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UNIVERSITY MATHEMATICS EDUCATION, COMPETENCIES AND THE FIGHTING OF SYLLABUSITIS

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Syllabusitis is a name for a disease that consists of identifying the mastering of a subject with proficiency related to a syllabus. In this paper I argue that using a set of mathematical competencies as the hub of mathematics education can be a means to fight syllabusitis. The introduction and thorough exemplification of this idea was the main outcome of the Danish KOM project. Furthermore, a two-dimensional structuring of the relation between subject specific competencies and subject matter was suggested. As the analytic core of this paper I argue that such a two-dimensional structure has proven to be a crucial element when attempting to put the competency idea into educational practice, and exemplify how that can be done when it comes to mathematics education at university level.

INTRODUCTION

Syllabusitis is the name of an educational disease. It consists of identifying the mastering of a subject with proficiency related to a syllabus (Jensen, 1995, Blomhøj & Jensen, 2007). This is a sometimes convenient, but severely damaging reduction of complexity, among other things because it defocuses the teaching and learning of the subject. Everyone with a sense of mastering a subject will agree that there is much more to it than proficiency related to a syllabus, and this ‘much more’ is forgotten (or neglected) in a system infected by syllabusitis.

My impression is that syllabusitis is widely disseminated in mathematics education systems around the world. My intention here is not to defend the validity of this impression, but to use it as a framing of a more constructive analysis initiated by the questions: How can we describe the content of mathematics education in a way that supports the fight against syllabusitis in mathematics education? In particular: How can

such a description become a source of inspiration for teachers' work and professional development?

The first question was one of the dominating points of departure for the Danish KOM project, whose basic approach to the question of mathematical mastery I lay out in the next section, cf. Højgaard (2012) of which the following sections of this paper is an edited version. In section three I address the second question by arguing that a two-dimensional structure has proven to be a crucial element when attempting to put the competency idea into educational practice, and in section four I present a specific example: using the two-dimensional structure to challenge and focus the planning of mathematics education at university level. In section five I finish with some general perspectives for future curriculum development.

MATHEMATICAL COMPETENCIES AS 'GUIDING STARS'

Syllabusitis and curriculum structure

In many countries, Denmark being one of them, the traditional way of specifying a mathematics curriculum is structured around the following components (cf. Niss & Højgaard, to appear):

- The *purpose* of the teaching.
- A *syllabus*, i.e. an outline of the topics (concepts, procedures, results etc.) to be covered.
- The instruments of *assessment and testing*.

Sometimes the purpose is determined first and used as a basis for the establishment of the syllabus and the modes of assessment and testing. Often, though, the syllabus is determined first and the purpose added as a sort of politically oriented foreword, the modes of assessment and testing only referring to the syllabus-specific goals and the formal settings ('A four-hour written test' etc.). The establishment of the syllabus becomes the hub of curriculum development and, consequently, the central arena for discussions between the teachers, the recipient bodies and institutions and the curriculum developers.

In such a system, syllabusitis is systematically fertilised, not only at the system level of curriculum development, but also – and this is the core of the problem – in the minds

and practices of mathematics teachers. To my experience, it is a generally accepted claim that the main channel of communication between ‘the system’ and the mathematics teachers regarding the content and orientation of the teaching is a list of guiding tasks for the written exam. If this list is structured mainly as an attempt to cover the syllabus, because it is constructed by people who are in the heart of a system infected by syllabusitis, then it should be no surprise that the minds and practices of the teachers are formatted in the same way.

Mathematical competencies as ‘the missing link’

One of the causes for the syllabus-focused curriculum structure is that we have nothing else to turn to when searching for a structure that is well suited for communication between the different bodies involved in mathematics education. The purpose of the teaching, being the natural alternative, is a much more general kind of statement with no direct relation to the planning and orchestration of the teaching. This creates a ‘missing link’ between the purpose and planning of a structure for classroom activities, and only a minority of the teachers are able to create this link themselves.

This problem was one of the reasons for the initiation of the Danish KOM project (‘Kompetencer Og Matematikl ring’, Danish for ‘Competencies and Mathematical Learning’), which took place in the years 2000-2002 (cf. Niss & Jensen, 2002, Niss & H jgaard, to appear) and was directed by Mogens Niss for whom I acted as academic secretary. Based on previous work by Niss (1999) the important analytical steps carried out in this project were (cf. H jgaard, 2010) to:

- move from a general understanding of the concept *competence*, which I – in semantic accordance with the KOM project – take to be someone’s insightful readiness to act in response to the challenges of a given situation (Blomh j & Jensen, 2007),
- to a focus on *a mathematical competency* defined as someone’s insightful readiness to act in response to a certain kind of mathematical challenge in a given situation (ibid.),
- and then identify, explicitly formulate and exemplify *a set of mathematical competencies* that can be agreed upon as independent dimensions in the spanning of what it means to master mathematics, cf. figure 1.

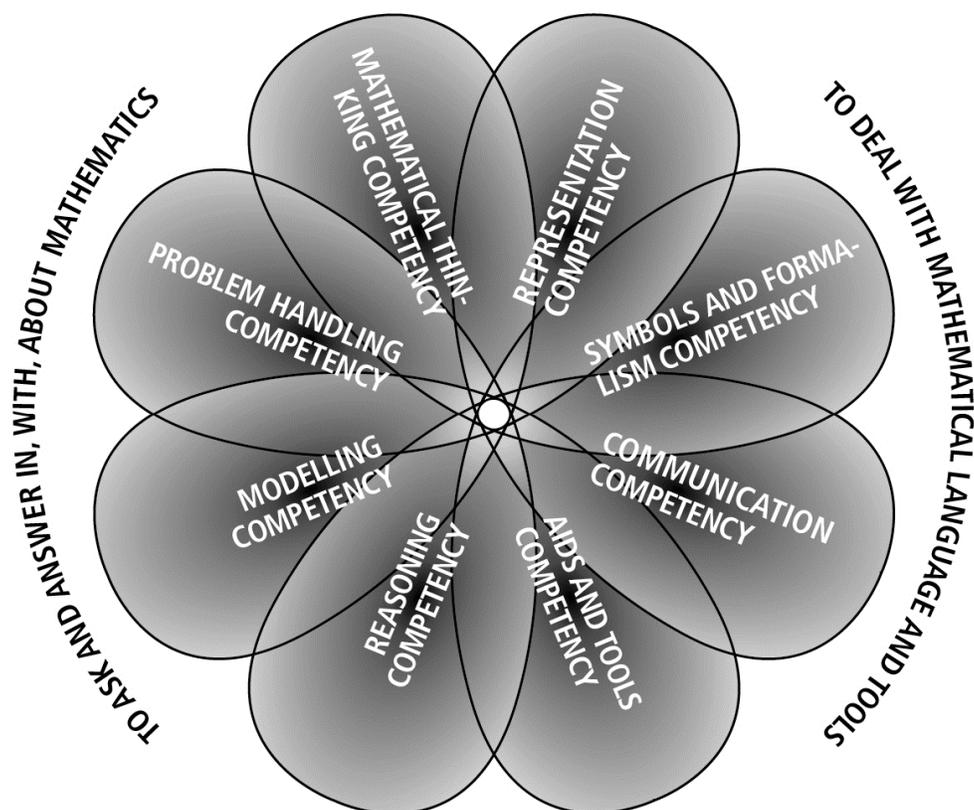


Figure 1. A visual representation of the eight mathematical competencies presented and exemplified in the KOM report (Niss & Højgaard, to appear).

In short, these competencies can be described as someone's insightful readiness to ...

Mathematical thinking competency: ... carry out and have a critical attitude towards mathematical thinking.

Problem tackling competency: ... formulate and solve both pure and applied mathematical problems and have a critical attitude towards such activities.

Modelling competency: ... carry out and have a critical attitude towards all parts of a mathematical modelling process.

Reasoning competency: ... carry out and have a critical attitude towards mathematical reasoning, comprising mathematical proofs.

Representing competency: ... use and have a critical attitude towards different representations of mathematical objects, phenomena, problems or situations.

Symbol and formalism competency: ... use and have a critical attitude towards mathematical symbols and formal systems.

Communicating competency: ... communicate about mathematical matters and have a critical attitude towards such activities.

Aids and tools competency: ... use relevant aids and tools as part of mathematical activities and have a critical attitude towards the possibilities and limitations of such use.

Such a set of mathematical competencies has the potential to replace the syllabus as the hub of the development of mathematics education, because it offers a vocabulary for a focused discussion of the aims of mathematics education that can make us feel comfortable for the same reasons that we are comfortable with the traditional specificity of the syllabus (Blomhøj & Jensen, 2007).

A TWO-DIMENSIONAL STRUCTURE FOR THE CONTENT OF MATH EDUCATION

The work with mathematical competencies laid out in the KOM report is one of many attempts to generate a more broad and ambitious framing of the design of mathematics curricula, where ‘something more’ than just mere syllabus reproduction is expected. The most well-known example is probably the American “Principles and Standards for School Mathematics” (National Council of Teachers of Mathematics, 2000, 2003), but many other countries have been through similar developmental processes during the last two to three decades.

For me the most recent experience was when I had the opportunity during a sabbatical leave in 2009 to get acquainted with a by then ongoing process of developing a new national curriculum for general education in Australia, including a new framework for mathematics education ([Australian] National Curriculum Board, 2009). The main feature of this framework is to distinguish between content strands, which I read as parallel to a syllabus, and proficiency strands, which I (cf. National Research Council, 2001) read as the chosen approach to fill out the ‘something more’ part of the

curriculum. In the short version the approach chosen in the Australian framework are described as follows:

The content strands describe the ‘what’ that is to be taught and learnt while the proficiency strands describe the ‘how’ of the way content is explored or developed i.e. the thinking and doing of mathematics. *Each of the ‘content descriptions’ in the mathematics curriculum will include terms related to understanding, fluency, problem solving or reasoning.* ([Australian] National Curriculum Board, 2009, p. 7, my emphasis)

The emphasised part of the quote is an unusually clearly stated example of what I based on unsystematic experience believe to be a very common way to relate the syllabus and the ‘something more’ part of a curriculum. It can be described as supplementing the syllabus with specific subject-related goals (‘acquire computational fluency with ...’, ‘develop a general understanding of ...’ etc.), and is also used in the American Standards mentioned above.

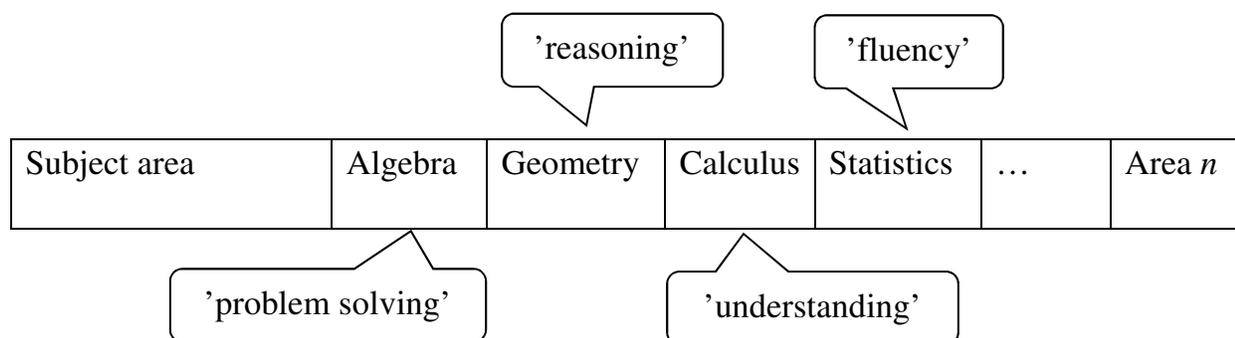


Figure 2. A ‘syllabus with comments’ structuring of the content of mathematics education.

The problem with this ‘syllabus with comments’ approach depicted in figure 2 is that it does not once and for all fundamentally depart from the tradition of writing up the content of mathematics education in a linear fashion, it just ‘dresses it up’ with some new ambitions. Hence, it is still possible to forget or neglect the new ambitions of the curriculum and carry out the much too easy transformation of the linearly described content into a by nature also linear plan for the teaching of the content: ‘Algebra’ as the heading for part one of the plan, ‘Geometry’ as the heading for part two, etc.

In the KOM project we suggested the alternative of using a matrix structure for incorporating mathematical competencies in mathematics curricula (Niss & Højgaard, to appear). Figure 3 is an adapted version of this model with the combinatorial cell structure left out. One of the consequences of this approach is that it makes didactical considerations necessary when planning from a two-dimensional content structure to a timewise one-dimensional teaching plan.

Subject area	Algebra	Geometry	Calculus	Statistics	...	Area n
Competency						
Math. think. comp.						
Probl. handl. comp.						
Modelling comp.						
Reasoning comp.						
Representation comp.						
Symbols & form. comp.						
Communication comp.						
Aids and tools comp.						

Figure 3. A two-dimensional structuring of the content of mathematics education.

USING THE TWO-DIMENSIONAL STRUCTURE – AN EXAMPLE FROM UNIVERSITY LEVEL

In the last decade or so, the two-dimensional structuring of content suggested here has been used for various curriculum developments in Denmark from primary school to university (cf. Niss & Højgaard, to appear). In this paper I will exemplify how that can be approached at university level, based on the elaborated presentation in Højgaard & Jankvist (2015).

The School of Education at Aarhus University in Denmark offers a master's programme in mathematics education, parallel to several other educational master's programmes (cf. Aarhus University, 2016). I have been the person responsible for that programme since its latest major educational re-design in 2005, of which I was the main architect. That process took place only a few years after the intense work with the KOM project, so it is

of little surprise that the educational design is not least framed by a competency approach to mathematics education. There is, however, also a more substantial reason for this choice made by my colleagues and I in the mathematics education group at Aarhus University: We acknowledge the fundamental challenge of fighting syllabusitis in mathematics education across educational levels, and we agree that a set of mathematical competencies is a useful developmental tool to address this challenge.

Hence, we have chosen to use the framework of mathematical competencies as a basis for the design of the mathematically focused part of the master's programme. That part covers two compulsory course modules; *Mathematics in a Didactical Perspective I and II* (cf. Aarhus University, 2016). For each of these modules two mathematical competencies and concepts from two mathematical subject areas have been chosen as the two-dimensional mathematical content. To maintain a focus on mathematics education in the entire master's programme, this content is put into a didactical perspective, giving us a three-dimensional content model for each of the two modules, of which the first is visualized in figure 4 (cf. Højgaard & Jankvist, 2013, which on request can be emailed).

Didactics of Mathematics Perspectives		
Concept Area	Calculus	Stochastics
Competency		
Mathematical Modeling Competency		
Mathematical Problem Tackling Competency		

Figure 4. A visualization of the content of the educational module: 'Mathematics in a Didactical Perspective I' (Højgaard & Jankvist, 2015, p. 87).

SOME GENERAL REMARKS

The generalizability of this model can and should be tested and challenged. To make a long story short, my experience from participating in several research and development projects framed by a two-dimensional mathematical content description is twofold (cf. Højgaard & Jankvist, 2015):

- a) One of the main advantages of using a two-dimensionally structured competency perspective on mathematics education is that it inherently fosters reflections among the teachers involved about the foci of the different mathematical competencies involved in the two-dimensional content structure (Højgaard, 2010).
- b) Such reflections promote the more ambitious kind of work processes aimed at in mathematics education, if the teachers involved get the necessary time and support (as was the case with the example given above) to learn how to use the two-dimensional structure as a developmental tool (cf. Hiebert, 2003).

As for the third dimension of the model in figure 4, it has proven both important and helpful for my colleagues and I when communicating with each other and with the students about the focus of the different modules. The didactics of mathematics perspective reminds us all that this is the context in which the mathematical content should be taught, learned and assessed, and I see no reason why that could not be replaced with “engineering perspective”, “biology perspective”, etc., to form a useful developmental tool in other educational settings.

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