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On minimizing the influence of the noise tail of correlation functions in operational modal analysis

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Abstract

In operational modal analysis (OMA) correlation functions are used by all classical time-domain modal identification techniques that uses the impulse response function (free decays) as primary data. However, the main difference between the impulse response and the correlation functions estimated from the operational responses is that the latter present a higher noise level. This is due to statistical errors in the estimation of the correlation function and it causes random noise in the end of the function and this is called the noise tail. This noise might have significant influence on the identification results (random errors) when the noise tail is included in the identification. On the other hand, if the correlation function is truncated too much, then important information is lost. In other to minimize this error, a suitable truncation based on manual inspection of the correlation function is normally used. However, in automated OMA, an automated procedure is needed for the truncation. Based on known theoretical solutions from the literature, a model is proposed in this paper to automatically truncate the correlation function at the point where it starts to get dominated by the noise tail. The accuracy of the proposed truncation procedure is studied using a three degree of freedom simulation case.

Keywords: Operational Modal Analysis; Correlation Function; Noise Tail

1. Introduction

In operational modal analysis (OMA) the common practice is to identify modal parameters by use of the spectral density functions in the frequency domain or the correlation functions in the time domain. We treat the transposed correlation function matrix as a free decay and we are able to extract the modal parameters of the system using any time domain identification technique from modal analysis [1–3].

However, the experimental correlation function matrix is an estimate due to a finite time length and this introduces statistical errors. These errors are random and they increase as the time lags advance while the correlation function

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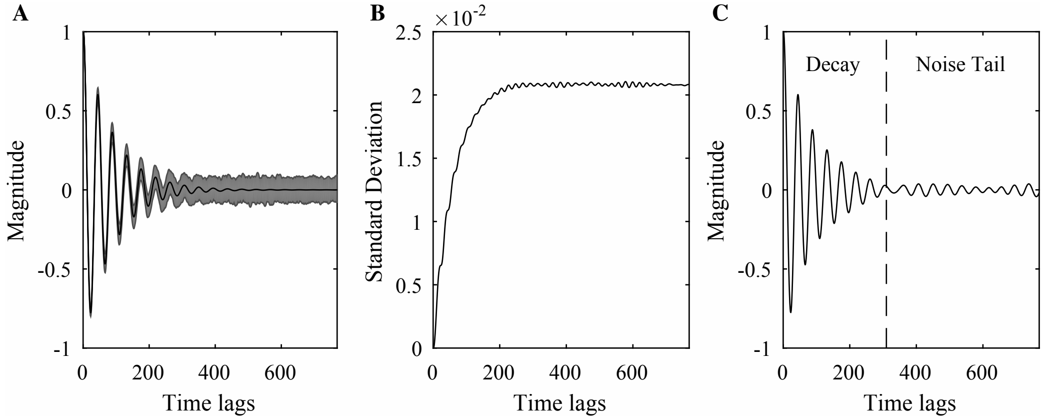

Fig. 1. A) 10^4 iterations of the estimated correlation function (*grey*) and the mean correlation function (*black*), B) Standard deviation of the estimated correlation functions, C) Illustration of a noise tail on a random iteration of the estimated correlation function

approaches zero. At a certain time lag, these errors will dominate the correlation function and it results in part dominated by noise. In order to reduce the statistical errors this part needs to be disregarded. Hereby, we want to estimate the modal parameter from the part of the correlation function matrix related to the physics of the system.

The phenomenon of the noise tail is only scarcely investigated in the literature. In [4] it is shown that statistical errors are introduced by estimating the correlation function. Additional, [5] shows the effect of random errors in the correlation function estimation for OMA. The analytic expression for the variance of the estimated correlation function is shown for a single degree-of-freedom in [6] and the author shows that the tail region of the estimated correlation function has erratic behaviour. In this paper, we focus on the phenomenon of the erratic behaviour in the tail region and how we can reduce the statistical errors on the correlation function matrix by detecting this noise tail.

2. Theory

2.1. Noise Estimation

In operational modal analysis we assume that the loading has white noise characteristics and the system is linear. This gives us a random response vector, $\mathbf{y}(t)$, which is measured in N_d degree of freedom on the structure. We presume that the response is ergodic and it has zero mean. The correlation function matrix is defined as [4]

$$\mathbf{R}_y(\tau) = \mathrm{E}\left[\mathbf{y}(t)\mathbf{y}^T(t+\tau)\right] \quad (1)$$

Since the signal is ergodic then we can use *time averaging* instead of *ensemble averaging*. However the length of the signal must tend towards infinite if the *time averaging* should be equal to the expected value [4].

$$\mathbf{R}_y(\tau) = \lim_{T\to\infty} \frac{1}{T} \int_0^{T-\tau} \mathbf{y}(t)\mathbf{y}^T(t+\tau)\, dt \quad (2)$$

In reality, a recorded system response has a finite length and this introduces statistical errors since we must estimate the correlation function matrix instead [4].

$$\hat{\mathbf{R}}_y(\tau) = \frac{1}{T-\tau} \int_0^{T-\tau} \mathbf{y}(t)\mathbf{y}^T(t+\tau)\, dt, \quad 0 \le \tau < T \quad (3)$$

The finite time length introduces statistical errors that depend on the modal parameters, time length and time lags. We can illustrate the error by a Monte-Carlo simulation where we simulate an arbitrary SDOF system 10,000 times and calculate the correlation function for each iteration by the discrete version of Eq. (3). The simulation uses the Fourier transformed superposition or convolution integral for a linear system. The initial amplitude of a correlation

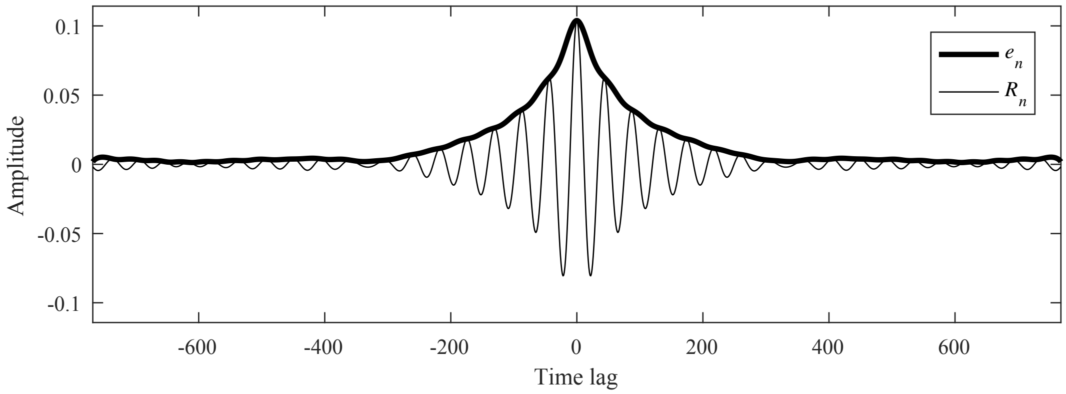

Fig. 2. Envelope detection on full correlation function matrix

function is not related to the modal parameters but it is equal to the covariance of the system responses, [2]. We scale each estimated correlation function so they have an initial amplitude of 1, see Fig. 1 A). The standard deviation of the estimated correlation function is related to the statistical errors and it is illustrated on Fig. 1 B). The standard deviation increases as we increase the time lag until it reaches a certain value and flattens, see Fig. 1 B). The statistical errors from this flat part result in a noise dominated part in the correlation function matrix, which is often referred to as the *noise tail*, see Fig. 1 C).

2.2. Decomposition of modal components

Envelope detection using Hilbert transformation works for a single modal component, but it fails for a signal influenced by multiple modal components. We would like to isolate the influence of the individual modes so we can apply a Hilbert transformation to detect the envelope of the correlation functions. One solution is to use modal decomposition to decorrelate the correlation function matrix by an initial estimated mode shape matrix, $\hat{\mathbf{B}}$. In the latter approach, we use the *Frequency Domain Decomposition* technique to estimate the mode shapes [7]. Then we calculate the decorrelated correlation function matrix for the modal coordinates and we use the autocorrelation functions of this matrix to detect the envelopes.

$$\mathbf{R}_q(\tau) = \hat{\mathbf{B}}^{+}\mathbf{R}_y(\tau)\left(\hat{\mathbf{B}}^{+}\right)^H \tag{4}$$

2.3. Envelope Detection

In Signal Processing, we can use the Hilbert transformation as envelope detection. The Hilbert transformation transforms a real signal to a 90-degree phase-shifted version of the original signal [8]. Consider that we have a real time domain signal, R_j, which in our case is an full auto-correlation function. When we apply the Hilbert transformation to the signal then we obtain a phase shifted signal, $\mathcal{H}\left[R_j\right]$. We introduce the analytic signal, A_j

$$A_j = R_j + i \cdot \mathcal{H}\left[R_j\right] \tag{5}$$

where $\mathcal{H}$ denotes the Hilbert transform. The absolute value of this analytic signal yields an approximate envelope of the original signal, see Fig. 2.

$$e_j = \left|A_j\right| = \sqrt{R_j^2 + \left(i \cdot \mathcal{H}\left[R_j\right]\right)^2} \tag{6}$$

We know from the theory that the analytic correlation function is a free decay and the statistical errors are constant in the noise tail. Therefore, the envelope function for the correlation function is a negative exponential function for

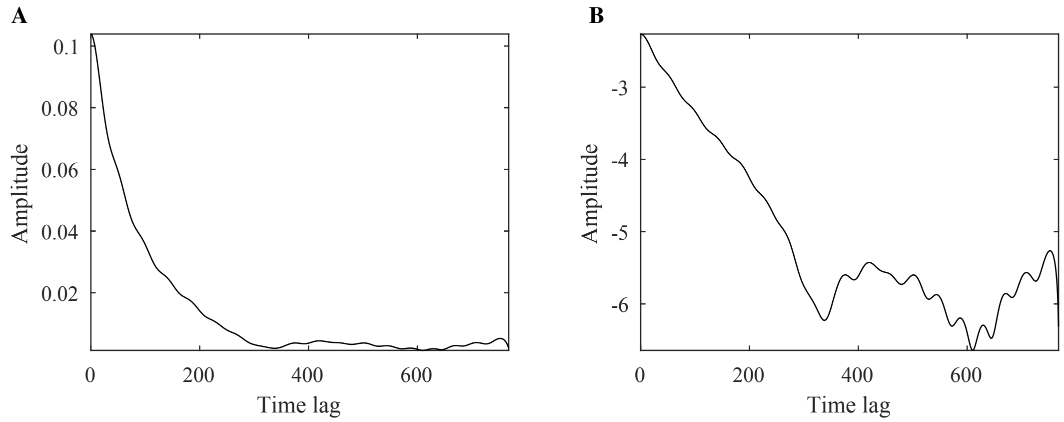

Fig. 3. A) Estimated envelope, B) Logarithmic scale of estimated envelope

the physical part whereas the envelope function is approximately constant in the noise tail, see Fig. 3 A. We use the natural logarithmic scale of the envelope function so the physical part turns into a line with negative slope and the noise tail is a horizontal line with fluctuation, see Fig. 3 B.

2.4. Curve Fitting

The envelope has two parts: an exponential decay governed by the physics of the system and a ”flat” part dominated by noise. This paper introduces two mathematical models to describe each of these parts by fitting to the logarithmic envelope. The first model is a negatively sloping line, which describes the physics of the system, and the second model is a constant value for the noise tail. The two models are used in continuation with a variable transition, n, between the models. We start by dividing our logarithmic envelope into two vectors

$$\begin{aligned} \mathbf{m_1} &= \log\left(\begin{bmatrix} e_1 \; e_2 \; \dots \; e_n \end{bmatrix}\right)^T \\ \mathbf{m_2} &= \log\left(\begin{bmatrix} e_{n+1} \; e_{n+2} \; \dots \; e_N \end{bmatrix}\right)^T \end{aligned} \tag{7}$$

where $\mathbf{m_1}$ is related to the first model and $\mathbf{m_2}$ is related to the second model. We use linear regression to fit the two models to the envelope [9]. Then, we have the two models $\hat{\mathbf{m}}_1$ and $\hat{\mathbf{m}}_2$ that we add together in one vector to express the estimated envelope

$$\hat{\mathbf{e}} = \exp\left(\begin{bmatrix} \hat{\mathbf{m}}_1 \\ \hat{\mathbf{m}}_2 \end{bmatrix}\right) \tag{8}$$

We repeat this process for all possible values of n. The coefficient of determination [9] is used for each iteration of n as a quality measurement.

$$r^2(n) = 1 - \frac{\sum_{k=1}^{N} (e_k - \hat{e}_k)^2}{\sum_{k=1}^{N} (e_k - \mathrm{E}[\mathbf{e}])^2} \tag{9}$$

This is a function of the n, which we used to separate the two models by. The two different models have different errors for different values of n. The first model has a relative unchanged error until we move n into the noise tail. In this region the error increases as we increase the value of n. The second model has a high error when n is located outside the noise tail and it has a relative unchanged error in the noise tail. This is illustrated on Fig. 4.

The highest value of $r^2(n)$ indicates the best fit of the two models and thereby we know the value of n that correspond to the start of the noise tail.

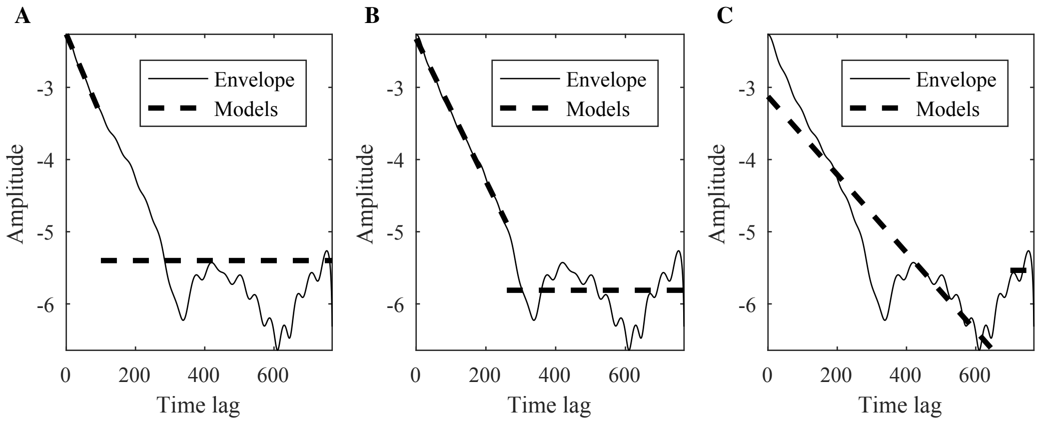

Fig. 4. A) n in exponential decay , B) n between decay and noise tail, C) n in noise tail

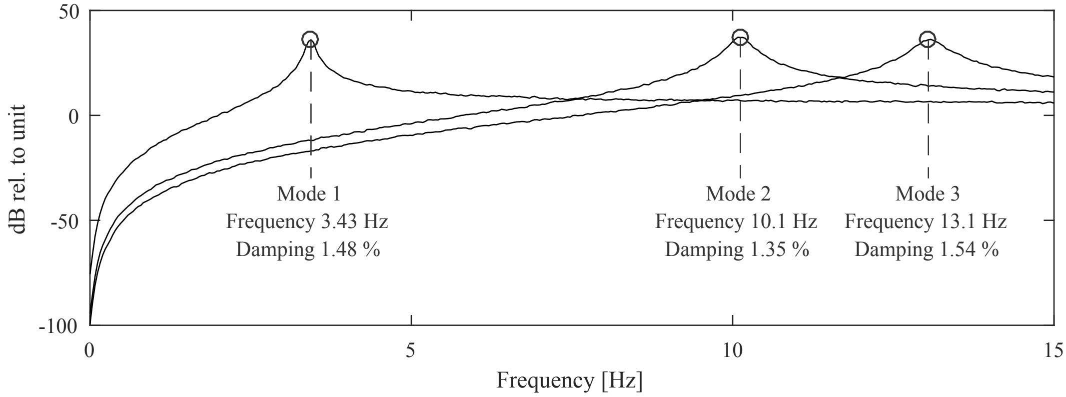

Fig. 5. Singular Values of Spectral Density Matrix

3. Case Study

We will show the proposed detection of the noise tail on a simulated 3DOF system. Fig. 5 shows the singular values of the spectral density matrix of the simulated system.

The mode shapes are known from the Frequency Domain Decomposition technique and these are used to decorrelate the correlation functions, see Fig. 6. In this case, we use the correlation function with the fastest decay, $\mathbf{R}_{\mathbf{q33}}$, to find the noise tail and thereby reduce the statistical errors.

We apply the Hilbert transformation and obtain the envelope. We calculate the coefficient of determination, Eq. (9), for each time lag in the correlation function matrix. The coefficient is shown on Fig. 7 A. The best fit of the two mathematical models are shown on Fig. 7 B. We use the best fit of the two mathematical models to estimate the start of the noise tail and this is illustrated on Fig. 7 C.

4. Conclusion

This paper presents a manner of detecting the noise tail, which is caused by statistical errors in the estimation of the correlation function matrix. The proposed method is demonstrated on a simulated system with promising results, that resembles a manual inspection of the noise tail. The method is a step toward a general approach for reduction of statistical errors but there are still issues, which need further development.

In practise, the correlation function matrix contains multiple modes and each decorrelated mode have different noise tails. For this case study, we chose to pick the autocorrelation function with the fastest decay to reduce random

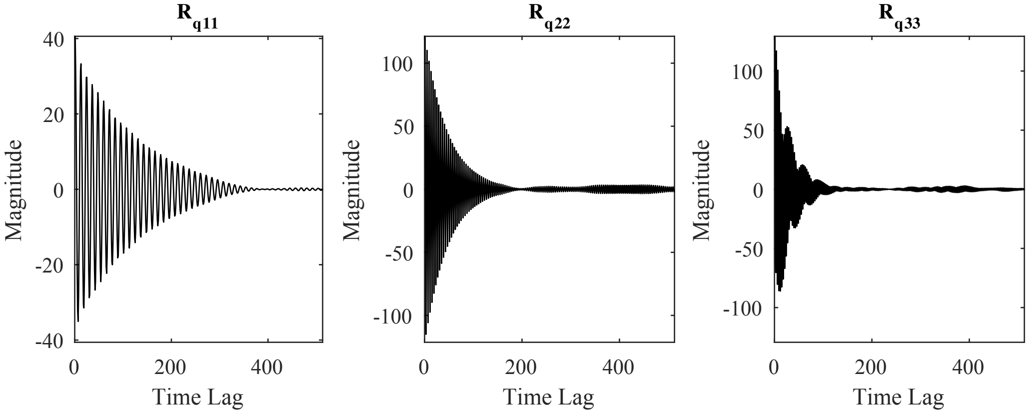

Fig. 6. Decorrelated autocorrelation Functions

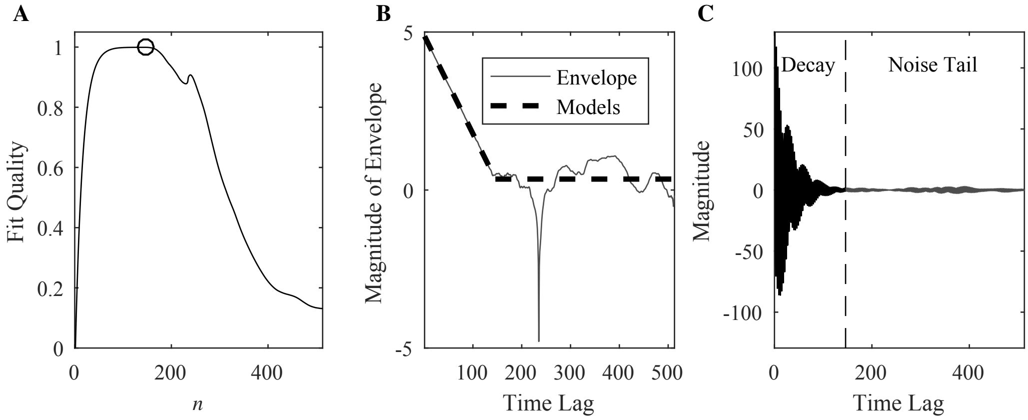

Fig. 7. A) Coefficient of Determination, B) Best Fit of the two models, C) Detection of noise tail

errors in an identification process but this would result in a loss of information for the other modes. This should be studies further.

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