The Characteristics of Magnetotelluric Responses for anisotropic conductivity

Bin Xiong, Guilin University of Technology; Hongzhu Cai*, University of Utah; Tianya Luo, Guilin University of Technology

Summary

In this paper, we assume a 2D earth structure with anisotropic conductivity. One of the principal anisotropy axes is parallel to the striking direction and another axes is perpendicular to the surface of structure layer. By solving the variational problem with assigned boundary condition, we can compute the apparent resistivity for magnetotelluric (MT). Based on the forward modeling of MT field, we will analysis the difference of EM field for isotropic and anisotropic medium for TE_z polarization model. Our study shows that the MT field can be significantly affected by anisotropic medium and should be taken into consideration in many geophysical applications. We also analyzed the effect of anisotropic parameter (ration of horizontal conductivity to vertical conductivity) and dipping layer on MT field.

Keywords: finite element method; magnetotelluric; anisotropic medium; forward modeling

Introduction

We usually assume an isotropic earth structure model in MT forward modeling. However, the conductivity property of rocks varies in many directions due to temperature, pressure and some other physical and chemical effect (Shi, 1985). The anisotropic effect of conductivity will cause the electric field deviated from the direction of current. In oil and gas exploration, the anisotropic effect can significantly affect the evaluation of oil/gas saturation of rocks. The effect of anisotropy on MT method was first discussed by Mann (1965). 2D MT forward modeling for anisotropic conductivity medium with finite element method was introduced by Xu and Zhao in 1985. In this paper, we compared the MT response of isotropic and anisotropic medium for TE_z polarization based on the study of Xu and Zhao. We analyzed the impact of anisotropic parameter on MT field.

2D boundary value problem for MT field in anisotropic medium

We consider 2D geoelectric model in Cartesian coordinate system. In our study, we assume the positive y axis directs upward; x and z axes are in transverse direction. The conductivity in transverse and longitudinal directions are denoted by $\sigma_{\parallel}$ and $\sigma_{\perp}$. It is well known that the MT field can be separated into TM_z and TE_z model (Allen and Susan, 2000). In this case, the TM_z model with anisotropic conductivity shares the same partial differential equations

with the TM_z model in isotropic medium with conductivity of $\sigma_{\parallel}$ (Zhao and Xu, 1983). In this paper, we will focus our discussion on TE_z model.

The MT problem for the model shows in figure 1 under the assumption of TE_z polarization is described by the following equations (Xu, 1994):

Figure 1: This Figure shows study domain for TE_z model.

$$\nabla \cdot (\tau \nabla u(\mathbf{r})) + \lambda u(\mathbf{r}) = 0, \mathbf{r} \in \Omega, \quad (1)$$

$$u(\mathbf{r}) = 1, \mathbf{r} \in AB, \quad (2)$$

$$\frac{\partial u(\mathbf{r})}{\partial n} = 0, \mathbf{r} \in AD, BC, \quad (3)$$

$$\frac{\partial u(\mathbf{r})}{\partial n} + ku(\mathbf{r}) = 0, \mathbf{r} \in CD, \quad (4)$$

where:

$$u = H_z, \lambda = i\omega\mu, \tau = \begin{bmatrix} \tau_{11} & \tau_{12} \\ \tau_{21} & \tau_{22} \end{bmatrix}, \quad (5)$$

$$\tau_{11} = \left(\frac{1}{\sigma_{\parallel} - i\omega\epsilon} \sin^2 \alpha + \frac{1}{\sigma_{\perp} - i\omega\epsilon} \cos^2 \alpha \right), \quad (6)$$

$$\tau_{22} = \left(\frac{1}{\sigma_{\parallel} - i\omega\epsilon} \cos^2 \alpha + \frac{1}{\sigma_{\perp} - i\omega\epsilon} \sin^2 \alpha \right), \quad (7)$$

$$\tau_{12} = \tau_{21} = -\frac{1}{2} \left(\frac{1}{\sigma_{\parallel} - i\omega\epsilon} - \frac{1}{\sigma_{\perp} - i\omega\epsilon} \right) \sin(2\alpha). \quad (8)$$

In the previous equations, ∇ is the Hamilton operator for 2D problem, $k = \sqrt{-i\omega\mu\sigma_{\parallel}}$, Ω is our study domain, ϵ is the dielectric constant, μ is the magnetic permeability, i is the imaginary unit, ω is the angular frequency for

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Figure 2: Four triangular element in a rectangular grid.

Varitional problem for MT field in anisotropic medium

The equivalent variational formulation for equation 1 to 4 is given as:

$$F(u) = \iint_{\Omega} \left[\frac{1}{2} \nabla u \cdot (\tau \nabla u) - \frac{1}{2} \lambda u^2 \right] d\Omega + \frac{1}{2} \int_{CD} \frac{k}{\sigma_{\parallel} - i\omega\epsilon} u^2 d\Gamma, \quad (9)$$

$$u|_{AB} = 1, \quad (10)$$

$$\delta F(u) = 0. \quad (11)$$

The variational problem described from equation 9 to 11 can be solved numerically by using finite element method.

Finite element method

In our study, we discretized the 2D plane into a grid of rectangular grids and each rectangular cell is divided into four triangular elements by the two diagonal lines (Luo and Zhang 1987; Xiong and Luo, 2006) which is shown in figure 2 and figure 3. For each triangular element, the linear interpolating function is given as follows (Lv, et al., 2011):

$$u = N_i u_i + N_j u_j + N_m u_m, \quad (12)$$

where the subscripts i, j, m are the nodes index for each triangular element in an anticlockwise manner. N_i, N_j, N_m are the linear basis function.

In a discretized form, equation (9) can be written as follows:

$$F(u) = \sum_{\Omega} \iint_e \left[\frac{1}{2} \nabla u \cdot (\tau \nabla u) - \frac{1}{2} \lambda u^2 \right] d\Omega_e + \sum_{CD} \int_{e_{CD}} \frac{1}{2} \frac{k}{\sigma_{\parallel} - i\omega\epsilon} u^2 d\Gamma_e, \quad (13)$$

We can compute each element's contribution to equation 13 and get a 4×4 local stiffness matrix by eliminate the

middle node. The local stiffness matrix can be expanded to a global stiffness matrix after assembling. The boundary value of field on AB is substituted into the global equation to solve the field H_z in each node. The apparent resistivity on the earth's surface can be calculated from the field.

Figure 3: A triangular element with nodes index.

Analysis of forward modeling result

Model 1: syncline conductive body

Figure 4 shows an illustration of this model. For this model we considered both isotropic conductivity and anisotropic conductivity. The isotropic conductivity is given as the geometric mean of anisotropic conductivities ($\rho = \sqrt{\rho_{\parallel} \cdot \rho_{\perp}}$). By taking this value, we can prove that the different MT response for the isotropic and anisotropic model is mainly caused by the perturbation of physical properties in different direction rather the value of resistivity. Figure 5 shows the numerical result for different frequency. As we can see that for the first frequency, the MT response for isotropic and anisotropic medium are 24 times difference. Therefore, we find an necessity to consider anisotropic effect in the processing and interpretation of MT data. As the frequency increase, the anisotropic effect will decrease monotonically.

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Figure 5: This figure shows the comparison of MT response between isotropic medium and anisotropic medium for model 1. Panel (a) to (d) shows the behavior of different frequency. The solid line indicates the MT response for anisotropic medium and the line with circle shows the MT response for isotropic medium.

Model 2 : dipping conductive body

The model is shown in figure 6 with different values for α to be 0° , 45° and 90° . Figure 7 shows the change of apparent resistivity for different α for TE_z polarization. As we can see that there is some artificial anomaly around our conductive body with resistivity higher than the wall-rock. As the angle α increase, the artificial anomaly starts to diminish.

Figure 6: An illustration for model 2.

Model 3: Rectangular model

In this model, we assume that the background resistivity is $\rho_2 = 1000 \Omega \cdot m$. A rectangular body is embedded in this

Figure 4: An illustration for model 1.

background. Figure 8 is an illustration of this model. For this model, we have two submodels. For submodel a, we fix the horizontal resistivity of the anomalous body and change the vertical resistivity from $10 \Omega \cdot m$ to $10^6 \Omega \cdot m$. For submodel b, we fix the vertical resistivity of the anomalous body and change the horizontal resistivity in a similar manner as submodel a. The parameters of these two submodels are shown in table 1.

Figure 7: Apparent resistivity calculated for model 2. (a) $\alpha = 0^\circ$, (b) $\alpha = 45^\circ$, (c) $\alpha = 90^\circ$

Figure 8: An illustration for model 3.

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Figure 9: Apparent resistivity for submodel *a* in model 3. Pannel (a) to (e) shows the result for different $\rho_{1\perp}$.

Submodel	Resistivity ($\Omega \cdot m$)						
	$\rho_{1\parallel}$	$\rho_{1\perp}$					
<i>a</i>	1	10	10^2	10^3	10^4	10^5	10^6
<i>b</i>	$\rho_{1\perp}$	$\rho_{1\parallel}$					
	1	10	10^2	10^3	10^4	10^5	10^6

Table 1. Resistivity parameter for model 3.

The apparent resistivity for submodel *a* and submodel *b* are shown in figure 9 and figure 10. From figure 9, we can see that if we fix the horizontal resistivity and increase the vertical resistivity, the apparent resistivity always shows a low resistivity anomaly compared to the background (wall-rock resistivity) even if the anomalous resistivity is higher than the wall-rock resistivity in our model.

For figure 10, the vertical resistivity of the anomalous body is fixed and we will increase the horizontal anomalous resistivity from a value lower than the wall-rock resistivity to another value larger than the wall-rock resistivity. We can identify the low resistivity anomaly, compared to the wall-rock, from the apparent resistivity figure if the horizontal resistivity of the body is lower than the wall-rock resistivity in our model. If the horizontal resistivity of the body is higher than the wall rock in our model, we are able to identify the higher resistivity anomaly from the apparent resistivity figure. However, if the horizontal resistivity of the the anomalous body is equal to the wall-rock resistivity, we will not be able to find the anomalous body from the apparent resistivity figure. In an anisotropic medium, the horizontal resistivity has a larger impact on the effective resistivity than the vertical resistivity for our model study.

Conclusion

In this study, we compared the MT response for a isotropic medium and an anisotropic medium. We also studied the impact of anisotropic parameter on MT method. Based on our study, we find that the anisotropic effect cannot be simply negelect in practical application MT method especially for low frequency.

We also analyzed the effect of the dipping layer angle to the MT response for an anisotropic medium. As we can see from our model study, the horizontal dimension of the low resistivity body in the apparent resistivity figure decrease as the dipping angle increase. The area nearby the anomalous body in the apparent resistivity figure is charactized by some artificial anomaly with the resistivity higher than the wall-rock reistivity and this effect is more serious for small dipping angle.

In an anisotropic medium, the MT response is primarily affected by the horizontal resistivity than the vertical resistivity.

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EDITED REFERENCES

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