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Procedia Engineering 172 (2017) 922 – 929

Procedia Engineering

www.elsevier.com/locate/procedia

Modern Building Materials, Structures and Techniques, MBMST 2016

Cracking in flexural reinforced concrete members

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Abstract

The system of cracks developing in reinforced concrete is in many aspects essential when modelling structures in both serviceability- and ultimate limit state. This paper discusses the behavior concerning crack development in flexural members observed from tests and associates it with two different existing models. From the investigations an approach is proposed on how to predict the crack pattern in flexural members involving two different crack systems; primary flexural cracks and local secondary cracks. The results of the approach is in overall good agreement with the observed tests and captures the pronounced size effect associated with flexural cracking in which the crack spacing and crack widths are approximately proportional to the depth of the member.

Peer-review under responsibility of the organizing committee of MBMST 2016

Keywords: , Comparison with tests, crack systems in flexural members, modeling of reinforced concrete, serviceability.

1. Introduction

Cracking in reinforced concrete members is of a complex nature. On the one side it involves various mechanisms and parameters related to the interaction between concrete and reinforcement, geometry, type of loading and support conditions. These are all factors that, to a certain extent, can be understood, controlled and manipulated. On the other hand, these deterministic factors are all superimposed by a certain level of randomness owing to the fluctuations of material parameters and the heterogeneous composition of concrete. Nevertheless, the process of cracking and the resulting crack systems has been widely studied during the years, striving to create physically transparent models which are able to predict the structural behavior. These studies have however mainly been

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Peer-review under responsibility of the organizing committee of MBMST 2016
doi:10.1016/j.proeng.2017.02.102

restricted to the case of either pure tension or flexure. Such studies are of course desirable due to the inevitable fact that when modelling, and thus predicting, the behavior of reinforced concrete, all important aspects are, obviously, influenced by the presence and continuous development of various types of cracks. It is, for instance, well known that various size effects are closely related to relation between crack development and structural scale. Important phenomena that require distinct considerations of the process of cracking as well as the actual crack location and shape could, among other, be; shear capacity of members without shear reinforcement, rotational capacity of plastic hinges and, more or less, all aspects associated with serviceability behavior (crack widths, stiffness and reinforcement stresses).

When consulting existing literature it appears that most models, addressing the influence of cracks for the case of members subjected to primarily flexure, are based upon tension tie models. When applying such approaches, the flexural member, be it a slab or a beam, is assumed to be composed of a fictitious tensile zone, localized around the longitudinal tensile reinforcement. This tensile zone is then treated as a tie subjected to a uniform state of tension stresses. The application of models like these is tempting in the sense that, besides being easy to apply, they introduces a mechanical description of the system of cracks and establishes an analytical link between this system and apparent governing parameters. Nonetheless, although applicable, this approach appears somehow over-simplistic, at least in some cases.

In the following cracking in flexural members is investigated through the observation of selected tests. Subsequently these observations are sought implemented through a combination of two different existing models; the Tension Chord Model [1] and a simple empirical expression, based on observations of members subjected to pure bending [2].

2. Experimental observations

In Fig. 1(a) the average spacing of cracks, S, relatively to the effective depth d, is plotted as a function of the applied load in the case of a reinforced concrete beam tested by Sherwood [3]. The crack spacing is indicated at two different levels of the beam depth; at the level of the concentrated tensile reinforcement and at mid-height of the member. The applied load is represented by the average shear stress across a section in the shear span. The tested beam was provided with a concentrated reinforcement in two layers near the bottom-side (ρ=0.83%) and no stirrups. Ultimately, the beam failed in shear.

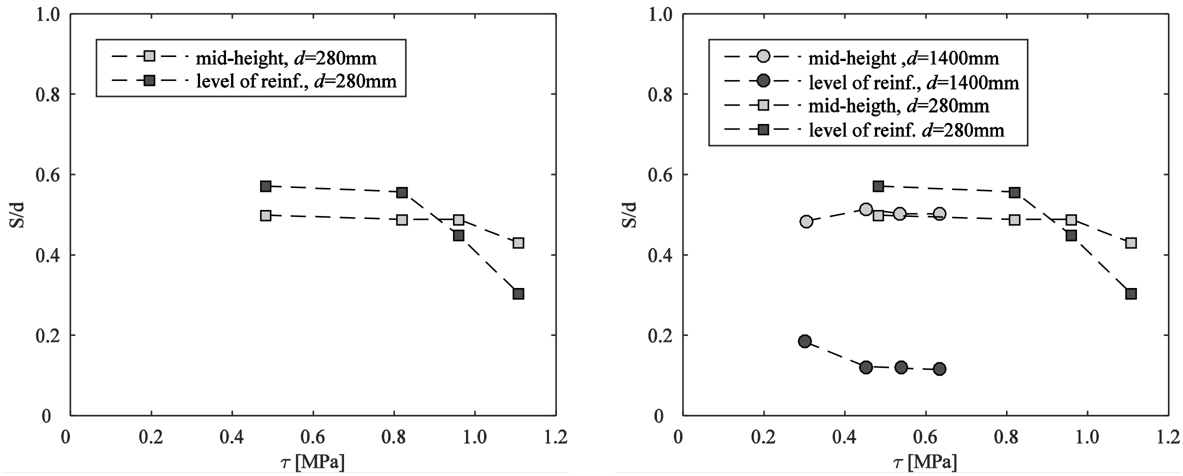

Fig. 1. Tests by Sherwood (a) relative crack spacing for small beam; (b) for large and small beam.

In Fig. 1(a), attention is given to the observation that the average crack spacing is of the same order at the two different levels, and roughly equals one half of the effective depth. This observation holds at least for as long as the member is subjected to what must be regarded as service load intensity; say an average shear stress less than approximately 0.8 MPa. The spacing at mid-height is seen to be slightly smaller than at reinforcement-level. This is primarily due to the somewhat curved shape of the cracks in the shear span. When the applied load is increased beyond 0.8 MPa, the crack spacing at reinforcement level is seen decrease. This is due to a gradual development of more cracks, locally around the reinforcement.

In Fig. 1(b) similar measurements have been included for a large beam from the same test series. The effective depth of this large member equaled five times that of the smaller one just considered. The slenderness, concrete strength and reinforcement ratio was, on the other hand, identical. What regards the relative crack spacing at mid-height, it is seen that the ratio (*S/d*) remains approximately the same for the two beams, despite the great difference in effective depth. In the case of the large beam, it thus makes sense to distinguish between at least two different systems of cracks; minor secondary cracks, that are located in the close vicinity of the concentrated reinforcement, and a system of primary cracks that penetrates all the way to the position of the neutral axis.

In Fig. 2(a) the average, relative crack-spacing for the same two levels of height is shown for the complete test series, involving a total of eight small beams and seven large, and with the depth being the only intended variation. The crack measurements all refer to a load stage just prior to failure, and basically confirms the tendency just described with respect to the spacing of primary flexural cracks; an apparent linear relation between the absolute depth of the member and the spacing of primary cracks.

The direct consequence of this relation is illustrated in Fig. 2(b). The plot shows the measured maximum crack widths along the shear span for a small and a large beam, respectively, and includes a linear fit for each of the two sets of measurements. For the same relative load level, the expected maximum crack widths in the large beam are seen to be approximately five times greater than for the case of the small beam.

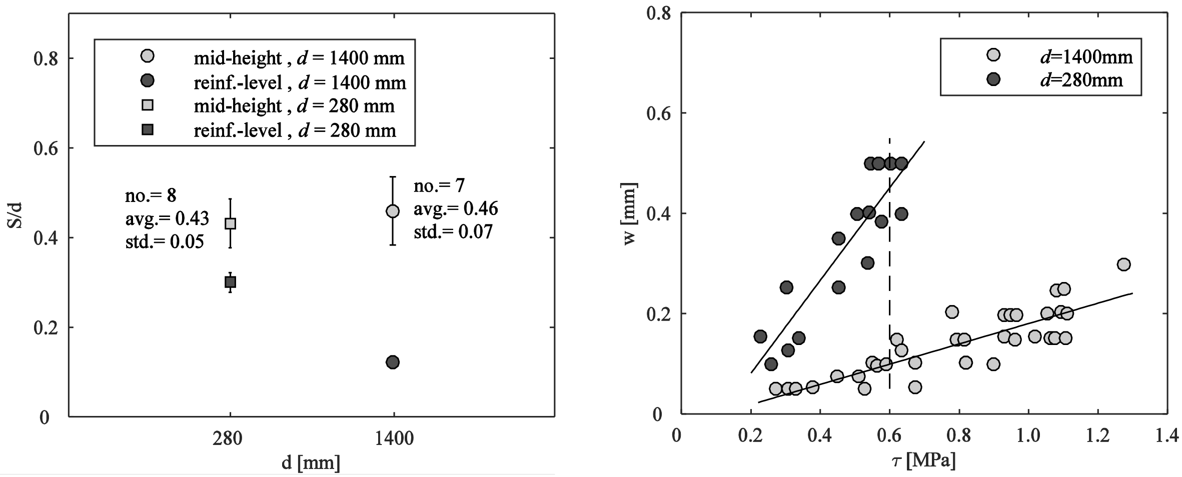

Fig. 2. Test by Sherwood (a) average crack spacing for complete test series; (b) measured crack widths in small and large beams.

3. Existing models

The well-known Tension Chord Model [1] gives a mechanical description of the system of cracks in tensile ties or chord, assuming a crack to occur when the tensile strength of the concrete is exceeded. The spacing of cracks is derived from the length required for the concrete stresses to reach the tensile strength, also referred as the *transfer length*. The tensile stresses at the location of a primary crack are then carried by the reinforcement alone, whereas the concrete adjacent to a crack is able to carry tension which is transferred from the reinforcement by means of bond stresses. From the study of pull-out tests by Shima *et al* [4] the intensity of these bond stresses are assumed to be constant along the length of the reinforcement and equal to twice the tensile strength of the concrete, $f_{ct,}$, at least before any yielding is introduced. This results in linear variations of concrete- and steel stresses, as illustrated in Fig. 3(a). From considerations of equilibrium, the minimum crack spacing, or transfer length, can be derived.

Even though the Tension Chord Model is derived from physical assumptions associated with tension members it has also been widely used on flexural members, where the fictitious tension bar is assumed located symmetrically around the tensile reinforcement. Several other models for flexural members rely on a similar physical approach, although with variations in the quantification of different parameters; the size of the effective concrete area, bond stress intensity, ratio of minimum, maximum and average crack spacing etc. [5]. A fundamentally different approach has, however, been proposed by Beeby [6], defying the presence of slip between reinforcement and concrete.

In the specific case of flexural cracking Reineck [2] proposed a very simple approach, where the spacing between flexural cracks is estimated in accordance with the expression given in Fig. 3(b). Contrary to Tension Chord like-models, this empirical relation offers no apparent physical insight.

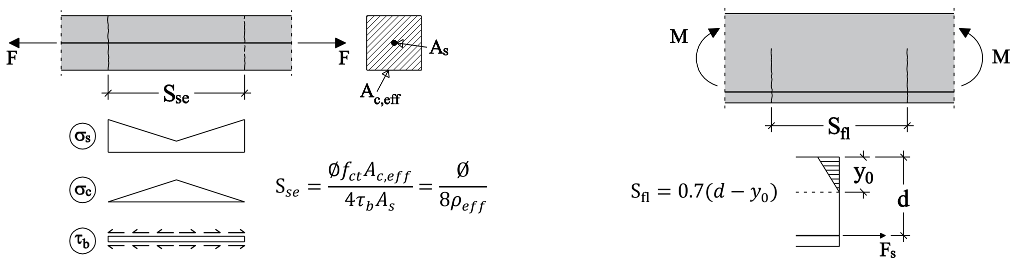

Fig. 3. (a) Tension Chord Model for crack spacing; (b) Estimation of crack spacing in flexural members by Reineck.

4. Proposal for modelling of crack development in beams

In the following an approach is outlined on how crack systems in flexural members can be modelled. The approach is exemplified for the case of a simply supported beam subjected to three-point-bending as illustrated in Fig. 4(a). The approach involves three different steps; description of the system of crack, estimation of reinforcement stresses and, finally, prediction of crack widths from summation of steel strains.

4.1. Crack pattern

The overall crack pattern is assumed to be a combination of two different crack systems; one being the system of cracks, previously described as the flexural cracks, with an individual spacing derived from the approach proposed by Reineck, and a second system of cracks, previously described as the secondary cracks, derived from the approach of the Tension Chord model.

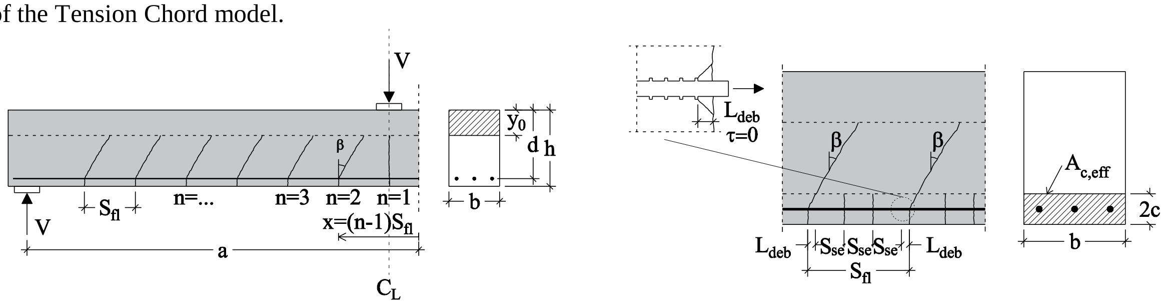

Fig. 4. (a) Assumed flexural crack pattern; (b) Assumed crack pattern of secondary cracks in fictitious tension bar and debonded zone.

The flexural cracks basically forms within a zone where bending moment exceeds the sectional cracking moment, and the spacing between them, S_{fl}, is estimated by the expression in Fig. 3(b). An example of the pattern is shown in Fig. 4(a) where the cracks are also numbered as $n=[1;N_{fl}]$, with N_{fl} being the total number of cracks along a shear span. The flexural cracks are assumed to propagate from the surface of the tensile zone to the neutral axis, being vertical from the surface to the location of the longitudinal reinforcement and continuing with a constant inclination, β. The inclination of the crack located at the center of the beam is zero while, as a simplification, all other cracks are assumed to have an inclination of $\beta=30^\circ$ [7].

In the absolute vicinity of flexural cracks a zone with no bond is assumed to exist. The presence of this debonded zone is illustrated in Fig. 4(b). The extension of the zone will, for reasons of simplicity, be estimated as a constant value of $L_{deb}=0.65c$, where c is the cover of the reinforcement [8].

In between the flexural cracks a system of secondary is allowed to develop. Whether or not these secondary cracks will develop depends entirely upon the spacing between flexural cracks. That is if the spacing is large enough for the bond to transfer stresses corresponding to the tensile force capacity of the fictitious concrete tie. The limit for secondary cracks to exist is;

$$2S_{se} < S_{fl} - 2L_{deb} \quad (1)$$

The secondary cracks are vertical with an individual spacing of S_{se} determined by the expression in Fig. 3(a).

4.2. Distribution of forces in between cracks

The illustrations in the upper part of Fig. 5(a) and (b) show the forces within three flexural cracks. The entire sectional shear force, V, is in the following assumed to be distributed solely along the inclined crack, denoted as V_{incl}.

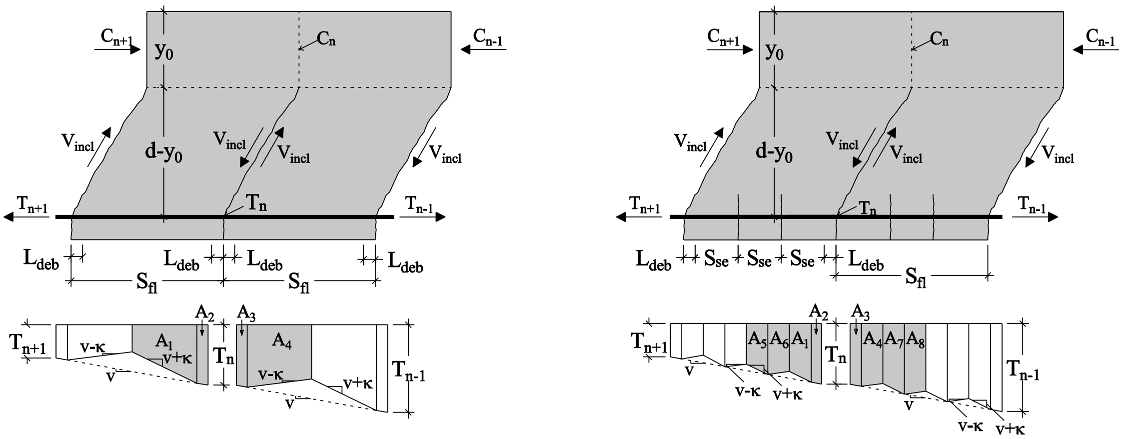

Fig. 5. Forces within three flexural cracks (a) only flexural cracks occurring; (b) both flexural and secondary crack occurring.

From considerations of equilibrium in the n'th crack, the force in the reinforcement can be determined as the sum of a contribution from flexure and the horizontal projection of the inclined shear force;

$$T_n = V\left(\frac{a-(n-1)S_{fl}}{d-\frac{1}{3}y_0} + tan(\beta)\right) \quad (2)$$

The variation of the force in the reinforcement is illustrated on the lower part of Fig. 5(a) and (b). The principal rate, v, of variation is determined by the variation of the bending moment between two cracks;

$$v = \frac{\Delta T}{S_{fl}} = \frac{T_{n-1} - T_n}{S_{fl}} = \frac{V}{d - \frac{1}{3}y_0} \quad (3)$$

while the secondary rate of variation, κ, is determined by the strength of the bond. Hence, the rate of change due to bond stresses becomes;

$$\kappa = \Sigma \emptyset \pi \tau_b \quad (4)$$

4.3. Estimation of cracks widths

From the knowledge of the variation of the reinforcement forces, the width of cracks can be determined as the sum of slip from both sides adjacent to the considered crack. The concrete strains are minor and therefore neglected. When flexural cracks are occurring alone, the flexural crack width at the level of the reinforcement can be determined from the integration of strains over the length of $½S_{fl}$ on both sides of the crack. This is corresponds to the sum of the grey shaded areas in Fig. 5(a) divided by the area of reinforcement and the elastic modulus, that is;

$$w_{fl} = \frac{\sum_{i=1}^{4} A_n}{E_s A_s} \quad (5)$$

When secondary cracks are present, the width of the flexural cracks will be decreased at the level of the reinforcement. At this level the width of the *n*'th flexural crack can still be evaluated using expression (5). However, the sub areas A_1-A_2 will now be located at illustrated in Fig. 5(b). Due to the presence of secondary cracks the width of flexural cracks must vary along the height of the web. Away from the tensile tie the secondary crack closes and from consideration of compatibility it must be reasonable to assume that the width of flexural cracks thus increases accordingly. For the specific case considered in Fig. 5(b) the maximum flexural crack width can therefore be estimated as;

$$w_{fl} = \frac{\sum_{i=1}^{8} A_n}{E_s A_s} \quad (6)$$

5. Preliminary results and discussion

In the following, the approach for estimating the crack pattern of flexural members is applied. Results for the spacing of cracks and crack widths and their variation with respect to important parameters are illustrated.

Fig. 6 illustrates the relative cracks spacing, *S/d*, of the flexural and secondary cracks with respect to varying effective depth, *d*, for three different reinforcement ratios; *0.35%*, *0.7%* and *1.05%*.

The plot reveals that for beams with an effective depth greater than, roughly, 250-300mm two different crack systems will develop. When adopting the simple expression proposed by Reineck, the spacing between the flexural cracks is seen to be linear proportional to the effective depth of the member. In other words, the ratio (S_{fl}/d) is constant when plotted against the effective depth. This is in accordance with the findings in the investigation by Sherwood, see Fig. 2(a). Furthermore the absolute value of this ratio is also in good agreement with the measurements. In the proposal by Reineck the crack spacing is estimated as 70% of the height of the tensile zone in a cracked elastic section based on members subjected to pure flexure. In the case of members subjected to combined flexure and shear it might be argued that this percentage could assume a value of 50-60%.

With respect to the spacing of the secondary crack, approximated by the Tension Chord Model, it can be observed that the ratio *S/d* exhibits a pronounced hyperbolic appearance; hence the crack spacing is not apparently influenced by the variation in effective depth of the member. This result is obvious, since the effective concrete area of the tensile chord includes no reference to the effective depth. Additionally, when comparing the plot to the observations carried out by Sherwood it appears that the spacing between secondary cracks becomes too small with the increase of the effective depth. This could of course be corrected by adjusting a number of the governing parameters of the Tension Chord Model. However it appears intuitively reasonable that the effective concrete area, in this regard, is critical due to the lack of dependence of the effective depth.

Fig. 7(a) illustrates the relations between crack spacing and the reinforcement ratio for members of three different effective depths; *280mm, 800mm* and *1400mm.* All other parameters are scaled in order to approximately maintain geometrical similiude. As also indicated in Fig. 6, spacing between flexural cracks is seen to be rather insensitive towards variation of the reinforcement ratio, especially for members of an effective depth, say less than 800mm.

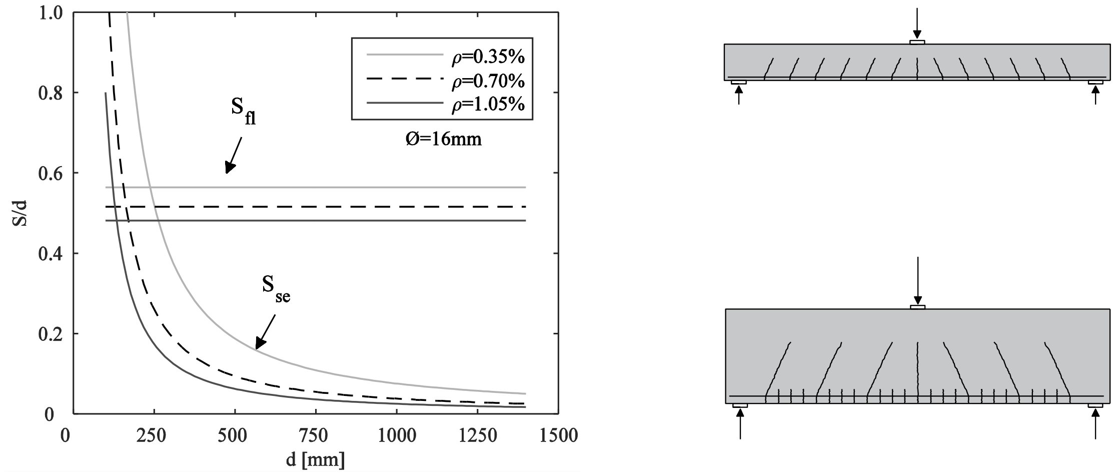

Fig. 6. Relations between relative crack spacing and effective depth, for reinforcement ratios; ρ=0.35%, ρ=0.7% and ρ=1.05%.

Lastly Fig. 7(b) illustrates the relation between the effective depth and the scaling of crack widths. The crack widths are shown relative to the flexural crack width of a member with the effective depth of 280mm. This is done in order to be able to specifically compare the results with the observations by Sherwood. All geometrical parameters are therefore taken indentical to the tested members. The position of the actual crack is irrelevant as it is not related to the scaling. For the beams with an effective depth of less than 300mm only flexural cracks exist due to the fact that the crack spacing from the Tension Chord Model yields greater values than the estimate proposed by Reineck. As seen, the results represents an almost complete replica of the test results reported by Sherwood.

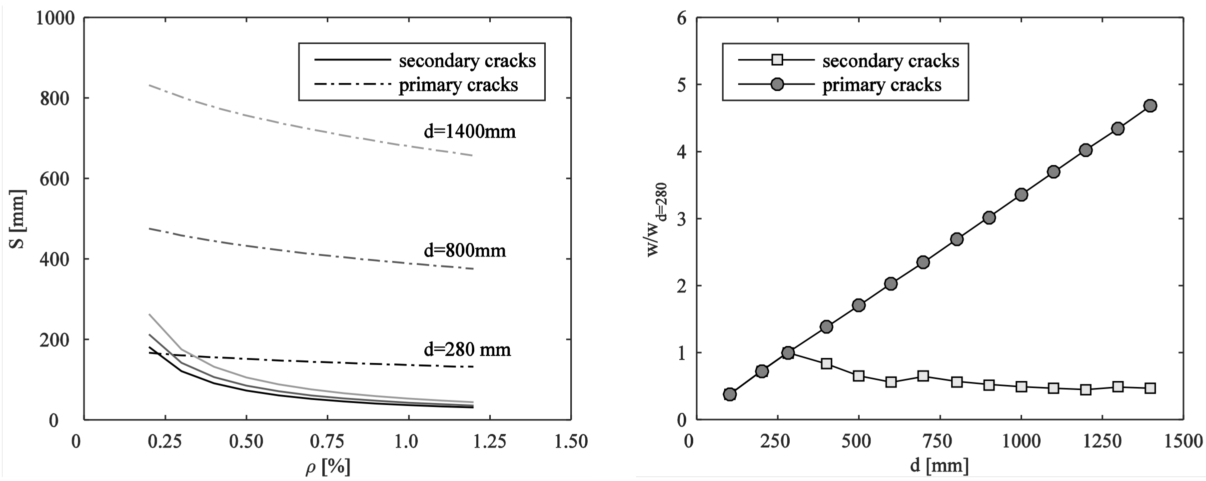

Fig. 7. (a): Relations between relative crack spacing and reinforcement ratio, for effective depths of d=280mm, d=800mm and d=1400mm

(b) Relations between relative crack width and effective depth. Plotted relatively to the case of d=280mm.

6. Conclusion

In this paper the crack pattern in flexural members has been investigated through experimental observations by Sherwood. From analysis of the experimental observations an approach is proposed for predicting crack systems, and more importantly crack widths in beams. The approach assumes the initial existence of flexural cracks the spacing between which is estimated by a simple empirical relation proposed by Reineck. For a certain configuration of the depth, cover, reinforcement ratio etc. secondary cracks are allowed to develop in between the flexural cracks. The spacing between the latter type of cracks is estimated by the Tension Chord Model. Currently the variation of the stresses in the reinforcement is addressed in a simplified way.

Preliminary results are shown for the proposed approach which shows good agreement with the reported test results where it can be observed that for beams with an effective depth of more than, roughly, 250mm the secondary crack system will appear. The theoretical results show that the spacing between the flexural cracks shows a linear proportionality to the effective depth of the member with only a minor variation with respect to the reinforcement ratio, neglectable for beams with an effective depth smaller than about 800mm. This is supported by the experimental observations. On the contrary, the secondary cracks indicate no apparent dependency of the variation in effective depth which is seemingly a consequence of the fact that the effective concrete area implemented in the Tension Chord Model exhibits a constant value independent of the member's depth.

When comparing the results of the estimated crack widths to the measurements that approach it is seen to precisely reproduce the observed size effect in terms of the relative increase. With respect to the absolute values of the estimated crack widths a fairly good agreement is also seen.

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