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Detection of additive outliers in seasonal time series

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Abstract

The detection and location of additive outliers in integrated variables has attracted much attention recently because such outliers tend to affect unit root inference among other things. Most of these procedures have been developed for non-seasonal processes. However, the presence of seasonality in the form of seasonally varying means and variances affect the properties of outlier detection procedures, and hence appropriate adjustments of existing methods are needed for seasonal data. In this paper we suggest modifications of tests proposed by Shin *et al.* (1996) and Perron and Rodriguez (2003) to deal with data sampled at a seasonal frequency and the size and power properties are discussed. We also show that the presence of periodic heteroscedasticity will inflate the size of the tests and hence will tend to identify an excessive number of outliers. A modified Perron-Rodriguez test which allows periodically varying variances is suggested and it is shown to have excellent properties in terms of both power and size.

KEYWORDS: Additive outliers, outlier detection, integrated processes, periodic heteroscedasticity, seasonality.

JEL CLASSIFICATION: C12, C2, C22,

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1 Introduction

Franses and Haldrup (1994), and Shin *et al.* (1996), (for non-seasonal time series), and Haldrup *et al.* (2005), (for seasonal data), show that the presence of additive outliers (AO) affects the limiting distribution of Dickey-Fuller type tests which tend to overreject the unit root null hypothesis in this case. The intuition behind these results is that the AOs introduce an MA-type autocorrelation component which distorts the size of the tests. As a consequence, it is necessary to check for the presence of outliers prior to testing for unit roots and subsequently to modify the unit root testing procedure. With respect to the first step, i.e. the testing for the presence of outliers in $I(1)$ variables, Shin *et al.* (1996), Vogelsang (1999), and Perron and Rodriguez (2003) have proposed tests based on iterative procedures. See however, Haldrup and Sanso (2008) regarding some caveats of the Vogelsang tests. Concerning the second aspect, i.e. the correction for the outliers when testing for unit roots, this has been considered by Franses and Haldrup (1994), Haldrup *et al.* (2005), Shin *et al.* (1996) and Vogelsang (1999). One of the suggestions of the last author is to use modified Phillips-Perron (1988) tests, see Perron and Ng (1996). These tests were originally designed to deal with dependent errors but also turn out to successfully deal with dynamics generated from outliers. Franses and Haldrup proposed to extend the auxiliary regression by including dummy variables to control for the AOs whilst Shin *et al.* (1996) suggested to consider the observation affected by AOs as a missing observation and replace this by its expected value under the hypothesis of a unit root. These procedures necessarily have to identify the location of outliers.

In this paper we will be concerned with the first step, i.e. the outlier detection problem for both stationary and non-stationary integrated processes. Most outlier detection procedures assume non seasonal data. However, the presence of seasonality in the form of seasonally varying means and variances easily interfere with outlying observations and hence affects the properties of outlier detection procedures when there is strong seasonality in the data. It is also important to consider how to deal with outlying observations in order not to affect the seasonal periodicity and the autocorrelation structure of the data. Therefore appropriate adjustments of existing methods are needed for seasonal data. In this paper we suggest modifications of tests proposed by Shin *et al.* (1996) and Perron and Rodriguez (2003) to deal with the seasonal case. It turns out that especially the observations in the beginning and the end of a sample need to be given a particular treatment. The modified version of the Perron-Rodriguez test appears to perform the best in terms of both power and size. One particular form of seasonality concerns the possibility of periodically varying variances, see also Burridge and Wallis (1990), Burridge and Taylor (2001), and Franses (1996). Periodic heteroscedasticity appears to generate inflated size distortion with respect to the identification of additive outliers and hence too many outliers are likely to be identified. Fortunately a simple (further) modification of the Perron-Rodriguez test statistic can be easily constructed to alleviate these problems.

In section 2 we review the tests proposed by Perron and Rodriguez (2003) and Shin *et al.* (1996), and we extend their tests in different ways to allow data observed at a seasonal frequency. Also we suggest a modification of the Perron Rodriguez test that allows for periodically varying variances. In section 3 the new tests are compared in a Monte Carlo experiment and we conclude in general that the extensions of the Perron-Rodriguez test perform excellently. However, when periodic heteroscedasticity is present the extensions of the test to allow for this feature will be necessary to control size. The modified Shin *et al.* tests are generally found to perform poorly. Section 4 presents an empirical application before we conclude.

2 Testing for additive outliers in integrated time series

Consider the univariate seasonal process generated by

$$y_t = y_{t-s} + u_t, \quad (1)$$

where u_t is a general I(0) process and s indicates the number of observations per year. For example, u_t can be a linear process of the form $u_t = \varphi(L)e_t$ with

$$\varphi(L) = \sum_{i=0}^{\infty} \varphi_i L^i \quad \sum_{i=0}^{\infty} i^2 \varphi_i^2 < \infty.$$

Additive outliers can be introduced in different ways. For instance, the observed variable may read

$$z_t = \mu_t + y_t + \delta \pi_t \quad (2)$$

where μ_t collects the deterministic terms (e.g. a constant, trend, and seasonal dummy variables) and $\delta \pi_t$ is the additive outlier. π_t is a Bernoulli-type variable independent of u_t , such that $P(\pi_t = 1) = P(\pi_t = -1) = p/2$, $P(\pi_t = 0) = 1-p$, $0 \leq p < 1$ and δ is the (fixed) magnitude of outliers. The size of outliers may also be considered to be stochastic. Alternatively, the location of additive outliers may be assumed fixed, e.g. like $\delta_j \pi_t^j$ where δ_j is the magnitude of outlier j with fixed location $\pi_t^j = 1$ for $t = T_j$ and $\pi_t^j = 0$ otherwise. Accordingly, z_t is an integrated process subject to AOs. We will consider simple procedures to detect outliers in integrated processes and suggest their modification to accommodate seasonal data. For the sake of simplicity of the exposition we initially assume that $\mu_t = 0$.

2.1 The Shin- Sarker-Lee (1996) test

The test due to Shin *et al.* (1996) (SSL hereafter) tests the null hypothesis $\delta = 0$ in equation (2), and is given by,

$$\tau^{SSL} = \sup_{T_{ao}} |t^{SSL}(T_{ao})|$$

where,

$$t^{SSL}(T_{ao}) = \frac{\Delta z_{T_{ao}+1} - \Delta z_{T_{ao}}}{\sqrt{2\hat{\sigma}}}$$

for $T_{ao} \in \{2, \dots, T-1\}$, $\hat{\sigma}^2 = (T-3)^{-1} \left[\left(\sum_{t=2}^T \Delta z_t^2 \right) - \Delta z_{t'}^2 - \Delta z_{t'+1}^2 \right]$, $\Delta z_t = z_t - z_{t-1}$ and t' is the time point at which $\max\{|d_t| : |d_t| > \max\{|\Delta z_t|, |\Delta z_{t+1}|\}\}$ and $d_t = 2^{-1/2} (\Delta z_{t+1} - \Delta z_t)$.

The test can be easily extended to seasonal data, in which case special attention must be taken regarding the outliers located at the beginning and at the end of the sample. Assume that a single outlier is located at $T_{ao} \leq s$, in which situation all the information about δ is contained in $\Delta_s z_{T_{ao}+s} = y_{T_{ao}+s} - y_{T_{ao}} - \delta = u_{T_{ao}+s} - \delta$ where $\Delta_s = 1 - L^s$ is the seasonal difference operator. If the outlier is not located in the tails of the sample, all the information about δ is contained in $\Delta_s z_{T_{ao}+s} = u_{T_{ao}+s} - \delta$ and $\Delta_s z_{T_{ao}} = u_{T_{ao}} + \delta$, whereas if $T_{ao} > T-s$ all the information is contained in $\Delta_s z_{T_{ao}} = u_{T_{ao}} + \delta$. Hence, under the assumption that $u_t \sim iid N(0, \sigma_u^2)$ and $\delta \neq 0$,

$$\begin{cases} \Delta_s z_{T_{ao}+s} \sim iid N(-\delta, \sigma_u^2) & \text{for } T_{ao} \leq s \\ \Delta_s z_{T_{ao}+s} - \Delta_s z_{T_{ao}} \sim iid N(-2\delta, 2\sigma_u^2) & \text{for } s < T_{ao} \leq T-s \\ \Delta_s z_{T_{ao}} \sim iid N(\delta, \sigma_u^2) & \text{for } T_{ao} > T-s \end{cases}$$

and thus, the test statistic for seasonal data is given by

$$\tau_s^{SSL} = \sup_{T_{ao}} |t_s^{SSL}(T_{ao})| \quad (3)$$

where

$$t_s^{SSL}(T_{ao}) = \begin{cases} \hat{\sigma}^{-1} \Delta_s z_{T_{ao}+s} & \text{for } T_{ao} \leq s \\ 2^{-1/2} \hat{\sigma}^{-1} (\Delta_s z_{T_{ao}+s} - \Delta_s z_{T_{ao}}) & \text{for } s < T_{ao} \leq T-s \\ \hat{\sigma}^{-1} \Delta_s z_{T_{ao}} & \text{for } T_{ao} > T-s \end{cases}$$

With respect to the deterministic terms, these can be dealt with by prior regression of $\Delta_s z_t$ on the deterministic terms (including seasonal dummy variables) and proceeding the analysis by using residuals. A robust estimate of the variance can be computed as

$$\hat{\sigma}^2 = \begin{cases} (T-s-1-k)^{-1} \left[\left(\sum_{t=s+1}^T \Delta_s z_t^2 \right) - \Delta_s z_{T_{ao}+s}^2 \right] & \text{for } T_{ao} \leq s \\ (T-s-2-k)^{-1} \left[\left(\sum_{t=s+1}^T \Delta_s z_t^2 \right) - \Delta_s z_{T_{ao}}^2 - \Delta_s z_{T_{ao}+s}^2 \right] & \text{for } s < T_{ao} \leq T-s \\ (T-s-1-k)^{-1} \left[\left(\sum_{t=s+1}^T \Delta_s z_t^2 \right) - \Delta_s z_{T_{ao}}^2 \right] & \text{for } T_{ao} > T-s \end{cases} \quad (4)$$

where k is the number of deterministic regressors. Other robust estimators of the variance can be used, such as the median absolute deviation (MAD) and the trimmed standard deviation.

Once an outlier is detected, Shin *et al.* (1996) suggest to treat this as a missing observation and replace it with its forecast under the null hypothesis of a random walk. That is, assume that an AO is identified at time

T_{ao} , then the contaminated observation $z_{T_{ao}}$ has to be replaced by: $\widehat{z}_{T_{ao}} = E(z_{T_{ao}} | z_{T_{ao}-1}, z_{T_{ao}-2}, \dots) = z_{T_{ao}-s}$. Next, the new series with the corrected observation must be checked for the presence of new outliers and the corresponding observations replaced by its forecast. The iterative procedure stops when no additional outlier is detected.

2.2 The Perron-Rodriguez (2003) test

The Perron and Rodriguez (2003) test (PR in the sequel) uses the differenced data and is based on the (non-seasonal) auxiliary regression:

$$\Delta z_t = \delta [D(T_{ao})_t - D(T_{ao})_{t-1}] + v_t \quad (5)$$

where $D(T_{ao})_{t-j} = 1$ when $t = T_{ao} + j$ and 0 otherwise, so that, under the null hypothesis of $\delta = 0$ the OLS estimator of (5) is given by

$$\begin{aligned} \widehat{\delta}(T_{ao}) &= \begin{cases} \frac{1}{2}(\Delta z_{T_{ao}} - \Delta z_{T_{ao}+1}) & T_{ao} \leq T-1 \\ \Delta z_{T_{ao}} & T_{ao} = T \end{cases} \\ &= \begin{cases} \frac{1}{2}(u_{T_{ao}} - u_{T_{ao}+1}) & T_{ao} \leq T-1 \\ u_{T_{ao}} & T_{ao} = T \end{cases} . \end{aligned}$$

Perron and Rodriguez propose to estimate the variance of $\widehat{\delta}$ as: $var[\widehat{\delta}(T_{ao})]_{PR} = \frac{1}{2}(\widehat{R}(0) - \widehat{R}(1))$ where $\widehat{R}(j) = T^{-1} \sum_{t=s+j+1}^T \widehat{v}_t \widehat{v}_{t-j}$ and \widehat{v}_t are the OLS residuals from (5). Defining $t^{PR}(T_{ao}) = \sqrt{2} \widehat{\delta}(T_{ao}) (\widehat{R}(0) - \widehat{R}(1))^{-1/2}$, the test statistic now reads

$$\tau^{PR} = \sup_{T_{ao}} |t^{PR}(T_{ao})|. \quad (6)$$

Note that for $T_{ao} = T$ the t -statistic to be computed should be $t^{PR}(T_{ao}) = \widehat{\delta}(T_{ao}) \widehat{R}(0)^{-1/2}$.

We now extend the PR test to the seasonal case by considering the auxiliary regression

$$\Delta_s z_t = \begin{cases} -\delta D(T_{ao})_{t-s} + v_t & T_{ao} \leq s \\ \delta [D(T_{ao})_t - D(T_{ao})_{t-s}] + v_t & T_{ao} > s \end{cases} \quad (7)$$

in which case,

$$\widehat{\delta}(T_{ao}) = \begin{cases} -u_{T_{ao}+s} & T_{ao} \leq s \\ \frac{1}{2}(u_{T_{ao}} - u_{T_{ao}+s}) & s < T_{ao} \leq T-s \\ u_{T_{ao}} & T_{ao} > T-s \end{cases} ,$$

and,

$$var[\widehat{\delta}(T_{ao})] = \begin{cases} \widehat{R}(0) & T_{ao} \leq s \\ \frac{1}{2}(\widehat{R}(0) - \widehat{R}(s)) & s < T_{ao} \leq T-s \\ \widehat{R}(0) & T_{ao} > T-s \end{cases} .$$

Similarly,

$$t_s^{PR}(T_{ao}) = \begin{cases} \widehat{\delta}(T_{ao}) \widehat{R}(0)^{-1/2} & T_{ao} \leq s \\ \sqrt{2} \widehat{\delta}(T_{ao}) \left(\widehat{R}(0) - \widehat{R}(s) \right)^{-1/2} & s < T_{ao} \leq T - s \\ \widehat{\delta}(T_{ao}) \widehat{R}(0)^{-1/2} & T_{ao} > T - s \end{cases}$$

and the test statistic is given by

$$\tau_s^{PR} = \sup_{T_{ao}} |t_s^{PR}(T_{ao})|. \quad (8)$$

If a single outlier is located amongst the initial observations, $T_{ao} \leq s$, then $\Delta_s z_{T_{ao}+s} = u_{T_{ao}+s} - \delta$. On the contrary, if the outlier lies in $s < T'_{ao} \leq 2s$, then $\Delta_s z_{T'_{ao}} = u_{T'_{ao}} + \delta$ and $\Delta_s z_{T'_{ao}+s} = u_{T'_{ao}+s} - \delta$. Hence, one way to determine whether an outlier lies in $T_{ao} \leq s$ or in $T_{ao} + s$ is to compare $|t_s^{PR}(T_{ao} + s)| = \left| \sqrt{2} \widehat{\delta}(T_{ao} + s) \left(\widehat{R}(0) - \widehat{R}(s) \right)^{-1/2} \right|$ with $|t_s^{PR}(T_{ao})| = \left| \widehat{\delta}(T_{ao}) \widehat{R}(0)^{-1/2} \right|$. If $|t_s^{PR}(T_{ao})| > |t_s^{PR}(T_{ao} + s)|$ the possible outlier lies in $T_{ao} \leq s$ and in $T_{ao} + s$ otherwise.

Concerning the treatment of deterministic terms, consider the auxiliary regression

$$\Delta_s z_t = F(t/T) + \delta [D(T_{ao})_t - D(T_{ao})_{t-s}] + v_t$$

where $F(t/T)$ contains deterministic terms such as a constant, a trend, and seasonal dummy variables. OLS estimation of this equation is equivalent to

$$\Delta_s z_t^* = \delta [D(T_{ao})_t - D(T_{ao})_{t-s}]^* + v_t$$

where $\Delta_s z_t^*$ and $[D(T_{ao})_t - D(T_{ao})_{t-s}]^*$ are the residuals from the regression of $\Delta_s z_t$ and $[D(T_{ao})_t - D(T_{ao})_{t-s}]$ on $F(t/T)$, respectively. But note that for $F(t/T)$ being a constant or $F(t/T) = \sum_{q=1}^s D_{qt}$ being seasonal dummy variables, we have that if $T_{ao} > s$, then $[D(T_{ao})_t - D(T_{ao})_{t-s}]^* = [D(T_{ao})_t - D(T_{ao})_{t-s}]$. Hence we can use the auxiliary regression

$$\Delta_s z_t^* = \delta [D(T_{ao})_t - D(T_{ao})_{t-s}] + v_t$$

that is, to use the demeaned variable. Given that under our assumption $\Delta_s z_t$ is stationary, demeaning will not affect the critical values. Hence, it is enough to compute the critical values for the most simple regression.

Once an outlier has been detected, Perron and Rodriguez (2003) suggest to drop the corresponding observation. With seasonal data, this procedure cannot be followed given that it will distort the seasonal autocorrelation structure of the data. For instance, eliminating one observation in one quarter will mean that the corresponding year will have only three quarters. Hence, we suggest to follow the procedure suggested by Shin *et al.* (1996) and substitute the observation of the outlier by its forecast under the hypothesis of a seasonal random walk with deterministic components.

We have simulated the critical values associated with the test (8). Without reporting these, it occurs that the fractiles are practically identical to those of Perron and Rodriguez (2003) where it is the total number of observations that matters for the relevant distribution. These findings apply regardless of the deterministic that have been conditioned upon in the construction of the test.

2.3 Periodic heteroscedasticity

In several empirical studies it has been documented that periodic heteroscedasticity often characterizes economic data, see e.g. Burridge and Taylor (2001) for a review. As we shall see later in section 3, both the SSL and the PR tests suffer from size distortion in this case. However, it is possible to adjust the PR test to account for this distortion.

Suppose that the underlying process is a seasonal random walk with periodic heteroscedasticity: $y_t = y_{t-s} + u_t$, with $u_t \sim iid(0, \sigma_{t(\text{mod } s)}^2)$. That is, each season follows a random walk with seasonally varying variances. This process has been considered by Burridge and Wallis (1990), Burridge and Taylor (2001) and Franses (1996) among others. The PR test statistics can be modified according to the periodic nature of the variances. Define the statistic:

$$\tau_{PH}^{PR} = \sup_{T_{ao}} |t_{PH}^{PR}(T_{ao})| \quad (9)$$

where

$$t_{PH}^{PR}(T_{ao}) = \begin{cases} \widehat{\delta}(T_{ao}) \widehat{R}_q(0)^{-1/2} & T_{ao} \leq s \\ \sqrt{2} \widehat{\delta}(T_{ao}) \left(\widehat{R}_q(0) - \widehat{R}_q(1) \right)^{-1/2} & s < T_{ao} \leq T - s \\ \widehat{\delta}(T_{ao}) \widehat{R}_q(0)^{-1/2} & T_{ao} > T - s \end{cases} .$$

$q = t(\text{mod } s)$ is the season with the convention that $q = 0$ corresponds to $q = s$. $\widehat{R}_q(j) = [T/s]^{-1} \sum_{n=j+1}^{[T/s]} \widehat{v}_{(n-1)s+q} \widehat{v}_{(n-1-j)s+q}$, and $\widehat{\delta}(T_{ao})$ is the OLS estimate of δ in (5). That is, the variance and autocovariances are estimated using only the observations corresponding to the same season where the (possible) additive outlier is located. The distribution of t_{PH}^{PR} will be different from the distribution reported by Perron and Rodriguez because the periodic nature of the test implies a reduction in the effective number of observations and the fact that the sup of tests across seasons is defining the statistic. Table 1 reports the critical values for this case.

Another strategy to follow is to pretest for periodic variances, e.g. by an LM test, and if periodicity is detected, then compute τ_{PH}^{PR} , otherwise, compute τ_s^{PR} . Let us denote τ_{P-PH}^{PR} the supremum statistic computed with this procedure.

Insert Table 1 about here

3 Monte Carlo Experiments

In this section we study the finite sample performance of the outlier detection tests presented above, i.e. the τ_s^{SSL} and τ_s^{PR} tests and the tests accounting for periodic variances, τ_{PH}^{PR} and τ_{P-PH}^{PR} .

The Monte Carlo experiment conducted here is similar to that of Perron and Rodriguez (2003) for the non-seasonal case. The data generating process (DGP) is given by:

$$\begin{aligned} z_t &= \sum_{j=1}^m \delta_j \pi_t^j + y_t & (10) \\ (1 - L^4)^d y_t &= v_t \\ v_t &= \rho v_{t-4} + \varepsilon_t + \theta \varepsilon_{t-4} \\ \varepsilon_t &\sim iid N(0, 1) \end{aligned}$$

Hence the series z_t is following a seasonal ARIMA₄ possibly contaminated with $m = 4$ additive outliers and with fixed locations T_j (i.e. $\pi_t^j = 1$ for $t = T_j$ and $\pi_t^j = 0$ otherwise) where $j = 1, 2, 3, 4$. The j 'th outlier has magnitude δ_j . Note that when $\delta_j = 0$ for all j then no additive outliers are present, i.e. the case of interest in analyzing the size of the tests. To analyze powers we have chosen an experimental design where $\delta_j = \{5, 3, 2, 2\}$ and $T_j = \{30, 55, 77, 100\}$. Hence, the first outlier is expected to be more easily detected than the subsequent three outliers. Note also that the last two outliers may be difficult to detect given that these have a magnitude of only two standard errors. The sample size in the experiments is $T = 120$ corresponding to 30 years of quarterly data. We have also considered $T = 200$ and a design with different values of δ_j and the location of outliers. In particular, we have considered experiments where the location of outliers is either in the beginning or in the end of the sample size. To economize the space, these results are not reported here, but are available from the authors upon request. However, we do comment on the conclusions following these extended experiments. In this set up, the size and power can be analyzed under different assumptions about the dynamics describing v_t which follows a seasonal ARMA₄(1,1) model. We separate AR and MA dynamics and simulate the cases $\theta = \{-.8, -.4, 0, .4, .8\}$ and $\rho = \{-.8, -.4, .4, .8\}$. Note that by choice of d in (10) we have a non-stationary seasonal process for $d = 1$ or a stationary process for $d = 0$.

Sizes and powers are reported in Table 2 and 3. In both tables Panel A and B correspond to non-stationary seasonal processes with MA and AR dynamics, respectively, and Panel C corresponds to a stationary seasonal process with AR dynamics.

First, we focus on sizes, Table 2. It is remarkable, that in all cases where v_t exhibits autocorrelation the τ_s^{SSL} test is seriously size distorted which makes the test useless in practice. For positive autocorrelation the test is very conservative whereas for negative autocorrelation it is heavily over sized. Also for the

stationary case the τ_s^{SLL} test will be heavily over sized implied by differencing when constructing the test and the implied negative autocorrelation induced.

Insert Table 2 about here

The Perron and Rodriguez based tests appear to have almost the correct size for both stationary and non-stationary processes. One exception is the τ_{PH}^{PR} test in the non-stationary case when strong positive autocorrelation is present in v_t . In this case the test is slightly conservative.

Next, turn to the powers reported in Table 3. Given the poor size of the τ_s^{SLL} test, we do not comment on powers or size-adjusted powers for this case. In detecting the first outlier, all the three Perron and Rodriguez tests perform very similarly and show good power in the case of non-stationary seasonality, see Panels A and B.. The tests loose some power, however, in the case with negative autocorrelation. But overall power seems fine. Turning to the second outlier, results are equally similar but obviously detection of the second outlier is less powerful mainly because the magnitude of this outlier is somewhat smaller, i.e. 3 standard deviations instead of 5. Also in this case negative autocorrelation decreases power. Essentially the outlier is hidden by the negative autocorrelation pattern and clearly this is most apparent as the magnitude of the outlier becomes smaller. This general pattern extends to outlier detection of outlier 3 and 4 with the modification, however, that the τ_s^{PR} test has better power than the equivalent tests correcting for periodic heteroscedasticity.

In the stationary case, Panel C, the rejection probabilities for the three PR tests are again similar. The performance in detecting the first outlier is generally fine but deteriorating w.r.t. the subsequent outliers following the same line of arguments as given above. However, when ρ becomes small the overdifferencing implied by the construction of the tests again induces negative autocorrelation which will hidden the outliers in a similar fashion as discussed in relation to panels A and B.

In the previous simulations the data generating mechanism assumed outliers to be located centrally amongst the observations. Simulation results not reported here but available upon request seem to indicate that in general outliers at the very end of the sample yield test powers similar to those reported here. However, for outliers in the beginning of the sample some loss of power is detected.

Table 4 presents results for a data generating process with periodic heteroscedasticity, which has also been considered by Burrige and Taylor (2001). The data generating mechanism corresponds to (10) with $d = 1$ and $\theta = \rho = \delta_j = 0$, $j = 1, 2, 3, 4$, but with $\varepsilon_t \sim iid N(0, \sigma_{t(\text{mod } s)}^2)$ with the convention that $\sigma_0 = \sigma_4$. Hence z_t follows a seasonal random walk with variances depending upon the season. As it is clear from Table 4 the τ_s^{PR} test is seriously distorted in this case (as is the τ_s^{SLL} test). However, it can be seen that both the tests correcting for heteroscedasticity perform nicely in terms size and hence is generally recommendable when periodic heteroscedasticity is suspected.

Insert Table 3 about here
Insert Table 4 about here

4 Empirical applications

In order to illustrate the performance of the procedures for outlier detection, we have applied the tests to the analysis of US money demand. To that end, we have selected the most liquid definition of money demand, considering both the currency component of the US money stock, measured by M1, as well as the currency in circulation in the US economy. We will refer to these as CCM1 and CC, respectively. The variables have been made real by using the US consumer price index as deflator. The monthly data covers the period 1947:1-2004:2 and the data are from the Board of Governors of the Federal Reserve System (see <http://www.forecasts.org/data>).

Figures 1-3 display the variables and their first regular and first seasonal differences, respectively. These figures show that the variables exhibit similar behavior. Also, the series do not seem stationary, they exhibit a clear seasonal component and finally, they take values abnormally high in some periods. More precisely, we can relate these abnormal behaviour to the end of 1999 and the first half of 2001 episodes. Thus, it will be interesting to see whether we can identify these as being outliers using the various tests.

Table 5 reports the values of the τ_{PH}^{PR} , and τ_s^{PR} tests. Formal F-tests for non-periodic heteroscedasticity could not be rejected and hence the results for τ_{PH}^{PR} and τ_s^{PR} are expected to be similar. For the currency component of the US money stock, CCM1, it can be seen that the τ_{PH}^{PR} and τ_s^{PR} tests imply the presence of 4 additive outliers (identically dated). Two of the outliers are clearly associated with the Y2K-effect: December 1999, and January 2000.

Insert Table 5 about here

The results of the currency in circulation, CC, are rather similar, although slight modifications exist. First, we observe that the τ_{PH}^{PR} statistic detects the existence of 6 outliers whilst the τ_s^{PR} tests identifies 5 outliers. The tests generally agree about the four outliers from November 1999 through January 2000. In fact, the τ_{PH}^{PR} test suggests the outlier episode to start in October 1999. The February 2002 outlier is common to all tests (as for CCM1) as are the June 2001 observation associated with the τ_{PH}^{PR} and τ_s^{PR} tests.

Insert Figure 1 about here
Insert Figure 2 about here
Insert Figure 3 about here

5 Conclusions

The presence of outlying observations in seasonal time can seriously affect inference and hence robust detection of outliers and their location is of utmost

importance. Seasonal time series appear to be especially problematic when detecting outliers because both the means and variances are likely to be seasonally varying. In this paper we show how existing procedures for outlier detection for non-seasonal data can be modified when analyzing seasonally unadjusted data. In particular, we shown how tests originally suggested by Perron and Rodriguez (2003) can be modified to the seasonal case and we demonstrate that size and power generally will be excellent in most cases. Periodic heteroscedasticity is generally a problem concerning the size of the tests, but we show how appropriately calculated estimates of the periodic variances and a correction of the test statistic will alleviate these problems. In practice, pretesting for periodic heteroscedasticity is recommended as an integral part of the outlier detection procedure.

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6 Appendix

Table 1: Critical values for the τ_{PH}^{PR} test

Years	$s = 4$				$s = 12$			
	10%	5%	2.5%	1%	10%	5%	2.5%	1%
10	6.695	7.864	9.142	11.074	7.781	8.869	9.976	11.590
20	5.348	6.019	6.656	7.425	8.570	10.082	11.518	13.406
30	5.532	6.206	6.807	7.572	8.251	9.500	10.647	12.095
40	5.851	6.554	7.177	7.885	7.923	8.949	9.922	11.155
50	6.163	6.919	7.562	8.279	7.884	8.864	9.808	11.014
60	6.494	7.323	7.982	8.734	8.094	9.084	9.993	11.140
70	6.811	7.654	8.402	9.172	8.274	9.324	10.245	11.353
80	7.123	8.021	8.752	9.511	8.440	9.539	10.494	11.600
90	7.412	8.370	9.149	9.953	8.652	9.701	10.676	11.735
100	7.675	8.633	9.478	10.308	8.901	9.998	10.952	12.039
150	8.872	10.030	10.990	11.969	9.900	11.126	12.203	13.394

Notes: DGP: $(1 - L^s)z_t = \varepsilon_{t-s}$, $\varepsilon_t \sim iidN(0, 1)$. 50 000 replications.

Table 2: Sizes of the different outlier detection tests: Probability to find one or more outliers.

PANEL A, Non-stationary seasonality: ARIMA ₄ (0,1,1); $\delta_j = 0 \forall j$								
θ	n_1	$n_{>1}$	n_1	$n_{>1}$	n_1	$n_{>1}$	n_1	$n_{>1}$
	τ_s^{SSL}		τ_s^{PR}		τ_{PH}^{PR}		τ_{P-PH}^{PR}	
-0.8	0.299	0.075	0.047	0.002	0.078	0.002	0.050	0.002
-0.4	0.201	0.030	0.050	0.004	0.064	0.002	0.046	0.002
0	0.053	0.001	0.054	0.003	0.046	0.001	0.052	0.002
0.4	0.009	0.000	0.035	0.004	0.022	0.001	0.041	0.002
0.8	0.005	0.000	0.020	0.007	0.004	0.000	0.019	0.002
PANEL B, Non-stationary seasonality: ARIMA ₄ (1,1,0); $\delta_j = 0 \forall j$								
ρ	n_1	$n_{>1}$	n_1	$n_{>1}$	n_1	$n_{>1}$	n_1	$n_{>1}$
	τ_s^{SSL}		τ_s^{PR}		τ_{PH}^{PR}		τ_{P-PH}^{PR}	
-0.8	0.383	0.202	0.031	0.004	0.030	0.000	0.025	0.000
-0.4	0.230	0.048	0.058	0.002	0.076	0.003	0.055	0.003
0.4	0.003	0.000	0.030	0.007	0.023	0.001	0.037	0.001
0.8	0.000	0.000	0.031	0.036	0.003	0.000	0.025	0.001
PANEL C, Stationary seasonality: ARIMA ₄ (1,0,0); $\delta_j = 0 \forall j$								
ρ	n_1	$n_{>1}$	n_1	$n_{>1}$	n_1	$n_{>1}$	n_1	$n_{>1}$
	τ_s^{SSL}		τ_s^{PR}		τ_{PH}^{PR}		τ_{P-PH}^{PR}	
0.9	0.056	0.003	0.053	0.003	0.050	0.001	0.051	0.001
0.7	0.092	0.006	0.051	0.002	0.047	0.001	0.046	0.003
0.5	0.144	0.017	0.056	0.002	0.056	0.003	0.047	0.003
0.3	0.206	0.039	0.055	0.001	0.061	0.003	0.056	0.004
0	0.308	0.097	0.053	0.002	0.078	0.002	0.055	0.002

Note: DGP: $z_t = \sum_{j=1}^m \delta_j \pi_t^j + y_t$, where $\pi_t^j = 1$ for $t = T_j$ and 0 otherwise. $(1 - L^4)^d y_t = v_t$, $v_t = \rho v_{t-4} + \varepsilon_t + \theta \varepsilon_{t-4}$, $\varepsilon_t \sim iidN(0, 1)$. 3.000 replications. $T = 120$. 5% critical values. n_i stands for the frequency of detecting the i -th outlier.

Table 3: Sizes of the different outlier detection tests: Probability to find one or more outliers.

PANEL A: ARIMA(0,1,1); $\delta_j = \{5, 3, 2, 2\}$ for $T_j = \{30, 55, 77, 100\}$										
θ	n_1	n_2	n_3	n_4	$n_{>4}$	n_1	n_2	n_3	n_4	$n_{>4}$
			τ_s^{SSL}					τ_s^{PR}		
-0.8	0.945	0.556	0.204	0.059	0.011	0.766	0.191	0.026	0.002	0.000
-0.4	0.993	0.700	0.268	0.078	0.015	0.945	0.410	0.077	0.006	0.000
0	0.998	0.755	0.278	0.045	0.004	0.998	0.679	0.219	0.043	0.003
0.4	1.000	0.621	0.133	0.016	0.001	1.000	0.827	0.370	0.100	0.016
0.8	0.987	0.323	0.027	0.002	0.000	0.999	0.769	0.309	0.088	0.041
			τ_{PH}^{PR}					τ_{P-PH}^{PR}		
-0.8	0.716	0.156	0.017	0.002	0.000	0.732	0.188	0.024	0.002	0.000
-0.4	0.936	0.379	0.059	0.006	0.000	0.947	0.426	0.084	0.013	0.000
0	0.997	0.662	0.161	0.014	0.001	0.999	0.697	0.240	0.040	0.004
0.4	1.000	0.823	0.275	0.039	0.003	1.000	0.839	0.368	0.095	0.009
0.8	0.999	0.740	0.218	0.026	0.002	0.998	0.775	0.282	0.068	0.010
PANEL B: ARIMA(1,1,0); $\delta_j = \{5, 3, 2, 2\}$ for $T_j = \{30, 55, 77, 100\}$										
ρ			τ_s^{SSL}					τ_s^{PR}		
-0.8	0.811	0.429	0.173	0.063	0.021	0.370	0.051	0.002	0.000	0.000
-0.4	0.991	0.695	0.288	0.085	0.015	0.932	0.380	0.075	0.007	0.000
0.4	1.000	0.641	0.125	0.013	0.001	1.000	0.867	0.445	0.134	0.029
0.8	0.945	0.119	0.004	0.000	0.000	1.000	0.927	0.618	0.412	0.340
			τ_{PH}^{PR}					τ_{P-PH}^{PR}		
-0.8	0.428	0.049	0.003	0.000	0.000	0.363	0.048	0.004	0.000	0.000
-0.4	0.920	0.340	0.048	0.004	0.000	0.930	0.367	0.075	0.007	0.000
0.4	1.000	0.864	0.335	0.050	0.001	1.000	0.875	0.429	0.119	0.007
0.8	1.000	0.883	0.374	0.063	0.002	1.000	0.878	0.408	0.097	0.005
PANEL C: ARIMA(1,0,0); $\delta_j = \{5, 3, 2, 2\}$ for $T_j = \{30, 55, 77, 100\}$										
ρ			τ_s^{SSL}					τ_s^{PR}		
0.9	0.998	0.718	0.259	0.049	0.004	0.996	0.637	0.179	0.030	0.002
0.7	0.995	0.651	0.208	0.040	0.005	0.975	0.502	0.117	0.014	0.001
0.5	0.979	0.601	0.184	0.035	0.005	0.941	0.373	0.064	0.007	0.000
0.3	0.957	0.520	0.168	0.037	0.007	0.844	0.248	0.038	0.004	0.000
0	0.881	0.444	0.164	0.048	0.011	0.619	0.117	0.009	0.001	0.000
			τ_{PH}^{PR}					τ_{P-PH}^{PR}		
0.9	0.994	0.614	0.126	0.009	0.000	0.993	0.656	0.204	0.035	0.002
0.7	0.973	0.451	0.076	0.007	0.000	0.973	0.488	0.109	0.017	0.001
0.5	0.922	0.307	0.039	0.002	0.000	0.930	0.354	0.056	0.005	0.001
0.3	0.807	0.209	0.023	0.001	0.000	0.827	0.256	0.038	0.004	0.000
0	0.595	0.108	0.009	0.001	0.000	0.613	0.123	0.015	0.001	0.000

Note: See table 2.

Table 4: Size of tests with periodic heteroskedastic random walks

σ_q^2	n_1 $n_{>2}$		n_1 $n_{>2}$		n_1 $n_{>2}$		n_1 $n_{>2}$	
	τ_s^{SSL}		τ_s^{PR}		τ_{PH}^{PR}		τ_{P-PH}^{PR}	
{3, 1, 3, 1}	0.204	0.039	0.213	0.041	0.053	0.001	0.058	0.002
{3, 1, 1, 1}	0.309	0.073	0.309	0.080	0.047	0.001	0.054	0.001
{3, 3, 1, 1}	0.222	0.038	0.226	0.043	0.048	0.001	0.066	0.003

Notes: DGP: $y_t = y_{t-4} + u_t$, $u_t \sim iidN(0, \sigma_q^2)$ with $q = t(\text{mod } s)$ and the convention $\sigma_0^2 = \sigma_4^2$. 3000 replications. $T = 120$. 5% critical values are used. n_i stands for the frequency of detecting the $i - th$ outlier.

Table 5: Detected outliers for US M1, CCM1, and the US currency in circulation, CC.

	τ_{PH}^{PR}		τ_s^{PR}		τ_s^{SSL}	
CCM1	7.17	1999 : 12	5.43	1999 : 12	4.55	1999 : 12
	7.08	2000 : 1	6.19	2000 : 1	5.31	2000 : 1
	5.89	2001 : 2	4.91	2001 : 2	4.02	2001 : 2
	5.11	2001 : 6	4.57	2001 : 6		
CC	6.12	1999 : 10				
	9.77	1999 : 11	6.93	1999 : 11	5.83	1999 : 11
	16.03	1999 : 12	10.26	1999 : 12	9.33	1999 : 12
	10.59	2000 : 1	8.77	2000 : 1	7.61	2000 : 1
	5.85	2001 : 2	4.97	2001 : 2	4.09	2001 : 2
	5.08	2001 : 6	4.61	2001 : 6		

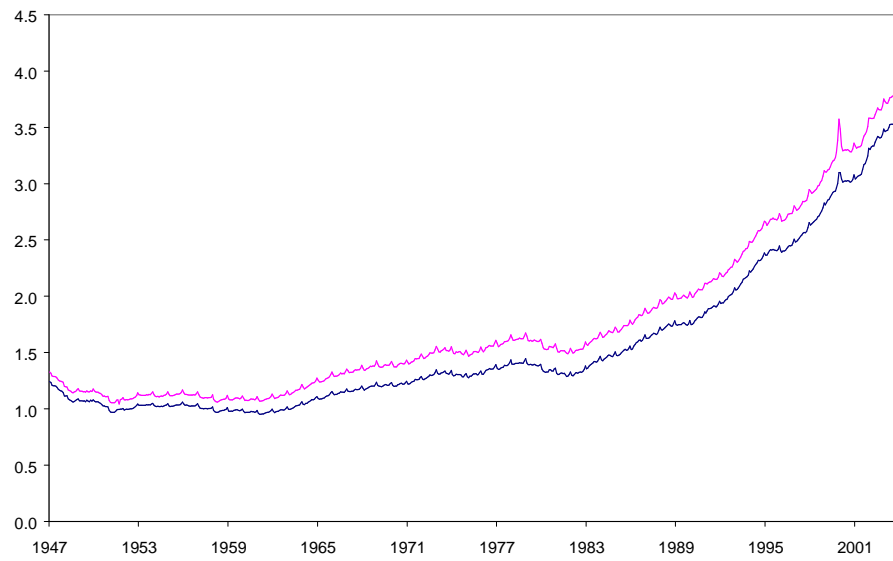


Figure 1: M1 and Currency in Circulation in the USA. Monthly data.: 1947-2003.

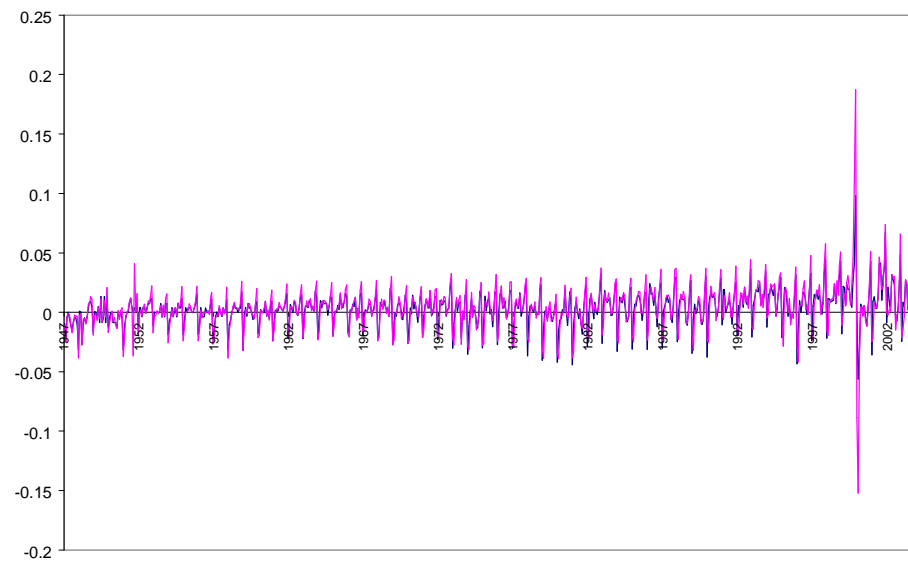


Figure 2: M1 and Currency in Circulation in the USA. Monthly data.: 1947-2003. First differences.

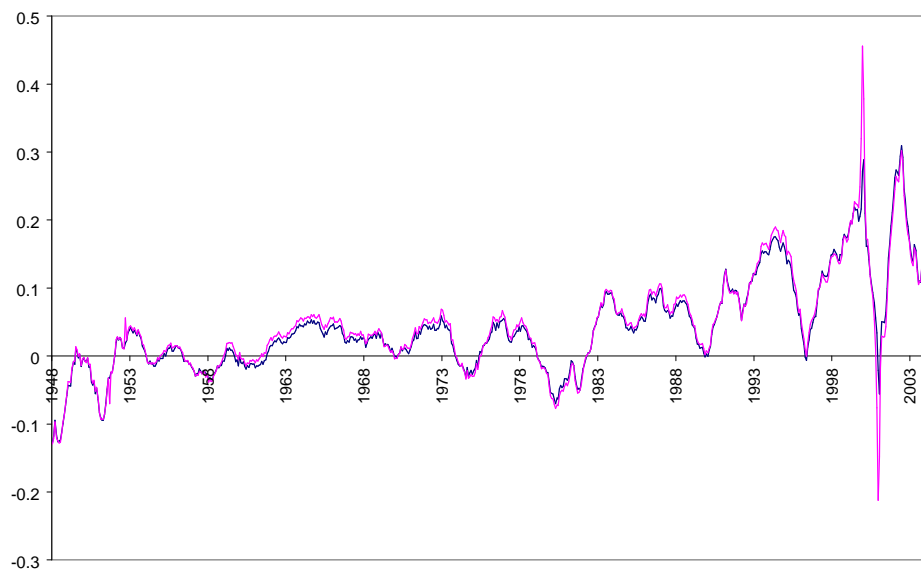


Figure 3: M1 and Currency in Circulation in the USA. Monthly data.: 1947-2003. Seasonal differences.

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