt; i.e. public consumption is disregarded. The first-order conditions for the policy choice read

$$\Omega_b \equiv -u_c(y - RD_{t-1} - b_t + D_t - s_t) + \phi u_c(Rs_{t-1} + b_t) = 0 \quad (30)$$

$$\Omega_D \equiv u_c(y - RD_{t-1} - b_t + D_t - s_t) + \delta u_c(Rs_t + b_{t+1}) \frac{\partial b_{t+1}}{\partial D_t} = 0 \quad (31)$$

Suppose a past without any pension - would there be political support to introduce the pension? This would require (assume initial debt is zero)

$$\phi u_c(Rs_{t-1}) > u_c(y - s_t)$$

Since individual foc for savings implies $u_c(y - s_t) = \delta R u_c(Rs_t)$, this condition can be rewritten

$$\phi u_c(Rs_{t-1}) > \delta R u_c(Rs_t)$$

Since the initial situation is characterized by $s_{t-1} = s_t$, it follows that introduction of the pension requires

$$\phi > \delta R$$

In the following this assumption is assumed fulfilled, and we consider the dynamics of a system with positive pensions and debt, cf. (30) and (31). Note first that since $\frac{\partial b_{t+1}}{\partial D_t} = -R$, cf. main text, it follows that

$$u_c(Rs_{t-1} + b_t) = \frac{\delta R}{\phi} u_c(Rs_t + b_{t+1})$$

and since $\phi > \delta R$, it must be the case that

$$u_c(Rs_t + b_{t+1}) < u_c(Rs_{t-1} + b_t) \quad (32)$$

and since $u_{cc}(\cdot) < 0$, it follows that along the equilibrium trajectory

$$Rs_{t-1} + b_t > Rs_t + b_{t+1}$$

From the first-order condition for savings, we find (recall that effects via debt do not matter given the equivalence)

$$u_c(y - RD_{t-1} - b_t + D_t - s_t) = R\delta u_c(Rs_t + b_{t+1})$$

and hence $\frac{\partial s_t}{\partial b_{t+1}} = -\frac{R\delta u_{cc}(\cdot)}{u_{cc}(\cdot) + R^2\delta u_{cc}(\cdot)} < 0$ implying that $\frac{\partial [Rs_t + b_{t+1}]}{\partial b_{t+1}} = -\frac{R^2\delta u_{cc}(\cdot)}{u_{cc}(\cdot) + R^2\delta u_{cc}(\cdot)} + 1 = \frac{u_{cc}(\cdot)}{u_{cc}(\cdot) + R^2\delta u_{cc}(\cdot)} > 0$. Therefore the above inequality only holds if $b_{t+1} < b_t$. From (32), one can also infer that the equilibrium path displays a decreasing sequence of c^0 . Note that there is no well-defined steady-state equilibrium in this case, but this finding can be attributed to the separability assumption and does not generalize. Note that the above finding also applies in the general case $v_g(g_t) \neq 0$, since (15) and (17) imply

$$\phi u_c(Rs_{t-1} + b_t) = R\delta u_c(Rs_t + b_{t+1})$$

D Tax distortions

Define total tax revenue as

$$T(\tau_t) = \tau_t y(\tau_t)$$

where it is assumed that $\frac{\partial T(\tau_t)}{\partial \tau_t} > 0$ (ruling out that the economy is on the downward sloping part of the Laffer curve). The public budget constraint reads

$$g_t = D_t - RD_{t-1} + T(\tau_t)$$

The tax rate can therefore be written

$$\tau_t = T^{-1}(RD_{t-1} + g_t - D_t)$$

which $\frac{\partial \tau_t}{\partial RD_{t-1} + g_t - D_t} > 0$. Disposable income can now be written

$$\begin{aligned} y(\tau_t) - T(\tau_t) &= y(T^{-1}(RD_{t-1} + g_t - D_t)) - [RD_{t-1} + g_t - D_t] \\ &\equiv \chi(RD_{t-1} + g_t - D_t) \end{aligned}$$

where

$$\frac{\partial \chi(RD_{t-1} + g_t - D_t)}{\partial [RD_{t-1} + g_t - D_t]} = y_\tau \frac{\partial \tau_t}{\partial [RD_{t-1} + g_t - D_t]} - 1$$

Utility for the young can now be written

$$\begin{aligned} \Omega_t^u &= u(y(\tau_t) - T(\tau_t) - s_t) + v(g_t) + \delta [u(Rs_t) + v(g_{t+1})] \\ &= u(\chi(RD_{t-1} + g_t - D_t) - s_t) + v(g_t) + \delta [u(Rs_t) + v(g_{t+1})] \end{aligned}$$

It is readily apparent that this has the same structure as the base model in Appendix A, and hence qualitatively works in the same way.