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Reservation policies of advance orders in the presence of multiple demand classes

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Abstract:

We consider an inventory system where customers provide advance order information. Specifically each customer order contains two attributes: a request date when the order is received and a due date specifying when the customer wants the order delivered. We assume M different customer classes whose demands are generated by independent Poisson processes. All customers in the same class have the same constant demand lead-time (the difference between the due date and the request date), and all demand lead-times are less than the replenishment lead-time of the inventory system. The inventory system is operated as a base-stock system where a replenishment order is issued instantaneously upon the receipt of a customer order. A key characteristic is that customers with a long demand lead-time can be overtaken by customers with a shorter demand lead-time. Therefore, a potentially important issue is: should items on the inventory be reserved for specific order requests, and if so, when? Here it is understood that after the reservation, the item cannot be used to fill other customer requests. We describe a general reservation policy, meaning that to each customer class a given reservation delay arises between receipt of an order and the reservation. Assuming that the revenue of an order depends on the demand lead-time, we propose a profit optimization model where the expected profit is the difference between the expected revenue and the expected inventory holding costs. We derive the profit function for the general reservation policy and find the optimal base stock level and the reservation policy by exhaustive search. In our numerical results we find cases where, by using the general reservation policy, there are improvements by more than 5% compared to either making no reservations at all, or making a complete reservation (that is, reservation immediately upon receipt) of items to incoming orders. Our numerical results also show that instead of using a general reservation policy, one can, without much loss of optimality, resort to a simpler reservation policy, denoted Backward Delay, BD .

Keywords: Inventory, advance demand information, heterogeneous demand

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1. Introduction

We consider an inventory system where customers do not necessarily expect instantaneous delivery (like if they showed up physically at the inventory system). Rather they submit their order requests (for instance by e-mail or via a web-page organized by the inventory system), and when submitting their order requests they also specify a future point in time when they want to receive their order. Such a situation is realistic in a business to business (B2B) context. The inventory system could be placed somewhere upstream in a supply chain, say a central warehouse serving different independent ‘business entities’. It might be that there are different levels of communication exchange between these business entities and the central warehouse. In some cases there is a very close cooperation such that the business entities well in advance, for instance by giving the central warehouse access to their information systems, specify future order requests, in combination with customer orders that are freezed when entering a certain time window. With others there is less information exchange and therefore customer orders are received with a shorter notice. The situation could also occur in a business to consumer (B2C) setting when sales are organized through the internet.

Advance order information or advance demand information, as often denoted in the literature, means that if a request for an order occurs, say at time τ , the customer also specifies a point in time, say at time $y + \tau$, when the order should be delivered. Thus, y can be interpreted as the **demand lead-time** of the customer. Several textbooks on inventory control, like Silver et al. (1998) and Zipkin (2000) tacitly assume that $y = 0$ (though the issue is raised in Exercise 7.8 of Silver et al. (1998) and Exercise 6.1 of Zipkin (2000)). One of the first papers to make a systematic analysis of advance demand information in inventory control systems is Hariharan and Zipkin (1995). Here all customers are assumed to have the same demand lead-time. The analysis of this system is equivalent to the analysis of a conventional system (with no advance demand information) but with an equal reduction in the replenishment lead-time. Of course, although the concept advance demand information is rather new, the general subject matter: to study inventory systems with some ‘early warning signals’ is not new and has been studied earlier than the contribution of Hariharan and Zipkin (1995). See for instance the analysis in Brown et al (1971). Another study is done in Gallego and Özer (2001) assuming a periodic review model where customers can have different demand lead-times. Contrary to Hariharan and Zipkin (1995) they assume that demand lead-times can be longer than the constant replenishment lead-time. They prove that a state dependent (s, S) policy is optimal. Lu et al. (2003) and Marklund (2006) extend the model of Hariharan and Zipkin (1995) by studying multi-level systems; the first studies an assembly system while the second studies a divergent distribution system. In particular Marklund (2006) is a source of inspiration for this paper because he introduces the idea of a reservation policy to be applied at the upper level echelon. The papers above only consider the inventory control side. However, if customers use different demand lead-times, the revenues of the orders should also depend on the demand lead-time. Except for a work by Chen (2001) studying advance demand information and market segmentation, the revenue side has not been linked to the inventory side when studying inventory control in the presence of advance order information. Our aim is to make a more easily accessible performance evaluation than that of Marklund (2006). Furthermore our aim is to make a link to a revenue model, acknowledging that the revenue varies with the demand lead-time. The ultimate aim, of course, is to develop a profit maximization model to find the optimal base stock level and the optimal reservation policy. For other contributions on advance demand information see also Tan (2008), Wang and Toktay (2008), Gayon et al (2009), Papier and Thoneman (2009), Benjafar et al (2011),

Huang and Van Mieghem (2013) and Bernstein and DeCroix (2015). In finalizing this literature review we should also mention another line of research studying which sort of inventory control policies should be applied in the presence of heterogeneous demand. When examining the literature (see Topkis (1968), Evans (1968), Kaplan (1969), Frank et al. (2003) and Tempelmeier (2006) for periodic review models and Nahmias and Demmy (1981), Dekker et al. (1998, 2002), Melchioris et al. (2000), Melchioris (2003) and Deshpande et al. (2003) for continuous review models) they all assume that the customer classes can be differentiated by introducing a rationing policy. A rationing policy means that low priority customers are denied access to the inventory when the on-hand level is critically low. The reservation policies we consider in this paper do have some resemblance to rationing. But the implementation is not based on critical numbers on the on-hand inventory but rather on time instead. This will later be illustrated in Section 3.

Our inventory system serves in all M customer classes where all customers belonging to customer class i ($i=1,...,M$) have a constant demand lead-time y_i . The demand in each customer class follows independent Poisson processes. We assume the inventory system is controlled by a base-stock system (implying negligible fixed replenishment costs) meaning any replenishment order is issued instantaneously upon the receipt of a customer order and having the same size (of size I) as the customer order. We assume complete backordering. In addition, we assume the lead-time of any replenishment order is a constant L and this replenishment lead-time is larger than any of the M demand lead-times. The aim is to find an optimal base-stock level in combination with setting the parameters of the reservation policy such that the expected profit is maximized. The key element in our paper is to study reservation policies. One option is to let an item on the inventory be reserved for the order immediately (and in case of no 'free' items on the inventory it will be the first unreserved item arriving to the inventory system). It means that it cannot be accessed by other customers, whose requests arrive later. This is denoted as **Complete Reservation (CR)** by Marklund (2006). The other extreme is just to allocate an item for the order upon its due date, that is, not in advance to reserve any item for the order. This is denoted as **No Reservation (NR)** in Marklund (2006). Another option is first to reserve an item after a certain time delay. We then denote it as a **Partial Reservation** policy. In general this can be modelled by a function $g()$, such that a customer making a request at time τ with a demand lead-time y will have an item from the inventory reserved for the order at time $\tau + g(y)$. The reservations are made on a FCFS (First Come First Served) basis. Logically, the function $g(y)$ is nonnegative and $g(y) \leq y$ for all possible values of y . Using a partial reservation policy instead of No Reservation seems reasonable in order to provide better service to those customers who provide information about their needs well ahead in time. Using a partial reservation policy instead of Complete Reservation policy may be sensible if the demand lead time of the customer is long that keeping some unreserved amount on inventory helps give adequate service also to customers with a short demand lead-time. The crucial thing, of course, is to investigate how to trade off these two conflicting considerations.

In Section 2 we will specify in more detail the assumptions behind the inventory system in general terms and specify the profit function. The profit function requires evaluation of the fill rate offered to customers of any class. It also depends on the average on-hand inventory. The mathematical derivations are a bit complex as the sequence of arrivals differs from the sequence in which customers are served. Therefore, we start in Section 3 by a numerical example to illustrate the basic ideas behind our derivations. Thereafter, we show in Section 4 how the expressions for the fill rates and the average on-hand inventory can be derived mathematically when assuming a general

reservation policy specified by the function $g(y)$. In Section 5 we will present the results of our numerical investigations. Finally, Section 6 contains some concluding remarks.

2. The mathematical model

We assume the inventory system serves in all M customer classes where $M \geq 1$. The demand in each customer class follows independent Poisson processes and the arrival/demand intensity of Class i is λ_i arrivals per time unit. Denote by $\Lambda = \sum_{i=1}^M \lambda_i$ the total demand/arrival rate. All customers of Class i have a demand lead-time y_i where $0 \leq y_1 < \dots < y_M$. That is, if an order request occurs at time τ and the customer has a demand lead-time y , then the customer wants to get the demand fulfilled at time $\tau + y$. We assume that the customer's preferences for demand lead-times are outside the control of the inventory system.

The inventory is controlled by a base-stock policy with base-stock level S (a non-negative integer). All un-filled demand is backlogged. We follow the standard assumption that replenishments, which all have a constant lead-time L , are made exactly at the point in time when an order request is made. We assume $y_M < L$, as any order, with a demand lead-time longer than the replenishment lead-time, can have its delivery timed exactly to the due date thereby incurring no costs. Denote by $\bar{y} = \frac{1}{\Lambda} \sum_{i=1}^M \lambda_i y_i$ the average demand lead-time. For any general reservation policy specified by the function $g()$, fulfilling $0 \leq g(y) \leq y$ for any value of y with $0 \leq y < L$, denote by $\bar{g} = \frac{1}{\Lambda} \sum_{i=1}^M \lambda_i g(y_i)$ the average delay from the receipt of an order until the reservation.

We assume that the inventory system gets different revenues per order depending on its demand lead-time. Specifically we assume it has a functional form $R_1(y)$. Furthermore this revenue is only collected if the order is fulfilled in due time. Otherwise the revenue function is $R_2(y)$, where $R_1(y) > R_2(y)$ for all $0 \leq y \leq L$. It seems natural to assume that those customers who offer a long demand lead-time are rewarded by being charged a lower price. Therefore we assume the revenue function $R_1(y)$ to be decreasing in y . A similar assumption is made in Chen (2001). However, contrary to Chen (2001) we use a function $R_2(y)$ instead of a constant, for modelling the revenues of non-fulfilled orders. In our opinion it is reasonable to assume that customers who provide a long demand-lead time will also be more disappointed if they do not get immediate fulfilment compared to the customers who provide a short demand lead-time. Therefore we will assume that $R_1(y) - R_2(y)$, which can be interpreted as an implicit backorder cost, is increasing in y . Denote by $FR_i^g(S)$ the fill rate to class i customers when using the general reservation policy (specified by the function $g()$) and using a base stock policy with base stock level S . Then the revenue per time unit is

$$\sum_{i=1}^M \lambda_i \left(FR_i^g(S) (R_1(y_i) - R_2(y_i)) + R_2(y_i) \right) \quad (1)$$

We assume a purchase cost c per unit, and as we assume a backlog model the total purchase costs per time unit are therefore $c\Lambda$. So in principle this is just a fixed cost that does not affect the specification of the optimal policy. However, the unit purchase cost has significance for the specification of the inventory cost parameter h which is the inventory cost per unit and per time unit. Because under a commonly applied convention, see Silver et al (1998), one assumes an annual inventory carrying charge rate, say 20% per year, and then specifies h (when measured per unit per

year) as this percentage of the unit cost c , for the example above meaning $h=0.2c$. This convention will be applied later in our numerical analysis. Denote by $I^g(S)$ the average on-hand inventory when using the general reservation policy and using a base stock policy with base-stock level S . Therefore, the profit function (measuring profit per time unit) is:

$$\Pi^g(S) = \sum_{i=1}^M \lambda_i \left(FR_i^g(S) (R_1^n(y_i) - R_2^n(y_i)) + R_2^n(y_i) \right) - hI^g(S) \quad (2)$$

Here $R_1^n(y_i) = R_1(y_i) - c$ and $R_2^n(y_i) = R_2(y_i) - c$ are the net-revenue functions. The aim is to solve optimization problem:

$$\min\{\Pi^g(S): S \geq 0, 0 \leq g_i \leq y_i, i = 1, \dots, M\} \quad (3)$$

The optimal values $g_i = g(y_i)$ specify the general reservation policy. So we do not compute an optimal delay function $g()$. Rather we compute for each customer class, its optimal reservation delay. The optimization of (3) is conducted through exhaustive search.

After presenting an illustrative numerical example in the next section, we mathematically derive the fill rates and the average on-hand inventories under a general reservation policy in Section 4.

3 An illustrative numerical example

The details of the numerical example can be seen in Table 1. In the first four columns are the data of the arrival process: the arrival number, when the arrival occurs, which type of customer and when will the order be reserved. Inspired by Axsäter (1990) we use the terms ‘considered-order’ and ‘considered-item’ to illustrate the interrelated movements in and out of the inventory. To each ‘considered-order’ (with data in the first 4 columns) is a ‘considered-item’ which is the replenishment of this ‘considered-item’ that is used for serving the ‘considered-order’.

<Table 1 about here>

As the system starts with a full inventory of 6 units (because the base-stock level is 6), the ‘first’ 6 customers will be served from the initial inventory. It concerns arrival no 4, 1, 2, 3, 6 and 11. These are the first in the sense that they are the first to be reserved. Here we have denoted ‘considered-item’ as zero, meaning these ‘considered orders’ are served from the initial stock and the ‘considered-item’ designated to serve the ‘considered-order’ has a sojourn time at the inventory which is the value of the arrival time plus the demand lead-time of the ‘considered order’. For all the remaining customer orders, the ‘considered-item’ will be the replenishment of an earlier received customer order. As an example let arrival no. 22 play the role of a ‘considered-order’. As being of Class 1 it will surpass all Class 2 orders received after time $25.40 + 2 - 16 = 11.40$. In the example this number is 5. It surpasses also all Class 3 orders received after time $25.40 + 2 - 7 = 20.40$. In the example this number is 1. Therefore, the ‘considered-order’ is no. $22 - 5 - 1 = 16$ in the priority list, and it will be served from the replenishment of customer order no. $16 - 6 = 10$. We subtract 6 due to the base-stock level of 6. Customer order no. 10 was received at time 12.06 and the replenishment will therefore arrive to the inventory at time 32.06. As the ‘considered-order’ has its due date at time $25.40 + 10 = 35.40$, it means this ‘considered-item’ will sit in inventory in

$35.40 - 32.06 = 3.34$ time units. Another example is if the ‘considered-order’ is arrival no. 16, arriving at time 19.66, which is a Class 2 customer. It will be surpassed by a total of 5 Class 1 customers arriving in the time interval from 19.66 until $19.66 + 16 - 2 = 33.60$ and it will be surpassed by a total of 3 Class 3 customers arriving in the time interval from 19.66 until time $19.66 + 16 - 7 = 28.66$. Therefore, this ‘considered-order’ has reservation no. $16 + 6 + 3 = 25$ and it is served by the replenishment of customer no. $25 - 6 = 19$ which arrives at time 23.72. So here we have an example where the ‘considered-item’ is first dispatched to the inventory system after the ‘considered order’ arrived. This, of course, is due to fact that Class 2 has low priority and is passed in priority by future incoming Class 1 and Class 3 orders. So in this case the ‘considered-order’ will have to wait until being served by its ‘considered-item’ which is why its inventory sojourn time is zero. Finally, we make an illustration of the case where the ‘considered-order’ is of Class 3. Here we use arrival no. 19 as example. It is surpassed by a total of 3 Class 1 customers arriving in the time interval from 23.72 until $23.72 + 7 - 2 = 28.72$ and it surpasses by a total of 3 Class 2 customers arriving after time $23.72 + 7 - 16 = 14.72$. Therefore, it has reservation no. $19 + 3 - 3 = 19$ and is served by the replenishment of the demand of customer no. $19 - 6 = 13$ occurring at time 15.49. This ‘considered-item’ arrives to the inventory at time 35.49 slightly before the due date of the ‘considered-order’ at time $23.72 + 12 = 35.72$, meaning the ‘considered-item’ will have a sojourn inventory time of $35.72 - 35.49 = 0.23$ time units.

From the example we see that none of the Class 2 customers will be served in due time. This is because this class is most often surpassed by future Class 1 and Class 3 customers. We also see that Class 1 enjoys from the presence of Class 2 customers, because their arrivals generate a supply to the inventory system that Class 1 can exploit.

4 Derivation of fill rates and inventory costs

Consider a customer of Class i which makes a request for an order at time τ with a due date $\tau + y_i$ and a reservation date $\tau + g(y_i)$. Like in the previous section, we denote it the ‘considered-order’. As we do not know the exact point in time when the triggering of the replenishment of the ‘considered-item’ occurred, we define that the triggering took place at the random point in time: $\tau - W_i^g(S)$. The random variable $W_i^g(S)$ specifies the time interval from when the triggering took place and the arrival time of the ‘considered-order’. In the last column of Table 1 we have for our sample path recorded the observations of these random variables $W_i^g(S)$ (stated with blanks where the ‘considered item is 0). As an example look at arrival no. 22 (the ‘considered order’) in Table 1 which is a customer order of Class 1 that arrived at time 25.40. It will be served by the replenishment of arrival no 10 (the ‘considered-item’) that arrived at time 12.06. Therefore in this case the random variable $W_1^g(S)$ attains the value $25.40 - 12.06 = 13.34$. Note that $W_i^g(S)$ can be negative as well as zero (the last case occurs when the replenishment of the ‘considered-item’ is triggered by the demand of the ‘considered-order’, see examples for this in Table 1). But obviously for the specification of fill rates and inventory costs, it is only of interest to consider when $W_i^g(S)$ is positive. As the replenishment of the ‘considered-item’ will arrive to the inventory at time $\tau - W_i^g(S) + L$, it now follows that the ‘considered-order’ is served due time if $\tau - W_i^g(S) + L \leq \tau + y_i$ which is equivalent to $W_i^g(S) \geq L - y_i$. This implies that $P(W_i^g(S) \geq L - y_i)$ is the fill rate of Class i customers. Furthermore, the total time the replenishment of the ‘considered-item’ sits in inventory is specified by $E[W_i^g(S) - L + y_i]^+$, where x^+ is the usual notation for $\max\{0, x\}$.

We introduce some notation. For a Poisson process of intensity λ and $s < t$ denote by $N_\lambda[s, t)$ the number of arrivals in time interval $[s, t)$ which is Poisson distributed with mean $\lambda(t-s)$. Also, define by $Po(\lambda)$ a random variable that is Poisson distributed with mean λ .

In order to specify the probability distribution of $W_i^g(S)$ we use an argument that resembles the analysis of a single stage model with guaranteed service from Graves and Willems (2000). We focus on establishing when $W_i^g(S) \geq t$ where t is some given value. The total accumulated supply to the inventory system from time 0 to time $\tau - t + L$ is

$$S + N_A[0, \tau - t) \quad (4)$$

Here we assume starting inventory is the base stock level S (if not, a replenishment will be triggered to raise the inventory level to S). Using that the total arrival process is an aggregation of M independent arrival processes, we can also write (4) as

$$S + \sum_{j=1}^M N_{\lambda_j}[0, \tau - t) \quad (5)$$

The total accumulated demand from time 0 until time $\tau + g(y_i)$ that will be reserved ahead of the ‘considered-order’ is

$$\sum_{j=1}^M N_{\lambda_j}[0, \tau + g(y_i) - g(y_j)) \quad (6)$$

Therefore $W_i^g(S) \geq t$ if (5) exceeds (6). In order to make a more operational expression for this, we need, for each customer class j , to compare the ‘supply part’ $N_{\lambda_j}[0, \tau - t)$ to the ‘demand part’ $N_{\lambda_j}[0, \tau + g(y_i) - g(y_j))$. We introduce some more notation. Define for each customer class i and for any scalar t the set $J(i, t)$ as $J(i, t) = \{j=1, \dots, M: g(y_j) \leq t + g(y_i)\}$. When $t \geq 0$, this set is non-empty as it then contains i . Define by $J(i, t)^c$ the complementary set, that is, the customer classes outside subset $J(i, t)$. Furthermore, define $\Lambda_{J(i, t)} = \sum_{j \in J(i, t)} \lambda_j$, $\Lambda_{J(i, t)^c} = \sum_{j \in J(i, t)^c} \lambda_j$, $\bar{g}_{J(i, t)} = \frac{1}{\Lambda_{J(i, t)}} \sum_{j \in J(i, t)} \lambda_j g(y_j)$ and $\bar{g}_{J(i, t)^c} = \frac{1}{\Lambda_{J(i, t)^c}} \sum_{j \in J(i, t)^c} \lambda_j g(y_j)$. Therefore when $j \in J(i, t)$ the ‘demand part’ exceeds the ‘supply part’, while the opposite holds when $j \notin J(i, t)$. Therefore the specification of the probability $P(W_i^g(S) \geq t)$ can be summarized into the following expression using that the summation of independent Poisson distributions is a Poisson distribution with a mean that is the summation of the means:

$$P(W_i^g(S) \geq t) = P(Po(\Lambda_{J(i, t)}(t + g(y_i) - \bar{g}_{J(i, t)})) - Po(\Lambda_{J(i, t)^c}(\bar{g}_{J(i, t)^c} - t - g(y_i))) \leq S - 1) \quad (7)$$

Enabling that the two Poisson distributions inside the right-hand side of (7) are independent, we can rewrite (7) to

$$P(W_i^g(S) \geq t) = \sum_{x=0}^{\infty} P\left(Po\left(\Lambda_{J(i,t)}(t + g(y_i) - \bar{g}_{J(i,t)})\right) \leq S - 1 + x\right) P\left(Po\left(\Lambda_{J(i,t)^c}(\bar{g}_{J(i,t)^c} - t - g(y_i))\right) = x\right) \quad (8)$$

For the case where $J(i,t) = \{1, \dots, M\}$ we can simplify (7) considerably. Because we can rewrite it as

$$P(W_i^g(S) \geq t) = P(Po(\Lambda(t + g(y_i) - \bar{g})) \leq S - 1) = e^{-\Lambda(t + g(y_i) - \bar{g})} \sum_{n=0}^{S-1} \frac{(\Lambda(t + g(y_i) - \bar{g}))^n}{n!} \quad (9)$$

For the general case, the fill rate of the Class i , $FR_i^g(S)$, is

$$FR_i^g(S) = P(W_i^g(S) \geq L - y_i) \quad (10)$$

It can be computed by using (8) with $t = L - y_i$. From (7) we see that customer class i benefits from a high fill rate if the set $J(i, L - y_i)^c$ is non-empty due to a ‘negative’ part (the random variable $Po\left(\Lambda_{J(i,t)^c}(\bar{g}_{J(i,t)^c} - t - g(y_i))\right)$) inside the right-hand side of (7). This was exactly what was observed in the numerical example of Section 3 where we pointed out that Class 1 benefits from the presence of Class 2 customers.

In the case where $J(i, L - y_i) = \{1, \dots, M\}$ we get from (9)

$$FR_i^g(S) = e^{-\Lambda(L - y_i + g(y_i) - \bar{g})} \sum_{n=0}^{S-1} \frac{(\Lambda(L - y_i + g(y_i) - \bar{g}))^n}{n!} \quad (11)$$

If we have $g(y) = 0$, that is, complete reservation (note that, in this case we have $J(i, L - y_i) = \{1, \dots, M\}$), then we get from (11):

$$FR_i^{CR}(S) = e^{-\Lambda(L - y_i)} \sum_{n=0}^{S-1} \frac{(\Lambda(L - y_i))^n}{n!} \quad (12)$$

This corresponds to the result in Hariharan and Zipkin (1995). If we have $g(y) = y$, that is, no reservation (note that, in case we have $J(i, L - y_i) = \{1, \dots, M\}$), then we get from (11):

$$FR_i^{NR}(S) = e^{-\Lambda(L - \bar{y})} \sum_{n=0}^{S-1} \frac{(\Lambda(L - \bar{y}))^n}{n!} \quad (13)$$

As the right-hand side of (13) does not depend on i , we get for the case of no reservation that all customer classes have the same fill rate. So we do not care about the probability distribution of the demand lead-time. Only the mean value matters. This has also been shown in Hariharan and Zipkin (1995) and is a consequence of Palm’s theorem (for more on Palms theorem, see Palm (1938) and Zipkin (2000, p. 247)).

In order to assess the average on-hand inventory level, we need, as explained in the beginning of the section, to compute

$$E[W_i^g(S) - L + y_i]^+ = \int_{L-y_i}^{\infty} P(W_i^g(S) \geq t) dt \quad (14)$$

For the general case where $J(i, L-y_i)^C$ is non-empty, meaning that $\max_j\{g(y_j)\} - g(y_i) > L - y_i$, the computation of the integral in (14) is very involved as we need to integrate the function in (8). Instead, we do the following approximation. We subdivide the interval from $L - y_i$ to $\max\{g(y_j):j\} - g(y_i)$ into G number of grids. We make an interval size Δ_i defined as

$$\Delta_i = (\max_j\{g(y_j)\} - g(y_i) - L + y_i)/G \quad (15)$$

We can now approximate $E[W_i^g(S) - L + y_i]^+$ as follows

$$\begin{aligned} E[W_i^g(S) - L + y_i]^+ &= \Delta_i \sum_{u=0}^{G-1} P(W_i^g(S) \geq L - y_i + u\Delta_i) \\ &+ \int_{\max_j\{g(y_j)\} - g(y_i)}^{\infty} P(Po(\Lambda(t + g(y_i) - \bar{g})) \leq S - 1) dt \end{aligned} \quad (16)$$

The first part of the right-hand side of (16) is the approximation. Because $P(W_i^g(S) \geq t)$ is decreasing as a function of t , the approximation is an overestimation of the integral $\int_{L-y_i}^{\max_j\{g(y_j)\} - g(y_i)} P(W_i^g(S) \geq t) dt$. So this term will decrease as the grid size G increases and it will converge to the exact integral. The last part (the exact part) of the right hand side of (16) can be rewritten, see for instance (14) in Axsäter and Zhang (1996), to

$$\begin{aligned} &\int_{\max\{g(y_j:j) - g(y_i)\}}^{\infty} P(Po(\Lambda(t + g(y_i) - \bar{g})) \leq S - 1) dt \\ &= \frac{e^{-\Lambda((\max\{g(y_j:j) - \bar{g})})}}{\Lambda} \sum_{n=0}^{S-1} \frac{S-n}{n!} \left(\Lambda((\max\{g(y_j:j) - \bar{g})}) \right)^n \end{aligned} \quad (17)$$

In the case where $J(i, L-y_i) = \{1, \dots, M\}$ we can again simplify

$$\begin{aligned} E[W_i^g(S) - L + y_i]^+ &= \int_{L-y_i}^{\infty} P(Po(\Lambda(t + g(y_i) - \bar{g})) \leq S - 1) dt \\ &= \frac{e^{-\Lambda((L-y_i+g(y_i)-\bar{g}))}}{\Lambda} \sum_{n=0}^{S-1} \frac{S-n}{n!} \left(\Lambda((L-y_i+g(y_i)-\bar{g})) \right)^n \end{aligned} \quad (18)$$

For the case of complete reservation, (18) simplifies further to

$$E[W_i^{CR}(S) - L + y_i]^+ = \frac{e^{-\Lambda(L-y_i)}}{\Lambda} \sum_{n=0}^{S-1} \frac{S-n}{n!} (\Lambda(L-y_i))^n \quad (19)$$

and for the case of no reservation, (18) simplifies to

$$E[W_i^{NR}(S) - L + y_i]^+ = \frac{e^{-\Lambda(L-\bar{y})}}{\Lambda} \sum_{n=0}^{S-1} \frac{S-n}{n!} (\Lambda(L-\bar{y}))^n \quad (20)$$

For the general reservation policy, the average on-hand inventory can now be assessed as

$$I^g(S) = \sum_{i=1}^M \lambda_i E[W_i^g(S) - L + y_i]^+ \quad (21)$$

We have now mathematically specified all the terms in the profit expression (2). All terms involve summations and are therefore possible to compute relatively easily. The infinity sign in (8) can be replaced by a sufficiently large finite integer without causing any significant numerical error.

5. Numerical results

We have set up a numerical experiment to investigate the impact of using a reservation policy. The following data are kept constant in our experiment. We have in all $M = 4$ customer classes. We let L be 20 days. Finally we fix Λ , the total demand rate across all customer classes, to be $\Lambda = 1$ per day. We combine the following scenarios:

3 revenue scenarios R1 – R3:

- R1: $R_1^n(y) = 10$, and $R_2^n(y) = 10 - 0.5y$
 R2: $R_1^n(y) = 20$, and $R_2^n(y) = 15 - 0.75y$
 R3: $R_1^n(y) = 30$, and $R_2^n(y) = 20 - y$

When going from R1 to R3, we see an increasing level of revenues in combination with higher penalty costs for not delivering on time.

2 scenarios of demand lead-times DMLT1 – DMLT2:

- DMLT1: $(y_1, y_2, y_3, y_4) = (0, 6, 12, 18)$
 DMLT2: $(y_1, y_2, y_3, y_4) = (4, 8, 12, 16)$

DMLT1 represents a scenario where some customers expect to be served immediately, while DMLT2 represents a setting where all customers provide advance demand information.

3 arrival scenarios A1 – A3:

- A1: $(\lambda_1, \lambda_2, \lambda_3, \lambda_4) = (0.4, 0.3, 0.2, 0.1)$
 A2: $(\lambda_1, \lambda_2, \lambda_3, \lambda_4) = (0.25, 0.25, 0.25, 0.25)$
 A3: $(\lambda_1, \lambda_2, \lambda_3, \lambda_4) = (0.1, 0.2, 0.3, 0.4)$

When going from A1 to A3, we see a higher fraction of customers giving an early warning signal about their needs.

2 purchase cost scenarios

C1: $c = 300$

C2: $c = 1000$

When going from *C1* to *C2*, the profit margins become smaller

2 scenarios for the level of the inventory cost parameter h

H1: An annual inventory carrying charge of 10%

H2: An annual inventory carrying charge of 20%

Assume a year consists of 300 days. This gives an inventory cost per unit per day, for scenario *H1* and *H2* to be $c/3000$ and $c/1500$, respectively, where c is the unit purchase price.

We have investigated all combinations of (*R1*-*R3*), (*DMLT 1* - *DMLT2*), (*A1* - *A3*), (*C1* - *C2*) and (*H1* - *H2*), in all 72 combinations, and in each of these combinations, we find the combined optimal base-stock level and reservation policy. However, we have restricted the values of $g(y_i)$, $i=1,...,M$ to be multiples of 0.5. We have used a grid size parameter $G = 10$. As mentioned in finalizing Section 4, we replace the infinity sign in (8) by a finite number. We use the number 40. Numerical investigations showed these restrictions to be sufficiently accurate for this particular experiment, see the appendix. The results of our optimizations are reported in Tables 2 – 9.

<Tables 2-9 about here>

When moving horizontally (from *A1* to *A3*) in the tables, we see that it is generally optimal to lower the base-stock levels as more customers provide long demand lead-times. This is in accordance with the results of Hariharan and Zipkin (1995), because the effective lead-time (the replenishment lead-time minus the demand lead-time) is on average lowered, thereby making it optimal to lower the base-stock level. When moving vertically (from *R1* to *R3*) in the tables we see that the optimal base-stock level in general is increased. This is due to the fact that the profit margin increases and the backlog penalty increases as well. Therefore a higher service level is optimal, and this leads to an increased base-stock level. Obviously, the general reservation policy dominates the two ‘pure’ policies: either no reservation (*NR*) or complete reservation (*CR*). In most of the cases the complete reservation policy is significantly underperforming compared to the general reservation policy, while the no reservation policy is generally more competitive. So when it comes to a choice between the two ‘pure’ policies it is better not to make any reservations at all. However, also cases occur where both ‘pure’ policies simultaneously perform significantly worse than the general reservation policy. See, the cases *DMLT2/C2/H2/R1/A2-A3* in Table 8 where the general policy improves both ‘pure’ policies by more than 5%. In general we see the most benefit from the general reservation policy when we look at scenario *H2* with the high inventory carrying charge of 20%. As remarked in Section 1, Marklund (2006) has worked with a model somewhat similar to ours. Though, in this reference a two-level system is analysed (most often with only 2 retailers in the numerical study) with a backorder cost structure that is different from ours. In addition, the profit side is not modelled. Yet we do reach similar conclusions. Figure 6 of Marklund (2006) also reveals little difference in performance between the optimal general policy and the two “pure” policies. Moreover, Marklund (2006) concludes that what he coins “last minute allocation”, which is the policy *NR*, is generally a close contender to the optimal general reservation policy.

The general reservation policy might be too complex to work with and one could consider restricting the shape of the $g()$ function to be within a specific class. Following that idea, Du (2011) proposes various partial reservation policies which are easier to implement. One of them is the Backward Delay *BD* reservation policy which is specified by a parameter d where $0 \leq d \leq L$ with $g(y) = \max(y-d, 0)$. The logic behind the policy is to prioritize orders in accordance with their actual needs. If y is small, $y \leq d$, it means an item is reserved for the order immediately when receiving the request. Some of the results using the general reservation policy seem to suggest that the optimal general reservation policy has a shape similar to that of $\max(y-d, 0)$. See for instance case *DMLT2/C1/H2/R3/A1-A3* in table 7 where the optimal reservation policies across the three arrival scenarios are $g(y_1) = 0.5$, $g(y_2) = 3.5$, $g(y_3) = 7$, $g(y_4) = 11$. The optimal *BD* policy which has optimal d parameter (when restricting to integer values of d) $d = 5$, which means $g(y_1) = 0$, $g(y_2) = 3$, $g(y_3) = 7$, $g(y_4) = 11$. These numbers are rather close to the optimal general reservation policy. Findings like this motivate a more thorough investigation of the reservation policy *BD*. Therefore, we compute the optimal *BD* policy for all the datasets and compare it to the optimal general policy. Tables 2-9 (the last entries in here) reveal that the *BD* policy performs quite well, giving almost as good results as the general reservation policy. The policy *BD* is based on using the due-date as an anchoring point and the offsetting backward in time, to some extent similar to an MRP methodology, to find the point in time for when reservation should be made. Apparently, this methodology works well.

6. Concluding remarks

We have developed a model to evaluate reservation policies for a base-stock inventory system with multiple customer classes having different demand lead-times. The technical derivations are rather challenging. Our numerical analysis supports only to some extent the idea of using a general reservation policy. Because it seems like the no reservation policy is rather robust. However, we can also see improvements at about 5 - 10 % when using a general reservation policy compared to the no reservation policy, and if the business environment is very competitive with focus on too high inventory investments, such improvements might matter, as well. One can generalize the mathematical models developed in the paper to the case with infinitely many customer classes by using integral calculus, see Du (2011).

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Tables

Table1: Sample path of a case with three customer classes with $\lambda_1 = \lambda_2 = 0.25$, $\lambda_3 = 0.5$, $y_1 = 10$, $y_2 = 19$, $y_3 = 12$, $g(y_1) = 2$, $g(y_2) = 16$, $g(y_3) = 7$. The base stock level is $S = 6$ which is also the starting inventory. The replenishment lead-time is 20. All time numbers are rounded to two decimals.

Arrival no (‘considered- order’)	Arrival- time	Class	Reservation- time	Reservation no.	‘Considered- item’ is a replenishment of order no.	Arrival time of repl.	Sojourn time at the inventory of ‘considered- item’	$W_i^g(S)$ obs.
1	2.26	3	9.26	2	0	0	14.26	
2	3.06	3	10.06	3	0	0	15.06	
3	4.67	3	11.67	4	0	0	16.67	
4	5.36	1	7.36	1	0	0	15.36	
5	6.27	2	22.27	11	5	26.27	0	0.00
6	6.38	3	13.38	5	0	0	18.38	
7	6.72	2	22.72	13	7	26.72	0	0.00
8	9.00	3	16.00	7	1	22.26	0	6.74
9	9.11	3	16.11	8	2	23.06	0	6.05
10	12.06	3	19.06	9	3	24.67	0	7.39
11	13.61	1	15.61	6	0	0	23.61	13.61
12	14.78	2	30.78	20	14	36.25	0	-1.47
13	15.49	3	22.49	12	6	26.38	1.11	9.11
14	16.25	2	32.25	22	16	39.66	0	-3.41
15	18.50	1	20.50	10	4	25.36	3.14	13.14
16	19.66	2	35.66	25	19	43.72	0	-4.06
17	20.12	3	27.12	15	9	29.11	3.01	11.01
18	23.50	1	25.50	14	8	29.00	4.50	14.50
19	23.72	3	30.72	19	13	35.49	0.23	8.23
20	24.73	2	40.73	28	22	45.40	0	-0.67
21	25.22	2	41.22	31	25	47.67	0	-2.45
22	25.40	1	27.40	16	10	32.06	3.34	13.34
23	25.40	1	27.40	17	11	33.61	1.79	11.79
24	25.79	3	32.79	23	17	40.12	0	5.67
25	27.67	1	29.67	18	12	34.78	2.89	12.89
26	29.56	2	45.56	36	30	53.12	0	-3.56
27	29.70	1	31.70	21	15	38.50	1.20	11.20
28	31.38	2	47.38	39	33	57.18	0	-5.80
29	32.94	1	34.94	24	18	43.50	0	9.44
30	33.12	3	40.12	27	21	45.22	0	7.90
31	33.96	1	35.96	26	20	44.73	0	9.23

32	36.72	3	43.72	33	27	49.70	0	7.02
33	37.18	3	44.18	35	29	52.94	0	4.24
34	38.21	2	54.21	40	34	58.21	0	0.00
35	39.15	1	41.15	29	23	45.40	3.75	13.75
36	39.16	1	41.16	30	24	45.79	3.37	13.37
37	39.36	3	46.36	37	31	53.96	0	5.42
38	39.74	3	46.74	38	32	56.72	0	0.00

Table 2: The case DMLT1/C1/H1. The first entry in each cell specifies the characteristics of the optimal solution of the general reservation policy: The resulting profit contribution, the optimal base stock level and 4 optimal values of $g(y_i)$, $i=1,2,3,4$. The second entry specifies the characteristics of the No Reservation policy (NR) policy: The resulting expected daily profit contribution, the optimal base stock level and the percentage of improvement by using the general reservation policy compared to policy NR. The third entry which concerns the Complete Reservation policy (CR) is organized similarly. Finally the fourth entry specifies the characteristics of the Backward Delay (BD): The resulting expected daily profit contribution, the optimal base stock level, the optimal d parameter of this policy and the percentage of improvement by using the general reservation policy compared to policy BD.

	A1	A2	A3
R1	Gen: 9.44;17;(0,0,3.5,9) NR: 9.16;20; 3.06% CR: 9.32;18;1.29% BD: 9.44;17;9;0.00%	Gen: 9.35;15;(0,0,3.5,9) NR: 9.15;18;2.19% CR: 9.11;16;2.63% BD: 9.35;15;9;0.00%	Gen: 9.29;13;(0,0,4,9.5) NR: 9.19;15;1.09% CR: 8.94;15;3.91% BD: 9.29;13;8;0.00%
R2	Gen: 18.96;23;(0, 5,10.5,16) NR: 18.94;23;0.11% CR: 18.37;21;3.21% BD: 18.96;23;2;0.00%	Gen: 19.02;19;(0, 5,10.5,16) NR: 18.99;19;0.16% CR: 18.40;24;3.37% BD: 19.01;19;2;0.05%	Gen: 19.09;16;(0,5,10.5,16) NR: 19.08;16;0.05% CR: 18.37;21;3.92% BD: 19.09;16;1;0.00%
R3	Gen: 28.86;24;(0,5.5,11,16.5) NR: 28.85;24;0.03% CR: 28.38;28;1.69% BD: 28.86;24;1;0.00%	Gen: 28.94;20;(0,5.5,11,16.5) NR: 28.92;20;0.07% CR: 28.20;26;2.62% BD: 28.93;20;1;0.03%	Gen: 29.03;16;(0,5.5,11,17) NR: 29.02;16;0.03% CR: 28.16;23;3.09% BD: 29.02;16;1;0.03%

Table 3: The case DMLT2/C1/H1. For further comments, see the caption for Table 2.

	<i>A1</i>	<i>A2</i>	<i>A3</i>
<i>R1</i>	<i>Gen:</i> 9.18;18;(0,2,5,5,5,9) <i>NR:</i> 9.15;19;0.03% <i>CR:</i> 9.02;19;1.80% <i>BD:</i> 9.17;18;6;0.11%	<i>Gen:</i> 9.19;16; (0,2,5,5,9) <i>NR:</i> 9.16;17;0.03% <i>CR:</i> 8.94;17;2.80% <i>BD:</i> 9.18;16;6;0.11%	<i>Gen:</i> 9.21;14; (0.5,3,6.5,10) <i>NR:</i> 9.19;15;0.02% <i>CR:</i> 8.93;16;3.13% <i>BD:</i> 9.21;14;6;0.00%
<i>R2</i>	<i>Gen:</i> 18.98;21; (0,3.5,7,11) <i>NR:</i> 18.97;21;0.05% <i>CR:</i> 18.70;23;1.50% <i>BD:</i> 18.97;21;5;0.05%	<i>Gen:</i> 19.03;18;(0,3.5,7,11) <i>NR:</i> 19.02;18;0.05% <i>CR:</i> 18.60;22;2.31% <i>BD:</i> 19.03;18;5;0.00%	<i>Gen:</i> 19.09;16;(1,4.5,8,12) <i>NR:</i> 19.06;15;0.02% <i>CR:</i> 18.60;19;2.63% <i>BD:</i> 19.09;16;5;0.00%
<i>R3</i>	<i>Gen:</i> 28.90;21;(2,5,6,10,13.5) <i>NR:</i> 28.89;22;0.03% <i>CR:</i> 28.58;24;1.12% <i>BD:</i> 28.89;21;5;0.03%	<i>Gen:</i> 28.96;19;(0,3.5,7.5,11) <i>NR:</i> 28.96;19;0.00% <i>CR:</i> 28.48;23;1.69% <i>BD:</i> 28.96;19;5;0.00%	<i>Gen:</i> 29.02;16;(0,4,7.5,11.5) <i>NR:</i> 29.02;16;0.00% <i>CR:</i> 28.47;21;1.93% <i>BD:</i> 29.02;16;5;0.00%

Table 4: The case DMLT1/C2/H1. For further comments, see the caption for Table 2.

	<i>A1</i>	<i>A2</i>	<i>A3</i>
<i>R1</i>	<i>Gen:</i> 8.73;14; (0,0,2,5,7.5) <i>NR:</i> 8.11;17;7.64% <i>CR:</i> 8.53;13;2.34% <i>BD:</i> 8.73;14;10;0.00%	<i>Gen:</i> 8.45;13;(0,0,3,8.5) <i>NR:</i> 7.94;15;6.42% <i>CR:</i> 8.05;11;4.97% <i>BD:</i> 8.44;13;9;0.12%	<i>Gen:</i> 8.22;11;(0,0,3,8.5) <i>NR:</i> 7.95;12;3.40% <i>CR:</i> 7.63;10;7.73% <i>BD:</i> 8.21;11;9;0.12%
<i>R2</i>	<i>Gen:</i> 17.41;17;(0,4,5,9.5,15) <i>NR:</i> 17.24;20;0.99% <i>CR:</i> 16.47;20;5.71% <i>BD:</i> 17.32;20;2;0.52%	<i>Gen:</i> 17.41;17;(0,4,5,9.5,15) <i>NR:</i> 17.32;17;0.52% <i>CR:</i> 16.10;18;8.14% <i>BD:</i> 17.39;17;2;0.12%	<i>Gen:</i> 17.56;14;(0,4,5,10,15.5) <i>NR:</i> 17.51;14;0.30% <i>CR:</i> 16.08;15;9.20% <i>BD:</i> 17.54;14;2;0.11%
<i>R3</i>	<i>Gen:</i> 26.94;21;(0,5,10.5,16) <i>NR:</i> 26.28;21;0.22% <i>CR:</i> 25.71;23;4.78% <i>BD:</i> 26.93;21;2;0.04%	<i>Gen:</i> 27.10;18;(0, 5,10.5,16) <i>NR:</i> 27.04;18;0.22% <i>CR:</i> 25.26;21;7.28% <i>BD:</i> 27.08;18;2;0.07%	<i>Gen:</i> 27.30;15;(0,5.5,11,16.5) <i>NR:</i> 27.28;15;0.07% <i>CR:</i> 25.62;17;6.56% <i>BD:</i> 27.30;15;1;0.00%

Table 5: The case DMLT2/C2/H1. For further comments, see the caption for Table 2.

	<i>A1</i>	<i>A2</i>	<i>A3</i>
<i>R1</i>	<i>Gen:</i> 8.11;15;(0,1,4,7.5) <i>NR:</i> 7.96;16;1.88% <i>CR:</i> 7.90;14;2.66% <i>BD:</i> 8.10;15;9;0.12%	<i>Gen:</i> 8.05;13; (0,0,3,6.5) <i>NR:</i> 7.93;14;1.01% <i>CR:</i> 7.71;12;4.41% <i>BD:</i> 8.04;13;9;0.12%	<i>Gen:</i> 8.03;12;(0,1,4,7.5) <i>NR:</i> 7.95;12;1.01% <i>CR:</i> 7.63;11;5.24% <i>BD:</i> 8.03;12;9;0.00%
<i>R2</i>	<i>Gen:</i> 17.31;18;(0,5,4,7.5,11) <i>NR:</i> 17.28;18;0.17% <i>CR:</i> 16.64;19;4.03% <i>BD:</i> 17.31;18;5;0.00%	<i>Gen:</i> 17.40;16;(0,3.5,7,10.5) <i>NR:</i> 17.37;16;0.17% <i>CR:</i> 16.40;18;6.10% <i>BD:</i> 17.39;16;5;0.06%	<i>Gen:</i> 17.53;14;(1,4.5,8,12) <i>NR:</i> 17.51;14;0.11% <i>CR:</i> 16.49;15;6.31% <i>BD:</i> 17.52;14;5;0.06%
<i>R3</i>	<i>Gen:</i> 27.00;19;(0,3.5,7,11) <i>NR:</i> 26.98;19;0.07% <i>CR:</i> 26.17;21;3.17% <i>BD:</i> 26.99;19;5;0.04%	<i>Gen:</i> 27.13;17;(0,3.5,7,11) <i>NR:</i> 27.12;17;0.04% <i>CR:</i> 25.88;20;4.83% <i>BD:</i> 27.13;17;5;0.00%	<i>Gen:</i> 27.29;15;(0,3.5,7,11) <i>NR:</i> 27.28;15;0.04% <i>CR:</i> 25.95;17;5.16% <i>BD:</i> 27.29;15;5;0.00%

Table 6: The case DMLT1/C1/H2. For further comments, see the caption for Table 2.

	<i>A1</i>	<i>A2</i>	<i>A3</i>
<i>R1</i>	<i>Gen:</i> 9.08;15;(0,0,2.5,8) <i>NR:</i> 8.62;18; 5.34% <i>CR:</i> 8.91;15;1.91% <i>BD:</i> 9.08;16;9;0.00%	<i>Gen:</i> 8.91;14;(0,0,3.5,9) <i>NR:</i> 8.56;16;4.09% <i>CR:</i> 8.56;13;4.09% <i>BD:</i> 8.91;14;9;0.00%	<i>Gen:</i> 8.78;12;(0,0,4.5,9) <i>NR:</i> 8.60;13;2.09% <i>CR:</i> 8.28;12;6.04% <i>BD:</i> 8.77;12;9;0.11%
<i>R2</i>	<i>Gen:</i> 18.19;21;(0,4.5,9.5,15) <i>NR:</i> 18.13;21;0.33% <i>CR:</i> 17.54;23;3.71% <i>BD:</i> 18.19;21;2;0.00%	<i>Gen:</i> 18.27;18;(0,4.5,10,15.5) <i>NR:</i> 18.21;18;0.33% <i>CR:</i> 17.26;21;5.85% <i>BD:</i> 18.26;18;2;0.05%	<i>Gen:</i> 18.38;15; (0,5,10.5,16) <i>NR:</i> 18.35;15;0.16% <i>CR:</i> 17.24;18;6.03% <i>BD:</i> 18.37;15;2;0.05%
<i>R3</i>	<i>Gen:</i> 27.97;22; (0,5,10.5,16) <i>NR:</i> 27.93;22;0.21% <i>CR:</i> 27.12;25;3.13% <i>BD:</i> 27.96;22;1;0.04%	<i>Gen:</i> 28.09;19;(0,5.5,11,16.5) <i>NR:</i> 28.06;19;0.11% <i>CR:</i> 26.81;24;4.77% <i>BD:</i> 28.08;19;1;0.04%	<i>Gen:</i> 28.24;15; (0,5.5,11,16.5) <i>NR:</i> 28.22;16;0.07% <i>CR:</i> 26.78;20;5.45% <i>BD:</i> 28.23;15;1;0.04%

Table 7: The case DMLT2/C1/H2. For further comments, see the caption for Table 2.

	<i>A1</i>	<i>A2</i>	<i>A3</i>
<i>R1</i>	<i>Gen:</i> 8.64;16;(0,5,2.5,8.5) <i>NR:</i> 8.56;17;0.93% <i>CR:</i> 8.43;16;2.49% <i>BD:</i> 8.62;16;7;0.23%	<i>Gen:</i> 8.63;15; (0,2,5,8.5) <i>NR:</i> 8.56;15;0.82% <i>CR:</i> 8.30;15;3.98% <i>BD:</i> 8.62;15;7;0.12%	<i>Gen:</i> 8.65;13; (0,2,5,8.5) <i>NR:</i> 8.60;13;0.58% <i>CR:</i> 8.27;13;4.59% <i>BD:</i> 8.64;13;9;0.12%
<i>R2</i>	<i>Gen:</i> 18.20;19; (0,5,4,7.5,11) <i>NR:</i> 18.18;19;0.11% <i>CR:</i> 17.72;21;2.71% <i>BD:</i> 18.19;19;5;0.05%	<i>Gen:</i> 18.27;17;(0,3.5,7,11) <i>NR:</i> 18.26;17;0.05% <i>CR:</i> 17.26;21;5.85% <i>BD:</i> 18.26;17;5;0.05%	<i>Gen:</i> 18.36;15;(0,3.5,7,11) <i>NR:</i> 18.36;15;0.00% <i>CR:</i> 17.58;18;4.44% <i>BD:</i> 18.36;15;5;0.00%
<i>R3</i>	<i>Gen:</i> 28.02;20;(0,3.5,7,11) <i>NR:</i> 28.01;20;0.04% <i>CR:</i> 27.45;22;2.08% <i>BD:</i> 28.02;20;5;0.00%	<i>Gen:</i> 28.12;18;(0,3.5,7,11) <i>NR:</i> 28.11;18;0.04% <i>CR:</i> 27.27;21;3.12% <i>BD:</i> 28.12;18;5;0.00%	<i>Gen:</i> 28.22;15;(0,3.5,7,11) <i>NR:</i> 28.22;16;0.21% <i>CR:</i> 27.29;19;3.63% <i>BD:</i> 28.23;15;5;0.18%

Table 8: The case DMLT1/C2/H2. For further comments, see the caption for Table 2.

	<i>A1</i>	<i>A2</i>	<i>A3</i>
<i>R1</i>	<i>Gen:</i> 8.19;8; (0,6,0,4.5) <i>NR:</i> 7.40;14;14.96% <i>CR:</i> 7.95;10;3.02% <i>BD:</i> 8.17;10;13;0.24%	<i>Gen:</i> 7.74;8;(0,6,0,5) <i>NR:</i> 6.88;13;12.50% <i>CR:</i> 7.28;6;6.32% <i>BD:</i> 7.66;9;14;1.04%	<i>Gen:</i> 7.34;7;(0,6,0,5) <i>NR:</i> 6.73;11;9.06% <i>CR:</i> 6.75;5;8.74% <i>BD:</i> 7.22;9;13;1.66%
<i>R2</i>	<i>Gen:</i> 15.73;17;(0,2.5,7.5,13) <i>NR:</i> 15.49;18;1.55% <i>CR:</i> 14.69;16;7.08% <i>BD:</i> 15.69;18;3;0.25%	<i>Gen:</i> 15.74;15;(0,3.5,8.5,14) <i>NR:</i> 15.50;16;1.55% <i>CR:</i> 14.19;13;10.92% <i>BD:</i> 15.69;15;4;0.32%	<i>Gen:</i> 15.86;12;(0,3,8,13.5) <i>NR:</i> 15.73;13;0.82% <i>CR:</i> 14.06;11;12.80% <i>BD:</i> 15.83;12;7;0.19%
<i>R3</i>	<i>Gen:</i> 24.83;19;(0,4.5,9.5,15) <i>NR:</i> 24.70;20;0.52% <i>CR:</i> 23.02;20;7.86% <i>BD:</i> 24.81;19;2;0.08%	<i>Gen:</i> 24.99;16;(0,4.5,10,15.5) <i>NR:</i> 24.89;17;0.40% <i>CR:</i> 22.37;17;11.71% <i>BD:</i> 24.96;17;2;0.12%	<i>Gen:</i> 25.31;13;(0,4.5,10,15.5) <i>NR:</i> 25.23;14;0.32% <i>CR:</i> 22.51;14;12.44% <i>BD:</i> 25.28;13;2;0.12%

Table 9: The case DMLT2/C2/H2. For further comments, see the caption for Table 2.

	<i>A1</i>	<i>A2</i>	<i>A3</i>
<i>R1</i>	<i>Gen:</i> 7.36;11;(4,0,2.5,6) <i>NR:</i> 7.00;13;5.14% <i>CR:</i> 7.13;11;3.23% <i>BD:</i> 7.30;12;10;0.82%	<i>Gen:</i> 7.15;10; (4,0,2,5.5) <i>NR:</i> 6.80;12;5.15% <i>CR:</i> 6.83;9;4.69% <i>BD:</i> 7.07;11;11;1.13%	<i>Gen:</i> 6.96;7;(4,8,0,3.5) <i>NR:</i> 6.73;11;3.42% <i>CR:</i> 6.60;8;5.45% <i>BD:</i> 6.90;10;11;0.87%
<i>R2</i>	<i>Gen:</i> 15.54;16;(1,4,7.5,11) <i>NR:</i> 15.46;16;0.52% <i>CR:</i> 14.62;16;6.29% <i>BD:</i> 15.52;16;6;0.13%	<i>Gen:</i> 15.61;14;(0,3,6.5,10) <i>NR:</i> 15.55;15;0.39% <i>CR:</i> 14.32;14;9.01% <i>BD:</i> 15.59;15;5;0.13%	<i>Gen:</i> 15.76;13;(0,3.5,7,10.5) <i>NR:</i> 15.73;13;0.19% <i>CR:</i> 14.48;12;8.84% <i>BD:</i> 15.75;13;5;0.06%
<i>R3</i>	<i>Gen:</i> 24.85;18;(3.5,7,10.5,14) <i>NR:</i> 24.81;18;0.16% <i>CR:</i> 23.53;19;5.61% <i>BD:</i> 24.84;18;5;0.04%	<i>Gen:</i> 25.02;16;(0,3.5,7,11) <i>NR:</i> 24.99;16;0.12% <i>CR:</i> 23.12;17;8.22% <i>BD:</i> 25.01;16;5;0.04%	<i>Gen:</i> 25.25;13;(0,3.5,7,11) <i>NR:</i> 25.23;14;0.08% <i>CR:</i> 23.34;14;8.18% <i>BD:</i> 25.24;13;5;0.04%

Table A1: The case DMLT1/C1/H2. The characteristics of the optimal solution of the general reservation policy when using an enlarged search: The resulting profit contribution, the optimal base stock level and 4 optimal values of $g(y_i)$, $i=1,2,3,4$.

	<i>A1</i>	<i>A2</i>	<i>A3</i>
<i>R1</i>	9.08;15;(0,0,2.75,8)	8.91;14;(0,0,3.5,8.75)	8.78;12;(0,0,3.75,9.25)
<i>R2</i>	18.19;21;(0,4.5,9.75,15.25)	18.27;18;(0,4.5,10,15.5)	18.38;15; (0,4.75,10.25,16)
<i>R3</i>	27.97;22; (0,5,10.5,16)	28.09;19;(0,5.25,10.75,16.5)	28.24;15; (0,5.25,10.75,16.5)

Table A2: The case DMLT2/C1/H2. For further comments, see the caption for Table A1.

	<i>A1</i>	<i>A2</i>	<i>A3</i>
<i>R1</i>	8.64;16;(0.25,1.75,4.75,8.25)	8.63;15; (0,2,5.25,8.75)	8.65;13; (0.25,2.25,5.5,9)
<i>R2</i>	18.20;19; (0,3.5,7,10.75)	18.27;17;(0,3.5,7,10.75)	18.36;15;(0,3.5,7.25,11)
<i>R3</i>	28.02;20;(0.5,4,7.75,11.5)	28.12;18;(0,3.5,7.25,11)	28.23;15;(0,3.75,7.5,11.25)

Table A3: The case DMLT1/C2/H2. For further comments, see the caption for Table A1.

	<i>A1</i>	<i>A2</i>	<i>A3</i>
<i>R1</i>	8.19;8; (0,6,0,4.5)	7.74;8;(0,6,0,5)	7.35;7;(0,6,0,4.75)
<i>R2</i>	15.73;17;(0,2.75,7.5,13)	15.74;15;(0,3.25,8.25,13.75)	15.86;12;(0,3,8,13.5)
<i>R3</i>	24.83;19;(0,4.5,9.75,15.25)	25.00;16;(0,4.5,9.75,15.25)	25.31;13;(0,4.75,10.25,15.75)

Table A4: The case DMLT2/C2/H2. For further comments, see the caption for Table A1.

	<i>A1</i>	<i>A2</i>	<i>A3</i>
<i>R1</i>	7.36;11;(4,0,2.5,6)	7.15;10; (4,0,2,5.25)	6.96;7;(4,8,0,3.5)
<i>R2</i>	15.54;16;(0,3,6.25,9.75)	15.61;14;(0,2.75,6.25,9.75)	15.76;13;(0,3.25,6.75,10.5)
<i>R3</i>	24.85;18;(0,3.5,7,10.75)	25.02;16;(0,3.5,7,10.75)	25.26;13;(0.5,4,7.5,11.25)

Appendix: Testing accuracy of the parameter choices for our numerical experiment

We here repeat the numerical study for all instances involving scenario H2. We restrict the values of $g(y_i)$, $i=1,...,M$ to be multiples of 0.25 (0.5 in the main experiment). We use a grid size parameter $G = 15$ (10 in the main experiment) and we replace the infinity sign in (8) by the finite number 50 (40 in the main experiment). For each instance we report the optimal solution of the general reservation policies in Tables A1-A4, below.

<Tables A1 – A4 about here>

When comparing the results reported in these tables to the results of the general reservation policy in the Tables 6 – 9 we see only small discrepancies, why we conclude that the parameter choices in numerical experiment of Section 5 are made with sufficient accuracy.