Synthetic study of 2.5-D ATEM based on finite element method

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Summary

Based on the regular triangular dissection for finite element method, we implemented the forward modeling of 2.5-D airborne transient electromagnetic method. The 3-D EM field was firstly transformed into Laplace domain and after that we will apply Fourier transform to reduce the dimension from 3-D to 2.5-D. We can obtain the EM field solution in Laplace domain by applying finite element method. The inverse Laplace transform is applied to our solution which finally leads to the airborne EM response in time domain. In compared to the traditional method, we apply our finite element method to the anomalous field which can avoid the singularity problem caused by the source which can excite the anomalous EM field. The EM source can be imposed to our process by incorporate the background EM field. The computation error can be accumulated due to the large variation of EM field and it can also be amplified in the process of Laplace transform and Fourier transform. In order to get accurate result, the error should be well controlled in every procedure. The induced electromagnetic force can be computed accurately from vertical magnetic component by applying Lagrange interpolation. The synthetic model study shows that our numerical solution fits well with the analytical solution for homogeneous and layered earth model. The study also demonstrated the effectiveness of our numerical method.

Introduction

Recent years, we observed a grow interest of apply airborne EM method (ATEM) as an exploration tool for geophysicists. This method can be effectively applied for the area that traditional method cannot work well such as forests, mountains with complex topography, marsh areas, and shallow sea and so on. With the development of modern airborne system, this method can be applied to a large area for deeper target.

The current dominant interpretation method for airborne EM is based conductivity-depth imaging and 1-D inversion. Such methods include: The "Mirror source method" or the "smoking ring theory" (Nabighian, *et al.*, 1979), horizontal current filament transformation (Fullagar, 1989), sheet-like

approach (Keating and Crossley, 1990; Zhdanov and Pavlov, 2002), the pseudo layer half space model (Huang, 2008), and layered earth inversions (Paterson, *et al.*, 1983; Huang and Palacky, 1991; Sattel, 2005; Li Yongxing *et al.* 2010), which can be described as the approximate interpretation methods. However, these 1-D interpretation methods can leads to wrong interpretation for complex geological structure. Higher dimensional interpretation technique is still a challenge due to the bottleneck of computation cost.

Stoyer and Greenfield (1976) discussed the 2.5-D time domain electromagnetic forward modeling problem excited by a magnetic dipole. The 2.5-D situation is more suitable, from the perspective of real application, in spite of high computation cost. Another solution of the ATEM 2.5-D forward modeling is to compute the electromagnetic response in frequency domain, then transform the solution into time domain due to step-function excitation using digital Hankel filtering algorithm (Raiche, 1998). So far, the computation accuracy and efficiency are still a problem for 2.5-D ATEM. In our study, we applied the method of anomalous field computation which can improve the accuracy of the field nearby source.

Methodology

The conventional equations derived from Maxwell's equations for the land source cannot be applied to the airborne EM field modeling for the reason that the source is located in the air. To get rid of the effect of source, we will separate the total transient EM field into a background part and an anomalous part as follows:

$$\mathbf{e} = \mathbf{e}_b + \mathbf{e}_a, \mathbf{h} = \mathbf{h}_b + \mathbf{h}_a, \quad (1)$$

where $\mathbf{e}$ and $\mathbf{h}$ are the electric and magnetic field, the subscripts a and b stand for anomalous and background part respectively. Here the anomalous refers to the difference between the total field and the field purely caused by background structure. According to the Maxwell' equation in the Cartesian coordinate system, we can write the anomalous electric and magnetic components equations in Laplace Fourier domain as follows

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$$\begin{cases} -imE_{zs} - \frac{\partial E_{ys}}{\partial z} = -\mu s H_{xs} \\ \frac{\partial E_{xs}}{\partial z} + \frac{\partial E_{zs}}{\partial x} = -\mu s H_{ys} \\ \frac{\partial E_{ys}}{\partial x} + imE_{xs} = -\mu s H_{zs} \end{cases}, \quad (2)$$

$$\begin{cases} -imH_{zs} - \frac{\partial H_{ys}}{\partial z} = (\epsilon s + \sigma) E_{xs} + J_{xs} \\ \frac{\partial H_{xs}}{\partial z} - \frac{\partial H_{zs}}{\partial x} = (\epsilon s + \sigma) E_{ys} + J_{ys} \\ \frac{\partial H_{ys}}{\partial x} + imH_{xs} = (\epsilon s + \sigma) E_{zs} + J_{zs} \end{cases}$$

where the y and z stand for the strike-along and vertical direction, i is the imaginary number unit, ϵ is the dielectric constant, μ is the magnetic permeability, s is the laplacian variable, m is the fourier variable, $\mathbf{J}$ is the current density vector with the expression

$$\mathbf{J}_s = \sigma_s \mathbf{E}_b. \quad (3)$$

These equations indicate that the anomalous field is excited by the background field and anomalous conductivity and it is not related with the true airborne source. As a result, we can solve the background field first and then solve the anomalous EM field based on solving equations 2 with the obtained background field. The total EM response can be obtained from equation 1.

From equation 2, we can eliminated the x and y component of the field and obtained a PDE system for y component. The corresponding Ritz variational problem is described as follows:

$$\begin{aligned} J(E, H) = & \iint_{\Omega} \left\{ a_1 \left[\left(\frac{\partial E}{\partial x} \right)^2 + \left(\frac{\partial E}{\partial z} \right)^2 \right] + a_3 \left[\left(\frac{\partial H}{\partial x} \right)^2 + \left(\frac{\partial H}{\partial z} \right)^2 \right] + \right. \\ & a_4 E^2 + a_5 H^2 + 2a_2 \left(\frac{\partial E}{\partial x} \frac{\partial H}{\partial z} - \frac{\partial E}{\partial z} \frac{\partial H}{\partial x} \right) + \\ & 2b_4 E_b E + 2b_1 \left(\frac{\partial E_b}{\partial x} \frac{\partial E}{\partial x} - \frac{\partial E_b}{\partial z} \frac{\partial E}{\partial z} \right) + \\ & 2b_3 \left(\frac{\partial H_b}{\partial x} \frac{\partial H}{\partial x} - \frac{\partial H_b}{\partial z} \frac{\partial H}{\partial z} \right) + 2b_2 \left(\frac{\partial H_b}{\partial z} \frac{\partial H}{\partial x} - \frac{\partial H_b}{\partial x} \frac{\partial H}{\partial z} \right) \left. \right\} + \\ & 2b_2 \left(\frac{\partial E_b}{\partial x} \frac{\partial H}{\partial z} - \frac{\partial E_b}{\partial z} \frac{\partial H}{\partial x} \right) \left. \right\} d\Omega + \frac{1}{2} \oint_l \left(\eta^H H^2 + \eta^E E^2 \right) dl \\ \rightarrow \min \end{aligned} \quad (4)$$

where Ω is the study region of interest, l is the boundary of Ω , $\mathbf{E}$ is the y component of electric field in Laplace Fourier domain, $\mathbf{H}$ is the product of imaginary number i and y component of magnetic field in Laplace Fourier domain, $a_1, a_2, a_3, a_4, a_5, a_{1b}, a_{2b}, a_{3b}, a_{4b}, a_{5b}, b_1, b_2, b_3, b_4, \eta^E, \eta^H$ are coefficients related to the petrophysical properties and the wavenumbers of Laplace Fourier domain. The functional problem gets the extreme value when

$$\delta J(E, H) = 0. \quad (5)$$

This problem can be solved using the finite element method. The process can be described as the following steps. 1. Discretize the study domain into a grid of elements. 2. Do the integral of the functional after assign interpolation function for each element. By doing this, the continuous function problem becomes discrete form of every node point. After that, we will summarize the equations for each element into a global linear system of equations which can be solved effectively to obtain the solution at each node due to the sparseness of the equations.

By using the expression:

$$H_{zs} = -a_1 \frac{\partial E}{\partial x} - a_2 \frac{\partial H}{\partial z}, \quad (6)$$

the vertical anomalous magnetic component can be solved in time domain using weighted central difference within the closest units.

In the step of solving the vertical induced electromagnetic voltage, the conventional differential algorithm might lead to unstable errors for the large variation of magnitude. We introduced a Lagrange interpolation for numerical differential which has higher accuracy than conventional method.

Case study

The Fortran program has been developed according to the algorithm to accomplish the finite element procedure using regular triangular mesh dissection. The 2.5-D ATEM responses due to horizontal central loop configuration of typical case are computed for the following modes.

Figure 1 shows the numerical modeling result for the homogeneous model and layered earth model. The homogeneous resistivity $\rho=100\Omega\cdot\text{m}$, and the resistivity for the layered model is: $\rho_1=100\Omega\cdot\text{m}$, $\rho_2=10\Omega\cdot\text{m}$, $\rho_3=100\Omega\cdot\text{m}$; the depth of each layer are $h_1=50\text{m}$, $h_2=100\text{m}$. The side length of the rectangular transmitter is $a=b=20\text{m}$, with

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current intensity $I=1\text{A}$, the equivalent area of receiver is 50m^2 .

It is obvious that the vertical induced voltage value for the homogeneous model decays logarithmically with time increasing from 1 to $1000\mu\text{s}$. The attenuation slows down in the later time. For layered model, the decay pattern is similar as the homogeneous model. The difference is that the induced electrodynamic voltage increases at time 0.1 to 10ms. The curve concave downward in this part. The result

indicates that the effect of the second layer is incorporated into our solution.

For the typical 2-D case, a inclined tabular conductor is displayed in figure 2 with the resistivity of $0.1\Omega\cdot\text{m}$, and the resistivity of wall rock is $100\Omega\cdot\text{m}$. The top of target is 6m beneath the surface and extend 27 m in vertical direction and 18m in horizontal direction. The transmitter side length is $a=b=20\text{m}$ and its flying elevation is 10m. The exciting current $I=1\text{A}$ and the equivalent area of receiver is 50m^2 .

Figure 1 Induced voltage decay curves of homogeneous (left) and layered (right) earth with transmitter on specific flight attitude

Figure 2 Illustration of the ATEM measuring (left) and the modeling response (right) on the tabular conductor anomaly

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It is shown from the ATEM section that the induced voltage curves is distorted drastically, with the maximum value points migrating from some early channels ($1.29\mu s$) to the right side to the left down. There are a few maximum points emerging in time window $129.42\text{--}514.53\mu s$, and the peak value descends soon afterwards. The feature of the voltage reflects the dipping direction of the target. The maximum point corresponds to the barycenter of the tabular body, which demonstrated that the 2.5-D modeling program gives the proper transient response on complex model situation.

Conclusions

Based on the variational problem for anomalous in Laplace Fourier domain, the ATEM 2.5-D finite element forward modeling algorithm is developed for the anomalous EM field solving to avoid the effect of source, which makes the procedure more feasible. The source information is supplied

in the numerical process by computing the synthetic homogeneous model electric and magnetic response. The resultant response of ATEM 2.5-D modeling shows consistency with 1-D case. More importantly, the local quality details of the anomaly can be indicated from pseudosection, such as shape, attitude, etc. of the targets, which consequently demonstrates the validity of the algorithm.

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