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Comparison of Conventional Closed-Loop Controller with an Adaptive Controller for a Disturbed Thermodynamic System

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***Abstract*— Non-adaptive proportional controllers suffer from the ability to handle a system disturbance leading to a large steady-state error and undesired transient behavior. On the other hand, they are easy to implement and tune. This article examines whether an adaptive controller based on the MIT and Lyapunov principle leads to a more robust and accurate regulation. Both controllers have been tested on a thermodynamic system exposed to a disturbance. The experiment shows that the adaptive controller handles the disturbance faster and more accurate.**

Keywords—MIT, Lyapunov, Closed-loop proportional controller, Thermodynamic system, external disturbance, MRAC

I. INTRODUCTION

A couple of decades ago, engineers and scientists were satisfied with the conventional control systems (feedback and feed forward). The need for rethinking into the designing of new control system came into play with the design of autopilots aircraft with some sophisticated controller in the early 1950s. Engineers were challenged to create a controller in the context of many uncertainties, and unforeseen large changes in the system parameters. Due to the high complexity and the lack of advanced technology at the time, the idea of adaptive controller design diminished [3]. However, the high demand for a sophisticated advanced adaptive controller became consistent and the booming interest became inevitable due to the incompleteness of the conventional controllers in some specific conditions [4]. Following the stability of the adaptive controllers in 80's and the robustness in the 90's, several efforts have been made to improve the transient performance [13][14].

But the interest in the design of an adaptive controller instead of a conventional controller started booming due to some of the following reasons [4] [8] [13]:

- Unpredictability of the system changes
- External and internal unexpected disturbances
- Due to the possibility of the sudden faults.
- Nonlinear behavior in cases of complex systems
- Unknown parameters.

However, [12] closed loop proportional controllers have an easily implementable controller algorithm, which is stated as:

$$e(t) = K \cdot (T_{ref} - T_{measure}) \quad (1)$$

and for lower order systems with adequately powerful actuators,

$$P_{out}(t) = K_{out} \cdot e(t) \quad (2)$$

they are rather stable [15], since the regulator does not saturate, keeping the regulated system responsive and non-erratic.

As stated in [2] an adaptive controller is a system, where in addition to the basic closed loop feedback controller, explicit measures are taken to compensate for variations in the process dynamics or for variations in the disturbances, in order to maintain the optimal performance and robustness of the system. Slotine and Li [9] argues that adaptive controller enjoys the same level of robustness to dynamics as a conventional feedback controller, yet obtain much better tracking accuracy than the conventional feedback controllers. Furthermore, its superior performance for high-speed operations, in the presence of parametric and nonparametric un-predictabilities, and its relative computational simplicity make it an interesting option for the industrial applications. However, there are some drawbacks related to the adaptive controllers [3]: mathematically complicated design (system modelling, Laplace- and z-transformations), large unpredictable error/fault may lead to the deterioration of the robustness and performance or lead to complete system failure, and adaptive strategy may happens sometimes too fast resulting oscillation or too slowly resulting irrelevant application.

In this paper, an attempt is made to compare the robustness and the performance accuracy of the adaptive controller with the non-adaptive controller in the context of heating an aluminium plate in the presence of external disturbance.

This paper is organized as follows. The section II introduces the methodology employed in the research. Section III presents the literature review with the theories that are used in the paper. Experiment and results are presented in section IV. Discussion and conclusions are given in sections V & VI.

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II. METHODOLOGY

A heating system has been developed to compare the robustness and the performance between the ordinary proportional controller and the direct proportional adaptive controller. In this paper, the MIT rule and the Lyapunov rule of the direct proportional adaptive controller have been developed and discussed. The system has been built as shown in the figure 5 to implement the ordinary and the adaptive controllers to evaluate as to which controller is efficient and robust in the face of external disturbance. The reference input for different gains has been chosen as 30^0C.

III. LITERATURE REVIEW

This section provides a detail description of the theories that have been employed in the paper. Since the primary objective of this paper is not the proof of adaptive theories, yet brief description of each theory is presented in the paper.

A. Closed Loop Proportional Controller (Non-Adaptive)

The proportional controller output is proportional to the current error and this error can be minimized by multiplying a constant positive gain which is called proportional gain as illustrated in figure 1. The relation can be seen in the following time domain equation [11]

$$u_c(t) = K_P e(t) \quad (3)$$

Where $u_c(t) -$ output of the controller $e(t) -$ systems error $K_P -$ proportional gain

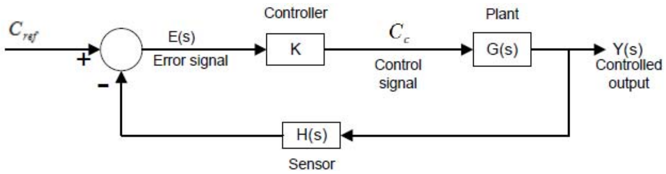

Figure 1: Conventional closed loop proportional controller

Transfer function for the closed loop proportional controller is

$$G_{CL}(s) = \frac{G(s)K(s)}{1 + H(s)G(s)K(s)} \quad (4)$$

Increasing the proportional gain to a large value may lead the system unstable whereas decreasing the proportional gain may lead the system to be less responsive and sluggish to the systems external disturbance [10]. However, the proportional gain has to increase to a larger value, if the systems error is very large. Yet an extensively large gain may lead to instability.

B. Componets of Model Reference Adaptive Controller (MRAC)

MRAC strategy is used to develop and design adaptive controller that is capable of adjusting the controller parameters so as to keep the output of the plant track the output of the reference model [7]. MRAC is composed of two loops. An inner (controller) loop which is a conventional control loop consisting of the plant and the controller. An outer loop is a parameter adaptation loop that adjusts the parameters in such a way to minimize the error between the plant and the reference model to zero. MRAC has the following components [8]:

Reference model: it is used to specify the ideal output response of the adaptive controller. The ideal behaviour of the reference model must be obtainable by the adaptive controller.

Controller: It is commonly defined by a set of adjustable parameters. In this paper, the control parameter is a proportional gain K. The value of K is primarily dependent on the adaptation gain.

Adaptation mechanism: The adaptation law constantly adjusts the parameters such that the output response of the actual plant is same as the reference model output. The adaptation law is designed with the objective of stabilizing the control parameters and convergence of minimising error to zero. The adaptation mechanism can be developed using MIT rule, Lyapunov rule and augmented error theory [8]. In this paper, MIT rule and Lyapunov rule are utilized for this purpose.

C. The MIT Rule

The primary principle behind MIT rule is to minimize the tracking error; *e* as shown in figure 2. The error is calculated as the difference between the plant and the reference model [5]

$$e = y_{plant} - y_{Model} \quad (4)$$

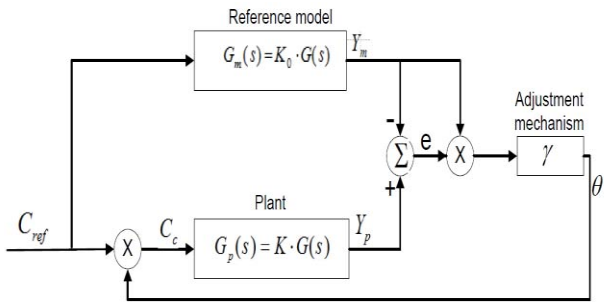

Figure 2: Adaptive controller based on MIT Rule

A cost function is defined as the function of theta (θ) which will be an adaptive parameter in the controller in order to minimize the error. Since the objective is to minimize the cost function to zero, the parameter θ has to follow the "steepest descent", that is to move in the direction of the negative gradient of J with a positive adaptation gain α [1]

$$J(\theta) = \frac{1}{2} \cdot e^2(\theta) \quad (5)$$

The change in J is proportional to the change in θ according to the equation 4. Therefore derivative of θ will be as follows [10]:

$$\frac{d\theta}{dt} = -\alpha \cdot \frac{\delta J}{\delta \theta} = -\alpha \cdot e \cdot \frac{\delta e}{\delta \theta} \quad (6)$$

The partial derivative $\frac{\delta e}{\delta\theta}$ is called "sensitivity derivative". This shows the correlation between e and θ; change of error with respect to θ. This also shows that the cost function J can be minimized to zero. Therefore the MIT rule is all about the relation between the cost function J and the change of θ.
The update law for the MIT rule is achieved through the derivation is as follows [1] [10]

$$\frac{d\theta}{dt} = -\gamma \cdot e \cdot \frac{k_o}{s+a} \cdot C_{ref} \tag{7}$$

where γ is an adaptive gain.
From this equation (6), the MIT rule function for minimising the error can be deducted to:

$$\frac{d\theta}{dt} = -\gamma \cdot e \cdot y_m \tag{8}$$

D. The Lyapunov Rule

A new method of designing MRAC by making use of the stability theory was suggested by Parks [3] as shown in the figure 3. The update law is same as that of MIT rule, but the sensitivity derivative is represented by another function.

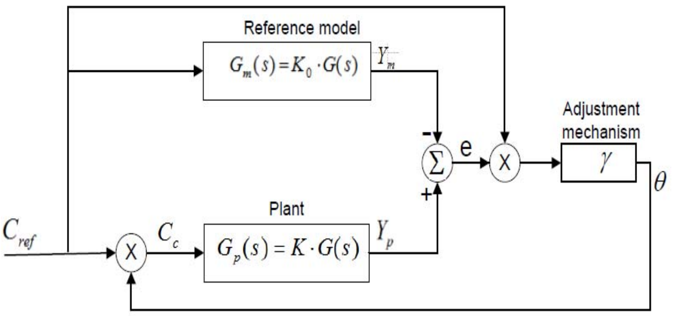

Figure 3: Adaptive controller based on Lyapunov Rule

The Lyapunov function is as follows according to Astrom and Wittenmark [3]

$$V = \frac{1}{2}\gamma e^2 + \frac{1}{2}k(\theta - \frac{k_o}{k})^2 \tag{9}$$

When the time derivative of the Lyapunov function $\frac{dV}{dt}$ yields as

$$\frac{dV}{dt} = \gamma e\dot{e} + k\dot{\theta}(\theta - \frac{k_o}{k}) \tag{10}$$

The negative gradient of the function will guarantee the converging error to zero. And the update function for the adaptive controller according to Lyapunov theory is

$$\frac{d\theta}{dt} = -\gamma e u_{ref} \tag{11}$$

IV. EXPERIMETS AND RESULTS

The heating system has been built using aluminium (Al) plate since Al has a very high specific heat (900 J/kg.^{o}C) and relatively high heat-conductivity (238 W/m.^{0}C). The schematic diagram illustrated in the figure 4 shows how the components are connected. The proportional controller or adaptive MIT rule/ Lyapunov rule controller is programmed in mbed (LPC 1768) in C++ language.

Depending on the analog input (P20) from the sensor, the mbed microprocessor will control the power MOSFET on how much power should be provided to the resistor to heat the aluminium plate. If PMW is zero, there is no power output to the resistor whereas when PWM is one, the power is fully supplied to the resistor to heat the plate. Otherwise the power supply to the resistor is rated 0 to 1 depending on the sensor measurement.

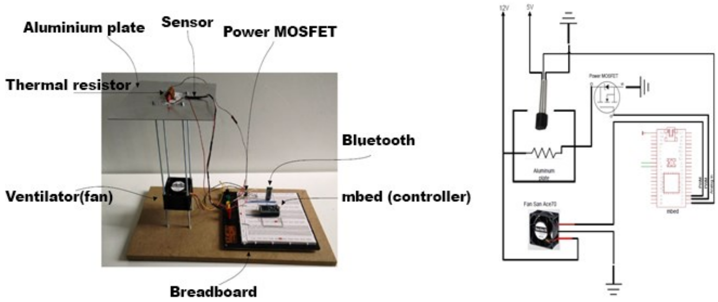

Figure 4: The heating system with its schematic diagram

The Fan San Ace70 is the external disturbance to the aluminium plate. When the fan works at the highest speed, it will blow a great amount of air and increase the convection and radiation. Thus the temperature of the aluminium plate will fall relatively low while the controller in the mbed will try to keep the temperature of the aluminium plate at the reference set-point.
The heat capacity **C** of any material is defined as the quantity of energy required to increase the temperature by 1 ^{o}C. If the change of temperature ΔT is achieved by giving Q energy, it can be stated by using the heat capacity as

$$Q = C \cdot \Delta T \tag{12}$$

The specific heat c of a material is the heat capacity C per unit mass [6]. Therefore the specific heat c can be expresses as follows:

$$c = \frac{Q}{m \cdot \Delta T} \Leftrightarrow \quad Q = m \cdot c \cdot \Delta T \tag{13}$$

The equation can be rewritten as follows in Power (watt)

$$\frac{dQ}{dt} = P(W) = m \cdot c \cdot \frac{dT}{dt} \tag{14}$$

The figure 5 illustrates the process of heat transfer of the aluminium plate. Once the power (P_{in}) is supplied in the form of heat on the plate, temperature of the plate (T) increases and the radiation of heat ($Q'_{radiation}$) and convection of heat ($Q'_{convection}$) begin to rise. Moreover, the atmospheric temperature (T_{atm}) has an impact on the system's heating process.

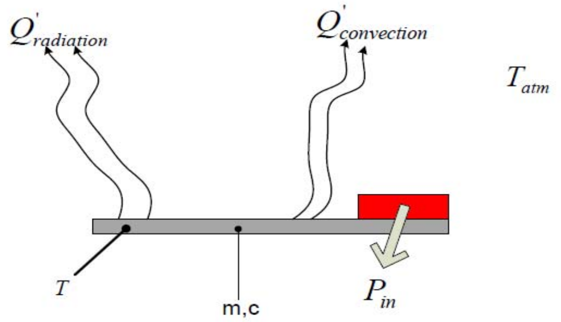

Figure 5: Thermodynamic s process of the aluminium plate

According to 1st law of Thermodynamics $\Delta E_{\text{int}} = Q + W$ [6], the following assumption can be taken

$$\Delta E = Q \Rightarrow \quad m \cdot c \cdot \frac{dT}{dt} = Q^0 = P_{in} - Q^0_{radiation} - Q^0_{convection} \tag{15}$$

Ignoring radiation since the heat convection much more dominant due to the fan, the system equation becomes:

$$m \cdot c \cdot \frac{dT}{dt} = P_{in} - h \cdot A \cdot (T - T_{atm}) \;\Rightarrow\; \frac{dT}{dt} + \frac{h \cdot A}{m \cdot c} \cdot T = \frac{P_{in}}{m \cdot c} + \frac{h \cdot A}{m \cdot c} \cdot T_{atm} \tag{16}$$

A. Open Loop and Closed Loop Controller

This section outlines the comparison of closed and open loop systems.

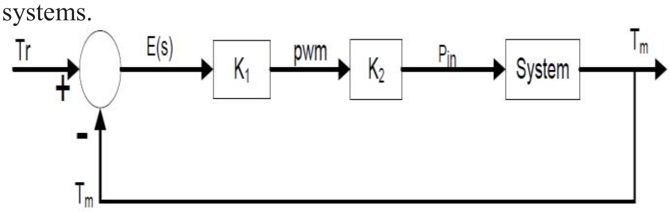

Figure 6: Closed loop controller for the heating system

Solution to this 1st order linear differential equation for the open loop system is:

$$T(t) = \frac{P_{in}}{h \cdot A} + T_{atm} + \left[T_0 - \frac{P_{in}}{h \cdot A} - T_{atm} \right] \cdot e^{-\frac{h \cdot A}{m \cdot c} \cdot t} \tag{17}$$

(15)

Solution to the eq. 15 can be simplified as:

$$y(t) = T_m = \frac{q}{p} + \left[T_m - \frac{q}{p} \right] \cdot e^{-pt} \tag{18}$$

Where $p = \dfrac{hA}{mc}$ and $q = \dfrac{P_{in} + hAT_a}{mc}$

Closed loop controller can be computed as follows:

Controller: $pwm = K_1 [T_r - T_m]$ where $K_1 = \dfrac{pwm_{\max}}{\Delta T_{\max}}$

Power: $P_{in} = K_2 \cdot pwm$ where $K_2 = \dfrac{E^2 / R}{pwm_{\max}}$

System:

$$m \cdot c \cdot \frac{dT}{dt} = P_{in} - h \cdot A \cdot (T_m - T_a) \tag{19}$$

Combining:

$$m \cdot c \cdot \frac{dT_m}{dt} = K_1 K_2 [T_r - T_m] - h \cdot A \cdot (T_m - T_a) \tag{20}$$

Differential equation:

$$\frac{dT_m}{dt} + \frac{K_1 K_2 + hA}{mc} \cdot T_m = \frac{K_1 K_2 T_r + hAT_a}{mc} \tag{21}$$

Solution:

$$\frac{dy}{dt} + py = q \; ; \; y(0) = y_0 \Rightarrow \quad y(t) = \frac{q}{p} + \left[y_0 - \frac{q}{p} \right] \cdot e^{-pt} \tag{22}$$

Where $p = \dfrac{K_1 K_2 + hA}{mc}$ and $q = \dfrac{K_1 K_2 T_r + hAT_a}{mc}$

If the heating system is applied with the real values for closed loop time response:

$K_1 = 5; K_2 = 0.681; h = 10; A = 0.04; m = 0.112;$

$c = 900; T_r = 30; T_a = 0$

When these parameter values are computed $p = 0.038$ and $q = 1.013$, then $T_m(t) = \dfrac{q}{p} + \left[T_0 - \dfrac{q}{p} \right] \cdot e^{-pt}$ can be computed for the time $t = 0,1..200$ with the error $err(t) = T_r - T_m(t)$.

For the open loop system with $p_o = 0.004$, $q_o = 0.676$ and $P_{in} = \dfrac{15^2}{3.3} = 68.182$, $T_{mo}(t) = \dfrac{q_o}{p_o} + \left[T_0 - \dfrac{q_o}{p_o} \right] \cdot e^{-p_o t}$ can be computed.

Figure 7 illustrates the open loop and closed loop systems time constant response versus temperature. From figure 7, the following observations can be inferred: closed loop controller becomes much faster than open loop controllers, secondly, the parameter that increases the closed loop systems dynamics is the power gain. Time constant of the closed loop controller as the explicit equation of the system gives a deep insight in the understanding of the systems dynamics: $\tau = \dfrac{1}{p} = \dfrac{mc}{K_1 K_2 + hA}$ which confirms that the power gain and the mass of the material increase the systems dynamic.

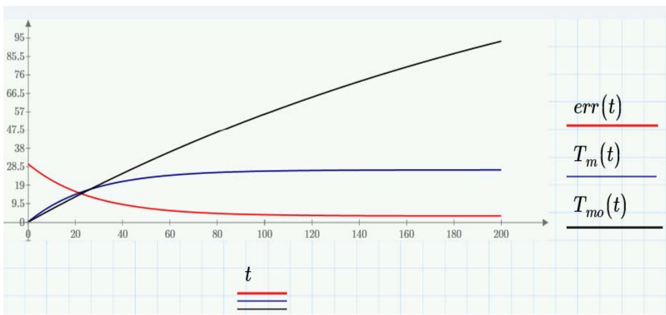

Figure 7: Time constant vs temperature

B. *Experimental Results*

As the experiment (figure 4) results shown in the figure 8, the closed loop feedback proportional controller is tested in the presence of the disturbance (left) and without the disturbance (right) for different gains.

As illustrated in the figure 8 (right), where there is no disturbance, the proportional controller regulates the system significantly well for higher gains as the adaptive controller performs in lower gain. When the disturbance is high, the time responses of both the proportional and the adaptive controller are comparable, but the steady state error is much higher for the proportional controller.

The figure 9 illustrates the output responses of the MIT adaptive controller for the gain range k=0.001- 5 in the presence of external disturbance at fan=50%.

The adaptive controller performs as expected with minimum steady-state error and there is no overshoot in the transient response whereas the non-adaptive proportional controller is completely unstable as shown in figure 8 in the same range. Furthermore, the fluctuations of the MIT adaptive controller found at lower gains significantly improve as the gain increases.

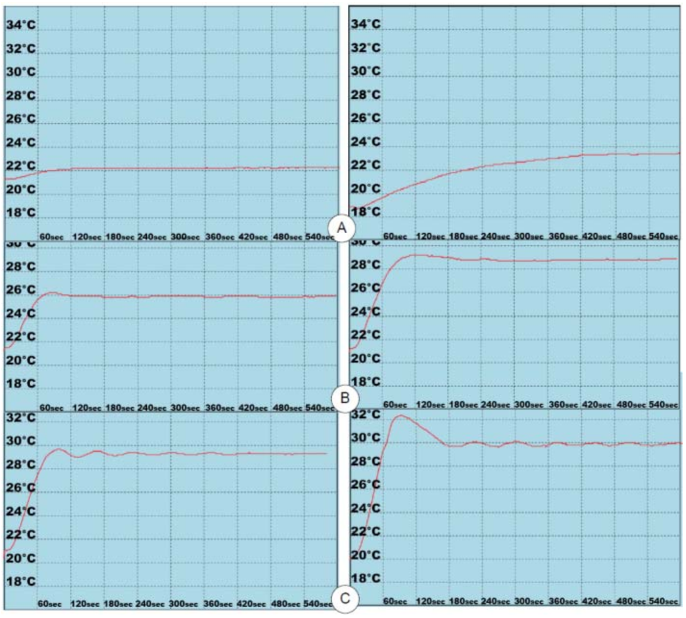

Figure 8: The response of non-adaptive controller at gain A) k=0.001 , B) k=0.5, C) k=1 with the external disturbance at fan=50%(left) and without any external disturbance at fan=0% (right).

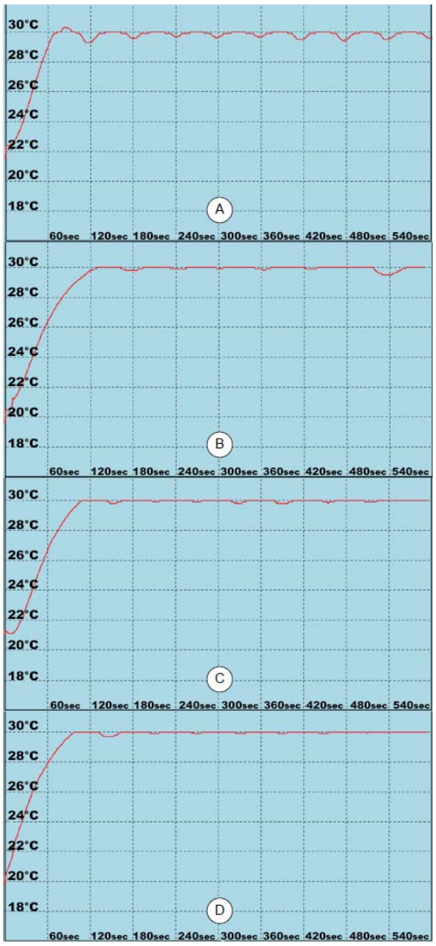

Figure 9: The response of adaptive controller MIT at gain A) k=0.001 , B) k=0.5, C) k=1 and D) k=5 with the external disturbance at fan=50%

V. DISCUSSION

There are two types of stability in any control systems [11]:

1. Absolute stability: it is all about the system as a whole to see whether the system is in a stable condition or not.
2. Relative stability: it is all about how much the system is deviating from the absolute stability by observing two main parameters:
    - Transient response of the system: the response of the system until it becomes stable; rising time, overshooting ,damping level, settling time
    - Steady state response of the system: the response of the system after it has settled: steady state error, steady state tolerance limit, settling patterns

The main focus in this paper is given to both stability issues in the discussion. Designing a system with both stability issues to be at optimal is a challenge. By looking at table 1 from the responses of the experiment, the following observations can be made:

The adaptive controllers take approx. 120 -130 sec to settle down to the stability when there is no external disturbance. There is always overshoot in the transient response. It is due to the fact that the power MOSFET provides 100% power to the thermal resistor until the temperature is very close to the reference set-point 30^0C. Even though the power MOSFET doesn't provide any power to thermal resistor at 30^0C, the temperature of the aluminium plate keeps on increasing approx. 3 ^{0}C. The increment in the settling time is due to the overshoot. The MIT and the Lyapunov are comparatively performing very well.

The stability is achieved in the range of approx. 60 -80 sec for the adaptive controllers when there is an external disturbance. This is exactly expected of the adaptive controllers in the presence of uncertainty and disturbance. Looking at the settling time for different gains, it can be confirmed that the gain is not a matter for the faster transient response at all. However, increment of gains improves the fluctuation level within the steady state tolerance band. Higher the gain is the less fluctuation the system tends to depict.

Another observation is that there is no overshooting effect when there is external disturbance. This is the reason for the shorter settling time for the system. It is due to the fact that the adaptive controllers try hard at bringing the system to the reference set-point. In doing that, the cool air from the fan brings the temperature down. As soon as the temperature strikes at 30^0C, the power MOSFET doesn't stop providing power whereas it finds a good balance to keep the temperature at the reference set-point in the presence of disturbance.

Non-adaptive controller cannot achieve the stability at the lower gain especially when there is/ isn't external disturbance and even though the non-adaptive controller is at the higher gains, there is a very clear tendency towards instability. The fluctuation becomes higher at the steady-state tolerance band. However, increment of proportional gain makes the settling time faster and improves a bit steady state error.

TABLE 1: COMPARISON OF STEADY STATE ERROR OF THE 3 CONTROLLERS

Detail of Gain and Disturbance	Non-Adaptive Controller	MIT Rule	Lyapunov Rule
K=0.01 & fan=0	84.38%	3.125%, fluctuations within the tolerance band	2.5%, fluctuations within the tolerance band
K=0.01 & fan=50	100%	5%, fluctuation response	5%, decaying fluctuations
K=0.1 & fan=0	13.75%	5%, fluctuations within the tolerance band	5%, fluctuations within the tolerance band
K=0.1 & fan=50	51.25%	2.25%, very small fluctuation	2% very small fluctuations
K=0.5 & fan=0	5%	5%, fluctuations within the tolerance band	5%, fluctuation within the tolerance band
K=0.5 & fan=50	21.25%	1.50%, very small fluctuation	1.25%, very small fluctuation
K=1 & fan=0	3.7%	5%, fluctuations within the tolerance band	5% , fluctuation within the tolerance band
K=5 & fan=50	8.75%	1.25%, fluctuations at the start and no fluctuation after 500s	1.25%, very few fluctuations
K=20 & fan=50	8%	1.2%, very small fluctuation at the start and vanishes at the end	1.2%, almost no fluctuation

By observing the steady-state error at the table 1, it can be inferred that the adaptive controllers are stable and the steady state error is within the tolerance band for any gain (0.01- 20). Steady state error is very small. The response fluctuation within the tolerance band is decreasing gradually and comes to zero in the course of time.

Non-adaptive proportional controller cannot work well for the lower proportional gains at the range [0.01 – 0.1] even when there is no external interference. The steady state error is very high [84.38% - 13.75%]. However, the performance of the non-adaptive controller in the presence of external disturbance is extremely high divergent [100% - 51.25%].

As the proportional gain increases, the steady state error decreases, but cannot be eliminated [11].

VI. CONCLUSION

It can be inferred that the adaptive proportional controllers such as MIT rule and Lyapunov approaches are exceptionally good at handling the external disturbance and the steady state error is remarkably minimal even at the lower gain. The settling time of the system is sufficiently small. Overall performance and robustness of the adaptive controllers are overwhelming and amazing.

It can also be deduced that the non-adaptive proportional controller are sluggish and unstable in the presence of disturbance. The settling time for the response takes longer time even at the higher gain. However, the fluctuation of the steady state error is high and leads to undesired oscillation.

Comparing the performances and the robustness of both controllers, adaptive controllers outperform the non-adaptive controllers in many different ways.

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