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Comparison of Cole-Cole and Constant Phase Angle modeling in time-domain induced polarization

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SUMMARY

The Cole-Cole model and the constant phase angle (CPA) model are two prevailing phenomenological descriptions of the induced polarization (IP), used for both frequency domain (FD) and time domain (TD) modeling. The former one is a 4-parameter description, while the latest one involves only two parameters. Choosing between a Cole-Cole description and a CPA one to invert a specific frequency domain data set is easy, since a look at the data is enough to estimate their spectral content. This is, however, not the case with TDIP data. This work aims at understanding how the spectral content is reflected in TDIP data, and therefore, at identifying (1) if and when it is possible to distinguish, in time domain, between a Cole-Cole description and a CPA one, and (2) if features of time domain data exist in order to know, from a simple data inspection, which model will be the most adapted to the data. Synthetic forward responses were computed for homogeneous Cole-Cole models, varying both time range of the modeled IP data and Cole-Cole parameters. Subsequently, CPA inversions were carried out on the Cole-Cole data. The inversion results show that it is generally possible to distinguish CPA and Cole-Cole models in time domain, except when the Cole-Cole frequency exponent is small (below 0.1) or for specific combinations of the Cole-Cole parameters. The distinctness increases with the time range of the IP data, but usually two decades in time are sufficient to distinguish the two models. Furthermore, forward modeling of quadrupolar sequences on 1D and 2D heterogeneous CPA models shows that the CPA decays differ among each other only by a multiplication factor. Consequently, the inspection of field data in log-log plots gives insight on the modeling needed for fitting them: the CPA inversion cannot reproduce the shape variability of the IP decays. Field examples of this latter result are presented.
Keywords: Cole-Cole, CPA, time-domain, spectral inversion

INTRODUCTION

The Induced Polarization of rocks and soils can be described with a frequency-dependent complex resistivity. Several models are used to describe the induced polarization of geomaterials, but the most used are the Cole-Cole model presented by Pelton et al. (1978) and the constant phase angle model (CPA), as described for instance in Van Voorhis et al. (1972).

The CPA model is suitable if no or negligible variation of the phase shift is observed in the complex resistivity data. Thus, the choice of using the Constant Phase Angle (CPA) model instead of the Cole-Cole model to describe a specific set of frequency domain IP data is straightforward. This is, however, not the case with time domain IP data. Being able to understand how the Cole-Cole description differs from the CPA description in time domain will allow us to judge more easily, which description will manage best to describe the induced polarization of a specific studied area.

TDIP forward responses of homogeneous half-spaces have been computed, using the Cole-Cole modeling and varying the acquisition time ranges. Each synthetic decay has then been inverted using the CPA modeling, in order to test to what extent the CPA inversion is able to fit Cole-Cole data. Finally, a field data set has been inverted using both models, to assess in a real 2D situation their ability to explain data.

METHOD AND RESULTS

The Cole-Cole and CPA models are the two principal phenomenological models used to describe the induced polarization of rocks and soils. The complex resistivity $\zeta_{Cole\text{-}Cole}$ of the Cole-Cole model takes the form:

$$\zeta_{Cole-Cole} = \rho\left(1 - m_0\left(1 - \frac{1}{1+(i\omega\tau)^C}\right)\right) \quad (1)$$

where ρ is the direct current resistivity, m_0 is the intrinsic chargeability, τ is the time constant, C is the frequency exponent and i is the imaginary unit.

The CPA model is much simpler, and describes the complex resistivity using only two parameters:

$$\zeta_{CPA} = K(i\omega)^{-b} \quad (2)$$

where b is a positive fraction, $\varphi = -\frac{\pi}{2}b$ represents the phase shift and completely defines the IP response, K is a constant and i is the imaginary unit. In the CPA model the DC resistivity cannot be defined, because the complex resistivity increases indefinitely at low frequencies. For this reason Van Voorhis et al. (1973) introduced the Drake model:

$$\zeta_{Drake} = K(i\omega + \omega_L)^{-b} \quad (3)$$

where in comparison with the CPA model a low frequency pole ω_L is introduced and the DC resistivity can be defined as $\rho = K{\omega_L}^{-b}$. In our implementation of the time-domain forward response, we used the Drake model of equation (3) with a fixed value for the low frequency pole $\omega_L = 10^{-5}$ Hz. In this way, the inversion is set up in terms of the model parameters ρ and φ.

We computed synthetic time domain data using the Cole-Cole description of the induced polarization, and tested for different types of acquisition to what extent the CPA inversion was able to fit the synthetic data. We chose to simulate data from homogeneous half-spaces, to be able to interpret the results of the tests easily. The computations of forward responses and inversions have been realized using the algorithm presented in Fiandaca et al. (2012). We simulated different data, changing the Cole-Cole model parameters C and τ, at fixed m_0 and ρ values. In particular, we chose as synthetic models every possible combination of the following parameters: ρ = 100 Ωm, m_0 = 40 mV/V, C = [0.1, 0.3, 0.5], τ = [0.001, 0.01, 0.1, 1, 10] s. Different acquisition ranges have been investigated, starting from a reference acquisition with 40 log-increasing gates ranging from 1 millisecond to 10 seconds. The reference acquisition range has been reduced by (Figure 1): decreasing the T_{on}=T_{off} values (T_{on} and T_{off} being the current on-time and off-time, respectively), and consequently the time of the last gate (range-type 1); increasing the delay after the current turn-off m_{dly}, and consequently by increasing the time of the first gate (range-type 2);increasing m_{dly} and decreasing T_{on} at the same time (range-type 3).

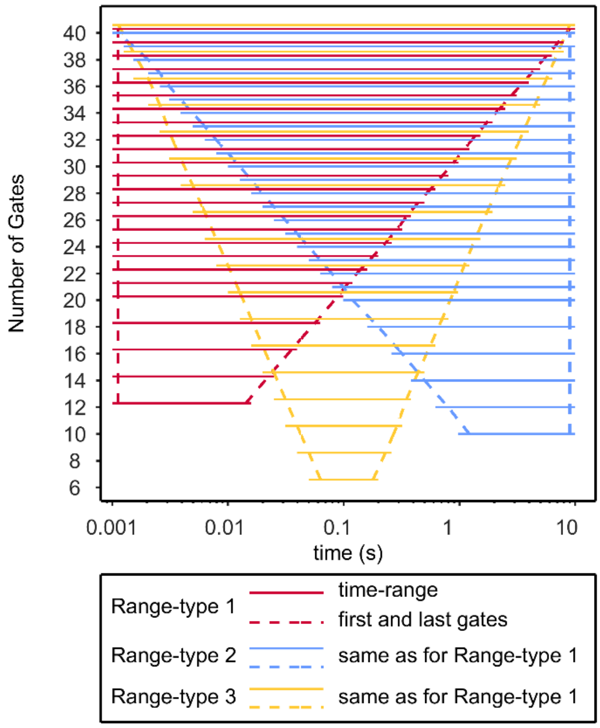

Figure 1. Definition of the three acquisition range-types. Range-type 1: the time length of the decay is increased by adding gates at the end of the acquisition time, keeping m_{dly} = 1 ms (red lines). Range-type 2: the time length of the decay is increased by adding gates at early times and keeping T_{on}=T_{off} = 10 s (blue lines). Range-type 3: the length of the decays is increased by adding gates both at the late and at the early times (yellow lines).

Figure 2 shows exemplary fits of Cole-Cole decays with CPA modeling when only 10 gates (one decade in time) are used in the acquisition range for all the three different range-types. The CPA inversion manages to explain the 10 gates-long curves for any C values, and it is not possible to distinguish CPA and Cole-Cole modeling. Figure 3 presents the CPA fits of three different 40-gates Cole-Cole decays, for different τ values (0.01 and 1 seconds) and C values (0.3 and 0.5). The shape of the Cole-Cole forward responses changes significantly when varying τ and C, while the shape of the CPA modeling in log-log scale remains practically unchanged, the only difference being a translation along the y axis. Practically, the shape of the CPA decays in log-log scale is univocally defined by the current waveform (in terms of current on-time T_{on}, current off-time T_{off} and stack size). For specific combinations of τ and C parameters (e.g. τ=1 s and C=0.3) the CPA and Cole-Cole decays are really similar, but in general the decays differ significantly.

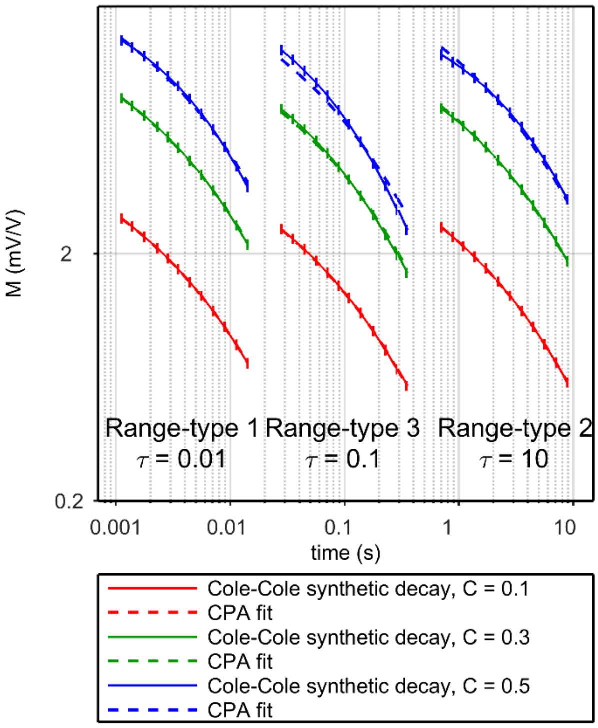

Figure 2. CPA fits of Cole-Cole decays (5% error bars) when varying models and range-types, but keeping constant the number of gates (10 gates).

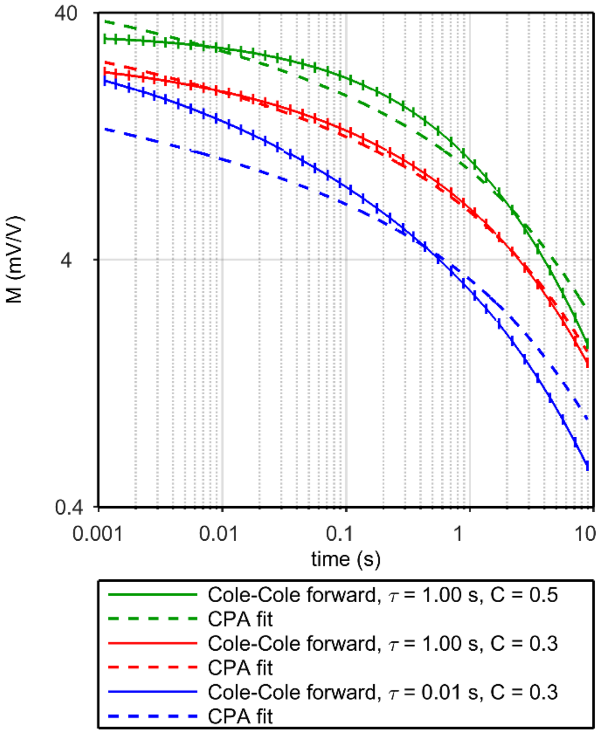

Figure 3. Exemplary CPA fits of Cole-Cole decays (5% error bars, 40 gates) when varying Cole-Cole parameters.

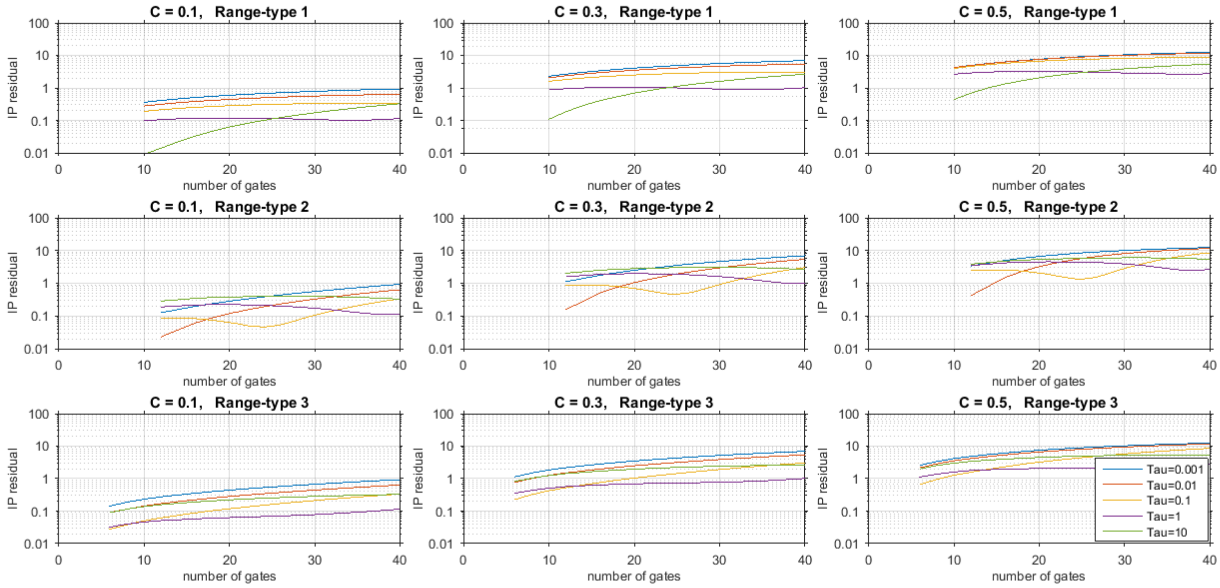

Figure 4. Complete results of the synthetic tests in terms of inversion residuals (χ values, 5% error bars). The results have been sorted according to the acquisition range-type (row), the frequency exponent (column) and the time constant (line color). For each case, the inversion residuals are displayed as a function of the number of gates in the synthetic data.

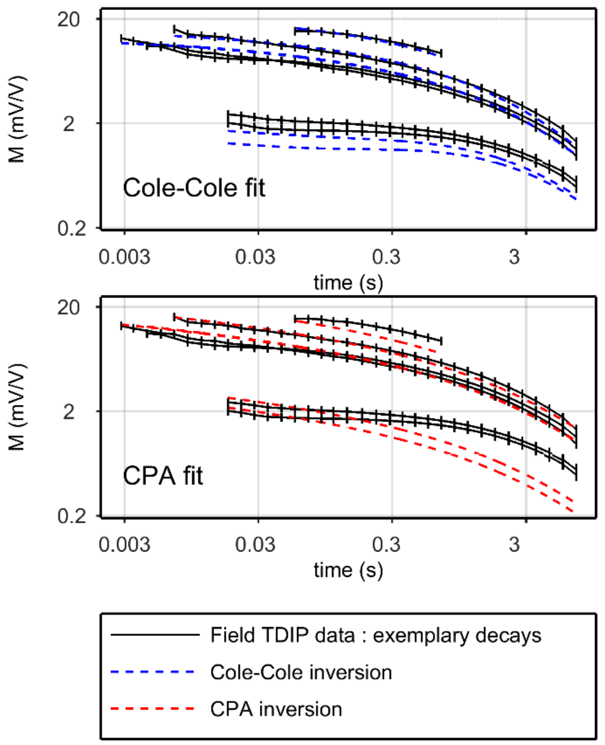

Figure 5. Examples of field decays, along with their Cole-Cole and CPA fits obtained through a 2D inversion.

Figure 4 shows the inversion residuals (χ values, 5% error bars) of the CPA inversions carried out on Cole-Cole forward decays, when varying model, range-type and number of gates. All the models with C=0.1 present misfit below/equal to one, regardless of the number of gates. This is easily understood considering that the Cole-Cole model tends to the CPA model when C goes to zero. On the other hand, with C=0.5 the two modeling are almost always distinguishable, except for specific models when less than two decades are used in the acquisition time range. In particular, it is more difficult to distinguish the CPA and Cole-Cole models for high τ values and range-type 1 when too few gates are used (i.e. when we miss the late times). With range-type 2 the CPA and Cole-Cole models are more difficult to distinguish with low τ values. The results with C=0.3 are similar to the results with C=0.5, except that the inversion misfits is smaller. Interestingly, with τ=1 s the CPA and Cole-Cole models are more difficult to differentiate, and the CPA inversion often fits the Cole-Cole data within 5% also with 40 gates.

Finally, Figure 5 shows the comparison of CPA and Cole-Cole modeling of field data, inverted in 2D following Fiandaca et al. (2013). The field data were acquired at Grindsted, Denmark, with a Terrameter LS (ABEM Instrument). We used an on-time and an off-time of 8 s both, and 10 gates per decade (re-gating the full-waveform data and applying the de-noising scheme described by Olsson et. al (2016)). The data quality was generally good, and after processing, most of the decay curves had still ~ 30 gates. As for the synthetic modeling for homogeneous halfspace, the shape of the CPA forward responses does not change in log-log plots. Consequently, the CPA description cannot explain the variety of shapes present in the data. On the contrary, the Cole-Cole modeling is able to retrieve the shape of the field decays.

CONCLUSIONS

The synthetic results show that it is generally possible to distinguish CPA and Cole-Cole models in time domain, except when the Cole-Cole frequency exponent is small (below/equal to 0.1) or for specific combinations of the Cole-Cole parameters. The distinctness increases with the time range of the IP data, but usually two decades in time are sufficient to distinguish the two models. Furthermore, the shape of the CPA forward responses in log-log plots is univocally defined by the current waveform, also for 2D modeling. Consequently, the inspection of field data in log-log plots gives insight on the modeling needed for fitting them: the CPA inversion cannot reproduce shape variability of the IP decays, as verified on field examples.

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