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Decompositions of injection patterns for nodal flow allocation in renewable electricity networks

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Abstract. The large-scale integration of fluctuating renewable power generation represents a challenge to the technical and economical design of a sustainable future electricity system. In this context, the increasing significance of long-range power transmission calls for innovative methods to understand the emerging complex flow patterns and to integrate price signals about the respective infrastructure needs into the energy market design. We introduce a decomposition method of injection patterns. Contrary to standard flow tracing approaches, it provides nodal allocations of link flows and costs in electricity networks by decomposing the network injection pattern into market-inspired elementary import/export building blocks. We apply the new approach to a simplified data-driven model of a European electricity grid with a high share of renewable wind and solar power generation.

1 Introduction

In view of climate change and the finite time horizon and political instability of fossil fuel supply, the transition towards a sustainable energy system is one of the main challenges for our modern society. As of today, wind and solar power generation are mature technology options which can provide electricity on a large scale at increasingly competitive costs [1]. Nevertheless, the fluctuating nature of these renewable generation technologies requires a new energy system design compared to past and present infrastructures, which are mostly based on centralised dispatchable fossil fuel power generation. Elaborate computational models seek to capture the respective relevant physical, technological and economical boundary conditions [2].

Applied Theoretical Physics can contribute another perspective to this challenge. By focussing on simplified, more abstract models it is often possible to identify the fundamental relationships and key dynamics of larger complex systems, and to develop and test new methods and concepts.

One focus for this kind of approach has been on network representations of electricity systems, which assess for instance structural properties [3] or the topological robustness of power grids [4,5]. Another strong line of research investigates synchronisation properties of power

grid models [6–9], for instance under decentralisation [10] or link removal [11].

In contrast, the modelling approach which we term “complex renewable energy networks” addresses the analysis and design of energy systems with a high share of renewable generation. The renewable generation as well as the electricity consumption in these models is based on realistic data, whereas the system representation (constituents and network structure), the backup power dispatch and the power flow equations are simplified. In this context, the optimal mix between solar and wind power generation [12], the influence of storage and an associated phase transition [13,14], or the benefit of power transmission have been investigated [15,16].

The transmission needs associated with the smoothing of spatial fluctuations of renewable power generation [15] provide a strong motivation to analyse the flow patterns occurring in complex renewable energy networks. The necessary transmission infrastructure crucially depends on the geographical distribution of load and generation capacities, and on the spatio-temporal correlations in the renewable generation patterns, in particular for wind power generation [17,18]. Flow allocation methods assign a share of the total power flow to the individual importing and/or exporting nodes in the system, thus providing valuable information about their role in the overall flow pattern and an estimate of their usage of transmission capacity. These techniques are to some degree related to measures which relate the collective dynamics of a network to small parameter changes of the system [19], in particular

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Fig. 1. Flow tracing: illustration of the partial flows of Germany (DE), Norway (NO), and Spain (ES) at an exemplary time step in a simplified model of the European electricity system. The nodes are colour-coded according to the magnitude of their exports (green) and imports (red) at this time, respectively. The left figure shows the total power flows in the system. The partial power flows assigned to Germany (blue), Norway (orange), and Spain (purple) based on the flow tracing method are displayed in the right figure. The size of the arrows for partial flows are scaled to the maximal partial flow of the respective country. For clarity only arrows for partial flows larger than a small threshold equal to 5% of the largest partial flow occurring in the system are shown.

with respect to robustness against link failures [11]. The method of flow tracing for instance has been discussed in the context of the transmission system operator compensation mechanism regarding transient cross-border flows in the European power grid [20,21]. In [22] this allocation method, also known as average participation, has been applied to a simplified model of the European electricity grid. From a complex networks perspective, flow tracing can be understood as a directed diffusion process on the directed acyclic flow graph [22–24]. The partial flows assigned to the respective exporting or importing nodes are thus always oriented in the same direction as the total power flow. This might be in contradiction with individual market transactions between importers and exporters, which can produce counter flows in the opposite direction.

Figure 1 illustrates this limitation of the flow tracing approach. In the depicted system state, Germany (DE) is a net exporter of electricity, whereas France (FR) is a net importer. This situation suggests the possibility of a trade contract between these countries, leading to an associated *partial* power flow from Germany to France. Since the *total* power flow is oriented from France to Germany, such a contract and the associated partial flow cannot be represented by the flow tracing method. In this article we propose an alternative approach to flow and transmission capacity allocation based on decompositions of injection patterns. These decompositions allow to associate partial flows to individual transactions between importers and exporters, and factor in explicitly the patterns of nodal imports and exports. Here we use the decomposition method to analyse power flows in highly renewable electricity systems, but this technique can also be adapted to other models of networked systems, in which line flows

depend linearly on nodal injections (for instance hydraulic networks [25], vascular networks [26], or network diffusion [27]).

This article is organized as follows: after briefly reviewing the linearised power flow equations, Section 2 provides a description of the flow tracing method and introduces the decomposition method of injection patterns. Two specific implementations (proportional and random contract decomposition) are described, and the conceptual differences to flow tracing are discussed. Section 3 compares flow tracing and the decomposition method of injection patterns in the context of a simplified model of a highly renewable European electricity network, which is specified in more detail in Appendix A. A conclusion and outlook is given in Section 4.

2 Methods of flow allocation

Flow allocation methods attribute a share of the total power flow on a network to the net power in- or outflow associated with a specific node or subset of nodes [20]. Due to the indistinguishability of the electrical power flow in the network, such an attribution in general is not unique and has to be based on intuitive algorithms. In the following sections we briefly state the linearised power flow equations [28] and review the tracing method of flow allocation [22,29–31]. We then introduce decompositions of injection patterns as an alternative way of allocating power flows, which is based on a decomposition of the net power inflows and outflows, and discuss this method in a comparison with the flow tracing approach.

2.1 The DC power flow equations

The majority of today's power grid infrastructure can be represented as AC networks. Although the power flow in these AC networks is governed by the full AC power flow equations, for the stable network operation the so-called DC approximation can be applied [28]. The directed active power flow $F_{m \rightarrow n}$ on a link from node m to n is then given by

$$F_{m \rightarrow n} = b_{mn} (\theta_m - \theta_n), \quad (1)$$

with θ_n the voltage angle at node n and b_{mn} the line susceptance of the link. The voltage phase angles have to fulfill the linear equations

$$P_n = \left(\sum_m b_{nm} \right) \theta_n - \sum_m b_{nm} \theta_m = \sum_m B_{nm} \theta_m, \quad (2)$$

where we have introduced the nodal susceptance matrix B_{nm} with

$$B_{nm} = \begin{cases} -b_{nm} & \text{if } n \neq m \\ \sum_m b_{nm} & \text{if } n = m, \end{cases} \quad (3)$$

and P_n denotes the net power injection at node n . In the present article we consider a coarse-grained model of the European electricity grid and for simplicity set $b_{nm} = 1$ for all links [32]. In this case, the nodal susceptance matrix B_{nm} corresponds to the Laplacian of the network [24].

In the DC approximation P_n represents the input to the calculation of the power flows. Solving equation (2) then yields the voltage phase angles, which via equation (1) determine the power flow on the links of the network. The resulting linear relationship between injection pattern and power flow can be expressed using the matrix of *power transfer distribution factors* H_{ln} (PTDF matrix):

$$F_l = \sum_n H_{ln} P_n. \quad (4)$$

Here we use the index $l = l(m, n)$ for the link between nodes m and n . The PTDF matrix $\mathbf{H}$ can be calculated as

$$\mathbf{H} = \mathbf{\Omega} \mathbf{K}^T \mathbf{B}^\dagger, \quad (5)$$

where $\mathbf{B}^\dagger$ denotes the Moore-Penrose pseudo inverse of the nodal susceptance matrix $\mathbf{B}$, the diagonal matrix $\mathbf{\Omega}$ contains the line susceptances on the links l , and $\mathbf{K}^T$ is the transposed incidence matrix with

$$K_{ln}^T = \begin{cases} 1 & \text{if link } l \text{ starts at node } n, \\ -1 & \text{if link } l \text{ ends at node } n, \\ 0 & \text{otherwise.} \end{cases} \quad (6)$$

Note that equation (2) with equation (1) yields the law of flow conservation at the nodes:

$$P_n = \sum_m b_{nm} (\theta_n - \theta_m) \quad (7)$$

$$= \sum_m F_{n \rightarrow m} - \sum_k F_{k \rightarrow n}. \quad (8)$$

The left part of Figure 1 shows the DC power flows at a specific point in time for a simplified model of the European transmission network (see Appendix A). Note that the power flows form a directed acyclic network without loops, which transports power from the exporting nodes (sources) to the importing nodes (sinks) of the system.

2.2 Flow tracing

The method of flow tracing dissects the total flow pattern into strands originating from the different generators [22,29–31]. If one visualises the power flow as a water flow from source nodes (net generators) to sink nodes (net loads), a specific colour can be added to the flow originating from the individual generators. These colour flows then will propagate and mix downstream throughout the total flow, until they end up in the loads at the different sink nodes. The colour mix on the connecting links then represents the partial flows attributed to the generators at the source nodes. This process corresponds to a directed diffusion process on the directed acyclic graph representing the power flows in the network [22,24]. The right part of Figure 1 illustrates the share of the total power flow allocated to two exporting countries in a simplified European transmission network using this flow tracing approach. Note that these partial flows always point in the same direction as the total flow.

Using a more formal description of this method, one seeks the share $P_{n,s}^-$ of the load P_n^- and the share $F_{k \rightarrow n}^{(s)}$ of the total power flow $F_{k \rightarrow n}$, which can be traced back to the generation P_s^+ at node s . Here the notation $P_n^+ = P_n$ for an exporting generator n with $P_n > 0$, and $P_n^- = -P_n$ for an importing load n with $P_n < 0$ is used. That is, the total flow pattern $F_{k \rightarrow n}$ and the injection pattern P_n containing the nodal exports and total imports is the input for this method. The tracing algorithm then yields as outputs the partial flows $F_{n \rightarrow k}^{(s)}$ and the composition of the imports $P_{n,s}^-$ according to the origin of the corresponding power flow. For the method of flow tracing one first assumes partial flow conservation:

$$\delta_{n,s} P_n^+ + \sum_k F_{k \rightarrow n}^{(s)} = P_{n,s}^- + \sum_m F_{n \rightarrow m}^{(s)}. \quad (9)$$

Additionally it has to be determined how these incoming or locally generated shares of the total flow are distributed to the local load and to the downstream neighbours. By following the principle of *proportional sharing* it is usually assumed that all incoming fractions are equally mixed and passed on [29]. This can be written as

$$\delta_{n,s} P_n^+ + \sum_k q_k^{(s)} F_{k \rightarrow n} = q_n^{(s)} P_n^- + \sum_m q_n^{(s)} F_{n \rightarrow m}, \quad (10)$$

where $q_n^{(s)}$ is what we term – following the colour flow visualization – the colour mix passed on at node n , and $P_{n,s}^- = q_n^{(s)} P_n^-$. The colour mixtures can be calculated

iteratively starting from pure sources without any inflows, or by interpreting equation (10) as a matrix equation and solving for $q_n^{(s)}$ [23,33]. Following this method of tracing based flow decomposition the partial flow on line l to the generator n is attributed as

$$F_l^{(n)} = q_{t(l)}^{(n)} F_l. \quad (11)$$

For the notation it is assumed that the directed links l are orientated along the total power flow F_l , and $t(l)$ denotes the tail of the directed link l , that is the node at its origin.

The description of flow tracing in the last paragraphs attributes partial flows to the net generating nodes (exporters) in the network. Flow allocation to importers can be obtained by inverting both the injection pattern $P_n \rightarrow -P_n$ and the flow pattern $F_l \rightarrow -F_l$, and then apply the flow tracing algorithm to these inverted patterns. It is also straightforward to consider a generalisation of the flow tracing method which follows the power originating from specific generation technologies or inflow distributions at the source nodes [23,33].

2.3 Decomposition of injection patterns into import/export patterns

The method of flow tracing dissects the *total power flow* in the network, making use of a directed diffusion process following the principles of flow conservation and proportional sharing. The linearity of the power flow equations in equation (4) suggests an alternative decomposition of the power flow pattern, based instead on a decomposition of the *injection pattern*. For this purpose we decompose the injection pattern into balanced elementary patterns. Associated with the index α , an elementary injection pattern $P_n^{(\alpha)}$ with $\sum_n P_n^{(\alpha)} = 0$ leads to the partial flow pattern

$$F_l^{(\alpha)} = \sum_n H_{ln} P_n^{(\alpha)}. \quad (12)$$

We call a collection of elementary injection patterns $P_n^{(\alpha)}$ a decomposition of a given injection pattern P_n , if $\sum_\alpha P_n^{(\alpha)} = P_n$. The linearity of equation (4) then ensures that the superposition of all partial flows $F_l^{(\alpha)}$ gives the total flow on link l as $F_l = \sum_\alpha F_l^{(\alpha)}$. The input to this method is the PTDF matrix H_{ln} representing the solution of the linearised power flow equations for the network under consideration, and a collection of elementary injection patterns $P_n^{(\alpha)}$ which sum up to the given total injection pattern P_n . Equation (12) then yields as the output the corresponding partial power flows $F_l^{(\alpha)}$. Note that the use of the general index α allows arbitrary decompositions of the injection pattern, for instance according to individual contracts, or identifying the electricity mix (conventional, wind, solar, etc.) injected into the grid at a specific node or subset of nodes [33,34].

For our investigation in this article we select a class of decompositions with an elementary injection pattern associated to every exporting or importing node. At a specific

instant in time, we partition the set of nodes $\mathcal{N}$ into the exporting nodes $\mathcal{N}^+$ with $P_n > 0$, and the set of importing nodes $\mathcal{N}^-$ with $P_n < 0$. If there is a transient node with $P_n = 0$ we add it to the set of exporting nodes. An export decomposition is given by the set of elementary injection patterns $\{P_n^{(s)}\}_{s \in \mathcal{N}^+}$ with

$$P_n^{(s)} = \begin{cases} P_s^+ \delta_{ns} & \text{if } n \in \mathcal{N}^+ \\ -P_{s \rightarrow n}^{\text{ex}} & \text{if } n \in \mathcal{N}^-. \end{cases} \quad (13)$$

The elementary injection pattern $P_n^{(s)}$ thus describes the total power P_s exported by node s through the network, with each sink $n \in \mathcal{N}^-$ covering the share $P_{s \rightarrow n}^{\text{ex}}$ of its respective deficit P_n^- by imports from node s . Given an injection pattern P_n , the components $P_{s \rightarrow n}^{\text{ex}}$ thus have to fulfill the conditions

$$\sum_{s \in \mathcal{N}^+} P_{s \rightarrow n}^{\text{ex}} = P_n^-, \quad (14)$$

$$\sum_{n \in \mathcal{N}^-} P_{s \rightarrow n}^{\text{ex}} = P_s^+. \quad (15)$$

Analogously we define an import decomposition as the set of elementary injection patterns $\{P_n^{(b)}\}_{b \in \mathcal{N}^-}$ with

$$P_n^{(b)} = \begin{cases} -P_b^- \delta_{nb} & \text{if } n \in \mathcal{N}^- \\ P_{n \rightarrow b}^{\text{im}} & \text{if } n \in \mathcal{N}^+. \end{cases} \quad (16)$$

In the following we will consider two specific implementations for export/import decompositions, which are each motivated by a different variant of electricity market trading. *Proportional decomposition* can be interpreted as a stylised model of a centralised market-clearing procedure, in which the participants interact via a central authority, which equates supply and demand based on the submitted bids. In the corresponding decomposition scheme the pooling nature of this market is represented by a homogeneous distribution of imports and exports in the system. In contrary, *random contract decomposition* serves as a coarse model of over-the-counter trading. In this interpretation, the participants engage directly in bilateral transactions without an intermediate central authority. The collection of all these contracts then determines the individual imports and exports. In the following we will consider these two schemes separately, but a mixture of both approaches would be a straightforward generalisation. Although for both implementations our general definition of import/export decompositions allows to define $P_{s \rightarrow b}^{\text{ex}} \neq P_{s \rightarrow b}^{\text{im}}$, the respective entries of the import and export decompositions coincide for a given injection pattern.

2.3.1 Proportional decomposition

In this case the share $P_{s \rightarrow n}^{\text{ex}}$ of exports from node s to the importing node n is given by the share of this node's export with respect to all exports in the system:

$$P_{s \rightarrow n}^{\text{ex}} = \frac{P_s^+}{P^+} P_n^-, \quad (17)$$

with

$$P^+ = \sum_{s \in \mathcal{N}^+} P_s^+ = P^- = \sum_{b \in \mathcal{N}^-} P_b^- \quad (18)$$

the total exports and imports for a balanced system. This elementary injection pattern corresponds to the situation that the exporting node s serves all sinks in the system, with their imports uniformly scaled down in such a way that the pattern is balanced. Analogously we define the import decomposition as

$$P_{n \rightarrow b}^{\text{im}} = \frac{P_b^-}{P^-} P_n^+. \quad (19)$$

With $P^+ = P^-$ it is easy to see that $P_{s \rightarrow b}^{\text{ex}} = P_{s \rightarrow b}^{\text{im}}$.

2.3.2 Random contract decomposition

In this scenario we assume a situation where the import and export decompositions are based on randomly assigned contracts between importers and exporters. In order to ensure that the conditions equations (14) and (15) are fulfilled while circumventing any complications due to finite block sizes of these contracts, we implement the following random assignment algorithm. We start with the node which has the lowest value of imports or exports. For now we assume that this node is an importing node b . We randomly choose one exporter n , which by definition is able to completely cover the respective import of the node first chosen. This procedure defines a contract between this node n and the importing node b with size $c_{n \rightarrow b} = P_b^-$. We now remove the node b from the set of importing nodes $\mathcal{N}^-$, and update the export of node n as $P_n \rightarrow (P_n - c_{n \rightarrow b})$. Then we repeat this procedure until no node is left. If the node with the lowest import or export at some iteration of the algorithm is an exporter s , we randomly choose an importing node n and assume that all (remaining) exports from node s go to this importer n . Note that during this algorithm often contracts between smaller countries are selected initially, with the imports and exports of the larger countries covered among each other in the final iterations. This bias in the distribution is build in on purpose to obtain an import/export pattern which is statistically different from the proportional decomposition discussed above.

After processing the algorithm, we define the import and export compositions based on this random assignment of contracts as follows:

$$P_{s \rightarrow b}^{\text{ex}} = c_{s \rightarrow b} = P_{s \rightarrow b}^{\text{im}}. \quad (20)$$

2.4 Flow tracing vs. decompositions of injection patterns

We want to emphasize some fundamental methodological differences between flow allocation techniques based on decompositions of injection patterns and the tracing based approach. In the former case one defines partial injection patterns and calculates partial flows via the linearised power flow equations. Both the amount of partial

inflow at the net generators and outflow at the net loads are inputs to the calculation. In the latter case of flow tracing the inflow from the generators at the source nodes and the total power flow pattern are the inputs to the algorithm, which yields both the partial flows on the links and the corresponding shares of outflow at the sink nodes. Furthermore, since flow tracing dissects the total flow pattern into different strands from source nodes to sink nodes, the partial flows always follow the orientation of the total flow. In contrast, the superposition of partial flow patterns under the decomposition method allows partial flows which are opposite to the total flow. These partial counter-flows mitigate the total power flow on a transmission line.

As an example, compare Figure 2 to Figure 1, that is the illustration of some partial flows for the simplified model of the European electricity system obtained via the method of flow tracing (Fig. 1) and based on proportional and random contract decomposition (Fig. 2). We observe that flow tracing leads to a decomposition of the import/export pattern based on geographically localised partial flows. The elementary injection patterns obtained from proportional decomposition on the contrary lead to partial flows spanning the whole system. For the random contract decomposition we observe a pattern with partial flows from Germany covering the south, whereas the partial flows from Norway are mostly restricted to the north. Note that for both implementations of decomposing injection patterns numerous counter-flows occur in the system.

3 Power flow decomposition in a simplified model of a highly renewable European electricity network

As a test case we apply the methods from the last section to a simplified model of a highly renewable European electricity system (see Appendix A for details about the model), and discuss the import/export transfer functions as well as the nodal usage of transmission link capacities. Five years of data from 2010 to 2014 with hourly resolution for the load and projected renewable generation are used, leading to 43 822 different injection patterns $P_n(t)$ and resulting flow patterns $F_l(t)$.

3.1 Export and import transfer functions

Both methods, flow tracing and decomposition into elementary patterns, do not only lead to a decomposition of the power flow in the system, but also to a partition of the injection pattern which associates the exports from one country to the imports of another. Whereas for the decomposition method this partition is an input into the method which yields a decomposition of the flow pattern, for the method of flow tracing these import/export patterns are a result of the algorithm (see Sect. 2.4).

Given the whole time series, we quantify these import/export patterns using the following export transfer

Fig. 2. Decomposition of injection patterns: illustration of the partial flows of Germany (DE) and Norway (NO) at an exemplary time step in a simplified model of the European electricity system (see the left part of Fig. 1 for the total power flow). The left figure shows the partial flows assigned to Germany (blue) and Norway (orange) according to the proportional decomposition scheme, whereas the right figure follows the random contract decomposition scheme. The scaling of arrow sizes and the respective threshold are shown in this figure. To allow for a clearer visualisation, the partial flows associated with Spain are not shown in this figure.

Fig. 3. Export transfer function based on flow tracing (blue), proportional decomposition (green), and random contract decomposition (red). Top left: Germany (DE). Top right: Great Britain (GB). Bottom left: Denmark (DK). Bottom right: Lithuania (LT). The transfer functions resulting from the application of the flow tracing method emphasize transfers between geographically close countries, whereas the proportional decomposition yields a more homogeneous, delocalised distribution. The random contract decomposition scheme leads to transfer functions showing stronger exports among the larger countries as well as among the smaller countries, respectively, resulting from an implicit bias in the underlying contract algorithm.

function [22]:

$$\mathcal{E}_{n \rightarrow m} = \frac{\langle q_m^{(n)} P_m^- \rangle}{\langle P_n^+ \rangle}, \quad \mathcal{E}_{n \rightarrow m} = \frac{\langle P_{n \rightarrow m}^{\text{ex}} \rangle}{\langle P_n^+ \rangle}. \quad (21)$$

Here the first definition refers to the method of flow tracing, whereas the second definition corresponds to the decomposition method. This export transfer function gives the share of the average exports of a country n which are associated with imports from a country m . An import

transfer function can be defined analogously, but gives very similar results for the set-up chosen for our simplified system with $\langle P_n^+ \rangle = \langle P_n^- \rangle$ (see Appendix A).

In Figure 3 we display the export transfer function based on flow tracing and the two injection pattern decomposition schemes for four selected countries: Germany (DE), Great Britain (GB), Denmark (DK), Lithuania (LT). We observe that, as expected, the method of flow tracing yields an export transfer function which emphasizes transfers between geographically close countries,

Fig. 4. Statistics of total power flow and partial flow attributed to Germany for the link from France to Germany (top) and from France to Great Britain (bottom). The partial flows are obtained using the method of flow tracing (left) and the decomposition method with proportional decomposition (middle) and random contract decomposition (right). The method of flow tracing yields partial flows which are always oriented in the same direction as the total flow, whereas partial flows based on proportional or random contract decomposition can be oriented opposite to it. These counter flows are emphasised with red colour in the figure.

see for instance exports from Denmark to Sweden (SE), Norway (NO) and Finland (FI), or from Great Britain to France (FR). The corresponding localisation of the partial flows has already been illustrated in Figure 1. The elementary injection patterns based on proportional decomposition yield a more homogeneous, delocalised distribution, which is mainly determined by the correlation between the imports and exports of the respective countries:

$$\mathcal{E}_{n \rightarrow m} = \frac{\langle P_{n \rightarrow m}^{\text{ex}} \rangle}{\langle P_n^+ \rangle} = \frac{\langle \frac{P_n^+ P_m^-}{P_n^+} \rangle}{\langle P_n^+ \rangle}. \quad (22)$$

The results for the random contract decomposition mirror the bias in the underlying contract algorithm: Germany as a country with a comparatively large load often exports to other large countries in the system, that is France, Italy (IT), or Great Britain, whereas Lithuania shows an export distribution characterised by exports to other smaller countries. Also consult Figure 2 for exemplary partial power flow patterns which illustrate the import/export patterns obtained via proportional and random contract decomposition.

3.2 Usage of transmission capacity

The patterns of imports and exports discussed in the last section are associated with power flow patterns, which

make use of the transmission capacity of the grid. Analogously to [15,22] we define the transmission capacity $\mathcal{K}_l$ of a link l as the 99% quantile of the flow distribution $\mathcal{P}(f_l)$:

$$0.99 = \int_0^{\mathcal{K}_l} \mathcal{P}(f_l) df_l. \quad (23)$$

Here $f_l = |F_l|$ denotes the absolute flow on a link l . We want to compare methods of capacity allocation which assign a usage share $\mathcal{K}_l^{(n)}$ of the total link capacity to the different nodes n of the network. We use the usage measure introduced in [22], which is based on the ensemble of total and partial power flows $(F_l, F_l^{(n)})$:

$$\mathcal{K}_l^{(n)} = \int_0^{\mathcal{K}_l} \frac{d\mathcal{K}}{1 - \mathcal{P}_l^C(\mathcal{K})} \int_{\mathcal{K}} \mathcal{P}_l(f_l) \left\langle \frac{F_l^{(n)}}{F_l} \middle| f_l \right\rangle df_l. \quad (24)$$

This measure incorporates correlations between the associated partial flows and the total power flows on a link and thus provides a fairer capacity allocation compared to simplified approaches using the average load, average import/export or average partial flow [22,23]. In Figure 4 we illustrate the statistics of the total power and the partial power flow attributed to Germany for the link from France

Fig. 5. Partial transmission capacities $\mathcal{K}_l^{(n)}$ of all links in the simplified European electricity network assigned to the different countries according to the network usage measure defined in equation (24). The underlying partial flows $F_l^{(n)}$ have been obtained using the method of flow tracing (left) and proportional decomposition (right). The random contract decomposition leads to results which are very similar to the proportional decomposition and are not shown. The transmission capacities are less distributed for the flow tracing method due to a stronger localisation of the partial flows. The occurrence of counter flows leads to the allocation of some negative partial capacities under the proportional decomposition method, for instance on the link between Norway (NO) and Denmark (DK).

to Germany and from France to Great Britain. The results are based on the different methods of flow decomposition discussed in the previous sections. For simplicity, here we only consider situations in which Germany is an exporter. Considering the flow tracing method, we observe that Germany only gets assigned partial flows in the direction towards France. This is due to the consideration of exports only, which will never be oriented in the opposite direction. In contrast, for the more remote link between France and Great Britain, Germany gets assigned partial flows in both directions. By definition, using the flow tracing method, we never observe counter flows, or partial flows larger than the total flow on a link. This is different for the partial flows based on the decomposition methods. Situations in which Germany is exporting power to France despite a total power flow in the opposite direction are represented as partial counter flows in these schemes. We also observe partial power flows exceeding the total flow. In this case, the partial flows associated with Germany are dominating the total power flow, which is reduced by opposing contributions from other countries. The same features are visible for the more remote link between France and Great Britain. With respect to the difference between the proportional and random decomposition scheme, we observe a more scattered distribution for the latter. These larger partial flows result from random contracts exceeding the respective proportional decompositions.

Based on the respective total and partial power flow statistics, in Figure 5 we show the share of transmission capacity $\mathcal{K}_l^{(n)}/\mathcal{K}_l$ assigned to the different nodes for each link in the network, based on the different methods of flow decomposition discussed in the previous sections. For each method we calculate the usage measure according to imports and exports separately, and then display the average over both these results. Overall we observe that despite the significant differences in the underlying patterns at individual time steps, the statistical measure in equa-

tion (24) leads to similar results for the capacity allocation of the different links for the three methods. However, as for the import/export pattern the flow tracing based decomposition leads to more localised partial flows and thus a less distributed attribution of transmission usage. A noteworthy feature of the capacity allocation derived from the decomposition method is the occurrence of *negative* capacity allocations. This negative allocation results from a high share of counter-flows assigned to the respective countries, which lead to a reduction of the total power flow and thus savings in transmission capacity. These negative allocations are observed for both proportional and random contract decomposition, most prominently on the link between Norway (NO) and Denmark (DK). We finally compare the total transmission capacity costs allocated to the system nodes according to equation (24) for the different methods of flow allocation. We approximate the cost of a transmission line $l = m \rightarrow n$ as the transmission capacity $\mathcal{K}_l$ and the distance d_l between the capitals of the countries represented by node n and m . The total transmission cost in the network is then given by

$$M = \sum_l d_l \mathcal{K}_l. \quad (25)$$

A share of this total transmission cost is associated with country n due the capacity allocation $\mathcal{K}_l^{(n)}$:

$$M_n = \sum_l d_l \mathcal{K}_l^{(n)}. \quad (26)$$

In Figure 6 we compare the cost allocation M_n derived from the flow decomposition according to the three different methods presented in this article with a simplified allocation proportional to the average import/export of the individual nodes:

$$M_n = \frac{\langle |P_n| \rangle}{\sum_k \langle |P_k| \rangle} M. \quad (27)$$

Fig. 6. Top: share of the total transmission system cost allocated to the different nodes of the network. The cost allocation is based on the usage measure in equation (24), with the partial flows determined either via flow tracing or the method of decomposing injection patterns (proportional and random contract decomposition). As a comparison, a simplified measure based on the average import/export of the respective nodes is also displayed. Bottom: this figure shows the same results as the top figure, but now as the relative difference compared to the simplified average import/export measure.

We observe that these aggregated statistical measures of transmission capacity based on different flow allocation methods yield very similar results. This accordance contrasts with the significantly different partial flows according to the flow tracing method or decomposition method at individual time steps (see Figs. 1 and 2). After statistical aggregation over the whole time series and the whole network, for the considered methods the average import/export of a country is the most determining factor for the allocation of network usage. The variances visible in Figure 6 due to the application of different decomposition methods depend on the geographical location of the respective country in the network, and in particular on the spatio-temporal correlations in the injection and resulting power flow patterns. While it is possible to trace back the influences underlying these differences by considering link specific partial and total power flow statistics (see Fig. 4) or individual system snapshots (see Figs. 1 and 2), it is difficult to deduce general trends for the aggregated usage measure. An exception is the localisation of partial flows for the flow tracing method as illustrated in Figure 1, which benefits four of the five largest importers/exporters, while assigning a higher share of transmission capacity to smaller countries at the periphery like Finland (FI), Serbia (RS) or the baltic countries [22].

4 Conclusion

Flow tracing uses a directed diffusion process on the total flow graph to assign power flows to generators or loads in electricity networks [22,29,30]. Although the downstream

dissection of the total flow into different strands emerging from the individual source nodes represents an intuitive technique for the flow allocation problem, this approach implies limitations for possible economic interpretations: the application to the total flow graph does not allow to associate power flows to individual trade contracts or other bilateral import/export patterns. In particular, partial power flows due to commercial exchanges which are not aligned with the overall total power flow (as for example a possible contract between Germany and France in Fig. 1) cannot be represented by this method. In this contribution, we propose an alternative method of flow allocation, which decomposes the overall injection pattern into elementary injection patterns. These patterns can be associated with individual contracts or other commercial exchange patterns. Two different variants of this decomposition method are compared with the flow tracing technique by applying them to a simplified model of a highly renewable European electricity system. We observe that the respective partial flow patterns differ significantly for individual network flow events. These differences are still visible in the resulting partial flow statistics evaluated for a time series of fluctuating injection patterns. For the event-averaged import/export patterns and an aggregated statistical measure of total network capacity usage, the methods studied in this contribution yield similar results, which are mostly driven by the respective average imports and exports of the individual countries.

The decomposition method introduced in this contribution depends on the linearity of the equations describing the power flow in the network. Accordingly, for the application of this method, the accuracy of the DC

approximation to the full AC power flow equations has to be assured. Whereas this usually is the case for the stable operation in high-voltage grids, for the power flow in distribution grids, in particular with infeed from renewable generation, one often has to consider voltage support or reactive power management not represented in the DC approximation [35]. On the other hand, the linear relationship between link flow and nodal injection is not exclusive to the DC power flow model, but also occurs for instance in models of network diffusion [27], hydraulic networks [25] or vascular networks [26]. Accordingly, the decomposition method can also be straightforwardly applied to such models, with a suitable choice of elementary injection patterns depending on the specifications of the system.

With respect to the application to electricity systems, it would be interesting to compare the decomposition method of injection patterns with further methods of flow allocation, for instance with marginal participation methods which also make use of the matrix of power transfer distribution factors [34,36,37]. The application to a more detailed model of the electricity system, in particular with a more realistic network representation and market-based dispatch schemes, could allow to learn more about the economical implications of the methods of flow allocation presented in this contribution. We emphasise that the analysis presented in this paper assumes an ex-post allocation of usage or costs to the users of the transmission system, that is the injection pattern itself is not influenced by this allocation. For an efficient design of the electricity system, both the short-term operational and the long-term investment decisions of the system users should incorporate the resulting transmission costs. This feedback process proposes interesting research questions for simplified models of networked electricity systems, incorporating methods from game theory or control theory. We also expect that for such a feedback loop the influence of the specific choice of the allocation method increases significantly. If for example congestion-relieving counter-flows are rewarded, the network nodes are incentivised to cause flows opposite to the flows attributed to the remaining nodes. From the system perspective, this concept then shows some similarity to the principles found in minority games [38].

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Author contribution statement

M.S. and M.G. developed the method introduced in the article and wrote the paper. S.H. performed the numerical calculations under supervision of S.S. in early stage of the project. B.T. performed the numerical calculations in the later stage of the project and created all figures.

Appendix A: A simplified model of a highly renewable European electricity network

We aggregate the European system to one node per country, with links between the nodes representing transmission capacity between the respective countries. This leads to a network with 30 nodes and 52 links; see Figure 1. The depicted network topology was used, among others, in [15,16,22], and is based on the layout reported by ENTSO-E (European Network of Transmission System Operators for Electricity) [39]. The links between Sweden and Lithuania, and between Poland and Lithuania have been commissioned at the end of 2015 and were added in this network compared to the topology used in [15,16,22].

We demand that for every node n at any time t the following balancing condition is fulfilled:

$$G_n^R(t) - L_n(t) = P_n(t) + B_n(t). \quad (\text{A.1})$$

In this expression $G_n^R(t)$ denotes the renewable generation and $L_n(t)$ the load. The term $B_n(t)$ refers to the nodal balancing, and $P_n(t)$ to the import or exports injected into or withdrawn from the network. The left side of this equation is data-driven: the aggregated load per country $L_n(t)$ at hour t is given by historical data provided by the European Transmission System Operators [40], whereas the renewable generation $G_n^R(t)$ is based on data from a Renewable Energy Atlas [12,41]. Both data sets comprise the years from 2010 to 2014 with hourly resolution, leading to 43 822 different time steps. The renewable generation data is obtained by converting spatio-temporal wind velocity and solar radiation fields into wind and solar power generation time series, which are scaled in such a way that *on average* the load in each country is completely covered:

$$\langle L_n \rangle = \langle G_n^R \rangle = \langle G_n^W \rangle + \langle G_n^S \rangle. \quad (\text{A.2})$$

Here G_n^W denotes the wind power generation, whereas G_n^S gives the solar power generation from photovoltaics. For the modeling underlying the results in the present article a fixed wind fraction of $\langle G_n^W \rangle / \langle G_n^R \rangle = 0.7$ has been used. Other fluctuating forms of renewable generation are neglected, or assumed to be included into the backup generation capacity. Despite the relation in equation (A.2), at a specific hour t in general there will be a mismatch $\Delta_n(t)$ between the renewable generation $G_n^R(t)$ and the load $L_n(t)$:

$$\Delta_n(t) = G_n^R(t) - L_n(t). \quad (\text{A.3})$$

This mismatch has to be balanced by a combination of nodal power import or export $P_n(t)$, and nodal balancing $B_n(t)$. The latter describes dispatchable backup power generation $G_n^B(t) = -\min(B_n(t), 0)$ when $B_n(t) < 0$, or curtailment $C_n(t) = \max(B_n(t), 0)$ of excess power when $B_n(t) > 0$. The net power injection $P_n(t)$ refers to exports $P_n^+(t)$ for $P_n(t) > 0$, or to imports $P_n^-(t) = -P_n(t)$ for $P_n(t) < 0$.

Given a time series of renewable generation and load, different combinations of nodal balancing power and imports and exports will fulfill the balancing equation (Eq. (A.1)). For simplicity and to allow a comparison with previous studies we follow the approach of *synchronised balancing* [32], where the global mismatch is distributed to the individual nodes proportional to their average load:

$$B_n(t) = \left[\sum_m \Delta_m(t) \right] \frac{\langle L_n \rangle}{\sum_k \langle L_k \rangle}. \quad (\text{A.4})$$

This choice of a balancing scheme fixes the power in- and outflows $P_n(t)$ at the individual nodes as

$$P_n = \Delta_n(t) - \alpha_n \Delta(t), \quad (\text{A.5})$$

where we have used the following abbreviations:

$$\Delta(t) = \sum_n \Delta_n(t), \quad (\text{A.6})$$

$$\alpha_n = \frac{\langle L_n \rangle}{\sum_k \langle L_k \rangle}. \quad (\text{A.7})$$

Note that this set-up with equations (A.2) and (A.4) leads to $\langle P_n \rangle = 0$ or equivalently $\langle P_n^+ \rangle = \langle P_n^- \rangle$, that is the average power imports and exports of every node coincide [23].

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