Models for Product Line Design with Customization - Considering Factors in Marketing and Operations

PhD Dissertation

Parisa Bagheri Tookanlou

Supervisor: Hartanto Wijaya Wong

Aarhus BSS, Aarhus University
Department of Economics and Business Economics

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Parisa Bagheri Tookanlou

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Summary

The central theme of this dissertation is product line design that involves customization. The theme covers a range of strategic decisions for manufacturing firms that adopt mass customization. The dissertation consists of three papers, each of which addresses different issues related to the product line design problem where customized products are offered. The three papers all present stylized models that take marketing and operations related factors into account.

In the first paper, we study a manufacturer’s product line design problem when consumers are heterogeneous in their valuation of the product customization level and lead time. We extend the standard quality-based segmentation problem by considering the degree of product customization and lead time as the important attributes of product quality that affect a consumer’s purchase decision. Contrary to the most existing studies that neglect inventories in the production system, this paper explicitly considers the inventory positioning decision that is interdependent with the product line decisions including pricing, product customization level, and lead time. We develop a model that provides insights into the conditions under which the optimal decision for the manufacturer is to implement the two-product strategy with two different customization levels or the one-product strategy with a single customization level. The results of the numerical study show that the costs of holding safety stock and pipeline inventory have an influence on the optimal choice of customization levels. The optimal customization levels tend to be lower when the inventory costs are considered.

The focus in the second paper is on determining the optimal product line design in a market with vertical and horizontal consumer heterogeneity. In particular, we are interested in examining the effect of offering a customized product in the product line. We consider two comparison scenarios. In the first scenario, we use the single-product strategy as the baseline strategy and examine the conditions under which a horizontal product line extension through the offering of the customized product is preferable to a vertical product line extension or quality-based segmentation strategy. In the second scenario, we focus on examining the effect of offering the customized product to an
existing quality-based segmentation strategy. The main results show that the horizontal product line extension may result in a higher increase in the channel profit as long as the investment to accommodate flexibility is not too costly. Offering the customized product to an existing quality-based segmentation strategy may also help increase the channel profit. The results show that the channel structure is influential. The preference for the horizontal product line extension is stronger in a decentralized channel than in a centralized channel.

In the third paper, the impact of demand uncertainty on product line design with customization is studied. This paper considers horizontal product differentiation, and the product line studied consists of a standard product offered in a make-to-stock fashion and a customized product offered in a make-to-order fashion. Due to the demand uncertainty, the stocking level for the standard product must be determined before the selling period starts, and consequently the problem is framed in a newsvendor setting. The focus of the study is on examining the effects of demand uncertainty on the optimal product line design and on how those effects are different when the products are sold in a centralized channel or in a decentralized channel. In the centralized channel, the results show that the presence of demand uncertainty drives the prices for the standard and customized products as well as the customization level down. In the decentralized channel, the retail price for the standard product in the case with demand uncertainty is lower than the price in the case with deterministic demand and the numerical example shows that the optimal customization level decreases by increasing the uncertainty in demand and also compared to when the demand is deterministic. This study also explores the possibility of employing a buy-back contract as a way to improve the channel profit while providing benefits for both the retailer and the manufacturer.

The dissertation advances the existing literature on product line design problem, especially the one that considers customized products as an important offering in the product line. Furthermore, it provides useful insights for practitioners regarding the effects of the different marketing and operations factors and their interactions on the optimal product line decisions.
Resumé


I den første artikel ser vi på en producents problemer ift. produktlinje-design, når kunderne er heterogene i deres vurdering af produkttilpasning og leveringstid. Den normale model for kvalitetsbaseret segmentering udvides ved at betragte graden af produkttilpasning og leveringstid som væsentlige parametre for kundens købsbeslutning. I modsætning til de fleste nuværende studier, der undlader at tage højde for lagerbeholdning i produktionssystemet, ser denne artikel eksplicit på beslutninger vedrørende lagerbeholdning, der er gensidigt afhængig af beslutninger vedrørende produktlinje, inklusiv prisfastsættelse, produkttilpasning og leveringstid. Vi udvikler en model, der viser, hvilke forhold der danner baggrund for virksomhedens beslutning om at indføre henholdsvis en to-produkt strategi med to forskellige tilpasningsniveauer og en et-produkt strategi med et enkelt tilpasningsniveau. Resultaterne af den numeriske undersøgelse viser, at omkostningerne ved at have et sikkerhedslager og et pipeline-lager har indflydelse på det optimale valg med hensyn til tilpasningsniveauer. De optimale tilpasningsniveauer har en tendens til at være lavere, når lageromkostninger tages i betragtning.

I afhandlingens anden artikel er fokus på at bestemme det optimale produktlinje-design i et marked med vertikal og horisontal kunde-heterogenitet. Vi er specielt interesserede i at undersøge effekten af at tilbyde et kundetilpasset produkt i produktlinjen. Vi sammenligner to scenarier. I det første scenarie tager vi udgangspunkt i en ét-produkt strategi og undersøger, under hvilke forhold en horisontal udvidelse af produktlinjen via tilbud om det kundetilpassede produkt er at foretrække fremfor en


Denne afhandling bidrager til den eksisterende litteratur om problemer ift. produktlinje-design, specielt ift. de tilfælde hvor kundetilpassede produkter udgør en vigtig del af produktlinjen. Derudover bidrager afhandlingen med vigtig viden til virksomhederne vedrørende effekterne af forskellige markedsforings- og driftsfaktorer og deres betydning for beslutninger med henblik på at opnå det optimale produktlinje design.
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Chapter 1

Introduction
1.1 Background and Motivation

This PhD dissertation considers product line design problems that involve the offering of customized products. Designing a product line is a strategic decision for manufacturing firms and is the very essence of every business. The firms, faced with a heterogeneous market in which consumers may have their own ideal product, must decide on the number of products offered, product quality, and price to maximize their profits. Customization is an important feature in the product line design problems studied in this dissertation, and is concerned with offering tailored products to individual consumers.

There is no doubt that designing a product line that involves customization will require a joint perspective on marketing and operations decisions. The traditional view suggesting that marketing and operations just focus on revenues and costs is outdated and will not give the best results (Tang 2010). In the context of product line design with customization, one important and relevant decision that manufacturing firms need to make concerns the degree of customization offered to consumers. This decision should be made based on careful consideration of marketing and operations related factors regarding consumers’ preferences and manufacturing or operational capabilities, and it is also important for the firms to enhance their understanding of the interaction among those factors.

As the offering of customized products represents an important part of the product line design problems studied in this dissertation, the concept of mass customization (MC) is of relevance. Mass customization refers to a manufacturing strategy aimed at providing sufficient product and service variety so that nearly all consumers find exactly what they want without significantly compromising cost efficiency. The concept of mass customization was first introduced by Davis (1987) and later developed by Pine (1993), and remains perceived as a promising strategy that may help companies gain competitive advantage (Gandhi et al. 2013). A study conducted by Bain and Company (Spaulding and Perry 2013) shows the promising market opportunity for individualized customization. According to their survey, 25 to 30% of online shoppers are interested in buying customized products. To achieve mass customization, manufacturing firms will need to be equipped with advanced manufacturing and information technologies and unique operational capabilities (Salvador et al. 2009). Implementing MC remains a challenging issue for many manufacturing firms due to increasing costs, uncertainty, and complexity of the manufacturing processes (Lai et al. 2012).

In an ideal world, firms implementing MC strategy should be able to provide customized products at a comparable price and speed of equivalent standardized products. However, empirical evidence
suggests that the customization is not ‘free’ and needs to be traded-off against factors such as lead
time and cost (Squire et al. 2006). This dissertation adopts this more pragmatic view. We are also in
agreement with Zipkin (2001) in that MC has its limits that depend on several factors such as the
availability of highly flexible technology and the existence of a potential mass market for customized
attributes. Different products (industries) may have different capabilities in meeting those
requirements. Clothing/apparel, footwear, computers, and furniture are some examples of products
that seem to have high capabilities in meeting the requirements (Zipkin 2001), for which this study is
quite relevant.

1.2 Focus of This Study

Product line design has been a key topic in the economics/marketing literature for decades, and more
recently, the integration of product line and operational decisions has become a central issue in the
literature. The literature on product line design can, in general, be classified into two categories.
Articles in the first category focus on the analytical results and the models presented are stylized and
rely on some restrictive assumptions. Nevertheless, they provide valuable managerial insights. Early
contributions in this category include e.g. Mussa and Rosen (1978) and Moorthy (1984). More recent
studies in this category that provide extensions to the earlier models include e.g. Kim and Chhajed
(2002), Netessine and Taylor (2007), Shi et al. (2013) and Jerath et al. (2017). Articles in the second
category present models in which the assumptions are less restrictive, but analytical and even
numerical solutions are difficult to obtain. The typical focus of those articles is on the development
of heuristic methods. With the help of data obtained through market surveys, the models developed
in the second category can be used for developing a decision support tool. Some earlier work is e.g.
studies are presented in e.g. Belloni et al. (2008) and Wang and Curry (2012). The models developed
in this dissertation are more closely related to and contributes to extending the literature in the first
category.

One of the important aspects in product line design is concerning the market structure characterized
by consumers’ heterogeneity. Some studies in the literature (e.g. Mussa and Rosen 1978, Moorthy
1984 and Moorthy and Png 1992) consider the market where consumers are heterogeneous in their
valuation of product quality so that they focus on the quality-based segmentation or vertical product
differentiation strategies. The notion of quality adopted in this literature is represented by a set of
product attributes, on which consumers agree in their preference ordering, i.e., every consumer
prefers higher values on the attributes to lower values (or vice versa), ceteris paribus (Moorthy 1984;
Moorthy and Png, 1992). Examples of such attributes are screen size and image resolution in the case of televisions, gas-mileage and towing capacity in the case of cars, and stitch count in the case of garment products. In these studies, firms implement vertical product differentiation by offering a product line comprised of multiple products with different quality levels serving a market with heterogeneous segments. We note that this definition of quality in the product line design literature is perhaps rather narrow compared to the broader definition used in the operations management literature. The American Society for Quality provides two perspectives in defining quality, namely the value perspective and the conformance perspective (Bozarth and Handfield 2016). More relevant to this study is the value perspective which is defined as the characteristics of a product or service that bear on its ability to satisfy stated or implied needs of consumers. Thus, the broad definition of quality also includes the implied needs of consumers that are hard to express and measure.

In contrast to the above stream of literature, some papers consider the market where consumers are heterogeneous in their taste or aesthetic attributes of the product such that the focus is on the horizontal product differentiation (e.g. De Groote 1994 and Lancaster 1990). Unlike the vertical attributes, the merits and demerits of the taste or aesthetic attributes may be evaluated differently by consumers. Some consumers may prefer a certain color but other consumers may prefer a different one. In the case of ice cream, some consumers may prefer vanilla flavor, but others may prefer chocolate. With the horizontal product differentiation, firms choose to offer products that have the same quality level (vertical attribute) but are different in the taste or aesthetic attributes. However, from the broad definition of quality perspective, customizing the taste or aesthetic attributes of a product will contribute to enhancing consumer satisfaction, and hence, increases the product quality.

In relation to the discussion on market structure and mass customization, an interesting question is how one should view customization as a product attribute, or in other words, whether customization should be associated with the vertical or horizontal product differentiation. The literature can be split into two categories in this regard. Papers in the first category (e.g. Gaur and Honhon 2006, Jiang et al. 2006, Alptekinoglu and Corbett 2008, and Mendelson and Parlakturk 2008a and 2008b) associate the degree of customization with the number of product variants that the consumers may choose from to best match their individual preference but they do not imply that one product variant is better in terms of quality (vertical attribute) than the others. Thus, the degree of customization represents the extent of the horizontal product differentiation.

Papers in the second category (see e.g. Franke and Schreier 2008, Michel et al. 2009, Franke et al. 2010, and Wong and Lesmono 2013) seem to be in agreement with Lampel and Mintzberg (1996)
and Duray et al. (2000) who suggest that the relative degree of product customization is determined by the extent to which consumers are involved in the production cycle. Those papers take a different view by arguing that increased customization may result in enhanced perceived values regarding uniqueness, utility, self-expressiveness, etc., which will in turn increase the consumers’ willingness to pay. When viewed from the value perspective of quality (Bozarth and Handfield 2016), this suggests that increased customization contributes to the improvement in product quality. This view on customization is particularly useful when considering a product line design that involves multiple customization levels. This dissertation acknowledges the importance of these two different views and the models developed in this dissertation are designed to be differentiated depending on which of the two views is adopted.

In studying product line design strategies, it is also important to pay attention to distribution channel structure used to sell the products. Channel structure can be differentiated by e.g. how many players involved, and in the case of more than one players, who the focal player is. This dissertation focuses on the setting where the manufacturer is the focal player and faced with the product line design problem. We follow the mainstream literature on product line design that also focuses on such a setting, and argue that the setting is commonly observed in practice, and hence of high relevance. In a centralized channel, the manufacturer sells the products directly to the end consumers, i.e., the manufacturer is the only player making all decisions. In contrast, in a decentralized channel, the manufacturer depends on intermediate players such as retailers in selling the products to end consumers. As the manufacturer’s interest is not always aligned with that of the retailers, it is expected that the manufacturer’s optimal product line design will be influenced by the presence of channel efficiency loss prevalent in a decentralized distribution channel. Of particular interest in this dissertation is a dual channel that could be seen as a variant of decentralized channel. In a dual channel, the manufacturer sells the standard products through the retailer, and sells the customized products directly to end consumers via an online channel owned by the manufacturer (Chiang et al. 2003 and Dumrongsiri et al. 2008). The way in which the product line design decisions are affected by the distribution channel structure represents an important line of inquiry examined in this dissertation.

As a note, we are aware of the possible existence of other settings to which the results presented in this dissertation may not be directly applicable. Consider, for example, a distribution channel where other players, e.g. retailers, play a more dominant role in the channel. In some cases, retailers are the players who make product line design decisions (see e.g. Dukes et al. 2014). There are also examples
where both the manufacturer and retailer are involved in designing the product attributes (Kolay 2015).

Motivated by the fact that most articles in the literature on product line design focus heavily on the marketing aspects and less so on the operations aspects, this dissertation examines enhancements by developing models that incorporate some of the important issues that are widely studied in the operations literature. By doing this, this dissertation contributes to developing more integrated models in the interface of marketing and operations. One important simplification in the existing literature on product line design is the assumption that demand is deterministic. This assumption certainly facilitates neat analytical results, but those results are questionable when considering many real settings in which demand uncertainty is actually prevalent. To cope with demand uncertainty, the manufacturing firm will need to hold inventories, which will, in turn, influence the product line design decisions.

Common to all the models developed in this study is the monopolistic setting assumption. That is, we focus on product line design problems faced by a manufacturing firm without considering any competition with other manufacturing firms. This allows us to focus on examining how a manufacturing firm considering mass customization should design a product line when dealing with the different issues explained above.

1.3 Structure of Dissertation

This dissertation consists of three self-contained papers prepared for publication in international journals. As outlined above, these papers all address the product line design problems where the customized product is part of the offering. Figure 1.1 summarizes the main issues addressed in each of the three papers and highlights how each paper differs from the other papers. The three papers can be differentiated based on: (a) consumers’ heterogeneity – whether it is in the horizontal dimension or vertical dimension or both; (b) demand – whether it is deterministic or stochastic; and (c) channel structure – whether we deal with a centralized channel or decentralized channel.
1.4 Highlights of Each Paper and Main Contributions

Paper 1: Determining the optimal customization levels, lead times and inventory positioning in vertical product differentiation

In the first paper, we study a product line design problem where products are customized and sold in a market where consumers are heterogeneous in their valuations of product quality. We take the view that the relative degree of product customization is determined by the extent to which consumers are involved in the product’s value chain, and that their reservation prices are likely to increase as the degree of customization increases. Given this view, we consider products for which the quality level is driven by the degree of customization. Further, in relation to the customization level, we explicitly consider lead time as an important parameter that may influence the consumer’s purchase decision. An important feature included in this study is the consideration of pipeline and safety stocks in the production processes that may influence the optimal product line design. By integrating both the marketing and production related factors, we consider the trade-offs among customization, lead times, and manufacturing and inventory costs, and examine how these trade-offs should be addressed in a market where consumers are differentiated by their valuation of customization level and lead time. The main contribution of this study is that we extend the literature on product line design by integrating the marketing and operations decisions. To the best of our knowledge, this study is the
first study to analyze the product line design problem that addresses the trade-offs between customization, lead times, and manufacturing and inventory costs.

**Paper 2: Product line design with vertical and horizontal consumer heterogeneity: the effect of distribution channel structure on the optimal quality and customization levels**

In this paper, we study a product line design problem where consumers are heterogeneous in both quality and aesthetic component of the products. In this study, offering customization in the product line is considered as a way of matching the product’s aesthetic component to the consumers’ ideal preferences. The study focuses on examining the preference of different product line extension strategies and how this preference is influenced by the type of distribution channel adopted. In particular, we are interested in examining the effect of offering the customized product in the product line. Using the single-product strategy as the baseline strategy, we first examine when a vertical product line extension strategy or quality-based segmentation is preferable to a horizontal product line extension strategy. In the vertical product line extension strategy, two products are offered with different quality, and each product is targeted to each segment. In contrast, in the horizontal product line extension strategy, a customized product is offered in addition to the standard product. We then examine the effect of adding a customized product to the existing quality-based segmentation strategy.

The main contribution of this chapter is as follows. We are the first to consider the product line design problem in which quality and customization level are decision variables. To the best of our knowledge, this is also the first time that two directions for the product line extension strategy are compared. This contribution is enhanced further by the examination of how the type of distribution channel may play a role in determining the best product line extension strategy.

**Paper 3: Product line design with customization: the effects of demand uncertainty and distribution channel structure**

In this paper, the impact of demand uncertainty on the optimal product line design decisions is investigated in a market where consumers are heterogeneous in the taste or preference of the products. A stylized model is developed to analyze how the optimal product line design regarding price and customization level is influenced by the risk of supply and demand mismatch, and how the influence differs between the centralized and decentralized channels. An attempt is also made to improve the channel profit in the decentralized channel by applying a supply chain contract. The work presented in Chapter 4 adds to the existing literature in the following respects. First, it extends the relatively
scant literature on product line design with customization in the presence of demand uncertainty. For the setting where the product line consists of the standard and customized products that are sold through a dual channel, this work is the first to examine the effect of demand uncertainty. Second, this work also contributes to filling the void in the literature on supply chain coordination in the context of product line design that especially involves marketing and operations decisions.

1.5 Managerial Implications

This PhD dissertation provides useful information for manufacturing firms interested in implementing mass customization in (re)-designing their product line. The results clearly show the importance of integrating operations and marketing related factors in product line design. Ignoring factors that are pertinent in operations such as inventory costs and demand uncertainty may result in sub-optimal product line design decisions. Furthermore, the impact of lead time cannot be ignored. While lead time reduction initiatives are always welcome, the approach presented in this dissertation provides helpful information for firms in determining the manufacturing processes where those lead time reduction initiatives will be most beneficial.

The results show that offering the customized product in the product line may enhance profitability, and manufacturing firms could consider offering customized products as an alternative to quality-based segmentation or as a complement to the existing quality-based segmentation strategy. However, this is not without conditions and depends on several factors. For example, in order to make the offering of customized products appealing, there is a need for a manufacturing system with a sufficiently high flexibility. In other words, the cost of enhancing flexibility that is driven by the customization level cannot be too high. This finding can motivate the firms to consider the possible adoption new advanced technologies (e.g. additive manufacturing), known for the high level of flexibility without necessarily incurring significantly higher costs (Rylands et al. 2016; Durach et al. 2017).

It is also important that manufacturing firms take into account the type of channel they are operating. This study provides a better understanding of the limitations inherent in a decentralized channel and provides guidelines for product line designers to determine the appropriate customization level in a decentralized channel in contrast to a centralized channel. In this study, we show that the presence of demand uncertainty has an influence on product line decisions. Hence, it highlights the importance of coordination between marketing and operations in product line design. Furthermore, this study should motivate the firms operating in a decentralized channel to be more proactive in designing channel contracts since we show that such contracts may improve channel profitability. While the use
of contracts is widespread in supply chain management practices, such contracts are less common when it comes to product line design. Nevertheless, this study shows the importance of channel coordination in product line design when the manufacturing firms rely on the retailer in selling their product.
Chapter 2

Paper 1: Determining the optimal customization levels, lead times and inventory positioning in vertical product differentiation

History: This paper has been presented at: ISIR 2016-Conference of International Society for Inventory Research, August 2016, Budapest, Hungary. The paper is currently under revisions for possible future publication in International Journal of Production Economics.
Determining the optimal customization levels, lead times and inventory positioning in vertical product differentiation

Parisa Bagheri Tookanlou and Hartanto Wong

Department of Economics and Business Economics

School of Business and Social Sciences, Aarhus University

Fuglesangs Allé 4, 8210 Aarhus, Denmark

Abstract

In this paper we study a manufacturer’s product line decisions when selling customized products in a market where consumers are heterogeneous with respect to their valuation of quality. The product has two attributes based on which a consumer’s product valuation is determined. The first attribute is customization level, defined by the extent to which consumers are involved in the value chain. The second attribute is delivery lead time which is influenced by the customization level. The vertically differentiated market is represented by two segments. Different from most existing work in the product line design literature that neglects the operational aspects attached to production, in this paper we consider the inventory positioning decision along the production processes which is interdependent with the customization level and lead time decisions. The model developed provides insights into the conditions under which the manufacturer should opt for the two-product strategy with two different customization levels or the one-product strategy with a single customization level. Our model can also be used to help determine which production process should be given the highest priority when the manufacturer initiates a lead time reduction project.

Keywords: Product Line Design; Mass Customization; Safety Stock Positioning; Quality-based Segmentation.
2.1 Introduction

Since the concept was first coined by Davis (1987) and further popularized due to Pine (1993), Mass Customization (MC) has been one of the central themes both practitioners and scholars focus on when discussing future manufacturing strategies. MC has always been perceived as a promising strategy that may help companies gain competitive advantage, increase revenue, and reduce waste through on-demand production, see e.g., Gandhi et al. (2013). However, the successful implementation of MC has proved hard to achieve; although there are successes, there are also many costly failures.

A study conducted by Bain and Company (Spaulding and Perry 2013) shows that the market opportunity for individualized customization appears to be significant. The results of their survey suggest that 25 to 30% of online shoppers are interested in trying the customization option. They also discover that individualized customization can increase the consumer loyalty and engagement because consumers who had experience in customizing a product online seem to be more willing to engage more with the company and visit its website more frequently. This opportunity has been made possible by the development of new technologies that not only allow firms to respond to consumers’ unique needs, but also to enhance the products’ affordability. Gandhi et al. (2013) states that some key enabling technologies include online configuration technologies, 3-D digital modeling, dynamically programmable robotic systems, etc.

Customized products can generally be categorized into two different groups. In the first group, consumers are allowed, at one or more stages in the production processes, to configure the product by choosing from an extensive set of choices. Examples of such products are many: bags (e.g., Timbuk), watches (e.g., Fossil), shoes (e.g., Nike and Adidas), and laptops (e.g., Dell). In recent years, we have witnessed a growing list of products that belong to the second group in which manufacturing firms offer individualized customization by letting consumers become involved in creating their own unique product. For example, Brooks Brothers and Stitch Fix offer custom suits made to fit one’s body shape and size. Regardless of the group to which a customized product belongs, firms must be careful when deciding the right customization level to offer.

In this paper, we adopt the concept of customization presented in Lampel and Mintzberg (1996) and Duray et al. (2000). That is, the relative degree of product customization is determined by the extent to which consumers are involved in the product’s value chain. A product can be considered to have a higher customization level when consumers are more deeply involved in the product’s value chain, e.g., in the design phase, as opposed to the assembly phase. Consumers in the market certainly may have different preferences for customization levels. Take, for example, customized furniture. Some
consumers may be satisfied with a particular sofa design offered by a manufacturer but may want to have customized fabric colors. Others might prefer a more unique sofa design instead. The differences in customization level preference are, to a great extent, related to the preference for product delivery lead time. Consumers who want a unique sofa design are willing to wait longer than those who only want customized fabric colors. Ideally, as also widely discussed in the literature, mass customization should allow consumers to buy a customized product at a price close to the price for the standard and mass-produced product. In reality, however, many innovative customized products offered either by start-ups or by established firms are still targeted at niche market segments. The reason is twofold. First, the firms must still incur additional costs for manufacturing and delivering those customized products. Second, the consumers in these niche segments, mostly conscious of product innovativeness, are those who are willing to pay a price premium for such customized products. Several studies (e.g. Tu et al. 2001, Franke and Schreier 2008, Michel et al. 2009, and Franke et al. 2010) support this notion as they show that when consumers perceive a product’s uniqueness as enhanced due to their involvement in the production process, the consumers’ reservation prices are likely to increase. Trentin et al. (2012) provides empirical support to the notion as they show that increased customization achieved through the use of product configurators improves product quality. The important role of product configurators in enhancing product quality is also highlighted in Zhang (2014). Based on the above discussion, product quality in our model is primarily driven by the customization level. We consider a product as a bundle of several attributes, but our focus is on two interacting attributes: customization level for which higher is better, and product delivery lead time for which shorter is better.

Given the differences in the consumers’ preferences as regards customization level and delivery lead times, manufacturers are confronted with some choices: should they offer single or multiple customization levels and which customization level(s) should they offer? When making those decisions, the manufacturers must also consider the potential cannibalization of the product with high customization level by the product with lower customization level. The manufacturers’ decisions on customization levels also have some inter-dependence with their production/inventory system. Depending on where the customization level is set, firms may still need to carry some product materials or components inventories. When the customization level is higher, there are less materials kept in the inventory compared to the situation where firms offer a lower customization level. Furthermore, the work-in-process or pipeline inventories are also important and therefore cannot be ignored.
Our study is motivated by the fact that literature addressing the aforementioned problems is scarce. In this paper, we frame the manufacturers’ product customization decisions as a product line design problem in which the quality attributes are represented by the customization level and delivery lead time. In contrast to most studies within product line design literature, we consider the interdependence of the decisions on those two quality attributes and the decisions on inventory held in the production system. More specifically, we aim to answer the following questions:

(i) Which customization level(s) should firms offer when taking into consideration the marketing factors (consumers’ appreciation of customization level and lead time) as well as inventories in the production system?

(ii) How the answer to the above question is affected by the changes in different marketing and production related parameters?

While focusing on the questions above, this paper also offers a method that can be used to establish priorities across all the main production processes such that firms can decide which process to focus on when implementing lead time reduction initiatives. This issue is particularly important given the fact that addressing the so-called customization-responsiveness squeeze is indeed crucial to the success of mass customization strategies (McCutcheon et al. 1994 and Tu et al. 2001).

The rest of the paper is organized as follows: In the next section, we present a survey of the relevant literature. Section 2.3 presents the model that integrates production-inventory and marketing decisions. In Section 2.4, we present the numerical results and discuss the main managerial insights. Finally, in Section 2.5 we wrap up the paper with a concluding discussion and some suggestions for future research.

2.2 Literature Review

Our paper is primarily related to four streams of literature. The first one is the literature considering product line design problems in a vertically differentiated market. Mussa and Rosen (1978) and Moorthy (1984) are among the first authors to introduce quality differentiation into the marketing literature. In an influential paper, Moorthy (1984) considers a monopolist serving multiple consumer segments that are different in their valuations of quality. In that work, the monopolist offers a menu of products with higher-quality products priced higher. The classical results from his study suggest that the presence of cannibalization forces the monopolist to only offer consumers in the highest valuation segment their optimal quality level, while all other segments receive products at the non-optimal quality level. Unlike Moorthy (1984), who considered discrete consumer valuations, Mussa
and Rosen (1978) present a model with a continuous distribution of consumer valuations. With continuous distribution, the choice between full market coverage and partial market coverage becomes relevant and has an impact on the optimal product prices and quality levels. Over the years several authors have extended the above classical results. Moorthy and Png (1992) extend the above classical results by considering the timing of product introductions. They show that under some conditions the sequential product introductions can alleviate cannibalization. Kim and Chhajed (2002) consider a product line design problem with multiple quality attributes. Choudhary et al. (2005) use the product line design models to study the effect of personalized pricing on the firm’s choices with regard to quality. Other papers examine the effect of channel structure in product line design problems with quality differentiation (see e.g., Villas-Boas 1998, Liu and Cui 2010, and Chung and Lee 2015), an issue we do not explore here.

What is common in all the literature above is that the authors do not consider relevant operational costs beyond variable production costs. This has been the motivation for several studies included in the second stream of literature related to our paper, a stream that integrates product line design and operational decisions. Several authors, e.g., Karmarkar and Kekre (1987) and De Groote (1994), examine the operational implications of product variety. In these papers, however, the authors consider horizontal product differentiation. Below, we discuss several papers that consider vertical product differentiation like we do in this paper. Netessine and Taylor (2007) study the impact of production technology on the optimal product line design. They combine the standard product line design problem with the classical EOQ (Economic Order Quantity) inventory model to capture the problem faced by the manufacturer when balancing production setups with accumulation of inventories in the presence of economies of scale. They show that the integration of the production-inventory model and product line design leads to interesting results that challenge the established results in the marketing literature. Their study reveals that more expensive production technology characterized by higher inventory-related cost parameters leads to higher quality products and lower product prices. Similar to their paper, we also integrate the production-inventory and product line decisions. However, our model differs from theirs in many respects. First, they assume deterministic demand such that the standard EOQ model is applicable whereas we consider demand uncertainty such that safety stock is necessary. Second, they consider a simple production setting characterized by a single stage production system, whereas we consider more complex production settings characterized by multiple stages. And finally, as we consider customized products, the manufacturer in our model is assumed to work in a make-to-order (MTO) or assemble-to-order (ATO) manner. Hence, lead time is an important factor in our model. In Netessine and Taylor (2007), the authors
consider make-to-stock production settings in which lead time is not relevant. Chayet et al. (2011) study the problem of simultaneously optimizing the product line design and the capacity investment. Our model is similar to theirs in that consumers make product choices to maximize a linear utility function of price, quality level, and waiting cost. The focus of their analysis is on the choice between dedicated or flexible production facilities for processing all product variants in the product line. They use a simple $M/M/1$ queuing model to represent the production system. Our research differs from their research as our primary focus is on the effect of inventory costs on the product line design decisions, and we capture more realistic production settings in which inventory can be held at different stages in the production processes.

The other stream of literature related to our paper includes papers that consider product line design problems in the context of mass customization. Most of those papers consider horizontal product differentiation, i.e., they compare the conventional strategy offering mass-produced standard products and the mass customization strategy offering unlimited and customized products (see e.g. Jiang et al. 2006 and Alptekinoglu and Corbett 2008). None of the papers explicitly consider consumers’ waiting costs incurred when buying customized products. Models considering the trade-off between customization and lead times are presented in e.g. Xia and Rajagopalan (2009), Alptekinoglu and Corbett (2010), and Mendelson and Parlakturk (2008a, 2008b). Xia and Rajagopalan (2009) study the standardization and customization decisions of two firms in a competitive setting and consider lead time along with product variety and price decisions. They show that product variety and lead time may play strategic roles in the competition. Alptekinoglu and Corbett (2010) present a model to optimize product line design while explicitly considering the trade-off between meeting individual preferences and incurring the longer lead times associated with customized products. Mendelson and Parlakturk (2008a, 2008b) consider competition between a mass customizer and a mass producer. In their paper, only the mass producer carries inventories whereas the mass customizer does not. While the main difference between our paper and theirs is that we consider vertical rather than horizontal differentiation, our model also facilitates detailed analysis of a multi-stage production network whereas they consider a simplified MTS system that produces standard products and a MTO system that produces customized products. Wong and Lesmono (2013) consider customization vs. lead time trade-off and vertical differentiation, and their paper is therefore the one that is most closely related to our paper. Our paper is in line with their paper as we also consider the quality attributes as customization level and lead time, but while they study the optimal conditions for offering a single product or two products with different customization levels as we do in this paper, we extend their work in two ways. Firstly, unlike their paper considering a simple queuing system to represent the
production processes, we consider a more realistic production network that includes multiple stages with interdependences among them. As a consequence of their modeling approach, the customization level is assumed to be continuous in their model. Although such an approach provides useful qualitative insights, it cannot be directly used by manufacturing firms for determining the optimal customization level(s). Our model captures the fact that there is always a limited set of points in which final customization can take place. Secondly, they do not include the question of inventories in their study whereas we explicitly consider both safety and work in progress inventories. In a recent study, Wong and Lesmono (2017) extends their previous work (Wong and Lesmono 2013) by examining the effect of distribution channel structure on the product line decisions, but they do not consider the operational aspects related to the production processes.

Finally, our work is also related to the literature studying assembly network operations that produce customized products. Several authors, e.g., Lee and Tang (1997), Swaminathan and Tayur (1998), Su et al. (2005) and Wong et al. (2009), present models to analyze the concept of postponement or delayed product differentiation. When implementing postponement, manufacturing firms delay the point at which the final customization of the product is to be configured. The concept has been perceived as a cost-effective way to reduce the operating costs associated with managing proliferating product variety (Van Hoek 2001 and Yang and Burns 2003). The literature on postponement is also closely related to the literature on customer order decoupling point (CODP) (see e.g. Giesberts and van den Tang 1992, Rudberg and Wikner 2004, van Donk and van Doorne 2016), defined as the point in the value-adding material flow that separates decisions made under uncertainty from decisions made under certainty concerning (customized) customer demand. Although we also investigate the optimal point in the production processes for making the final customization of the product, there are several important differences. None of the above papers associate the customization point with the perceived quality level. In the literature on postponement and CODP, the final products are assumed to have the same quality level, whereas we presume that customization level is related to perceived quality and thereby influences the consumers' willingness to pay. Another main difference is that all the above mentioned papers focus solely on the operations aspect of customization whereas we consider the integration of both the marketing and operations decisions.

Even though the above mentioned papers do not pay attention to the detailed operations of the production processes, some authors do. Shao and Ji (2008) present a model for evaluating different postponement strategies in mass customization with service guarantees. Shao and Dong (2012) compare the order fulfilment performance in MTS and MTO systems taking into account an inventory cost budget constraint. Lu et al. (2012) present a model to evaluate delayed differentiation strategies.
They consider multiple points of product differentiation and develop a cost model that explicitly includes the operational delay cost and the penalty cost. Our model is also related to these papers. For example, we adopt service guarantees like in Shao and Ji (2008) although our inventory model differs. In line with Lu et al. (2012), our model also allows more than one point of product differentiation. Nevertheless, the main difference stems from the fact that, as we consider a product line design problem, we use a profit maximization model whereas all these papers use a cost minimization model.

2.3 Models

In this section, we present our models. We first introduce the manufacturer’s decisions related to the marketing factors. Then, we present the inventory model focusing on the safety stock placement optimization for a given customization level. Finally, we present the integrated model. In Table 2.1 below, we summarize the model’s notation.

Table 2.1: The model’s notation

<table>
<thead>
<tr>
<th>Production/Inventory notations</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$A$</td>
<td>Set of arcs in the production network</td>
</tr>
<tr>
<td>$a$</td>
<td>Base cost of producing the standardized product</td>
</tr>
<tr>
<td>$b$</td>
<td>The production cost factor related to customization level</td>
</tr>
<tr>
<td>$C$</td>
<td>Set of all the main processing nodes in the network</td>
</tr>
<tr>
<td>$n$</td>
<td>Customization node</td>
</tr>
<tr>
<td>$C_n$</td>
<td>Set of all stages where customization takes place given that node $n$ is the customization node</td>
</tr>
<tr>
<td>$h_j$</td>
<td>Unit holding cost at stage $j$</td>
</tr>
<tr>
<td>$N$</td>
<td>Set of all stages in production network</td>
</tr>
<tr>
<td>$S_j$</td>
<td>Outbound service time at stage $j$</td>
</tr>
<tr>
<td>$SI_j$</td>
<td>Inbound service time at stage $j$</td>
</tr>
<tr>
<td>$T_j$</td>
<td>Processing time at stage $j$</td>
</tr>
<tr>
<td>$W_j$</td>
<td>Working-in-process (pipeline) inventory at stage $j$</td>
</tr>
<tr>
<td>$\mu_j$</td>
<td>Mean of demand at stage $j$</td>
</tr>
<tr>
<td>$\sigma_j$</td>
<td>Standard deviation of demand at stage $j$</td>
</tr>
<tr>
<td>$\varphi_{ij}$</td>
<td>Number of units of upstream component $i$ that are required per downstream unit $j$.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Marketing notations</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$\omega_k$</td>
<td>Consumer’s waiting cost at segment $k$ ($k = L, H$)</td>
</tr>
<tr>
<td>$\theta_k$</td>
<td>Consumer’s valuation on customization level at segment $k$ ($k = L, H$)</td>
</tr>
<tr>
<td>$\mu_k$</td>
<td>Mean of (external) demand at segment $k$ ($k = L, H$)</td>
</tr>
<tr>
<td>$\sigma_k$</td>
<td>Standard deviation of (external) demand at segment $k$ ($k = L, H$)</td>
</tr>
<tr>
<td>$s$</td>
<td>Guaranteed service time for one-product strategy</td>
</tr>
<tr>
<td>$s_k$</td>
<td>Guaranteed service time at segment $k$ ($k = L, H$) for the two-product strategy</td>
</tr>
<tr>
<td>$p$</td>
<td>Price of customized product for one-product strategy</td>
</tr>
<tr>
<td>$p_k$</td>
<td>Price of customized product at segment $k$ ($k = L, H$) for the two-product strategy</td>
</tr>
<tr>
<td>$p_0$</td>
<td>Reservation price for standardized product</td>
</tr>
<tr>
<td>$q$</td>
<td>Customization level for one-product strategy</td>
</tr>
<tr>
<td>$q_k$</td>
<td>Customization level at segment $k$ ($k = L, H$) for the two-product strategy</td>
</tr>
</tbody>
</table>
2.3.1 The Marketing Model

Here we present the marketing model that focuses on the product line design problem faced by the manufacturer. The manufacturer produces and sells customized products in a market where consumers are heterogeneous in their valuation of product quality. As stated earlier, the product quality in our model is mainly represented by two quality attributes that are relevant in the customization context: customization level and delivery lead time. The customization level is determined by the extent to which consumers are involved in the production network. In the setting we consider, the product delivery lead time is not independent of the customization level. Since the final product is produced in a make-to-order or assemble-to-order fashion, there is a minimum lead time incurred to make or to assemble the final products. This minimum delivery lead time is longer when the customization level is higher.

The heterogeneity of consumer’s preferences is represented by two segments in the market denoted by the high-valuation \( (H) \) segment and the low-valuation \( (L) \) segment. Consumers in each segment are homogeneous in their valuation for attributes. Considering two segments in the market with homogeneous valuation for attributes are common in the product line design literature (see e.g. Moorthy and Png 1992, Netessine and Taylor (2007), and Kim et al. 2013). However, different from the classical literature on product line design that assumes deterministic demand, in this paper, we consider a different setting where the manufacturer faces demand uncertainty. Uncertain demand in product line design is also considered in e.g. Chayet et al. (2011) and Wong and Lesmono (2013). In our model, demand uncertainty is represented by the randomness in the size of each segment. Let \( \mu_k \) and \( \sigma_k \) \( (k = H, L) \) denote the mean and standard deviation of demand in segment \( k \), respectively. The randomness in the size of each segment arising in our context can be due to the fact that the number of potential consumers in each segment is uncertain at the time that decisions are made. We are aware that the approach we use to model consumers’ heterogeneity and demand uncertainty may represent one of the simplest approaches, but we believe it is sufficient for the purpose of our study.

In the context of mass customization, it is relevant to consider the scenario where a consumer who highly values the customization level is less concerned about the product delivery lead time (Squire et al. (2006); Wong and Lesmono (2013)). We assume that consumers who belong to the high-valuation segment are more concerned with the product customization level than consumers in the low-valuation segment. We denote \( \theta_k \) \( (k = H, L) \) as the consumer’s valuation on customization level, where \( \theta_H > \theta_L \). Conversely, consumers who belong to the high-valuation segment are less concerned with delivery lead time than consumers in the low-valuation segment. We define \( \omega_k \) \( (k = H, L) \) as
the cost per time unit incurred by every consumer in segment \( k \) waiting for the product to be delivered, with \( \omega_H < \omega_L \).

With the presence of two segments, the manufacturer aims to sell two different products, one for each segment. We term this as the manufacturer’s two-product strategy. Depending on the production and marketing related factors, it is also possible that the manufacturer adopts the one-product strategy in which only one product is offered in the market. Under the two-product strategy, the manufacturer offers a product with customization level, delivery lead time and price \( (q_H, s_H, p_H) \) designed for the high-valuation segment and product \( (q_L, s_L, p_L) \) designed for the low-valuation segment. In the case of single product strategy, one product \( (q, s, p) \) is offered in the market.

For product \( (q, s, p) \) offered in the market, a consumer from segment \( k \) has a utility function \( u_k(q, s, p) = p_0 + \theta_k q - \omega_k s - p, (k = H, L) \). We define the constant \( p_0 \) as the reservation price for a completely standardized product, and all consumers in the market are the potential buyers of the customized product as long as their utility is nonnegative. Following Wong and Lesmono (2013), we focus on the product line design problem where the customization level is the main attribute of interest, and the completely standardized product, i.e., \( q = 0 \), is therefore not included in our analysis.

### 2.3.2 The Production-Inventory Model

The value chain (production system) producing the customized products in our model can be considered as a multi-stage network. Each stage in the network is a potential location for holding a certain amount of safety stock to meet the uncertain demand of its consumers. In this paper, the guaranteed-service model (Graves and Willems 2000) is adopted to determine the optimal amount and placement of safety stocks in the network for a given customization level. The model allocates and determines safety stocks across the supply chain to achieve a target service level at minimum total safety stock costs. Although we acknowledge the possibility of applying other multi-stage inventory models, we have chosen the guaranteed-service model for two reasons. Firstly, the model is suitable for coping with complex multi-echelon inventory systems and it has been widely applied in many real-world supply chains across different industries (Eruguz 2015). Secondly, the service measure used in the model is the so-called guaranteed service time, which implies that the system must guarantee (with 100% probability) that the service time promised to consumers will be met. We believe that this service measure is appropriate for mass customization strategies given that there is an increasing demand for manufacturing firms that offer customized products to be more responsive and reliable at the same time (McCutcheon et al. 1994, Tu et al. 2001, and Rondeau et al. 2003).
In the standard guaranteed-service model, all stages operate with a periodic-review base-stock policy. External demands occur at the most downstream stages called the demand nodes in the network. Each demand node $j$ faces the end-consumer’s demand that comes from a stationary process with mean $\mu_j$ and standard deviation $\sigma_j$. Depending on which segment is targeted, the demand node $j$ may face external demand with mean $\mu_k$ and standard deviation $\sigma_k$ ($k = H, L$). Internal demands at internal stages depend on the units ordered by its downstream nodes. The average demand rate at stage $i$ is represented by $\mu_i = \sum_{(i,j) \in A} \mu_j \phi_{ij}$, where $\phi_{ij}$ is the number of units of the upstream component $i$ that are required per downstream unit $j$.

The production lead time at stage $j$ ($T_j$) is deterministic and includes processing, waiting, and transportation time to place the processed items for production at its downstream stages. Each stage quotes a deterministic outbound service time ($S_j$) to its consumers and guarantees that the demand occurring at time $t$ is fulfilled at time $S_j + t$, and each stage quotes the same service time to its consumers. There is an inbound service time ($SI_j$) at stage $j$ that represents the time that should elapse to get supplies from its direct upstream stages to start the production process. There is no stock-out in the system and demand is bounded at each stage by an increasing concave function. The demand bound function represents the amount of demand that should be satisfied by safety stocks over the interval time ($\tau = SI_j + T_j - S_j$) or net replenishment time, if it is positive. The demand bound is a meaningful upper bound on demand, and the safety stock is set to cover all demand realizations that fall within the upper bounds. Extra ordinary cases occur when demand surpasses the upper bounds, and managers must use other approaches such as expediting, overtime, or subcontracting to meet the excess demand. The expected inventory level at stage $j$ is defined by $D_j(\tau) - (\tau)\mu_j$, where $D_j(\tau)$ represents the demand bound function in terms of net replenishment time.

In principle, the guaranteed-service model does not require any assumptions about the distribution of demand. In specifying the demand bounds, a manager indicates how demand variation should be managed, i.e., what range is covered by safety stock and what range should be dealt with by other actions or responses. For the purposes of positioning safety stock at demand node $j$, a manager might, for example, specify the demand bound as $D_j(\tau) = \tau\mu_j + z\sigma_j\sqrt{\tau}$, which is derived based on the assumption that the demand at demand node $j$ is normally distributed with mean $\mu_j$ and standard deviation $\sigma_j$. In this expression, $z$ reflects the percentage of time that the safety stock covers the demand variation. For each internal node $i$, demand bound can be determined from $D_i(\tau) = \tau\mu_i + r \left( \sum_{(i,j) \in A} (\phi_{ij}(D_j(\tau) - \tau\mu_j))^r \right)$, where $r \geq 1$ is a given constant. Larger values of $r$ correspond to
more risk pooling. We refer the reader to Graves and Willems (2000) for the more detailed explanation of the demand bound.

The expected inventory level represents the safety stock held at stage \( j \), and depends on the net replenishment time and the demand bound. In addition to safety stock, there is also pipeline inventory at stage \( j \) that depends on processing time \( T_j \), and the expected pipeline inventory level at stage \( j \) can be written as \( E(W_j) = T_j \mu_j \).

In the guaranteed-service model, the outbound and inbound service times are decision variables and there are three main constraints in this optimization problem: (i) \( S_j \geq S_i \), to ensure that stage \( j \) starts its process after receiving all essential inputs from its upstream node \( i \); (ii) \( S_j + T_j \geq S_f \) for all stages \( j \), necessary for meeting the demand over the net replenishment time; and (iii) \( S_j \leq s_j \) for all demand nodes \( j \), with \( s_j \) representing the maximum service time promised to consumers.

Graves and Willems (2000 and 2003) develop a dynamic programming algorithm to determine the optimal safety stock placement. Their algorithm is applied for a relabeled spanning-tree network. The reader is referred to Graves and Willems (2000 and 2003) for a more detailed description of the guaranteed-service model and the solution algorithm. In this paper, we need to slightly modify the model because in our model, the service time promised to consumers depends on the customization level. We explain the modified algorithm below.

*The one-product strategy*

Let \( N \) denote the set of all the nodes in the production network, and \( A \) denote the set of all the arcs in the network. Let us also define \( C \) as a subset of \( N \) including all main processing nodes in the production network where customization may take place. As customization at some nodes may be impractical or even impossible, some nodes (production stages) in the network may not be customization nodes. The production network is characterized by \((n; s)\) where \( n \in C \) is the customization node and \( s \) is the maximum service time promised to the end consumers. Customization node \( n \) is defined as the most upstream stage in the production network where customization takes place. In our model, we focus on the setting where all the main processing nodes form a serial production line, but the whole production network is not restricted to a serial production network. When the production network has customization node \( n \), customization also takes place at all the other nodes \( j \in C \) located downstream of node \( n \). We define \( C_n \subseteq C \) as a set containing all the stages in the production network where customization is carried out, including customization node \( n \).
For each stage \( j \in C_n \), there is no need to keep safety stock, and the time required to supply its downstream consumers corresponds to its processing time. Note that some nodes that are not included in \( C \) and for which safety stocks are held may exist downstream of customization node \( n \). The pipeline inventory is maintained at all the nodes in the production network. Figure 2.1 below illustrates an example of production network with customization.

![Figure 2.1. The production network with customization](image)

Let \( h_j \) be the unit holding cost per time unit at stage \( j \). The inventory optimization problem for the production network characterized by \((n; s)\) is as follows:

\[
\text{Min}_{S_j, SL_j} \bar{IC}_{(n,s)} = \sum_{j \in N} h_j \{ D_j (SL_j + T_j - S_j) - (SI_j + T_j - S_j)\mu_j + E(W_j) \} 
\]

Subject to

\[
\begin{align*}
S_j - SL_j & \leq T_j & \text{For all } j \in N - C_n & (1.1) \\
S_j - SL_j & = T_j & \text{For all } j \in C_n & (1.2) \\
SI_j - S_i & \geq 0 & \text{For all } (i, j) \in A & (1.3) \\
S_j & \leq s - \sum_{t \in C_n} T_t & \text{For all } j, (j, n) \in A & (1.4) \\
S_j, SL_j & \geq 0 & \text{Integer for } j \in N & (1.5)
\end{align*}
\]

The objective function \( \bar{IC}_{(n,s)} \) is the expected total inventory cost per time unit. Although the expected pipeline inventory is included in the objective function, it is not relevant for finding the optimal safety stock placement for a given \((n; s)\) since its value is constant. However, as will be shown later, the expected pipeline inventory level becomes relevant when we optimize the customization level. The pipeline inventory becomes more costly as the products become more customized. The importance of pipeline inventory is also highlighted in e.g. Graves and Willems (2000) and Hopp and Spearman.
(2008). Constraint (1.1) indicates that the net replenishment times are nonnegative for all the nodes not carrying out customization. Constraint (1.2) assures that the net replenishment times at the nodes carrying out customization are zero so that no safety stocks are held, and this constraint represents the first modification to the standard guaranteed-service model. Constraint (1.3) ensures that the inbound service time at each stage is no less than the service times of its supplier stages. Constraint (1.4), which is the second modification to the standard model, limits the service times at the nodes directly supplying customization node $n$ such that the guaranteed service time promised to consumers is satisfied. The expected total inventory cost that corresponds to the optimal safety stock placement is denoted by $\overline{IC}^*_{(n,s)}$. We use the dynamic programming algorithm of Graves and Willems (2000 and 2003) to determine the optimal safety stock placement in the production network. To apply the algorithm for a production network with customization, some modifications are necessary. We present a more detailed description of the dynamic programming algorithm in Appendix B.

The two-product strategy

In the two-product strategy, the manufacturer offers two customization levels, targeted at the low-valuation segment and the high-valuation segment, respectively. We assume that there are two parallel production networks, each with its customization level. We capture the scenario where, despite the similarity of the production steps of the two products, the high quality product may require more complex customizing operations than the low quality product, which can be achieved by employing e.g. more skillful operators or more advanced machines. One could, alternatively, consider a single production network that contains two customization levels. The main difference between the two approaches stems from the omission of the pooling effect in the two parallel networks. We choose to consider the first approach as the resulting model is simpler, and moreover, the resulting pooling effect from two demand streams coming from the two segments seems to be limited.

The two independent parallel networks are characterized by $(n_L; s_L)$ and $(n_H; s_H)$ for the low and high customization levels, respectively. For the first network producing the low-quality product, we define $C_{n_L} \subseteq C$ as a set containing all the stages in the production network where customization is carried out, including customization node $n_L$. Similarly, we define $C_{n_H} \subseteq C$ in the second network producing the high-quality product. The total expected inventory cost that corresponds to the optimal safety stock placements in the two parallel networks is denoted by $\overline{IC}^*_{(n_L,s_L;n_H,s_H)} = \overline{IC}^*_{(n_L,s_L)} + \overline{IC}^*_{(n_H,s_H)}$. 
2.3.3 The Integrated Model

We now present the integrated model for the manufacturer’s product line design problem. In the standard product line design problem with two market segments, the manufacturer must make decisions on the optimal quality levels and prices for the high-quality and low-quality products taking into account the potential cannibalization of the high-quality product by the low-quality product (see Appendix A). In this paper, we extend the standard product line design problem in the following respects. Firstly, the product quality in our paper is represented by two main attributes: customization level and product delivery lead time, and the two attributes are interdependent of each other. Secondly, we include the cost of holding safety stock and pipeline inventory in the manufacturer’s profit function, and the cost of holding safety stock and pipeline inventory is influenced by the choice of customization levels and product delivery lead times.

Given that there is a limited number of stages in the production network where customization can take place, customization level is a discrete decision variable in our model. This also highlights another difference of our model compared to the standard product line design model. The ratio of the total customizing time to the total processing time is used as a proxy for the customization level. Let us consider a production network with customization node \( n \). Then, the customization level is defined as:

\[
q = \frac{\sum_{i \in \mathcal{C}} c_i T_i}{T_0} \quad (0 < q \leq 1)
\]

where \( T_0 \) is the sum of total processing times of the main production stages. We assume that the higher the customization level, the higher the marginal production cost of the customized product. This assumption is common in the product line design literature, and captures the scenario where offering a higher quality product leads to a higher manufacturing complexity. Following the literature (e.g. Syam and Kumar 2006; Wong and Lesmono 2013), we assume that the unit production cost is an increasing and quadratic function in customization level and takes the form of \( a + bq^2 \), where \( a \) represents the base cost of producing a standard product.

The Two-Product Strategy

Under the two-product strategy, the manufacturer offers two products with different customization levels. The product with the low customization level is targeted at the low-valuation segment whereas the product with the high customization level is targeted at the high-valuation segment. Like in the standard product line design problem, the manufacturer must find a way to avoid the potential cannibalization as consumers in the high-valuation segment may switch downward and purchase the product with the low customization level whenever this product gives them a higher utility. This potential cannibalization occurs because the manufacturer does not possess full information about
individual consumers’ preferences (whether an individual belongs to the high or low valuation segment), and it is difficult to deliver marketing programs to individual market segments without ‘leakage’ to other segments (Moorthy 1984; Netessine and Taylor 2007). In other words, it is almost impossible for the manufacturer to directly address individual segments and achieve the perfect market segmentation such that products do not cannibalize each other. Hence, the manufacturer must design the product line such that consumers in each segment buy the product targeted at them.

The manufacturer’s product line design problem in the two-product strategy can be formulated as follows:

$$\text{Max}_{q_L,q_H,s_L,s_H,p_L,p_H} Z_{L,H}^{\text{two}} = \left( p_L - (a + bq_L^2) \right) \mu_L + \left( p_H - (a + bq_H^2) \right) \mu_H - \overline{IC} (n_L,s_L,n_H,s_H)$$

Subject to

$$\begin{align*}
    p_0 + \theta_L q_L - \omega_L s_L - p_L &\geq 0 \quad (2.1) \\
    p_0 + \theta_H q_H - \omega_H s_H - p_H &\geq 0 \quad (2.2) \\
    \theta_L q_L - \omega_L s_L - p_L &\geq \theta_L q_H - \omega_H s_H - p_H \quad (2.3) \\
    \theta_H q_H - \omega_H s_H - p_H &\geq \theta_H q_L - \omega_L s_L - p_L \quad (2.4) \\
    s_L &\geq \sum_{x \in C_L} T_x \quad (2.5) \\
    s_H &\geq \sum_{y \in C_H} T_y \quad (2.6) \\
    s_L, s_H, q_L, q_H, p_L, p_H &> 0 \quad (2.7)
\end{align*}$$

The objective function is the expected profit per time unit. Constraints (2.1) and (2.2) are the participation constraints ensuring that consumers in each segment do not have negative utility by buying the product offered to them. Constraints (2.3) and (2.4) are the self-selection constraints that are needed because the manufacturer is unable to directly address individual segments as previously explained. A consumer may find a product designed for another segment more attractive than the product designed for her own segment, and so the firm faces a cannibalization problem. To prevent this cannibalization, the two self-selection constraints must be included. If the manufacturer were able to do perfect market segmentation, the self-selection constraints will not be required, and each segment can be treated independently. Constraints (2.5) and (2.6) state that the guaranteed service times that the manufacturer promise to consumers cannot be shorter than the sum of all the processing times at the stages in the production network where customization is carried out, which depend on the customization levels. Note that constraints (2.5) and (2.6) will actually be satisfied by constraint
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(1.4) in the inventory optimization problem. However, since $s_L$ and $s_H$ are decision variables in the product line design problem while they are treated as given parameters in the inventory optimization problem, we present the two constraints ((2.5) and (2.6)) above to explicitly show the feasible values for $s_L$ and $s_H$. For reference, we also present the standard product line design problem that exclude the lead time and inventory in Appendix A.

The following propositions summarize how the manufacturer must set the prices and guaranteed service times.

**Proposition 1:** In the two-product strategy, the manufacturer sets the prices for the two products such that the participation constraint of the low-valuation segment (Constraint (2.1)) and the self-selection constraint of the high-valuation segment (Constraint (2.4)) are binding. The prices are equal to: $p_L = p_0 + \theta_L q_L - \omega_L s_L$ and $p_H = \theta_H (q_H - q_L) - \omega_H (s_H - s_L) + p_L$.

**Proposition 2:** In the two-product strategy, the guaranteed service times for the two products are restricted by:

(i) $\sum_{x \in C_{nL}} T_x \leq s_L < \frac{\theta_L q_L + p_0}{\omega_L}$,

(ii) $\sum_{y \in C_{nH}} T_y \leq s_H < \frac{\theta_H (q_H - q_L) - s_L (\omega_L - \omega_H) + \theta_L q_L + p_0}{\omega_H}$

Proofs of the two propositions are presented in Appendix C.

In Proposition 1, we show that the price setting method used in the standard product line design problem is also valid for the product line design problem in which customization level and service time are the two quality attributes. Given specific customization levels and service times, it is optimal for the manufacturer to set the price of the low-quality product such that consumers in the low-valuation segment are indifferent between buying the low-quality product and not buying. In order to prevent cannibalization, the price of the high-quality product needs to be reduced such that consumers in the high-valuation segment are indifferent between buying the high-quality product and buying the low-quality product. The conditions in Proposition 2 show the ranges for the guaranteed service times for the two products that lead to feasible solutions. These ranges help in reduce the search space in the optimization algorithm.

**The One-Product Strategy**

Instead of selling two products with two different customization levels, the manufacturer may also be interested in selling only one product. There are two possible options for the manufacturer to target
the market under the one-product strategy. Firstly, the manufacturer may target both the high-
valuation and low-valuation segments. Secondly, the manufacturer may customize its product based
on the consumers’ preferences in the high-valuation segment. If he chooses the latter option, the
manufacturer extracts all surpluses of consumers in the high-valuation segment.

The problem under the one-product strategy targeting both segments can be formulated as follows:

\[
\text{Max}_{\mathbf{p}, \mathbf{q}, \mathbf{s}} \ Z_{L,H}^{\text{one}} = (p - (a + b q^2)) (\mu_L + \mu_H) - IC^*(n; s) \tag{3}
\]

Subject to
\[
\begin{align*}
\mathbf{p} & = \mathbf{p}_0 + \theta_L \mathbf{q} - \omega_L \mathbf{s} - \mathbf{p} \geq 0 \tag{3.1} \\
\mathbf{p} & = \mathbf{p}_0 + \theta_H \mathbf{q} - \omega_H \mathbf{s} - \mathbf{p} \geq 0 \tag{3.2} \\
\mathbf{s} & \geq \sum_{x \in C_n} T_x \tag{3.3} \\
\mathbf{q}, \mathbf{s}, \mathbf{p} & > 0 \tag{3.4}
\end{align*}
\]

Notice that when both segments are targeted, the manufacturer sets the price such that the low-
valuation segment gets zero surpluses, i.e. by making (3.1) binding. This will make the participation
constraint for the high-valuation segment redundant.

The problem under the one-product strategy targeting only the high-valuation segment can be
formulated as follows:

\[
\text{Max}_{\mathbf{p}, \mathbf{q}, \mathbf{s}} \ Z_{H}^{\text{one}} = (p - (a + b q^2)) \mu_H - IC^*(n; s) \tag{4}
\]

Subject to
\[
\begin{align*}
\mathbf{p} & = \mathbf{p}_0 + \theta_H \mathbf{q} - \omega_H \mathbf{s} - \mathbf{p} \geq 0 \tag{4.1} \\
\mathbf{s} & \geq \sum_{x \in C_n} T_x \tag{4.2} \\
\mathbf{q}, \mathbf{s}, \mathbf{p} & > 0 \tag{4.3}
\end{align*}
\]

Under the two-product strategy as well as under the one-product strategy the manufacturer’s product
line design problem is a nested optimization problem. Under the two-product strategy, the
manufacturer first decides on the customization levels \( q_L \) and \( q_H \). Then, all the feasible values of \( s_L \)
and \( s_H \) are determined. Next, for a given \( q_L \) and \( q_H \ (q_L < q_H) \) and \( s_L \) and \( s_H \), we determine both the
optimal prices and safety stock placements. Next, we search over all the feasible values for \( s_L \) and \( s_H \)
to find the optimal service times. And finally, we optimize the customization levels \( q_L \) and \( q_H \). A
similar procedure is applied for the one-product strategy. In Appendix D we provide a more detailed
description of the optimization algorithm.
2.4 Numerical Results and Discussion

In this section, we present the results of our numerical investigation. We will focus on two aspects that are related to our research questions. Firstly, we are interested in studying how the optimal customization levels are influenced by different manufacturing and marketing factors. In relation to this, we also compare the performances of the three product line strategies. Secondly, we show how our model can be used by firms to identify the process in the production network that should be given priority when they intend to initiate a lead time reduction project.

In our numerical study, we use a hypothetical production network as depicted in Figure 2.2. The dynamic programming algorithm presented in Graves and Willems (2000, 2003) that we use requires that the production network is a spanning tree. The network consists of 18 nodes, each of which might represent part of the manufacturing processes including, fabrication, sub-assembly, assembly, packaging, etc. In the figure, the processing time for each node is given above the node, and the value given in each arc represents the number of units required per downstream unit. The nodes in the network can be classified into two groups. The first group consists of all the nodes that represent the main processing stages where customization may take place. The second group consists of all the other nodes that represent the supporting processes, and these nodes are excluded as candidates for the customization point. Following the model notation, the main processing stages are included in set \( C = \{5, 8, 10, 11, 14, 15, 17, 18\} \).

The unit holding cost for each stage in the network may depend on the customization level offered. Since a product with a higher customization level is more valuable, the unit holding cost is also higher, and this applies to all the nodes where customization takes place. For the nodes located before the customization node (where customization does not take place), the unit holding costs considered correspond to the standardized product without customization. Furthermore, to capture the fact that higher values are accumulated as the materials move downstream, we also consider that the unit holding cost for a node is higher when its location is closer to the demand node (see Appendix E).
Table 2.2 lists all parameter values used in the experiment. The values have been chosen based on the results of some preliminary experiments conducted to ensure that it is, for example, reasonable to consider all the nodes in the first group as candidates for the customization node. While the values of the production related parameters are easier to estimate in practice, we are aware that the estimation of the values for the marketing related parameters may require a great deal of efforts. Marketing research techniques such as conjoint analysis are often used to estimate the consumers’ preference of a certain quality attribute, i.e. to estimate the values of $\theta_H, \theta_L, \omega_H$, and $\omega_L$ (see Cattin and Wittink (1982) for a survey of conjoint analysis applications).

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>$(\mu_H, \mu_L)$</td>
<td>(30,10), (20, 20), (10,30) (in unit per day)</td>
</tr>
<tr>
<td>$(\sigma_H, \sigma_L)$</td>
<td>(4.5, 4.5)</td>
</tr>
<tr>
<td>$h_j$</td>
<td>20%, 30%, and 40% of the production cost (annual rate)</td>
</tr>
<tr>
<td>$a$</td>
<td>300</td>
</tr>
<tr>
<td>$b$</td>
<td>550, 700, 850</td>
</tr>
<tr>
<td>$\theta_H$</td>
<td>1500, 1800, 2100, 2400</td>
</tr>
<tr>
<td>$\theta_L$</td>
<td>1400</td>
</tr>
<tr>
<td>$\omega_H$</td>
<td>15, 20, 25</td>
</tr>
<tr>
<td>$\omega_L$</td>
<td>30</td>
</tr>
<tr>
<td>$p_0$</td>
<td>400</td>
</tr>
<tr>
<td>$z$</td>
<td>1.8</td>
</tr>
<tr>
<td>$r$</td>
<td>2</td>
</tr>
</tbody>
</table>

There are in total 324 problem instances in our experiment. For each problem instance, we determine the optimal customization levels for the three product line strategies: the one-product strategy targeted at two segments, the one-product strategy targeted only at the high-valuation segment, and two-product strategy targeted at two segments.
The importance of considering inventory costs

Here, we present the results of our analysis of how the optimal customization levels are influenced by the inclusion of inventory costs and by the different production and marketing related parameters. In order to examine the effects of considering production network inventory costs, we determine the optimal customization levels for each problem instance in two different scenarios: one with inventory costs and one without inventory costs. The main results are depicted in Figures 2.3, 2.4, and 2.5.

Figure 2.3: Average optimal customization level as a function of $b$
Figure 2.4: Average optimal customization level as a function of $\frac{\theta_H}{\theta_L}$.
The results demonstrate the significant role of inventory costs on the optimal customization levels under each product line design strategy. As illustrated in the figures, the average optimal customization levels decrease when the inventory costs are considered, regardless of the choice of different production or marketing related parameters. This leads to the main conclusion that when making decisions regarding the customization level(s), firms need to consider both the production and marketing related factors. If they ignore the amount and positioning of inventory in the production network, the result may be non-optimal customization levels due to overestimation of the manufacturer’s profit.

The effects of the unit holding cost rate on the optimal customization levels are summarized in Tables 2.3, 2.4, and 2.5. From our numerical experiment it is evident that the average customization levels
tend to decrease as the unit holding cost increases, although the rate of decrease is less pronounced than the one we observed in the comparison between the two scenarios: without and with inventory costs.

Table 2.3: The average optimal customization level for each strategy as influenced by $\frac{\omega_H}{\omega_L}$

<table>
<thead>
<tr>
<th>Strategies</th>
<th>$\frac{\omega_H}{\omega_L}$ = 0.5</th>
<th>$\frac{\omega_H}{\omega_L}$ = 0.67</th>
<th>$\frac{\omega_H}{\omega_L}$ = 0.83</th>
</tr>
</thead>
<tbody>
<tr>
<td>One-product strategy- two segments</td>
<td>0.50/0.50/0.45*</td>
<td>0.50/0.50/0.45</td>
<td>0.50/0.50/0.45</td>
</tr>
<tr>
<td>One-product strategy- segment H</td>
<td>0.90/0.89/0.84</td>
<td>0.88/0.85/0.80</td>
<td>0.84/0.80/0.78</td>
</tr>
<tr>
<td>Two-product strategy-high quality product</td>
<td>0.85/0.84/0.76</td>
<td>0.75/0.69/0.66</td>
<td>0.67/0.65/0.61</td>
</tr>
<tr>
<td>Two-product strategy-low quality product</td>
<td>0.31/0.30/0.28</td>
<td>0.38/0.36/0.36</td>
<td>0.46/0.44/0.41</td>
</tr>
</tbody>
</table>

*The sequence represents the average optimal customization level where rate of inventory cost is 20%/30%/40%.

Table 2.4: The average optimal customization level for each strategy as influenced by $\frac{\theta_H}{\theta_L}$

<table>
<thead>
<tr>
<th>Strategies</th>
<th>$\frac{\theta_H}{\theta_L}$ = 1.071</th>
<th>$\frac{\theta_H}{\theta_L}$ = 1.29</th>
<th>$\frac{\theta_H}{\theta_L}$ = 1.5</th>
<th>$\frac{\theta_H}{\theta_L}$ = 1.714</th>
</tr>
</thead>
<tbody>
<tr>
<td>One-product strategy- two segments</td>
<td>0.50/0.50/0.45*</td>
<td>0.50/0.50/0.45</td>
<td>0.50/0.50/0.45</td>
<td>0.50/0.50/0.45</td>
</tr>
<tr>
<td>One-product strategy- segment H</td>
<td>0.69/0.64/0.58</td>
<td>0.85/0.84/0.79</td>
<td>0.95/0.91/0.89</td>
<td>1.099/0.95</td>
</tr>
<tr>
<td>Two-product strategy-high quality product</td>
<td>0.69/0.64/0.59</td>
<td>0.79/0.76/0.71</td>
<td>0.77/0.74/0.72</td>
<td>0.77/0.75/0.70</td>
</tr>
<tr>
<td>Two-product strategy-low quality product</td>
<td>0.30/0.28/0.27</td>
<td>0.32/0.30/0.28</td>
<td>0.42/0.41/0.39</td>
<td>0.49/0.48/0.44</td>
</tr>
</tbody>
</table>

*The sequence represents the average optimal customization level where rate of inventory cost is 20%/30%/40%.

Table 2.5: The average optimal customization level for each strategy as influenced by $b$

<table>
<thead>
<tr>
<th>Strategies</th>
<th>$b = 550$</th>
<th>$b = 700$</th>
<th>$b = 850$</th>
</tr>
</thead>
<tbody>
<tr>
<td>One-product strategy- two segments</td>
<td>0.60/0.60/0.45*</td>
<td>0.45/0.45/0.45</td>
<td>0.45/0.45/0.45</td>
</tr>
<tr>
<td>One-product strategy- segment H</td>
<td>0.97/0.95/0.93</td>
<td>0.88/0.85/0.81</td>
<td>0.78/0.74/0.68</td>
</tr>
<tr>
<td>Two-product strategy-high quality product</td>
<td>0.88/0.84/0.83</td>
<td>0.76/0.73/0.65</td>
<td>0.63/0.61/0.55</td>
</tr>
<tr>
<td>Two-product strategy-low quality product</td>
<td>0.44/0.42/0.40</td>
<td>0.39/0.38/0.34</td>
<td>0.32/0.31/0.29</td>
</tr>
</tbody>
</table>

*The sequence represents the average optimal customization level where rate of inventory cost is 20%/30%/40%.
The effect of production and marketing factors

Next, we discuss the effect of the different marketing and production factors in more detail. The results are summarized in Figures 2.6(a)-(d). Figure 2.6(a) indicates that the average customization levels consistently decrease with \( b \) for all three product line strategies. Figure 2.6(b) then shows the effect of the ratio between the customization valuation of the high-valuation segment and the low-valuation segment. We see that for the one-product strategy targeting both segments, the customization level stays the same because the customization level for this strategy only depends on the consumers’ valuation in the low segment, which was set constant in the experiment. For the one-product strategy targeting only the high-valuation segment, the average customization level increases as the ratio increases. This is not surprising as the manufacturer can increase its expected profit by offering products with higher customization levels and charge higher prices. The results for the two-product strategy seem to be more surprising. In the standard product line design literature, it is widely known that the cannibalization effect becomes more severe as the ratio increases. If the same experiment scenario is applied to the standard product line design problem that does not consider lead time and inventory cost, the expected result is that the low quality level will decrease whereas the high quality level will increase. The reader is referred to Appendix A for the solution of the standard product line design problem. However, Figure 2.6(b) shows a different result. The average customization level of the product targeted at the high-valuation segment does not seem to keep increasing as the ratio increases. In fact, the average customization level of the product targeted at the low-valuation segment can also increase. Several factors may contribute to this result. When the customization level increases, the price of the product with high customization level will not necessarily increase because of the longer lead time (as shown in Proposition 1, the price of the product with high customization is
\[
p_H = \theta_H(q_H - q_L) - \omega_H(s_H - s_L) + p_L.
\]
This is especially true when the waiting cost \( \omega_H \) is high such that the contribution of the increase in customization valuation to the price can be outweighed by the negative contribution of the increase in waiting cost. This effect is not present in the standard product line design literature. As a way to compensate for this effect, the manufacturer may be motivated to increase the customization level of the product targeted at the low-valuation segment instead. On one hand, increasing the customization level of the low-quality product reduces the difference of the two customization levels, and hence negatively affects the price of the high quality product. On the other hand, it also reduces the difference of the two lead times, which has a positive effect to the price. The effect of the latter becomes stronger in the case with higher waiting cost in the high-valuation segment, \( \omega_H \).
In Figure 2.6(c), we show the effect of the ratio between the waiting cost of the high-valuation segment and the low-valuation segment. The results can be explained by the same arguments as were used in relation to Figure 2.6(b). For the two-product strategy, in particular, increasing the ratio would result in smaller differences between the two customization levels. We also study the effect of the ratio of mean demand between the high-valuation segment and the low-valuation segment on the customization levels, as depicted in Figure 2.6(d). As expected, the average customization levels are not affected in the one-product strategies. Changing the relative sizes of the two segments in the one-product strategies will affect the profit, but not the customization levels. For the two-product strategy, we see a different result as the low customization level may also decrease.

Figure 2.6: Average optimal customization level as a function of $b$, $\frac{\theta_H}{\theta_L}$, $\frac{\omega_H}{\omega_L}$, and $\frac{\mu_H}{\mu_L + \mu_H}$.
The best product-line strategy

Our numerical study shows that it is possible for each of the three product line strategies to be optimal. In Figure 2.7, we show the frequency distributions for each product line strategy being the optimal strategy and we also show how the frequency distributions are affected by the proportion of the high-valuation segment in the market. The frequency of the one product strategy targeted at both segments to be optimal is relatively low compared to the frequency computed for the other two strategies. It can be seen that the effect of increasing the proportion of the high-valuation segment is mainly observed for the two-product strategy and one-product strategy targeted only at the high-valuation segment. When demand in the high-valuation segment increases, the manufacturer would be more motivated to offer a highly-customized product targeted only at consumers in the high-valuation segment.

![Figure 2.7: Optimal strategy as a function of proportion of demand in the high-valuation segment](image)

Prioritization in lead time reduction

Reducing lead times is always desirable as it will enhance the success rate of the mass customization strategy. Hence, firms must proactively find ways to reduce lead times. Given the limited resources they have, there is certainly a need for firms to spend time and money in the most effective way possible. To this end, our model can be used to identify the processes in the production network that should be given priority when firms initiate a lead time reduction project. The highest priority should be given to the process in the production network for which a reduction of its processing time will yield the highest increase in profit. Below, we use a numerical example to illustrate how this is achieved.
Consider a setting with the following parameter values: $\mu_H = 20; \mu_L = 20; \theta_H = 1500$; unit holding cost rate = 30%; $\omega_H = 15; \omega_L = 30; b = 700$. Under the one-product strategy (targeted at the high-valuation segment), the optimal customization level is at node 10. Under the two-product strategy, the optimal customization levels are at nodes 10 and 15. In Figure 2.8 below, we plot the profit improvements for different nodes obtained when the manufacturer is able to shorten the processing time of the corresponding node by one day. In this example, as shown in the figure, the manufacturer should give priority at node 8 to maximize the benefits of the lead time reduction project. As also shown in the figure, reducing the lead times of the nodes located upstream of the customization node appears to yield higher profit improvements. A plausible explanation is that when lead times for those nodes are reduced, the manufacturer may benefit both from offering a shorter delivery time to consumers and from reducing the total inventory costs. This explanation can be directly related to Constraint (1.4) in the inventory optimization problem. In the case where the constraint is binding, we can clearly see the direct effect of reducing the lead time at node 8 located upstream the customization node (node 10). Reducing the processing time at node 8 will result in a lower safety stock level carried out by this node, and will allow the manufacturer to reduce the service time to the customers. A similar, although smaller, effect can be observed in Figure 2.8 for the two-product strategy at nodes 14 and 15, respectively. As for the nodes located downstream from the customization node, the benefit is more limited since no safety stocks are held there.

![Figure 2.8: Manufacturer’s profit improvement under budget constraints](image)

2.5 Conclusions

In this paper, we examine how manufacturing firms can make optimal decisions regarding customization levels in a vertically differentiated market. We extend the standard product line design
problem by considering two interdependent product attributes, namely customization level and product delivery lead time. In addition, we incorporate the importance of safety stock and pipeline inventory costs in determining the optimal customization levels. We integrate the guaranteed service model with the product line design model to determine the optimal customization levels, product delivery lead times, prices, and safety stock placements in the production system.

Our numerical investigation reveals several interesting and important insights. We show that the cost of holding safety stock and pipeline inventory influences the optimal choice of customization levels. The optimal customization levels tend to be lower when the inventory costs are considered compared to the case where the inventory costs are ignored. This finding highlights the importance of considering operational factors other than the production cost when making product line design decisions. Ignoring the amount and positioning of inventory in the production network may lead to non-optimal customization levels due to the profit overestimation. Another interesting observation is that the effect of consumer valuation difference between the two segments on the optimal customization levels is affected by whether or not the lead time and inventory cost are included. In the standard product line design problem in which the lead time and inventory cost are ignored, when the quality valuation ratio of the high segment to the low segment increases, the low quality product will be more distorted downward while the high quality level will increase. However, when the lead time and inventory cost are included, the observation is different. Our numerical results show that the high quality level does not always keep increasing, and the low quality level may also increase. Our exploration on the two-product and one-product strategies shows that in most cases, the primary choice for the manufacturer is either to offer only one customization level for the high-valuation segment or to offer two customization levels, one for each segment. Offering one product (one customization level) to the high-valuation segment will be attractive when the size of the high-valuation segment is relatively larger than the size of the low-valuation segment.

We have also demonstrated how our model can be used for identifying the process in the production system that should be given priority when firms intend to initiate a lead time reduction project. More specifically, our model is able to identify the process for which a lead time reduction yields the highest increase in profit. Not all processes should be given the same priority and we have proven that marketing factors and inventory costs both have an impact on profits.

We acknowledge some limitations of this paper and would like to suggest several topics for future research. First, our model assumes that consumers’ preferences are homogeneous within each market segment such that each segment is represented by a single quality valuation value. The extent to which
the optimal customization levels and service times in our results will change when consumers are also heterogeneous within each segment is still unclear. Hence, a future research avenue could be to consider a market where consumers’ preferences are continuously distributed such that market segmentation is also part of the manufacturer’s decisions. In relation to this, one could also consider a different approach in defining demand uncertainty. For example, one could model demand uncertainty that is due to the probabilistic nature of the minimum and/or maximum values of the consumers’ preferences.

Second, our model implicitly assumes that the manufacturer sells its customized products directly to end consumers, i.e., no explicit consideration is given to the distribution channel. It would be interesting to extend our study by considering the effect of channel structure. One interesting research question could be, for example, what the manufacturer’s optimal customization strategy would be if the manufacturer uses a dual channel by selling the products through a retailer as well as through an online channel.
Appendix A

Here, we introduce the standard product line design problem where a manufacturer sells two products targeted to two segments. The manufacturer sells the low quality product with quality level $q_L$ and price $p_L$ targeted at the low-valuation segment, and the high-quality product with quality level $q_H$ and price $p_H$ targeted at the high-valuation segment. The size of the low-valuation and the high-valuation segments are $\mu_L$ and $\mu_H$, respectively. The manufacturer’s problem is as follows:

$$\max_{p_L p_H q_L q_H} (p_L - b q_L^2) \mu_L + (p_H - b q_H^2) \mu_H$$

(A)

Subject to

$$\theta_L q_L - p_L \geq 0 \quad (A-1)$$

$$\theta_H q_H - p_H \geq 0 \quad (A-2)$$

$$\theta_L q_L - p_L \geq \theta_L q_H - p_H \quad (A-3)$$

$$\theta_H q_H - p_H \geq \theta_H q_L - p_L \quad (A-4)$$

Constraints (A-1) and (A-2) denote the participation constraints for the two segments, and constraints (A-3) and (A-4) are the self-selection constraints for the two segments.

It is optimal for the manufacturer to set the price for the low-quality product by making constraint (A-1) binding, i.e. $p_L = \theta_L q_L$, and to set the price for the high-quality product by making constraint (A-4) binding i.e. $p_H = p_L + \theta_H (q_H - q_L)$. The optimal quality levels of the two products are $q_H^* = \frac{\theta_H}{2b}$ and $q_L^* = \frac{\theta_L}{2b} (1 - R)$, where $R = \frac{\mu_H}{\mu_L} \left( \frac{\theta_H}{\theta_L} - 1 \right)$, is a measure of potential cannibalization and $R < 1$ (Moorthy and Png 1992).
Appendix B

The modified dynamic programming algorithm for a production network with one customization level is presented as follows:

Run the dynamic programming algorithm\(^1\) for a given production network, \(n\), and \(s\) to optimize \(\overline{\text{TC}}_{(n,s)}\) problem:

- **Step 1:** Define a set \(\text{ADJ}\) including the most downstream nodes connected to the customization node
- **Step 2:** Labeling the production network\(^2\)
  1. **Step 2.1:** Consider all nodes in an unlabeled set, \(\mathcal{U}\)
  2. **Step 2.2:** Set \(\text{label} = 1\)
  3. **Step 2.3:** Find a node \(i\) that is connected to at most one other node in \(\mathcal{U}\)
  4. **Step 2.4:** Label node \(i\) with label and remove node \(i\) from set \(\mathcal{U}\).
  5. **Step 2.5:** \(\text{label} = \text{label} + 1\)
  6. **Step 2.6:** If \(\mathcal{U}\) is empty
     - Stop
     - Else
     - \(\text{label} = \text{label} + 1\) and go to step 2.3
   - End
- **Step 3:** \(s' = s - \sum_{i \in \mathcal{C}_n} T_i\)
- **Step 4:** Execute the modified dynamic programming algorithm:
  1. **Step 4.1:** For \(j = 1 \text{ to } N - 1\)
     1. **Step 4.1.1:** If \(P(j)\) is downstream of \(j\), determine the feasible values for \(S\) and evaluate \(f_j(S)\)
        - If \(j^* \in \mathcal{C}_n\) \((j^*\) is the label of node \(j\) before renumbering)
          - \(S := \sum_{i \in \mathcal{C}_n, i \leq j} T_i, \ldots, M_j\).
        - End If
        - If \(j^* \geq n\) and \(j^* \in N - C\)
          - \(S := 0, \ldots, M_j\).
        - End If
        - If \(j^* < n\) and \(j^* \in \text{ADJ}\)
          - \(S := 0, \ldots, s'\)
        - Else If \(j^* < n\) and \(j^* \in N - \text{ADJ}\)

---

\(^1\) The proposed dynamic programming algorithm in (Graves and Willems 2000, 2003) includes some functions and terms as follow:

(i) \(f_j(S) = \min_{S_I} \{c_j(S, S_I)\}\) w.r.t. outlet service time and \(g_j(S_I) = \min_{S_I} \{c_j(S, S_I)\}\) w.r.t. inbound service time are defined functional equations.

(ii) \(c_j(S, S_I) = h_j \left[ D_j(S + T_j - S_I) - (S_I + T_j - S) \mu_j \right] + \sum_{(i,j) \in A} \min_{S \leq S_I} (f_i(x)) + \sum_{(i,j) \in A} \min_{S \leq S_I} \left( g_j(y) \right) \).

(iii) \(P(j)\) is one of the adjacent nodes to \(k\) with higher label.

(iv) The maximum replenishment time for node \(j\): \(M_j = T_j + \max \{ \{ M_i | i \in A \} \} \)

\(^2\) The algorithm for labeling the network is based on the algorithm presented in Graves and Willems (2000)
\[ S := 0, \ldots, M_j, \]
End If

Step 4.1.2: Evaluate \( f_j(S) \)

Step 4.1.3: Else If \( P(j) \) is upstream of \( j \), determine the feasible values for \( SI \) and evaluate \( g_j(SI) \)
If \( j^* \in \{n\} \cup (N - C_n) \)
\[ SI := 0, \ldots, M_j^* - T_j^* \]
Else
\[ SI := \sum_{i \in C_n, i < j^*} T_i, \ldots, M_j^* - T_j^* \]
End

Step 4.1.4: Evaluate \( g_j(SI) \)
End For

Step 4.2: Determine the feasible values for \( SI \) for node \( N \) and evaluate \( g_N(SI) \)

Step 4.3: Minimize \( g_N(SI) \) to find the optimal value for \( IC_{(n,s)} \).

Appendix C

Proof of Proposition 1

We shall derive the maximum prices the manufacturer can charge in the two-product strategy given that the customization levels and delivery lead times have been set. Following Kim and Chhajed (2002), the manufacturer tries to extract all the consumers’ surplus in one segment in the optimal solution. Therefore, three cases are analyzed: (i) consumers belong to both segments get zero surpluses; (ii) only consumers in the high-valuation segment obtain positive surplus; and (iii) only consumers in the low-valuation segment obtain positive surplus.

Case 1: No segment obtain positive surplus

The manufacturer charges the maximum prices to both segments, i.e. extracts all the consumers’ surplus in both segment. This is done by making the two participation constraints (2.1) and (2.2) binding, i.e. \( p_H = p_0 + \theta_H q_H - \omega_H s_H \) and \( p_L = p_0 + \theta_L q_L - \omega_L s_L \). By substituting \( p_L \) in the right term of constraint (2.4), the net utility of consumers in the high valuation segment when buying the low-quality product becomes \( q_L (\theta_H - \theta_L) + s_L (\omega_L - \omega_H) \), which is always positive. Thus, with this pricing strategy, consumers in the high-valuation segment will find it attractive to buy the low-quality product designed for the low-valuation segment. In other words, this pricing strategy cannot avoid the cannibalization problem.
Case 2: Only the low-valuation segment obtains positive surplus

In this case, the manufacturer extracts all the consumers’ surplus in the high-valuation segment by making the participation constraint of the high-valuation segment (constraint (2.2)) binding. As a result, the participation constraint (2.1) and self-selection constraint (2.3) provide two different upper bounds for $p_L$: $p_L \leq p_0 + \theta_L q_L - \omega_L s_L$ and $p_L \leq p_0 + \theta_L q_L - \omega_L s_L + q_H(\theta_H - \theta_L) + s_H(\omega_L + \omega_H)$. The first upper bound for $p_L$ is tighter than the second upper bound. However, if we set $p_L$ to equal the first upper bound then the manufacturer faces the cannibalization problem.

Case 3: Only the high-valuation segment obtains positive surplus

In this case, the manufacturer only charges the maximum price to the low-valuation segment: $p_L = p_0 + \theta_L q_L - \omega_L s_L$. Constraints (2.2) and (2.4) provide two different upper bounds for $p_H$: $p_H \leq p_0 + \theta_H q_H - \omega_H s_H$ and $p_H \leq \theta_H(q_H - q_L) - \omega_H(s_H - s_L) + p_L$. By substituting $p_L$, the second upper bound can be written as $\theta_H(q_H - q_L) - \omega_H(s_H - s_L) + p_L$, which is tighter than the first upper bound. Therefore, $p_H$ can be set as $\theta_H(q_H - q_L) - \omega_H(s_H - s_L) + p_L$, and this is sufficient to avoid the cannibalization problem. This case provides the pricing solutions to the manufacturer’s product line design problem with customization.

Proof of Proposition 2

Since the manufacturer set $p_L = p_0 + \theta_L q_L - \omega_L s_L$ and $p_H = \theta_H(q_H - q_L) - \omega_H(s_H - s_L) + p_L$, $s_L$ should be less than $\frac{p_0 + \theta_L q_L}{\omega_L}$ and $s_H$ should be less than $\frac{\theta_H(q_H - q_L) - s_L(\omega_L - \omega_H) + \theta_L q_L + p_0}{\omega_H}$, to ensure that the prices are positive. Additionally, both $s_L$ and $s_H$ are bounded from below by the total processing time required to customize the products. Thus, we need the following condition: for segment $k (k = L, H)$, $s_k \geq \sum_{l \in C_{nk}} T_l$. 
Appendix D

The optimization algorithm for two-product strategy is as follows:

Find all possible customization nodes \((n_H, n_L)\) so that \((q_H > q_L)\) and store in a set.

\(\text{counter} := 1\)

While \(\text{counter} \leq \text{(number of members of the set containing all \((n_H, n_L)\) )}\)

Create \(C_{n_L} \subseteq N\), and \(C_{n_H} \subseteq N\) for a given \((n_H, n_L)\)

Find the values of customization levels:

\[ q_L = \frac{\sum_{i \in C_{n_L}} T_i}{T_0} \quad \text{and} \quad q_H = \frac{\sum_{i \in C_{n_H}} T_i}{T_0} \]

Evaluate the production costs for two product lines:

\[ (a + bq_H^2) \quad \text{and} \quad (a + bq_L^2) \]

Determine the maximum and minimum values of the promised lead time at segment \(L\):

\[ s_{L_{min}} = \sum_{i \in C_{n_L}} T_i \quad \text{and} \quad s_{L_{max}} = \frac{p_0 + \theta_L q_L}{a_L} \]

Initialize \(l_1 := 1\) (The iteration counter)

For \(s_L = s_{L_{min}} \) to \(s_{L_{max}}\)

Set \(p_L = p_0 + \theta_l q_L - \omega_L s_L\)

Run the dynamic programming algorithm for given production network \( n_L\) and \(s_L\) to optimize \(\mathcal{I}(n_L; s_L)\)

Find the manufacturer’s profit in segment \(L\) for given \(q_L, s_L\):

\[ Z_{L, l_1} = (p_L - (a + bq_L^2))\mu_L - \mathcal{I}^*_{L}(n_L; s_L) \]

Determine the maximum and minimum values of the promised lead time at segment \(H\) with a given \(s_L\) and \(p_L\).

\[ s_{H_{min}} = \sum_{i \in C_{n_H}} T_i \quad s_{H_{max}} = \frac{\theta_H (q_H - q_L) - s_L(\omega_L - \omega_H) + \theta_L q_L + p_0}{\omega_H} \]

\(l_2 := 1\) (The iteration counter)

For \(s_H = s_{H_{min}} \) to \(s_{H_{max}}\)

Set \(p_H = \theta_H (q_H - q_L) - \omega_H (s_H - s_L) + p_L\)

Run the dynamic programming algorithm to optimize \(\mathcal{I}^*_{H}(n_H; s_H)\) problem.

Find the maximum manufacturer’s profit at segment \(H\) for given \(q_L, s_L, q_H, s_H\)

\[ Z_{H, l_2} = (p_H - (a + bq_H^2))\mu_H - \mathcal{I}^*_{H}(n_H; s_H) \]

\(l_2 := l_2 + 1\)

End For

\(Z_H := \text{Find the maximum value of profit at segment } H \text{ for a given } q_H\)

\(Z_{L,H,l_1} := Z_{L,l_1} + Z_H\)

\(l_1 := l_1 + 1\)

End For

\(Z_{L,H,\text{counter}} := \text{Find the maximum value of profit at both segments for a given } q_H \text{ and } q_L\)

\(\text{counter} := \text{counter} + 1\)

End While

\(Z_{L,H}^* := \text{Find the maximum of } Z_{L,H,\text{counter}}\)

\((q_H^*, q_L^*) := \text{The customization levels for the nodes in product lines where the total maximum profit occurs.}\)
Appendix E

In Table E-1, we present the unit holding cost for each node when the annual rate is 30% of the production cost and \( b = 700 \).

Table E-1: Annual per-unit holding cost at each node for different customization nodes

<table>
<thead>
<tr>
<th>Nodes</th>
<th>Customization nodes</th>
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<tbody>
<tr>
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<td>( n = 5 )</td>
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<td>1</td>
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<td>2</td>
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Chapter 3

Paper 2: Product line design with vertical and horizontal consumer heterogeneity: the effect of distribution channel structure on the optimal quality and customization levels

History: This paper has been presented at: OR59 Annual Conference -Conference of OR Society, September 2017, Loughborough, United Kingdom. The paper is submitted to IEEE Transactions on Engineering Management.
Product line design with vertical and horizontal consumer heterogeneity: the effect of distribution channel structure on the optimal quality and customization levels

Parisa Bagheri Toonkamlou and Hartanto Wong

Department of Economics and Business Economics
School of Business and Social Sciences, Aarhus University
Fuglesangs Allé 4, 8210 Aarhus, Denmark

Abstract

In this paper, we investigate a product line design problem for a manufacturer selling the products in a market where consumers are heterogeneous in two aspects. First, consumers are vertically heterogeneous with respect to their valuation of quality. Consumers in the high-valuation segment value quality more than consumers in the low-valuation segment. Second, consumers are horizontally heterogeneous with respect to their preference on the aesthetic component of the product. We compare a number of product line strategies that can be chosen by the manufacturer. In particular, we are interested in examining the effect of offering a customized product in the product line. Using the single-product strategy as the baseline strategy, we examine when it is best for the manufacturer to consider the product line extension in a vertical direction or in a horizontal direction. We also consider the quality-based segmentation as the baseline strategy, and examine whether offering the customized product will enhance the profitability, and whether the customized product should be offered with low or high quality. Furthermore, we study how the choice of distribution channel in the supply chain affects the optimal product line decision. First, we consider a centralized channel where the manufacturer sells all the products directly to the end consumers. Second, we consider a dual channel where the manufacturer sells the standard product through a retailer while the customized product is sold through a direct (online) channel.

Keywords: product line design; customization; distribution channel; horizontal differentiation; vertical differentiation.
3.1 Introduction

It can be argued that one of the most important decisions within product line design is the length of product line, which is defined as the number of product offerings in a product line. Offering a larger product variety would allow firms to increase both demand and market share (Kotler 2000). Thus, product line extension has always been an integral part of marketing managers’ strategies. Motivated by the fact that consumers are heterogeneous in their valuation of product quality, firms often differentiate their product lines vertically by offering a product line comprised of multiple products with different quality levels. Quality-based segmentation is commonly practiced in marketing by offering a product line comprised of multiple products with different quality levels, serving a market comprised of heterogeneous segments. Consider, for example, garment products such as suits, pants, sweaters, etc., that are offered in different materials based on which the prices are differentiated. Apparel companies such as Land’s End and Levi Strauss offer a variety of jeans that are differentiated by the quality of the fabric (organic or conventional; pure cotton or a synthetic mix), the treatment of denim (washed or unwashed), and the quality of construction (high stitch count or low stitch count).

Alternatively, firms may choose to differentiate their product lines horizontally by offering products that have the same quality level but different characteristics or aesthetic attributes such as taste, color or flavor. Garment manufacturing firms usually offer products that are differentiated by color and size but sold at the same price. In relation to the horizontal product line differentiation, we witness an increasing number of firms offering the customized product to their consumers today. The two examples above (Land’s End and Levi Strauss) offers customized shirts, jeans, and jackets by allowing consumers to choose their preferred styles, colors, inseam, etc. There are abundant examples of firms offering customized t-shirts where consumers have the possibility to choose the most preferred color and/or pictures (Spreadshirt and Rush order Tees companies). Sisal Rugs Direct (http://www.sisalrugsdirect.com) allows consumers to choose the shape, colors and pattern of their rugs.

The motivation for firms to offer such customized products is that they may increase both their consumer base and revenues. However, this motivation does not come without costs as some investments are necessary to enhance flexibility in the production facility such that customization can be accommodated. An interesting but less understood question related to the firms’ product line extension decision is which extension direction is more desirable: vertical product line extension, or horizontal product line extension with customization. Our literature review suggests that none of the existing studies has ever compared the potential benefits offered by the horizontal product line
extension strategy that accommodates customization to the benefits achieved from the vertical product line extension strategy. Firms that are implementing quality-based segmentation may want to make another interesting and related assessment on whether additional benefits can be obtained by adding customized offerings to the product line, i.e., by combining both vertical and horizontal product line extension strategies. In the case of firms offering the customized product, the question of which customization level should be offered also becomes relevant.

Furthermore, manufacturing firms must also consider the type of distribution channel used to sell their products when designing a product line. When the manufacturer sells directly to consumers i.e. uses a centralized distribution channel, it has full control over the ultimate targeting of the products to the different consumer segments through its pricing and quality level decisions. In a decentralized channel, however, the manufacturer depends on intermediate parties, e.g. retailers, who may choose a market coverage strategy that differs from what is intended by the manufacturer. For example, in the case where the manufacturer intends to cover the whole market, the retailer may prefer to cover the market only partially. This potential misalignment of the manufacturer’s and the retailer’s strategies may create channel inefficiency that results in a reduction of channel profitability.

Inspired by observations in practice, in the case where the manufacturer chooses the horizontal product line extension and sells the customized product, we also consider a dual channel which can be seen as a variant of a decentralized channel. That is, the standard product is sold through the retailer, and the customized product is sold online directly by the manufacturer. While the issue of downward quality distortion in the decentralized channel has been studied previously, none of the existing studies examines how the manufacturer’s adoption of dual channel strategy will affect the extent of the quality distortion.

In this paper, we are interested in investigating the preference for different product line extension strategies and how this preference is influenced by the type of distribution channel adopted.

More specifically, we aim to answer the following questions:

(1) **When is the horizontal product line extension strategy preferable to the vertical product line extension or quality-based segmentation strategy?**

(2) **What is the effect of adding a customized product to the product line that consists of products with different quality? Which quality level should be chosen for the customized product?**
(3) How are the answers to the above questions influenced by the type of distribution channel and by the operations and marketing factors?

In answering these research questions, we develop stylized models that extend and integrate the existing models in the product line design literature. We limit the analysis to a monopolistic setting. Though we consider competition between the retailer and the online channel in our model, we do not consider competition between manufacturers as well as between retailers. The choice of this setting is conventional in the literature studying channel structure in product line design (see e.g. Villas-Boas 1998, Liu and Cui 2010, and Jerath et al. 2018), and allows us to isolate the impact of operations and marketing factors on the relative performances of the different product line extension strategies without any interference from the market competition. Our model should be seen as a building block for developing an extended model considering market competition.

The remainder of this paper is structured as follows. Section 3.2 surveys the related literature. In Section 3.3, we present two comparison scenarios for both the centralized and decentralized channels. This section starts with the models and results for the first comparison scenario where the focus is on contrasting the horizontal and vertical product line extension strategies. The section continues with the second comparison scenario where the focus is on examining the effects of offering a customized product to the existing quality-based segmentation strategy. In Section 3.4, we present the results of our sensitivity analysis in relation to some of the assumptions used in our model. We conclude the paper in Section 3.5.

3.2 Literature Survey

This paper is related to four streams of literature. The first stream of research that is related to our paper is the stream considering quality-based segmentation or vertical product differentiation. Mussa and Rosen (1978) and Moorthy (1984) are among the first introducing quality differentiation into the marketing literature. The classical result from their studies is that due to potential cannibalization, only consumers with the highest valuation for quality receive their optimal quality level, whereas consumers in the other segments receive quality levels that are lower than their optimal quality levels. Subsequent related-papers analyze the product line design problem with quality differentiation in various settings. Netessine and Taylor (2007) study the impact of production technology on the optimal quality and price decisions. Lauga and Ofek (2011) study a duopoly model in a market where consumers are vertically heterogeneous with respect to the quality level of a product with two attributes. They show that when the production cost for providing the quality is sufficiently low they use only one attribute to differentiate the product. Several authors e.g. Kim and Chhajed (2000), Desai
et al. (2001), and Heese and Swaminathan (2006) analyze quality-based segmentation with commonality where it is possible to use common components in both the high and low quality products.

The second stream of research considers product line design problems with horizontal product differentiation. In this stream of research, quality is assumed to be given and therefore, the reservation price of consumers is assumed to be constant. A common feature, as we also use in this paper, is that consumers are assumed to be uniformly distributed over Hotelling’s line that is characterized by a closed interval of product space $[0, 1]$, (Hotelling, 1929). Each point on the line represents the consumer’s preference (or location) for the aesthetic attribute(s) of the product. De Groote (1994) studies the role of flexibility in accommodating product line extension in the horizontal direction. Several papers in this stream particularly study the comparison between mass production offering a limited set of standard products and mass customization offering unlimited and customized products (see e.g. Gaur and Honhon 2006, Jiang et al. 2006, Alptekinoğlu and Corbett 2010, and Wong and Eyers 2011). Several authors, e.g. Syam and Kumar (2006), Alptekinoğlu and Corbett (2008), Mendelson and Parlaktürk (2008), and Xia and Rajagopalan (2009) examine the competition between the standard and customized products in a duopoly market. Takagoshi and Matsubayashi (2013) present a model considering a two-attribute space where one attribute indicates a characteristic in relation to the function of the product and the other attribute indicates a taste or flavour of the product. They define customization as a continuous extension of their product line from the core product only along the function attribute, and study a competition between two firms. Some authors, e.g. Dewan et al. (2003) and Xiao et al. (2014) model the product variety in a circular spatial market instead of the traditional Hotelling model. Our paper differs from all the above papers in that they do not consider product quality as a decision variable as we do in our paper.

Our literature review suggests that there are only a few papers in the literature on the third stream of research studying product line design that considers vertical and horizontal consumer heterogeneity, thereby lending support to the analysis presented in this paper. Desai (2001) considers a market where consumers are two-dimensionally heterogeneous. They focus on the effect of cannibalization on product line design and do not pay attention to product line extension. Lacourbe et al. (2009) study the optimal product portfolio positioning in a market where consumers are two-dimensionally heterogeneous. In our paper, we also consider vertical and horizontal consumer heterogeneity similar to those two papers, but our paper is different in the two following respects. First, they consider horizontal product differentiation on Hotelling’s line but does not relate them to the customization level as we consider in our model. Second, the issue of distribution channel is not considered in their
models whereas it is an important topic in our paper. Shi et al. (2013) consider a market where consumers can be horizontally or vertically differentiated, but they do not look into the quality-based segmentation problem.

The final stream of research considers product line design that takes into account the choice of distribution channel structure. Villas-Boas (1998) considers a product line design problem in a distribution channel that consists of a manufacturer selling a product line through a retailer. The paper only considers a vertically differentiated market. They find that a manufacturer offers products that are more differentiated in a decentralized channel than in a centralized channel. Chung and Lee (2014) extend the work by Villas-Boas (1998) by considering consumer segments as clusters of somewhat heterogeneous consumers. Hua et al. (2011) considers a problem similar to Villas-Boas (1998) in that they only consider vertical product differentiation. Their focus, however, is on designing a contract between the manufacturer and retailer in order to improve the performance of the channel. They find that a revenue-sharing contract can perfectly coordinate the distribution channel on product design decisions. Unlike Villas-Boas (1998), Hua et al. (2011) do not consider quality as part of the manufacturer’s decision. In a recent study, Wong and Lesmono (2017) examine the effect of distribution channel structure on product line decisions in settings where a product has a two-conflicting attributes. While all the above four papers consider vertical product differentiation, we consider both vertical and horizontal product differentiation. In contrast to the above four papers, Liu and Cui (2010) study a product line design problem in a distribution channel where products are differentiated horizontally. They focus on the optimal number of products in a product line. They show that a decentralized channel can be an efficient structure of distribution by providing a product line length that is optimal from a social welfare perspective. Their analysis, however, does not consider quality-based segmentation and is only limited to the case of two different products without paying attention to the choice of customization level. The paper most closely related to our work is by Li et al. (2015). They consider competition between the manufacturer’s online customization channel and the conventional retailer. Similar to their model, our modeling framework also includes a dual channel in the case when both standard and customized products are offered in the market. In their paper, however, as quality-based segmentation is not considered, there is only one product quality level that is assumed to be given and constant. Hence, their main focus is on the effect of extending the product line in the horizontal direction by offering a customized product in addition to the existing standard product. We extend their model by considering quality-based segmentation as an integral part of the manufacturer’s product line strategy. Such an extension allows us to compare the horizontal product line extension to the vertical product line extension. In particular, our study is
also able to examine how the adoption of dual channel may influence the extent of quality distortion prevalent in the decentralized channel. In conclusion, our paper contributes to advancing the existing literature by considering more comprehensive product line extension strategies and channel structures.

3.3 Models

3.3.1 Notation and Assumptions

We consider a market where consumers are two-dimensionally heterogeneous. We assume that consumers belong to either the high-valuation segment or to the low-valuation segment, indexed by \( i = \{ L, H \} \) (see Table 3.1 for the notation of the model). Consumers in segment \( i \) have valuation \( \theta_iq \) for the product, where \( \theta_i \) represents consumers’ quality valuation in segment \( i \), and \( q \) represents the quality level of the product. Consumers in the high-valuation segment have a higher valuation per unit of quality than consumers in the low-valuation segment. That is, \( \theta_H > \theta_L > 0 \). In the case where two products of different quality are offered, the quality level of the product targeted at segment \( i = \{ L, H \} \) is denoted by \( q_i \). The size of each segment is denoted by \( \alpha_i, i = \{ L, H \} \). Without loss of generality, we assume that the total market size of both segments is normalized to be one, i.e., \( \alpha_L + \alpha_H = 1 \).

Table 3.1: Notations

<table>
<thead>
<tr>
<th>Decision variables</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>( p_{ij} )</td>
<td>Retail price of product ( j ), ( j = S, C ) targeted at segment ( i ), ( i = { L, H } ).</td>
</tr>
<tr>
<td>( x_i )</td>
<td>Customization level of the customized product targeted at segment ( i ), ( i = { L, H } ).</td>
</tr>
<tr>
<td>( q_i )</td>
<td>Quality of the product targeted at segment ( i ), ( i = { L, H } ).</td>
</tr>
<tr>
<td>( w_i )</td>
<td>Wholesale price of the standard product targeted at segment ( i ), ( i = { L, H } ).</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Marketing parameters</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>( t_i )</td>
<td>Unit mismatch cost for consumers in segment ( i ), ( i = { L, H } ).</td>
</tr>
<tr>
<td>( \theta_i )</td>
<td>Consumer’s quality valuation in segment ( i ), ( i = { L, H } ).</td>
</tr>
<tr>
<td>( y_i )</td>
<td>Consumer’s location along the Hotelling’s line in segment ( i ), ( i = { L, H } ).</td>
</tr>
<tr>
<td>( \alpha_i )</td>
<td>Size of segment ( i ), ( i = { L, H } ).</td>
</tr>
<tr>
<td>( D_{ij} )</td>
<td>Portion of consumers in segment ( i ), ( i = { L, H } ) buying product ( j ), ( j = S, C )</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Production parameters</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>( a )</td>
<td>Coefficient of the production cost</td>
</tr>
<tr>
<td>( b )</td>
<td>Coefficient of the fixed investment cost or flexibility cost to accommodate customization</td>
</tr>
<tr>
<td>( c )</td>
<td>Additional marginal cost for the customized product</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Others</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \pi_{SC,c}^{(k)} )</td>
<td>Total profit in the centralized channel for Strategy ( k )</td>
</tr>
<tr>
<td>( \pi_{SC,d}^{(k)} )</td>
<td>Total profit in the decentralized channel for Strategy ( k )</td>
</tr>
<tr>
<td>( \pi_R^{(k)} )</td>
<td>Retailer’s profit in the decentralized channel for Strategy ( k )</td>
</tr>
<tr>
<td>( \pi_M^{(k)} )</td>
<td>Manufacturer’s profit in the decentralized channel for Strategy ( k )</td>
</tr>
</tbody>
</table>
To model the horizontal consumer heterogeneity, we assume that consumers in each of the two segments are uniformly located on Hotelling’s line that is characterized by a closed interval of product space [0, 1]. Each consumer in segment \( i = \{L, H\} \) is represented by his/her aesthetic preference of the product, which is denoted as \( y_i \in [0, 1] \). Following Li et al. (2015), when the manufacturer decides to offer a customized product, the customization level of the product is represented by \( x \in (0,1) \), which measures the fraction of aesthetic attributes of the product that the manufacturer chooses to customize. A standard product is represented by the left-end point, i.e., \( x = 0 \), whereas a fully customized product is represented at the right-end point, i.e., \( x = 1 \). Suppose a standard product is sold to consumers with price \( p \) and quality \( q \). The utility derived by a consumer in segment \( i \) located at \( y_i \in [0, 1] \) is 
\[
    u_i = \theta_i q - t_i y_i - p.
\]

A customized product with customization level \( x \) can satisfy the preference for consumers in segment \( i \) located at \( y_i \in [0, x] \). However, consumers in segment \( i \) located at \( y_i \in (x, 1] \) incurs a mismatch cost \( t_i \) per unit deviation between their preference and that of the purchased product. We assume that consumers in the high-valuation segment are more sensitive to this deviation than consumers in the low-valuation segment, i.e. \( t_H > t_L > 0 \). This assumption is also used in Desai (2001). We are aware that this assumption may not always reflect the true market characteristic. The situation where, for example, consumers in the high-valuation segment are less sensitive to the deviation between their preference and that of the purchased product than the low-valuation segment is also possible. In Section 4, we provide a discussion of the effect of altering the assumption, and show that some of the structural results of this study still hold under the assumption that \( t_L > t_H > 0 \).

In the product line extension strategies we consider, the manufacturer may offer a customized product with low quality or high quality alongside the standard product. For notational convenience, we define \( x_i, i = \{L, H\} \), as the customization level targeted at segment \( i \). Following the standard assumption in the product line design literature (see e.g. Moorthy 1984; and Desai 2001), the manufacturer’s unit production cost is a quadratic function of product quality, \( q \), and equal to \( aq^2 \).

To capture the fact that offering a higher customization level will imply higher production complexity and therefore require higher flexibility, we follow Li et al. (2015) and assume that the manufacturer incurs a fixed flexibility cost \( bx^2 \) when a customized product with customization level \( x \) is offered in the product line. For example, a customized-rug manufacturer needs to invest in a new production system or reconfigure the existing one that can produce the rugs with different patterns and/or color. The quadratic form of the flexibility cost reflects the situation where it becomes increasingly difficult
for a manufacturer to extend its customization ability, which may require a larger amount of machine and labor resources.

We note several aspects of flexibility that deserve further elaboration. First, flexibility in the context of this study should be interpreted more broadly as operations flexibility rather than manufacturing flexibility. The reader is referred to e.g. Slack (1987, 2005) and Coronado and Lyons (2007), and the references therein, for a comprehensive discussion of flexibility. This is especially true when considering the customized product that is offered through an online channel where the fulfilment of a customer order not only relies on manufacturing or production but also on order processing and execution (pre-production) and distribution and transportation (post-production). Second, in relation to manufacturing flexibility, the flexibility cost in our model is more closely related to the notion product and mix flexibility. Product flexibility is defined as the ability to introduce new products or modify the existing ones whereas mix flexibility is defined as the ability to change the range of products (Slack 1987). Once the manufacturer decides to customize an aesthetic attribute of the product, she must be able to produce a large number of product variants differentiated by this attribute. The higher the customization level offered, the larger the number of product variants is.

We consider situations where the unit production cost is primarily driven by product quality, i.e., there is no difference in the unit production cost between the standard product and the customized product with the same quality level. However, as the fulfilment of the customized product may require additional order processing, we assume that there is an additional marginal cost $c$ for the customized product. Suppose a customized product is sold to consumers with price $p$, quality $q$, and customization level $x$. The utility derived by a consumer in segment $i$ located at $y_i \in [0, x]$ is $u_i = \theta_i q - p$. The utility derived by a consumer in segment $i$ located at $y_i \in (x, 1]$ is $u_i = \theta_i q - t_i(y_i - x) - p$. A consumer will make a purchase as long as her utility is non-negative. We use index $j = S, C$, to differentiate the standard product(s) from the customized product.
3.3.2 Comparison of Product Line Strategies

Below, we present the formulations and results for different product line extension strategies. In Figure 3.1, we show that there are five product line strategies considered in this study.

*Strategy 1:* offering one standard product (with low quality) targeted at both segments.

*Strategy 2:* offering one standard product and one customized product with the same quality.

*Strategy 3:* offering to standard products with two different quality levels, each targeted at each of the two segments.

*Strategy 4a:* offering two standard products with two different quality levels and one customized product with low quality.

*Strategy 4b:* offering two standard products with two different quality levels and one customized product with high quality.

Since the effect of offering the customized product is our primary focus, our comparison of product line extension strategies will be based on two scenarios. In the first scenario, we examine how the horizontal product line extension strategy offering customization (Strategy 2) performs in comparison to the vertical product line extension strategy (Strategy 3). The baseline strategy that offers one standard product (Strategy 1) will be used as a benchmark for these two product line extension strategies. In the second scenario, we consider a manufacturer that implements quality-based segmentation (Strategy 3) as a benchmark, and examine the effects of adding the customized product with low quality (Strategy 4a) or with high quality (Strategy 4b) in the product line. In each comparison scenario, we also examine the effect of channel structure, and hence the comparisons will be made under the centralized channel as well as the decentralized channel.
We compare three product line strategies (Strategies 1, 2 and 3). We first investigate product line strategies in a centralized channel, and then we continue with the analysis in a decentralized channel.

The centralized channel

Under a centralized distribution channel, the manufacturer is the only player involved. This could also be seen as equivalent to the setting where the manufacturer sells its product directly to consumers without any intermediaries.
Strategy 1 (one standard product)

As described in the standard literature on product line design (see e.g. Moorthy and Png 1992), the manufacturer basically offers a low-quality product targeted at the two segments and sells a standard product with quality level $q_L$ and price $p_{LS}$. We consequently use index $L$ in this strategy. The manufacturer thus solves the following optimization problem:

$$
\max_{p_{LS}, q_L} \pi_{SC,C}^{(1)} = (p_{LS} - a q_L^2) (\alpha_L D_{LS} + \alpha_H D_{HS})
$$

In (1), the term in the first parentheses on the right hand side denotes the manufacturer’s marginal profit from selling the standard product with low-quality level. The term in the second parentheses represents the total demand of the standard product in both the high-valuation and low-valuation segments. Note that we use $SC$ in the subscript of the profit since we are particularly interested in the channel or supply chain profit in both the centralized and decentralized channels. Figure 3.2 below is provided to illustrate the consumers’ utility in the two segments.

Demands in the low-valuation and high-valuation segments can be expressed as $\alpha_L D_{LS}$ where $D_{LS} =\min\left(\frac{\theta_L q_L - p_{LS}}{t_L}, 1\right)$, and $\alpha_H D_{HS}$ where $D_{HS} = \min\left(\frac{\theta_H q_L - p_{LS}}{t_H}, 1\right)$, respectively. Note that in this paper, we focus on the situation where the whole market is covered for all product line strategies, i.e., $D_{LS} = \frac{\theta_L q_L - p_{LS}}{t_L} = 1$ and $D_{HS} = \frac{\theta_H q_L - p_{LS}}{t_H} = 1$. To impose the whole market coverage in this baseline strategy, we need the following condition:

$$
\beta \left( \frac{4\theta_L}{t_L} - \frac{3(\alpha_L \theta_H t_{HL} + \alpha_H \theta_H t_{LH})}{t_L(\alpha_H t_{HL} + \alpha_H t_{LH})} \right) \geq 1, \ i = \{L, H\}, \text{ where } \beta = \frac{\alpha_L \theta_L t_{HL} + \alpha_H \theta_H t_{LH}}{8a(\alpha_H t_{HL} + \alpha_L t_{LH})} \text{ (see Appendix A, for the derivation of this condition).}
$$

The condition for the whole
market coverage can be justified in settings where the mismatch cost, \( t_l \), is relatively small compared to the quality valuation, \( \theta_l \). To limit the scope, we also focus on settings under the condition \( \frac{4\theta_H - 3(\alpha_l \theta_H - \theta_L)}{t_H - (\alpha_H t_H + \alpha_L t_L)} \geq \frac{4\theta_L - 3(\alpha_l \theta_L t_H + \alpha_H t_H)}{t_L - (\alpha_H t_H + \alpha_L t_L)} \), which implies that when it is optimal to cover the whole market in the low-valuation segment, it is also optimal to cover the whole market in the high-valuation segment. With the conditions above, the manufacturer sets the price such that \( \frac{\theta_L q_L - p_{LS}}{t_L} = 1 \iff p_{LS} = \theta_L q_L - t_L \). The problem in (1) is solved by substituting \( p_{LS} \) with \( \theta_L q_L - t_L \) and by setting \( D_{LS} = D_{HS} = 1 \). The optimal results for strategy 1 are \( q_L^* = \frac{\theta_L}{2a} \), \( p_{LS}^* = \frac{\theta_L^2}{2a} - t_L \), and \( \pi_{SC,c}^{(1)}(1) = \frac{\theta_L^2}{4a} - t_L \). The reader is referred to Table 3.2 and Table 3.3 for the solutions to all the strategies in the centralized and decentralized channels and to Appendix B for the complete derivation of the solutions.

Table 3.2: The optimal solutions for all strategies in the centralized channel

<table>
<thead>
<tr>
<th>Strategies</th>
<th>Optimal solutions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Strategy 1</td>
<td>( q_L^* = \frac{\theta_L}{2a} ) ( \pi_{SC,c}^{(1)} = \frac{\theta_L^2}{4a} - t_L )</td>
</tr>
<tr>
<td>Strategy 2</td>
<td>( q_L^* = \frac{\theta_L}{2a} - \frac{\alpha_H(\theta_H - \theta_L)}{2aa_L} ), where ( \frac{\theta_L}{\theta_H} &gt; \alpha_H ) ( p_{LS}^* = \frac{\theta_L^2 - \alpha_H \theta_H t_L}{2aa_L} - \frac{c}{2} - t_L + \frac{\theta_L^2}{2b} - \frac{\alpha_H t_H t_L}{2b} ) ( \pi_{SC,c}^{(2)}(1) = \frac{\theta_L^2}{4a} - c - t_L + \frac{\theta_L^2}{4b} + \frac{c^2}{4(\alpha_H^2 + \alpha_L^2)} )</td>
</tr>
<tr>
<td>Strategy 3</td>
<td>( q_L^* = \frac{\theta_L}{2a} - \frac{\alpha_H(\theta_H - \theta_L)}{2aa_L} ), where ( \frac{\theta_L}{\theta_H} &gt; \alpha_H ) ( q_H^* = \frac{\theta_H}{2a} ) ( \pi_{SC,c}^{(3)}(1) = \frac{\theta_L^2}{4a} - t_L + \frac{\alpha_H(\theta_H - \theta_L)^2}{4aa_L} )</td>
</tr>
<tr>
<td>Strategy 4a</td>
<td>( q_L^* = \frac{\theta_L}{2a} - \frac{\alpha_H(\theta_H - \theta_L)}{2aa_L} ), where ( \frac{\theta_L}{\theta_H} &gt; \alpha_H ) ( q_H^* = \frac{\theta_H}{2a} ) ( X_L^* = \frac{t_L}{2b} - \frac{\alpha_H t_L}{2b} ), where ( \frac{t_L}{\alpha_H} &gt; \alpha_H ) ( p_{LS}^* = \frac{\theta_L^2 - \alpha_H \theta_H t_L}{2aa_L} - \frac{c}{2} - t_L + \frac{\theta_L^2}{2b} - \frac{\alpha_H t_H t_L}{2b} ) ( \pi_{SC,c}^{(4a)}(1) = \frac{\theta_L^2}{4a^2a_L} - \frac{1}{4aa_L} + \alpha_H \frac{\theta_L^2}{4a_L} + \frac{t_L^2}{4b} - \left( 1 + \frac{\alpha_H t_L}{2b} \right) t_L - \alpha_H c \left( + \frac{\theta_L^2}{2b} - \frac{\alpha_H t_L t_H}{2a_L} + \frac{\alpha_H t_H^2}{4b} - \frac{\alpha_H^2 - \alpha_L^2}{4a^2a_L} \right) )</td>
</tr>
<tr>
<td>Strategy 4b</td>
<td>( q_H^* = \frac{\theta_H}{2a} ) ( q_L^* = \frac{\theta_L}{2a} - \frac{\alpha_H(\theta_H - \theta_L)}{2aa_L} ), where ( \frac{\theta_L}{\theta_H} &gt; \alpha_H ) ( X_H^* = \frac{\theta_H(c + t_H)/2}{b + \alpha_H t_H} ) ( \pi_{SC,c}^{(4b)}(1) = \frac{\theta_L^2}{4a} + \alpha_H \frac{(\theta_H - \theta_L)^2}{4a_L} - \alpha_H \frac{c}{2} \left( 1 + \frac{b}{(\alpha_H t_H + b)} \right) - t_L + \frac{\alpha_H^2(t_H + c)^2}{4(\alpha_H t_H + b)} )</td>
</tr>
</tbody>
</table>
Table 3.3: The optimal solutions for all strategies in the decentralized channel

<table>
<thead>
<tr>
<th>Strategies</th>
<th>Optimal solutions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Strategy 1</td>
<td>$q_L^* = \frac{2\theta_L - \gamma}{2a}$, where $\gamma = \frac{a_H \theta_L}{4aL} - \frac{\theta_H}{t_L} &gt; \gamma$. $\pi_L^*(1) = \frac{(2\theta_L - \gamma)^2}{4a} - 2t_L$</td>
</tr>
<tr>
<td></td>
<td>$q_R^* = \frac{\theta_L(2\theta_L - \gamma)}{2a} - \frac{(2\theta_L - \gamma)^2}{2a} + t_L$ $\pi_R^*(1) = \frac{\theta_L(2\theta_L - \gamma)}{2a} - \frac{(2\theta_L - \gamma)^2}{4a} - t_L$</td>
</tr>
<tr>
<td>Strategy 2</td>
<td>$q_L^* = \frac{\theta_L}{2a}$ $\pi_L^*(2) = \frac{\theta_L^2}{4a} - \frac{2c - t_L + \frac{c^2}{8} + \frac{a_H \theta_L + \alpha_L}{t_L}}{t_L}$</td>
</tr>
<tr>
<td></td>
<td>$q_R^* = \frac{\theta_L}{2a}$ $w_L^* = \frac{\theta_L^2}{2a} + \frac{c^2}{2b} - \frac{c}{2} - t_L$</td>
</tr>
<tr>
<td>Strategy 3</td>
<td>$q_L^* = \frac{\theta_L}{2a} - \frac{a_H (\theta_H - \theta_L)}{a_L}$ $q_R^* = \frac{\theta_H}{2a}$ $x_L^* = \frac{t_L}{2b}$</td>
</tr>
<tr>
<td></td>
<td>$\pi_L^<em>(3) = \frac{\alpha_H \theta_L(\theta_H - \theta_L)}{a_L} - \frac{\alpha_H^2 (\theta_H - \theta_L)^2}{2a_L} + \frac{t_L}{a_L}$ $\pi_R^</em>(3) = \frac{\alpha_H \theta_L(\theta_H - \theta_L)}{a_L} - \frac{\alpha_H^2 (\theta_H - \theta_L)^2}{2a_L} + \frac{t_L}{a_L}$</td>
</tr>
<tr>
<td>Strategy 4a</td>
<td>$q_L^* = \frac{\theta_L}{2a} - \frac{a_H (\theta_H - \theta_L)}{a_L}$, where $\frac{\theta_L}{\theta_H} &gt; \frac{2a_H}{1 + a_H}$. $q_R^* = \frac{\theta_H}{2a}$ $x_L^* = \frac{t_L}{2b} - \frac{a_H (2t_H - 3t_L)}{2b}$. $w_L^* = \frac{(1 + a_H) \theta_L^2}{2a_L} + \frac{2a_H \theta_L^2}{b} + \frac{2a_H^2 t_L^2}{b} - \left(1 - \frac{a_H}{a_L}\right) t_L - \frac{c}{2} + \frac{a_H t_L^2}{2b} - \frac{a_H \theta_H t_L}{a_L}$ $\pi_L^*(4a) = \frac{(1 + a_H) \theta_L^2}{2a_L} + \frac{a_H (\theta_H - \theta_L)^4}{4a_L} + \frac{a_H^2 (\theta_H - \theta_L)^2}{a_L} + \frac{a_H^3 (t_H - 2t_L)^2}{b} - \frac{a_H^2 (t_H - 2t_L)^2}{b} + \frac{a_H^2 t_L^2}{4b}$</td>
</tr>
<tr>
<td></td>
<td>$\pi_R^*(4a) = \frac{(1 + a_H) \theta_L^2}{2a_L} + \frac{a_H (\theta_H - \theta_L)^4}{4a_L} + \frac{a_H^2 (\theta_H - \theta_L)^2}{a_L} + \frac{a_H^3 (t_H - 2t_L)^2}{b} - \frac{a_H^2 (t_H - 2t_L)^2}{b} + \frac{a_H^2 t_L^2}{4b}$</td>
</tr>
<tr>
<td>Strategy 4b</td>
<td>$q_L^* = \frac{\theta_L}{2a} - \frac{a_H (\theta_H - \theta_L)}{2a_L}$ $q_R^* = \frac{\theta_R}{2a}$ $x_L^* = \frac{a_H (c + t_H)}{2(b + 2a_H t_H)}$</td>
</tr>
<tr>
<td></td>
<td>$\pi_L^<em>(4b) = \theta_L^2 + \frac{a_H (\theta_H - \theta_L)^2}{4a}$ $\pi_R^</em>(4b) = \frac{\theta_L^2}{4a} + \frac{a_H (\theta_H - \theta_L)^2}{4a} - t_L - \frac{\theta_H}{2a} + \frac{a_H^2 t_H}{4(b + 2a_H t_H)} + \frac{a_H^2 t_H^2}{4(b + 2a_H t_H)}$</td>
</tr>
</tbody>
</table>
**Strategy 2 (One standard product and one customized product)**

In Strategy 2, the manufacturer extends the product line in the horizontal direction. Under Strategy 2, the manufacturer offers a standard product and a customized product with the same quality level and they are offered to the two segments. The price of the customized product is denoted as $p_{LC}$. Introducing the customized product alongside the standard product will certainly create competition between the two products, i.e. the demands from the two segments will now be divided over the two products. This will give the manufacturer a possibility to raise the price of the standard product and gain additional revenues from a portion of consumers who will be more interested in buying the customized product offered at a price that is higher than the price of the standard product. In other words, offering a customized product will give the manufacturer the possibility to sell the standard product at a higher price than the price in Strategy 1 while the full market is still covered. In the derivation of the solution to Strategy 2, we show that the manufacturer will be better off by increasing the price of the standard product so that it is higher than the price in Strategy 1 (see Appendix B).

The manufacturer solves the following optimization problem:

$$\max_{p_{LS}, p_{LC}} \pi_{SC,c}^{(2)} = (p_{LS} - a q_{L}^2) (\alpha_{L} D_{LS} + \alpha_{H} D_{HS}) + (p_{LC} - a q_{L}^2 - c) (\alpha_{L} D_{LC} + \alpha_{H} D_{HC}) - b x_{L}^2$$

(2)

In (2), the first term denotes the manufacturer’s profit from selling the standard product with low quality to both segments. The second term represents the profit from selling the customized product with low-quality level to both segments, and the last term is the fixed investment cost that the manufacturer incurs for offering the customization. As shown in Figure 3.3, the manufacturer needs to determine the customization level, $x_{L} > 0$, in such a way that the consumer’s utility in the low-valuation segment located at $y_{L} = 1$ is equal to zero when buying the customized product. Since whole market is covered, the manufacturer sets the price for the customized product equal to $p_{LC} = \theta_{L} q_{L} - (1 - x_{L}) t_{L}$. We denote $y_{L}^0 = \frac{p_{LC} - p_{LS}}{t_{L}}$ as the location of the consumer who is indifferent between buying the standard product and buying the customized product in the low valuation segment. The demands from the low-valuation segment can be expressed as $D_{LS} = y_{L}^0 = \frac{p_{LC} - p_{LS}}{t_{L}}$ for the standard product, and $D_{LC} = 1 - y_{L}^0$ for the customized one. With similar explanation, the demands from the high-valuation segment can be expressed as $D_{HS} = \frac{p_{LC} - p_{LS}}{t_{H}}$ for the standard
product, and $D_{HC} = 1 - \frac{P_{LC} - P_{LS}}{t_H}$ for the customized one. The problem in (2) reduces to determining the optimal $q_L, p_{LS}$ and $x_L$.

\[\max \pi^{(3)}_{SC,c} = (p_{LS} - aq_L^2)\alpha_L D_{LS} + (p_{HS} - aq_H^2)\alpha_H D_{HS},\]  

Subject to

\[
\begin{align*}
\theta_L q_L - p_{LS} &\geq \theta_L q_H - p_{HS} \\
\theta_H q_H - p_{HS} &\geq \theta_H q_L - p_{LS}
\end{align*}
\]

In (3), the first term represents the profit from selling the standard product with low quality level to the low-valuation segment, and the second term denotes the profit from selling the standard product with high quality level to the high-valuation segment. The two self-selection constraints (3.1) and (3.2) are necessary to prevent product cannibalization. Following the standard product line design model, the optimal solution is obtained by making (3.2) binding, which gives $p_{HS} = p_{LS} + \theta_H(q_H - q_L)$. Therefore, the manufacturer has to set the price for the high quality product lower than the maximum price that could be charged to cover the whole market in the high-valuation segment.
As in Strategy 1, the price of the low-quality product is \( p_{LS} = \theta L q_L - t_L \), and \( D_{LS} = D_{HS} = 1 \).

\[
\begin{align*}
\theta H q_H - p_{HS} \\
\theta L q_L - p_{LS}
\end{align*}
\]

\( y_l \)

\( D_{IS} \)

Figure 3.4: Consumer’s utility and demand in Strategy 3

Discussion

The optimal results show that the optimal quality level offered in Strategy 1 is not affected when the manufacturer extends the product line horizontally in Strategy 2. This quality level also represents the efficient quality level for the low-valuation segment obtained in the market with perfect information in which there is no risk of cannibalization (Moorthy and Png 1992, and Netessine and Taylor 2007). As expected, the optimal customization level increases in the mismatch cost \( t_L \) and decreases in the fixed flexibility cost to accommodate customization \( b \). The difference in profit between Strategy 2 and Strategy 1 is equal to \( \pi_{SC,c}^{(2)*} - \pi_{SC,c}^{(1)*} = \frac{t_L^2}{4b} + \frac{c^2}{4} \left( \frac{\alpha_H}{t_H} + \frac{\alpha_L}{t_L} \right) - c \), which can be negative or positive. This suggests that the horizontal product line extension strategy is not always desirable. Under the vertical product line extension strategy (Strategy 3), while the high-valuation segment gets the efficient quality level, the low-valuation segment now gets the quality that is lower than its efficient quality level. This downward quality distortion occurs due to the existence of potential cannibalization of the high-quality product by the low-quality product. In line with the results in the product line design literature, as long as the optimal quality level of the low-end product is positive, quality-based segmentation always results in higher profits compared to the baseline strategy offering only one standard product. The difference in profit between Strategy 3 and Strategy 1 is equal to \( \pi_{SC,c}^{(3)*} - \pi_{SC,c}^{(1)*} = \frac{\alpha_H (\theta_H - \theta_L)^2}{a \alpha_L} > 0 \).

We now make a comparison between Strategy 2 and Strategy 3, and examine if the horizontal product line extension strategy can be used as an alternative to the vertical product line extension strategy,
and if so, which conditions are necessary. Several parameters may influence the conditions and hence, there are several ways to represent the condition for the preference for Strategy 2 over Strategy 3. We choose to focus on the condition based on the flexibility cost $b$ incurred in Strategy 2 and summarize the main finding in Proposition 1.

**Proposition 1:** In the centralized channel, the horizontal product line extension strategy with customization is preferred to the vertical product line extension strategy if $b < b_{2-3Central}^*$, where

$$
b_{2-3Central}^* = \frac{t_H^2}{4c - c^2(\frac{a_L}{t_L} + \frac{a_H}{t_H}) + a_H(\theta_H - \theta_L)^2}.\]

(Proofs for all propositions are provided in Appendix C).

In Proposition 1, we show that if the flexibility cost to accommodate customization $b$ is not too high, manufacturing firms may consider the horizontal product line extension strategy as a better option than the quality-based segmentation strategy. This finding complements the existing results in the product line design literature that predominantly focuses solely either quality-based segmentation or horizontal product line extension. In Table 3.4, we present the effects of parameters $t_H$, $a$, and $(\theta_H - \theta_L)$ qualitatively on the preference for Strategy 2 over Strategy 3.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>$t_H$</th>
<th>$(\theta_H - \theta_L)$</th>
<th>$a$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Influence on the preference for Strategy 2</td>
<td>Negative</td>
<td>Negative</td>
<td>Positive</td>
</tr>
</tbody>
</table>

Note: the effect of the rest of parameters rely on the value chosen for the other parameters.

**Decentralized channel**

In the decentralized channel, the manufacturer sells the standard product(s) through an independent retailer and in the case where a customized product is offered, the customized product is sold through a direct (online) channel owned by the manufacturer. The general sequence of decisions is as follows:

i) The manufacturer decides on the quality level(s) of the standard product(s) and the corresponding wholesale prices charged to the retailer. In the case where a customized product is offered, the manufacturer also decides on the customization level and the price of the customized product.

ii) Given the quality level(s) and wholesale price(s) offered by the manufacturer, the retailer decides on the retail price(s) of the standard product(s). We consider the scenario in which the manufacturer is powerful so that she can make a *take-it-or-leave-it* offer to the retailer. This
means that the retailer will carry the standard product(s) offered by the manufacturer. This scenario is also used in e.g. Liu and Cui (2010) and Wong and Lesmono (2017). In Section 4, we examine a different scenario where the retailer has the option to only carry one of the two products in the case of the manufacturer offering two standard products.

**Strategy 1 (One standard product)**

For given $q_L$ and $w_L$ offered by the manufacturer, the retailer solves the following problem:

$$
\max_{p_{LS}} \pi_R^{(1)} = (p_{LS} - w_L)(\alpha_L D_{LS} + \alpha_H D_{HS})
$$

As in the centralized channel, $D_{LS} = \min\left(\frac{\theta_L q_L - p_{LS}}{t_L}, 1\right)$, and $D_{HS} = \min\left(\frac{\theta_H q_L - p_{LS}}{t_H}, 1\right)$. With full market coverage, $D_{LS} = 1$ and $D_{HS} = 1$, and the price set by the retailer is determined by the equation $D_{LS} = \frac{\theta_L q_L - p_{LS}}{t_L} = 1$, which gives $p_{LS} = \theta_L q_L - t_L$. It can be shown that the retailer’s optimal pricing decision when the demands are represented as $\frac{\theta_L q_L - p_{LS}}{t_L}$ and $\frac{\theta_H q_L - p_{LS}}{t_H}$ is $p_{LS}^* = \frac{w_L}{2} + \gamma_1 \frac{q_L}{2}$, where $\gamma_1 = \frac{\theta_L}{t_L} \frac{\theta_H}{t_H}$. The manufacturer knows the optimal response of the retailer and consequently sets the wholesale price equal to $w_L = (2\theta_L - \gamma_1)q_L - 2t_L$ using backward induction. The fact that the wholesale price set by the manufacturer is lower than the retail price shows the existence of double marginalization in the decentralized channel. The manufacturer sets the wholesale price lower than the retail price to ensure that the retailer is willing to cover the whole market as intended by the manufacturer. The manufacturer solves the following problem:

$$
\max_{p_{LS}} \pi_{M,d}^{(1)} = ((2\theta_L - \gamma_1)q_L - 2t_L - aq_L^2)(\alpha_L D_{LS} + \alpha_H D_{HS})
$$

The optimal solutions for all strategies in the decentralized channel are presented in Table 3.3.

**Strategy 2 (One standard product and one customized product)**

In this strategy, the retailer must determine the retail price of the standard product, given that she knows the quality, the wholesale price of the standard product, and the price of the customized product offered by the manufacturer. As in the centralized channel, $D_{LS} = \frac{p_{LS} - p_{PLS}}{t_L}$ and $D_{HS} = \frac{p_{PLS} - p_{HS}}{t_H}$. The retailer’s problem is:
\[
\max_{p_{LS}} \pi_R^{(2)} = (p_{LS} - w_L) \left( \alpha_L \frac{p_{LC} - p_{LS}}{t_L} + \alpha_H \frac{p_{LC} - p_{LS}}{t_H} \right)
\]

It can be shown that the retailer’s optimal pricing decision is \( p_{LS}^* = \frac{w_L + p_{LC}}{2} \). The manufacturer solves:

\[
\max_{w_L,q_L,x_L,p_{LC}} \pi_M^{(2)} = (w_L - \alpha q_L^2) \left( \frac{p_{LC} - p_{LS}}{t_L} + \alpha_H \frac{p_{LC} - p_{LS}}{t_H} \right) + (p_{LC} - \alpha q_L^2 - c) \left( \alpha_L \left( 1 - \frac{p_{LC} - p_{LS}}{t_L} \right) + \alpha_H \left( 1 - \frac{p_{LC} - p_{LS}}{t_H} \right) \right) - bx_L^2
\]

By substituting \( p_{LC} = \theta_L q_L - (1 - x_L) t_L \), and \( p_{HS}^* = \frac{w_L + p_{LC}}{2} \), the above problem reduces to determining the optimal \( q_L, x_L \), and \( w_L \).

**Strategy 3 (Two standard products)**

Under this strategy, the retailer solves

\[
\max_{p_{LS}} \pi_R^{(3)} = (p_{LS} - w_L) \alpha_L D_{LS} + (p_{HS} - w_H) \alpha_H D_{HS},
\]

Subject to

\[
\theta_L q_L - p_{LS} \geq \theta_L q_H - p_{HS} \quad (8.1)
\]

\[
\theta_H q_H - p_{HS} \geq \theta_H q_L - p_{LS} \quad (8.2)
\]

and the manufacturer solves:

\[
\max_{q_L, q_H, w_L, w_H} \pi_M^{(3)} = (w_L - \alpha q_L^2) \alpha_L D_{LS} + (w_H - \alpha q_H^2) \alpha_H D_{HS}
\]

In the case of full market coverage, \( D_{LS} = D_{HS} = 1 \), and the retail price of the low-quality product is set to \( p_{LS} = \theta_L q_L - t_L \). To prevent cannibalization, the retail price of the high-quality product is set to \( p_{HS} = p_{LS} + \theta_H (q_H - q_L) \). It can be shown that the retailer’s optimal pricing decision when the demands are represented as \( \frac{\theta_L q_L - p_{LS}}{t_L} \) and \( \frac{\theta_H q_H - p_{HS}}{t_H} \) is \( p_{LS}^* = \frac{\theta_L q_L + w_L}{2} \) and \( p_{HS}^* = \frac{\theta_H q_H + w_H}{2} \). Using backward induction, the manufacturer needs to set the wholesale prices equal to \( w_L = 2p_{LS} - \theta_L q_L \) and \( w_H = 2p_{HS} - \theta_H q_H \). These equations on the wholesale prices will ensure that the wholesale prices are not too high such that the retailer is willing to cover the whole market. By substituting \( w_L \),
$w_H, p_{LS}, p_{HS}$ to the manufacturer’s objective function, the manufacturer’s problem reduces to optimizing the quality levels $q_H$ and $q_L$.

Discussion

In Strategy 1, the optimal quality in the decentralized channel is lower than the quality in the centralized channel. It is interesting to see that downward quality distortion still occurs despite the fact that we assume a take-it-or-leave-it offer from the manufacturer. This distortion is a consequence of the potential misalignment of the manufacturer’s and the retailer’s market coverage decisions.

An interesting observation can be made about the results in Strategy 2 as the optimal quality and customization levels are the same as those obtained in the centralized channel, which shows that the quality distortion force is reduced when the manufacturer offers customization in the product line. From the optimal results, we see that the wholesale price for the standard product in the decentralized channel is as high as the retail price in the centralized channel, which explains why the quality level remains the same as in the centralized channel. As a result, the product must be sold in the decentralized channel at a higher retail price than the price in the centralized channel. The reduction of quality distortion in Strategy 3 can be explained by the fact that even though the whole market is covered by the two products, the offering of the customized product allows the retailer to increase the retail price of the standard product and sell it only to a fraction of the market. The optimal results in Strategy 3 show that the quality of the low-end product is lower than the quality level in Strategy 2, and also lower compared to the quality level in the centralized channel under the same strategy. In line with the standard results in product line design literature, the optimal quality of the high-end product is not affected and the high-valuation segment always get the efficient quality level.

Similar to the case of centralized channel, we compare the channel profits of the three strategies and examine the preference for Strategy 2 over Strategy 3, or vice versa. The following proposition summarizes the condition for the preference for Strategy 2 over Strategy 3.

Proposition 2: In the decentralized channel, the horizontal product line extension strategy with customization is preferred to the vertical product line extension strategy if $b < b_{2-3Decentralize} = \frac{t_L^2}{4c^2} \left(\frac{a_H}{a_L} - \frac{a_H^3}{a_L^3} - \frac{a_H a_H^2}{a_L} + \frac{a_H a_H^2}{a_L^2} - \frac{2a_H a_H}{a_L} a_H a_H^2 a_H^2 a_L\right)$. 

In Proposition 2, we show that the horizontal product line extension may be preferable to the vertical product line extension in the decentralized channel like in the centralized channel. It can be shown
that the effects of the other parameters on the preference for Strategy 2 over Strategy 3 in the decentralized channel remain the same as in the centralized channel.

Table 3.5. The effect of parameters the preference for Strategy 2 in the decentralized channel

<table>
<thead>
<tr>
<th>Parameters</th>
<th>( t_H )</th>
<th>( \theta_H - \theta_L )</th>
<th>( a )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Influence on the preference for Strategy 2</td>
<td>Negative</td>
<td>Negative</td>
<td>Positive</td>
</tr>
</tbody>
</table>

Note: the effect of the rest of parameters rely on the value chosen for the other parameters.

We now examine if the channel structure affects the preference for Strategy 2 over Strategy 3. By comparing the threshold values of the flexibility cost \( b \) in the centralized and decentralized channels, it can be concluded that if \( c^2 < \frac{\alpha_L a_H \theta_H^2 + a_H (\theta_H - \theta_L)^2 - a_L a_H (\theta_H - 2 \theta_L)^2}{a \alpha_L (t_L - t_H)} \) then \( b_{2-3 \text{Decentralized}} > b_{2-3 \text{Centralized}} \). When \( b_{2-3 \text{Decentralized}} > b_{2-3 \text{Centralized}} \), there is a more stringent requirement regarding the flexibility cost in the centralized channel than in the decentralized channel. In other words, the manufacturer can be more interested in offering the customized product in the decentralized channel. We provide Figure 3.5 below to illustrate how the threshold values of the flexibility cost \( b \) in the two channel structures are affected by \( c \) and \( \alpha_L \). In this numerical example, the parameter values are: \( b = 0.5, \theta_H = 5, \theta_L = 4, t_H = 1.0, t_L = 0.8, a = 1 \).

For \( c = 0.2 \), \( b_{2-3 \text{Decentralized}} > b_{2-3 \text{Centralized}} \) for \( \alpha_L = 0.4 \) up to \( \alpha_L = 0.8 \), but \( b_{2-3 \text{Decentralized}} < b_{2-3 \text{Centralized}} \) for \( \alpha_L = 0.9 \). For \( c = 0.8 \), \( b_{2-3 \text{Decentralized}} > b_{2-3 \text{Centralized}} \) for \( \alpha_L = 0.4 \) up to \( \alpha_L = 0.6 \), but \( b_{2-3 \text{Decentralized}} < b_{2-3 \text{Centralized}} \) for \( \alpha_L > 0.6 \). Figure 3.5 shows that both threshold values increase in the size of the low valuation segment, but the rate of increase for the centralized channel is higher. The preference for Strategy 2 over Strategy 3 in the decentralized channel seems to be strong when the size of the low-valuation segment is small, and gets weaker as the size of the low-valuation segment increases. Increasing the size of the low-valuation segment reduces the extent of quality distortion in the low-end product but also reduces the profit from the high-valuation segment. Since the quality distortion in the decentralized channel is more severe, the increase in the size of the low-valuation segment will give more compensation to balance the profit loss from the high-valuation segment. The figure also shows the effect of the additional marginal cost of selling the customized product, \( c \). With a higher \( c \) value (\( c = 0.8 \) as opposed to \( c = 0.2 \)), we see a larger region where the manufacturer should be more aggressive in offering customization in the centralized channel than in the decentralized channel. The marginal cost of selling the customized product negatively affects the price of the low quality product, and the effect is more pronounced in the centralized channel than in the decentralized channel. The above discussion clearly shows that
the preference for the two product line extension strategies is influenced by the type of distribution channel in which firms are operating.

![Figure 3.5](image)

Figure 3.5: Impact of $c$ and $\alpha_L$ on threshold values of $b$ in the centralized and decentralized channels

### 3.3.2.2 Comparison Scenario 2 – the role of offering the customized product in the quality-based segmentation strategy

The focus in the second comparison will be on the effect of adding a customized product to the existing product line that consists of two standard products of different quality. We intend to assess the role played by the horizontal product line extension offering the customized product when firms already apply quality-based segmentation. We compare two new strategies that differ in whether the customized product is designed with high or low quality, and we use Strategy 3 as a baseline strategy.

**The centralized channel**

**Strategy 4a (two standard products and one customized product with low quality)**

In this strategy, the manufacturer offers two standard products and also a customized product targeted at the low-valuation segment. In order to make quality-based segmentation meaningful, the manufacturer needs to ensure that the consumers in the high-valuation segment prefer buying the high quality product rather than buying the standard low quality product or buying the customized product targeted at the low-valuation segment. The manufacturer solves the following problem:

$$
\begin{align}
\max_{q_L, p_{LS}, q_{LS}, \alpha_L, p_{LC}, q_{LC}, \alpha_L, p_{HS}, q_{HS}, \alpha_H} & \quad \pi_{SC,c}^{(4a)} = (p_{LS} - aq_L^2)\alpha_L D_{LS} + (p_{LC} - aq_L^2 - c)\alpha_L D_{LC} + \\
& \quad (p_{HS} - aq_H^2)\alpha_H D_{HS} - bx_L^2
\end{align}
$$

Subject to:
\[
\begin{align*}
\theta_L q_L - p_{LS} & \geq \theta_L q_H - p_{HS} & (10.1) \\
\theta_H q_H - p_{HS} & \geq \theta_H q_L - p_{LS} & (10.2) \\
\theta_H q_H - p_{HS} & \geq \theta_H q_L + t_h x_L - p_{LC} & (10.3)
\end{align*}
\]

In (10), the first and second terms represent the manufacturer’s profit from selling the standard and customized products, respectively, to the low-valuation segment. The third term is the profit from selling the standard product with high quality to the high-valuation segment, and the last term denotes the fixed investment cost for offering the customization. There are three self-selection constraints in the above problem formulation. The first two constraints are necessary to prevent cannibalization between the two standard products. The third self-selection constraint is required to prevent consumers in the high-valuation segment from buying the customized product. Constraint (10.3) will give the first upper bound on \( p_H \), \( \bar{p}_H = p_{LC} + \theta_H (q_H - q_L) - t_h x_L \), and constraint (10.2) will give the second upper bound \( \bar{p}_H = p_{LS} + \theta_H (q_H - q_L) \). Since \( \bar{p}_H < \bar{p}_H \), we make constraint (10.3) binding. As in Strategy 2, the price of the customized product is \( p_{LC} = \theta_L q_L - (1 - x_L) t_L \). Following Strategy 2, with full market coverage, we have \( D_{LS} = \frac{p_{LC} - p_{LS}}{t_L} \), \( D_{LC} = 1 - \frac{p_{LC} - p_{LS}}{t_L} \), and \( D_{HS} = 1 \).

Figure 3.6: Consumer’s utility and demand in Strategy 4a

**Strategy 4b (two standard products and one customized product with high quality)**

This strategy is similar to Strategy 4a but the customized product is now offered with high quality and targeted at the high-valuation segment. The customization level and price of the customized product are denoted by \( x_H \) and \( p_{HC} \), respectively. As in Strategy 2, the manufacturer needs to determine the customization level, \( x_H \), such that the utility of the consumer in the high-valuation...
segment located at $y_H = 1$ is equal to zero when buying the customized product. The manufacturer’s profit function can be written as:

$$\max_{q_L, q_{LS}, q_H, y_H, x_H, p_{HC}} \pi_{MC}^{(a,b)} = (p_{LS} - a q_L^2)\alpha_L D_{LS} + (p_{HS} - a q_H^2)\alpha_H D_{HS} + (p_{HC} - a q_H^2 - c)D_{HC} - b x_H^2$$

Subject to:

$$\theta_L q_L - p_{LS} \geq \theta_L q_H - p_{HS} \tag{11.1}$$
$$\theta_H q_H - p_{HS} \geq \theta_H q_L - p_{LS} \tag{11.2}$$
$$\theta_L q_L - p_{LS} \geq \theta_L q_H + t_L x_H - p_{HC} \tag{11.3}$$

In contrast to Strategy 4a and also Strategy 2 where it is possible for the manufacturer to increase the price of the standard (low-quality) product, the manufacturer in Strategy 4b needs to maintain the price of the standard (high-quality) product based on the self-selection constraint (11.2), to prevent the consumers in the high-valuation segment from switching to the low-quality product.

By binding constraint (11.2), we have $p_{HS} = p_{LS} + \theta_H (q_H - q_L)$ where the price of the low quality product is $p_{LS} = \theta_L q_L - t_L$. When the manufacturer sets the price for the customized product equal to $p_{HC} = p_{HS} + t_H x_H$, the consumers in the high-valuation segment located at $y_H \in [0, x_H]$ will buy the standard product, whereas the consumers located at $y_H \in [x_H, 1]$ will be indifferent between buying the standard product and the customized one, i.e., $D_{HS} = x_H$ and $D_{HC} = 1 - x_H$. It can be shown that with the above price for the customized product, constraint (11.3) will always be satisfied, (see Appendix D).

![Figure 3.7: Consumer’s utility and demand in Strategy 4b](image-url)
Discussion

By comparing the results in Strategy 3 and Strategy 4a, we can see that offering the customized product in Strategy 4a does not affect the optimal quality of the existing standard products. We also note that the optimal customization level in Strategy 4a is lower than the customization level offered in Strategy 2, which shows the existence of distortion in both quality and customization levels. The additional distortion in the customization level occurs as a result of the manufacturer’s action to prevent cannibalization of the high-quality standard product by the customized product. The extent of distortion in customization level is increasing in the difference of the mismatch costs of consumers in both segments. A notable advantage of moving from Strategy 3 to Strategy 4a can be drawn from what we have observed when moving from Strategy 1 to Strategy 2. That is, the manufacturer has the possibility to increase the profit from the low-valuation segment by introducing the customized product with low quality, and this possibility can be justified when, for instance, the fixed flexibility cost is not too high. However, there is also a detrimental effect due to the reduction in the price of the high-quality product to prevent cannibalization by the customized product. Thus, there is no guarantee that the manufacturer is better-off by offering the customized product to the existing quality-based segmentation strategy. In the following proposition, we define the threshold value of the flexibility cost.

**Proposition 3:** In the centralized channel, offering a customized product with low quality to the existing quality-based segmentation strategy will increase the profit if

\[ b < \frac{\alpha_H t_H t_L (\alpha_H t_H - 2t_L) + t_L^2}{\alpha_L c (4t_L - c)} \]

When we compare Strategy 3 and Strategy 4b, we find no difference in the profit obtained from the low-quality product offered to the low-valuation segment. Hence, the difference comes only from the profit generated in the high-valuation segment. Since the prices of the standard products are the same for the two strategies, the difference is then solely due to the profit generated from a fraction of consumers who switch from buying the standard product in Strategy 3 to buying the customized product in Strategy 4b. In contrast to Strategy 4a where the customization level is dependent on the mismatch costs of consumers in both segments, \( t_H \) and \( t_L \), the optimal customization level in Strategy 4b is only influenced by the mismatch cost in the high-valuation segment, \( t_H \). For a fixed \( t_L \), the optimal customization level in Strategy 4a (Strategy 4b) is decreasing (increasing) in \( t_H \). As in Strategy 4a, the preference for Strategy 4b over Strategy 3 cannot be guaranteed, and depends on factors such as the flexibility cost associated with the customized product.
Proposition 4: In the centralized channel, offering a customized product with high quality to the existing quality-based segmentation strategy will increase the profit if $b < \frac{\alpha_H(\theta - t_H)^2}{4c}$.

In Table 3.6, we present the effects of some other parameters qualitatively on the preference for Strategy 4b over Strategy 3. For Strategy 4a, the effect of each parameter on the preference for Strategy 4a over Strategy 3 rely on the value chosen for the other parameters.

Table 3.6: The effects of parameters on preference for Strategy 4b over Strategy 3 in the centralized channel

<table>
<thead>
<tr>
<th>Parameters</th>
<th>$a$</th>
<th>$t_L$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Influence on the preference for Strategy 4b</td>
<td>Without effect</td>
<td>Without effect</td>
</tr>
</tbody>
</table>

Note: the effect of the rest of parameters rely on the value chosen for the other parameters.

In Figure 3.8, we illustrate how the profits in Strategy 4a and Strategy 4b are influenced by the fixed flexibility cost $b$ using a numerical example with the following parameter values: $\alpha_H = 0.2, \alpha_L = 0.8, \theta_H = 5.0, \theta_L = 4.0, t_H = 1.5, t_L = 0.8, c = 0.2, a = 1$.

Figure 3.8: The effect of the flexibility cost $b$ on channel profit for Strategies 3, 4a and 4b in the centralized channel

The profit in Strategy 3 is also included for comparison purposes. In this numerical example, we show how the best strategy shifts from Strategy 4a to Strategy 4b by increasing the flexibility cost, before these two strategies are dominated by the incumbent Strategy 3. A more interesting observation is that the profit in Strategy 4a is decreasing in the flexibility cost with a rate higher than that of Strategy 4b, i.e., Strategy 4a is more sensitive to the flexibility cost than Strategy 4b. This can be explained by the fact that changing the customization level in Strategy 4b will only affect the profit
generated from the high-valuation segment, as a consequence of changing the fraction of consumers in this segment who buy the standard or customized products. Recall that the prices of the standard products also remain the same in this strategy. But changing the customization level in Strategy 4a is more impactful since it affects the profits in both segments and the prices of the three products.

The decentralized channel

**Strategy 4a: Two standard products and one customized product with low quality**

Based on the demands derived in the centralized channel, in this strategy, the retailer solves

\[
\max_{p_{LS}, p_{HS}} \pi^{(4a)}_R = (p_{LS} - w_L)\alpha_L(D_{LS}) + (p_{HS} - w_H)\alpha_H D_{HS}
\]

Subject to:

\[
\begin{align*}
\theta_L q_L - p_{LS} & \geq \theta_L q_H - p_{HS} \\
\theta_H q_H - p_{HS} & \geq \theta_H q_L - p_{LS} \\
\theta_H q_H - p_{HS} & \geq \theta_L q_L + t_H x - p_{LS}
\end{align*}
\]

The retailer’s optimal prices are \( p_{LS}^* = \frac{w_L + p_{LC}}{2} \) and \( p_{HS}^* = \frac{\theta_H q_H + w_H}{2} \) (see Appendix E). As in the centralized channel, we have \( p_{LC} = \theta_L q_L - (1 - x_L) t_L \), and \( p_{HS} = p_{LC} + \theta_H (q_H - q_L) - t_H x_L \), \( D_{LS} = \frac{p_{LC} - p_{LS}}{t_L} \) and \( D_{HS} = 1 \). The manufacturer’s objective function can be written as:

\[
\max_{q_L, q_H, x_L, w_L, w_H} \pi^{(4a)}_M = (w_L - aq_L^2)\alpha_L D_{LS} + (w_H - aq_H^2)\alpha_H D_{HS} + (p_{LC} - aq_L^2 - c)\alpha_L D_{LC} - b x_L^2
\]

By substituting \( w_H \) and \( p_{LC} \), the above problem reduces to determining the optimal \( q_L, q_H, w_L \) and \( x_L \).

**Strategy 4b (two standard products and one customized product with high quality)**

In this strategy, the retailer solves:

\[
\max_{p_{LS}, p_{HS}} \pi^{(4b)}_R = (p_{LS} - w_L)\alpha_L D_{LS} + (p_{HS} - w_H)\alpha_H D_{HS}.
\]

Subject to:
\begin{align*}
\theta_L q_L - p_{LS} &\geq \theta_L q_H - p_{HS} \\
\theta_H q_H - p_{HS} &\geq \theta_H q_L - p_{LS} \\
\theta_L q_L - t_L x_H - p_{LS} &\geq \theta_L q_H - p_{HC} 
\end{align*}

(14.1) \quad (14.2) \quad (14.3)

$$D_{LS} = 1 \text{ and } D_{HS} = \frac{p_{HC} - p_{HS}}{t_H}.$$ The retailer’s optimal responses to the manufacturer’s offer are $p_{LS}^* = \frac{\theta_L q_L + w_L}{2}$ and $p_{HS}^* = \frac{p_{HC} + w_H}{2}$. Like in the centralized channel, $p_{LS} = \theta_L q_L - t_L$, and by binding constraint (14.2), we have $p_{HS} = p_{LS} + \theta_H(q_H - q_L)$. The manufacturer’s problem is:

$$\max_{q_L q_H x_H w_L w_H p_{HC}} \pi_M^{(ab)} = (w_L - a q_L^2)\alpha_L D_{LS} + (w_H - a q_H^2)\alpha_H D_{HS} + (p_{HC} - a q_H^2 - c)\alpha_H D_{HC} - b x_H^2$$

(15)

Like in the centralized channel, the manufacturer sets the price for the customized product equal to $p_{HC} = p_{HS} + t_H x_H$. Knowing the retailer’s optimal responses, the manufacturer sets the wholesale prices equal to $w_L = 2p_{HS} - p_{HC}$ and $w_H = 2p_{LS} - \theta_L q_L$ (see Appendix E). By substitutions, $D_{HS} = x_H$ and $D_{HC} = 1 - x_H$, and $p_{HS}$ and $p_{LS}$ the manufacturer’s problem reduces to determining the optimal $q_L, q_H$, and $x_H$.

**Discussion**

Like in the centralized channel, offering the customized product with low quality in Strategy 4a in the decentralized channel does not affect the optimal quality of the existing standard products in Strategy 3. However, it is interesting to see that offering the customized product with high quality in Strategy 4b in the decentralized channel increases the optimal quality level of the low-quality product, while there is no change in the centralized channel. In fact, the optimal quality level of the low-quality product in Strategy 4b in the decentralized channel is the same as the optimal quality level in the centralized channel. In other words, offering the customized product with high quality can alleviate the quality distortion effect in the decentralized channel.

When the customized product is offered with low quality in the decentralized channel (Strategy 4a), the optimal customization level can be higher or lower compared to the customization level offered in the centralized channel. It can be shown that when $\frac{t_H}{t_L} < 3$, the customization level in the decentralized channel is higher. It would also be interesting to make a comparison with the customization level offered in Strategy 2. The previous analysis has shown that the customization level offered in Strategy 4a in the centralized channel is always lower than the customization level.
offered in Strategy 2. In the decentralized channel, however, under the condition \( \frac{t_H}{t_L} < 1.5 \), it is optimal for the manufacturer to offer a customization level that is higher than the optimal customization level in Strategy 2. In all cases, when it is of the manufacturer’s interest to offer the customized product with lower quality, the customization level offered should be higher when the difference of the mismatch costs of the two segments becomes smaller.

In Strategy 4b, when the customized product is offered with high quality, the optimal customization level offered in the decentralized channel is lower than that of the centralized channel. Different from Strategy 4a, the customization level in Strategy 4b is only influenced by the mismatch cost of the high-valuation segment rather than the mismatch costs of the two segments. This can be explained by the fact that the self-selection constraint preventing the low-valuation segment from buying the customized product is redundant in Strategy 4b.

Like in the centralized channel, offering the customized product to the existing quality-based segmentation strategy provides the opportunity to increase the channel profit. However, there is no guarantee that introducing the customized product in the product line will always increase the channel profit. The preference for Strategy 4a or Strategy 4b over Strategy 3 depends on the flexibility cost and other parameters. In the following propositions, we define the threshold values of the flexibility cost.

**Proposition 5:** In the decentralized channel, offering customized product with low quality to the existing quality-based segmentation strategy will not change the channel profit when

\[
b = \frac{2\alpha_H a_L (t_H - 2t_L) - a_H^2 t_L^2}{4 \left( \frac{1+3\alpha_H}{4a} \theta_L^2 + \frac{a_L^2}{4a} \theta_L^2 \right)}.
\]

**Proposition 6:** In the decentralized channel, offering customized product with high quality to the existing quality-based segmentation strategy will not change the channel profit when

\[
b = \frac{a_H (c + t_H) \sqrt{a_L (a_L (c^2 + t_H^2) - 4\alpha_H t_H (\theta_H - \theta_L)^2)}}{2a_H (4a + a_L^2 - a_H (\theta_H - \theta_L)^2)} - \alpha_H t_H.
\]

In Tables 3.7 and 3.8, we present the effects of some other parameters qualitatively on the preference for Strategy 4a and Strategy 4b over Strategy 3.
Table 3.7: The effects of parameters on preference for Strategy 4a over Strategy 3 in the decentralized channel

<table>
<thead>
<tr>
<th>Parameters</th>
<th>( t_H )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Influence on the preference for Strategy 4a</td>
<td>Negative</td>
</tr>
</tbody>
</table>

Note: the effect of the rest of parameters rely on the value chosen for the other parameters.

Table 3.8: The effects of parameters on preference for Strategy 4b over Strategy 3 in the decentralized channel

<table>
<thead>
<tr>
<th>Parameters</th>
<th>((\theta_H - \theta_L))</th>
<th>( a )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Influence on the preference for Strategy 4b</td>
<td>Positive</td>
<td>Negative</td>
</tr>
</tbody>
</table>

Note: the effect of the rest of parameters rely on the value chosen for the other parameters.

We use the numerical example \((\alpha_H = 0.1, \alpha_L = 0.9, \theta_H = 8, \theta_L = 5.5, t_H = 1, t_L = 0.6, c = 0.3, a = 1)\) to illustrate how the profits for the three strategies change by increasing the flexibility cost \(b\). As we can see in Figure 3.9, similar to the centralized channel case, the total profit channel for strategies 4a and 4b are decreasing in the flexibility cost and Strategy 4a is more sensitive to the flexibility cost than Strategy 4b.

Figure 3.9: The effect of the flexibility cost \(b\) on channel profit for Strategies 3, 4a and 4b in the decentralized channel

3.4 Sensitivity Analysis of Some Assumptions

We have analyzed how the horizontal product line extension with customization could be used to enhance profitability. We used a stylized model that allows us to obtain several insights that extend our understanding of when the horizontal product line extension with customization is preferable to the vertical product line extension, or when offering a customized product to the existing quality-based segmentation strategy generates additional profits. Like all models, our model relies on some
assumptions, and hence, the results and insights generated may be affected in case some of the assumptions do not hold. In this section, we therefore discuss how the results will change when we modify some of the assumptions. We focus on three assumptions in our model which we believe worthy of further discussions. First, our model assumes that consumers in the high-valuation segment are more sensitive to the deviation between their preference and that of the purchased product than the low-valuation segment. We are interested in examining how the results are affected when the opposite scenario, i.e., consumers in the high-valuation segment is less sensitive than the low-valuation segment. Second, our model assumes that quality redesign is not costly such that the manufacturer can always design the optimal quality levels in all product line extension strategies. We are interested in examining how the results are affected when it is costly to redesign the existing quality. Finally, we assume a powerful manufacturer who gives a take-it-or-leave-it offer to the retailer such that the retailer always sells all the products offered by the manufacturer. In some practical settings, this assumption may be questionable since retailers may also have the option to choose only a subset of the manufacturer’s product line. We focus our analysis in this section only on the first comparison scenario.

a. The case with $t_L > t_H$

In this section, we discuss how the models and results may change when assuming that $t_L > t_H$. Figure 3.9 below gives an illustration of how the consumer utility in both segments in the case with $t_L > t_H$ is different from that of the case with $t_H > t_L$ in Strategy 1. It can be shown that the previous models and optimal results for the three strategies in the first comparison scenario are not affected by changing the assumption. In the case of full market coverage, $t_H$ does not play any role in the results for Strategies 1 and 3. The new assumption, however, has an influence on the domain in which full market coverage is optimal. Under the new assumption, the domain becomes larger as we now only need the condition for full market coverage in the low-valuation segment. When it is optimal to cover the whole market in the low-valuation segment, it is also optimal to cover the whole market in the high-valuation segment.

In Strategy 2, although the optimal price, quality, and customization level are not affected, the magnitude of the profit will be affected. For the same value of $t_L$, changing the assumption will increase (decrease) the portion of demand for the standard (customized) product in the high-valuation segment.
The preference for Strategy 2 over Strategy 3 certainly depends on both values of $t_L$ and $t_H$. For the same value of $t_L$, as shown in Tables 3.4 and 3.5, the preference for the horizontal product line extension (Strategy 2) over the vertical product line extension (Strategy 3) will become stronger when consumers in the high-valuation segment has a lower mismatch cost than consumers in the low-valuation segment.

![Figure 3.10: Consumer’s utility in Strategy 1 under the assumptions $t_H > t_L$ and $t_H < t_L$](image)

$b. \ The \ case \ with \ costly \ quality \ redesign$

We consider a new scenario in which the manufacturer is not interested in changing the quality offered in Strategy 1 when considering horizontal product line extension (Strategy 2) or vertical product line extension (Strategy 3). Recall that the optimal quality in Strategy 1 is $q_L^* = \frac{\theta_L}{2a}$ in the centralized channel. The previous results in the centralized channel show that while there is no quality distortion in Strategy 2, quality distortion does occur in Strategy 3 due to the potential cannibalization problem.

The optimal quality for the low-quality product in the case where it is not costly for the manufacturer to redesign quality is $q_L^* = \frac{\theta_L}{2a} - \frac{\alpha_H(\theta_H - \theta_L)}{2a a_L}$. Thus, if the manufacturer needs to maintain the existing quality of the low-quality product in Strategy 1, i.e., $q_L^* = \frac{\theta_L}{2a}$, the resulting profit in Strategy 3 becomes lower than the profit obtained in the case where it is not costly for the manufacturer to redesign quality. The profit in Strategy 2, however, is not affected. Hence, the manufacturer will be more motivated to extend the product line horizontally than to extend it vertically. In other words, the manufacturer should be more motivated in offering a customized product in situations where quality redesign is costly. It can be shown that the threshold value of the flexibility cost in the case where
quality redesign is costly is equal to $b_{2-3}^{* \text{ centralized}} = \frac{t_H^2}{4c-c^2 \left( \frac{a_H}{t_H} + \frac{a_L}{t_L} \right)^2 + \frac{a_H(\theta_H-\theta_L)^2}{a}}$, which is larger than the threshold value in the case where quality redesign is not costly.

We now consider a similar scenario in the decentralized channel and maintain the quality level obtained in Strategy 1 in the two product line extension strategies. Recall that the optimal quality in Strategy 1 is $q_L^* = \frac{2\theta_L - \gamma}{2a}$, where $\gamma = \frac{a_H(\theta_H-\theta_L)}{t_H + \frac{a_H}{t_L}}$. In contrast to the case of the centralized channel, Strategy 2 is now disadvantaged by maintaining the optimal quality in Strategy 1 in the decentralized channel. Recall that when quality is a free decision variable, offering the customized product in Strategy 2 can actually increase the quality of the low-end product. The manufacturer can no longer enjoy the same benefit when she has to maintain the quality obtained in Strategy 1. In Strategy 3, the optimal quality of the low-end product in the case where quality design is not costly is equal to $q_L^* = \frac{\theta_L}{2a} - \frac{a_H(\theta_H-\theta_L)}{a a_L} < \frac{2\theta_L - \gamma}{2a}$. Thus, if the manufacturer needs to maintain the same quality as in Strategy 1, the channel benefits from the reduction of quality distortion, but this will be disadvantageous for the manufacturer who now earns a lower profit. Since we focus on channel profit comparisons, the preference for Strategy 3 over Strategy 2 in the decentralized channel increases in comparison to the case where quality design is not costly. This finding is interesting as we see the opposite effects of the costly quality redesign on the preference for the horizontal or vertical product line extension in the centralized and decentralized channels. It can be shown that the threshold value of the flexibility cost in the case where quality redesign is costly is equal to $b_{2-3}^{* \text{ decentralized}} = \frac{t_H^2}{4c-c^2 \left( \frac{a_H}{t_H} + \frac{a_L}{t_L} \right)^2 + \frac{a_H(\theta_H-\theta_L)^2}{a}}$, which is smaller than the threshold value in the case where quality redesign is not costly.

c. The case with the retailer having more options in a decentralized channel

We consider a different scenario where the retailer now has the option of not carrying the whole product line offered by the manufacturer. This scenario is introduced in Villas-Boas (1998). Note that under this scenario, modifications are only relevant for Strategy 3, i.e., the results for Strategy 1 and Strategy 2 will not be affected. Under Strategy 3, while the manufacturer intends to sell two products, the retailer has three options: (a) to sell two products as intended by the manufacturer; (b) to sell only the low-quality product targeted at both segments; or (c) to sell only the high-quality product targeted at the high-valuation segment. This shows that in the decentralized channel, the manufacturer’s
targeting strategy is not always aligned with the retailer’s targeting strategy. Hence, the manufacturer must ensure that the retailer chooses the first option rather than the other two options. The retailer’s profit when selling two products (option a) can be written as:

\[ \pi_R(a) = (p_{LS} - w_L)(\alpha_L) + (p_{HS} - w_H)(\alpha_H) \]  

(16)

where \( p_{HS} = p_{LS} + \theta_H (q_H - q_L) \).

The retailer’s profit when selling only the low-quality product targeted at both segments (option b):

\[ \pi_R(b) = (p_{LS} - w_L)(\alpha_L + \alpha_H) \]  

(17)

The retailer’s profit when selling only the high-quality product targeted at the high-valuation segment (option c):

\[ \pi_R(c) = (p^*_{HS} - w_H)(\alpha_H) \]  

(18)

where \( p^*_{HS} = \theta_H q_H - t_H \)

The manufacturer then solves the following optimization problem:

\[ \max_{q_L, q_H, w_L, w_H} \quad \pi_M = (w_L - a q_L^2)(\alpha_L) + (w_H - a q_H^2)(\alpha_H) \]  

(19)

Subject to:

\[ \pi_R(a) \geq \pi_R(b) \Leftrightarrow w_H \leq w_L + \theta_H (q_H - q_L) \]  

(19.1)

\[ \pi_R(a) \geq \pi_R(c) \Leftrightarrow w_L \leq \frac{q_L(\theta_L - \alpha_H \theta_H) + \alpha_H t_H - t_L}{\alpha_L} \]  

(19.2)

The two constraints above give the upper bounds on the wholesale prices. Constraint (19.2) states that the wholesale price of the low quality product must not be too high, and constraint (19.1) ensures that the difference of the two wholesale prices cannot be too large. We derive the optimal solution by binding the two constraints. The optimal quality levels are \( q_L^* = \left( \frac{1}{\alpha_L^2} \right) \frac{\theta_L}{2a} - \left( \frac{\alpha_H (1 + a_L)}{\alpha_L^2} \right) \frac{\theta_H}{2a} \) and \( q_H^* = \frac{\theta_H}{2a} \). In Appendix F, we show that the channel profit decreases due to further quality distortion in the low-quality product. This implies that, since the profit for Strategy 2 is not affected, there is now a
higher motivation to consider the horizontal product line extension strategy. From this analysis, we show how the power structure in the channel may play a role in determining the best product line extension strategy.

3.5 Conclusions

In this article, we examine the role that offering the customized product may play in product line design. We consider a market where consumers are heterogeneous with respect to their valuation of quality and preference on the aesthetic component of the product. While the standard quality-based segmentation is used for addressing the consumers’ heterogeneity in their valuation for quality, offering the customized product is used for addressing the consumers’ heterogeneity in their preference of the aesthetic component of the product. The effect of offering the customized product in the product line is evaluated in two comparison scenarios. In the first scenario, we use a single product strategy as a baseline strategy and examine the preference for the horizontal product line extension strategy that offers the customized product of the same quality to the vertical product line extension strategy that offers two products of different quality. In the second scenario, we use the quality-based segmentation strategy as a baseline strategy and examine the effect of adding the customized product to the existing product line. Furthermore, we examine if the channel structure affects the results in those two comparison scenarios. For this purpose, we extend the standard models in the literature on product line design by considering a joint decision on quality and customization levels.

Some of the most important results are summarized as follows. When the single-product strategy is used as the baseline strategy, adding a new product with a higher quality, i.e., applying the vertical product line extension strategy, always increases the profit. The effect of adding the customized product, i.e., applying the horizontal product line extension strategy, depends on several factors. One notable factor is the fixed flexibility cost for accommodating the customization. The horizontal product line extension strategy may increase the profit if this cost is not too high. Interestingly, our study also shows that the horizontal product line extension strategy that offers the customized product can be used as an alternative to the vertical product line extension strategy. Provided that the fixed cost is not too high, offering the customized product with low quality may be preferred to the quality-based segmentation strategy that offers two products of different quality. The main benefit of the horizontal product line extension strategy comes from the increase in profit from the low-valuation segment. Even though this result regarding the possible preference for the horizontal product line extension strategy is observed in both the centralized and decentralized channels, we show that the
channel structure is influential. That is, the preference for the horizontal product line extension strategy is more pronounced in the decentralized channel than in the centralized channel. In the decentralized channel, the vertical product line extension strategy suffers from the channel efficiency loss because of the potential misalignment of the manufacturer’s and the retailer’s coverage strategies. The manufacturer needs to keep the wholesale price sufficiently low such that the retailer is motivated to apply full instead of partial market coverage, which results in downward quality distortion in the low-quality product. This finding suggests that when manufacturing firms rely on retailers to sell their products, the manufacturers have stronger incentives to offer the customized product in their product lines.

The results in the second comparison scenario show that offering the customized product in addition to an existing product line that consists of two products of different quality may increase the channel profit. In line with the results in the first scenario, the fixed cost for accommodating customization plays an important role. When the manufacturer offers the customized product targeted at the low-valuation segment, we notice that the optimal customization level is lower compared to the customization level offered in the single-product strategy in the first comparison scenario. The reason is the need for preventing cannibalization of the high-quality product by the customized one. While the optimal customization level offered to the low-valuation segment is influenced by both the mismatch cost of the two segments, the optimal customization level offered to the high-valuation segment is only influenced by the mismatch cost of the high-valuation segment. Another interesting result is that offering the customized product targeted at the high-valuation segment may alleviate the quality distortion effect, and hence, motivates the manufacturer to increase the quality level of the low-quality product.

The results of our sensitivity analysis related to the first comparison scenario show that the market characteristic regarding the mismatch costs of consumers in both segments do not affect the optimal results. The cost of quality redesign has an influence on the optimal product line design. In the centralized channel, there is a higher motivation to offer the customized product when the quality redesign is costly, thereby increasing the preference for a horizontal product line extension over a vertical product line extension. Interestingly, we see the opposite effect in the decentralized channel, i.e., the preference for the horizontal product line extension over the vertical product line extension is diminished. Furthermore, we show that the bargaining position of the retailer in the distribution channel is also influential. When the retailer has the option of selling one of the two products offered in the product line, the horizontal product line extension strategy could be considered as an appealing
option for overcoming the potential misalignment of the manufacturer’s and the retailer’s targeting strategies.

The present study has a number of limitations and we consequently suggest several topics for future research. First, our study only considers the full market coverage, and hence, further research is needed in order to see how the results presented in this paper can be generalized to the case with partial market coverage. Additional results for the case with partial market coverage will give more complete information on the benefits of offering the customized product in the product line. Considering the case with partial market coverage, however, would require an extensive numerical study since the analytical results are hard to obtain in most of the product line strategies studied in this paper. Second, like most of the existing studies in the product line design literature, our model setup does not capture the possible presence of demand uncertainty. In many realistic settings, demand uncertainty cannot be neglected and may have an influence on the optimal product line decisions. In the presence of demand uncertainty, the manufacturer must also determine an optimal production or inventory level in addition to the product quality and customization levels, and there might be interdependence between them. In the case of a decentralized channel, there is an even a more interesting and complex problem because for instance the retailer’s order quantity decision may influence the pricing decision, and vice versa. We further foresee a coordination issue between the manufacturer and the retailer that must be investigated in order to maximize the channel profit. Finally, as the main insights in this paper are generated from a stylized model, it would be useful to conduct an empirical study that may reveal, for example, the heterogeneity of consumers’ valuation for quality and their preference of the aesthetic component of the product. Such an empirical study will be very valuable when considering the development of decision tools for product line managers.
Appendix A

We show how to set the condition in Strategy 1 under which covering the whole market in the two segments is optimal:

In the baseline strategy, we have:

$$\max_{p_{LS}, q_L} \pi_{SC,c}^{(1)} = (p_{LS} - aq_L^2)(\alpha_l \frac{\theta_L q_L - p_{LS}}{t_L} + \alpha_l \frac{\theta_H q_L - p_{LS}}{t_H})$$ (A1)

The first and second partial derivatives of the manufacturer’s profit function are as follows:

$$\frac{\partial \pi_{SC,c}^{(1)}}{\partial p_{LS}} = -(p_{LS} - aq_L^2)\left(\frac{\alpha_H}{t_H} + \frac{\alpha_l}{t_L}\right) - \frac{\alpha_l (p_{LS} - q_l \theta_L)}{t_L}$$ (A2)

$$\frac{\partial^2 \pi_{SC,c}^{(1)}}{\partial p_{LS}^2} = -2\frac{\alpha_H}{t_H} - 2\frac{\alpha_l}{t_L} < 0$$ (A3)

$$\frac{\partial \pi_{SC,c}^{(1)}}{\partial q_L} = (p_{LS} - aq_L^2)\left(\frac{\alpha_H \theta_L q_L}{t_H} + \frac{\alpha_l \theta_L}{t_L}\right) + 2aq_L\left(\frac{\alpha_H (p_{LS} - q_l \theta_L)}{t_H} + \frac{\alpha_l (p_{LS} - q_l \theta_L)}{t_L}\right)$$ (A4)

$$\frac{\partial^2 \pi_{SC,c}^{(1)}}{\partial q_L^2} = -2aq_L\left(\frac{\alpha_H (q_L \theta_L - p_{LS})}{t_H} + \frac{\alpha_l (q_L \theta_L - p_{LS})}{t_L}\right) - 4aq_L\left(\frac{\alpha_H \theta_L}{t_H} + \frac{\alpha_l \theta_L}{t_L}\right)$$ (A6)

Since \(\frac{\alpha_H (q_L \theta_L - p_{LS})}{t_H} + \frac{\alpha_l (q_L \theta_L - p_{LS})}{t_L}\) represents the total demand for the standard product in the market, we can conclude that \(\frac{\partial^2 \pi_{SC,c}^{(1)}}{\partial q_L^2} < 0\).

From the first-order conditions, there are two stationary points:

\[(p_{LS_1}, q_{L_1}) = \left(\frac{3(\alpha_l t_H \theta_L + \alpha_l t_L \theta_H)^2}{8a(\alpha_l t_H + \alpha_l t_L)^2}, \frac{\alpha_l t_H \theta_L + \alpha_l t_L \theta_H}{2a(\alpha_l t_H + \alpha_l t_L)}\right)\] (A7)

\[(p_{LS_2}, q_{L_2}) = \left(\frac{\alpha_l t_H \theta_L + \alpha_l t_L \theta_H}{a(\alpha_l t_H + \alpha_l t_L)^2}, \frac{\alpha_l t_H \theta_L + \alpha_l t_L \theta_H}{a(\alpha_l t_H + \alpha_l t_L)}\right)\] (A8)

\(\pi_{SC,c}^{(1)}(p_{LS_1}, q_{L_1}) > 0\), where \(\sigma_1 = \frac{3(\alpha_l t_H \theta_L + \alpha_l t_L \theta_H)^2}{8a(\alpha_l t_H + \alpha_l t_L)^2}\).
\[ \pi_{SC,c}^{(1)}(p_{LS}, q_{L}) = 0. \] The determinant of Hessian matrix at the first stationary point \((p_{LS_1}, q_{L_1})\) is
\[ \frac{(3a-2)(a_Ht_H + a_\theta H + a_\theta H)}{2at_Ht_L^2} > 0, \] where \(a > \frac{2}{3}.\) Consequently, the stationary point \((p_{LS_1}, q_{L_1})\) is a global maximum for the optimization problem (A1). If the parameter values are set such that
\[ D_{IS}(p_{LS}, q_{L_1}) = 1 \text{ where } i \in \{L, H\}, \text{ then } D_{IS} = 1, i \in \{L, H\}. \]

**Appendix B**

In this part, we provide the derivatives for determining the optimal solutions in all the strategies, for both the centralized and decentralized channels.

**Strategy 1- Centralized channel:**

\[ \frac{\partial \pi_{SC,c}^{(1)}}{\partial q_L^2} = -2a < 0 \] implies that \(\pi_{SC,c}^{(1)}\) is concave in \(q_L.\) Therefore, solving \(\frac{\partial \pi_{SC,c}^{(1)}}{\partial q_L} = \theta_L - 2aq_L = 0\) gives us the optimal quality level of the standard product.

**Strategy 2- Centralized channel:**

Before providing derivatives for Strategy 2, we show how demand for the standard product will be affected by introducing a customized product. Suppose the manufacturer offers both a standard product and a customized one in the market. The manufacturer’s profit function is as follows:

\[ \pi_{SC}^{(2)} = (p_s - aq^2)\alpha D_s + (p_c - aq^2 - c)\alpha D_c - bx^2 \quad \text{(B1)} \]

where \(D_s = \frac{p_c - p_s}{t}\) and \(D_c = 1 - \frac{p_c - p_s}{t}.\)

Using transformation \(p_s = p_c - D_s t,\) the resulting manufacturer’s profit is written as:

\[ \pi_{SC}^{(2)*} = (p_c - D_s t - aq^2)\alpha D_s + (p_c - aq^2 - c)\alpha(1 - D_s) - bx^2 \quad \text{(B2)} \]

Since \(\frac{\partial^2 \pi_{SC}^{(2)*}}{\partial D_s^2} = -2at < 0,\) \(\pi_{SC}^{(2)*}\) is concave in \(D_s.\) From the first-order condition, the stationary point of \(\pi_{SC}^{(2)*}\) is \(D_s^* = -\frac{p_c - aq^2 - c}{t} < 0.\) Since \(D_s \) should be positive, the feasible solution for \(D_s \) is
represented on interval $\left(0, \frac{2c}{t}\right]$. $\pi^{(2)^*}_{SC}$ is a decreasing function on this interval. Therefore, by decreasing $D_s$, $\pi^{(2)^*}_{SC}$ will increase. Under the full market coverage, $p_c$ will be set equal to $\theta q - (1 - x)t$, and an increase in $p_s$ leads to a decrease in $D_s$ which consequently increases $\pi^{(2)^*}_{SC}$. Thus, the manufacturer is better off by decreasing the demand for standard product. Figure B-1 shows the demand for the standard product with and without accommodating a customized product.

![Figure B-1: Changing demand by introducing a customized product](image)

Following Wu et al. (2012), since $\frac{\partial \pi^{(2)}_{SC,c}}{\partial q^2_L} = -\frac{2\theta^2_L(\alpha_L t^H + \alpha_H t^L) + 2a t^H t^L}{t^H t^L} < 0$, $\pi^{(2)}_{SC,c}$ is a concave function w.r.t. $q_L$ for given $p_{LS}$ and $x_L$. Therefore, by first solving for the optimal $q_L$ and substituting into the original profit function, the problem can be reduced to solving for two variables, $p_{LS}$ and $x_L$. The resulting profit function $\pi^{(2,1)}_{SC,c}$ is jointly concave w.r.t. $p_{LS}$ and $x_L$. The sufficient conditions of the resulting profit function are as follows: $\frac{\partial^2 \pi^{(2,1)}_{SC,c}}{\partial x^2_L} < 0$ , $\frac{\partial^2 \pi^{(2,1)}_{SC,c}}{\partial x^2_L} = \frac{a^2}{\theta^4_L} > 0$ . Thus, from the first-order conditions of $\pi^{(2,1)}_{SC,c}$ the optimal results can be found.

**Strategy 3- Centralized channel:**

The conditions $\frac{\partial^2 \pi^{(3)}_{SC,c}}{\partial q^2_L} = -2a \alpha_L < 0$, $\frac{\partial^2 \pi^{(3)}_{SC,c}}{\partial q^2_H} = -2a \alpha_H < 0$, and $\left(\frac{\partial^2 \pi^{(3)}_{SC,c}}{\partial q^2_L}\right)\left(\frac{\partial^2 \pi^{(3)}_{SC,c}}{\partial q^2_H}\right) - \left(\frac{\partial^3 \pi^{(3)}_{SC,c}}{\partial q_L \partial q_H}\right)^2 = 4a^2 \alpha_L \alpha_H > 0$ denote that $\pi^{(3)}_{SC,c}$ is jointly concave in $q_L$ and $q_H$. Thus, the stationary point obtained from the first-order conditions $\frac{\partial \pi^{(3)}_{SC,c}}{\partial q_L} = \alpha_L (\theta_L - 2a q_L) - \alpha_H (\theta_H - \theta_L) = 0$ and $\frac{\partial \pi^{(3)}_{SC,c}}{\partial q_H} = \alpha_H (\theta_H - 2a q_H) = 0$ give us the optimal result.
Strategy 4a Centralized channel:

In this strategy, \( p_{LS} \), \( q_L \), \( q_H \), and \( x_L \) are decision variables. The second partial derivatives of the profit function w.r.t. the decision variables are as follows: 

\[
\frac{\partial^2 \pi_{SC,c}^{(4a)}}{\partial p_{LS}^2} = -\frac{2aL}{t_L} < 0, \quad \frac{\partial^2 \pi_{SC,c}^{(4a)}}{\partial q_L^2} = -\frac{2aL(\theta_L^2 + a_{tL})}{t_L} < 0, \quad \frac{\partial^2 \pi_{SC,c}^{(4a)}}{\partial q_H^2} = -2b - 2a_L t_L < 0. 
\]

Also, the determinant of the Hessian matrix is 

\[
\frac{16a^2b_\alpha a_H^2}{t_L} > 0. 
\]

Therefore, the stationary point that comes from the first-order conditions is the optimal solution.

Strategy 4b Centralized channel:

The manufacturer’s profit function \( \pi_{SC,c}^{(4b)} \) is not concave in \( q_L \), \( q_H \), and \( x_L \). However, 

\[
\frac{\partial^2 \pi_{SC,c}^{(4b)}}{\partial q_H^2} = -2a_\alpha < 0 
\]

implies that, for given values of \( q_L \) and \( x_L \), the manufacturer’s profit function is concave in \( q_H \). Thus, we first solve for \( q_H \) for given \( q_L \) and \( x_L \) then substitute the optimal \( q_H \) into the profit function. The resulting expected profit function \( \pi_{M,c}^{(4b,1)} \) is jointly concave in \( q_L \) and \( x_L \), because we have 

\[
\frac{\partial^2 \pi_{SC,c}^{(4b,1)}}{\partial L^2} = -2a_L < 0, \quad \frac{\partial^2 \pi_{SC,c}^{(4b,1)}}{\partial x_L^2} = -2b - 2a_L t_H < 0, \quad \text{and} \quad 
\]

\[
\left( \frac{\partial^2 \pi_{SC,c}^{(4b,1)}}{\partial q_L \partial x_L} \right)^2 = 4a_\alpha (b + a_H t_H) > 0. 
\]

From solving the first-order conditions of \( \pi_{M,c}^{(4b,1)} \), the optimal solutions for \( q_L \) and \( x_L \) will be derived.

Strategy 1- Decentralized channel:

Since 

\[
\frac{\partial \pi_{M,d}^{(1)}}{\partial q_L^2} = -2a < 0, 
\]

solving 

\[
\frac{\partial \pi_{M,d}^{(1)}}{\partial q_L} = 2\theta_L - \lambda = 2a q_L = 0 
\]

for \( q_L \) results in the optimal quality level.

Strategy 2- Decentralized channel:

Using a backward induction approach, we first maximize the retailer’s profit function. Since 

\[
\frac{\partial \pi_{R}^{(2)}}{\partial p_{LS}^2} = -2\frac{\pi_H}{t_H} - 2\frac{a_L}{t_L} < 0, 
\]

the first-order condition gives us the retailer’s optimal pricing decision: 

\[
p_{LS} = \frac{w_L}{2} + \frac{\theta_L q_L}{2} + \frac{t_L (x_L - 1)}{2} 
\]

for given values of \( w_L \), \( q_L \), and \( x_L \). By knowing the retailer’s best response on retail price for the standard product, the manufacturer’s profit function can be written as a function of \( w_L \), \( q_L \), and \( x_L \). The determinant of the Hessian matrix of \( \pi_{M,d}^{(2)}(w_L, q_L, x_L) \) is not negative definite.
Therefore, we first solve for $q_L$ because $\frac{\partial \pi_{M,d}^{(2)}}{\partial q_L^2} = -\frac{(a_L t_H + a_H t_H) t_H^2}{2 t_H} < 0$, i.e., the manufacturer’s profit function is concave in $q_L$. Next, we substitute the optimal $q_L$ into the original manufacturer’s profit function. The sufficient conditions of the resulting profit function $\pi_{M,d}^{(2,1)}$ indicate that it is jointly concave in $w_L$ and $q_L$.

**Strategy 3- Decentralized channel:**

Solving $\frac{\partial \pi_{M,d}^{(3)}}{\partial q_L} = a_L (\theta_L - 2a q_L) - 2a_H (\theta_H - \theta_L) = 0$ and $\frac{\partial \pi_{M,d}^{(3)}}{\partial q_H} = a_H (\theta_H - 2a q_H) = 0$ for $q_L$ and $q_H$ respectively, gives us the optimal solutions because the sufficient conditions are $\frac{\partial^2 \pi_{M,d}^{(3)}}{\partial q_L^2} = -2a\alpha_L < 0$, and $\frac{\partial^2 \pi_{M,d}^{(3)}}{\partial q_H^2} = -2a\alpha_H < 0$; and $\left(\frac{\partial^2 \pi_{M,d}^{(3)}}{\partial q_L \partial q_H}\right)^2 = 4a^2\alpha_L\alpha_H > 0$, which implies that the profit function is jointly concave in $q_L$ and $q_H$.

**Strategy 4a- Decentralized channel:**

$w_L, q_L, q_H, x_L$ are decision variables. The determinant of the Hessian matrix is $\frac{16a^2 b \alpha_H a_L^2}{t_L} > 0$. Also, we have $\frac{\partial^2 \pi_{M,d}^{(4a)}}{\partial w_L^2} = -\frac{2a_L}{t_L} < 0$, $\frac{\partial^2 \pi_{M,d}^{(4a)}}{\partial q_L^2} = -\frac{2a_L (q_L^2 + a_L t_L)}{t_L} < 0$, $\frac{\partial^2 \pi_{M,d}^{(4a)}}{\partial q_H^2} = -2a\alpha_H < 0$, $\frac{\partial^2 \pi_{M,d}^{(4a)}}{\partial x_L^2} = -2b - 2a\alpha_L t_L < 0$. Therefore, the first-order conditions give us the optimal solutions.

**Strategy 4b- Decentralized channel:**

The determinant of the Hessian matrix of $\pi_{M,d}^{(4b)}(q_L, q_H, x_L)$ is not negative definite. However, $\pi_{M,d}^{(4b)}$ is concave in $q_H$ as $\frac{\partial^2 \pi_{M,d}^{(4b)}}{\partial q_H^2} = -2a\alpha_H < 0$. First we find the optimal solution for $q_H$ and substitute the result back into $\pi_{M,d}^{(4b)}$ to find $\pi_{M,d}^{(4b,1)}(q_L, x_H)$. The derivatives $\frac{\partial^2 \pi_{M,d}^{(4b,1)}}{\partial x_H^2} = -2b - 4a_H t_H < 0$, $\frac{\partial^2 \pi_{M,d}^{(4b,1)}}{\partial q_L^2} = -2a\alpha_L < 0$ and $\left(\frac{\partial^2 \pi_{M,d}^{(4b,1)}}{\partial q_L^2}\right) \left(\frac{\partial^2 \pi_{M,d}^{(4b,1)}}{\partial x_H^2}\right) - \left(\frac{\partial^2 \pi_{M,d}^{(4b,1)}}{\partial q_L \partial x_H}\right)^2 = 4a\alpha_L (b + 2a_H t_H)$ show that the resulting profit function is jointly concave in $q_L$ and $x_H$. 
Appendix C

In this section, we prove Propositions 1 to 4. The solution approaches applied to prove these propositions are the same. First, we create a comparison function that represents the difference between the profits of the two strategies compared. Second, we show that the comparison function is strictly increasing w.r.t. $b$. Therefore, for all $b$ that is less than the threshold value, the manufacturer is better-off moving from the first strategy to the second one. The threshold value of $b$ is the single root of the comparison function.

For Propositions 5 and 6, the single root of the comparison function only indicates that there is no difference between the two strategies.

Table C-1: proofs for Propositions 1 to 4

<table>
<thead>
<tr>
<th>Proposition</th>
<th>Comparison Function</th>
<th>Threshold Value</th>
<th>Increasing Function</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$\Lambda_{c1}^3 - \Lambda_{c1}^2 = \pi_{SC,c}^{(3)} - \pi_{SC,c}^{(2)} = \frac{t_L^2}{4b} + \rho_1$, where $\rho_1 = \frac{a_H(\theta_H - \theta_L) + c}{4a_L} \frac{c}{4} + \frac{a_H}{t_H} + \frac{a_L}{t_L}$</td>
<td>$\frac{\partial \Lambda_{c1}(b)}{\partial b} = \frac{t_L^2}{4b^2} &gt; 0$</td>
<td>Increasing function</td>
</tr>
<tr>
<td>2</td>
<td>$\Lambda_{d1}^3 - \Lambda_{d1}^2 = \pi_{SC,d}^{(3)} - \pi_{SC,d}^{(2)} = \frac{t_L^2}{4b} + \rho_2$, where $\rho_2 = \frac{a_H(\theta_H - \theta_L) + c}{4a} \frac{c}{4} + \frac{a_H}{t_H} + \frac{a_L}{t_L}$</td>
<td>$\frac{\partial \Lambda_{d1}(b)}{\partial b} = \frac{t_L^2}{4b^2} &gt; 0$</td>
<td>Increasing function</td>
</tr>
<tr>
<td>3</td>
<td>$\Lambda_{c2}^{-3} - \Lambda_{c2}^{-4} = \pi_{M,c}^{(3)} - \pi_{M,c}^{(4)} = \frac{a_H t_H - t_L}{4b}$</td>
<td>$\frac{\partial \Lambda_{c2}^{-3}(b)}{\partial b} = \frac{a_H t_H - t_L}{4b^2} &gt; 0$</td>
<td>Increasing function</td>
</tr>
<tr>
<td>4</td>
<td>$\Lambda_{c2}^{-4b} = \pi_{M,c}^{(4b)}$</td>
<td>$\frac{\partial \Lambda_{c2}^{-4b}(b)}{\partial b} = \frac{2a_H^2 t_H + a_L^2 (c^2 + t_L^2)}{4(a_H t_H + b)^2} &gt; 0$</td>
<td>Increasing function</td>
</tr>
</tbody>
</table>

Appendix D

In this section, we prove that when the manufacturer offers a customized product with the high quality level, there is no cannibalization between the standard product with the low-quality and the customized one. Consumers in the high-value segment are indifferent between buying the standard product and the customized one when $p_{HC} = p_{HS} + t_H x_H$. From this, $p_{HC} - p_{HS} > t_L x_H$.

$\Rightarrow \theta_L q_H - t_L y_L - p_{HS} > \theta_L q_H - t_L(y_L - x_H) - p_{HC} \Rightarrow \theta_L q_L - t_L y_L - p_{LS} > \theta_L q_H - t_L(y_L - x_H)$.
\( x_H - p_{HC} \). Therefore, we can conclude that there is no cannibalization between the standard low-quality product and the customized product targeted at the high-valuation segment.

**Appendix E**

In this part, we show how to set the wholesale prices in strategies 3, 4a, and 4b in the decentralized channel:

**In Strategy 3:**

The retailer’s profit function is as follows:

\[
\pi_R^{(3)} = (p_{LS} - w_L)\alpha_L \left(\frac{\theta_L q_L - p_{LS}}{t_L}\right) + (p_{HS} - w_H)\alpha_H \left(\frac{\theta_H q_H - p_{HS}}{t_H}\right)
\]  

(E1)

Since \( \frac{\partial^2 \pi_R^{(3)}}{\partial p_{LS}^2} \frac{\partial^2 \pi_R^{(3)}}{\partial p_{HS}^2} - \left(\frac{\partial^2 \pi_R^{(3)}}{\partial p_{HS} \partial p_{LS}}\right)^2 = \frac{4\alpha_L\alpha_H}{t_{LL}t_{HH}} > 0 \), the first-order conditions \( \frac{\partial \pi_R^{(3)}}{\partial p_{LS}} = 0 \) and \( \frac{\partial \pi_R^{(3)}}{\partial p_{HS}} = 0 \) give us the retailer’s optimal pricing decisions:

\[
p_{LS}^* = \frac{\theta_L q_L}{2} + \frac{w_L}{2}
\]

(E2)

\[
p_{HS}^* = \frac{\theta_H q_H}{2} + \frac{w_H}{2}
\]

(E3)

Then, the wholesale prices in Strategy 3 are \( w_L = 2p_{LS} - \theta_L q_L \) and \( w_H = 2p_{HS} - \theta_H q_H \).

**In Strategy 4a:**

The retailer’s profit function is as follows:

\[
\pi_R^{(4a)} = (p_{LS} - w_L)\alpha_L \left(\frac{p_{LC} - p_{LS}}{t_L}\right) + (p_{HS} - w_H)\alpha_H \left(\frac{\theta_H q_H - p_{HS}}{t_H}\right)
\]  

(E4)

The first-order and sufficient conditions are determined as:

\[
\frac{\partial \pi_R^{(4a)}}{\partial p_{LS}} = \alpha_L \left(\frac{p_{LC} - 2p_{LS}}{t_L} + \frac{w_L}{t_L}\right)
\]

(E5)

\[
\frac{\partial \pi_R^{(4a)}}{\partial p_{HS}} = \alpha_H \left(\frac{\theta_H q_H - 2p_{HS}}{t_H} + \frac{w_H}{t_H}\right)
\]

(E6)
\[
\frac{\partial^2 \pi_R^{(4a)}}{\partial p_L^2} = -\frac{2\alpha_L}{t_L} < 0, \quad \frac{\partial^2 \pi_R^{(4a)}}{\partial p_H^2} = -\frac{2\alpha_H}{t_H} < 0, \quad \text{and} \quad \frac{\partial^2 \pi_R^{(4a)}}{\partial p_L^2} \frac{\partial^2 \pi_R^{(4a)}}{\partial p_H^2} - \left( \frac{\partial^2 \pi_R^{(4a)}}{\partial p_H \partial p_L} \right)^2 = \frac{4\alpha_L \alpha_H}{t_L t_H} > 0 \quad (E7)
\]

Consequently, the retailer’s optimal pricing decisions are as follows:

\[
p_L^* = \frac{p_{LC}}{2} + \frac{w_L}{2} \quad (E8)
\]

\[
p_H^* = \frac{\theta_H q_H}{2} + \frac{w_H}{2} \quad (E9)
\]

Thus, \( w_L = 2p_L - p_{LC} \) and \( w_H = w_L + \theta_H (q_H - 2q_L) + \theta_L q_L + (3t_L - 2t_H)x_L - t_L \).

In strategy 4b:

The retailer’s profit function is as follows:

\[
\pi_R^{(4b)} = (p_L - w_L)\alpha_L (\frac{\theta_L q_L - p_L}{t_L}) + (p_H - w_H)\alpha_H (\frac{p_{HC} - p_H}{t_H}) \quad (E10)
\]

From the first-order and sufficient conditions, we have:

\[
p_L = \frac{\theta_L q_L}{2} + \frac{w_L}{2} \quad (E11)
\]

\[
p_H = \frac{p_{HC}}{2} + \frac{w_H}{2} \quad (E12)
\]

Since \( p_L = \theta_L q_L - t_L \), therefore, \( w_L = \theta_L q_L - 2t_L \) and \( w_H = \theta_L q_L - t_L + \theta_H (q_H - q_L) - x_H t_H \).

Appendix F

In this part, we compare the channel profits in the case where the manufacture has no option in purchasing the standard products (Case 1) with the case when the retailer has more options in a decentralized channel (Case 2). The optimal quality level for the standard product targeted at the low-valuation segment is distorted downward because:

\[
q_{L-\text{cas}2} < q_{L-\text{cas}1} \quad \iff \quad \left( \frac{1}{\alpha_L^2} \right) \frac{\theta_L}{2a} - \left( \frac{\alpha_H (\theta_H - \theta_L)}{2a} \right) \frac{\theta_H}{2a} < \left( 1 - \alpha_L^2 - \frac{2\alpha_L \alpha_H}{a} \right) \frac{\theta_L}{2a} < \alpha_H^2 \frac{\theta_H}{2a} \iff \alpha_L^2 (\theta_H - \theta_L) > 0 .
\]

Next, we show that the optimal channel profit in case 1 is bigger than that of Case 2.
\[ \pi_{SC,d}^{(3\text{-case 1})} - \pi_{SC,d}^{(3\text{-case 2})} = (p_{L\text{-case 1}} - aq_{L\text{-case 1}}^2)\alpha_L + (p_{HS\text{-case 1}} - aq_H^2)\alpha_H - \]
\[ (p_{L\text{-case 2}} - aq_{L\text{-case 2}}^2)\alpha_L - (p_{HS\text{-cas 2}} - aq_H^2)\alpha_H = (\theta_L q_{L\text{-case 1}} - aq_{L\text{-cas 1}}^2)\alpha_L + \]
\[ (\theta_L q_{L\text{-cas 1}} - \theta_H q_{L\text{-cas 1}})\alpha_H - (\theta_L q_{L\text{-case 2}} - aq_{L\text{-cas 2}}^2)\alpha_L - (\theta_L q_{L\text{-case 2}} - \theta_H q_{L\text{-case 2}})\alpha_H \]
\[ \quad \text{(F1)} \]

Since \( q_{L\text{-case 2}}^1 < q_{L\text{-case 1}}^1 \), by substituting \( q_{L\text{-cas 2}}^1 = q_{L\text{-cas 1}}^1 - \varepsilon \), where \( \varepsilon > 0 \), \( \pi_{SC,d}^{(3\text{-cas 1})} - \pi_{SC,d}^{(3\text{-case 2})} \) can be rewritten as
\[ \pi_{SC,d}^{(3\text{-case 1})} - \pi_{SC,d}^{(3\text{-case 2})} = \alpha_L \theta_L \varepsilon + \alpha_L a \varepsilon^2 - 2a \alpha_L q_{L\text{-cas 1}}^1 \varepsilon - \alpha_H (\theta_H - \theta_L) \varepsilon \]
\[ \quad \text{(F2)} \]

By substituting the optimal value of \( q_{L\text{-cas 1}}^1 \) into equation (F2), we have:
\[ \pi_{SC,d}^{(3\text{-case 1})^*} - \pi_{SC,d}^{(3\text{-case 2})^*} = a\alpha_L \varepsilon^2 + \alpha_H (\theta_H - \theta_L) \varepsilon > 0 \Rightarrow \pi_{SC,d}^{(3\text{-case 1})^*} > \pi_{SC,d}^{(3\text{-case 2})^*} \]
\[ \quad \text{(F3)} \]
Chapter 4

Paper 3: Product line design with customization: the effects of demand uncertainty and distribution channel structure

History: This paper was initiated during my stay abroad at the University of Nottingham in August–September, 2017. This paper is expected to be submitted in May 2018.
Product line design with customization in a horizontally differentiated market: the effects of demand uncertainty and distribution channel structure

Parisa Bagheri Tookanlou

Department of Economics and Business Economics

School of Business and Social Sciences, Aarhus University

Fuglesangs Allé 4, 8210 Aarhus, Denmark

Abstract

In this paper, I analyze a product line design problem faced by a manufacturing firm where the product line consists of a customized product in addition to a standard product, and is offered in a market in which consumers are heterogeneous on the aesthetic attributes of the product. The product line decisions include the prices of the two products and the customization level for the customized product. The customization level of a product is defined by the fraction of aesthetic attributes of the product that the manufacturer chooses to customize. In contrast to the existing literature on product line design which predominantly assumes deterministic demand, I consider the presence of demand uncertainty and frame the product line design problem in a single period (newsvendor) setting. With the presence of demand uncertainty, the order or production quantity is also a relevant decision variable. I examine the effect of demand uncertainty on product line decisions. Furthermore, I examine how product line decisions are influenced by channel structure. While I use the centralized channel as a benchmark, I consider the decentralized channel where the customized product is sold through an online channel owned by the manufacturer and the standard product is sold through a retailer. I introduce a supply contract between the manufacturer and the retailer with the purpose of improving channel profitability and coordinating the distribution channel. The analytical results and numerical examples provide interesting insights into the importance of taking into account demand uncertainty in designing a product line. This study also demonstrates the potential of using a supply contract in improving channel profitability, and hence, should motivate firms to implement such a contract in the context of product line design.

Keywords: product line design, demand uncertainty, customization level, distribution channel.
4.1 Introduction

In view of the rapid growth of e-commerce and mass customization technologies, manufacturing firms, nowadays, cannot ignore the possibility of modifying their product line by offering customized products alongside the standard or non-customized products. Such a product line extension may require manufacturing firms to consider the adoption of a dual channel in selling their products. They may often need to involve a channel partner, e.g., retailer, especially in selling the standard product. However, selling customized products in a direct or online channel is obviously a viable option since consumers may prefer to buy personalized goods online (Gandhi et al. 2013). Companies such as Nike and Adidas implement the customization strategy where, in addition to selling the standard trainers at their retail outlets, they also allow customers to personalize their trainers at their websites. Timbuk2 offers customized bags online and also sells more standard bags through resellers. Dell and Gateway also sells customized laptops in their online channel alongside the more standard laptops sold at their retail stores. When employing such a product line design in a dual channel, a manufacturer must make multiple decisions, including the customization level of the customized products, the price to be charged to consumers, and the price to be charged to the retailer for the standard product. The retailer must make decisions on the retail price for the standard product, and on the quantity to order from the manufacturer. The latter decision made by the retailer is particularly relevant in the case where demand uncertainty is present and the products are sold in a relatively short selling season such that the retailer will only place one order prior to demand realization.

In this paper, I examine the product line design strategy that offers both the standard and customized products, sold through a dual channel. This paper advances the existing literature primarily by considering the presence of demand uncertainty. While product line design has long been a central topic in the economics, marketing, and operations literature, most of the existing work considers deterministic demand, thereby neglecting the possible supply-demand mismatch caused by the demand uncertainty that is prevalent in many real settings. Hence, little light has been shed on how demand uncertainty influences product line decisions. The deterministic demand assumption is acceptable in situations where the firm operates in a make-to-order (MTO) or an assemble-to-order (ATO) manner, i.e., the firm makes the ordering or production decision after the consumers make their purchasing requests. However, the assumption is questionable in the situations where the number of consumers is uncertain. For example, in the apparel industry, developing accurate forecasts remains a major challenge, especially for any seasonal introduction (Fisher et al., 1994 and Kuksov and Wang 2013). In such situations, production or ordering decision must be made prior to the selling season, i.e., the firm operates in a make-to-stock (MTS) manner. As well illustrated by the classical
newsvendor problem, the firm must pay the costs associated with the supply-demand mismatch caused by the demand uncertainty.

Another basic problem analyzed in this paper is related to the adoption of a distribution channel where the manufacturer must rely on the retailer to sell the standard product. As different parties along a distribution channel may not share the same interest, the distribution channel is often not fully coordinated. In the decentralized channel, the retailer who faces demand uncertainty must make a joint decision on price and order quantity while taking into account the presence of the customized product offered by the manufacturer via the online channel. This situation is different from the case of a centralized channel where the manufacturer is the only channel member involved. In the light of this, this paper also aims to uncover the extent to which the existence of a retailer in a distribution channel and its strategic behavior influence the manufacturer’s product line decisions. As it is expected that the channel’s performance in a decentralized distribution channel is inferior to that of a centralized channel, this article also examines how channel coordination can be used to improve the channel profitability. More specifically, I aim to answer the following questions:

1. How does demand uncertainty affect product line design decisions when the product line consists of a standard product and a customized product, and how are these decisions affected by the type of distribution channel?

2. How can channel profitability be improved to the benefit of the channel members by a supply chain contract such as a buy-back contract?

The rest of this paper is organized as follows. In the next section, a review of the relevant literature is presented. In Section 4.3, I present the models and results for the centralized and decentralized channels. Section 4.4 focuses on the evaluation of a buy-back contract for improving the channel’s profitability. In Section 4.5, I summarize the conclusions and suggest some directions for future research.

4.2 Literature Review

This research contributes to the literature on product line design in a market where consumers are horizontally differentiated. A common feature of the existing literature is that consumers are assumed to be uniformly distributed over Hotelling’s line, and one of the main central questions is how many products to be offered in the product line. De Groote (1994) studies the role of flexibility in accommodating product line extension in the horizontal direction. Several authors analyze the attractiveness of mass customization strategy relative to the standard mass production strategy (see
e.g. Gaur and Honhon 2006, Jiang et al. 2006, Alptekinoğlu and Corbett 2010, and Wong and Lesmono 2011). All these papers consider product line design from the manufacturer’s point of view, so that competition and channel structure are not relevant. There are a number of papers that examine the competition between the standard and customized products in a duopoly market. Dewan et al. (2003) evaluate product customization and price discrimination in monopoly and duopoly markets. They show that a manufacturer may obtain the highest profits by offering both standard and fully customized products and that the manufacturer can increase prices for the standard and customized products when customization and information technologies enhance. In their model, however, the customization level is not a decision variable. Studies that consider customization level are presented in e.g. Syam and Kumar (2006), Alptekinoğlu and Corbett (2008), Mendelson and Parlaktürk (2008a and 2008b), and Xia and Rajagopalan (2009). Although these papers consider customization level, they do not investigate the effect of the distribution channel structure. A study on the product line design problem in a distribution channel where consumers are only horizontally differentiated is presented in Liu and Cui (2010). They show that a decentralized channel can be an efficient structure of distribution by providing a product line length that is optimal from a social welfare perspective. Their analysis is, however, limited to the case of two different products and does not pay attention to the choice of customization level, as is done in this paper. Furthermore, while they consider a standard distribution channel that consists of a manufacturer and a retailer, I consider a dual channel where the standard product is sold through a retailer and the customized products are sold in an online channel owned by the manufacturer.

The product line design problem in a dual channel that involves customization is studied by Li et al. (2015). They consider competition between the manufacturer’s online customization channel and the conventional retailer, which is also adopted in this paper. In their paper, however, demand is assumed to be deterministic. We extend their model by considering demand uncertainty and its effects on the product line design decisions.

Most studies in the literature on product line design assume deterministic demand. Only few papers consider product line design problems in the case with demand uncertainty, and they all consider a market where consumers are vertically differentiated. Xiao and Xu (2014) focus on examining how the risk aversion in the supply chain affects the product line decisions when information asymmetry is not present. Rong et al. (2015) consider demand uncertainty in a product line design problem with information asymmetry. They illustrate the interplay between two effects: risk pooling that mitigates the supply-demand mismatch due to demand uncertainty and market segmentation that facilitates consumer differentiation. While they consider product line design problems in a vertically
differentiated market, this paper studies product line design in a horizontally differentiated market where customized products are offered in addition to the standard product.

This paper is also related to the vast literature on dual channel supply chains (e.g. Chiang et al. 2003, Tsay and Agrawal 2004, Chiang and Monahan 2005, Huang and Swaminathan 2009, and Hua et al., 2010) that all considers a market without demand uncertainty. Few papers study dual channel supply chains with demand uncertainty. Chiang (2010) investigates the impact of consumers’ stock-out based substitution on the product availability and the channel efficiency of a dual-channel supply chain. In his paper, there is only a single product sold through a direct channel and an independent retailer, and the two channels apply a base stock inventory policy. This paper differs in that the products distributed through the two channels are different, and the inventory or order quantity decision for the standard product is modelled as a newsvendor problem. Dumrongsiri et al. (2008) investigate the impact of demand variability on equilibrium prices. They consider a dual channel where the manufacturer uses a traditional channel to sell its product and introduces a direct channel to sell the product in a make-to-order fashion. They show that the dual channel is better than a single channel when demand uncertainty, wholesale price, consumer valuation, and retailer’s marginal cost are low. Although the dual channel setup considered in this paper is similar to that of Dumrongsiri et al. (2008), there are some differences. First, they do not analyze a product line design problem in the sense that none of product design decisions are considered, whereas in this paper, the manufacturer must determine an optimal customization level. Second, this paper considers channel coordination between the manufacturer and the retailer, an issue that is not considered in their paper.

Finally, this paper also contributes to the literature on channel coordination in product line design. Although many studies analyze the effects of distribution channel structure on product line strategies, literature focusing on channel performance improvements by means of channel coordination mechanisms is sparse. Many authors study channel coordination in contexts other than product-line design. Jeuland and Shugan (1983), Moorthy (1987), Ingene and Parry (1995), and Raju and Zhang (2005) use quantity discounts or two-part tariffs as the coordination mechanism. Cachon and Lariviere, (2005) examine revenue-sharing contracts as a mechanism to motivate supply chain members to cooperate on ordering decisions. Some authors (e.g. Pasternack 1985, Song et al. 2008, and Shao et al. 2013) examine the use of buy-back contracts to coordinate the supply chain. In this paper, I also use the buy-back contract as a mechanism to improve the channel profitability. Channel coordination mechanisms in the context of product line design are studied by Hua et al. (2011) and Jerath et al. (2017). They both consider the product line design problem with vertical product differentiation. Under the assumption of deterministic demand and exogenous quality, Hua et al.
(2011) show that a revenue-sharing contract can perfectly coordinate the distribution channel on product design decisions. The work of Jerath et al. (2017) is more related to this paper as it considers demand uncertainty. They demonstrate that quality is decreasing in demand uncertainty, and that a wholesale price contract cannot coordinate the channel. This paper, however, differs because I consider the product line design problem in which consumers are horizontally differentiated and the customization level is a key decision. Furthermore, while they consider a standard distribution channel that involves a manufacturer and a retailer, this paper considers a dual channel in which the retailer is also in competition with the manufacturer. To the best of the author’s knowledge, this paper is the first studying channel coordination in the context of product line design in a horizontally differentiated market where demand uncertainty is present.

4.3 Models

The product line considered in this paper consists of two products, a standard product and a customized product, indexed by \( i = \{s, c\} \). It is assumed that the two types of products have the same quality level, which is determined exogenously. The product line decisions include the prices of the two products and the customization level for the customized product. This study evaluates two channel structures. In a centralized channel, the manufacturer directly sells both the standard and customized products to the end consumers. In a decentralized channel, the standard product is sold through a retailer, and the customized product is sold directly by the manufacturer. Consumers are heterogeneous in their valuation of the aesthetic dimensions of the product, and their preferences are represented on Hotelling’s line. Hotelling’s line is the interval \([0,1]\) such that each point represents customer’s preference (or location). In other words, each consumer is represented by his/her aesthetic preference of the product, which is denoted by \( y \in [0,1] \). The size of the market is normalized to one. Note that since the focus of this paper is on horizontal differentiation, consumers’ heterogeneity in their quality valuation is excluded. In other words, the product line design problem considered in this paper does not include quality-based segmentation.

Each consumer incurs a per unit mismatch cost \( t \) for any deviation between her best preference and that of the purchased product. The manufacturer offers the customized product with customization level \( x \), which is defined as the fraction of aesthetic attributes of the product that she chooses to customize. Following Li et al. (2015), a standard product (with \( x = 0 \)), is represented by the left-end point of the Hotelling’s line, whereas a fully customized product (with \( x = 1 \)), is represented by the right-end point of the Hotelling’s line. The utility of the consumer with preference \( y \) for the standard product sold at price \( p_s \) is \( u_s = v - ty - p_s \), where \( v \) represents the consumer’s quality valuation of
the product. For a customized product offered with customization level \( x \) and price \( p_c \), the utility of the consumer with preference \( y \in [0, x] \) is \( u_c = v - p_c \). In this case, her preference is completely satisfied and no mismatch cost is incurred. However, for the consumer with preference \( y \in (x, 1] \), the utility from buying the customized product is \( u_c = v - t(y - x) - p_c \). In other words, the customization level \( x \) is also equivalent to the proportion of consumers to whom their preference is completely satisfied by the customized product.

The marginal production and selling costs for the standard and customized products are denoted by \( c_s \) and \( c_c \), respectively, and it is assumed that \( c_c > c_s \). This assumption represents realistic settings where there could be additional order processing costs incurred for the customized product. Without loss of generality, the manufacturer’s fixed production cost (excluding the fixed flexibility cost for accommodating customization) is normalized to zero. The consumer located at \( \bar{y} = \frac{p_c - p_s}{t} \) will be indifferent between purchasing the two products (see Figure 4.1). Consumers located in the interval \([0, \bar{y})\) will purchase the standard product as \( u_s > u_c \), whereas those located in the interval \([\bar{y}, 1]\) will purchase the customized product as \( u_s \leq u_c \). This paper will only focus on the case with full market coverage. That is, the whole market will be served by the two products. Since serving the whole market is optimal, the manufacturer charges the price for the customized product such that the utility of the consumer located at \( y = 1 \) is equal to zero. Thus, \( p_c = v - (1 - x)t \), i.e., the price of the customized product is directly related to the customization level.

![Figure 4.1: Consumer’s utility and demand](image)

The deterministic (riskless) demand for the standard product is \( \bar{D}_s = \frac{p_c - p_s}{t} = \frac{v - (1 - x)t - p_s}{t} \), and the demand for the customized product is \( \bar{D}_c = 1 - \bar{D}_s \).

In the setting considered in this paper, the standard product is offered in a make-to-stock (MTS) fashion whereas the customized product is offered in a make-to-order (MTO) or assemble-to-order
(ATO) fashion. The newsvendor problem is applied for the standard product so that the inventory for
the standard product must be determined before the selling period starts. Thus, in this problem, the
relevant decision variables include the prices of the standard and customized products, the
customization level for the customized product, and the inventory for the standard product.

Although demand uncertainty is present for both products, demand uncertainty for the customized
product will be excluded as it is assumed to be offered in a MTO or an ATO fashion. This assumption
is also adopted in Dumrongsiri et al. (2008). The model developed here is probably more suitable for
the ATO setting because the customization offered is mainly related to the aesthetic attributes of the
product while the basic components are the same as in the standard product. Furthermore, since the
consumer utility function in my model does not consider the waiting time consumers must incur, the
validity of the model is less questionable in the ATO setting since the lead time is shorter than the
MTO setting. However, the model could also be a good representation of the MTO setting as long as
the lead time is not so excessive or consumers are not so sensitive to lead time.

Note that in this paper, I focus on the effect of demand uncertainty on the optimal inventory of the
standard product that may have an influence on the product line design decisions, including prices
and customization level. The effect of demand uncertainty on the stocking of materials or
components, especially in the ATO setting, is excluded. This may represent one limitation of the
model, but it can, to a certain extent, be justified in real settings. Consider, for example, manufacturing
firms that offer standard and customized printed t-shirts. The supply of the basic material for the two
products, i.e., the blank t-shirts, is easier to manage because demand uncertainty is lower due to the
pooling effect and its life cycle may last much longer than the printing design.

To capture demand uncertainty for the standard product, randomness is included in addition to the
deterministic demand function explained above. Following Petruzzi and Dada (1999), randomness is
assumed to be price independent, and can be modelled either in an additive or a multiplicative fashion.
The additive fashion is considered in this paper because it makes the model more tractable as demand
variability becomes independent of the price charged. The choice of additive demand is also common
in the literature (e.g. Dumrongsiri et al. 2008). The stochastic demand function for the standard
product can be written as:

\[ D_s = \bar{D}_s + \theta_s, \]  

(1)
where \( \theta_s \in [A, B] \) is a random variable with mean \( \mu_s \) and standard deviation \( \sigma_s \), and is independent of the price charged to the consumers. The probability density function, cumulative distribution function, and inverse cumulative distribution function of \( \theta_s \) are denoted by \( f(\cdot) \), \( F(\cdot) \), and \( F^{-1}(\cdot) \), respectively.

The stochastic demand function for the customized product can be written as:

\[
D_c = \bar{D}_c + \theta_c,
\]  

(2)

where \( \theta_c \) is a random variable with mean \( \mu_c \) and standard deviation \( \sigma_c \). As previously explained, since the customized product is offered in an MTO or ATO fashion, the manufacturer will always satisfy the demand for the customized product without the risk of having the supply-demand mismatch, i.e., \( \sigma_c \) does not play any role. Without loss of generality, it is assumed that both \( \mu_s \) and \( \mu_c \) are equal to zero. This assumption implies that the randomness in demand does not have any influence on the riskless demands for both products, i.e., the riskless demands are only influenced by the product line decisions. Hence, the riskless demand for the customized product is equal to \( \bar{D}_c \).

For the standard product, the number of units stocked at the beginning of the selling period, \( Q \), must be determined prior to the selling period. If demand during the selling period does not exceed \( Q \), then the revenue is \( p_s D_s \), and the per-unit inventory or disposal cost \( h \) is incurred for each of the \( Q - D_s \) leftovers. Alternatively, if demand exceeds \( Q \), then the revenue is \( p_s Q \), and the per-unit penalty cost \( s_c \) is incurred for each of the \( D_s - Q \) shortages. Note that my model assumes that there is no substitution between the two products. This implies that consumers who are willing to purchase the standard product but face stock-out will not be interested in purchasing the customized product. This assumption allows for an examination of the most severe impact of demand uncertainty on the channel profitability.

Following Petruzzi and Dada (1999), the transformation of variables is applied by defining \( z = Q - \bar{D}_s \), as the stocking level to fulfill the stochastic portion of the demand. The order quantity is equal to \( Q = z + \bar{D}_s \). The notations regarding the parameters and decision variables are presented in Table 4.1.
Table 4.1: Notations

**Decision variables**

- $p_i$: Price of product $i$, $(i = s, c)$
- $x$: Customization level offered by the manufacturer
- $w$: Wholesale price in the decentralized channel for the standard product
- $w_B$: Wholesale price for the standard product under the buy-back contract
- $b_B$: Buy-back price

**Marketing parameters**

- $D_i$: Demand for product $i$, $(i = s, c)$
- $v$: Consumer’s quality valuation of the product
- $t$: Unit mismatch cost for consumers
- $\mu_i$: Mean of the uncertain part of demand for product $i$, $(i = s, c)$
- $\sigma_i$: Standard deviation of the uncertain part of demand for product $i$, $(i = s, c)$
- $\theta_i$: Uncertain part of demand for product $i$, $(i = s, c)$
- $y$: Consumer’s location along Hotelling’s line

**Production parameters**

- $a$: Cost coefficient of the production cost
- $b$: Coefficient of the fixed investment cost to accommodate customization
- $c_s$: Marginal production cost for the standard product
- $c_c$: Marginal production cost for the customized product
- $z$: Stocking level
- $Q$: Total order quantity for the standard product
- $s_c$: Per-unit penalty cost for shortages
- $h$: Per-unit holding/disposal cost for unsold products

**Others**

- $\pi_{SC}$: Channel’s profit
- $\pi_R$: Retailer’s profit
- $\pi_M$: Manufacturer’s profit

### 4.3.1 Centralized Channel

I first consider the problem in the centralized channel where the manufacturer sells the two products directly to the end consumers and makes decisions on the customization level $x$, the price for the
standard product $p_s$, the price for the customized one $p_c$, and stocking level $z$ to maximize the expected profit.

The manufacturer’s expected profit can be written as follows:

$$E[\pi_{SC}(z, p_s, x)] = (p_s - c_s)D_s + (p_c - c_c)(1 - D_s) - (c_s + h)O(z) - (p_s + s_c - c_s)S(z) - bx^2$$

where $p_c = v - (1 - x)t$; $O(z) = \int_A^B (z - u) f(u) du$ represents the expected overages at the end of the selling period; and $S(z) = \int_z^B (u - z) f(u) du$ represents the expected shortages at the end of the selling period.

The first term at the right hand side of (3) represents the manufacturer’s expected profit from producing and selling the standard product. The second term represents the expected profit from producing and selling the customized product. The next two terms represent the expected overage and underage costs for the standard product, and the last term is the fixed investment cost corresponding to the customization level offered. Maximizing the manufacturer’s expected profit function over the decision variables characterizes the manufacturer’s optimal customization level, prices, and quantity ordered, that can be summarized in the following proposition.

(Proofs for all propositions in this section are provided in Appendix A).

**Proposition 1:** The manufacturer’s optimal decisions in the centralized channel: $x^*$, $p_s^*$, and $z^*$, satisfy the following three equations simultaneously:

$$p_s^* = p_0 + xt - \frac{t}{2} S(z)$$

$$F(z^*) = \frac{p_s^* + s_c - c_s}{p_s^* + s_c + h}$$

$$x^* = \frac{1}{2(b + t)}(2p_s + x_0), where x_0 = c_c - c_s + 3t - 2v$$

The optimal solutions for a newsvendor problem with price decision are uniquely determined if the condition $2H(z)^2 + \frac{dH(z)}{dz} > 0$ is fulfilled, where $H(z) = \frac{f(z)}{1 - F(z)}$ denotes the hazard rate function. This condition is satisfied by many distributions commonly used in the literature on inventory modelling, such as the Normal and Uniform distributions (Dumrongsiri et al.
2008). Equation (5) also represents what is called the critical ratio, i.e., the ratio of the unit underage cost to the sum of the underage and overage costs. In Appendix A, I show that \( x^*, p_s^*, \) and \( z^* \) are uniquely determined if \( F(\cdot) \) satisfies the condition \( 2H(z)^2 + \frac{dH(z)}{dz} > 0 \). The optimal selling price of the customized product can be directly determined once the optimal customization level is obtained, i.e., \( p_c^* = v - (1 - x^*)t \).

Before analyzing the effect of demand uncertainty on the product line decisions, it is useful to discuss how the decision variables influence each other. Propositions 2(a) to 2(c) summarize the results.

**Proposition 2:** By assuming ceteris paribus:

(a) The optimal price for the standard product \( p_s^* \) is increasing in the stocking level \( z \).

(b) Under the following condition: \( H(z^* > \frac{t}{2(c_s+h)}(1 - F(z^*))^2 \), the optimal stocking level \( z \) is increasing in the customization level \( x \).

(c) The optimal stocking level \( z \) is increasing in the price \( p_s \).

The following discussion will focus on the effect of demand uncertainty on the optimal product line decisions. The main findings on the effect of demand uncertainty are summarized in the following propositions.

**Proposition 3:** In the centralized channel, the optimal customization level for the customized product and the price for the standard product in the case with demand uncertainty are lower than those in the case with deterministic demand.

**Proposition 4:** In the centralized channel, the riskless demand for the standard product in the presence of demand uncertainty is higher than that of the case with deterministic demand, and vice versa for the customized product.

The two propositions above clearly show that the presence of demand uncertainty influences the product line decisions. In the case with deterministic demand, there is no risk of overstocking or understocking so that the optimal decisions on price and customization level are obtained by solving the deterministic part of the manufacturer’s profit function. When demand is uncertain, there is the risk of overstocking and understocking and hence, the optimal decisions obtained in the deterministic setting need to be adjusted to reduce that risk. The manufacturer has the opportunity to reduce the risk through her decision on the price for the standard product. As shown in Proposition 3, the price
for the standard product in the stochastic setting is lower than the price in the deterministic setting. This course of action will increase the riskless demand of the standard product. This result is in line with Petruzzi and Dada (1999) in that the optimal pricing decision is such that the demand coefficient of variation is reduced. In the additive case where a pricing decision cannot change demand variance, a reduction in the price will increase the demand, which leads to the reduction in the coefficient of variation. The price reduction for the standard product can also be seen as the manufacturer’s way of reducing the unit underage cost associated with the risk of understocking. In fact, that is the only possible way for the manufacturer to reduce the unit underage cost caused by demand uncertainty. However, this course of action also has a downside as it reduces the revenue from the customized product. The manufacturer has the opportunity to alleviate this downside by offering a lower customization level, so that it may limit the reduction in the demand for the customized product. As shown in Proposition 4, those two driving forces eventually result in the optimal solution with the higher riskless demand for the standard product compared to the demand in the absence of uncertainty.

4.3.2 Decentralized Channel

In this section, I study the case where the manufacturer sells the standard product to the end consumers through a retailer who faces the uncertain demand. As in the centralized channel, the customized product is sold in the direct channel owned by the manufacturer. The analysis is carried out under the scenario in which the manufacturer first determines the customization level $x$, the selling price for the customized product $p_c$, and the wholesale price for the standard product $w$. The retailer then determines the order quantity $Q$ and the retail price $p_o$. As in the centralized channel, the transformation of variable is applied by defining $z = Q - D_s(p_s, x)$, as the stocking level to fulfill the stochastic portion of the demand. Since the manufacturer designs the customized product and full market coverage is considered, the price of the customized product can be determined using the same expression as in the centralized channel: $p_c = v - (1 - x)t$. Note, however, that as will be shown later, the optimal price of the customized product in the decentralized channel can be different from that in the centralized channel due to the difference in the optimal customization level.

The retailer’s expected profit is as follows:

$$E(\pi_R(p_s, z)) = (p_s - w)\bar{D}_s - (w + h)O(z) - (p_s + s_c - w)S(z)$$ (7)

where $\bar{D}_s = \frac{p_c - p_s}{t}$, $p_c = v - (1 - x)t$, $O(z) = \int_{A}^{z}(z - u)f(u)du$, and $S(z) = \int_{z}^{B}(u - z)f(u)du$. 
The following propositions characterize the retailer’s optimal decisions for the given values of the manufacturer’s decisions on \( w, x, \) and \( p_c \).

**Proposition 5:** The retailer’s optimal decisions \( p_s^* \) and \( z^* \) satisfy the following two equations simultaneously:

\[
p_s^* = p_1 + \frac{t}{2}x + \frac{w}{2} - \frac{t}{2}S(z), \quad \text{where } p_1 = \frac{v-t}{2} \quad (8)
\]

\[
F(z^*) = \frac{p_x + s_c - w}{p_s + s_c + h} \quad \text{or } z^* = F^{-1}\left(\frac{p_x + s_c - w}{p_s + s_c + h}\right) \quad (9)
\]

(Proofs for all propositions in this section are provided in Appendix B).

Similar to the centralized channel, the optimal solutions are uniquely determined if the condition

\[
2H(z)^2 + \frac{dH(z)}{dz} > 0
\]

is fulfilled (see Appendix B).

The following two propositions show how the retailer’s optimal order quantity is influenced by the customization level and how the price for the standard product charged by the retailer in the stochastic setting is different from the price offered in the deterministic setting. The results shown in the two propositions are consistent with those in the centralized channel. Due to the risk of overstocking or understocking, the retailer will have an incentive to increase the riskless demand for the standard product by reducing the selling price in order to lower the demand coefficient of variation.

**Proposition 6:** In the decentralized channel, the retailer’s optimal stocking level \( z^* \) is increasing in the customization level \( x \).

**Proposition 7:** In the decentralized channel, the price for the standard product in the case with demand uncertainty is lower than the price in the case with deterministic demand.

The analysis of the manufacturer’s problem is presented below. The manufacturer’s expected profit function can be written as follows:

\[
E(\pi_M(w, x)) = (w - c_s)(\bar{D}_s + x) + (p_c - c_c)(1 - \bar{D}_s) - bx^2 \quad (10)
\]

In principle, the problem for the manufacturer can be solved using backward induction approach. That is, knowing the retailer’s optimal decisions for the given values of the manufacturer’s decisions, the manufacturer optimizes her decisions. Due to the complexity of the manufacturer’s optimization
problem in determining the optimal wholesale price, customization level, and the optimal selling price for the customized product that needs to take into account the retailer’s decisions on the retail price and order quantity under demand uncertainty, the numerical approach will be used to solve the manufacturer’s optimization problem. The analytical result can only be obtained for the deterministic setting. An algorithm for solving the manufacturer’s optimization problem, i.e., finding the optimal solutions \((x^*, w^*, p_c^*)\), is presented below. By knowing the optimal customization level \(x^*\), \(p_c^*\) can be directly determined from: \(p_c^* = v - (1 - x^*)t\). The derivation for the stopping criterion in the algorithm is presented in Appendix C.
Algorithm 1 for finding the manufacturer’s optimal decisions in a decentralized channel

Step 1: Start with the initial value of \( w = c_s + \varepsilon_0 \)
Step 2: Set the initial value for \( d = M \), \( M \) is a large number
Step 3: Set \( j = 1 \)
Step 4: Set the initial value for \( x = \varepsilon_x \)
Step 5: While \((d > 0.001)\)
   Step 5.1: set \( i = 1 \)
   Step 5.2: If \((x \leq 1)\)
      Step 5.2.1: Solve the equations (8) and (9) simultaneously for \( z^* \) and \( p_s^* \) for given \( x \) and \( w \)
      Step 5.2.2: Set \( p_c = v - (1 - x)t \) and find \( \bar{D}_s \)
      Step 5.2.3: If \((0 < \bar{D}_s < 1 \) and \( w < p_s^* \))
         Calculate the expected retailer’s and the manufacturer’s profits \( E(\pi_M) \) and \( E(\pi_R) \) for given \( w \) and \( x \)
         \( \pi_{1,R}(i) = E(\pi_R) \)
         \( \pi_{1,M}(i) = E(\pi_M(w, x)) \)
         Increase \( x \): \((x = x + \varepsilon_x)\) and go to Step 5.2
         \( i = i + 1 \)
      Else
         Increase \( w \): \((w = w + \varepsilon_w)\)
         Set \( x = \varepsilon_x \) and go to step 5.1
   End If
   Else
      Step 5.2.4: Calculate \( \pi_2(j) = \max(\pi_{1,M}) \) or the maximum of \( E(\pi_M) \) for given \( w \)
      Step 5.2.5: Find \( z_{w,x=1}^* \) for given \( w \) from solving equations (8) and (9)
      Step 5.2.6: Calculate \( d = dist(z_{w,x=1}^*) - dist(A) \)
      Step 5.2.7: Increase \( w \): \((w = w + \varepsilon_w)\), and go to Step 4
      Step 5.2.8: \( j = j + 1 \)
   End If
End While
Step 6: Find \( E(\pi_M^*) = \max(\pi_2) \) and the corresponding \( w, x, E(\pi_R^*) \)
Step 7: Calculate the optimal supply chain’s profit \( \pi_{SC}^* = E(\pi_M^*) + E(\pi_R^*) \)

Note that the formulation of \( dist(z^* ) \) is shown in Appendix C and to implement this algorithm, \( \varepsilon_0 = 0.1, \varepsilon_w = 0.01, \) and \( \varepsilon_x = 0.01 \) are considered.

Below, I present numerical examples to illustrate the effect of demand variability on the optimal values of customization level, stocking level, and selling price in the centralized and decentralized channels. In this numerical study, the stochastic part of the demand is assumed to follow a uniform distribution with mean equals to zero. The parameter values used are: \( v = 8, t = 1.4, a = 1, b = 0.8, \)
$c_c = 4.2, c_s = 3.5, \ h = 0, \ s = 0$. The standard deviation of the uniform distribution is used to represent the demand variability and the following five values are used: 0, 0.01, 0.02, 0.03 and 0.04, 0.05, 0.06, 0.07 where the zero value represents the deterministic setting. Note that although the numerical study is based on this set of parameter values, most qualitative insights discussed here are not parameter specific. The effects observed are, to a large extent, due to the relative differences between parameters rather than the absolute differences. The choice of the parameter values is also made by assuring that the feasibility conditions are met, i.e., $0 < x < 1$, and $0 < \bar{D}_s < 1$.

![Figures 4.2(a) – 4.2(d) show the effect of demand variability on a number of measures in both the centralized and decentralized channels. Figure 4.2(a) shows the difference in the optimal selling price and optimal customization level, Figure 4.2(b) shows the optimal channel profit and optimal stock level, and Figure 4.2(c) shows the optimal channel profit and optimal stock level, respectively.]
customization level between the centralized and decentralized channels. From the analytical results for the centralized channel, it is known that the customization level will decrease in the presence of demand uncertainty. While the analytical results are not available for the decentralized channel, Figure 4.2(a) indicates that the optimal customization level is also reduced in the decentralized channel, and this reduction seems to be more pronounced than in the centralized channel. The double marginalization of the standard product motivates the manufacturer to reduce the customization level and thus increase the portion of demand for the customized product.

Figure 4.2(b) shows that the optimal selling price for the standard product in the stochastic setting is lower than the price in the deterministic setting, and this is true for both types of channel (see Propositions 2 and 7). The figure also shows that the selling price is lower as demand variability increases in both the centralized and decentralized channels. As explained previously, reducing the price for the standard product reduces the risk measured by the demand coefficient of variation. Thus, it is reasonable that a larger price reduction is observed when demand variability increases. In the absence of demand uncertainty, the selling price in the decentralized channel is always higher than the centralized channel due to the double marginalization. However, as shown in Figure 4.2(b), the selling price in the decentralized channel can also be lower than the selling price in the centralized channel. Figure 4.2(c) depicts the optimal stocking levels in the two channels. For the parameter values chosen in this study, the optimal stocking levels are positive in the centralized channel but negative in the decentralized channel. The reason is that the values of the critical ratio in the centralized channel are above 0.5, and since the uniform distribution has a mean of zero, the values of $z$ are positive. In contrast, due to the double marginalization, the values of the critical ratio in the decentralized channel are below 0.5, resulting in the negative stocking levels. Note that a positive (negative) value of $z$ means that the order quantity is above (below) the riskless demand or the deterministic part of the demand. Also note that the positive or negative values of $z$ are parameter specific, and hence, cannot be directly generalized to other settings. And finally, Figure 4.2(d) shows the effect of demand variability on the channel profit. As expected, the channel profit in the decentralized channel is lower than the channel profit in the centralized channel, and both channel profits are decreasing in demand variability.

In view of the above results, manufacturing firms should be more conservative in offering customization in the presence of demand uncertainty, i.e., there is a motivation to lower the customization level compared to the case where the demand is deterministic. The channel structure is also influential. The motivation to lower the customization level is higher when the manufacturing firms have to rely on retailers in selling their standard products.
The next section will explore the possible use of channel coordination mechanisms such as a buy-back contract to improve the channel profit while ensuring that the manufacturer and the retailer are not made worse off.

Figure 4.3: The impact of demand variability on the riskless demand and optimal order quantity in the centralized and decentralized channel.

Figure 4.3(a) and Figure 4.3(b) depict the riskless demand for the standard product $D_s$ and the optimal order quantity $Q^* = D_s + z^*$ for the standard product in the centralized channel and decentralized channel, respectively. The two figures show that the riskless demand for the standard product in the two channels is increasing in the demand variability.

4.4 Channel Coordination

The previous analysis showed that the channel profit in the decentralized channel is lower than that in the centralized channel. Deterioration in channel performance is common in most supply chain management problems, and the product line design problem in the decentralized channel considered in this paper is no exception. Previous studies on channel coordination show that to improve the channel performance, the upstream partner can provide incentives to the downstream partner such that the downstream partner’s decisions are more aligned with what the upstream partner wishes. Such incentives are offered through supply chain contracts such as buy-back, revenue sharing, wholesale price contracts, and so forth. An important condition for channel coordination is that both partners should certainly benefit from such incentives.
This section will therefore focus on exploring possibilities to improve the channel performance in the product line design problem considered in this paper. I have chosen to focus on the buy-back contract for two reasons. Firstly, it is one of the contracts widely studied in the literature (see e.g. Pasternack 1985, Wu 2013 and the references therein). Secondly, return contracts are commonly applied in some industries such as cosmetics, fashion and apparel (Li et al. 2012). Padmanabhan and Png (1995) states that return contracts play important roles for the sellers of seasonal or style goods to coordinate their distribution channels. Buy-back contract is a simple return contract wherein suppliers specify a wholesale price for the ordered items and a buy-back price for the unsold items at the end of selling season. The exploration of using the buy-back contract in this paper can serve as a building block or departure point for future research that examines the use of other contracts in the context of product line design with demand uncertainty.

Under the buy-back contract, the manufacturer charges the retailer a wholesale price \( w_B \) for each unit of the standard product ordered and offers a buy-back price \( b_B \), \((b_B < w_B)\) for each unit of the leftovers at the end of the selling period. The subscript \( B \) is added to denote the decentralized channel with the buy-back contract. It is assumed that the retailer still needs to pay the holding cost \( h \) for each unit of the leftovers at the end of the selling period. The expected profits for the manufacturer and the retailer under the buy-back contract are as follows:

\[
E(\pi_{RB}(p_s, z)) = p_s \Gamma(p_s, x, z) - w_B(\bar{D}_s + z) - (h - b_B)(\bar{D}_s + z - \Gamma(p_s, x, z)) - s_c S(z) \quad (11)
\]

\[
E(\pi_{MB}(w_B, x, b_B)) = (w_B - c_s)(\bar{D}_s + z) + (p_c - c_c)(1 - \bar{D}_s) - b_B(\bar{D}_s + z - \Gamma(p_s, x, z)) - bx^2 \quad (12)
\]

where \( \Gamma(p_s, x, z) = \bar{D}_s + z - \int_{A}^{Z} F(u) \, du \) denotes the retailer’s expected sales (see Appendix B).

The following propositions characterize the retailer’s optimal decisions for the given values of the manufacturer’s decisions on \( w_B, x, \) and \( b_B \).

**Proposition 8:** The retailer’s optimal decisions \( p_s^* \) and \( z^* \) with the buy-back contract can be determined by solving the below equations simultaneously.

\[
p_s^* = \frac{v - t}{2} + x\bar{t} + \frac{w_B}{2} + \frac{t}{2}(z - \int_{A}^{Z} F(u) \, du) \quad (13)
\]
\[ F(z^*) = \frac{p_e+S_c-w_B}{p_e+S_c+(h-b_B)} \]  \hfill (14)

The optimal solutions are uniquely determined if the condition \(2H(z)^2 + \frac{dH(z)}{dz} > 0\) is fulfilled (see Appendix B). It is shown in Proposition 8 that the retailer’s problem can be solved analytically. From (14), it is obvious that the buy-back price offered by the manufacturer will reduce the overage cost incurred by the retailer, and hence, the retailer will be motivated to increase the stocking level. Given this optimal response from the retailer, the manufacturer now solves her optimization problem that maximizes her profit while ensuring that the retailer is not made worse-off compared to the case when the contract is not implemented. Like in the case of the decentralized channel without contract, the manufacturer’s optimal decisions on the wholesale price, customization level, and buy-back price will be determined numerically. An algorithm for solving the manufacturer’s optimization problem, i.e., finding the optimal solutions \((x^*, w_B^*, b_B^*, p_c^*)\) is presented below.
Algorithm 2 for finding the manufacturer’s optimal decisions under the buy-back contract

Step 1: Run Algorithm 1 to find $E(\pi_R^i)$ and $E(\pi_M^i)$
Step 2: Set the initial value for $w_B = c_s + \epsilon_0$
Step 3: Set the initial value for $d_B = M$, where $M$ is a very large number
Step 4: Set $j = 1$
Step 5: While ($d_B > 0.001$)
  Step 5.1: Set the initial value for $b_B = \epsilon_{B_B}$
  Step 5.2: While ($b_B < w_B$)
    Step 5.2.1: set $i = 1$
    Step 5.2.2: Set the initial value for $x = \epsilon_x$
    Step 5.2.3: While ($x \leq 1$)
      Solve the equations (13) and (14) simultaneously for $z^*$ and $p_s^*$ for given $x, w_B$, and $b_B$
      Set $p_c = v - (1 - x)t$ and find $D_s$
      If ($0 < D_s < 1$ and $w_B < p_s^*$)
        Calculate the expected retailer’s and manufacturer’s profits for given $w_B$, $b_B$, and $x$ by using the equations (11) and (12)
        $\pi_{1,R}(i) = E(\pi_{R_B}(p_s^*, z^*))$
        $\pi_{1,M}(i) = E(\pi_{M_B}(w_B, x, b_B))$
        Increase $x$: ($x = x + \epsilon_x$) and go to Step 5.2.3
        $i = i + 1$
      Else
        Increase the buy-back price $b_B$: ($b_B = b_B + \epsilon_{B_B}$) and go to Step 5.2
      End
    End
  End While
  Step 5.2.4: Calculate $\pi_{M,B,1}^* = \max(\pi_{1,M})$ or the maximum of manufacturer’s profits for given the $w_B$ and $b_B$ and find corresponding index $i_M$
  Step 5.2.5: Find the expected retailer’s profit $E(\pi_{R,B,1}) = \pi_{1,R}(i_M)$
  Step 5.2.6: If ($E(\pi_{R,B,1}) > E(\pi_M^*)$ and $E(\pi_{M,B,1}^*) > E(\pi_M^*)$)
    $\pi_{2,R}(j) = E(\pi_{R,B,1})$
    $\pi_{2,M}(j) = E(\pi_{M,B,1}^*)$
    $j = j + 1$
  End If
Step 5.2.7: Increase the buy-back price $b_B$: ($b_B = b_B + \epsilon_{B_B}$) and go to Step 5.2
End While
Step 5.3: Find $z_{w_B,b_B=w_B-\epsilon_{B_B},x=1}$ for given $w_B$ from solving equations (13) and (14)
Step 5.4: Calculate $d_B = dist_B(z_{w_B,b_B=w_B-\epsilon_{B_B},x=1}) - dist_B(A)$
Step 5.5: Increase $w$: ($w_B = w_B + \epsilon_w$), and go to Step 5
End While
Step 6: Calculate $\pi_{M,1}^* = \max(\pi_{2,M})$ and find the corresponding $w_B, b_B, x$, and index $i_B$
Step 7: Calculate the optimal supply chain’s profit: $\pi_{SC}^* = \pi_{M,1}^* + \pi_{2,R}(i_B)$
Note that the formulation of $dist_B(z^*)$ is shown in Appendix C and to implement this algorithm, $\epsilon_0 = 0.1, \epsilon_{B_B} = 0.01, \epsilon_w = 0.01$, and $\epsilon_x = 0.01$ are considered.
This algorithm is applied to the same numerical example that is used in the decentralized channel without a contract. The discussions below focus on the effects of the buy-back contract. As shown in Figures 4.4 and 4.5, the buy-back contract is able to increase the channel profit as well as the retailer’s and the manufacturer’s profits. Under the buy-back contract, the retailer is motivated to place a larger order quantity, as depicted in Figure 4.6. Also, increases in the channel profit and order quantity due to the contract appears to be more pronounced when demand variability increases.

These results should give motivation to manufacturing firms, especially those operating a dual channel and sell both the standard and customized products, to be more proactive in initiating a supply contract with the retailers so that they achieve a win-win situation. This is especially true when they face high demand uncertainty. Furthermore, they should also see that such a contract does not solely affect the inventory or order quantity decision, but also the product line design decisions. This should motivate the firms to enhance the integration of marketing (product line design) and operations (inventory) decisions.

Figure 4.4: The impact of demand variability on the retailer and manufacturer’s profits with and without the buy-back contract
4.5 Conclusions

This paper studies the product line design problem that includes the standard and customized products. The issues of primary interest are the examination of the effects of demand uncertainty on product line decisions, and how these effects are different between the centralized and decentralized channels. A stylized model that integrates the product line design problem with customization and the newsvendor problem is developed to analyze if the supply demand mismatch due to demand uncertainty has important implications for the price, customization level and inventory choices in the centralized and decentralized channels.
In a centralized channel, where the manufacturer makes all decisions, the presence of demand uncertainty drives the prices for the standard and customized products as well as the customization level down. These effects become more pronounced when the demand variability increases. Thus, this paper shows that demand uncertainty can have a significant influence on the product line decisions. The analytical results derived show the mutual dependence of the product line and order quantity decisions. By lowering the price for the standard product, the manufacturer can reduce the demand coefficient of variation and lessen the impact of demand uncertainty.

In a decentralized channel, where the manufacturer sells the standard product through a retailer, the effect of demand uncertainty resembles what is observed in the centralized channel. One notable difference is that due to double marginalization, the retailer tends to order the quantity that is lower than that of the centralized channel. Consequently, the channel profitability deteriorates in the decentralized channel, and even more when the demand variability is higher.

The existing literature on supply chain management points out the importance of coordination, which has motivated us to examine the possibility of designing a coordination mechanism to improve channel profitability in the decentralized channel. In this paper, the author has attempted to apply the buy-back contract, which is one of the contracts widely studied in the literature. The results show that, even though full coordination seems difficult to achieve, the buy-back contract is able to make both the manufacturer and the retailer better off, and that the improvements seem more pronounced when demand variability increases.

To address the limitations of this paper, I provide some suggestions for future research. First, the number of numerical examples studied in this paper is rather limited. For some results that cannot be derived analytically, more numerical examples are required to evaluate whether those results presented in this paper can be more generalized. Second, this paper only considers consumers’ heterogeneity in product taste or preference, and hence, it excludes product quality from the product line design problem. However, product quality is often an important consideration in product line design. Further research can study a richer problem where consumers are heterogeneous in both product taste and quality. Third, another interesting avenue for future research could be to carry out a more comprehensive examination of the channel contracts that may result in a channel profit that is closer to that achieved under a fully coordinated channel. This paper only explores the possibility of a buy-back contract for improving the channel performance. Contracts that allow the retailer’s initiatives to influence the manufacturer’s decisions could be an interesting topic to consider. Finally, this paper analyzes a monopolistic setting where the manufacturer is not in competition with other
manufacturers. Thus, the effects of competition are neglected. To consider a more realistic setting, it would be interesting to consider the presence of more than one manufacturers (e.g. a duopoly setting). In such a setting, the product line design decisions of each manufacturer, for example, regarding the customization level, will depend on the other manufacturer’s decisions in addition to the response from the retailer. Considering the competition at the retailer level is also a possible topic for future research.
Appendix A

In this section, I provide the derivatives for the problem in the case of a centralized channel. Before providing proofs for Propositions 1 and 2, I need to demonstrate that the expected shortage function \( S(z) \) is a decreasing and convex function in \( z \). By applying the integration by parts, the following outcome is derived:

\[
\int_z^B uf(u)du = B - zF(z) - G(z) \tag{A1}
\]

where \( G(z) \) is defined as \( \int_z^B F(u) \, du \). Since \( \frac{dG(z)}{dz} = -F(z) \leq 0 \) and \( \frac{d^2G(z)}{dz^2} = -f(z) \leq 0 \), \( G(z) \) is a decreasing and concave function w.r.t. \( z \).

By applying equation (A1), \( S(z) \) can be written as follows:

\[
S(z) = \int_z^B (u - z) f(u)du = \int_z^B uf(u)du - z \int_z^B f(u)du = B - G(z) - z \tag{A2}
\]

Since \( \frac{dS(z)}{dz} = -(1 - F(z)) < 0 \), \( \frac{d^2S(z)}{dz^2} = f(z) > 0 \), \( S(z) \) is a strictly decreasing and convex function in \( z \). Moreover, \( S(z) \) is a nonnegative function because it is a decreasing function and \( S(B) = 0 \).

Proof of Proposition 1:

The first and second partial derivatives are as follows:

\[
\frac{\partial E[\pi_{SC}(z,p_s,x)]}{\partial p_s} = \frac{2}{t} (v - (1 - x)t) - \frac{2}{t} p_s - \frac{1}{t} (c_c - c_s) - S(z) \tag{A3}
\]

\[
\frac{\partial^2 E[\pi_{SC}(z,p_s,x)]}{\partial p_s^2} = -\frac{2}{t} < 0 \tag{A4}
\]

\[
\frac{\partial E[\pi_{SC}(z,p_s,x)]}{\partial x} = (p_s - c_s) + t \left( 1 - \frac{v-(1-x)t-p_s}{t} \right) - (v - (1 - x)t - c_c) - 2bx \tag{A5}
\]

\[
\frac{\partial^2 E[\pi_{SC}(z,p_s,x)]}{\partial x^2} = -2b < 0 \tag{A6}
\]

\[
\frac{\partial E[\pi_{SC}(z,p_s,x)]}{\partial z} = -(c_s + h)F(z) + (p_s + s_c - c_s)(1 - F(z)) = -(c_s + h) + (p_s + s_c + h)(1 - F(z)) \tag{A7}
\]
\[
\frac{\partial^2 E[\pi_{SC}(z,p_s,z)]]}{\partial z^2} = -(p_s + s_c + h) f(z) < 0 \tag{A8}
\]

By applying the first-order conditions, I obtain the three equations (4), (5), and (6). By substituting \(x^*\) that comes from equation (6) into equation (4), \(p_s^*\) can be written as in the below equation:

\[
p_s^* = p_s(z) = p_{1,c} - \frac{t(b+t)}{2b} S(z), \text{ where } p_{1,c} = \frac{b+t}{b} (p_0 + \frac{t}{2(b+t)} x_0) \tag{A9}
\]

The first-order conditions reduce to equations (5) and (A9). Petruzzi and Dada (1999) states that \(p_s^*\) and \(z^*\) are uniquely determined if \(F(\cdot)\) is a cumulative distribution satisfying the condition \(2H(z)^2 + \frac{dH(z)}{dz} > 0\) is fulfilled, where \(H(z) = \frac{f(z)}{1-F(z)}\). Below, I show that this condition is valid for the optimization problem in the centralized channel:

\[
R_c(z) \text{ is defined as } \frac{dE[\pi_{SC}(z,p_s(z),x(p_s))]}{dz} = -(c_s + h) + \left( p_{1,c} - \frac{t(b+t)}{2b} S(z) + s_c + h \right) (1 - F(z))
\]

\[
\frac{dR_c(z)}{dz} = -\frac{t(b+t)}{2b} f(z) \left( \frac{2b}{t(b+t)} \left( p_{1,c} + s_c + h \right) - S(z) - \frac{1-F(z)}{H(z)} \right)
\]

\[
\frac{d^2R_c(z)}{dz^2} \bigg|_{R_c(z)=0} = -\frac{t(b+t)}{2b} f(z) \frac{1-F(z)}{H^2(z)} (2H^2(z) + \frac{dH(z)}{dz})
\]

If \(2H(z)^2 + \frac{dH(z)}{dz} > 0\), \(R_c(z)\) is monotone or unimodal. Therefore, \(R_c(z)\) has at most two roots. \(R_c(B) = -(c_s + h) < 0\). If \(R_c(z)\) has one root, it implies that the sign of \(R_c(z)\) changes from positive to negative at the root. Thus, \(\frac{dR_c(z)}{dz} \bigg|_{R_c(z)=0} < 0\) which means that the root is the local maximum of \(\pi_{SC}(z,p_s(z),x(p_s))\). If there are two roots for \(R_c(z)\), it implies that two changes occur for the sign of \(R_c(z)\), first from negative to positive at the smaller root and then from positive to negative at the bigger root of \(R_c(z)\). Therefore, the bigger root is local maximum of \(\pi_{SC}(z,p_s(z),x(p_s))\). In addition, since \(\frac{\partial^2 E(\pi_{SC})}{\partial p_s^2} < 0\) and \(\frac{\partial^2 \pi_{SC}}{\partial z^2} < 0\), the stationary point for the retailer’s profit function cannot be a minimum point and it can be concluded that \(R_c(z)\) has at most one root. Consequently, \((p_s^*, z^*, x^*)\) from the first-order conditions maximizes the profit function.
Proof of Proposition 2:

2(a):
As mentioned earlier, by substituting \( x^* \) that comes from equation (6) into equation (4), \( p^*_s \) can be written as equation (A9). Since \( S(z) \) is a decreasing function in the stocking level \( z \), the optimal retail price will increase.

2(b):
When I substitute \( p^*_s \) from equation (4) into equation (5) the new equation below will be derived:

\[
F(z^*) = 1 - \frac{c_s + h}{p_0 + xt - \frac{1}{2}S(z^*) + s_c + h}
\] (A10)

Equation (A10) can be written as \( \Lambda = F(z^*) + \frac{c_s + h}{p_0 + xt - \frac{1}{2}S(z^*) + s_c + h} - 1 = 0 \). I need to find \( \frac{\partial \Lambda}{\partial x} \) and \( \frac{\partial \Lambda}{\partial z^*} \) to use \( \frac{dz^*}{dx} = -\frac{\partial \Lambda}{\partial x} \) formulation, \( \frac{\partial \Lambda}{\partial x} = -\frac{t(1-F(z^*))}{p_0 + xt - \frac{1}{2}S(z^*) + s_c + h} < 0 \) and \( \frac{\partial \Lambda}{\partial z^*} = f(z^*) - \frac{t(1-F(z^*))^2}{2\alpha(p_0 + xt - \frac{1}{2}S(z^*) + s_c + h)} \). If \( f(z^*) > \frac{t(1-F(z^*))}{2\alpha(p_0 + xt - \frac{1}{2}S(z^*) + s_c + h)} \Rightarrow H(z^*) > \frac{t(1-F(z^*))}{2\alpha} \Rightarrow H(z^*) > \frac{t}{2(c_s + h)}(1 - F(z^*))^2 \), \( \frac{\partial \Lambda}{\partial z^*} \) will be positive and it can be concluded that \( \frac{dz^*}{dx} > 0 \).

2(c):
The manufacturer’s decisions on \( z^* \) and \( p^*_s \) are the intersection point of equations (A9) and (5). To prove this part of Proposition 2, \( \frac{dz^*}{dp_s} \) should be calculated. From equation (5), \( \gamma = F(z^*) + \frac{c_s + h}{p_0 + s_c + h} - 1 = 0 \). Accordingly, \( \frac{\partial \gamma}{\partial z^*} = f(z^*) > 0 \) and \( \frac{\partial \gamma}{\partial p_s} = -\frac{c_s + h}{(p_0 + s_c + h)^2} \), therefore, \( \frac{dz^*}{dp_s} = -\frac{\partial \gamma}{\partial p_s} > 0 \).

Proof of Proposition 3:

Superscripts \( i \) and \( j \) are used in the optimal solutions to distinguish the case with deterministic demand from the case with demand uncertainty. Without considering demand uncertainty, the manufacturer’s profit function can be written as:

\[
\pi^i_{SC}(p^i_s, x^i) = (p^i_s - c_s)\overline{D}_s + (p^i_e - c_e)(1 - \overline{D}_s) - b x^{(i)2}
\] (A11)
where \( p_c^i = v - (1 - x^i)t \).

From the first-order conditions, the stationary point \( p_s^j = \frac{t^2}{2b} - t - \frac{(c_c - c_d)}{2} + v \) and \( x^i = \frac{t}{2b} \) can be achieved. Since \( \frac{\partial^2 \pi_{购物中心}^i}{\partial p_s^j} = -\frac{2}{t} < 0 \), \( \frac{\partial^2 \pi_{购物中心}^i}{\partial x^i} = -2b - 2t < 0 \), and determinant of Hessian matrix of \( \pi_{购物中心}^i \) is \( 4b > 0 \), \( \pi_{购物中心}^i \) will be maximized globally at the stationary point \( (p_s^i, x^i) \). The optimal riskless price can be written as \( p_s^i = p_0 + x^i t \) where \( p_0 = v - \frac{(c_c - c_d)}{2} - t \). Also, from equation (4), the selling price under demand uncertainty is \( p_s^j = p_0 + x^j t - \frac{t}{2} S(z^*) \). By substituting the optimal selling price \( p_s^j \) that is derived from equation (4) into equation (6), \( x^j \) can be written as \( x^j = \frac{t - \frac{t}{2} S(z^*)}{2b} \). Since \( S(z^*) \) is a nonnegative function, \( x^j < x^i \) and consequently \( p_s^j < p_s^i \).

**Proof of Proposition 4:**

As mentioned earlier, in the case with demand uncertainty, the demand function consists of two parts: the deterministic part \( D_2 \) and the uncertain part \( \theta_s \). Here, I show that the deterministic part of demand in the case when demand is uncertain is bigger than when demand is deterministic. \( \overline{D}_2(p_s^i, x^i) = \frac{p_l^i - p_d^i}{t} \), where \( p_l^i = v - (1 - x^i)t \) and \( l \in \{i, j\} \). Superscripts \( i \) and \( j \) are used to distinguish the case with deterministic demand from the case with demand uncertainty. Hereafter, by knowing the manufacturer’s best response on price and customization level, I substitute them into \( \overline{D}_2 \): \( \overline{D}_2 (p_s^i, x^i) = \frac{(c_c - c_d)}{2t} + \frac{S(z^*)}{2} \) and \( \overline{D}_2 (p_s^j, x^j) = \frac{(c_c - c_d)}{2t} \). Since \( S(z^*) \) has a nonnegative value, \( \overline{D}_2^i > \overline{D}_2^j \) and consequently \( \overline{D}_2^i < \overline{D}_2^j \).

**Appendix B**

In this section, the derivatives for the problem in the decentralized channel are provided.

**Proof of Proposition 5:**

The first partial derivatives of the retailer’s profit function are as follow:

\[
\frac{\partial E(\pi_R(p_s z))}{\partial p_s} = \frac{1}{t} p_c - \frac{2}{t} p_s + \frac{1}{t} w - S(z) \tag{B1}
\]

\[
\frac{\partial^2 E(\pi_R(p_s z))}{\partial p_s^2} = -\frac{2}{t} < 0 \tag{B2}
\]
\[
\frac{\partial E(\pi_R(p_s,z))}{\partial z} = -(w + h)F(z) + (p_s + s_c - w)(1 - F(z)) = -(w + h) + (p_s + s_c + h)(1 - F(z))
\]  \hspace{1cm} (B3)

\[
\frac{\partial^2 E(\pi_R)}{\partial z^2} = -(p_s + s_c + h)f'(z) < 0
\]  \hspace{1cm} (B4)

By applying the first-order conditions:

\[
\frac{\partial E(\pi_R(p_s,z))}{\partial p_s} = 0 \Rightarrow p_s^* = \frac{p_c}{2} + \frac{w}{2} - \frac{t}{2} S(z)
\]  \hspace{1cm} (B5)

\[
\frac{\partial E(\pi_R(p_s,z))}{\partial z} = 0 \Rightarrow F(z^*) = \frac{p_s^* + s_c - w}{p_s^* + s_c + h}
\]  \hspace{1cm} (B6)

For the given \( w \) and \( x \), \( p_s^* \) and \( z^* \) are uniquely determined because:

\[
R_d(z) = \frac{dE(\pi_R(p_s,z))}{dz} = -(w + h) + (p_{1,d} - \frac{t}{2} S(z) + s_c + h)(1 - F(z))
\]  \hspace{1cm} (B7)

where \( p_{1,d} = \frac{v - t}{2} + \frac{t}{2} x + \frac{w}{2} \)

Next, I find zeros of \( R_d(z) \):

\[
\frac{dR_d(z)}{dz} = -\frac{t}{2} f(z) \left( p_0 + s_c + h - S(z) - \frac{(1 - F(z))}{H(z)} \right)
\]  \hspace{1cm} (B8)

\[
\frac{d^2 R_d(z)}{dz^2} \bigg|_{R_d(z)=0} = -\frac{t}{2} f(z) \frac{1 - F(z)}{H(z)} \left( 2H^2(z) + \frac{dH(z)}{dz} \right)
\]  \hspace{1cm} (B9)

If \( 2H(z)^2 + \frac{dH(z)}{dz} > 0 \), \( R_d(z) \) is monotone or unimodal. Therefore, \( R_d(z) \) has at most two roots. \( R_d(B) = -(w + h) < 0 \). If \( R_d(z) \) has one root, it implies that the sign of \( R_d(z) \) changes from positive to negative at the root. Thus, \( \frac{dR_d(z)}{dz} \bigg|_{R_d(z)=0} < 0 \) which means that the root is local maximum of \( \pi_R(z,p_s^*(z)) \). If there are two roots for \( R_d(z) \), it implies that two changes occur for the sign of \( R_d(z) \), first from negative to positive at the smaller root and then from positive to negative at the bigger root of \( R_d(z) \). Therefore, the bigger root is local maximum of \( \pi_R(z,p_s^*(z)) \). Moreover, the second-order necessary conditions \( \frac{\partial^2 E(\pi_R)}{\partial p_s^2} < 0 \) and \( \frac{\partial^2 E(\pi_R)}{\partial z^2} < 0 \) imply that the stationary point for the retailer's profit function cannot be a minimum point and it can be concluded that \( R_d(z) \) has at most one root. Therefore, the retailer's profit function is quasi-concave that has only one peak and the stationary point represents the maximum point.
Proof of Proposition 6:

I need to calculate the first partial derivative of $z^*$ w.r.t. $x$.

First, from equation (9), it can be obtained the below expression:

$$p_s = \frac{w+h}{1-F(z^*)} - (s_c + h) \quad \text{(B10)}$$

Second, by equalizing the left-sides of equations (8) and (B10), there exists:

$$K_1 = p_2 + \frac{t}{2}x + \frac{w}{2} - \frac{t}{2}S(z^*) - \frac{w+h}{1-F(z^*)} = 0 , \text{ where } p_2 = \frac{v-t}{2} + s_c + h \quad \text{(B11)}$$

By applying the implicit differentiation method, then:

$$\frac{\partial z^*}{\partial x} = -\frac{\frac{\partial K_1}{\partial x}}{\frac{\partial K_1}{\partial z^*}} = \frac{1-F(z^*)}{\frac{t(w+h)H(z^*)-t(1-F(z^*))^2}{2}} \quad \text{(B12)}$$

The below condition for the retailer’s optimal decision on stocking level $z$ should be satisfied:

$$\frac{d^2 \pi_R}{dz^2} = \frac{t}{2} \left( \frac{1}{\alpha} \left( 1 - F(z^*) \right)^2 - \frac{2}{t} (w+h)H(z^*) \right) < 0 \Rightarrow \frac{t}{2} (w+h)H(z^*) - \frac{1}{\alpha} (1 - F(z^*))^2 > 0 \quad \text{(B13)}$$

Therefore, $\frac{\partial z^*}{\partial x} > 0$.

Proof of Proposition 7:

Without demand uncertainty, the retailer’s and manufacturer’s profit functions can be written as:

$$\pi_R = (p_s - w)\bar{D}_s \quad \text{(B14)}$$

$$\pi_M = (w - c_s)\bar{D}_s + (p_c - c_c)\bar{D}_c - bx^2 \quad \text{(B15)}$$

The first and second derivatives of the retailer’s profit function are as follows:

$$\frac{\partial \pi_R}{\partial p_s} = \frac{w}{t} + \frac{p_c}{t} - \frac{2p_s}{t} \quad \text{(B16)}$$
\[
\frac{\partial^2 \pi_R}{\partial p_s^2} = -\frac{2}{t} < 0
\]  
(B17)

The retailer’s profit function is concave in \( p_s \). Therefore, the retailer’s optimal pricing decision can be obtained from the first-order condition: 
\[
p_s^* = p_1 + \frac{xt}{z} + \frac{w}{z}, \quad \text{where} \quad p_1 = \frac{v - t}{2}.
\]
Since \( S(z) \) is nonnegative, a comparison of the optimal results for the selling price under demand certainty and uncertainty demonstrates a reduction in the optimal selling price in the decentralized channel.

**Buy-back contract**

First, the expected sales function is provided. By defining 
\[
D_s(p_s, x, \vartheta_s) = D_\theta(p_s, x) + \vartheta_s
\]
and applying the transformation 
\[
z = Q - D_s(p_s, x),
\]
the retailer’s expected sales can be expressed as follows:

\[
\Gamma(p_s, x, Q) = \begin{cases} 
D_s(p_s, x, \vartheta_s), & \text{if } Q > D(p_s, x, \vartheta_s) \\
Q, & \text{if } Q < D(p_s, x, \vartheta_s)
\end{cases}
\]

Thus, the retailer’s expected sales is defined as:

\[
E(\Gamma(p_s, x, Q)) = E[\min(Q, D_s(p_s, x, \vartheta_s))] = \int_{A}^{Z}(\Bar{D}_s(p_s, x) + u)f(u)du + \int_{Z}^{B}(\Bar{D}_s(p_s, x) + z)f(u)du
\]

\[
= \int_{A}^{Z}F(u)du + \int_{Z}^{B}F(u)du = Q - \int_{A}^{Z}F(u)du
\]

**Proof of Proposition 8:**

\[
\frac{\partial E(\pi_{RB})}{\partial p_s} = \left(\frac{p_c - p_s}{t} + z - \int_{A}^{Z}F(u)du\right) - \frac{p_s}{t} + \frac{w_B}{t}
\]

\[
\frac{\partial^2 E(\pi_{RB})}{\partial p_s^2} = -\frac{2}{t} < 0
\]  
(B21)

\[
\frac{\partial E(\pi_{RB})}{\partial z} = (p_s + (h - b_B) + s_c)(1 - F(z)) - (w_B + (h - b_B))
\]

\[
\frac{\partial^2 E(\pi_{RB})}{\partial z^2} = -f(z)(p_s + (h - b_B) + s_c) < 0
\]  
(B23)
First-order condition $\frac{\partial E(\pi_{B})}{\partial p_{s}} = 0$ gives $p_{s}^{*}(z) = p_{0B} + \frac{t}{2}(z - \int_{A}^{Z} F(u) du)$, where $p_{0B} = \frac{v-t}{2} + \frac{w_{B}}{2}$. In what follows, I show that $p_{s}^{*}$ and $z^{*}$ are uniquely determined if $F(\cdot)$ as a cumulative distribution function satisfies the condition $2H(z)^{2} + \frac{dH(z)}{dz} > 0$:

First, $R_{B}(z)$ is defined as follows:

$$R_{B}(z) = \frac{dE(\pi_{B}(x,p^{*}_{s}(z)))}{dz} = \left(p_{0B} + \frac{t}{2}(z - \int_{A}^{Z} F(u) du) + (h - b_{B}) + s_{c}\right)(1 - F(z)) - (w_{B} + (h - b_{B})),$$

(B24)

Next, I find zeros of $R_{B}(z)$:

$$\frac{dR_{B}(z)}{dz} = -\frac{t}{2}f(z)\left[\frac{1}{t}\left(p_{0B} + (h - b_{B}) + s_{c}\right) + (z - \int_{A}^{Z} F(u) du) - \frac{1-F(z)}{H(z)}\right]$$

(B25)

$$\frac{d^{2}R_{B}(z)}{dz^{2}} \bigg|_{dR_{B}(z)/dz = 0} = -\frac{t}{2} \frac{f(z)(1-F(z))}{H(z)^{2}} (2H(z)^{2} + \frac{dH(z)}{dz})$$

(B26)

If $2H(z)^{2} + \frac{dH(z)}{dz} > 0$, $R_{B}(z)$ is monotone or unimodal. Therefore, $R_{B}(z)$ has at most two roots. Let $w_{B} + h > b_{B}$, therefore $R_{B}(B) = -(w_{B} + (h - b_{B})) < 0$. If $R_{B}(z)$ has one root, it implies that the sign of $R_{B}(z)$ changes from positive to negative at the root. Thus, $\frac{dR_{B}(z)}{dz} \bigg|_{R_{B}(z) = 0} < 0$ which it means that the root is the local maximum of $\pi_{B}(z, p_{s}(z))$. If there are two roots for $R_{B}(z)$, two changes occur for sign of $R_{B}(z)$, first from negative to positive at the smaller root, and then from positive to negative at the bigger root of $R_{B}(z)$. Therefore, the bigger root is the local maximum of $\pi_{B}(z, p_{s}(z))$.

Since $\frac{\partial^{2}E(\pi_{B})}{\partial p_{s}^{2}} < 0$ and $\frac{\partial^{2}E(\pi_{B})}{\partial z^{2}} < 0$, the second order necessary condition is satisfied. It implies that the stationary point for the retailer’s profit function cannot be a minimum point and it can be concluded that $R_{B}(z)$ has at most one root. Therefore, the retailer’s profit function is quasi-concave that has only one peak and the stationary point represents the maximum point.
Appendix C

In this part, I derive the stopping criteria for both algorithms:

Algorithm 1:

By using the equations (B6) and (B8), there exists:

\[
\frac{\partial z^*}{\partial w} = -\frac{\partial K}{\partial z^*} = \frac{-F(z^*)-1}{2(w+h)H(z^*)-t(1-F(z^*))^2} < 0
\]  

(B27)

The equation (B27) implies that for a given customization level \( x \), increasing the wholesale price \( w \) will decrease the optimal stocking level \( z^* \). Equation (B13) states that for a given wholesale price \( w \), increasing the customization level \( x \) will increase the optimal stocking level \( z^* \). Also, since the demand should be positive and the wholesale price \( w \) should be less than the optimal retail price \( p_0^* \), the lower bound \( L(z^*) \) and the upper bound \( U(z^*) \) for given \( w \) and \( x \) can be defined as follows:

\[
L(z^*) = 2\left(\frac{v^3 t}{2} + \frac{xt}{2} + \frac{t}{2} S(z^*)\right)
\]  

(B28)

\[
U_1(z^*) = 2\left(\frac{v-t}{2} + \frac{xt}{2} - \frac{t}{2} S(z^*)\right)
\]  

(B29)

The upper bound and the lower bound are functions w.r.t. the optimal stocking level \( z^* \). Furthermore, the second upper bound can be defined. Because \( R_d(A) > 0 \Rightarrow \left(p_0 - \frac{t}{2} S(A) + s_c + h \right) - (w + h) > 0 \Rightarrow w < 2\left(\frac{v-t}{2} + \frac{xt}{2} - \frac{t}{2} S(A) + s_c \right) \Rightarrow U_2 = 2\left(\frac{v-t}{2} + \frac{xt}{2} - \frac{t}{2} S(A) + s_c \right). \) Then, it should be determined which one is tighter:

\[
U(z) = U_2 - U_1(z^*) = 2\left(-\frac{t}{2} S(A) + s_c \right) + 2\left(\frac{t}{2} S(z^*) \right) = t\left(S(z^*) - S(A) \right) + 2s_c.
\]  

(B30)

Since \( S(z) \) is a decreasing function in \( z \), I have \( S(z^*) < S(A) \). If \( 2s_c > t\left(S(A) - S(z^*) \right) \), the first upper bound is tighter than the second one.

Next, I define \( dist(z^*) \) as follows:
\[ \text{dist}(z^*) = \min\{U_2, U_1(z^*)\} - L(z^*) \]

\[ = \begin{cases} 
2t - 2tS(z^*) & \text{if } U_1(z^*) < U_2 \\
2t + 2s_c - tS(A) - tS(z^*) & \text{if } U_1(z^*) > U_2 
\end{cases} \] (B31)

Since \( \frac{d\text{dist}(z^*)}{dz^*} > 0 \), by decreasing the optimal stocking level \( z^* \), \( \text{dist}(z^*) \) will decrease. According to the equations (8) and (9), \( z^* \) is a function w.r.t the wholesale price \( w \) and the customization level \( x \). For a given wholesale price \( w \), the maximum of the optimal stocking level \( z^*_{w,x=1} \) occurs at \( x = 1 \). According to equation (B27), by increasing the wholesale price \( w \), \( \text{dist}(z^*_{\text{max}}) \) will decrease.

\[ A < z^*_{w,x=1} < B \Rightarrow \text{dist}(A) < \text{dist}(z^*_{w,x=1}) < \text{dist}(B) \Rightarrow 0 < \text{dist}(z^*_{w,x=1}) - \text{dist}(A) < \text{dist}(B) - \text{dist}(A). \] Thus, \( d(z^*_{w,x=1}) = \text{dist}(z^*_{w,x=1}) - \text{dist}(A) \) is a decreasing function w.r.t the wholesale price \( w \). In the algorithm, I increase the \( w \) value until the value of \( d(z^*_{w,x=1}) \) is sufficiently small.

**Algorithm 2:**

As in Algorithm 1, the upper bound and the lower bound for the wholesale price under the buy-back contract \( w_B \) can be defined as:

\[ L_B(z^*) = v - t + xt - t\left(z^* - \int_A^{z^*} F(u)du\right) - 2t \] (B32)

\[ U_{B_1}(z^*) = v - t + xt + t(z^* - \int_A^{z^*} F(u)du) \] (B33)

In addition, \( R_B(A) \) (see Appendix B) should be positive:

\[ R_B(A) > 0 \Rightarrow \left(p_{0_B} + \frac{At}{2} + (h - b_B) + s_c\right) - (w_{B} + (h - b_B)) > 0 \Rightarrow \] \( w_B < p_{0_B} + \frac{At}{2} + s_c \). By substituting \( p_{0_B} \), there exists the second upper bound for the wholesale price \( w_B \):

\[ w_B < U_{B_2}, \text{ where } U_{B_2} = v - t + xt + At + 2s_c. \] (B34)

Next, it should be determined which upper bound is tighter:

\[ \mathcal{U}_B(z^*) = U_{B_2} - U_{B_1}(z^*) = At + 2s_c - t(z^* - \int_A^{z^*} F(u)du) \]
\( U_B(A) = 2s_c > 0 \) and \( \frac{dU_B}{dz} = -t(1 - F(z^*)) < 0 \) implies that \( U_B(z^*) \) is a decreasing function in \( z^* \).

Therefore, there is a possibility that \( U(B) \) has a negative value. Under the condition \( z^* - \int_A^{z^*} F(u)du > A + \frac{2s_c}{t} \), \( U_{B_2} \) is tighter than \( U_{B_1} \), otherwise under the condition \( z^* - \int_A^{z^*} F(u)du < A + \frac{2s_c}{t} \), \( U_{B_1} \) is tighter than \( U_{B_2} \).

The distance function \( \text{dist}_B(z^*) \) is equal to:

\[
\text{dist}_B(z^*) = \min\{U_{B_1}(z^*), U_{B_2}\} - L_B(z^*) =
\begin{cases}
t \left( z^* - \int_A^{z^*} F(u)du \right) + 2t; & \text{if } U_{B_1}(z^*) < U_{B_2} \\
t \left( z^* - \int_A^{z^*} F(u)du \right) + (A + 2)t + 2s_c; & \text{if } U_{B_1}(z^*) > U_{B_2}
\end{cases}
\]

Next, I show that \( \text{dist}_B \) will decrease by increasing the wholesale price.

\[
\frac{\partial \text{dist}_B(z^*)}{\partial z^*} = t(1 - F(z^*)) > 0 \implies \text{the distance function is an increasing function in } z^*.
\]

When I substitute \( p_s^* \) from equation (13) into equation (14), I have \( K_2 = 0 \), where

\[
K_2 = \frac{v-t}{2} + \frac{x}{2} + \frac{w_B}{2} + \frac{t}{2} \left( z^* - \int_A^{z^*} F(u)du \right) - \frac{w_B + (h-b_B)}{1-F(z^*)} + (s_c + (h - b_B))
\]

Then, by using the implicit differentiation formulation:

\[
\frac{\partial z^*}{\partial w_B} = -\frac{\partial K_2}{\partial w_B} = -\frac{1+F(z^*)}{2(w_B + (h-b_B))H(z^*) - t(1-F(z^*))^2}
\]

\[
\frac{\partial z^*}{\partial x} = -\frac{\partial K_2}{\partial x} = \frac{t(1-F(z^*))}{2(w_B + (h-b_B))H(z^*) - t(1-F(z^*))^2}
\]

\[
\frac{\partial z^*}{\partial b_B} = -\frac{\partial K_2}{\partial b_B} = \frac{2F(z^*)}{2(w_B + (h-b_B))H(z^*) - t(1-F(z^*))^2}
\]

In what follows, I prove that \( \left( w_B + (h - b_B)H(z^*) - t \left( 1 - F(z^*) \right)^2 \right) > 0 \). First, I substitute \( p_s^* \) from equation (13) into (B22) and take derivative w.r.t \( z \):
\[
\frac{d^2\pi_{RB}(z,p^*_s(z))}{dz^2} = -f(z)(p^*_s(z) + (h - b_B) + s_c) + \frac{t}{2}(1 - F(z))^2
\]  \hspace{1cm} (B40)

Second, from equation (14), the optimal retail price can be written as:

\[
p^*_s(z) = \frac{w_B + (h - b_B)}{1 - F(z)} - (s_c + (h - b_B))
\]  \hspace{1cm} (B41)

Third, by substituting \(p^*_s(z)\) from equation (B41) into (B40), \(\frac{d^2\pi_{RB}(z,p^*_s(z))}{dz^2}\) is equal to:

\[
\frac{d^2\pi_{RB}(z,p^*_s(z))}{dz^2} = -f(z)\left(\frac{w_B + (h - b_B)}{1 - F(z)}\right) + \frac{t}{2}(1 - F(z))^2
\]  \hspace{1cm} (B42)

Since \(\frac{d^2\pi_{RB}(z,p^*_s(z))}{dz^2}\) should be negative, \(2\left(w_B + (h - b_B)H(z) - t(1 - F(z))^2\right) > 0\). Therefore, \\
\(\frac{\partial z^*}{\partial w_B} < 0, \frac{\partial z^*}{\partial b_B} > 0\) and \(\frac{\partial z^*}{\partial x} > 0\). For a given wholesale price \(w_B\), the optimal stocking level occurs at \(b_B = w_B - \epsilon_{b_B}\) and \(x = 1\). \(\frac{\partial z^*}{\partial w_B} < 0\) implies that \(z^*\) decreases by increasing the wholesale price \(w_B\) and therefore \(\text{dist}_B(z^*)\) will decrease by decreasing \(z^*\). I also know that \(A < z^*_{w_B,b_B=w_B-\epsilon_{b_B},x=1} < B \Rightarrow \text{dist}_B(A) < \text{dist}_B\left(z^*_{w_B,b_B=w_B-\epsilon_{b_B},x=1}\right) < \text{dist}_B(B) \Rightarrow 0 < \text{dist}_B\left(z^*_{w_B,b_B=w_B-\epsilon_{b_B},x=1}\right) - \text{dist}_B(A) < \text{dist}_B\left(z^*_{w_B,b_B=w_B-\epsilon_{b_B},x=1}\right) - \text{dist}_B(A)\). Therefore, the stopping criterion is determined based on \(d_B = \text{dist}_B\left(z^*_{w_B,b_B=w_B-\epsilon_{b_B},x=1}\right) - \text{dist}_B(A)\).


Declaration of co-authorship

Full name of the PhD student: Parisa Bagheri Tookanlou

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<td>Parisa Bagheri Tookanlou and Hartanto Wong</td>
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<tr>
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<td>Hartanto Wong</td>
<td>[Signature]</td>
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Date: January 31, 2018

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Full name of the PhD student: Parisa Bagheri Tookanlou

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| Authors: | Parisa Bagheri Tookanlou and Hartanto Wong |

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