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Noise properties of Fourier deconvolution for time-domain electromagnetic soundings

Søren Rasmussen¹, Nicklas Skovgaard Nyboe², Sune Mai³, and Jakob Juul Larsen¹

ABSTRACT

The transient electromagnetic method (TEM) is widely used for mapping the earth's subsurface resistivity structures, e.g., for groundwater or mineral exploration, using airborne and ground-based systems. Data from TEM surveys can be modified using Fourier deconvolution to correct for instrument drift, increase apparent bandwidth, or alter the apparent excitation waveform, e.g., to fuse or compare measurements from different systems, for visual presentation purposes, or to make the data compatible with a specific processing or inversion code. Although this method has been applied in several studies, little attention has been devoted to its properties with regard to noise. Using a generic analytical system model and examples featuring synthetic and field data, we perform a detailed analysis of the noise properties of Fourier deconvolution in the context of TEM. We find that although the effects from stationary noise are trivial, effects from nonstationary noise are less intuitive and more severe, e.g., causing on-time noise phenomena to degrade off-time data. In general, we observe that the method decreases the signal-to-noise ratio, and our recommendation is therefore that Fourier deconvolution should only be applied when it is desirable for presentation purposes or when strictly necessary for processing or inversion.

INTRODUCTION

The transient electromagnetic method (TEM) is a widely used technique for airborne and ground-based geophysical surveys, for example, in mineral exploration and groundwater mapping (McCracken et al., 1986; Nabighian and Macnae, 1991; Danielsen et al., 2003; Christiansen et al., 2006; Macnae, 2007). The working

principle of the method is that a current waveform is applied to a transmitter induction loop, thereby generating a magnetic primary field that interacts with the earth below. In response, the earth emits a time-varying magnetic secondary field that depends on the resistivity structure and the excitation waveform. The primary and secondary fields are picked up by a receiver, on which the output voltage is measured. Induction coil receivers, sensitive to B -field variations, are widely used, whereas magnetometers that are sensitive to B -field strength, such as the superconducting quantum interference device (SQUID) sensor, are less common (Keller, 1981; Spies and Frischknecht, 1991; Lee et al., 2002).

Due to design limitations, the receiver is always band limited; thus, the receiver transfer function affects the measurements. In mathematical terms, omitting the magnetic coupling factors and noise, the measured signal $m(t)$, is the sum of two terms: (1) the convolution of the time derivative of the transmitted waveform $i_T'(t)$ and the receiver impulse response $r(t)$ and (2) the convolution of $i_T'(t)$, the earth response $x(t)$, and $r(t)$. Often, the combined effects of the transmitter waveform and the receiver impulse response are referred to as the system response, $h(t) = [i_T' * r](t)$, where $*$ denotes convolution (Raiche, 1998):

$$m(t) = [i_T' * r](t) + [i_T' * x * r](t) = h(t) + [h * x](t). \quad (1)$$

TEM surveys are conducted using a wide range of different systems, using various waveforms, repetition rates, and hardware configurations (e.g., Nabighian and Macnae, 1991; Balch et al., 2003; Sørensen and Auker, 2004; Witherly et al., 2004; Allard, 2007; Thomson et al., 2007). Any specific combination of these parameters gives a unique system response, on which the measurements depend (Nabighian and Macnae, 1991).

Convolution in the time domain corresponds to multiplication in the frequency domain; thus, any missing or weak-frequency components in $h(t)$ will result in correspondingly weak or missing frequency components in $m(t)$. This naturally limits the conclusions

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that can be drawn from measured data. For example, early-time high-frequency signal components pertaining to shallow-depth resistivity features may be drowned out by noise due to bandwidth limitations in the system response. The goal of deconvolution is then to approximate the true earth response $x(t)$ by recovering some or all of these frequency components. However, because deconvolution of noisy signals is often an unstable process, some form of regularization, i.e., assumptions or limitations on the shape of the recovered earth response, must be applied to obtain a meaningful and stable result (Hansen, 2010).

In the published literature on deconvolution of TEM signals, two distinct approaches have been used. The first approach, based on the observation that, in some cases the ideal earth response can be modeled by a finite sum of exponential decays of varying amplitude and time constant, restricts the recovered earth response to reside in a subspace of such functions (Wolffgram and Karlik, 1995; Stolz and Macnae, 1998). The second approach uses the fact that periodic convolution in the discrete time domain corresponds to multiplication in the discrete frequency domain (Macnae and Lamontagne, 1982; Annan, 1986; Lane et al., 2000; Sattel et al., 2004; Macnae and Baron-Hay, 2010; Legault et al., 2012; Sattel and Battig, 2016). Thus, because the time-domain system response is normally periodic in practice, deconvolution corresponds to division in the frequency domain. Contrary to the former approach, fewer restrictions on the shape of the recovered earth response are imposed. Instead, stability is achieved by imposing a band limitation and removing certain frequency components from the recovered signal. This approach appears in the literature to be the most widely used of the two, and it is also the focus of this work. In the following, we refer to this method as the Fourier deconvolution method.

The result of applying Fourier deconvolution is to modify the data to appear as if it were obtained using another system response, rather than give an approximation of the true earth response. This may be desirable for several different reasons: (1) in some cases, especially the early-time part of the measurement is heavily affected by the system response, making it difficult to interpret visually. Here, deconvolution can be used to “push” these effects to even earlier times, making interpretation easier. (2) Measurements made using different systems or a multiwaveform system (Sørensen and Auken, 2004; Chen et al., 2015) can be deconvolved to share a common apparent system response and fused together, or otherwise compared. (3) Data can be modified into a more suitable form for an inversion code or processing code, e.g., if the code is unable to handle data obtained by a system with a given response (Green, 1998; Sattel et al., 2004). When deconvolved data are inverted or used for quantitative interpretation, it is important that uncertainties in the data are well-understood and tracked through the deconvolution process. Because we have been unable to find anything more than small comments or warnings on the noise properties in the literature, the purpose of this work is to provide a thorough analysis of the noise propagation through the Fourier deconvolution method.

FOURIER DECONVOLUTION IN TEM

The following is a description of the Fourier deconvolution method as used in TEM (Macnae and Lamontagne, 1982; Annan, 1986; Lane et al., 2000; Sattel et al., 2004; Macnae and Baron-Hay, 2010; Legault et al., 2012). The method relies on the fact that convolution in the time domain corresponds to multiplication in the frequency domain, so the frequency-domain counterpart of equation 1 is

$$M(\omega) = H(\omega)(1 + X(\omega)). \quad (2)$$

Here, and in the remainder of this work, capital-letter variables denote the Fourier-transformed frequency-domain counterparts of time-domain signals; e.g., $M(\omega) = \mathcal{F}\{m(t)\}$. Round brackets (e.g., $m(t)$ or $M(\omega)$) denote variables in the continuous time or frequency domains, whereas square brackets (e.g., $m[n]$ or $M[\omega]$) denote variables in the discrete domains. Throughout the text, we switch freely between denoting signals by their time- and frequency-domain representations, and we use whichever is more convenient.

The system response independent measurement would then ideally be computed as the inverse Fourier transform of $1 + X(\omega) = M(\omega)/H(\omega)$, but, in practice, several complications arise. Due to bandwidth limitations in any TEM system, $|H(\omega)|$ and thereby also $|M(\omega)|$ tend to zero as frequency increases. Thus, any noise at high frequencies would be highly amplified in an attempt to compute $x(t)$ using equation 2. Instead, $m(t)$ is deconvolved to appear as if it were measured using a system with the response $\hat{h}(t)$, which has desirable characteristics compared with $h(t)$, e.g., higher bandwidth or a different transmitter waveform:

$$\hat{M}(\omega) = \frac{\hat{H}(\omega)}{H(\omega)} M(\omega), \quad (3)$$

$$\hat{m}(t) = \mathcal{F}^{-1}\{\hat{M}(\omega)\} = [g * m](t), \quad (4)$$

where $\mathcal{F}^{-1}$ denotes the inverse Fourier transform. Above and in the following, we use a “hat” ($\hat{}$) to symbolize deconvolved and desired quantities. By selecting the apparent system response $\hat{h}(t)$ such that $|\hat{H}(\omega)|$ is small at frequencies, where $|M(\omega)|$ is dominated by noise, the amplification of noise can be controlled. We shall denote the function $G(\omega) = \hat{H}(\omega)/H(\omega)$, which is multiplied onto $M(\omega)$ to obtain $\hat{M}(\omega)$, and its time-domain counterpart $g(t) = \mathcal{F}^{-1}\{G(\omega)\}$, the deconvolution kernel.

Common for practically all TEM systems is that the transmitted waveform is opposite-sign periodic, consisting of alternating positive- and negative-sign replications of the same pulse, which makes $H(\omega)$ and $M(\omega)$ nonzero only at the waveform repetition frequency and its odd harmonics. By sign correcting and stacking the measured signals, sensitivity is concentrated at the odd harmonics of the repetition frequency, in which the signal power lies. For a thorough treatment of the effects of sign correction, stacking and gating, see the expositions by Macnae et al. (1984) and Nyboe and Sørensen (2012).

Typically, the repetition frequency is selected at odd multiples of 25 or 30 Hz, such that there is no sensitivity to powerline harmonics at multiples of 50 or 60 Hz. Consequently, at all other frequencies than the odd harmonics of their respective repetition frequencies, the numerator and denominator in equation 3 are zero. In the common case in which the system responses $\hat{H}(\omega)$ and $H(\omega)$ in equation 3 share the same repetition frequency and thus have overlapping odd harmonics, this means that the fraction is undefined at all other frequencies, leaving the result undefined. This is circumvented by only including the odd harmonics in the inverse Fourier transformation, effectively defining zero divided by zero as $0/0 := 0$, which is sensible from a physical point of view in this case. The case in which the system responses do not have the same repetition rate is treated in the section on fusing such data sets below. As a consequence of only

including the odd harmonics, the time-domain deconvolved signal becomes exactly opposite-sign periodic. Furthermore, some transmitter waveforms cause $H(\omega)$ to have zero or vanishing power at some of the spectral lines at the odd harmonics. The first case leaves the fraction in equation 3 undefined, and the second leaves the fraction close to singular. This situation should be avoided by proper choice of the transmitter waveform or by making sure that the desired system response $\hat{H}(\omega)$ has zeros at the same frequencies.

In practice, the system response $H(\omega)$ is derived from measurements at high altitude in which no earth response is present ($X(\omega) \simeq 0$), such that only the primary field is measured. Even in this setting, however, a response from the aircraft carrying the TEM system may contaminate the measurement. In production mode, temperature changes may cause the transmitted waveform, and thereby also the instantaneous system response, to drift away from the high-altitude reference. To mitigate this, an average transmitter current waveform $c_0(t)$ is measured at high altitude, and a drift-correction term $C_0(\omega)/C(\omega)$ is included in equation 3, where $C(\omega) = \mathcal{F}\{c(t)\}$ and $C(\omega)$ denotes the Fourier transform of an instantaneous production-mode current measurement $c(t)$ (Macnae and Baron-Hay, 2010). Furthermore, because the measurements consist of discrete-time samples, the discrete Fourier transform (DFT) and its inverse are used, rather than the continuous transforms, yielding the final expression for the deconvolved signal:

$$\hat{m}[n] = \text{DFT}^{-1} \left\{ \frac{C_0[\omega] \hat{H}[\omega]}{C[\omega] H[\omega]} M[\omega] \right\}. \quad (5)$$

The deconvolution is performed on blocks of measurements spanning one period of the transmitted waveform, containing a positive and a negative instance of the base waveform.

SYSTEM AND NOISE MODELS

In any practical scenario, the measured signal is contaminated by noise. To examine how noise in the measured signal $m(t)$ propagates to the deconvolved signal $\hat{m}(t)$, a more detailed model of the measurement process than the one defined in equation 2 is required. We define a model as illustrated in Figure 1. Changes in the transmitter current $i_T(t)$ induce a primary-field signal in the receiver and in the earth, which in turn induces a secondary-field signal in the receiver. The signal from the receiver coil is passed through a low-pass antialiasing (AA) filter and is sampled at a fixed rate using an analog-to-digital converter (ADC). External noise $n_{\text{ext}}(t)$ and internal noise $n_{\text{int}}(t)$ enter the measurements before and after the receiver, respectively. Geometric coupling factors for the primary and secondary fields are omitted because they do not affect the conclusions. We neglect any bias effects.

Omitting the AA filter, the system response is divided in two: a part relating to the transmitter $I_T(\omega) = \mathcal{F}\{i_T(t)\}$ and a part relating to the receiver $R(\omega)$ such that $H(\omega) = j\omega I_T(\omega)R(\omega)$, where j is the imaginary unit and multiplication by $j\omega$ corresponds to differentiation in the time domain. The function $M(\omega)$ can then be expressed as a combination of a primary-field component, a secondary-field component, and two noise components; $M_P(\omega)$, $M_S(\omega)$, $M_{N,\text{ext}}(\omega)$, and $M_{N,\text{int}}(\omega)$:

$$\begin{aligned} M(\omega) &= j\omega I_T(\omega)R(\omega) + j\omega I_T(\omega)X(\omega)R(\omega) \\ &\quad + N_{\text{ext}}(\omega)R(\omega) + N_{\text{int}}(\omega), \\ &= \underbrace{H(\omega)}_{M_P(\omega)} + \underbrace{H(\omega)X(\omega)}_{M_S(\omega)} \\ &\quad + \underbrace{N_{\text{ext}}(\omega)R(\omega)}_{M_{N,\text{ext}}(\omega)} + \underbrace{N_{\text{int}}(\omega)}_{M_{N,\text{int}}(\omega)}. \end{aligned} \quad (6)$$

Assuming that there is no drift ($C_0(\omega)/C(\omega) = 1$), we deconvolve using equation 3, which yields

$$\begin{aligned} \hat{M}(\omega) &= \underbrace{\hat{H}(\omega)}_{\hat{M}_P(\omega)} + \underbrace{\hat{H}(\omega)X(\omega)}_{\hat{M}_S(\omega)} \\ &\quad + \underbrace{G(\omega)N_{\text{ext}}(\omega)R(\omega)}_{\hat{M}_{N,\text{ext}}(\omega)} + \underbrace{G(\omega)N_{\text{int}}(\omega)}_{\hat{M}_{N,\text{int}}(\omega)}, \end{aligned} \quad (7)$$

where $G(\omega) = \hat{H}(\omega)/H(\omega)$. The primary- and secondary-field components are perfectly deconvolved to $\hat{M}_P(\omega)$ and $\hat{M}_S(\omega)$, whereas the noise components are modified by multiplication with $G(\omega)$.

The external noise $n_{\text{ext}}(t)$ is assumed to consist of stationary and nonstationary components, e.g., from radio signals, spherics, and motion-induced noise, or nonlinear effects such as amplifier saturation due to a strong primary-field component in the on time. Examples of typical noise spectra in the relevant frequency range can be found in Macnae et al. (1984), Nyboe and Sørensen (2012), and Macnae (2015). The internal noise $n_{\text{int}}(t)$, which is assumed to be stationary, consists of thermal noise (Kittel, 2004), electronics noise from oscillators and power supplies, and quantization noise in the ADC.

Stationary and nonstationary noise

For stationary noise, the amplification by $G(\omega)$ has an inconvenient although somewhat trivial effect; the spectrum of the ever-present stationary noise is altered. For nonstationary noise, the situation is different. In addition to altering the spectrum, $G(\omega)$ may “copy” time-local noise to other locations in the time domain. This happens when only the odd harmonics are included in the inverse transform in equation 5, thus forcing opposite-sign periodicity on all components of the deconvolved signal $\hat{m}(t)$, including the noise. As a result, a noise transient appearing once in $m(t)$ will appear twice in $\hat{m}(t)$ with opposite signs. For some choices of $\hat{H}(\omega)$, even more duplications occur, as is demonstrated in the “Synthetic examples” section below. These noise transients can be powerful enough to render the affected part of the signal useless (Macnae et al., 1984).

Figure 1. System- and noise-model diagram.

Time-domain properties of convolution

In the absence of nonstationary noise, the noise properties of the deconvolution method are sufficiently illuminated by equation 7. However, in the nonstationary case, we must look at convolution from a time-domain perspective. As we show in the sections below, the time-domain deconvolution kernels can be described as a composition of some characteristic features, namely, impulse-like functions, step-like functions, and smooth features. By recalling the effect of convolving a signal with each characteristic feature, we can gain some insight into what Fourier deconvolution actually does to the signal and noise components.

Convolution with an impulse at time t_0 , $\delta(t - t_0)$ results in a copy of the signal, time shifted by t_0 :

$$x(t) * \delta(t - t_0) = x(t - t_0). \quad (8)$$

The step function is the integral of an impulse, and thus convolution with a step at t_0 , $\theta(t - t_0)$, results in a time-shifted integration, which has a smoothing effect:

$$x(t) * \theta(t - t_0) = \int_{-\infty}^t x(\tau - t_0) d\tau. \quad (9)$$

Figure 2. Waveforms and resulting deconvolution kernels used in the deconvolution. (a) Time-domain waveforms. The sawtooth waveform is deconvolved to a boxcar and a full duty-cycle waveform. The repetition frequency is 275 Hz. (b) Frequency spectrum, with only the uneven harmonics of the repetition frequency plotted because the magnitude is zero elsewhere. (c) Time-domain deconvolution kernels. The arrows indicate impulses that rise to amplitudes of ± 0.5 .

Convolution with a smooth function results in a smearing of the signal, i.e., a low-pass filtering, the properties of which depends on the width and smoothness of the function.

SYNTHETIC EXAMPLES

To demonstrate the effects of applying deconvolution kernels to noise, we now turn to some synthetic examples of common applications.

Increasing apparent bandwidth

In some cases, it is desirable to enhance the high-frequency components of a measured signal by deconvolving it such that the passband of the apparent receiver extends to higher frequencies. In such cases, the apparent transmitter waveform is not altered, i.e., $\hat{H}(\omega) = j\omega I(\omega)\hat{R}(\omega)$, and thus we have

$$G(\omega) = \frac{\hat{R}(\omega)}{R(\omega)}. \quad (10)$$

Like the noiseless primary and secondary fields, the external noise now appears as if it was measured through $\hat{R}$ instead of R . Thus, in this situation, the deconvolution is particularly well behaved — a property also noted in Macnae and Baron-Hay (2010): “The method is virtually perfect for small bandwidth corrections and transmitter drift.” The word “small” is of importance here because a large increase in bandwidth would require amplification of frequency components where the signal-to-noise ratio (S/N) is too low to extract any useful information.

Changing apparent excitation waveform

When the method is pushed further, and the signal is deconvolved to appear as if it were obtained using a different transmitter waveform $\hat{I}_T(\omega)$, the situation is more complicated. To exemplify this, we examine the deconvolution kernels that are used to transform from a sawtooth waveform to a boxcar waveform and from a sawtooth to a full duty-cycle waveform, all periodic with the same base frequency. The waveforms are shown in Figure 2a. Transformations involving other waveforms, e.g., a half-sine, are naturally also possible.

Figure 2b and 2c shows the resulting frequency- and time-domain deconvolution kernels for the two examples, respectively. In the time-domain kernels, we see examples of impulse, step, and smooth features, as discussed in the “time-domain properties of convolution” section.

In the two examples here, convolution with the impulses in the deconvolution kernels serves to reproduce the effect of the sharp current steps that we see in the target waveforms by making copies of the sawtooth signal. The reason that the sawtooth → boxcar kernel (orange) has twice as many impulses as the sawtooth → full duty-cycle kernel (blue), is that the boxcar waveform contains twice as many current steps. The smooth features and the step features in the deconvolution kernels compensate for the remaining differences in the shapes of the waveforms by a smoothing effect and a differentiating effect. In these examples, the impulse components contain the same signal energy as the step components and smooth features, combined. Convolution of noise with the broad, low-amplitude features results in damping and broad smearing of the noise, whereas

convolution with the impulse features results in time-shifted copies of the noise for each impulse in the deconvolution kernel. An effect of this is that noise from the on-time intervals of the measurement, can be pulled into the off-time intervals of the deconvolved signal. This is important because noise conditions can be worse in the on time, as is discussed in later sections.

Fusing data sets with different waveform repetition rates

Signals recorded with identical system parameters, except for different repetition rates, can be compared to a certain degree, but they will differ, due to the difference in the overlap of early- and late-time data for successive pulses. In addition, systems with a faster repetition rate typically transmit at a lower dipole moment. A lower dipole moment can be achieved using fewer windings in the transmitter loop, which enables a faster current turnoff and thereby more high-frequency content in the excitation waveform (Sørensen and Auken, 2004).

When using Fourier deconvolution for fusing two data sets, recorded using different repetition rates of the excitation waveform, a fundamental limitation occurs. Let $H_{\text{fast}}[\omega]$ and $H_{\text{slow}}[\omega]$ denote the system responses of systems with repetition frequencies f_{fast} and f_{slow} , where $f_{\text{fast}} > f_{\text{slow}}$. Both excitation waveforms are assumed to be opposite-sign periodic, such that the spectra of $H_{\text{fast}}[\omega]$ and $H_{\text{slow}}[\omega]$ are only nonzero at the odd harmonics of their respective repetition frequencies, and because $f_{\text{fast}} \neq f_{\text{slow}}$, their nonzero spectral components do not always coincide. If none or few of the nonzero components coincide, a meaningful deconvolution kernel $G_{\text{fast} \rightarrow \text{slow}}[\omega]$ or $G_{\text{slow} \rightarrow \text{fast}}[\omega]$ cannot be computed. When $f_{\text{fast}} = n f_{\text{slow}}$, $n = [2, 3, 4, \dots]$, the coincidence of nonzero components is maximized, such that every n th nonzero component overlaps, as sketched in Figure 3.

Further, under the assumption that $f_{\text{fast}} = n f_{\text{slow}}$, deconvolution from a high to a low repetition rate is impossible because computation of $G_{\text{fast} \rightarrow \text{slow}}[\omega] = H_{\text{slow}}[\omega]/H_{\text{fast}}[\omega]$ involves division of a nonzero value by zero, whereas going the other way using $G_{\text{slow} \rightarrow \text{fast}}[\omega] = H_{\text{fast}}[\omega]/H_{\text{slow}}[\omega]$ does not cause such problems (see Figure 3). An intuitive view of this is that deconvolution using $G_{\text{slow} \rightarrow \text{fast}}[\omega]$ involves overlapping of early- and late-time data of the measurement with the lower repetition frequency, whereas deconvolution using $G_{\text{fast} \rightarrow \text{slow}}[\omega]$ would require separation of the data measured with the higher repetition frequency into an early- and a late-time part, which is not possible.

An example application for this type of data fusion is with the SkyTEM dual moment scheme (Sørensen and Auken, 2004), which uses low-moment (H_{275}) and high-moment waveforms (H_{25}) with typical repetition frequencies $f_{275} = 275$ Hz and $f_{25} = 25$ Hz ($n = 11$). Figure 4a and 4b shows the two waveforms and the corresponding system responses. The deconvolution kernel, $G(\omega) = \hat{H}(\omega)/H(\omega)$, has large amplification at high-frequency components due to $\hat{H}(\omega)$ having higher bandwidth than $H(\omega)$. Because the signal and the deconvolution kernel is likely to be dominated by noise at high frequencies, we cannot draw reliable information from these frequency components, and therefore we opt to remove them from $G_{25 \rightarrow 275}$. This is done by applying a filter (in this case a Gaussian zero-phase filter), which is responsible for the roll-off of the deconvolution kernel at high frequencies (Figure 4b black). The filtering has virtually no effect on the kernel functionality, and this results in a much cleaner looking time-domain deconvolution kernel with the

insignificant high-frequency contents removed. The filtered time-domain deconvolution kernel is shown in Figure 4c at two levels of detail. Because the deconvolution kernel is periodic with a period time of $1/275 \text{ s} \approx 3.6 \text{ ms}$, we only show a single repetition. The Gaussian shape of the impulses in the kernel (Figure 4c, right) is due to the Gaussian filter described above. As in the previous examples, the time-domain deconvolution kernels show distinct impulsive features, which give rise to duplication of noise and intermixing of on- and off-time data. In this case, however, the number of duplications is significantly higher than in the previous examples.

FIELD EXAMPLES

Field measurements were made using a ground-based custom configuration test setup, with an amplifier and digital oscilloscope as a receiver, sampling the TX current and the RX coil output

Figure 3. (Left) Nonzero frequency-domain components of H_{fast} and H_{slow} ($n = 11$). (Right) Examples of corresponding time-domain waveforms.

Figure 4. Waveforms and resulting deconvolution kernels. (a) Time-domain 25 and 275 Hz waveforms. (b) Frequency magnitude spectra of the waveforms and the filtered deconvolution kernel. (c) Filtered time-domain deconvolution kernel (left) and a magnification on a single pulse (right).

voltage at 2.5 MHz over intervals of 20 s. Two different transmitter waveforms, $i_1(t)$ and $i_2(t)$ with repetition rates of 275 and 25 Hz, and two different RX coils: one with low bandwidth and high sensitivity (A) and another with high bandwidth and low sensitivity (B) were used to measure three data series. The transmitter waveforms are shown in Figure 5, and transmitter and receiver details are given in Table 1. Table 2 lists the signals used in the field examples. The functions s_1 to s_3 are actual, measured signals, whereas s_4 and s_5 are synthetically generated for use in deconvolutions to “ideal” waveforms.

Because the transmitter and the receiver clocks were not synchronized, the data show a slight mismatch between the two. In addition, the 2.5 MHz sample rate was not ideal because it does not give an integer number of samples per period at a TX repetition rate of 275 Hz. Therefore, the signals were digitally compensated for clock mismatch and resampled at a rate of 2,499,750 Hz in an essentially lossless process, giving 99,990 and 9090 samples per period at 25 and 275 Hz repetition rates, respectively.

Figure 5. The 50% duty cycle transmitter waveforms used in the field examples to produce $s_1 - s_3$ (see Table 2).

Table 1. Details of the transmitter and receivers used in the field examples.

Transmitter	Effective area	Peak current	Waveform	f_{rep}
40×40	1600 m ² (square loop)	≈ 1 A	50% duty-cycle square wave	25 Hz or 275 Hz
Receiver	Effective area	Natural frequency	Filter cutoff	Filter order
RX (A)	40.3 m ²	34.3 kHz	322.0 kHz	Second
RX (B)	5 m ² (central loop)	547.0 kHz	322.0 kHz	Second

Table 2. Signals used in the field examples.

Signal	f_{rep}	RX coil	Waveform	Type
s_1	275 Hz	A	50% duty cycle	Measured
s_2	25 Hz	A	50% duty cycle	Measured
s_3	275 Hz	B	50% duty cycle	Measured
s_4	275 Hz	A	Full duty cycle	Synthetic
s_5	25 Hz	A	Full duty cycle	Synthetic

The ground-based setup, compared with an airborne system, has the disadvantage that the full system response cannot be directly measured because an earth response is always present. However, because the setup geometry is fixed, the earth response and coupling coefficients remain constant, and thus the deconvolution kernels can still be computed from averaged versions of the RX coil voltage signals and the TX waveforms. When transforming measurements from receiver X to appear as measurements from receiver Y , we have $G_{X,Y}(\omega) = \overline{M}_Y(\omega)/\overline{M}_X(\omega)$, and when transforming from the TX-waveform T to an alternative waveform Z , we have $G_{T,Z}(\omega) = \overline{I}_Z(\omega)/\overline{I}_T(\omega)$. An advantage of the ground-based setup is that the measurement geometry is completely fixed, so the noise statistics can be more accurately estimated than with a moving system because there is no ambiguity introduced by changes in the subsurface or system geometry.

Stationary noise

To show the effects of stationary noise, we examine the standard deviation for each gate for deconvolved and nondeconvolved measurements. Figure 6 shows such an analysis for various signals listed in Table 2. The left column shows error bars for the gated signals, distributed about the mean, such that for a signal $x[k]$ with standard deviation $\sigma_x[k]$ the solid lines trace $x[k] \pm \sigma_x[k]$, where k denotes the gate number. The right column shows the noise levels for each of the plots to the left. Our notation for the source signals and deconvolved signals is such that $d_{i \rightarrow j}$ denotes the result of deconvolving the source signal s_i to appear like s_j , where s_i and s_j are signals listed in Table 2. For example, $d_{1 \rightarrow 2}$ is the result of deconvolving s_1 to appear as if it were acquired using the same system as s_2 . The signals are gated using a logarithmic scheme with 26 box-car-shaped gates in the time range shown. Statistics are computed using the sign-corrected averages of the positive and negative halves of each period using 499 periods for signals with $f_{\text{rep}} = 25$ Hz and 5500 periods for signals where $f_{\text{rep}} = 275$ Hz. Results from experiments with alternative gate shapes did not significantly alter these results.

From Figure 6c and 6d, in which the deconvolutions are simple bandwidth corrections, we see that

- 1) The noise level of the deconvolved signals approaches that of their source signal at late times. This is due to the fact that at late-time low-bandwidth gates, there is essentially no signal correction, and the source signal is copied.
- 2) Deconvolving s_3 to appear like s_1 ($d_{3 \rightarrow 1}$) gives a consistently higher noise level than measuring s_1 directly. This is because the relatively high internal noise level in RX coil (B) used to measure s_3 gives lower S/N than RX coil (A) used to measure s_1 .
- 3) In the first eight gates, deconvolving s_1 to appear like s_3 ($d_{1 \rightarrow 3}$) gives an inferior result compared with measuring s_3 directly, due to damping of the early-time high-frequency signal components by RX coil (A), used to measure s_1 . In the remaining 18 gates, however, $d_{1 \rightarrow 3}$ is superior to s_3 because of the lower internal noise in RX coil (A).

From Figure 6e and 6f, in which signals are deconvolved to appear as if measured using band limited full duty-cycle square waveforms, we observe that

- 1) Deconvolving to a target signal with a higher repetition rate than the source signal ($d_{2 \rightarrow 4}$) gives significantly higher noise level than deconvolving to the same repetition rate ($d_{1 \rightarrow 4}$ and $d_{2 \rightarrow 5}$).
- 2) The noise level of the 275 Hz repetition rate $d_{1 \rightarrow 4}$ is approximately the same as for the 25 Hz repetition rate $d_{2 \rightarrow 5}$. However, because the acquisition time of $d_{2 \rightarrow 5}$ is 11 times longer than that of $d_{1 \rightarrow 4}$, one can argue that the comparison is unfair. By stacking 11 repetitions of $d_{1 \rightarrow 4}$, we get equal acquisition times, and the noise level of the 275 Hz repetition rate signal becomes approximately $\sqrt{11} \approx 3.3$ times lower than that of the 25 Hz repetition rate signal.
- 3) Note that the curves here only show gates spanning approximately 1 ms, which fits within the off time of the 275 Hz TX waveform used. The 25 Hz repetition rate signals have significantly longer off times, which is not shown in the plots.

DATA FUSION

One of the applications of deconvolution listed in the introduction is fusion of data measured using different system parameters. Take, for example, s_3 , which captures the early-time earth response well due to the high bandwidth of the receiver (RX B), but it

performs poorly at late times due to the receiver's high internal noise level. In a given scenario, the early-time data quality might be acceptable, but the late-time data quality is insufficient.

To improve the late-time S/N we could, for example, double the number of measurements with the same setup and double the stack depth, thus lowering the gate variance by approximately a factor two. However, because we were satisfied with the early-time performance, we can do better by taking the extra measurements using a lower bandwidth, lower noise receiver (RX A) and obtain s_1 . By deconvolving s_1 to $d_{1 \rightarrow 3}$ and fusing s_3 with $d_{1 \rightarrow 3}$ by means of a weighted stacking, we obtain $f_{3,1 \rightarrow 3} = \text{stack}_w(s_3, d_{1 \rightarrow 3})$, with slightly better early-time performance and significantly better late-time performance than s_3 . Under the assumption that the gate values of s_3 and $d_{1 \rightarrow 3}$ are independent and normally distributed with the same mean value, the optimum stacking weights are the inverse gate variances, such that

$$f_{3,1 \rightarrow 3}[i] = \frac{\bar{s}_3[i]\sigma_3^{-2}[i] + \bar{d}_{1 \rightarrow 3}[i]\sigma_{1 \rightarrow 3}^{-2}[i]}{\sigma_3^{-2}[i] + \sigma_{1 \rightarrow 3}^{-2}[i]}, \quad (11)$$

$$\sigma_{3,1 \rightarrow 3}^2[i] = \frac{1}{\sigma_3^{-2}[i] + \sigma_{1 \rightarrow 3}^{-2}[i]}, \quad (12)$$

where $\bar{s}[i]$ and $\bar{d}_{1 \rightarrow 3}[i]$ denote the i th stacked gate values of s and $d_{1 \rightarrow 3}$, whereas σ_3^2 and $\sigma_{1 \rightarrow 3}^2$ are the gate variances of s_3 and $d_{1 \rightarrow 3}$, respectively. Figure 7 compares the gate standard deviation for s_3 , $s_{1 \rightarrow 3}$, $f_{3,1 \rightarrow 3}$, and $\tilde{s}_3$, which approximates the effect of doubling the stack depth for s_3 . Relative to s_3 , $f_{3,1 \rightarrow 3}$ shows a slight early-time improvement and a significant late-time improvement, whereas $\tilde{s}_3$ shows moderate improvement at all times.

Nonstationary noise

Nonstationary noise enters the measurements from several sources. Some noise occurs randomly, such as spherics and radio bursts, whereas other noise comes from the TEM system itself. Examples of sources in the latter category include varying primary field coupling in the on time due to vibrations, amplifier saturation due to strong coupling with the primary field and transients induced by switching operations in the transmitter.

Traditionally, when deconvolution is not applied, the on-time data are discarded and the on-time noise is of no concern. When

Figure 6. Noise levels for measured and deconvolved signals. (a, c, e) The uncertainty bars show $\pm$ one standard deviation, distributed among the mean for each gate. (b, d, f) Standard deviations for each of the plots on the left.

Figure 7. Comparison of gate standard deviations for measured, deconvolved, and fused signals, as well as the approximate reduced standard deviation for the measured signal with double stack depth.

the signals are deconvolved, however, the time-domain representations of the deconvolution kernels (Figures 2 and 4) show that samples are copied and smeared in the time domain to produce a deconvolved signal. Thus, increased noise in the on time of the source signal is drawn into the off time of the deconvolved signal. A special case is deconvolution to the full duty-cycle waveform, in

Figure 8. Migration of noise from on to off time after deconvolution. Dark- and light-gray areas indicate on and off time, respectively. The signal shown with blue is a copy of the orange signal with the on-time noise amplified by a factor of two. (a) Source signals ($f_{\text{rep}} = 25$ Hz). (b) Result of deconvolving the measured signal (orange) and the signal with amplified on-time noise (blue) ($f_{\text{rep}} = 275$ Hz). (c) Mean noise standard deviation after gating the deconvolved signals, computed from 500 instances of the scenario.

Figure 9. On-time transient entering the off-time after deconvolution. Dark- and light-gray areas indicate on and off times, respectively. The signal shown with blue is a copy of the orange signal with a $200 \mu\text{s}$ wide boxcar-shaped disturbance added (see inset). (a) Source signals ($f_{\text{rep}} = 25$ Hz). (b) Result of deconvolving the measured signal (orange) and the signal with the superimposed boxcar disturbance (orange) ($f_{\text{rep}} = 275$ Hz). The black line shows the difference between the two deconvolved signals. (c) Mean noise standard deviation after gating the deconvolved signals, computed from 500 instances of the scenario.

which the concept of on and off time is abandoned, and all samples are used for interpretation. To show examples of these effects, we set up two semisynthetic deconvolution scenarios, both of which operate on measured pure noise signals, i.e., with no TEM signal, with and without a synthetic noise component added. In the first scenario (Figure 8), we show how the deconvolution process mixes artificially amplified noise from the on-time into the off-time of the deconvolved signal, and in the second scenario (Figure 9), we superimpose a $200 \mu\text{s}$ wide boxcar-shaped disturbance onto the measured signal to emulate a narrow noise burst and show how it affects the deconvolved signal. The figure for each scenario shows the source signals, the result of deconvolving each source signal and, last, the off-time noise levels, in the form of the mean standard deviation for each gate value, for the deconvolved signals (orange and blue). For the second scenario, the difference between the two deconvolved signals is shown in black.

The first scenario (Figure 8), in which the on-time noise is doubled, shows how deconvolution causes on-time noise to leak into the off-time of the deconvolved result. In this case, this leads to an increase in the gated off-time noise standard deviation by a factor of approximately 1.5–1.7, depending on the gate number (Figure 8b and 8c). In a real scenario, the increase naturally depends on the amplitude, frequency, and time characteristics of the on-time noise and of the deconvolution kernel.

The second scenario (Figure 9), in which a short boxcar-shaped disturbance is added to the measured noise in the beginning of the second on-time, clearly shows the time and frequency dependence mentioned above. Although the high-frequency features of the noise are damped due to the band limitation of the deconvolution kernel, the lower frequency features propagate through to the deconvolved result via smearing and time-shifted copying, contributing to the noise in a nonobvious way (Figure 9b, black). When gating is applied, Figure 9c shows how the first half of the gates, covering the interval $[0, 0.05]$ ms and the last gates in the interval $[0.4, 1]$ ms are virtually unaffected by the additional noise, whereas the noise level in the remaining gates is heavily affected by the oscillations starting at approximately 0.05 ms, seen as the black line of Figure 9b. Naturally, the examples are simplifications of reality, but they serve the purpose of demonstrating the effects of deconvolution well; namely, that on- and off-time data are mixed in a way that may cause non-obvious and nonnegligible increases in noise.

CONCLUSION

Through theoretical analysis and a combination of synthetic and field examples, we have investigated the noise properties of the Fourier deconvolution method as used for TEM data processing. An advantage of this deconvolution method over other methods used in TEM is that it does not rely on model assumptions relating to the subsurface. The method can be applied to enhance the visual presentation of time-domain data, to alter the apparent system response used to acquire the data, for fusing data sets acquired using different system responses, and for drift correction. An important note is that, except for a possible improvement due to drift correction, this type of deconvolution should not give improved inversion results because it does not reveal any new data. Nevertheless, it may be necessary to alter the apparent system response by deconvolution before inversion in some cases. Two examples are the following: If the inversion code cannot handle the given system response or if the

inversion code is unable to perform a joint inversion of two dissimilar data sets, in which case, they can be fused into a single data set.

In the synthetic examples, we have shown how the primary and secondary fields are perfectly deconvolved by the Fourier method to appear as desired. External noise enters the measurements in a band-limited form, filtered through the receiver system, whereas internal noise is not band limited. When deconvolution is applied to the noise components, high-frequency components of the internal noise may thus be strongly amplified. We have treated two different kinds of noise: stationary and nonstationary. For stationary noise, deconvolution alters the spectrum by amplifying or damping certain components. For nonstationary noise, however, the method imposes opposite-sign periodicity, which causes duplication of these noise components in the time domain. Fortunately, in a well-designed system, the internal wide-band noise should be stationary and of low amplitude compared with the external noise. External nonstationary noise enters the measurements from various sources, such as radio signals, spherics, and from the system itself, e.g., in the form of motion causing a variable coupling of the primary field, in the form of amplifier saturation due to strong primary field coupling, or from the transmitter electronics.

By applying the deconvolution method to a range of field measurements, we confirmed the theoretical work, and we provided examples of how deconvolution affects the noise levels of the real and semisynthetic signals that mimic real scenarios. Our main observations are that

- 1) Deconvolution decreases the S/N, compared with nondeconvolved measurements.
- 2) Deconvolution between repetition rates yields a significantly lower S/N than deconvolution to the same repetition rate.
- 3) Two or more data sets from different systems with different noise properties can be fused to produce a single data set with improved noise properties compared with each of the original data sets.
- 4) For typical applications, e.g., transformation to an alternative apparent TX waveform, increased on-time noise in the source signal affects the off time of the deconvolved signal in a nonobvious and nonnegligible manner.

In conclusion, our recommendation is that Fourier deconvolution should only be applied when necessary to improve the presentation of time-domain data, or to condition the data for use with a specific inversion code unable to handle the given system response. For generic inversion codes with the ability to model arbitrary system responses, deconvolution should not be necessary, except possibly for drift correction, and it is likely to yield inferior results. If Fourier deconvolution is applied, one should be aware of any nonideal on-time behavior of the system before applying deconvolution and uncertainties should be carefully tracked through the deconvolution process.

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