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**CREATES Research Paper 2009-29**

# Stochastic Volatility and DSGE Models

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July 7, 2009

**Abstract**

This paper argues that a specification of stochastic volatility commonly used to analyze the Great Moderation in DSGE models may not be appropriate, because the level of a process with this specification does not have conditional or unconditional moments. This is unfortunate because agents may as a result expect productivity and hence consumption to be inifinite in all future periods. This observation is followed by three ways to overcome the problem.

Keywords: Great Moderation, Productivity shocks, and Time-varying coefficients.
JEL: E10, E30

## 1 Introduction

Recent contributions have argued that the Great Moderation in the post-war US economy can be attributed to a decline in the volatility of the shocks hitting the economy (see for instance Primiceri (2005), Sims & Zha (2006), and Smets & Wouters (2007)). Fernández-Villaverde & Rubio-Ramírez (2007) and Justiniano & Primiceri (2008) illustrate this in Dynamics Stochastic General Equilibrium (DSGE) models extended with stochastic volatility. By including stochastic volatility the conditional variance of the structural shocks is allowed to change over time according to a stochastic process. The two papers document the presence of stochastic volatility in US data and show that this can explanation the Great Moderation.

This paper argues that a specification of stochastic volatility commonly used in DSGE models may not be appropriate, because it implies that the level of a process with stochastic volatility does not have conditional or unconditional moments. This is unfortunate because agents may as a result expect productivity and hence consumption to be infinite in all future periods. Such expectations are hard to justify based on microeconomic evidence. Throughout this paper we refer to this specification of stochastic volatility as SV1.

This paper suggests three ways to overcome the problem. Firstly, we derive a sufficient condition which ensures that all conditional and unconditional moments exist for the level of a

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process with SV1. This condition can thus justify the use of SV1 in DSGE models. However, this condition only imposes an upper bound on all moments and closed form solutions for either conditional or unconditional moments do not exist.

As an alternative solution, we therefore further improve SV1 such that the level of a process with stochastic volatility has closed form solutions for at least first and second moments, both conditionally and unconditionally. This specification is denoted SV2.

Finally, we suggest a specification of stochastic volatility denoted SV3 which is less standard but has some interesting implications in the context of DSGE models. For example, this specification implies that stochastic volatility can lead to time-varying persistency rates. Hence, SV3 illustrates that ignoring stochastic volatility in an otherwise standard DSGE model may lead to time-varying structural coefficients.

## 2 Analyzing SV1

SV1 is set in two equations. The first equation is given by

$$\ln\left(\frac{a_{t+1}}{a_{ss}}\right) = \rho \ln\left(\frac{a_t}{a_{ss}}\right) + \sigma_{t+1}\epsilon_{a,t+1}, \tag{1}$$

where $a_{ss}$ denotes the steady state value of $a_t$. In the present discussion, we let $a_t$ denote the level of productivity. Moreover, it is assumed that $\rho \in [-1, 1]$ and $\epsilon_{a,t} \sim \mathcal{NID}(0, 1)$. The important difference from the standard log-normal process is the subscript on the level of the conditional volatility $\sigma_t$, meaning that the conditional volatility is allowed to change over time.

The second equation specifies the law of motion for $\sigma_t$:

$$\ln\left(\frac{\sigma_{t+1}}{\sigma_{ss}}\right) = \rho_\sigma \ln\left(\frac{\sigma_t}{\sigma_{ss}}\right) + \epsilon_{\sigma,t+1}, \tag{2}$$

where $\rho_\sigma \in [-1, 1]$ and $\epsilon_{\sigma,t} \sim \mathcal{NID}(0, Var[\epsilon_{\sigma,t}])$. The innovations $\epsilon_{a,t+1}$ and $\epsilon_{\sigma,t+1}$ are assumed to be mutually independent at all leads and lags.

The crucial assumptions for our discussion are the log-transformations of both $a_t$ and $\sigma_t$. These transformations ensure that the level of productivity and the level of volatility are both strictly positive. We argue that log-transformations of both $a_t$ and $\sigma_t$ lead to an unfortunate implication. To realize this, consider the following proposition.

**Proposition 1** *Let $Z_1$ and $Z_2$ be independent. For $k \in \{1, 2, ...\}$, $Z_1 \sim N\left(0, \sigma_1^2\right)$, and $Z_2 \sim N\left(0, \sigma_2^2\right)$ the moment $E\left(\exp\left\{ke^{Z_1}Z_2\right\}\right)$ does not exist.*

**Proof.** $E\left(\exp\left\{ke^{Z_1}Z_2\right\}\right) = \sum_{\substack{i,j\in\{0,\infty\} \\ i\neq j}} \int_{-j}^{i} \sum_{\substack{n,m\in\{0,\infty\} \\ n\neq m}} \int_{-m}^{n} \frac{1}{2\pi\sigma_1\sigma_2} \exp\left\{g\left(z_1, z_2\right)\right\} dz_1 dz_2$ where $g\left(z_1, z_2\right) \equiv ke^{z_1}z_2 - 0.5\left(z_1/\sigma_1\right)^2 - 0.5\left(z_2/\sigma_2\right)^2$. Now, $g\left(z_1, z_2\right) \to \infty$ for $z_1 \to \infty$ because exponential functions grow faster than power functions. Hence

$\int_0^\infty \int_0^\infty \frac{1}{\sqrt{2\pi}\sigma_1} \frac{1}{\sqrt{2\pi}\sigma_2} \exp\left\{g\left(z_1, z_2\right)\right\} dz_1 dz_2$ is infinite. ■

Based on (1) and (2) we have

$$a_{t+1} = a_{ss}^{1-\rho} a_t^{\rho} \exp\left\{\sigma_{ss}^{1-\rho_\sigma} \sigma_t^{\rho_\sigma} \exp\left\{\epsilon_{\sigma,t+1}\right\} \epsilon_{a,t+1}\right\}. \tag{3}$$

From the result in proposition 1 it follows that $E_t\left[a_{t+1}^k\right] = \infty$, and by the law of iterated expectations, $E_t\left[a_{t+l}^k\right] = \infty$ and $E\left[a_t^k\right] = \infty$ for $k \in \{1, 2, ...\}$ and $l \in \{1, 2, ...\}$. This result is driven by the two log-transformations which give rise to the exponential of the term $\exp\{Z\}$, and with $Z \sim N\left(0, \sigma^2\right)$ such a term does not have moments. In a continuous time setting, Chernov, Gallant, Ghysels & Tauchen (2003) notice a similar property for a stochastic volatility model where $\sigma_t$ is log-transformed.

### 2.1 An illustration: The Neoclassical growth model with stochastic volatility

To illustrate the problem with SV1 for DSGE models, we consider the case where the neoclassical growth model is specified with SV1.[1]

A representative agent solves

$$\max_{c_t \geq 0} \quad U_t = E_t \sum_{l=0}^{\infty} \beta^l \log c_{t+l} \quad \text{s.t.} \quad k_{t+1} = (1-\delta) k_t + i_t \tag{4}$$

where $c_t$ is consumption and $\beta \in (0, 1)$. Capital $k_t$ depreciates with $\delta \in [0, 1]$ and $i_t$ is investments. Production $y_t$ is given by

$$y_t = a_t k_t^{\alpha}, \tag{5}$$

where $\alpha \in [0, 1]$ and $a_t$ is technology specified with SV1.

To simplify the presentation, we consider the case where $\delta = 1$. The model solution is then

$$c_t = (1 - \alpha\beta) a_t k_t^{\alpha} \tag{6}$$

$$k_{t+1} = \alpha\beta a_t k_t^{a} \tag{7}$$

As argued above, the process for $a_t$ does not have conditional moments, and this property carries over to the process for consumption. That is, the representative agent expects productivity to attain infinite levels in all future periods, and therefore, the expected level of consumption in the future is also infinite. Such expectations are hard to justify based on microeconomic evidence.

## 3 Three ways to overcome the problem

### 3.1 A sufficient condition for SV1

This section derives a sufficient condition which ensures that technology in SV1 has moments of any order. We start by considering the class of stochastic volatility models given by

$$\ln\left(\frac{a_{t+1}}{a_{ss}}\right) = \rho \ln\left(\frac{a_t}{a_{ss}}\right) + \sigma\left(v_{t+1}\right) \epsilon_{a,t+1} \tag{8}$$

[1]Note that the neoclassical model is the foundation of many DSGE models, including the ones considered by Fernández-Villaverde & Rubio-Ramírez (2007) and Justiniano & Primiceri (2008).

$$v_{t+1} = v_{ss}(1-\rho_\sigma) + \rho_\sigma v_t + \epsilon_{\sigma,t+1} \tag{9}$$

where $\sigma_t$ defines conditional volatility as a function of $v_t$. When $\sigma(v_t) \equiv \exp\{v_t\}$, we thus get SV1. The problem with SV1 is related to the volatility function $\sigma(v_t)$, and Chernov et al. (2003) therefore suggest to modify the volatility function by splicing the exponential function with another function for $v_t > \bar{v}$ to ensure a well-behaved process for $a_t$. Using this approach, we let

$$\sigma(v_t) \equiv \begin{cases} \exp\{v_t\} & \text{for } v_t \leq \bar{v} \\ \exp\{\bar{v}\}\sqrt{1+\bar{v}+v_t} & \text{for } v_t > \bar{v} \end{cases} \tag{10}$$

The lower part of this function is chosen to generate a smooth splicing at $\bar{v}$, where the level and the first derivative of the upper and lower part of the function in (10) are identical. With this specification of $\sigma(v_t)$, the variance of the conditional volatility is normally distributed for $v_t > \bar{v}$, and it is straightforward to show that this ensures finiteness of all conditional and unconditional moments for technology.

Chernov et al. (2003) suggest to choose the splicing point $\bar{v}$ above any empirical realistic level of $\sigma_t$. This implies that the somewhat arbitrary splicing of the volatility function does not have empirical effects. When perturbation methods are used to solve DSGE models, the less restrictive assumption that $\bar{v} > v_{ss}$ is sufficient to ensure that the splicing of the volatility function does not affect the approximated solution and have empirical effects. This is because all derivatives of $\sigma(v_t)$ with $\bar{v} > v_{ss}$ are identical to the case where $\sigma(v_t) \equiv \exp\{v_t\}$ for all $v_t$.

### 3.2 Specification SV2

As an alternative to SV1 amended with the assumption in (10), we present another specification of stochastic volatility suitable for DSGE models. This specification consists of two equations. The first equation describes the evolution of $a_t$ as

$$a_{t+1} = a_{ss}(1-\rho) + \rho a_t + \sigma_{t+1}\epsilon_{a,t+1}, \tag{11}$$

where $a_{ss} \geq 0$ and $\rho \in [0,1[$. The variable $\epsilon_{a,t}$ is assumed to be independent and identically distributed according to a probability distribution with only positive probability mass for $\epsilon_{a,t} \geq 0$. We further require the existence of first and second moments in this probability distribution and denote it by $\epsilon_{a,t} \sim IID^+(E[\epsilon_{a,t}], Var[\epsilon_{a,t}])$. The class of probability distributions with these properties include i) the gamma distribution, ii) the inverse gamma distribution, iii) the weilbull distribution among other. As is typical in stochastic volatility models, we suggest to normalize the variance of $\epsilon_{a,t}$ to a given level.

The second equation is the law of motion for the conditional volatility:

$$\sigma_{t+1} = (1-\rho_\sigma)\sigma_{ss} + \rho_\sigma \sigma_t + \epsilon_{\sigma,t+1}, \tag{12}$$

where $\sigma_{ss} \geq 0$, $\rho_\sigma \in [0,1[$, and $\epsilon_{\sigma,t} \sim IID^+(E[\epsilon_{\sigma,t}], Var[\epsilon_{\sigma,t}])$. The innovation $\epsilon_{\sigma,t}$ is assumed to be mutually independent of $\epsilon_{a,t}$ at all leads and lags.

The next proposition states sufficient conditions for the value of productivity in (11) to be non-negative.

**Proposition 2** *Let the starting value for $a_t$ be $a_0 \geq 0$, and let $\sigma_t \geq 0$ for all $t$. This implies $a_t \geq 0$ for all $t$.*

**Proof.** The value of $a_t$ is given by $a_t = \sum_{i=0}^{t-1} (1-\rho)\, a_{ss}\rho^i + \rho^t a_0 + \sum_{i=0}^{t-1} \rho^i \epsilon_{a,t-i}\sigma_{t-i}$ for all $t$. All terms in this expression are non-negative because $\rho \in [0,1[$ and $a_{ss}, a_0, \{\epsilon_{a,t}, \sigma_i\}_{i=1}^t \geq 0$ for all $t$. Hence, $a_t \geq 0$ for all $t$. ■

Two assumptions are essential to establish the non-negativety of $a_t$. Firstly, we only consider the case where $\rho \in [0,1[$ and thus rule out oscillating behavior in the technology process implied by $\rho \in ]-1,0[$. We do not consider this a particular restrictive assumption because estimated values of $\rho$ are always significantly larger than zero (see for instance Smets & Wouters (2007)).

Secondly, the assumptions we impose on $\epsilon_{a,t}$ imply that $E[\epsilon_{a,t}] > 0$, unless $Var[\epsilon_{a,t}] = 0$ and the process for $a_t$ degenerates. A mean value of $\epsilon_{a,t}$ different from zero may at first seem somewhat awkward, but this feature is in fact completely standard in DSGE models. To realize this, consider the standard log-normal process for technology, i.e.

$$a_{t+1} = a_{ss}^{1-\rho} a_t^{\rho} \exp\{\sigma\epsilon_{a,t+1}\}, \tag{13}$$

where $\epsilon_{a,t+1} \sim \mathcal{NID}(0,1)$. Here, the innovation is given by $\exp\{\sigma\epsilon_{a,t+1}\}$ where $E[\exp\{\sigma\epsilon_{a,t+1}\}] = \exp\{0.5\sigma^2\}$ which is clearly different from zero. Of course, applying the log-transformation in (13) leads to the familiar expression where the innovation $\sigma\epsilon_{a,t+1}$ has a mean value of zero. In a similar fashion, the process in (11) can easily be transformed by letting

$$\tilde{\epsilon}_{a,t+1} \equiv \epsilon_{a,t+1} - E[\epsilon_{a,t+1}] \quad \text{and} \quad \tilde{a}_{ss} \equiv a_{ss} + \frac{E[\epsilon_{a,t}]\, E[\sigma_t]}{1-\rho} \tag{14}$$

which implies

$$a_{t+1} = (1-\rho)\,\tilde{a}_{ss} + (\sigma_{t+1} - E[\sigma_{t+1}])\, E[\epsilon_{a,t+1}] + \rho a_t + \sigma_{t+1}\tilde{\epsilon}_{a,t+1}. \tag{15}$$

In this transformed version of (11), the innovation is $\tilde{\epsilon}_{a,t+1}$ which has a mean value of zero and $E_t[a_t] = \tilde{a}_{ss}$. As for the log-transformed representation of (13), the transformed expression in (15) is convenient when DSGE models are solved numerically.

Equation (15) also shows that the process of $a_t$ in (11) is very similar to models where the process of $\sigma_t$ also affects the conditional mean of $a_t$. Examples of such models are the "stochastic volatility in mean model" by Koopman & Uspensky (2002) and the various "GARCH in mean models" following the work of Engle, Lilien & Robins (1987).

Using a similar argument as in proposition 2, it is straightforward to show that SV2 also satisfies the non-negativity constraint for $\sigma_t$, provided $\sigma_0 \geq 0$.

The moments for the volatility process are standard, and the moments for technology are stated in Table 1. Note that $a_t$ is never less persistent than the corresponding AR(1) model, i.e. $corr(a_{t+l}, a_t) \geq \rho^l$. This is because the persistence in the volatility process may generate additional persistence in the process for $a_t$.

### 3.3 Specification SV3

Finally, we suggest the following specification of stochastic volatility as another way to overcome the problem with SV1:

$$a_{t+1} = a_{ss} + \sigma_{t+1} v_{t+1} \tag{16}$$

$$v_{t+1} = \rho v_t + \epsilon_{v,t+1} \tag{17}$$

$$\sigma_{t+1} = (1 - \rho_\sigma)\, \sigma_{ss} + \rho_\sigma \sigma_t + \epsilon_{\sigma,t+1} \tag{18}$$

where $a_{ss}, \sigma_{ss} \geq 0,\ \rho, \rho_\sigma \in [0,1[,\ \epsilon_{v,t} \sim IID^+ (E[\epsilon_{v,t}], Var[\epsilon_{v,t}])$, and $\epsilon_{\sigma,t} \sim IID^+(E[\epsilon_{\sigma,t}], Var[\epsilon_{\sigma,t}])$. The variance of $v_t$ should be normalized to a given value. This specification is very similar to SV2, but the persistence in the process for $a_t$ is moved to the innovation, denoted $v_t$. To see what impact this change has on the process for $a_t$, consider the following equivalent expression for $a_t$

$$a_{t+1} = a_{ss} + \rho \frac{\sigma_{t+1}}{\sigma_t} (a_t - a_{ss}) + \sigma_{t+1} \epsilon_{v,t+1}. \tag{19}$$

We emphasize three interesting properties in relation to (19). Firstly, the persistence in $v_t$ (i.e. $\rho > 0$) *and* the stochastic volatility $\left(\text{i.e. } \frac{\sigma_{t+1}}{\sigma_t} \neq 1\right)$ generate a sort of time-varying persistency rate in the process for $a_t$. That is, the effect of a technology shock depends on the "volatility regime". For instance, in periods of a stable evolution in $\sigma_t$ (i.e. $\frac{\sigma_{t+1}}{\sigma_t} \cong 1$), the persistency coefficient for $a_t$ is approximately equal to $\rho$. On the other hand, in periods with very changing levels of volatility (i.e. $\frac{\sigma_{t+1}}{\sigma_t} \neq 1$), the persistency coefficient is $\rho \frac{\sigma_{t+1}}{\sigma_t}$.

Secondly, the process for $a_t$ in SV3 is similar to the law of motion of $a_t$ in SV2 if $\rho$ is replaced with the time-varying coefficient $\rho_t \equiv \rho \frac{\sigma_{t+1}}{\sigma_t}$. Thus, if SV3 is the true data generating process but SV2 is estimated, then one could observe time-variation in the estimate of $\rho$ if allowed for. In other words, stochastic volatility can lead to time-varying parameters in DSGE models.

Thirdly, the process for technology could locally display properties similar to unstable processes if $\left| \rho \frac{\sigma_{t+1}}{\sigma_t} \right| \geq 1$. However, the processes for $v_t$ and $\sigma_t$ are stable and the process for $a_t$ is therefore also stable.

Using a similar argument as in proposition 2, it is straightforward to show that SV3 satisfies the non-negativity constraint for $a_t$ and $\sigma_t$, provided $v_0 \geq 0$ and $\sigma_0 \geq 0$.

The moments for technology are stated in Table 2. When $\rho_\sigma \leq \rho,$ we have $corr\,(a_{t+l}, a_t) \leq \rho^l$ and the process for technology is less persistent than its corresponding AR(1) model. On the other hand, if $\rho_\sigma \geq \rho$ the relation between $corr\,(a_{t+l}, a_t)$ and $\rho^l$ is indeterminate.

## 4 Conclusion

We have in this paper adopted a purely theoretical focus to examine suitable specifications of stochastic volatility in DSGE models. An interesting direction for future research would therefore be to adopt an empirical focus and examine the performance of SV1 compared to SV2 and SV3.

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**Table 1: Moments for SV2**

We only report the conditional variance for $a_{t+1}$ in the interest of space.

| | |
|---|---|
| $E_t\left[a_{t+l}\right]$ | $a_{ss}\left(1-\rho^l\right)+\rho^l a_t+E\left[\epsilon_{a,t}\right]\sum_{i=0}^{l-1}\rho^i E_t\left[\sigma_{\sigma,t+l-i}\right]$ |
| $Var_t\left[a_{t+1}\right]$ | $E\left[\epsilon_{\sigma,t}^2\right]E\left[\epsilon_{a,t}^2\right]-E\left[\epsilon_{\sigma,t}\right]^2E\left[\epsilon_{a,t}\right]^2$ |
| | $+\left[\left(\sigma_{ss}\left(1-\rho_\sigma\right)+\rho_\sigma\sigma_t\right)^2+2E\left[\epsilon_{\sigma,t}\right]\left(\sigma_{ss}\left(1-\rho_\sigma\right)+\rho_\sigma\sigma_t\right)\right]Var\left[\epsilon_{a,t}\right]$ |
| $E\left[a_t\right]$ | $a_{ss}+\frac{E[\epsilon_{a,t}]E[\sigma_t]}{1-\rho}$ |
| $Var\left[a_t\right]$ | $\frac{1}{1-\rho^2}\left[\left(Var\left[\sigma_t\right]+E\left[\sigma_t\right]^2\right)E\left[\epsilon_{a,t}^2\right]-\frac{1+\rho}{1-\rho}E\left[\sigma_t\right]^2E\left[\epsilon_{a,t}\right]^2\right]$ |
| | $+\frac{2\rho E[\epsilon_{a,t}]^2}{1-\rho^2}\left(\frac{Var[\sigma_t]\rho_\sigma}{1-\rho\rho_\sigma}+\frac{(E[\sigma_t])^2}{1-\rho}\right)$ |
| $corr\left(a_{t+l},a_t\right)$ | $\begin{cases}\rho^l+\rho_\sigma\frac{\left(E[\epsilon_{a,t}]E[\epsilon_{a,t}]\frac{Var[\sigma_t]}{1-\rho\rho_\sigma}\right)\frac{\rho_\sigma^l-\rho^l}{\rho_\sigma-\rho}}{Var(a_t)} & \text{for } \rho_\sigma\neq\rho \\ \rho^l\left(1+\frac{\left(E[\epsilon_{a,t}]E[\epsilon_{a,t}]\frac{Var[\sigma_t]}{1-\rho\rho_\sigma}\right)l}{Var[a_t]}\right) & \text{for } \rho_\sigma=\rho\end{cases}$ |

**Table 2: Moments for SV3**

| | |
|---|---|
| $E_t\left[a_{t+l}\right]$ | $a_{ss}+\left[E\left[\sigma_t\right]+\rho_\sigma^l\left(\sigma_t-E\left[\sigma_t\right]\right)\right]\left[\rho^l v_t+E_t\left[\epsilon_{v,t}\right]\frac{1-\rho^l}{1-\rho}\right]$ |
| $Var_t\left[a_{t+l}\right]$ | $\left(Var_t\left[\sigma_{t+l}\right]+E_t\left[\sigma_{t+l}\right]^2\right)Var_t\left[v_{t+l}\right]+Var_t\left[\sigma_{t+l}\right]E_t\left[v_{t+l}\right]^2$ |
| $E\left[a_t\right]$ | $a_{ss}+E\left[\sigma_t\right]\left(\frac{E_t[\epsilon_{v,t}]}{1-\rho}\right)$ |
| $Var\left[a_t\right]$ | $\left(Var\left[\sigma_t\right]+E\left[\sigma_t\right]^2\right)Var\left[v_t\right]+Var\left[\sigma_t\right]E\left[v_t\right]^2$ |
| $corr\left(a_{t+l},a_t\right)$ | $\frac{Var[\sigma_t]\left(\rho_\sigma^l\rho^l Var[v_t]+\rho_\sigma^l E[v_t]^2\right)+\rho^l E[\sigma_t]^2 Var(v_t)}{Var[\sigma_t]\left(Var[v_t]+E[v_t]^2\right)+E[\sigma_t]^2 Var[v_t]}$ |

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