Abstract: We analyze under what conditions competitive credit markets are efficient in providing loans to entrepreneurs who can start a new project after failure. An entrepreneur of uncertain talent chooses the riskiness of her project. If banks privately observe the entrepreneur’s risk choices, two equilibria coexist: (1) an inefficient equilibrium in which the entrepreneur realizes a low-risk project and has no access to finance after failure and (2) a more efficient equilibrium in which the entrepreneur first realizes high-risk projects and then, after continuous failures, a low-risk project. There is a non-monotonic relationship between bank information and potential credit market inefficiency. We discuss the implications for credit registers and entrepreneurial education.

Keywords: stigma of failure, entrepreneurship, credit markets, asymmetric information
JEL Classification: C73, G21, M13

1 Introduction

What determines the level of entrepreneurial activity in an economy? One key variable is the extent to which failed entrepreneurs are excluded from further entrepreneurial finance. European and Japanese financiers, for instance, are perceived to be more reluctant to finance a failed entrepreneur’s restart than their American counterparts. It therefore has become commonplace to praise the
US’ lower “stigma of failure” as the source of its higher entrepreneurship rates\(^1\) and consequently of its competitive edge in terms of the ability to innovate, commercialize and grow.\(^2\)

In this paper, we present endogenous risk choice as an explanation for why economies with identical institutional, legal and cultural constraints can experience different levels of the “stigma of failure”. If the “stigma of failure” is high and banks provide credit only to those who have never failed, entrepreneurs will choose low-risk projects. In this economy, failure indicates low entrepreneurial talent. If the “stigma of failure” is low and banks provide credit to failed entrepreneurs, new entrepreneurs will be more inclined to experiment with novel (and more risky) business ideas. Failure at the beginning of an entrepreneurial career then indicates bad luck rather than low entrepreneurial talent.

To get an intuition for the model, imagine an entrepreneur who can undertake one of two projects, a low-risk project or a high-risk project. Both projects have no investment costs. The low-risk project yields a safe net present value (NPV) of \(y_L > 0\). The high-risk project fails with probability \(p_H \in (0, 1)\) and then yields a NPV of 0. It succeeds with probability \(1 - p_H\) and then yields a NPV of \(y_H > y_L\). The expected NPV of the low-risk project is higher than the expected NPV of the high-risk project, \(y_L > (1 - p_H)y_H\). If the entrepreneur has only one chance to undertake a project, she should clearly choose the low-risk project. However, assume now that, in the case of success, she works on the project and enjoys its returns, while in the case of failure, she can start a new project and faces the same choice.\(^3\) Suppose for simplicity that there is no discounting. What project should the entrepreneur now choose? If she chooses the low-risk project, she will earn \(y_L\). If she always chooses the high-risk project, she will succeed for sure after finite time and earn \(y_H\). Therefore, it is optimal to go (a few times, if needed) for the high-risk project.

This logic is typical for optimal strategies in sequential search problems (Weitzman 1979). Projects where high payoffs occur with low probability may be investigated first, even though their expected value is smaller than that of projects with lower variance. As in the example above, the high-risk project

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\(^1\) GEM (2015) reports that in 2014, 13.8% of adults were engaged in early-stage entrepreneurial activity in the United States as compared to only 7.8% in the EU or 3.8% in Japan.


\(^3\) For example, both projects could be a business that has no value except for the human capital of the entrepreneur (such as a gourmet restaurant). Returns only materialize if the entrepreneur runs the project during her entire professional life. Another interpretation is that once the entrepreneur has established a successful business, she gets settled so that her costs of starting a new business become prohibitively large.
should be undertaken first, because if it succeeds, the highest possible payoff is obtained; and if it fails, one can still realize the low-risk project. In contrast, if one starts with the low-risk project immediately, the low payoff occurs with certainty.

In our model, there are two barriers to optimal search. First, there are investment costs that cannot be financed by the entrepreneur herself. She has to apply for a loan on a competitive credit market. Second, her entrepreneurial talent, which is either high or low, influences her probability of success, but is unknown to her and to banks. Only the distribution of talent is common knowledge. The entrepreneur and banks learn about her talent from the outcome of the project. However, learning depends on the project's risk: failure with a low-risk (high-risk) project implies a relatively low (high) probability of high talent. Given that the ex-ante probability of high talent is sufficiently large, the first-best outcome is as follows. The entrepreneur first undertakes high-risk projects. After continuous failures, she undertakes the low-risk project. Failure with the low-risk project discloses low talent; therefore, she stops undertaking projects at all.

Under what circumstances is the first-best an equilibrium outcome and what other equilibria exist? To answer this question, we consider three different informational settings: perfect information (PI), private information of banks (PIB) and imperfect information (IM). The equilibrium outcome is efficient when banks have perfect information about the riskiness of past and present projects. However, if banks only observe the risk of projects in their own loan portfolio (PIB), two equilibria may co-exist. First, an equilibrium in which the entrepreneur undertakes a low-risk project and, if it fails, does not get any more loans. Second, an equilibrium in which the entrepreneur first undertakes high-risk projects and then, after continuous failures, a low-risk project. The reason why the first assessment is an equilibrium is that only the lending bank observes the entrepreneur’s risk choice. Even if she undertakes a high-risk project, this bank becomes a monopolistic supplier of finance in case of failure and then can extract all future rents from her. This equilibrium is inefficient since the entrepreneur undertakes the low-risk project too early. Thus, countries with the same average entrepreneurial talent and institutional environment can experience different levels of the “stigma of failure”.

Under IM, banks never observe the risk of projects. This causes a moral hazard problem. The entrepreneur undertakes a low-risk project only if the expected payoff of the high-risk project is sufficiently small relative to that of the low-risk project. As under PIB, multiple equilibria may exist. However, in contrast to PIB, if the expected payoff of the high-risk project is close to that of the low-risk project, the entrepreneur will only undertake high-risk projects in equilibrium. If the ex-ante probability of high talent is sufficiently large,
multiple rounds of project financing must occur in equilibrium. Consequently, the relationship between bank information and potential credit market inefficiency can be non-monotonic. Under PI, the outcome is always efficient. Under PIB, there is always an inefficient equilibrium with only one period of project financing. This equilibrium may be strictly dominated by any equilibrium under IM since in these equilibria the entrepreneur realizes several high-risk projects. In other words, the opportunity to shift risks constrains the maximal credit market inefficiency.

Our model yields a number of implications. It shows that the credit market outcome may be inefficient even if there is no asymmetric information between the entrepreneur and the bank that finances her project. The inefficiency is caused by the fact that outside banks cannot observe the entrepreneur’s risk choice. The model therefore highlights the role of information sharing. If banks are required to share their evaluation of $E$'s project risk with the market, banks would again have perfect information and the equilibrium outcome would be efficient.

Even if banks only share information about the loan rates of previously chosen contracts through credit registers, this could have a positive effect. Such information transmits the entrepreneur’s risk choice to the extent that if the entrepreneur chose a contract with a very low loan rate, this may indicate that she undertook a low-risk project (otherwise, the bank that offered this contract would have made negative expected profits). In this case, banks stop offering loans. Hence, a credit register may ensure the existence of an equilibrium in which the entrepreneur undertakes projects as in the case of PI.

Finally, our results imply that the potential welfare gains from an increase in the population’s average entrepreneurial talent might not fully be realized. If banks do not adjust their financing policies, entrepreneurs may end up choosing inefficiently low-risk levels.

The paper that comes closest to ours is Landier (2006). In his model, banks cannot distinguish between entrepreneurs with low ability who start a new venture because the previous one failed and entrepreneurs with high ability who are not bankrupt, but would like to start a more promising project. This can generate equilibria in which few (many) high-ability entrepreneurs restart and loan rates are high (low). Our model differs significantly from Landier (2006). First, banks always can observe how many times the entrepreneur went bankrupt. This assumption seems appropriate since information about bankruptcy is shared among banks in most developed countries. Second, the multiplicity of equilibria is caused by the interaction of the entrepreneur’s risk choice with banks’ beliefs about her action (and not by asymmetric information about her talent). Consequently, our model generates a number of new results about the impact of institutions on credit market outcomes.
The remainder of the paper is organized as follows. Section 2 relates the paper to the literature. Section 3 introduces the model and characterizes the first-best outcome. Section 4 examines the equilibrium set under PI, Section 5 under PIB, and Section 6 under IM. Section 7 discusses the implications of the model. All proofs are in the Appendix.

2 Related Literature

A considerable empirical literature tries to explain cross-country differences in the “stigma of failure” by using persistent institutional, legal or cultural characteristics.\(^4\) Evidence confirms that fresh starts are affected by bankruptcy laws (Armour and Cumming 2008). The debate persists over which and how cultural traits shape attitudes towards entrepreneurial failure (Licht and Siegel 2006; Hayton, George, and Zahra 2002; Giannetti and Simonov 2004; Singh, Corner, and Pavlovich 2015).\(^5\) Our contribution is to present a model that endogenizes the “stigma of failure”.

Two other papers study models of entrepreneurial finance with asymmetric information and find multiple equilibria. Gromb and Scharfstein (2002) analyze an occupational choice model in which agents either become entrepreneurs or intrapreneurs. When there are few (many) entrepreneurs, the average quality of failed entrepreneurs is low (high). As a result, the external labor market (that offers jobs to failed entrepreneurs) is poor (good), so that few (many) agents start a venture on their own. Ghatak, Morelli, and Sjostrom (2007) consider a general equilibrium model in which wages for dependent labor are low (high) when there are many (few) untalented entrepreneurs, which implies that many (few) agents become entrepreneurs. In contrast to these papers, we endogenize the number of rounds banks are willing to finance an entrepreneur after failure. The driver of our results is the link between risk choice and bank lending. New entrepreneurs will choose risky (safe) projects provided that banks (do not) finance projects after failure. Consequently, the average talent of failed

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\(^4\) Most research aims to identify individual characteristics that determine the propensity to entrepreneurship per se, such as family background (Dunn and Holtz-Eakin 2000; Sørensen 2007a), work experience (Sørensen 2007b) or wealth (Hurst and Lusardi 2004).

\(^5\) For instance, Burchell and Hughes (2006) find that GDP growth is not related to failure tolerance, but positively related to second chancing willingness. Surveys, however, show more failure tolerance but less second chancing willingness in the United States than in the EU. In the United States, the stigma should thus be higher and entrepreneurship rates lower than in the EU, which contradicts empirical evidence.
entrepreneurs is high (low), such that multiple rounds of project financing (do not) occur in equilibrium.\(^6\)

We also contribute to the literature on entrepreneurial risk-taking. A common explanation why entrepreneurs often bear substantial undiversified risk despite the lack of a positive premium, is (over-)optimistic beliefs (see, e.g. Landier and Thesmar 2009). Vereshchagina and Hopenhayn (2009) interpret entrepreneurial risk-taking as a lottery over future wealth that is chosen by borrowing-constrained agents. Limited liability then makes poor and impatient agents (who have not yet saved much) take more risk. Campanale (2010) rationalizes entrepreneurial risk-taking on the grounds that entrepreneurs usually make small personal investments and gain large human capital from starting a business. In our model, risk-taking is determined by the willingness of banks to finance new projects after failure.

3 The Model

We consider an economy populated by an entrepreneur \(E\) and banks \(B_k, k \in \mathbb{N}\).\(^7\) All agents are risk-neutral. \(E\) can undertake a project, which requires an investment of 1. She has no wealth on her own. Thus, the project needs to be financed by a bank. If banks do not offer any loans, the game is over and agents’ payoffs are 0. Otherwise, \(E\) chooses a loan contract and nature decides on the project’s outcome. In case of success, project returns are realized, \(E\) pays the loan rate and the game is over. In case of failure, no payments are made and the game starts anew. Time is discrete and denoted by \(t \in \{1, 2, \ldots\}\), where period \(t > 1\) is reached if and only if \(E\) undertook \(t - 1\) times a project that failed. There is no discounting between periods.\(^8\)

3.1 The Entrepreneur

In period 0, nature decides on \(E\)’s entrepreneurial talent \(\theta_i\), which is high \((i = H)\) with probability \(\alpha_1 \in (0, 1)\) and low \((i = L)\) with probability \(1 - \alpha_1\). Talent is time

\(^6\) Moreover, a recent theoretical and empirical literature analyzes how past entrepreneurial performance influences future performance and labor market outcomes, see Gompers et al. (2010), Baptista, Lima, and Petro (2012), Gottschalk et al. (2014), and Canidio and Legros (2014). However, these papers do not discuss the extent to which entrepreneurs have access to finance after failure.

\(^7\) The results easily carry over to a continuum of entrepreneurs.

\(^8\) This assumption is not essential. All results remain valid if the discount factor is sufficiently close to unity.
invariant and unobservable to E and banks. Only $\alpha_1$ is commonly known. E can undertake projects with high ($j = H$) or low ($j = L$) risk of failure $p_j$. For convenience, we normalize $0 = p_L < p_H < 1$ and $0 \leq \theta_L < \theta_H = 1$. The project’s NPV is $y_j$ with probability $(1 - p_j)\theta_i$ and 0 with probability $1 - (1 - p_j)\theta_i$. The high-risk project yields a higher NPV in case of success, $y_L < y_H$. If E has high (low) talent, projects have positive (negative) expected NPV:

$$(1 - p_j)y_j > 1 \text{ and } (1 - p_j)\theta_i y_j < 1 \text{ for } j \in \{L, H\}. \quad [1]$$

### 3.2 Banks

Banks compete in a Bertrand manner by offering loan contracts. Their net refinancing costs are normalized to 0. Our solution concept is sequential equilibrium (SE). We therefore assume finite actions sets. Let the set of possible loan rates be given by $R(\varepsilon) = \{1 + \varepsilon, 1 + 2\varepsilon, \ldots, 1 + q\varepsilon\}$, where $q$ is large enough such that $1 + q\varepsilon > y_H$. We will think of $\varepsilon$ as a very small value so that it does not artificially rule out equilibria. A contract offer of $B_k$ appears as $(k, r, \bar{p})$, where $r \in R(\varepsilon)$ is the loan rate and $\bar{p} \in \{p_L, p_H\}$ the maximum riskiness of the project. If $E$ chooses a contract $(k, r, \bar{p})$ with $\bar{p} = p_L$, then $E$ can only undertake the low-risk project. If $\bar{p} = p_H$, then $E$ can undertake any project. We therefore allow banks to control the project’s risk (at a later stage, we drop this assumption). Each bank offers at most two contracts. Denote by $C^k_t$ ($C_t$) the set of all contract offers made by $B_k$ (by all banks) in period $t$. $C_t$ also contains the zero-contract $(0, 0, 0)$. If $E$ chooses this contract, the game is over and payoffs are 0 for every agent in this period. Denote by $C^k$ ($C$) the set of all possible $C^k_t$’s ($C_t$’s).

In period $t$, $E$ chooses at most one contract out of $C_t$ and, given its terms, the project risk $j$. If $E$ chose contract $(k, r, \bar{p})$ and risk $j$, then, in case of success, she earns $\max\{0, y_j - r\}$, while $B_k$ gets $\min\{y_j, r\}$.

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9 The implicit assumptions merit discussion. We could also allow for a type of contract that forces $E$ to realize the high-risk project. The assumption that banks cannot offer such a contract is made for convenience and does not affect our results. Next, we assume that banks can only offer short-term contracts. There exist several just for this assumption. First, arrangements that specify the details of a follow-up contract after failure do not exist in reality. Second, if banks’ refinancing rate or $E$’s future business opportunities are uncertain, it may be impossible to commit to a certain contract offer in the future. Finally, the restriction to debt contracts is standard in the literature (see Diamond 1991 for a justification of this assumption).
3.3 Strategies and Beliefs

Denote by $H^k_t (H^E_t)$ the history of $B_k (E)$ in period $t$, i.e. everything $B_k (E)$ observed up to the beginning of period $t$. Let $H^k_t (H^E_t)$ be the set of such histories. We will clarify the details of $H^k_t$ for each informational setting at a later stage. Throughout, we assume that banks never observe their competitors’ contract offers and always know how many times the entrepreneur failed. $E$ always recalls her previous choices and contract offers. $B_k$’s strategy $\sigma^k$ is a sequence of action functions $\sigma^k_t$ for every $t \in \{1, 2, \ldots\}$, where $\sigma^k_t$ gives $C^k_t$ as a function of $H^k_t$: $\sigma^k_t : H^k_t \rightarrow \Delta(C^k)$. Here $\Delta$ denotes the set of mixed actions. $E$’s strategy $\sigma^E$ is a sequence of action functions $\sigma^E_t$ for every $t \in \{1, 2, \ldots\}$, where $\sigma^E_t$ gives the contract choice and the project risk as a function of $H^E_t$ and $C_t$: $\sigma^E : H^E_t \times C \rightarrow \Delta(\mathbb{N} \times R) \times \{p_L, p_H\} \times \{L, H\}$. Denote by $\alpha^k_t (H^k_t)$ ($\alpha^E_t (H^E_t)$) the belief of $B_k (E)$ in period $t$ that $E$ has high talent conditional on $H^k_t$ ($H^E_t$). We will drop the reference to $H^k_t$ ($H^E_t$) whenever it is clear from the context. Define the expected level of talent for given belief $\alpha$ by $\theta(\alpha) = \alpha + (1 - \alpha)\theta_L$.

**Definition 1.** An SE (assessment), in which $E$ undertakes and banks finance the high-risk project in the first $\tau - 1$ periods and the low-risk project in period $\tau$, is called a $\tau - P$ SE (assessment).

3.4 The First-Best Outcome

Assume for a moment that $\alpha_1 = 1$ and banks are absent. Instead, $E$ has “deep pockets” and finances all projects by herself. Her expected payoff from always choosing the high-risk project, $V^H$, amounts to

$$V^H = (1 - p_H)(y_H - 1) + p_H(-1 + V^H).$$

Solving for $V^H$ yields $V^H = y_H - 1/(1 - p_H)$. Her expected payoff from choosing the low-risk project, $V^L$, is given by $V^L = y_L - 1$. We make two assumptions. First, if $E$ has only one chance to undertake a project, she will prefer the low-risk project. Second, if she has infinitely many opportunities to undertake a project, she will always choose the high-risk project, i.e. $V^H > V^L$.

**Assumption (A1).** $y_L > (1 - p_H)y_H$.

**Assumption (A2).** $y_H - 1/(1 - p_H) > y_L - 1$.

10 Note that (A1) and (A2) can be satisfied at the same time if and only if $y_L > 1$. This is the case as both projects have a positive value as long as $E$ has high skills.
Now imagine that $E$ has “deep pockets” and $\alpha_1 < 1$. In equilibrium, we have $\alpha^E_1 = \alpha_1$,
\[
\alpha^E_t = \frac{\alpha_1 p^E_t}{\alpha_1 p^E_t + (1 - \alpha_1) (1 - (1 - p_H) \theta_L)^{t-1}}
\]  
for each $t > 1$ if $E$ chose $j = H$ in all periods $\tau < t$, and $\alpha^E_t = 0$ for $t > 1$ if $E$ chose $j = L$ in at least one period $\tau < t$. $E$’s expected payoff from undertaking the low-risk project in period $t$ is non-negative if and only if the probability of high talent is sufficiently large,
\[
\alpha^E_t \geq \frac{1}{1 - \theta_L} \left( \frac{1}{y_L} - \theta_L \right) \equiv \bar{\alpha}(\theta_L, y_L).
\]
Since $\alpha_1 < 1$, repeated failure indicates low talent, i.e. we have $\alpha^E_t \to 0$ for $t \to \infty$. Thus, $E$ will not undertake high-risk projects in infinitely many periods. When $E$ believes that she very likely is the high type, she foresees many trials ahead and implements the high-risk project. However, as she keeps failing, her belief about her type decreases. When she foresees sufficiently few trials ahead, the value of the low-risk project exceeds that of the high-risk project. After failing with the low-risk project, she stops realizing projects.

**Lemma 1** There is a function $\bar{\alpha} : [\bar{\alpha}(\theta_L, y_L), 1) \to \mathbb{N}, \alpha_1 \to (\bar{\alpha}(\alpha_1))$ such that $E$ with “deep pockets” maximizes her expected payoff for given $\alpha_1$ (i) only if she chooses $j = H$ in all periods $t \leq \bar{\alpha}(\alpha_1) - 1$, and (ii) if she chooses $j = H$ in all periods $t \leq \bar{\alpha}(\alpha_1) - 1$ and $j = L$ in period $\bar{\alpha}(\alpha_1)$. For each $t \in \mathbb{N}$, there is a $\hat{\alpha}_1 < 1$, such that $\bar{\alpha}(\alpha_1) > t$ whenever $\alpha_1 > \hat{\alpha}_1$.

**Proof.** See Appendix. ■

We will refer to $\bar{\alpha}(\alpha_1)$ a number of times. Let $V(\alpha_1)$ be $E$’s expected payoff if she has “deep pockets” and chooses $j = H$ in all periods $t \leq \bar{\alpha}(\alpha_1) - 1$ and $j = L$ in period $\bar{\alpha}(\alpha_1)$. An equilibrium is efficient if and only if the total expected payoff in this equilibrium equals $V(\alpha_1)$.

**4 Perfect Information (PI)**

We first consider a setting with PI, in which banks can observe all of $E$’s risk choices. Note that if $\varepsilon$ is sufficiently small, there is a Nash equilibrium in which banks finance the project in period 1, but never thereafter. Given that $E$ has only one chance to undertake a project, she chooses $j = L$. If she fails with this project, banks know that $E$ has low talent. This information, in turn, justifies the banks’
strategy. However, the threat of no offers being made in period 2 if $E$ chose $j = H$ in period 1 is not credible. The reason is that all banks observe $E$'s decisions and update their beliefs via Bayes’ rule. If $E$ chose the high-risk project, it is still profitable for a bank to finance her after failure. As banks compete in a Bertrand manner, $E$ can undertake projects as if she had “deep pockets”.

**Proposition 1** If $\varepsilon$ is sufficiently small, then under PI, total expected payoffs are $V(a_1)$ in any SE.

### 5 Private Information of Banks (PIB)

#### 5.1 Multiple Equilibria

We now relax the assumption that banks can perfectly observe the riskiness of all past projects. Instead, a bank can only observe the risk of projects in its own loan portfolio. The risk of projects in other banks’ portfolios remains unknown. Therefore, a lending bank acquires private information about $E$.

Consider an assessment in which $E$ chooses $j = L$ in period 1 and no contract offers are made in period $t \geq 2$. This assessment now can be an SE even if $\bar{t}_{(a_1)} > 1$. To see why, assume that $E$ deviates and chooses a contract $(k, r, p_H)$ and $j = H$ in period 1 instead of $j = L$. If her project fails, $B_k$ updates its belief about $E$’s type, knowing that she chose the high-risk project. All other banks do not observe $E$’s deviation and assume that $E$ chose $j = L$ in period 1. Consequently, these banks refuse to finance $E$’s project in any period $t > 1$. This makes $B_k$ a monopolistic supplier of finance to $E$. It can extract (almost) all rents from $E$, leaving her with an expected payoff of at most $\varepsilon$. Therefore, it can be optimal for $E$ to undertake the low-risk project in period 1, which is inefficient whenever $\bar{t}_{(a_1)} > 1$.

Moreover, an efficient SE with $\bar{t}_{(a_1)}$ periods of project financing may not exist in this setting. Under PIB, banks not financing $E$’s project in period $t \leq \bar{t}(a) - 1$ have no information about $E$’s risk choice and the contract $E$ signed in period $t$. Thus, it could be profitable for $E$ to undertake a low-risk project in a period $t \leq \bar{t}(a_1) - 1$ and then, in period $t + 1$, switch to another bank that assumes that $E$ chose the high-risk project in period $t$. This cannot happen in equilibrium. Therefore, a $\bar{t}(a_1) - P$ SE might not exist. In the next section, we show that this inefficiency can be corrected by sharing the information about loan rates of chosen contracts.

There can also be equilibria with more than one period of project financing. Consider a $\bar{t} - P$ assessment for some $\bar{t} > 1$, in which the expected profit of each bank is $\varepsilon$ at most. If $\varepsilon$ is sufficiently small, $E$ will not deviate to $j = L$ in a period
as long as she is relatively certain that she has high talent. Thus, for $\alpha_1$ sufficiently close to 1, both a $1 - P$ and a $\tilde{t} - P$ assessment can be SEs when banks have private information about $E$’s risk choices. Due to this multiplicity of equilibria, credit markets with identical starting conditions may exhibit different levels of the “stigma of failure”.

**Proposition 2** Let $\tilde{t} \geq 2$ be given. If $\alpha_1$ is sufficiently large and $\varepsilon$ is sufficiently small, then under PIB, there exists a $\tau - P$ SE for any $\tau \in \{1, \tilde{t}\}$ and the total expected payoff in any $1 - P$ SE is strictly smaller than in any $\tilde{t} - P$ SE.

**Proof.** See Appendix. ■

### 5.2 Credit Registers

We now assume the existence of a credit register that informs all banks about the loan rates of $E$’s previously chosen loan contracts. When $E$ chooses contract $(k, r, p)$ in period $t$, then $r$ becomes publicly known. Again, $E$’s risk choice in period $t$ is observed only by $B_k$. We will see that in some cases an efficient equilibrium exists if and only if there is a credit register.

Consider a $\tilde{t}(\alpha_1) - P$ assessment. When there is no credit register, it may pay off for $E$ to realize the low-risk project in period $\tilde{t}(\alpha_1) - 1$ and finance it with a relatively cheap loan contract $(., r, p_L)$. If the project fails, $E$ chooses another bank to finance the low-risk project in period $\tilde{t}(\alpha_1)$. However, the bank that finances $E$ in period $\tilde{t}(\alpha_1)$ would then make a negative expected profit. Since this cannot happen in equilibrium, there may be no $\tilde{t}(\alpha_1) - P$ SE.

Next, let there be a credit register. If $E$ deviates and chooses a contract $(., r, p_L)$ in a period $t \leq \tilde{t}(\alpha_1) - 1$ to undertake the low-risk project, the credit register discloses $r$ to all banks. If $r$ is relatively low (i.e. the bank that offered this contract would make a negative expected profit if $E$ realizes the high-risk project in period $t$), other banks may infer that $E$ implemented the low-risk project in period $t$ and therefore refuse to finance her projects in future periods. Hence, the loan rate may signal $E$’s risk choice to other banks. To show that there exists a $\tilde{t}(\alpha_1) - P$ SE, we only have to rule out that it pays off for $E$ to realize a low-risk project and to finance it with a relatively expensive contract $(., r, p_H)$, where $r$ is such that the respective bank breaks even when $E$ chooses $j = H$. In the proof of the following result, we show that this is implied by (A2) if $y_L$ is not too large relative to the loan rate that must be paid for a high-risk project in equilibrium.

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11 In the United States, contract terms are not reported in credit registers, only success or failure.
**Proposition 3** Suppose that \( y_L < 2 / (1 - p_H) \). If \( \varepsilon \) is sufficiently small, then under PIB with credit register, there exists a \( \tau - P \) SE for any \( \tau \in \{1, \bar{t}(\alpha_1)\} \). In any \( \tau - P \) SE with \( \tau < \bar{t}(\alpha_1) \) (\( \tau = \bar{t}(\alpha_1) \)), the total expected payoff is strictly smaller than (equal to) \( V(\alpha_1) \).

**Proof.** See Appendix. ■

5.3 A Numerical Example

The following example illustrates these results (see the Appendix for details). Let \( y_L = 2.9, \ p_H = 0.35, \ \theta_L = 0.15, \ \alpha_1 = 0.9 \). Both (A1) and (A2) are satisfied if the profit of the high-risk project \( y_H \) is in the interval \([3.44, 4.46]\). Figure 1 displays the total expected payoff of the \( 1 - P, \ 2 - P \) and \( 3 - P \) assessment for each \( y_H \in [3.44, 4.00] \) (in this interval, one of these assessments maximizes the total expected payoff). Clearly, the higher \( y_H \), the larger the total expected payoff in the \( 2 - P \) and \( 3 - P \) assessments.

Let us consider the equilibrium set in this example. Under PI, the maximal expected payoff is realized: for small values of the high-risk project’s profit (\( y_H \in [3.44, 3.54] \)), any SE is a \( 1 - P \) SE; for intermediate values (\( y_H \in [3.55, 3.70] \)), any SE is a \( 2 - P \) SE; and for large values (\( y_H \in [3.71, 4.00] \)), any SE is a \( 3 - P \) SE. This changes when banks have private information about \( E \)’s risk choices. Under PIB, there exists a \( 1 - P \) SE for any value of \( y_H \) in the considered interval. This equilibrium is inefficient if \( y_H \) is sufficiently large and the welfare loss strictly increases in \( y_H \). For intermediate values of the high-risk project’s profit (\( y_H \geq 3.58 \)), both a \( 1 - P \) SE and a \( 2 - P \) SE exist; and for large values (\( y_H \geq 3.77 \)), there is a \( \tau - P \) SE for \( \tau \in \{1, 2, 3\} \).

A credit register that informs banks about the loan rates of chosen contracts can make a difference in the example. The requirement \( y_L < 2 / (1 - p_H) \) from Proposition 3 is satisfied so that the credit register ensures the existence of an efficient equilibrium. When there is no credit register, then in the interval \( y_H \in [3.55, 3.57] \) a \( 2 - P \) assessment maximizes total payoffs, but is not an equilibrium; similarly, in the interval \( y_H \in [3.71, 3.76] \) a \( 3 - P \) assessment maximizes payoffs, but is not an equilibrium. In Figure 1, these two intervals are marked with an asterisk.

In these intervals, \( E \) can deviate profitably by realizing a low-risk project in period \( \bar{t}(\alpha_1) - 1 \) (Bertrand competition among banks ensures that this project would be financed by a relatively cheap loan contract).
We now turn to the setting with IM, in which no bank can observe $E$’s risk choices. It is no longer possible for banks to restrict $E$’s risk choice in the contract. Therefore, banks can only offer contracts with $\bar{p} = p_H$ (nevertheless, in equilibrium it may still be optimal for $E$ to choose a low-risk project).

Under IM, banks face a moral hazard problem. $E$ may be inclined to shift risk and undertake the high-risk project when banks offer loan contracts that guarantee non-negative profits only if $E$ chooses the low-risk project. We will see that this moral hazard problem sometimes eliminates those inefficient equilibria where $E$ realizes the low-risk project too early.

Consider an assessment in which $E$ chooses $j = L$ in period $t$, all banks charge the loan rate $r$ in this period, and no bank offers credit to $E$ after period $t$. It is rational for $E$ to choose $j = L$ in period $t$ if and only if

$$\theta(a_t^E)(y_L - r) \geq \theta(a_t^E)(1 - p_H)(y_H - r). \tag{5}$$

Rearranging this inequality yields

$$r \leq \frac{y_L - (1 - p_H)y_H}{p_H}. \tag{6}$$
Banks make non-negative profits only if \( r > 1 \). Hence, there exists an equilibrium in which \( E \) chooses the low-risk project only if the following assumption on payoffs holds:

**Assumption (A3).** \( y_L - (1 - p_H)y_H > p_H \).

The interpretation of (A3) is that the expected NPV of the low-risk project is sufficiently large relative to the expected NPV of the high-risk project such that risk-shifting is not profitable. Note that (A3) implies (A1) and that (A2) and (A3) can hold simultaneously if \( y_H > 2 \). In the following, we analyze credit market equilibria in two scenarios: one in which (A3) does not hold and one in which (A3) holds.

### 6.1 Equilibria if (A3) does Not Hold

We just derived that when (A3) is violated and banks charge loan rates above 1 (which must be the case whenever banks make non-negative expected profits), \( E \) will always realize high-risk projects. Banks anticipate this and adjust their contract offers accordingly. \( B_k \) will make non-negative expected profits in period \( t \) only if \( E \)'s expected type is large enough so that \( (1 - p_H)\theta(a^k_t) \geq 1 \). We rewrite this inequality as

\[
\alpha^k_t \geq \frac{1}{1 - \theta_L} \left( \frac{1}{(1 - p_H)y_H} - \theta_L \right) = \bar{a}(\theta_L, y_H).
\]

As \( E \) keeps failing with high-risk projects, banks’ assessment of her ability worsens. We define the maximum number of periods banks are willing to offer loan contracts to \( E \) (given that \( E \) always chooses the high-risk project) by

\[
\bar{t}(\alpha_1) = \max \left\{ t \in \mathbb{N} \middle| \frac{\alpha_1 p_H^{t-1}}{\alpha_1 p_H^{t-1} + (1 - \alpha_1)(1 - (1 - p_H)\theta_L)^{t-1}} \geq \bar{a}(\theta_L, y_H) \right\}.
\]

We now can state:

**Proposition 4** If (A3) does not hold, \( \alpha_1 \in [\bar{a}(\theta_L, y_H), 1] \), and \( \varepsilon \) is sufficiently small, then in any SE under IM, \( E \) undertakes high-risk projects in \( \bar{t}(\alpha_1) - 1 \) or \( \bar{t}(\alpha_1) \) periods. \( E \) never undertakes a low-risk project. In any SE, the total expected payoff is strictly smaller than \( V(\alpha_1) \).
Clearly, an equilibrium in which $E$ always realizes high-risk projects is not efficient. In the period before banks stop offering loans, $E$ would prefer to undertake the low-risk project when the loan rate is adjusted accordingly. Unable to commit to choosing $j = L$, $E$ ends up realizing the high-risk project.

However, total expected payoffs in the equilibrium under IM can be much larger than in the inefficient $1 - P$ SE under PIB, in which banks’ equilibrium strategies force $E$ to undertake the low-risk project in the first period (see the example below). We therefore obtain a non-monotonic relationship between potential credit market inefficiency and bank information. When banks are perfectly informed about $E$’s past and present risk choices, the equilibrium is always efficient. When they observe the risk of projects in their own loan portfolio but not the risk of projects financed by other banks, there is always an equilibrium with only one period of project financing. The corresponding welfare loss is especially large if $\alpha_1$ is close to 1 and the trade-off between the project’s expected NPV and its maximal NPV is favorable (i.e. $(1 - p_H)y_H$ is close to $y_L$). However, when banks cannot observe the riskiness of any project, the inefficient $1 - P$ assessments are no longer equilibria. The informational asymmetry between $E$ and the bank that finances her project ensures that $E$ will never realize the low-risk project.

Consider again the example from Section 5.3. Figure 2 displays the total expected payoff in the $1 - P$, $2 - P$ and $3 - P$ assessment. In addition, it shows the total expected payoff of an assessment where $E$ realizes (and banks finance) the high-risk project in four periods, and no projects thereafter (the gray line). Call this the $4 - PH$ assessment. For sufficiently large values of the high-risk project’s payoff ($y_H \geq 3.88$), the lowest possible equilibrium loan rate $r$ (which depends on $\alpha_1$) violates [6] so that $E$ will always choose the high-risk project in equilibrium. The unique SE is then a $4 - PH$ assessment. Note that the total expected payoff in this SE is substantially larger than the total expected payoff in a $1 - P$ assessment.

### 6.2 Equilibria if (A3) holds

When (A3) holds, the trade-off between the expected NPV and the maximal NPV of the project is relatively unfavorable, i.e. $(1 - p_H)y_H$ is substantially smaller than $y_L$. Then [6] implies that risk-shifting does not pay off for $E$ when the loan rate is sufficiently close to 1. Consequently, if (A3) holds and $\alpha_1$ is sufficiently large, then again multiple equilibria can occur.

Consider first a $1 - P$ assessment where banks charge loan rates such that their expected profit from financing $E$’s project is non-negative and below $\varepsilon$. The threat of not providing any further loans is credible since banks do not observe $E$’s risk choice. Hence, it is optimal for $E$ to realize the low-risk project when $\alpha_1$ is
sufficiently large. Again, the $1 - P$ SE is inefficient since the low-risk project is realized too early. In the example above, the $1 - P$ SE exists for all values $y_H \leq 3.87$. Observe that the opportunity to shift risks limits the potential welfare loss: as soon as $y_H$ is sufficiently large, only a $4 - PH$ assessment can occur in equilibrium which yields a larger total expected payoff than the $1 - P$ assessment.

Next, consider a $\bar{t} - P$ assessment for some $\bar{t} > 1$ where again banks’ expected profit from financing $E$’s project is non-negative and below $\varepsilon$. If $a_1$ is sufficiently large, then, in all periods $t \leq \bar{t}$, $E$ is relatively certain that she has high talent. Hence, it does not pay off for her to realize the low-risk project in a period $t < \bar{t}$ (even if she finds a bank that offers credit at a low loan rate). Thus, there is a $\bar{t} - P$ SE if $a_1$ is sufficiently large. The multiplicity of equilibria therefore may persist if banks do not observe $E$’s risk choices.

Again, an efficient equilibrium may not exist under IM. The reason is that the common belief about $E$’s talent in period $\bar{t}(a_1)$ may be so unfavorable (and therefore individually rational loan rates so high) that [6] is violated. Then it is not rational for $E$ to realize the low-risk project in period $\bar{t}(a_1)$.

**Proposition 5** Let $\bar{t} \geq 2$ be given. If (A3) holds, $a_1$ is sufficiently large and $\varepsilon$ is sufficiently small, then under IM, there exists a $\tau - P$ SE for any $\tau \in \{1, \bar{t}\}$ and the total expected payoff in any $1 - P$ SE is strictly smaller than in any $\bar{t} - P$ SE.

**Proof.** See Appendix. ■

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**Figure 2:** Total expected payoff of the $1 - P$ (flat line), $2 - P$ (dotted line), $3 - P$ (solid line) and $4 - PH$ assessment (gray line).

![Figure 2: Total expected payoff](image-url)


7 Discussion

We have analyzed a model of entrepreneurial finance and risk-taking where the extent to which failed entrepreneurs are excluded from further financing is determined endogenously. The driver of our results is the evolution of banks’ beliefs about an entrepreneur’s talent and the interplay between these beliefs and her risk choices. If banks can perfectly observe the entrepreneur’s actions, then the first-best outcome is realized in any equilibrium: she first undertakes a number of high-risk projects and then, after continuous failure, undertakes a low-risk project.

When banks acquire private information about $E$’s risk choices, the first-best outcome is not the unique equilibrium outcome. Instead, there also exists an inefficient equilibrium in which the entrepreneur undertakes a low-risk project and becomes excluded from finance after failure. This inefficiency is due to the fact that banks may expect the entrepreneur to undertake the low-risk project. Failure of low-risk projects indicates low talent. Therefore, outside banks may refuse to finance her after failure. The bank that financed the project then becomes a monopolistic supplier of finance to the entrepreneur. Hence, it is rational for the entrepreneur to undertake a low-risk project.

We also saw that the opportunity to shift risks (when banks cannot control the project risk) may increase welfare as compared to the case when only the financing bank controls the project risk. If banks do not observe the riskiness of projects (i.e. banks cannot acquire private information about $E$) and the expected NPV of the high-risk project is sufficiently close to that of the low-risk project, $E$ always undertakes high-risk projects in equilibrium. This outcome is inefficient. Yet, it may be more efficient than the one-shot financing equilibrium when banks acquire private information. However, if the expected NPV of the high-risk project is sufficiently small relative to that of the low-risk project, then multiple equilibria with different levels of total expected payoffs again exist.

In the following, we discuss several implications of our model.

7.1 Asymmetric Information

The notion that asymmetric information can lead to an inefficient allocation of credit is well-established in economic theory. The financial contracting literature has focused on solutions of the moral hazard problem if banks cannot
directly observe $E$’s risk choice (via monitoring, collateral or incentive contracts). We have shown that the credit market equilibrium can be inefficient even if there is no asymmetric information between $E$ and the bank that finances her project. This inefficiency is caused by the feedback loop between risk-taking and outside banks’ beliefs. Policies that aim to change the nature of the equilibrium may not be effective, as both entrepreneurs’ actions and banks’ expectations would have to be changed simultaneously. Consider, for example, the approach of the European Commission (2000; 2007). It attempts to reduce the “stigma of failure” by advising entrepreneurs to choose higher risk levels. Entrepreneurs, however, will follow such advice only if financiers change their policy at the same time.

### 7.2 Sharing Information

Our model has novel implications for credit registers and their informational content. So far, the literature has shown that credit registers flagging failure increase credit market efficiency because they resolve the adverse selection problem (Pagano and Jappelli 1993; Padilla and Pagano 1997). We show that a credit register that merely reports bankruptcy history may not be enough to remove inefficient equilibrium outcomes. When banks acquire private information about $E$’s risk choices, an equilibrium arises where the bank that finances $E$’s project will hold up $E$ after failure, which in turn leads to the (inefficient) realization of low-risk projects at an early stage of $E$’s career.

A possible way out of this trap is to promote information sharing among banks. If the incumbent bank truthfully shares its evaluation of $E$’s project with the market, we are again back in the setting with perfect information where in equilibrium $E$ first experiments with risky business ideas before she realizes the low-risk project. Thus, information sharing can increase entrepreneurial activity. Note that information sharing would not impact on banks’ profits in our setting.

If the truthful communication of $E$’s previous risk choices is not feasible, the exchange of information about contract details through credit registers may also increase credit market efficiency. As we emphasized in Section 5, information about the loan rates of previous contracts can be crucial for ensuring the existence of more efficient credit market equilibria. This information enables banks to infer $E$’s previous risk choices from loan rates. An unpaid loan with a relatively low loan rate may indicate that the underlying project risk was low (otherwise, the bank would not have offered this loan to the entrepreneur).
Failure then discloses low entrepreneurial talent, preventing banks from granting further loans to her. In contrast, an unpaid credit with a high loan rate may indicate that the underlying project’s risk was high, suggesting that its failure owes more to bad luck rather than low entrepreneurial talent. In this case, \( E \) probably deserves another chance. However, note that the loan rate alone does not reveal \( E \)’s actions. Banks might also infer from a high loan rate that \( E \) undertook the low-risk project and – by mistake – chose an inappropriate contract. Therefore, the multiplicity of equilibria persists as long as banks do not perfectly observe \( E \)’s risk choices.

### 7.3 Improving Entrepreneurial Talent

Our model suggests that the maximal number of times an entrepreneur can start anew after bankruptcy increases in entrepreneurs’ average talent (see Lemma 1). Hence, one measure to increase entrepreneurial activity is the promotion of education that leads to the formation of relevant skills. Entrepreneurial education plays a substantial role both in economic development (Bjorvatn and Tungodden 2010; Klinger and Schündeln 2011) and the EU’s policy to increase entrepreneurship after failure (European Commission 2007). In terms of our model, these policies would increase the probability of having high entrepreneurial talent \( \alpha_1 \).

If banks have PI, then an increase in \( \alpha_1 \) has both a direct and an indirect effect on equilibrium welfare. The direct effect is that loan rates decrease in all periods. The indirect effect is that the number \( t(\alpha_1) \) of periods in which projects are financed (weakly) increases. Yet, if banks acquire private information about \( E \)’s risk choices, the indirect effect might not materialize. There always exists an equilibrium in which the entrepreneur undertakes a low-risk project and does not get financed after failure. Therefore, an increase in \( \alpha_1 \) does not necessarily entail a positive effect on entrepreneurial activity among those who fail. Unless the banks’ policies and entrepreneurs’ risk-taking behaviors become coordinated simultaneously to another equilibrium, only the direct effect unfolds.

### Appendix

#### A.1 Proof of Lemma 1

Take \( \alpha_1 \) as given. Whenever \( E \) chooses \( j = L \) and the project fails, \( E \) knows that her talent is low. Then she does not undertake any further projects. Consider the
set of assessments in which \( E \) chooses \( j = H \) in periods \( t \in \{1, \ldots, t^* - 1\} \), \( j = L \) in period \( t^* \), and no projects thereafter. Denote by \( V_t^{(r)} \) the expected payoff of \( E \) at the beginning of period \( t \in \{1, \ldots, t^*\} \) under the assessment with \( t^* \) periods of project realizations. We have

\[
V_t^{(r)} = \theta(\alpha_t^{E})y_L - 1, \tag{9}
\]

and for \( t \in \{1, \ldots, t^* - 1\} \):

\[
V_t^{(r)} = (1 - p_H)\theta(\alpha_t^{E})y_H - 1 + (1 - (1 - p_H)\theta(\alpha_t^{E}))V_{t+1}^{(r)}, \tag{10}
\]

where \( \alpha_t^{E} = \alpha_t^{1} \) and \( \alpha_t^{E} \) is given by eq. [3] for each \( t \in \{2, \ldots, t^*\} \). Note that there must be a \( t^* \) such that \( V_{t}^{(r)} \) is positive only if \( t^* \in \{1, \ldots, t^*\} \). We therefore find that

\[
\bar{\theta}(\alpha_t) = \min\{g \in \{1, \ldots, t^*\} | V_{t}^{(g)} \geq V_{t}^{(r)}, t^* \in \{1, \ldots, t^*\} \}. \tag{11}
\]

To prove the second claim, observe that

\[
\lim_{\alpha_t \to 1} V_{t}^{(r)} = (1 - p_H^{t-1}) \left( y_H - \frac{1}{1 - p_H} \right) + p_H^{t-1}(y_L - 1). \tag{12}
\]

(A2) then implies

\[
\lim_{\alpha_t \to 1} V_{t}^{(r)} > \lim_{\alpha_t \to 1} V_{t}^{(t)} \tag{13}
\]

for all \( t < t^* \). Note that \( V_{t}^{(r)} \) is continuous in \( \alpha_t \) for all \( t^* \in \mathbb{N} \). Thus, for each \( t \in \mathbb{N} \) there is a \( \tilde{\alpha}_t < 1 \) such that \( \bar{\theta}(\alpha_t) > t \) whenever \( \alpha_t > \tilde{\alpha}_t \).

### A.2 Proof of Proposition 2

Let \( \sigma = (\sigma^E, \sigma^1, \sigma^2, \ldots) \) be a strategy profile and \( \alpha = (\alpha^E, \alpha^1, \alpha^2, \ldots) \) a system of beliefs, where \( \alpha^E = \{a_i^E(H_k^E)\}_{i=1}^{\infty} \) and \( \alpha^k = \{a_i^k(H_k^k)\}_{i=1}^{\infty} \). The assessment \((\sigma, \alpha)\) is an SE if (i) in each period \( E \) and banks maximize expected payoffs for given beliefs and competitors’ strategies, and (ii) it is the limit of a sequence \( \{\sigma^n, \alpha^n\}_{n \geq 1} \), where \( \sigma^n \) is a totally mixed strategy profile and \( \alpha^n \) is uniquely defined from \( \sigma^n \) by Bayes’ rule. We first show the existence of a \( 1 - P \) SE and then the existence of a \( \bar{\theta} \) – \( P \) SE. The last statement of Proposition 2 directly follows from Lemma 1.

**1 - P SE.** We propose a \( 1 - P \) assessment and show that it is an SE. Define

\[
r(j, \alpha) = \min \left\{ r \in R(\varepsilon) \left| r \geq \frac{1}{(1 - p_j)\theta(\alpha)} \right\} \right.
\]

\[
\tag{14}
\]
and consider an assessment \((\sigma, \alpha)\) with the following properties:

- In period 1, a bank \(B_k\) offers contracts

\[
(k, r(L, \alpha_1), p_L) \text{ and } (k, r(H, \alpha_1), p_H).
\]

- A bank \(B_k\) that did not finance the project in period 1 has beliefs \(\alpha^k_t = 0\) in all periods \(t \geq 2\) and therefore does not offer any contracts.
- A bank that financed the project in period 1 has a belief in period 2 that is derived from Bayes’ rule. As long as it is profitable, this bank offers contracts with loan rates equal to \(\max \{r \in R(\varepsilon)\}\).
- \(E\) undertakes projects whenever possible.

In this assessment, the expected payoff of \(E\) in period 2 is 0 regardless of her choice in period 1. Hence, it is optimal for her to choose a contract \((k, r(L, \alpha_1), p_L)\) and \(j = L\) in period 1 if

\[
\theta(\alpha_1)(y_L - r(L, \alpha_1)) \geq (1 - p_H)\theta(\alpha_1)(y_H - r(H, \alpha_1)),
\]

which is implied by (A1) given that \(\varepsilon\) is sufficiently small. Facing Bertrand competition, no bank can deviate profitably in period 1. It remains to show that beliefs are consistent. Consider a sequence \(\{(\sigma^{[n]}, \alpha^{[n]})\}_{n \in \mathbb{N}}\) with \((\sigma^{[n]}, \alpha^{[n]}) \rightarrow (\sigma, \alpha)\) in which \(\sigma^{[n]}\) is such that \(E\) chooses \(j = L\) with probability \((n - 1)/n\) and \(j = H\) with probability \(1/n\) in period 1 whenever she chooses a contract that allows for high-risk projects. Clearly, we have \((\alpha^k_t(H^k_t))^{[n]} \rightarrow 0\) for any \(t > 1\) and any \(H^k_t\) that does not contain \(E\)’s actual risk choice in period 1.

\(\tilde{t} - P\ SE\). Let \(r(j, \alpha)\) be given by eq. [14]. Consider an assessment \((\sigma, \alpha)\) with the following properties:
- In each period \(t < \tilde{t}\), \(B^k\) offers contracts

\[
(k, r(L, \alpha^k_t), p_L) \text{ and } (k, r(H, \alpha^k_t), p_H),
\]

where \(\alpha^k_t = \alpha^E_t\) and \(\alpha^E_t\) is given by eq. [3], unless \(B^k\) observes that in a period \(\tau \in \{1, \ldots, \tilde{t} - 1\}\) \(E\) chose \(j = L\). In this case, it does not offer any contracts.
- In period \(\tilde{t}\), \(B^k\) offers contract \((k, r(L, \alpha^E_t), p_L)\) where \(\alpha^E_t = \alpha^E_t\) and \(\alpha^E_t\) is given by eq. [3], unless it observes that in a period \(\tau \in \{1, \ldots, \tilde{t} - 1\}\) \(E\) chose \(j = L\). In this case, it does not offer any contracts.
- Banks offer no contracts in any period \(t > \tilde{t}\).
- Whenever banks offer contracts as described above, \(E\) undertakes a high-risk project in all periods \(t \in \{1, \ldots, \tilde{t} - 1\}\) and a low-risk project in period \(\tilde{t}\).

To show that these can be the properties of an SE, assume that \(E\) deviates in period \(t^* < \tilde{t}\) and chooses a contract \((., r(L, \alpha^E_t), p_L)\). Given the banks’ strategy, \(E\)’s expected payoff at the beginning of period \(t^*\) is less than
\[ \theta(\alpha^E_t)(y_L - r(L, \alpha^E_{t})) + (1 - \theta(\alpha^E_t))(y_H - 1). \]  

For \( \alpha_1 \to 1 \) and \( \varepsilon \to 0 \) this term becomes \( y_L - 1 \), while \( E \)'s expected payoff at the beginning of period \( t^* \) under the original strategy for \( \alpha_1 \to 1 \) and \( \varepsilon \to 0 \) equals

\[ (1 - p^H_{t-t^*})(y_H - \frac{1}{1-p^H_t}) + p^H_{t-t^*}(y_L - 1). \]

Thus, (A2) ensures that \( E \) cannot profitably deviate in any period \( t < \bar{t} \) if \( \alpha_1 \) is sufficiently large and \( \varepsilon \) sufficiently low. (A1) ensures that the same is true for period \( \bar{t} \). Following the same steps as in the proof of existence of the \( 1-P \) SE, we can show that banks’ beliefs can be consistent in all periods. Facing Bertrand competition banks cannot profitably deviate.

### A.3 Proof of Proposition 3

The proof of existence of a \( 1-P \) SE proceeds as in the proof of Proposition 2. We prove the existence of a \( \bar{t}(\alpha_1) - P \) SE. Consider an assessment \( (\alpha, \alpha) \) with the following properties:

- In period 1, \( B_k \) offers the same contracts as in [15].
- In each period \( t \in \{2, ..., \bar{t}(\alpha_1) - 1\} \), \( B_k \) offers contracts

\[ (k, r(L, \alpha^k_t), p_L) \text{ and } (k, r(H, \alpha^k_t), p_H), \]

where \( \alpha^k_t = \alpha^E_t \) and \( \alpha^E_t \) is given by eq. [3], unless it observes that in a period \( \tau \in \{1, ..., t-1\} \), \( E \) chose \( j = L \) or did not chose contracts with loan rates equal to \( r(H, \alpha^E_{\tau}) \) where \( \alpha^E_{\tau} \) is given by eq. [3]. In these cases, it does not offer any contracts.

- In period \( t = \bar{t}(\alpha_1) \), \( B_k \) offers contract \( (k, r(L, \alpha^{k}_{\bar{t}(\alpha_1)}, p_L) \) where \( \alpha^{k}_{\bar{t}(\alpha_1)} = \alpha^{E}_{\bar{t}(\alpha_1)} \) and \( \alpha^{E}_{\bar{t}(\alpha_1)} \) is given by eq. [3], unless it observes that in a period \( \tau \in \{1, ..., t-1\} \), \( E \) chose \( j = L \) or did not choose contracts with loan rates equal to \( r(H, \alpha^E_{\tau}) \) where \( \alpha^E_{\tau} \) is given by eq. [3]. In these cases, it does not offer any contracts.

- Banks offer no contracts in any period \( t > \bar{t}(\alpha_1) \).
- Whenever banks offer contracts as described above, \( E \) undertakes a high-risk project in all periods \( t \in \{1, ..., \bar{t}(\alpha_1) - 1\} \) and a low-risk project in period \( \bar{t}(\alpha_1) \).

To show that these can be the properties of an SE, assume that \( E \) deviates in period \( t^* < \bar{t}(\alpha_1) \) and chooses a contract \( (k, r(L, \alpha^k_t), p_L) \). Note that \( r(L, \alpha^k_t) \neq r(H, \alpha^E_t) \). Given the banks’ strategy, the expected payoff of \( E \) in period \( t^* \) is

\[ \tilde{V}^{(r)}_{t^*} = \theta(\alpha^E_{t})(y_L - r(L, \alpha^E_{t})), \]
and in a period \( t \in \{1, \ldots, t^* - 1\} \) it is
\[
\tilde{V}^{(c)}_t = (1 - p_H)\theta(\alpha^E_t)(y_H - r(H, \alpha^E_t)) + (1 - (1 - p_H)\theta(\alpha^E_t))\tilde{V}^{(c)}_{t+1}.
\]  
[22]

Note that for \( \varepsilon \to 0 \) these expressions are identical to those in eqs. [9] and [10]. It follows from Lemma 1 that \( E \) has no incentive to deviate if \( \varepsilon \) is sufficiently small. Moreover, it does never pay off for \( E \) to choose a contract \( (., r(H, \alpha^E_t), p_H) \) and \( j = L \) in any period \( t < \tilde{t}(\alpha_t) \). If it paid off, then for some \( V \geq 0 \) we would have
\[
\theta(\alpha^E_t)(y_L - r(H, \alpha^E_t)) + (1 - \theta(\alpha^E_t))V > \theta(\alpha^E_t)(1 - p_H)(y_H - r(H, \alpha^E_t)) + (1 - \theta(\alpha^E_t)(1 - p_H))V
\]  
[23]
and
\[
\theta(\alpha^E_t)(y_L - r(H, \alpha^E_t)) + (1 - \theta(\alpha^E_t))V \geq \theta(\alpha^E_t)(y_L - r(L, \alpha^E_t)).
\]  
[24]

We can transform [23] into
\[
V < \frac{y_L}{p_H} - \frac{(1 - p_H)y_H}{p_H} - r(H, \alpha^E_t)
\]  
[25]
and [24] into
\[
V > \frac{\theta(\alpha^E_t)}{1 - \theta(\alpha^E_t)}(r(H, \alpha^E_t) - r(L, \alpha^E_t)).
\]  
[26]

Thus, it never pays off for \( E \) to choose a contract \( (., r(H, \alpha^E_t), p_H) \) and \( j = L \) in period \( t \) if the right-hand side of [26] exceeds the right-hand side of [25]. If \( \varepsilon \) is sufficiently small, this follows from
\[
\frac{p_H}{1 - p_H} > \frac{y_L}{p_H} - \frac{(1 - p_H)y_H}{p_H} - \frac{1}{(1 - p_H)\theta(\alpha^E_t)}.
\]  
[27]
Assumption (A2) implies
\[
y_H > y_L + \frac{p_H}{(1 - p_H)}.
\]  
[28]

This inequality can be used to show that [27] follows from \( y_L < 2/(1 - p_H) \). Due to Bertrand competition no bank can profitably deviate. The consistency of beliefs can be shown as in the proof of Proposition 2. Finally, the last claim of Proposition 3 directly follows from Lemma 1.

### A.4 Proof of Proposition 5

The proof is very similar to the proof of Proposition 2 and therefore omitted.
A.5 Details of the Numerical Example

PIB. We set $\varepsilon = 0.0001$, which guarantees that no equilibrium is ruled out artificially. Next, we calculate the loan rates $r(j, \alpha^k_t)$ for $t \in \{1, 2, 3, 4\}$ and $j \in \{L, H\}$ according to eq. [14] where $\alpha^k_t = \alpha^E_t$ and $\alpha^E_t$ is given by eq. [3]:

<table>
<thead>
<tr>
<th>$t$</th>
<th>$r(L, \alpha^k_t)$</th>
<th>$r(H, \alpha^k_t)$</th>
<th>$\alpha^E_t$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.0929</td>
<td>1.6814</td>
<td>0.9000</td>
</tr>
<tr>
<td>2</td>
<td>1.2335</td>
<td>1.8977</td>
<td>0.7773</td>
</tr>
<tr>
<td>3</td>
<td>1.5654</td>
<td>2.4082</td>
<td>0.5751</td>
</tr>
<tr>
<td>4</td>
<td>2.2594</td>
<td>3.4760</td>
<td>0.3442</td>
</tr>
</tbody>
</table>

We can verify for each pair $t, j$ that $B_k$ makes positive expected profits in period $t$ if it offers the contract $(k, r(j, \alpha^k_t), p_j)$, $\alpha^k_t = \alpha^E_t$, $E$ accepts this contract and realizes the project with risk $j$. From now on a $\tilde{t} - P$ assessment has the following features:

- In each period $t < \tilde{t}$, each bank $B_k$ offers the contracts
  $$(k, r(L, \alpha^k_t), p_L) \text{ and } (k, r(H, \alpha^k_t), p_H),$$
  unless $B_k$ observes that $E$ realized the low-risk project in a previous period (in this case, it offers no contracts).
- In period $\tilde{t}$, $B_k$ offers the contract $(k, r(L, \alpha^k_t), p_L)$, unless $B_k$ observed that $E$ realized the low-risk project in a previous period (it offers no contracts in this case).
- Banks offer no contracts in any period $t > \tilde{t}$.
- Whenever banks offer contracts as described above, $E$ undertakes a high-risk project in all periods $t \in \{1, ..., \tilde{t} - 1\}$ and a low-risk project in period $\tilde{t}$.

Since banks compete in Bertrand manner, the $\tilde{t} - P$ assessment is an SE if and only if $E$ has no incentive to realize the low-risk project in a period $t < \tilde{t}$. For each $\tilde{t} \in \{1, 2, 3\}$ we then can calculate $E$’s expected payoff in the $\tilde{t} - P$ assessment and her payoff for any possible deviation. This yields us the equilibrium set under $PIB$ without credit register.

IM. In any equilibrium, $E$ realizes the high-risk project (given that it gets financed) if the lowest possible equilibrium loan rate, $r(L, \alpha^k_1)$, violates [6]. This is the case if $y_H > 3.88$. For each value $y_H \in [3.50, 4.00]$, we have $\tilde{t}(a_1) = 4$. Hence, every SE is a $4 - PH$ assessment if $y_H \geq 3.88$. For $y_H \in [3.44, 3.87]$ the loan rate $r(L, \alpha^k_1)$ satisfies [6]. Hence, a $1 - P$ SE exists if $y_H \in [3.44, 3.87]$. 
References


Note: The views expressed herein are those of the authors and should not be attributed to the IMF, its Executive Board, or its management, or to the Federal Reserve Bank of Boston or Federal Reserve System. We thank Guido Friel (the editor) and an anonymous referee for useful advice; Viral Acharya, Bruno Biais, Patrick Bolton, Douglas Gale, Hans Peter Grüner, Markus Kinatered, Holger Müller, Nicola Persico, Rafael Repullo, Sebastian Schäfer, Christian Schmieders, Ernst-Ludwig von Thadden as well as seminar participants at Pompeu Fabra, Goethe-University Frankfurt, Mannheim University and Federal Reserve Bank of Kansas City for valuable discussions and helpful comments. All remaining errors are our own.