SCHOOL OF ECONOMICS AND MANAGEMENT
FACULTY OF SOCIAL SCIENCES
AARHUS UNIVERSITY

CREATES
Center for Research in Econometric Analysis of Time Series

CREATES Research Paper 2009-53

Forecasting long memory time series under a break in persistence

Florian Heinen, Philipp Sibbertsen and Robinson Kruse

School of Economics and Management
Aarhus University
Bartholins Allé 10, Building 1322, DK-8000 Aarhus C
Denmark

Forecasting long memory time series under a break in persistence*

Florian Heinen[†,1], Philipp Sibbertsen[†], and Robinson Kruse[‡]

[†]Institute of Statistics, Faculty of Economics and Management,
Leibniz University of Hannover, D-30167 Hannover, Germany

[‡]Center for Research in Econometric Analysis of Time Series,
School of Economics and Management,
Aarhus University, DK-8000 Aarhus C, Denmark

Abstract

We consider the problem of forecasting time series with long memory when the memory parameter is subject to a structural break. By means of a large-scale Monte Carlo study we show that ignoring such a change in persistence leads to substantially reduced forecasting precision. The strength of this effect depends on whether the memory parameter is increasing or decreasing over time. A comparison of six forecasting strategies allows us to conclude that pre-testing for a change in persistence is highly recommendable in our setting. In addition we provide an empirical example which underlines the importance of our findings.

JEL-Numbers: C15, C22, C53

Keywords: Long memory time series · Break in persistence · Structural change · Simulation · Forecasting competition

---

*We thank Niels Haldrup, Peter Boswijk, Matei Demetrescu, Helmut Lütkepohl and the participants at the Econometric Society European Meeting 2009 in Barcelona, Spain and at the Meeting of the German Statistical Association in Wuppertal, Germany for helpful comments. Robinson Kruse acknowledges support from CREATES funded by the Danish National Research Foundation.

[1]Corresponding author:
Phone: +49-511-762-5383
Fax: +49-511-762-3923
E-Mail: heinen@statistik.uni-hannover.de

1 Introduction

In the last two decades long memory models have become widely used in economics to model the persistence of many economic time series. Among these are financial variables like stock market volatility and macroeconomic data like inflation. Baillie (1996) provides a detailed survey about various applications of long memory in economics. Recent empirical and theoretical work focusses on the potential time-variation of the long memory parameter. Kumar and Okimoto (2007) report a decrease in the memory of US inflation, while Hassler and Nautz (2008) find an increase of memory in the spread of the European overnight rate index average (Eonia). Beran and Terrin (1996) propose a test for constancy of the fractional differencing parameter which has been extended by Horváth (2001) and Lavancier et al. (2009). Further theoretical contributions are Ray and Tsay (2002) who suggest a Bayesian method for break detection in long memory processes and Sibbertsen and Kruse (2009) who generalize a test for changing persistence in the classical $I(0)-I(1)$ framework with respect to fractional integration. Although forecasts of economic time series are often generated by fractionally integrated models, e.g. Bos et al. (2002), potential structural breaks in the memory parameter have been ignored. To the best of our knowledge, no one considered the implications of a changing memory parameter on forecasting so far. This paper an contains an extensive simulation study on this topic and provides practical recommendations.

It seems to be natural to expect that the knowledge about a break in the persistence of a process, i.e. the memory of a fractionally integrated process, can lead to superior forecasts. However, it is unclear whether how large these potential gains are and if they can be achieved in practice. Therefore, we consider the performance of six forecasting strategies which model or ignore a one-time break in the memory. The forecasting strategies we analyze are as follows: As a benchmark we assume that there is no break in persistence and estimate the memory parameter using the entire in-sample period. If the true data generating process (DGP) exhibits a break in persistence, then the estimated memory parameter is a convex combination of the pre- and the post-break memory parameters. A second strategy assumes a priori that the series has a break in the persistence and estimates the memory parameter before and after an estimated breakpoint. This strategy also includes a break in persistence even if the memory parameter is constant over time. The third strategy re-estimates the memory parameter and the forecasting coefficients in every forecasting step. This type of updating the forecasting coefficients may take some fluctuations of the parameters into account. As a fourth strategy, we apply a pre-test for constant versus changing persistence proposed by Sibbertsen and Kruse (2009). Depending on the test decision either the first or the second strategy will be applied. As the pre-test may give some wrong decisions due to non-zero type I- and type II-errors, this strategy can be seen as a compromise between the first two strategies. The fifth and sixth strategy fit short memory models to the series which are potentially capable to mimic long memory behavior. Namely as a fifth strategy we use an AR(1) process and as the sixth strategy we use an ARMA(2,1) model. A similar model has also been investigated for short term forecasts of long memory time series by Man (2003). For all strategies the forecasts are computed by using rolling windows with different window lengths and for different forecast horizons. It is important to note that this paper focuses on structural change in the memory of the DGP and thus we do not consider non-linear models in our subsequent analysis.

The paper is organized as follows. After introducing the model and the test briefly in section 2, section 3 describes the forecasting strategies in more detail. Section 4 contains an intensive Monte Carlo study showing the forecasting behavior of the different strategies. Section 5 provides a short empirical application of the afore studied strategies before section 6 concludes.

2 Model and Test

As a starting point for our forecasting study consider the simple ARFIMA $(0,d,0)$ process as introduced by Granger and Joyeux (1980)

$$(1-B)^d X_t = \varepsilon_t, \quad \text{for} \quad t = 1,\ldots,T,$$

where B denotes the usual backshift operator $B^k X_t \equiv X_{t-k}$ and ε_t is a martingale difference sequence. In order to avoid misinterpretations such as a change in an autoregressive parameter we omit any autoregressive and moving average terms here and concentrate only on the ARFIMA $(0,d,0)$ case. This is also in line with the test of Sibbertsen and Kruse (2009) which is constructed for the ARFIMA $(0,d,0)$ setup. In this model framework they test the hypothesis

$$H_0: d = d_0 \quad \text{for} \quad t = 1,\ldots,T,$$

where we assume $0.5 < d_0 < 1.5$ and test this against the following alternative

$$H_1 : \begin{cases} d = d_1 & \text{for} \quad t = 1, \ldots, [\tau T] \\ d = d_2 & \text{for} \quad t = [\tau T]+1, \ldots, T \end{cases}$$

where $0 \leq d_1 < 0.5$ and $0.5 < d_2 < 1.5$ and d_1 and d_2 can be interchanged. We denote by $[x]$ the biggest integer smaller than x and denote the relative breakpoint as $\tau \in \Lambda$ assuming $\Lambda \subset (0,1)$.Therefore, the alternative contains always a break from either the stationary to the non-stationary region or vice versa. The test statistic is based on a CUSUM of squares type test introduced by Leybourne et al. (2007) and is given by

$$R = \frac{\inf_{\tau \in \Lambda} K^f(\tau)}{\inf_{\tau \in \Lambda} K^r(\tau)} \tag{1}$$

with

$$K^f(\tau) = [\tau T]^{-2d_0} \sum_{t=1}^{[\tau T]} \widehat{v}_{t,\tau}^2$$

and

$$K^r(\tau) = (T - [\tau T])^{-2d_0} \sum_{t=1}^{T-[\tau T]} \tilde{v}_{t,\tau}^2.$$

Here, $\widehat{v}_{t,\tau}$ is the residual from the OLS regression of X_t on a constant $z_t = 1 \; \forall t$ based on the observations up to $[\tau T]$. This is

$$\widehat{v}_{t,\tau} = X_t - \bar{X}(\tau)$$

with $\bar{X}(\tau) = [\tau T]^{-1} \sum_{t=1}^{[\tau T]} X_t$. Similarly $\tilde{v}_{t,\tau}$ is defined for the reversed series $y_t = X_{T-t+1}$. Thus, it is given by

$$\tilde{v}_{t,\tau} = y_t - \bar{y}(1-\tau)$$

with $\bar{y}(1-\tau) = (T - [\tau T])^{-1} \sum_{t=1}^{T-[\tau T]} y_t$. The test can also be used when there are linear trends in the data by setting $z_t = [1, \; t]'$, which corresponds to linear de-trending. Sibbertsen and Kruse (2009) derive the limiting distribution of this test statistic for $0.5 < d_0 < 1.5$ and show that it depends on d_0. They provide response curves in order to compute critical values for different hypothetical memory parameters d_0. Moreover, the limiting distribution of this test statistic is degenerated in the sense that R converges in probability to 1 if $0 \leq d_0 < 0.5$. As a result, the test behaves conservatively in this parameter region and rejects the null hypothesis with asymptotic probability of zero. In order to avoid this problem, the time series X_t is integrated one if d_0 lies in the problematic region. The test is then simply applied to $Y_t := \sum_{i=1}^{t} X_t$. Although d_0 is unknown and has to be estimated consistently, this approach provides a satisfying test performance under H_0, see Sibbertsen and Kruse (2009). The authors also prove consistency of simple estimators for the breakpoint which we will use as well in our simulation study. These are given by

$$\widehat{\tau}^f = \arg \inf_{\tau \in \Lambda} K^f(\tau)$$

for increasing persistence and for the case of decreasing persistence by

$$\widehat{\tau}^r = \arg \inf_{\tau \in \Lambda} K^r(\tau) \; .$$

It is important to note that for the purpose of the simulation study in section 4 we use a broader definition of a change in persistence than formulated in the above alternative. We consider a break in persistence as a change in the order of integration d, i.e. the memory parameter, without restricting ourselves only to changes between stationary and non-stationary long memory or vice versa: $d_1 \neq d_2$ with $d_1, d_2 \in [0, 1.5]$.

3 Forecasting strategies

In our forecasting study we compare the following six different forecasting strategies:

Strategy 1: We do not take any possible change in persistence into account and estimate the memory parameter using the whole in-sample period. This means that we work under the null hypothesis of the test for breaks in persistence without employing the test. This strategy serves as a benchmark for the other approaches.

Strategy 2: It is always assumed that there is a break in the persistence and that the direction of a change is known. The breakpoint is estimated and according to this estimate the in-sample period is split in two parts. These two parts are used to estimate the memory parameters d_1 and d_2 in the pre- and the post-break period. This means that we work under the alternative of the test for breaks in persistence without employing the test.

Strategy 3: The memory parameter and the forecast coefficients are re-estimated in each step of the forecast.

Strategy 4: First, we apply the pre-test for a break in persistence and depending on the outcome we use either strategy 1 (non-rejection of the null) or strategy two (rejection of the null).

Strategy 5: As a fifth competitor in the forecast comparison we fit an AR(1) process to the data and perform the forecasts with this approximation.

Strategy 6: A different kind of approximation is done as a sixth strategy. We fit an ARMA(2,1) process and perform forecasts using this approximation. This is done since the sum of two AR(1) processes is an ARMA(2,1) process and the aggregation of stationary series can lead to a process that is able to show long memory patterns (see e.g. Granger, 1980 and Granger and Ding, 1996). All forecasts are done by a rolling window strategy with different window lengths as detailed in the next section. Further approximation schemes such as using automatic lag selection criterions for determining the lag length are thinkable but as was confirmed by simulation the consistent Schwarz information criterion (SIC) (Schwarz, 1978) chooses relatively few lags, usually between two and four, under the data generating process detailed in section 4. Thus, by using an ARMA(2,1) process strategy 6 captures such an approach implicitly but more parsimoniously.

The aim of this paper is to assess the effects of a break in persistence on the forecast performance of an ARFIMA process. Therefore, we work mainly under the alternative of the afore mentioned test and consider situations in which the persistence of the time series actually changes. However, for the sake of completeness we also perform simulations with a constant long memory parameter d and thus we also study the results under the null of the test proposed by Sibbertsen and Kruse (2009). As it is common practice, we compute the forecasts by using the autoregressive approximation of the long memory process. The advantage of this approximation approach is that it is also feasible when the process is non-stationary as we only assume invertibility of the process which requires $d > -0.5$ (e.g. Beran, 1995). Assuming invertibility we can rewrite the general ARFIMA (p,d,q) process as

$$\Theta(B)^{-1}\Phi(B)(1-B)^d X_t = \varepsilon_t \; .$$

or equivalently as

$$\Delta(B)X_t = \varepsilon_t \; .$$

with $\Delta(B) := \Theta(B)^{-1}\Phi(B)(1-B)^d$, $\Theta(B)^{-1} = (1-\theta_1 B - \theta_2 B^2 - \ldots - \theta_q B^q)^{-1}$ and $\Phi(B) = (1-\phi_1 B - \phi_2 B^2 - \ldots - \phi_p B^p)$. Since we exclude the moving-average component we have $\Theta(B)^{-1} = 1$ and the autoregressive coefficients can therefore be obtained as the coefficients of B^k in the expansion of

$$\Delta(B) = \Phi(B)(1-B)^d \; .$$

By its expansion as a Maclaurin series the fractional differencing operator $(1-B)^d$ can be written as

$$(1-B)^d = \sum_{k=0}^{\infty} \pi_k B^k$$

with

$$\pi_k = \frac{k-1-d}{k}\pi_{k-1} \quad \text{and} \quad \pi_0 = 1 \; .$$

Let $\overset{z}{\rightsquigarrow}$ denote the z transformation. For $\Phi(B)$ and $\sum_{k=0}^{\infty} \pi_k B^k$ we obtain

$$\Phi(B) \overset{z}{\rightsquigarrow} (1-\phi_1 z - \phi_2 z^2 - \ldots - \phi_p z^p)$$

and

$$\sum_{k=0}^{\infty} \pi_k B^k \overset{z}{\rightsquigarrow} \sum_{k=0}^{\infty} \pi_k z^k \; .$$

Multiplying these two polynomials yields

$$\begin{aligned} \Delta(z) &= (1-\phi_1 z - \phi_2 z^2 - \ldots - \phi_p z^p)(\pi_0 z^0 + \pi_1 z^1 + \pi_2 z^2 + \pi_3 z^3 + \pi_4 z^4 + \ldots) \\ &= \sum_{k=0}^{\infty} \pi_k z^k - \sum_{k=0}^{\infty} \phi_1 \pi_k z^{k+1} - \sum_{k=0}^{\infty} \phi_2 \pi_k z^{k+2} - \ldots - \sum_{k=0}^{\infty} \phi_p \pi_k z^{k+p} \; . \end{aligned}$$

Transforming the indices we have

$$\begin{aligned} \Delta(z) &= \sum_{k=0}^{\infty} \pi_k z^k - \sum_{k=1}^{\infty} \phi_1 \pi_{k-1} z^k - \sum_{k=2}^{\infty} \phi_2 \pi_{k-2} z^k - \ldots - \sum_{k=p}^{\infty} \phi_p \pi_{k-p} z^k \\ &= \sum_{k=0}^{\infty} (\pi_k - \phi_1 \pi_{k-1} - \phi_2 \pi_{k-2} - \ldots - \phi_p \pi_{k-p}) z^k \; . \end{aligned}$$

Consequently, reversion of the z - transformation leads to

$$\begin{aligned} \Delta(B) &= \sum_{k=0}^{\infty} \underbrace{(\pi_k - \phi_1 \pi_{k-1} - \phi_2 \pi_{k-2} - \ldots - \phi_p \pi_{k-p})}_{\equiv \delta_k} B^k \\ &= \sum_{k=0}^{\infty} \delta_k B^k \; . \end{aligned} \tag{2}$$

The forecast coefficients for the special case of an ARFIMA $(0,d,0)$ process are then computed by setting $p = q = 0$ which leads to the recursion (e.g. Boutahar, 2007)

$$\pi_j = \frac{j-1-d}{j}\pi_{j-1}$$

with $\pi_0 := -d$. The forecast equation for the h-step forecast is given by (e.g. Bhansali and Kokoszka, 2002)

$$\widehat{X}_{T+h} = -\sum_{j=1}^{h-1} \widehat{\pi}_j \widehat{X}_{h-j} - \sum_{j=h}^{k} \widehat{\pi}_j X_{T+h-j}. \tag{3}$$

Here, $\widehat{\pi}_j$ denotes the estimated forecast coefficient from the estimated underlying model and k is the number of lags after which the autoregressive approximation is truncated. We use different values of k for our simulations. Table 1 shows the forecast coefficients for the ARFIMA $(0,d,0)$ case. The last column shows the lag after which $|\pi_j| \leq$ 1e-04. Such a truncation order is in line with Bhardwaj and Swanson (2006).

— Table 1 here —

Whereas strategies 1 and 4, if the test does not reject the null, use the standard ARFIMA $(0,d,0)$ model we need to specify a change in persistence model for strategies 2 and 4 if the test does reject the null. This is given by

$$\begin{aligned}\widehat{X}_{T+h} &= \left(-\sum_{j=1}^{h-1}\widehat{\pi}_j^{(1)}\widehat{X}_{h-j} - \sum_{j=h}^{k}\widehat{\pi}_j^{(1)}X_{T+h-j}\right)1_{t\leq[\widehat{\tau}T]} \\ &+ \left(-\sum_{j=1}^{h-1}\widehat{\pi}_j^{(2)}\widehat{X}_{h-j} - \sum_{j=h}^{k}\widehat{\pi}_j^{(2)}X_{T+h-j}\right)\left(1-1_{t\leq[\widehat{\tau}T]}\right),\end{aligned} \tag{4}$$

where $\widehat{\tau}$ is the estimated relative breakpoint and $\widehat{\pi}_j^{(1)}$ and $\widehat{\pi}_j^{(2)}$ denote the forecast coefficients based on the estimated memory parameter before and after the estimated breakpoint.

4 Monte Carlo study

We start this section with details about the computational aspects of our Monte Carlo study. All simulations are computed in the statistical programming language R (2009). The number of replications is set to $M = 2000$ for each experiment and we consider sample sizes of $T = 700$ which we divide into an in-sample period of $T^* = 500$. The remaining 200 data points are used to generate forecasts as detailed above. In order to reduce the impact of starting values we simulate $200+T$ observations and discard the first 200 data points. The DGP under constant persistence is an ARFIMA $(0,d,0)$ model whose innovations are specified either as a white noise process or as a GARCH (1, 1) process:

$$(1-B)^d X_t = \varepsilon_t, \quad \text{with} \quad \varepsilon_t \sim N(0,1) \tag{5}$$

$$(1-B)^d X_t = \varepsilon_t, \quad \text{with} \quad \varepsilon_t \sim N(0,h_t) \quad \text{and} \quad h_t = 0.01 + 0.2\varepsilon_{t-1}^2 + 0.75h_{t-1}\,. \tag{6}$$

The reason for considering the first specification is that the white noise assumption is standard in the time series literature although it is well known that it is invalid for most economic or financial time series. The second specification is used to account for the occurrence of an integrated and nearly integrated GARCH behavior of many financial time series such as stock returns, exchange rates or interest rates (for a survey see Bollerslev et al., 1992). In addition to this well known property of financial time series Baillie et al. (1996) provide evidence of a long memory property of the daily returns of the nominal Deutschmark - U.S. dollar spot exchange rate. The data generating process above combines these two features of financial time series and we add the property of a changing persistence. In fact the chosen GARCH parameters are very close to the findings of Engle and Bollerslev (1986) for weekly U.S. dollar - Swiss franc exchange rates. It is important to note that the test for changing persistence suggested by Sibbertsen and Kruse (2009) maintains satisfactory size and power properties under such a data generating process and that the breakpoint estimator is also unaffected by GARCH disturbances.[2]

The long memory parameter d takes a broad range of values covering the stationary and non-stationary region and we consider changes from non-stationary to stationary long memory and vice versa as well as within the stationary and non-stationary region. We estimate the parameter d via log-periodogramm regression due to Geweke and Porter-Hudak (1983) with a rate of frequencies of $o(T^{0.8})$ which was shown to be MSE-optimal by Deo et al. (1998). Note that the performance of this estimator is unaffected by GARCH effects as shown by Cheung (1993). Under the alternative of changing persistence we consider the following DGPs:

$$(1-B)^{d_1}X_t = \varepsilon_t,\ t = 1,\ldots,[\tau T] \tag{7}$$

$$(1-B)^{d_2}X_t = \varepsilon_t,\ t = [\tau T]+1,\ldots,T \tag{8}$$

where we use the same specifications for the error variance as before. As relative breakpoints within the in-sample period we use $\tau = 0.3$, $\tau = 0.5$ and $\tau = 0.7$ corresponding to the beginning, the middle and the end of the in-sample period respectively. The forecasts are computed by using a rolling window forecasting scheme. We do not consider an expanding window forecasting scheme because of the simulation results in Pesaran and Timmermann (2005). They showed that a rolling window scheme produces superior forecasts compared to the expanding window scheme when a break in the process parameters is present. The decision about the length of the rolling window depends on the size of the break. Short windows work best for large breaks and long windows work best for small breaks. In

[2]Results are not reported in order save space, but they are fully available from the authors upon request.

order to deal with these findings we use different windows of length 500, 250 and 100. These different lengths should also serve to answer the question whether more information improve the forecasts or not and thus dealing with expanding windows implicitly. Smaller window lengths than 100 have not been considered as they do not seem reasonable when dealing with highly persistent time series (see Table 1).

We compare the forecasting strategies by employing the Diebold and Mariano (1995) test for predictive accuracy pairwise to our strategies. As loss function we use the quadratic MSE loss function. In order to avoid potential size distortions due to small samples and multi-step ahead forecasts we use a modified version of the Diebold Mariano test proposed by Harvey et al. (1997). As an estimator for the long run variance of the loss differential we use a heteroscedasticity and autocorrelation robust covariance estimator with VAR(1) pre-whitening and Bartlett kernel as described in Newey and West (1987, 1994). For all tests the level of significance is set to $\alpha = 0.05$. We conduct the simulations for de-meaned data only since preliminary results show virtually no difference for the de-meaned case and the de-trended case. For the multi-step ahead predictions we use 1-step, 5-step, 10-step and 20-step ahead forecasts.

Tables 2 to 8 show subsets of the simulation results.[3] Note that despite having tested each strategy pairwise we only report a subset which shows the overall effect of a break in persistence sufficiently. We also restrict our discussion to the case of a window length of 500 and white noise innovations. The results for smaller windows of 250 and 100 observations or GARCH(1,1) innovations lead to the same overall conclusions. Each cell shows the percentage of how often the strategy in the row dominated the strategy in the column significantly at the 5% level. We will start the discussion with the results under the null of the persistence change test from section 2 and look briefly at simulation results under a constant memory parameter d. The results are shown in Table 2.

— Table 2 here —

At a first glance no real difference is evident between strategies 1 to 4. A striking feature is that strategies 5 and 6 are clearly dominated by the other four. This effect slightly reduces for multi-step ahead predictions. Another factor that reduces the dominance of the strategies 1 to 4 seems to be the value

[3]The full set of simulation results is available from the authors on request.

of the long-memory parameter d. If $0 < d < 1$, the dominance is more pronounced compared to the cases where we have pure white noise (or GARCH) ($d = 0$) or a random walk ($d = 1$). Interestingly, strategies 2 and 4 are not dominated by strategy 1 which is the true data generating process. This is due to the fact that the GPH estimator before and after the break, for strategy 2, is roughly the same and thus we do not induce large errors. A similar argument holds for strategy 4. The test for a change in persistence seems to be a good guidance whether to accommodate the persistence change in the forecasting formula or not.

We conclude that standard short memory models are not capable to mimic long memory patterns as good as indicated from theoretical considerations for short term forecasts. For long term forecasts the differences between the six strategies become smaller but we conjecture that this is more a weakness of the forecasting strategies 1 to 4 inherent in long term forecasting than a strength of the strategies 5 and 6. In addition, the presence of nearly integrated GARCH innovations does not seem to change the results when forecasting the conditional mean. When we turn to the case of a change in persistence we first discuss the results for the increasing persistence. The results are shown in Tables 3 to 5.

— Table 3 here —

— Table 4 here —

— Table 5 here —

Clearly the benchmark strategy 1 is unable to dominate one of the strategies 2 to 4 considerably. A notable exception is the clear dominance over the strategies 5 and 6 regardless of the value of τ or the forecast horizon h. Once again, this dominance is reduced as the forecast horizon expands and for the break from $d_1 = 0$ to $d_2 = 1$. Importantly, we find in some settings strategy 1 to be not as inferior as one might expect under a change in persistence. Strategy 1 dominates strategy 2 in some settings over 20% of the time. Especially when the break is small and around the value of 0.5 such as $d_1 = 0.4$ to $d_2 = 0.6$. Recall that in the case where $\widehat{d_0} < 0.5$ holds, the time series X_t is integrated once and vice versa. Although this rule provides a correctly sized test, it may cause some unexpected results under the alternative. For a detailed discussion see Sibbertsen and Kruse (2009). The correct estimation of the breakpoint is a crucial premise for the model in (4) to yield good forecasts and, thus, wrongly dated breakpoints may lead to inferior forecasts of strategy 2 and 4 compared to the benchmark (for a related discussion see Dacco and Satchell, 1999).

In general for the case of an increasing persistence it is hard to identify a clear winner of the forecast competition. One result is that the short memory models are unable to produce satisfactory forecasts even for the short term. This result is also found in the case of a constant d. Moreover, the results indicate that incorporating the persistence change in the forecast model is more important for a late and large break in the memory parameter. For increasing persistence we find that the specification of the error term does not change the main findings. These findings partly change and partly become more severe when we turn to the case of a decreasing persistence in Tables 6 to 8.

— Table 6 here —

— Table 7 here —

— Table 8 here —

A first glance reveals that also in this setting strategies 5 and 6 are inferior to the other competitors and the remaining results for these two strategies do not change. An important finding in the case of a decreasing persistence is that the strategies which account for a break in persistence explicitly, namely strategies 2 and 4, are clearly superior to the naive benchmark strategy 1. Comparing strategies 2 and 4 one can see that strategy 2 that always fits a breakpoint model is even better than strategy 4. This is due to the test prior to the decision which model to use as the test gives some wrong decisions due to non-zero type I- and type II-errors. The flexible strategy 3 is also much better suited to deal with a changing persistence than strategy 1. However, strategy 3 is unable to dominate strategy 2 or 4 despite its flexibility. Strategies 2 and 4 in turn are often very well able to deliver better forecasts than strategy 3. A notable exception is the case of a large break and multi-step ahead forecasts. In such a setting strategy 3 outperforms strategy 4. This is due to the flexibility of the strategy to capture some degree of structural change by coefficient updating. Strategy 3 estimates the forecast coefficients in each step and thus can reduce the variance inflation in multi-step ahead forecasts which leads to more accurate forecasts. The results for the specification of the error variance do not change.

In the case of a decreasing persistence strategies dealing explicitly with a changing persistence deliver in most of the cases superior forecasts. For relatively large breaks in persistence strategy 4 outperforms the other contestants in often more than 50% of the time. The tendency here is, in contrast to an increase in persistence, that the relative superiority weakens the later the breakpoint lies in the sample and thus accounting for the changing persistence is more important the earlier the break occurs. Although the results for strategy 4 versus strategy 1 show a similar behavior as in the situation with an increasing persistence the overall impression implies the result that in a situation in which the persistence of a time series decreases the allowance for a changing persistence in the forecast formula becomes more important. Testing for such a change seems to be advisable as indicated by the good performance of strategy 4. The importance of these results for a change in persistence is emphasized by empirical findings that long memory is subject to structural breaks in several economic and financial time series.

One major difference between the case of increasing and decreasing persistence should be noted. In the case of an increase of persistence accounting for this change improves the forecast the larger the break and the *later* the breakpoint lies within the in-sample period. Given a situation with a decreasing persistence, forecasts using equation (4) improve the larger the break is and the *earlier* the break occurs. A possible explanation for this asymmetric outcome is the mixture of two phenomena. First, there is an overassessment of the low (high) frequencies of the time series depending on the fraction at which the break occurs which influences the log-periodogram regression heavily. Second, the problems are induced by the infinite variance of non-stationary time series.

We first discuss the scenario with an increasing persistence. We focus our discussion on the behavior of strategies 1 and 2 since they exhibit the feature we would like to assess, namely the changing persistence. In the case of an increasing persistence one can say that the later the breakpoint the more important is the allowance for the persistence change. In this case the later the breakpoint lies the more high frequencies are present induced by the relative large fraction of lower persistent data. This leads to an estimated d which is lower than appropriate for the out-of-sample period when the estimation is based on the whole in-sample period. Because the forecast coefficients depended solely on d such an approach will result in inferior forecasts as supported by our results. Thus, the later the break occurs the poorer are the estimated forecast coefficients resulting in inferior forecasts if one does not account for the change in persistence. On the contrary, if we estimate the breakpoint split the in-sample period and fit a breakpoint model such as (4) we will obtain more precise forecast coefficients and consequently better forecasts.

When we turn to the case of a decreasing persistence we observe that the earlier the break occurs the more important becomes accounting for the changing persistence. The argument follows similar lines as above. The later the break occurs the more biased is the estimate of d because of the disturbances caused by the very low frequencies of the highly persistent data. In particular the estimated forecast coefficients $\widehat{\pi}_j$ are inappropriately small and decay at a too high rate for the out-of-sample period because the long memory parameter d is overestimated in this situation. Hence, the application of a breakpoint model leads to more accurate coefficients and as a result to better forecasts.

This explains also why strategy 2 outperforms strategy 1 so clearly when we have a decreasing persistence compared to the case of an increasing persistence where both strategies behave rather similar. In the case of a decreasing persistence the overall memory parameter is in the non-stationary region in most situations leading to an infinite variance. On the contrary, the memory parameter after the break, which drives the forecasting coefficients, is in the stationary region implying a finite variance. As the true series is stationary with a finite variance the stationary model from strategy 2 leads to clearly superior forecasts. In the case of an increasing persistence this effect is minor as both memory parameters are in the non-stationary region with an infinite variance and so is the true process. Simulation evidence for this explanation is reported in Table 9 for selected values of d.

— Table 9 here —

The results clearly show that the low frequencies from the part of the process with higher persistence influence the estimation of the d parameter disproportionately and from Table 1 we see that even small changes in d lead to different coefficients. The use of less frequencies such as $m = o(T^{0.5})$ reduces the bias of the estimate but this is usually more than offset by the inflation of the variance of the estimator as indicated by the results of Hurvich and Beltrao (1994) and Deo et al. (1998). Also the use of modified estimators such as smoothed periodogram regression as proposed by Reisen (1994) or the approximate maximum likelihood estimator proposed by Beran (1995) do not change the general conclusion drawn from Table 9. This was confirmed via simulation but unreported to save space. Hence, such approaches lead to similar problems as above.

5 Empirical Illustration

The data set we use is the monthly US price inflation series from Stock and Watson (2005).[4] In particular we consider the first difference of the logarithmic implied price deflator for durable goods. This series has also been under investigation from Cavaliere and Taylor (2008) who found a change in persistence from $I(0)$ to $I(1)$. However, they did not consider the possibility of fractional integration in the series although inflation related time series are likely to show long memory behavior (see e.g. Hassler and Wolters, 1995).

The data spans from January 1959 to December 2003 which amounts to a sample size of $T = 538$. We split the data into an in-sample period from January 1959 to July 1995 ($T^* = 438$) which leaves an out-of-sample period of size $T = 100$ for computing forecasts (the light gray shaded area in figure 1). The series is depicted in figure 1.

— Figure 1 here —

As the out-of-sample period is rather small we reduce the maximal forecast horizon to $h = 10$ in order to obtain a reasonable amount of forecast errors. However, to get an impression about how the forecast performance changes with the forecast horizon we also include the intermediate steps and calculate forecasts for the horizons $h = 1, 2, 3, \ldots, 8, 9, 10$. Applying the persistence change test described in section 2 to empirical data we have do consider the possibility of short run correlations. To deal with this we estimate the long memory parameter d and the autoregressive parameters jointly by the approximate maximum likelihood estimator of Beran (1995) an choose the lag length by AIC. The estimation results are shown in Table 10.

— Table 10 here —

Performing the persistence change test we obtain a test statistic of $R = 0.039$ which is significant on the 5% level of significance for an increase in persistence as the critical value is 0.066. The estimated breakpoint is $[\widehat{\tau}T] = 100$ and is depicted as the dashed line in figure 1. The estimated long memory parameter d until this breakpoint is 0.176 and from the breakpoint until the end of the in-sample period

[4]The data can be downloaded from Mark Watson's website at: http://www.princeton.edu/∼mwatson/wp.html.

d is estimated as 0.561 which is a moderate incline in persistence from stationary to non-stationary long memory. This result is consistent with the results of Cavaliere and Taylor (2008) who found a change from $I(0)$ to $I(1)$ but without considering fractional integration. In order to deal with the short run correlations explicitly in the forecasting comparison we use the autoregressive approximation of the ARFIMA$(2, d, 0)$ process for computing the forecasts. The respective coefficients are obtained as detailed above in section 3. We measure the forecast performance by the standard measures mean squared forecast error (MSFE) and mean absolute forecast error (MAFE). The empirical results are shown in Table 11.

— Table 11 here —

Judged by these results the forecast performance of strategy 2 that includes the persistence change performs the best from one-step up to seven-step ahead forecasts (implying over half a year for monthly data). For longer forecast horizons strategy 2 performs clearly better than its competitors second only to strategy 6 the ARMA(2,1) approximation approach. These empirical findings support the results from the simulation study and show that the conclusions drawn above hold true even for more complicated data generating processes and empirical data. It is also noteworthy that dealing with the persistence change improves the forecast performance even though the change in persistence is only moderate (but never the less statistically significant).

6 Conclusion

In this paper we study the effect of a break in persistence on the forecast precision of long memory time series. We provide simulation evidence that accounting for such a break can improve forecasts significantly. This is somewhat surprising given that the change in the forecast coefficients depending on a change in the long memory parameter d is rather modest. We find a different behavior of the forecasts depending on whether the persistence increases or decreases due to an possible overweighing of the low periodogram frequencies and to variance effects. We conclude that accounting for a break in persistence is more important the larger the break is and the earlier (decrease in persistence) or the later (increase in persistence) the breakpoint lies. As a general result we draw the conclusion that testing for a break in persistence and embedding it in the forecast formula is often highly recommendable.

This becomes even more important when the persistence decreases. In an empirical application to US inflation data we show that these results carry over to empirical settings with more complex data generating processes and only moderate persistence changes. A further, more general, result is that standard short memory models do not yield satisfactory forecasts even for the short term when there is true long memory, i.e. $0 < d < 1$. This result strongly underlines the importance of accounting for long-range dependence in time series data even when the intention of the analysis is only a one-step ahead prediction.

References

Baillie, R.T. (1996): "Long memory processes and fractional integration in econometrics." *Journal of Econometrics* 73, 5 – 59.

Baillie, R.T., Bollerslev, T., Mikkelsen, H.O.(1996): "Fractionally integrated generalized autoregressive conditional heteroskedasticity." *Journal of Econometrics* 74, 3 – 30.

Banerjee, A., Lumsdaine, R. and Stock, J. (1992): "Recursive and sequential tests of the unit root and trend break hypothesis: theory and international evidence." *Journal of Business & Economic Statistics* 10, 271 – 288.

Beran, J. (1995): "Maximum likelihood estimation of the differencing parameter for invertible short and long memory autoregressive integrated moving average models." *Journal of the Royal Statistical Society. Series B* 57, 659 – 672.

Beran, J., Terrin, N. (1996): "Testing for a Change of the Long-Memory Parameter." *Biometrika* 83, 627 – 638.

Bhansali, R.J., Kokoszka, P.S. (2002): "Computation of the forecast coefficients for multistep prediction of long-range dependent time series." *Journal of Forecasting* 18, 181 – 206.

Bhardwaj, G., Swanson, N.R. (2006): "An empirical investigation of the usefulness of ARFIMA models for predicting macroeconomic and financial time series." *Journal of Econometrics* 131, 539 – 578.

Bos, C.S., Franses, P.H., Ooms, M. (2002): "Inflation, forecast intervals and long memory regression models." *International Journal of Forecasting* 18, 243 – 264.

Bollerslev, T., Chou, R.Y., Kroner, K.F. (1992): "ARCH modeling in finance - A review of the theory and empirical evidence." *Journal of Econometrics* 52, 5 – 59.

Boutahar, M. (2007): "Optimal prediction with nonstationary ARFIMA model." *Journal of Forecasting* 26, 95 – 111.

Busetti, F., Taylor, A.M.R. (2004): "Tests of stationarity against a change in persistence." *Journal of Econometrics* 123, 33 – 66.

Cavaliere, G., Taylor, A.M.R. (2008): "Testing for a change in persistence in the presence of non-stationary volatility." *Journal of Econometrics* 147, 84 – 98.

Cheung, Y.W. (1993): "Tests for fractional integration: A monte carlo investigation." *Journal of Time Series Analysis* 14, 331 – 345.

Dacco, R., Satchell, S. (1999): "Why do regime-switching models forecast so badly?" *Journal of Forecasting* 18, 1 – 16.

Deo, R., Hurvich, C.M., Brodsky, J. (1998): "The mean squared error of Geweke and Porter-Hudaks estimator of the memory parameter of a long-memory time series." *Journal of Time Series Analysis* 19, 19 – 46.

Diebold, F.X., Mariano, R.S. (1995): "Comparing predictive accuracy." *Journal of Business & Economic Statistics* 13, 253 – 263.

Engle, R.F., Bollerslev, T. (1986): "Modeling the persistence of conditional variances." *Econometric Reviews* 5, 1 – 50.

Geweke, J., Porter-Hudak, S. (1983): "The estimation and application of long-memory time series models." *Journal of Time Series Analysis* 4, 221 – 238.

Granger, C.W.J., Joyeux, R. (1980): "An introduction to long-range time series models and fractional differencing." *Journal of Time Series Analysis* 1, 15 – 30.

Granger, C.W.J. (1980): "Long memory relationships and the aggregation of dynamic models." *Journal of Econometrics* 14, 227 – 238.

Granger, C.W.J., Ding, Z. (1996): "Varieties of long memory models." *Journal of Econometrics* 73, 61 – 77.

Harvey, D., Leybourne, S., Newbold, P. (1997): "Testing the equality of prediction mean squared errors." *International Journal of Forecasting* 13, 281 – 291.

Hassler, U., Nautz, D. (2008): "On the persistence of the Eonia spread." *Economics Letters* 10, 184 – 187.

Hassler, U., Wolters, J. (1995): "Long memory in inflation rates: International evidence." *Journal of Business and Economic Statistics* 13, 37 – 45.

Horváth, L. (2001): "Change-point detection in long-memory processes." *Journal of Multivariate Analysis* 78, 218 - 234.

Hurvich, C.M., Beltrao, K.I. (1994): "Automatic semiparametric estimation of the memory parameter of a long-memory time series." *Journal of Time Series Analysis* 15, 285 – 302.

Kim, J. (2000): "Detection of change in persistence of a linear time series." *Journal of Econometrics* 95, 97 – 116.

Kim, J., Belaire Franch, J. and Badilli Amador, R. (2002): "Corrigendum to "Detection of change in persistence of a linear time series". "*Journal of Econometrics* 109, 389 – 392.

Kumar, M.S., Okimoto, T. (2007): "Dynamics of persistence in international inflation rates." *Journal of Money, Credit and Banking* 39, 1457 – 1479.

Lavancier, F., Philippe, A., Surgailis, D. (2009): "A two-sample test for comparison of long memory parameters." *mimeo*.

Leybourne, S., Kim, T., Smith, V. and Newbold, P. (2003): "Tests for a change in persistence against the null of difference stationarity." *Econometrics Journal* 6, 291 – 311.

Leybourne, S., Taylor, A.M.R. and Kim, T. (2007): "CUSUM of squares - based tests for a change in persistence." *Journal of Time Series Analysis* 28, 408 – 433.

Man, K.S. (2003): "Long memory time series and short term forecasts." *International Journal of Forecasting* 19, 477 – 491.

Newey, W.K., West, K.D. (1987): "A simple, positive semi-definite, heteroskedasticity and autocorrelation consistent covariance matrix." *Econometrica* 55, 703 – 708.

Newey, W.K., West, K.D. (1994): "Automatic lag selection in covariance matrix estimation." *The Review of Economic Studies* 61, 631 – 653.

Pesaran, M.H., Timmermann, A. (2005): "Small sample properties of forecasts from autoregressive models under structural breaks." *Journal of Econometrics* 129, 183 – 217.

R Development Core Team (2009): "R: A language and environment for statistical computing." *Available at www.r-project.org.*

Ray, B.K., Tsay, R.S. (2002): "Bayesian Methods for Change-Point Detection in Long-Range Dependent Processes." *Journal of Time Series Analysis* 23, 687 – 706.

Reisen, V.A. (1994): "Estimation of the fractional difference parameter in the ARIMA(p,d,q) model using the smoothed periodogram." *Journal of Time Series Analysis* 15, 335 – 350.

Schwarz, G. (1978): "Estimating the dimension of a model." *The Annals of Statistics* 6, 461 – 464.

Sibbertsen, P., Kruse, R. (2009): "Testing for a change in persistence under long-range dependencies." *Journal of Time Series Analysis* 30, 263 – 285.

Stock, J.H., Watson, M.W. (2005): "Implications od dynamic factor models for VAR analysis." *NBER Working Paper 11467*, Available at: http://www.nber.org/papers/w11467.

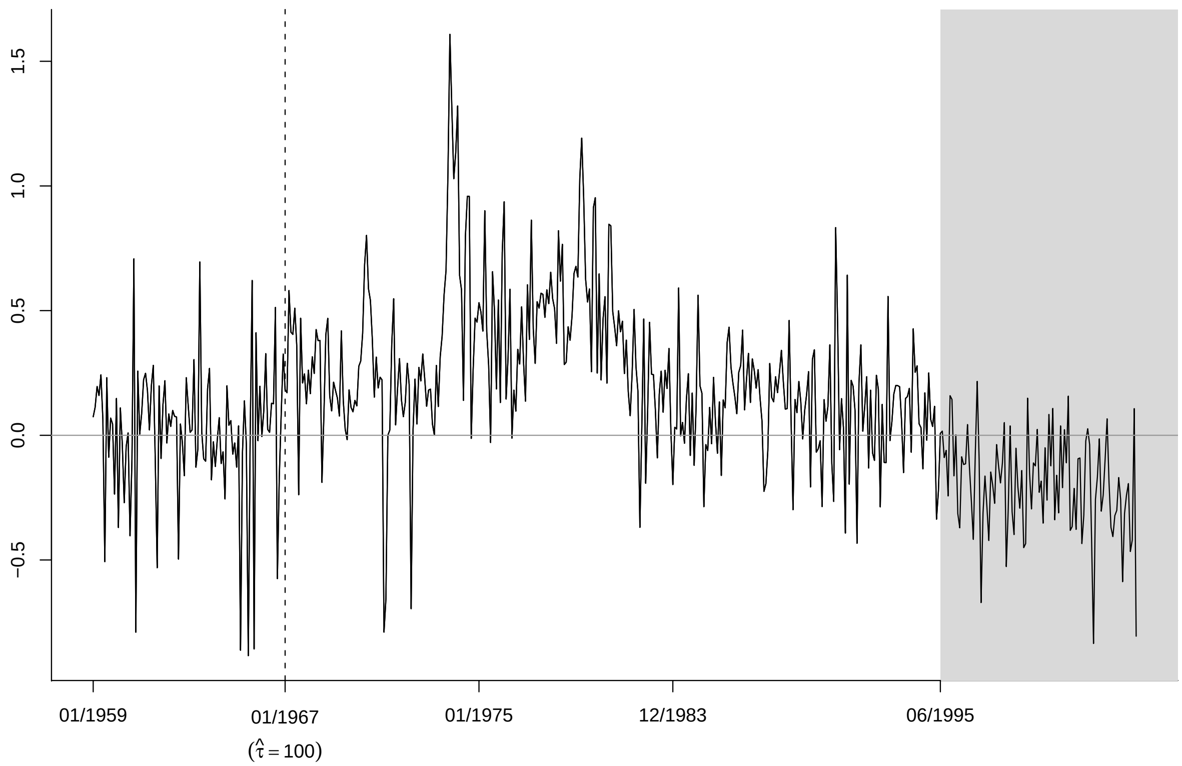

Figure 1: First difference of logarithmic price deflator for durable goods.

d	$j = 5$	$j = 10$	$j = 20$	$j = 25$	$j = 50$	$j = 75$	$j = 100$	$\lvert\pi_j\rvert \leq$ 1e-04
0.10	-0.0161	-0.0075	-0.0035	-0.0027	-0.0013	-0.0008	-0.0006	502
0.20	-0.0255	-0.0110	-0.0047	-0.0036	-0.0016	-0.0010	-0.0007	496
0.30	-0.0297	-0.0118	-0.0048	-0.0035	-0.0014	-0.0008	-0.0006	387
0.40	-0.0300	-0.0110	-0.0041	-0.0030	-0.0011	-0.0006	-0.0004	281
0.60	-0.0228	-0.0071	-0.0023	-0.0016	-0.0005	-0.0003	-0.0002	139
0.70	-0.0173	-0.0050	-0.0015	-0.0010	-0.0003	-0.0002	-	96
0.80	-0.0113	-0.0030	-0.0008	-0.0005	-0.0002	-	-	63
0.90	-0.0054	-0.0013	-0.0003	-0.0002	-	-	-	37

Table 1: Forecast coefficients of lag order j

h	*d*							
			Strategy 1	Strategy 2	Strategy 3	Strategy 4	Strategy 5	Strategy 6
	0.2	Strategy 1	-	13.65%	8.10%	1.40%	26.20%	93.85%
		Strategy 2	11.10%	-	8.75%	11.80%	24.50%	93.95%
		Strategy 3	7.65%	9.10%	-	8.45%	23.65%	94.15%
		Strategy 4	0.25%	12.80%	7.95%	-	24.95%	93.15%
		Strategy 5	1.95%	2.45%	1.50%	2.90%	-	91.50%
		Strategy 6	0.15%	0.10%	0.00%	0.20%	0.65%	-
			Strategy 1	Strategy 2	Strategy 3	Strategy 4	Strategy 5	Strategy 6
1	0.4	Strategy 1	-	16.50%	6.90%	0.80%	69.50%	99.60%
		Strategy 2	11.70%	-	7.25%	11.85%	68.00%	99.60%
		Strategy 3	7.70%	9.60%	-	8.10%	69.60%	99.65%
		Strategy 4	0.25%	15.90%	6.75%	-	67.55%	99.35%
		Strategy 5	0.10%	0.05%	0.00%	0.30%	-	97.70%
		Strategy 6	0.00%	0.00%	0.00%	0.00%	0.25%	-
			Strategy 1	Strategy 2	Strategy 3	Strategy 4	Strategy 5	Strategy 6
	0.8	Strategy 1	-	22.35%	7.85%	0.80%	55.25%	94.85%
		Strategy 2	12.70%	-	8.00%	12.90%	48.65%	93.65%
		Strategy 3	11.20%	17.95%	-	11.10%	52.50%	94.90%
		Strategy 4	0.45%	21.00%	7.60%	-	52.35%	93.90%
		Strategy 5	0.65%	3.60%	0.10%	0.95%	-	91.90%
		Strategy 6	0.20%	0.45%	0.05%	0.25%	1.65%	-
			Strategy 1	Strategy 2	Strategy 3	Strategy 4	Strategy 5	Strategy 6
	0.2	Strategy 1	-	7.70%	6.20%	0.70%	6.70%	40.00%
		Strategy 2	4.75%	-	6.10%	4.95%	6.55%	39.70%
		Strategy 3	4.90%	5.95%	-	5.20%	5.90%	39.60%
		Strategy 4	0.45%	7.50%	6.00%	-	6.70%	38.65%
		Strategy 5	5.15%	5.15%	4.85%	5.55%	-	31.15%
		Strategy 6	1.70%	1.55%	1.75%	1.80%	3.80%	-
			Strategy 1	Strategy 2	Strategy 3	Strategy 4	Strategy 5	Strategy 6
5	0.4	Strategy 1	-	7.95%	6.15%	0.25%	22.45%	71.20%
		Strategy 2	5.90%	-	6.00%	6.05%	21.65%	71.40%
		Strategy 3	5.50%	5.90%	-	5.40%	22.05%	71.15%
		Strategy 4	0.20%	7.85%	6.35%	-	22.30%	70.30%
		Strategy 5	2.25%	2.30%	2.15%	2.10%	-	49.00%
		Strategy 6	0.00%	0.05%	0.00%	0.05%	1.85%	-
			Strategy 1	Strategy 2	Strategy 3	Strategy 4	Strategy 5	Strategy 6
	0.8	Strategy 1	-	11.50%	6.25%	0.50%	29.30%	60.10%
		Strategy 2	7.55%	-	5.05%	7.75%	26.25%	58.75%
		Strategy 3	7.30%	11.00%	-	7.10%	29.05%	59.35%
		Strategy 4	0.65%	11.10%	6.25%	-	28.25%	58.60%
		Strategy 5	1.30%	2.65%	0.90%	1.40%	-	52.60%
		Strategy 6	0.70%	0.65%	0.25%	0.75%	6.30%	-
			Strategy 1	Strategy 2	Strategy 3	Strategy 4	Strategy 5	Strategy 6
	0.2	Strategy 1	-	11.90%	9.65%	0.95%	10.45%	16.40%
		Strategy 2	10.55%	-	10.80%	10.60%	10.40%	15.65%
		Strategy 3	9.35%	11.65%	-	9.50%	9.90%	16.30%
		Strategy 4	0.70%	11.75%	9.60%	-	10.35%	16.35%
		Strategy 5	12.20%	12.00%	12.15%	12.35%	-	12.05%
		Strategy 6	9.60%	9.60%	9.75%	9.80%	14.35%	-
			Strategy 1	Strategy 2	Strategy 3	Strategy 4	Strategy 5	Strategy 6
20	0.4	Strategy 1	-	10.20%	9.55%	0.40%	13.60%	30.45%
		Strategy 2	10.50%	-	10.75%	10.40%	13.45%	29.95%
		Strategy 3	10.20%	9.45%	-	10.25%	13.75%	30.00%
		Strategy 4	0.45%	10.25%	9.75%	-	13.65%	30.05%
		Strategy 5	9.75%	9.85%	9.50%	9.65%	-	21.30%
		Strategy 6	4.80%	4.45%	4.65%	4.85%	8.85%	-
			Strategy 1	Strategy 2	Strategy 3	Strategy 4	Strategy 5	Strategy 6
	0.8	Strategy 1	-	13.55%	8.20%	0.70%	22.55%	32.45%
		Strategy 2	10.15%	-	10.30%	10.00%	21.10%	31.15%
		Strategy 3	9.70%	12.90%	-	9.90%	22.35%	31.90%
		Strategy 4	1.15%	13.25%	8.55%	-	22.50%	31.80%
		Strategy 5	7.55%	9.05%	7.45%	7.65%	-	24.05%
		Strategy 6	4.35%	5.00%	4.15%	4.55%	14.05%	-

Table 2: Simulation results for a constant d

h	$d_1 \rightarrow d_2$		$\tau = 0.3$					
			Strategy 1	Strategy 2	Strategy 3	Strategy 4	Strategy 5	Strategy 6
	$0.1 \rightarrow 0.9$	Strategy 1	-	16.45%	6.75%	16.35%	31.80%	71.85%
		Strategy 2	16.30%	-	9.55%	0.00%	31.85%	75.15%
		Strategy 3	10.65%	11.50%	-	11.45%	30.65%	72.85%
		Strategy 4	16.30%	0.10%	9.55%	-	31.85%	75.20%
		Strategy 5	3.50%	7.50%	1.10%	7.45%	-	68.25%
		Strategy 6	0.50%	0.70%	0.30%	0.70%	7.55%	-
			Strategy 1	Strategy 2	Strategy 3	Strategy 4	Strategy 5	Strategy 6
1	$0.4 \rightarrow 0.6$	Strategy 1	-	21.90%	9.05%	4.90%	75.55%	98.80%
		Strategy 2	9.50%	-	7.70%	8.40%	71.60%	98.35%
		Strategy 3	9.00%	16.45%	-	11.50%	75.55%	98.90%
		Strategy 4	1.20%	17.00%	8.25%	-	73.80%	98.70%
		Strategy 5	0.15%	1.15%	0.15%	0.40%	-	95.45%
		Strategy 6	0.00%	0.05%	0.00%	0.00%	1.15%	-
			Strategy 1	Strategy 2	Strategy 3	Strategy 4	Strategy 5	Strategy 6
	$0.1 \rightarrow 0.4$	Strategy 1	-	21.70%	8.85%	6.90%	75.95%	98.65%
		Strategy 2	10.75%	-	7.45%	10.70%	65.55%	97.80%
		Strategy 3	9.90%	17.10%	-	10.20%	74.75%	98.75%
		Strategy 4	4.30%	21.85%	6.95%	-	74.40%	98.70%
		Strategy 5	0.00%	1.85%	0.05%	0.00%	-	95.80%
		Strategy 6	0.00%	0.10%	0.00%	0.00%	1.00%	-
			Strategy 1	Strategy 2	Strategy 3	Strategy 4	Strategy 5	Strategy 6
	$0.1 \rightarrow 0.9$	Strategy 1	-	10.90%	5.25%	10.90%	26.20%	23.05%
		Strategy 2	5.95%	-	4.15%	0.00%	22.45%	23.00%
		Strategy 3	5.65%	8.85%	-	8.85%	25.25%	22.30%
		Strategy 4	5.95%	0.00%	4.15%	-	22.40%	23.00%
		Strategy 5	2.40%	6.10%	2.15%	6.10%	-	16.35%
		Strategy 6	1.55%	3.00%	1.20%	3.00%	18.50%	-
			Strategy 1	Strategy 2	Strategy 3	Strategy 4	Strategy 5	Strategy 6
5	$0.4 \rightarrow 0.6$	Strategy 1	-	12.15%	5.75%	2.40%	39.25%	67.30%
		Strategy 2	5.40%	-	4.15%	4.75%	36.00%	67.90%
		Strategy 3	5.65%	10.25%	-	7.55%	39.20%	67.10%
		Strategy 4	1.00%	9.95%	5.75%	-	38.25%	66.85%
		Strategy 5	0.80%	1.80%	0.75%	1.05%	-	48.05%
		Strategy 6	0.05%	0.35%	0.05%	0.15%	6.80%	-
			Strategy 1	Strategy 2	Strategy 3	Strategy 4	Strategy 5	Strategy 6
	$0.1 \rightarrow 0.4$	Strategy 1	-	9.75%	4.70%	3.95%	31.00%	76.10%
		Strategy 2	5.55%	-	4.65%	6.50%	27.55%	74.85%
		Strategy 3	5.95%	8.65%	-	6.85%	30.00%	76.20%
		Strategy 4	3.15%	9.35%	4.35%	-	29.60%	75.30%
		Strategy 5	1.50%	2.30%	1.30%	1.35%	-	53.00%
		Strategy 6	0.05%	0.15%	0.10%	0.05%	1.90%	-
			Strategy 1	Strategy 2	Strategy 3	Strategy 4	Strategy 5	Strategy 6
	$0.1 \rightarrow 0.9$	Strategy 1	-	13.80%	9.20%	13.75%	25.70%	22.10%
		Strategy 2	8.95%	-	8.85%	0.00%	22.40%	19.10%
		Strategy 3	8.95%	12.95%	-	12.90%	24.15%	21.15%
		Strategy 4	8.95%	0.05%	8.95%	-	22.35%	19.15%
		Strategy 5	7.05%	10.55%	6.60%	10.55%	-	12.95%
		Strategy 6	5.85%	9.20%	6.60%	9.15%	22.40%	-
			Strategy 1	Strategy 2	Strategy 3	Strategy 4	Strategy 5	Strategy 6
20	$0.4 \rightarrow 0.6$	Strategy 1	-	13.95%	9.55%	4.20%	23.35%	32.20%
		Strategy 2	7.95%	-	8.85%	6.55%	22.25%	31.45%
		Strategy 3	8.60%	12.90%	-	10.80%	22.95%	31.45%
		Strategy 4	1.70%	10.00%	9.55%	-	22.65%	31.50%
		Strategy 5	6.05%	7.10%	6.10%	6.55%	-	19.25%
		Strategy 6	3.05%	3.90%	3.30%	3.35%	14.20%	-
			Strategy 1	Strategy 2	Strategy 3	Strategy 4	Strategy 5	Strategy 6
	$0.1 \rightarrow 0.4$	Strategy 1	-	13.05%	10.25%	5.90%	22.80%	38.25%
		Strategy 2	8.65%	-	9.75%	8.95%	22.45%	37.00%
		Strategy 3	9.40%	12.00%	-	10.50%	22.25%	38.20%
		Strategy 4	4.50%	12.45%	10.45%	-	22.50%	37.45%
		Strategy 5	7.80%	8.60%	7.85%	8.05%	-	24.45%
		Strategy 6	2.55%	3.05%	2.65%	2.65%	6.80%	-

Table 3: Simulation results for an increase in persistence I

h	$d_1 \rightarrow d_2$	$\tau = 0.5$						
			Strategy 1	Strategy 2	Strategy 3	Strategy 4	Strategy 5	Strategy 6
	$0.1 \rightarrow 0.9$	Strategy 1	-	10.50%	7.00%	10.50%	35.05%	63.80%
		Strategy 2	27.95%	-	16.55%	0.05%	41.70%	76.30%
		Strategy 3	24.80%	7.70%	-	7.75%	37.35%	65.90%
		Strategy 4	27.90%	0.00%	16.45%	-	41.70%	76.30%
		Strategy 5	4.75%	7.00%	2.45%	7.00%	-	58.90%
		Strategy 6	2.65%	1.35%	0.65%	1.35%	16.30%	-
			Strategy 1	Strategy 2	Strategy 3	Strategy 4	Strategy 5	Strategy 6
1	$0.4 \rightarrow 0.6$	Strategy 1	-	21.10%	6.80%	12.40%	72.10%	98.10%
		Strategy 2	10.35%	-	6.80%	7.50%	72.15%	98.35%
		Strategy 3	8.75%	16.35%	-	15.60%	73.10%	98.80%
		Strategy 4	3.15%	8.75%	5.90%	-	71.55%	97.80%
		Strategy 5	0.15%	1.10%	0.00%	0.40%	-	91.55%
		Strategy 6	0.00%	0.05%	0.00%	0.05%	1.85%	-
			Strategy 1	Strategy 2	Strategy 3	Strategy 4	Strategy 5	Strategy 6
	$0.1 \rightarrow 0.4$	Strategy 1	-	19.55%	7.20%	11.25%	74.20%	97.75%
		Strategy 2	13.50%	-	8.50%	13.90%	67.95%	97.70%
		Strategy 3	12.35%	16.20%	-	14.45%	75.65%	98.50%
		Strategy 4	9.85%	19.00%	8.50%	-	75.30%	97.60%
		Strategy 5	0.35%	1.75%	0.20%	0.50%	-	91.65%
		Strategy 6	0.15%	0.25%	0.00%	0.15%	1.25%	-
			Strategy 1	Strategy 2	Strategy 3	Strategy 4	Strategy 5	Strategy 6
	$0.1 \rightarrow 0.9$	Strategy 1	-	8.45%	3.65%	8.45%	37.70%	17.35%
		Strategy 2	9.45%	-	5.70%	0.05%	35.70%	20.70%
		Strategy 3	6.70%	6.35%	-	6.35%	37.70%	17.65%
		Strategy 4	9.45%	0.00%	5.70%	-	35.70%	20.65%
		Strategy 5	1.60%	4.50%	1.55%	4.50%	-	8.70%
		Strategy 6	3.05%	4.65%	2.15%	4.65%	33.45%	-
			Strategy 1	Strategy 2	Strategy 3	Strategy 4	Strategy 5	Strategy 6
5	$0.4 \rightarrow 0.6$	Strategy 1	-	11.10%	5.70%	5.80%	45.75%	62.65%
		Strategy 2	4.25%	-	3.75%	2.20%	42.35%	65.30%
		Strategy 3	4.50%	9.30%	-	6.80%	44.80%	63.25%
		Strategy 4	2.05%	5.35%	4.50%	-	44.00%	63.75%
		Strategy 5	0.65%	1.70%	0.75%	1.10%	-	34.85%
		Strategy 6	0.15%	0.25%	0.10%	0.20%	11.00%	-
			Strategy 1	Strategy 2	Strategy 3	Strategy 4	Strategy 5	Strategy 6
	$0.1 \rightarrow 0.4$	Strategy 1	-	10.65%	5.25%	6.85%	44.40%	66.80%
		Strategy 2	5.45%	-	4.90%	6.40%	39.60%	69.25%
		Strategy 3	4.30%	8.30%	-	6.75%	42.85%	68.10%
		Strategy 4	3.55%	9.10%	4.95%	-	42.45%	67.70%
		Strategy 5	0.50%	1.15%	0.40%	0.65%	-	29.85%
		Strategy 6	0.10%	0.30%	0.10%	0.10%	4.85%	-
			Strategy 1	Strategy 2	Strategy 3	Strategy 4	Strategy 5	Strategy 6
	$0.1 \rightarrow 0.9$	Strategy 1	-	11.95%	8.95%	11.95%	30.35%	21.45%
		Strategy 2	8.60%	-	9.85%	0.00%	28.20%	20.15%
		Strategy 3	8.30%	11.85%	-	11.85%	29.55%	20.60%
		Strategy 4	8.60%	0.00%	9.85%	-	28.20%	20.15%
		Strategy 5	5.50%	7.60%	5.00%	7.60%	-	8.45%
		Strategy 6	7.20%	10.70%	7.25%	10.70%	29.30%	-
			Strategy 1	Strategy 2	Strategy 3	Strategy 4	Strategy 5	Strategy 6
20	$0.4 \rightarrow 0.6$	Strategy 1	-	12.65%	9.65%	7.40%	30.85%	32.15%
		Strategy 2	7.75%	-	8.80%	4.15%	29.85%	31.35%
		Strategy 3	8.45%	12.55%	-	11.55%	30.65%	32.35%
		Strategy 4	3.65%	5.30%	8.35%	-	30.45%	32.10%
		Strategy 5	4.65%	5.30%	4.50%	4.90%	-	12.00%
		Strategy 6	3.10%	3.70%	2.95%	3.25%	20.40%	-
			Strategy 1	Strategy 2	Strategy 3	Strategy 4	Strategy 5	Strategy 6
	$0.1 \rightarrow 0.4$	Strategy 1	-	15.95%	9.15%	9.15%	28.75%	36.75%
		Strategy 2	8.40%	-	8.65%	9.75%	28.20%	35.05%
		Strategy 3	10.30%	14.50%	-	11.85%	28.05%	36.60%
		Strategy 4	6.05%	14.10%	9.25%	-	27.75%	36.80%
		Strategy 5	6.05%	6.80%	6.00%	6.40%	-	18.95%
		Strategy 6	2.40%	3.15%	2.60%	2.65%	12.15%	-

Table 4: Simulation results for an increase in persistence II

h	$d_1 \rightarrow d_2$	$\tau = 0.7$						
			Strategy 1	Strategy 2	Strategy 3	Strategy 4	Strategy 5	Strategy 6
	$0.1 \rightarrow 0.9$	Strategy 1	-	5.65%	5.50%	5.70%	57.60%	54.95%
		Strategy 2	50.05%	-	32.15%	4.45%	67.45%	79.50%
		Strategy 3	49.60%	4.85%	-	8.75%	61.05%	62.90%
		Strategy 4	46.65%	0.05%	28.65%	-	67.00%	77.05%
		Strategy 5	4.15%	3.40%	2.30%	3.45%	-	39.90%
		Strategy 6	11.75%	1.65%	2.65%	3.60%	37.80%	-
			Strategy 1	Strategy 2	Strategy 3	Strategy 4	Strategy 5	Strategy 6
1	$0.4 \rightarrow 0.6$	Strategy 1	-	19.30%	5.95%	12.25%	71.10%	97.05%
		Strategy 2	15.70%	-	7.10%	12.25%	72.05%	98.20%
		Strategy 3	16.30%	15.95%	-	22.20%	72.45%	98.00%
		Strategy 4	4.10%	7.30%	4.35%	-	68.60%	96.80%
		Strategy 5	0.25%	1.20%	0.05%	0.65%	-	84.15%
		Strategy 6	0.00%	0.05%	0.00%	0.00%	4.70%	-
			Strategy 1	Strategy 2	Strategy 3	Strategy 4	Strategy 5	Strategy 6
	$0.1 \rightarrow 0.4$	Strategy 1	-	21.15%	6.95%	11.40%	83.50%	93.80%
		Strategy 2	21.30%	-	11.75%	19.80%	76.45%	95.80%
		Strategy 3	23.05%	18.00%	-	20.45%	84.90%	96.05%
		Strategy 4	16.30%	21.65%	9.95%	-	83.30%	94.25%
		Strategy 5	0.30%	1.35%	0.00%	0.85%	-	77.40%
		Strategy 6	0.40%	0.35%	0.05%	0.30%	5.20%	-
			Strategy 1	Strategy 2	Strategy 3	Strategy 4	Strategy 5	Strategy 6
	$0.1 \rightarrow 0.9$	Strategy 1	-	5.60%	2.75%	5.60%	49.95%	13.65%
		Strategy 2	15.70%	-	7.20%	1.25%	51.65%	22.05%
		Strategy 3	12.90%	4.80%	-	5.55%	51.85%	16.20%
		Strategy 4	15.00%	0.05%	6.60%	-	51.40%	21.30%
		Strategy 5	0.85%	2.55%	0.85%	2.55%	-	3.25%
		Strategy 6	7.60%	6.75%	3.30%	7.05%	48.85%	-
			Strategy 1	Strategy 2	Strategy 3	Strategy 4	Strategy 5	Strategy 6
5	$0.4 \rightarrow 0.6$	Strategy 1	-	10.40%	5.65%	6.90%	55.25%	56.30%
		Strategy 2	5.25%	-	4.70%	3.00%	50.60%	62.10%
		Strategy 3	5.70%	8.95%	-	8.95%	55.40%	59.25%
		Strategy 4	2.40%	3.80%	4.35%	-	52.55%	56.90%
		Strategy 5	0.60%	1.45%	0.60%	0.90%	-	16.95%
		Strategy 6	0.15%	0.20%	0.10%	0.15%	20.95%	-
			Strategy 1	Strategy 2	Strategy 3	Strategy 4	Strategy 5	Strategy 6
	$0.1 \rightarrow 0.4$	Strategy 1	-	11.20%	3.95%	5.60%	55.90%	56.20%
		Strategy 2	7.05%	-	4.65%	6.65%	49.90%	61.15%
		Strategy 3	6.15%	10.05%	-	6.50%	54.25%	57.80%
		Strategy 4	6.65%	10.95%	5.40%	-	53.90%	58.75%
		Strategy 5	0.45%	1.25%	0.60%	0.65%	-	14.60%
		Strategy 6	0.25%	0.45%	0.15%	0.25%	16.50%	-
			Strategy 1	Strategy 2	Strategy 3	Strategy 4	Strategy 5	Strategy 6
	$0.1 \rightarrow 0.9$	Strategy 1	-	9.40%	7.65%	9.25%	35.45%	20.40%
		Strategy 2	10.85%	-	10.70%	1.05%	35.05%	21.65%
		Strategy 3	9.80%	9.70%	-	10.40%	35.80%	20.50%
		Strategy 4	10.10%	0.20%	10.35%	-	34.80%	21.45%
		Strategy 5	3.00%	3.55%	3.00%	3.50%	-	3.70%
		Strategy 6	7.25%	9.50%	6.05%	9.45%	37.35%	-
			Strategy 1	Strategy 2	Strategy 3	Strategy 4	Strategy 5	Strategy 6
20	$0.4 \rightarrow 0.6$	Strategy 1	-	15.25%	10.35%	8.30%	35.10%	29.90%
		Strategy 2	8.30%	-	9.05%	4.45%	32.70%	29.10%
		Strategy 3	8.30%	14.40%	-	11.85%	34.35%	30.25%
		Strategy 4	4.15%	7.25%	9.05%	-	34.40%	29.85%
		Strategy 5	4.35%	5.15%	4.25%	4.75%	-	9.75%
		Strategy 6	3.20%	4.30%	3.25%	3.80%	25.30%	-
			Strategy 1	Strategy 2	Strategy 3	Strategy 4	Strategy 5	Strategy 6
	$0.1 \rightarrow 0.4$	Strategy 1	-	15.65%	8.90%	9.00%	40.60%	30.10%
		Strategy 2	12.20%	-	9.70%	11.15%	37.40%	29.55%
		Strategy 3	11.65%	14.80%	-	11.55%	38.65%	30.45%
		Strategy 4	9.90%	14.50%	10.65%	-	39.30%	30.95%
		Strategy 5	4.10%	5.40%	4.35%	4.35%	-	11.20%
		Strategy 6	3.00%	3.45%	2.65%	3.00%	23.35%	-

Table 5: Simulation results for an increase in persistence III

h	$d_1 \rightarrow d_2$		$\tau = 0.3$					
			Strategy 1	Strategy 2	Strategy 3	Strategy 4	Strategy 5	Strategy 6
	$0.9 \rightarrow 0.1$	Strategy 1	-	5.05%	8.00%	24.20%	99.80%	99.85%
		Strategy 2	94.65%	-	89.30%	41.65%	99.85%	99.95%
		Strategy 3	77.00%	7.25%	-	28.05%	99.85%	99.90%
		Strategy 4	47.75%	28.55%	45.65%	-	69.40%	81.60%
		Strategy 5	0.10%	0.00%	0.00%	14.55%	-	95.45%
		Strategy 6	0.05%	0.00%	0.00%	8.75%	2.90%	-
			Strategy 1	Strategy 2	Strategy 3	Strategy 4	Strategy 5	Strategy 6
1	$0.6 \rightarrow 0.4$	Strategy 1	-	14.75%	11.70%	1.10%	95.05%	100.00%
		Strategy 2	40.05%	-	30.25%	14.45%	93.15%	99.95%
		Strategy 3	27.70%	12.05%	-	9.00%	95.00%	99.95%
		Strategy 4	24.25%	21.30%	26.85%	-	91.40%	99.95%
		Strategy 5	0.00%	0.00%	0.00%	0.05%	-	99.45%
		Strategy 6	0.00%	0.00%	0.00%	0.00%	0.00%	-
			Strategy 1	Strategy 2	Strategy 3	Strategy 4	Strategy 5	Strategy 6
	$0.4 \rightarrow 0.1$	Strategy 1	-	18.55%	13.45%	1.90%	84.80%	99.80%
		Strategy 2	59.00%	-	44.95%	50.80%	86.95%	99.80%
		Strategy 3	43.20%	17.35%	-	37.75%	85.55%	99.80%
		Strategy 4	2.75%	17.20%	13.20%	-	78.80%	99.30%
		Strategy 5	0.55%	0.35%	0.25%	0.95%	-	99.70%
		Strategy 6	0.00%	0.00%	0.00%	0.05%	0.00%	-
			Strategy 1	Strategy 2	Strategy 3	Strategy 4	Strategy 5	Strategy 6
	$0.9 \rightarrow 0.1$	Strategy 1	-	4.80%	3.30%	25.05%	83.10%	97.30%
		Strategy 2	54.55%	-	46.60%	41.55%	81.40%	95.05%
		Strategy 3	38.20%	4.50%	-	27.30%	83.80%	97.25%
		Strategy 4	11.30%	5.25%	10.00%	-	33.70%	55.00%
		Strategy 5	1.25%	0.45%	1.10%	12.35%	-	90.25%
		Strategy 6	0.10%	0.00%	0.00%	8.60%	2.05%	-
			Strategy 1	Strategy 2	Strategy 3	Strategy 4	Strategy 5	Strategy 6
5	$0.6 \rightarrow 0.4$	Strategy 1	-	6.90%	5.55%	0.95%	43.95%	90.75%
		Strategy 2	18.45%	-	13.25%	7.70%	43.70%	89.75%
		Strategy 3	9.90%	5.55%	-	4.00%	43.40%	90.45%
		Strategy 4	9.80%	9.85%	12.00%	-	40.70%	88.15%
		Strategy 5	1.40%	1.00%	1.05%	1.25%	-	82.35%
		Strategy 6	0.00%	0.00%	0.00%	0.00%	0.55%	-
			Strategy 1	Strategy 2	Strategy 3	Strategy 4	Strategy 5	Strategy 6
	$0.4 \rightarrow 0.1$	Strategy 1	-	5.80%	4.55%	2.10%	30.45%	90.60%
		Strategy 2	11.95%	-	9.50%	12.40%	33.75%	89.55%
		Strategy 3	9.25%	4.60%	-	9.90%	30.85%	90.10%
		Strategy 4	1.10%	5.15%	5.05%	-	26.15%	85.80%
		Strategy 5	3.70%	2.90%	3.75%	4.10%	-	85.20%
		Strategy 6	0.10%	0.05%	0.10%	0.10%	0.15%	-
			Strategy 1	Strategy 2	Strategy 3	Strategy 4	Strategy 5	Strategy 6
	$0.9 \rightarrow 0.1$	Strategy 1	-	5.30%	5.85%	30.35%	48.35%	62.95%
		Strategy 2	21.85%	-	20.35%	39.55%	45.20%	55.85%
		Strategy 3	17.60%	6.05%	-	32.05%	48.70%	62.50%
		Strategy 4	7.70%	5.05%	7.35%	-	21.55%	28.65%
		Strategy 5	2.60%	1.60%	2.35%	16.80%	-	48.80%
		Strategy 6	0.35%	0.45%	0.35%	11.80%	3.35%	-
			Strategy 1	Strategy 2	Strategy 3	Strategy 4	Strategy 5	Strategy 6
20	$0.6 \rightarrow 0.4$	Strategy 1	-	8.05%	8.10%	2.30%	21.70%	48.70%
		Strategy 2	16.25%	-	15.05%	9.80%	22.75%	47.75%
		Strategy 3	12.00%	8.95%	-	7.55%	22.00%	48.75%
		Strategy 4	8.70%	11.05%	12.95%	-	24.45%	46.55%
		Strategy 5	6.95%	6.85%	6.60%	6.35%	-	41.05%
		Strategy 6	1.25%	1.20%	1.35%	1.40%	3.45%	-
			Strategy 1	Strategy 2	Strategy 3	Strategy 4	Strategy 5	Strategy 6
	$0.4 \rightarrow 0.1$	Strategy 1	-	11.40%	9.45%	2.85%	24.25%	51.45%
		Strategy 2	12.85%	-	11.95%	14.25%	26.60%	50.25%
		Strategy 3	11.60%	12.75%	-	13.10%	24.70%	51.20%
		Strategy 4	1.55%	10.80%	9.35%	-	22.40%	47.50%
		Strategy 5	7.25%	6.20%	7.25%	7.85%	-	42.45%
		Strategy 6	0.70%	0.80%	0.90%	1.00%	2.25%	-

Table 6: Simulation results for a decrease in persistence I

h	$d_1 \to d_2$		$\tau = 0.5$					
			Strategy 1	Strategy 2	Strategy 3	Strategy 4	Strategy 5	Strategy 6
	$0.9 \to 0.1$	Strategy 1	-	24.05%	8.80%	48.55%	99.45%	97.80%
		Strategy 2	75.95%	-	58.55%	51.90%	99.75%	98.80%
		Strategy 3	76.95%	33.20%	-	49.95%	99.75%	98.55%
		Strategy 4	34.40%	29.55%	32.40%	-	43.80%	55.50%
		Strategy 5	0.55%	0.25%	0.00%	40.45%	-	90.70%
		Strategy 6	1.30%	0.60%	0.70%	31.85%	6.50%	-
			Strategy 1	Strategy 2	Strategy 3	Strategy 4	Strategy 5	Strategy 6
1	$0.6 \to 0.4$	Strategy 1	-	30.35%	13.90%	2.35%	97.40%	100.00%
		Strategy 2	45.05%	-	27.95%	23.30%	97.40%	100.00%
		Strategy 3	35.20%	25.60%	-	18.85%	97.20%	99.95%
		Strategy 4	21.90%	31.15%	26.30%	-	93.00%	99.75%
		Strategy 5	0.00%	0.00%	0.00%	0.15%	-	99.75%
		Strategy 6	0.00%	0.00%	0.00%	0.10%	0.05%	-
			Strategy 1	Strategy 2	Strategy 3	Strategy 4	Strategy 5	Strategy 6
	$0.4 \to 0.1$	Strategy 1	-	36.90%	18.15%	2.85%	91.60%	99.90%
		Strategy 2	54.15%	-	36.20%	49.25%	93.75%	99.90%
		Strategy 3	43.30%	31.80%	-	38.85%	92.95%	99.90%
		Strategy 4	2.30%	31.55%	16.55%	-	84.60%	98.95%
		Strategy 5	0.35%	0.25%	0.05%	1.65%	-	99.95%
		Strategy 6	0.00%	0.00%	0.00%	0.35%	0.00%	-
			Strategy 1	Strategy 2	Strategy 3	Strategy 4	Strategy 5	Strategy 6
	$0.9 \to 0.1$	Strategy 1	-	21.55%	4.25%	53.60%	91.70%	97.75%
		Strategy 2	65.40%	-	43.00%	56.70%	93.20%	97.95%
		Strategy 3	49.50%	22.30%	-	55.35%	93.00%	98.00%
		Strategy 4	10.25%	7.75%	9.00%	-	21.40%	32.10%
		Strategy 5	0.85%	0.30%	0.15%	41.50%	-	81.20%
		Strategy 6	0.15%	0.30%	0.05%	34.85%	6.20%	-
			Strategy 1	Strategy 2	Strategy 3	Strategy 4	Strategy 5	Strategy 6
5	$0.6 \to 0.4$	Strategy 1	-	14.00%	5.30%	1.55%	52.60%	92.30%
		Strategy 2	19.05%	-	12.00%	10.05%	52.35%	92.10%
		Strategy 3	12.95%	11.45%	-	7.65%	53.05%	91.70%
		Strategy 4	8.80%	13.20%	10.60%	-	48.95%	88.35%
		Strategy 5	1.10%	0.95%	0.90%	1.05%	-	81.50%
		Strategy 6	0.00%	0.00%	0.00%	0.00%	0.40%	-
			Strategy 1	Strategy 2	Strategy 3	Strategy 4	Strategy 5	Strategy 6
	$0.4 \to 0.1$	Strategy 1	-	9.60%	4.40%	3.30%	46.30%	90.80%
		Strategy 2	9.75%	-	6.25%	11.30%	47.30%	90.40%
		Strategy 3	8.60%	6.35%	-	10.60%	46.40%	90.40%
		Strategy 4	0.50%	8.75%	4.00%	-	40.85%	84.70%
		Strategy 5	2.70%	2.15%	2.50%	4.05%	-	71.40%
		Strategy 6	0.15%	0.10%	0.10%	0.45%	0.45%	-
			Strategy 1	Strategy 2	Strategy 3	Strategy 4	Strategy 5	Strategy 6
	$0.9 \to 0.1$	Strategy 1	-	10.50%	5.30%	53.35%	61.80%	66.65%
		Strategy 2	29.85%	-	21.10%	56.70%	61.50%	65.40%
		Strategy 3	20.55%	11.00%	-	54.85%	62.55%	66.25%
		Strategy 4	4.85%	4.30%	4.80%	-	15.55%	16.35%
		Strategy 5	1.95%	1.00%	0.95%	41.00%	-	37.10%
		Strategy 6	0.45%	0.50%	0.30%	36.55%	11.45%	-
			Strategy 1	Strategy 2	Strategy 3	Strategy 4	Strategy 5	Strategy 6
20	$0.6 \to 0.4$	Strategy 1	-	11.25%	9.00%	5.35%	30.60%	50.90%
		Strategy 2	14.70%	-	13.65%	13.20%	31.50%	50.00%
		Strategy 3	11.75%	11.40%	-	11.65%	30.95%	50.75%
		Strategy 4	7.80%	12.35%	12.85%	-	33.15%	46.70%
		Strategy 5	6.70%	6.05%	6.40%	5.85%	-	33.30%
		Strategy 6	1.30%	1.40%	1.25%	1.55%	6.50%	-
			Strategy 1	Strategy 2	Strategy 3	Strategy 4	Strategy 5	Strategy 6
	$0.4 \to 0.1$	Strategy 1	-	11.35%	9.05%	3.70%	36.65%	47.55%
		Strategy 2	11.90%	-	12.05%	13.50%	37.70%	47.35%
		Strategy 3	11.10%	11.95%	-	12.90%	37.20%	47.10%
		Strategy 4	2.05%	11.55%	9.40%	-	33.80%	43.10%
		Strategy 5	4.90%	4.45%	4.35%	5.90%	-	27.80%
		Strategy 6	1.35%	1.50%	1.45%	2.45%	6.90%	-

Table 7: Simulation results for a decrease in persistence II

h	$d_1 \to d_2$		$\tau = 0.7$					
			Strategy 1	Strategy 2	Strategy 3	Strategy 4	Strategy 5	Strategy 6
	$0.9 \to 0.1$	Strategy 1	-	27.35%	8.25%	58.90%	97.15%	90.80%
		Strategy 2	72.65%	-	51.25%	61.20%	98.30%	93.50%
		Strategy 3	78.05%	37.95%	-	60.90%	99.05%	92.90%
		Strategy 4	28.00%	25.30%	26.70%	-	32.95%	42.05%
		Strategy 5	2.60%	1.60%	0.20%	54.05%	-	83.95%
		Strategy 6	6.30%	4.00%	3.75%	47.15%	13.40%	-
			Strategy 1	Strategy 2	Strategy 3	Strategy 4	Strategy 5	Strategy 6
1	$0.6 \to 0.4$	Strategy 1	-	38.25%	11.70%	2.00%	98.65%	100.00%
		Strategy 2	48.90%	-	27.15%	41.25%	98.45%	100.00%
		Strategy 3	44.35%	38.20%	-	38.50%	98.85%	100.00%
		Strategy 4	10.60%	39.20%	17.70%	-	96.20%	99.70%
		Strategy 5	0.00%	0.00%	0.00%	0.25%	-	99.65%
		Strategy 6	0.00%	0.00%	0.00%	0.10%	0.15%	-
			Strategy 1	Strategy 2	Strategy 3	Strategy 4	Strategy 5	Strategy 6
	$0.4 \to 0.1$	Strategy 1	-	41.85%	12.30%	1.15%	95.00%	99.95%
		Strategy 2	52.90%	-	28.70%	47.75%	95.60%	99.90%
		Strategy 3	54.65%	43.90%	-	49.55%	96.35%	99.95%
		Strategy 4	2.60%	39.20%	13.05%	-	90.00%	99.50%
		Strategy 5	0.15%	0.30%	0.00%	0.80%	-	99.85%
		Strategy 6	0.00%	0.00%	0.00%	0.25%	0.05%	-
			Strategy 1	Strategy 2	Strategy 3	Strategy 4	Strategy 5	Strategy 6
	$0.9 \to 0.1$	Strategy 1	-	28.10%	4.70%	66.15%	92.75%	92.40%
		Strategy 2	65.85%	-	38.75%	69.10%	93.70%	94.00%
		Strategy 3	55.55%	31.55%	-	68.10%	94.90%	94.25%
		Strategy 4	9.40%	7.10%	7.65%	-	15.30%	22.45%
		Strategy 5	2.40%	1.50%	0.25%	59.40%	-	73.80%
		Strategy 6	1.90%	1.35%	0.45%	52.75%	12.45%	-
			Strategy 1	Strategy 2	Strategy 3	Strategy 4	Strategy 5	Strategy 6
5	$0.6 \to 0.4$	Strategy 1	-	17.05%	5.40%	1.40%	62.00%	92.20%
		Strategy 2	18.25%	-	9.35%	15.30%	61.75%	92.00%
		Strategy 3	16.20%	14.80%	-	13.80%	62.35%	91.95%
		Strategy 4	4.55%	17.20%	7.55%	-	59.35%	90.10%
		Strategy 5	1.05%	0.75%	0.85%	0.90%	-	74.10%
		Strategy 6	0.10%	0.10%	0.10%	0.15%	1.50%	-
			Strategy 1	Strategy 2	Strategy 3	Strategy 4	Strategy 5	Strategy 6
	$0.4 \to 0.1$	Strategy 1	-	9.65%	4.00%	1.20%	57.25%	92.90%
		Strategy 2	10.40%	-	5.85%	10.65%	58.40%	92.90%
		Strategy 3	9.60%	8.70%	-	10.20%	58.30%	92.70%
		Strategy 4	0.65%	9.15%	4.05%	-	54.05%	89.00%
		Strategy 5	1.95%	1.70%	1.75%	2.30%	-	56.35%
		Strategy 6	0.05%	0.05%	0.05%	0.10%	3.00%	-
			Strategy 1	Strategy 2	Strategy 3	Strategy 4	Strategy 5	Strategy 6
	$0.9 \to 0.1$	Strategy 1	-	15.25%	3.40%	66.25%	65.50%	68.35%
		Strategy 2	33.80%	-	20.60%	68.10%	66.25%	68.60%
		Strategy 3	24.80%	14.90%	-	67.40%	67.90%	69.20%
		Strategy 4	4.10%	3.05%	3.55%	-	8.85%	12.40%
		Strategy 5	2.10%	1.10%	0.65%	58.95%	-	34.40%
		Strategy 6	0.85%	0.50%	0.30%	53.95%	18.70%	-
			Strategy 1	Strategy 2	Strategy 3	Strategy 4	Strategy 5	Strategy 6
20	$0.6 \to 0.4$	Strategy 1	-	11.40%	8.40%	2.15%	43.55%	51.70%
		Strategy 2	13.20%	-	11.60%	12.55%	43.05%	51.40%
		Strategy 3	12.30%	11.70%	-	11.90%	44.00%	51.25%
		Strategy 4	3.35%	11.65%	9.25%	-	43.60%	49.80%
		Strategy 5	4.35%	3.90%	4.15%	4.15%	-	27.35%
		Strategy 6	0.85%	0.90%	0.90%	1.10%	13.75%	-
			Strategy 1	Strategy 2	Strategy 3	Strategy 4	Strategy 5	Strategy 6
	$0.4 \to 0.1$	Strategy 1	-	14.90%	13.05%	2.05%	48.70%	48.15%
		Strategy 2	13.90%	-	13.25%	14.15%	49.15%	48.05%
		Strategy 3	9.45%	13.00%	-	10.60%	49.60%	47.60%
		Strategy 4	1.20%	14.65%	12.35%	-	46.80%	45.50%
		Strategy 5	4.95%	4.80%	4.80%	5.30%	-	20.80%
		Strategy 6	0.95%	1.00%	1.00%	1.35%	15.25%	-

Table 8: Simulation results for a decrease in persistence III

	Increasing persistence				Decreasing persistence		
	τ				τ		
$d_1 \rightarrow d_2$	0.3	0.5	0.7	$d_1 \rightarrow d_2$	0.3	0.5	0.7
$0.4 \rightarrow 0.6$	0.590	0.561	0.524	$0.6 \rightarrow 0.4$	0.521	0.561	0.592
$0.3 \rightarrow 0.75$	0.717	0.661	0.586	$0.75 \rightarrow 0.3$	0.582	0.658	0.717
$0.6 \rightarrow 0.9$	0.752	0.729	0.691	$0.9 \rightarrow 0.6$	0.779	0.838	0.879
$0.1 \rightarrow 0.4$	0.399	0.355	0.313	$0.4 \rightarrow 0.1$	0.319	0.355	0.399

Table 9: d estimation under changing persistence

	Full sample	Until Breakpoint	After Breakpoint
	Estimate	Estimate	Estimate
μ	0.201	0.01	0.258
ϕ_1	-0.305	-0.438	-0.185
ϕ_2	-0.190	-0.263	-0.160
d	0.531	0.176	0.561

Table 10: NLS estimation results for ARFIMA $(2,d,0)$ process

MSFE						
h	Strat. 1	Strat. 2	Strat. 3	Strat. 4	Strat. 5	Strat. 6
1-step	0.056	0.053	0.056	0.053	0.071	0.073
2-step	0.078	0.069	0.078	0.069	0.118	0.099
3-step	0.096	0.083	0.096	0.083	0.152	0.121
4-step	0.099	0.081	0.097	0.081	0.169	0.091
5-step	0.092	0.078	0.091	0.078	0.154	0.093
6-step	0.131	0.107	0.132	0.107	0.219	0.120
7-step	0.068	0.054	0.068	0.054	0.125	0.091
8-step	0.114	0.090	0.114	0.090	0.189	0.084
9-step	0.097	0.083	0.098	0.083	0.148	0.191
10-step	0.189	0.155	0.190	0.155	0.279	0.101
MAFE						
h	Strat. 1	Strat. 2	Strat. 3	Strat. 4	Strat. 5	Strat. 6
1-step	0.184	0.181	0.186	0.181	0.209	0.216
2-step	0.213	0.198	0.215	0.198	0.281	0.243
3-step	0.262	0.240	0.261	0.240	0.332	0.278
4-step	0.250	0.226	0.249	0.226	0.352	0.256
5-step	0.234	0.218	0.233	0.218	0.317	0.263
6-step	0.310	0.276	0.312	0.276	0.423	0.254
7-step	0.204	0.176	0.204	0.176	0.312	0.259
8-step	0.296	0.259	0.298	0.259	0.406	0.240
9-step	0.263	0.249	0.264	0.249	0.329	0.396
10-step	0.374	0.327	0.376	0.327	0.479	0.247

Table 11: MSFE and MAFE from empirical forecast comparison.

Research Papers 2009

CREATES
Center for Research in Econometric Analysis of Time Series

2009-40: Niels Haldrup, Antonio Montañés and Andreu Sansó: Detection of additive outliers in seasonal time series

2009-41: Dennis Kristensen: Pseudo-Maximum Likelihood Estimation in Two Classes of Semiparametric Diffusion Models

2009-42: Ole Eiler Barndorff-Nielsen and Robert Stelzer: The multivariate supOU stochastic volatility model

2009-43: Lasse Bork, Hans Dewachter and Romain Houssa: Identification of Macroeconomic Factors in Large Panels

2009-44: Dennis Kristensen: Semiparametric Modelling and Estimation: A Selective Overview

2009-45: Kim Christensen, Silja Kinnebrock and Mark Podolskij: Pre-averaging estimators of the ex-post covariance matrix

2009-46: Matias D. Cattaneo, Richard K. Crump and Michael Jansson: Robust Data-Driven Inference for Density-Weighted Average Derivatives

2009-47: Mark Podolskij and Mathias Vetter: Understanding limit theorems for semimartingales: a short survey

2009-48: Isabel Casas and Irene Gijbels: Unstable volatility functions: the break preserving local linear estimator

2009-49: Torben G. Andersen and Viktor Todorov: Realized Volatility and Multipower Variation

2009-50: Robinson Kruse, Michael Frömmel, Lukas Menkhoff and Philipp Sibbertsen: What do we know about real exchange rate non-linearities?

2009-51: Tue Gørgens, Christopher L. Skeels and Allan H. Würtz: Efficient Estimation of Non-Linear Dynamic Panel Data Models with Application to Smooth Transition Models

2009-52: Torben G. Andersen, Dobrislav Dobrev and Ernst Schaumburg: Jump-Robust Volatility Estimation using Nearest Neighbor Truncation

2009-53: Florian Heinen, Philipp Sibbertsen and Robinson Kruse: Forecasting long memory time series under a break in persistence