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Pre-reading for Remich workshop (2)

WHAT IS MATHEMATICAL TOPOLOGY?

In the program for the Remich workshop and in the pre-readings there are many references to ”topology” or derived concepts as e.g. “mereotopology” and “finite topology” and many references to different pictorial or geometric concepts, areas, boundaries, “rubber sheet geometry”, etc. and many examples of intuitive, metaphorical and also some more formalized description of manifolds of objects or “points” in a visual or geometric frame or language. I think it must be a little confusing for the non-mathematical readers, and perhaps even to some mathematical ones, and I also think it sometimes must be hard to see the relevance for psychology, and especially for cultural psychology and semiotics. Of course we all know the usefulness of figures and diagrams with boxes, circles, arrows, barriers and borders etc. to illustrate relations, interactions, processes and forces etc. But what qualifies it as “topology”, and what is gained which could not just as well be illustrated by drawings, pictures, diagrams and figures as we know them? Perhaps the workshop will reveal that.

Sometimes “topology” is also used in a very broad sense as “qualitative mathematics” in opposition to “quantitative” or parametric mathematics which tries to represent or reduce psychological and other phenomena to sets of numbers, e.g. real numbers. Of course an alternative to this “number fetishism” is needed and one among an infinity of alternatives is to focus on the relation “being part of” which provides some order in the structure of different parts of connected objects, without using numbers. And this structure can be formalized in a mathematical way defining what has been named “mereotopology”. But why choose this special case among a plentitude of other formalized descriptions of structures in the world, which do not use numbers, and why is “topology” part of the name? And why is it relevant for psychology, and more relevant than non-formalized free drawings and descriptions? Perhaps the workshop will reveal that also, but I think it could be useful to clarify some concepts before that which might help with the pre-reading process also.

As some background for all these questions I think it can be useful to give a short introduction to the *mathematical concept of topology*. I also feel it necessary, to prevent misunderstandings, to present some basic properties of Standard Mathematical Topology also called Point Set Topology or Set Theoretical Topology or General Topology to which I shall refer in my own presentation, and which in many ways is different from the above examples, e.g. from mereotopology and “finite topologies”.

But before that we need a short introduction to Formal Logic and Theory of Sets.

Formal Logic and Theory of Sets

This introduction is rather trivial and well-known also to non-mathematicians and is only given as a background to the later introduction of topology. It lines up some close ties between relations in logic and sets. Sets are collections of elements, points or objects with some definition of which ones are included. The elements are different and they need not be ordered in any way. The definition can be a reference to some criterion or property which the elements shares, it could be some procedure selecting them, or it could just be a (finite) list with references to the individual elements or it could be less explicit. See my discussion of that in my first pre-reading introduction “Topology with a subject”.

A set can be finite or infinite as for instance the set of positive integers. But the elements need not be mathematical objects as e.g. numbers. They can as well be animals in the Zoo, if what we mean by “animals” and “the Zoo” is well-defined.

When we have a set with more than one element we can form different subsets be selecting some of the elements and not others. If the set is finite with N elements we can form 2^N different subsets if we include the set itself and as a special case the “empty” set with no elements, usually denoted with an Ø. If the set is infinite we can do the same, in principle. The complete set of subsets of a given set is called the *power set* of the set.

If A is a subset of a set P (“P” for the point set), we can form the *complement of A in P*, often denoted $P\backslash A$ which is the set of elements in P which are not elements in A. If we write $x \in A$ for the statement that the element x is a member of the set A this can be expressed as follows

$$x \in P\backslash A \equiv \text{NOT } (x \in A)$$

Here “≡” means “is the same as”, and it is presupposed that $x \in P$.

If we have two subsets A and B of P we can form their *intersection* denoted $A \cap B$ which is the set of elements in P which are members of both A and B. With same notations as above this can be expressed as

$$x \in A \cap B \equiv (x \in A) \text{ AND } (x \in B)$$

Finally, if we have two subsets A and B of P we can form their *union* denoted $A \cup B$ which is the set of elements in P which are members of A or B or both. With same notations as above this can be expressed as

$$x \in A \cup B \equiv (x \in A) \text{ OR } (x \in B)$$

Here “OR” is the so-called “inclusive or” also denoted the “and/or” logical operation.

It is seen that there is a correspondence between the logical operations (NOT, AND, OR) on statements and the set theoretical operations (complement, intersection, union) on sets.

As we in logic have the so-called *duality rules*

$$\text{NOT } (p \text{ AND } q) \equiv (\text{NOT } p) \text{ OR } (\text{NOT } q)$$

$$\text{NOT } (p \text{ OR } q) \equiv (\text{NOT } p) \text{ AND } (\text{NOT } q),$$

where p and q are statements, we have the corresponding *duality rules* in set theory

$P \backslash (A \cap B) \equiv P \backslash A \cup P \backslash B$

$P \backslash (A \cup B) \equiv P \backslash A \cap P \backslash B$

In other words: The complement of an intersection is the union of the complements, and the complement of a union is the intersection of the complements.

The operations on subsets forming complements, intersections and unions are called *set algebraic* operations. The two "coupling" operations intersection and union are here defined on pairs of subsets, but they can by so-called *infinite induction* be expanded to any set of subsets, also an *infinite* number of subsets. For example is the intersection of all open intervals on the real axis with the number 2 as an element just the number 2 itself, although none of the intervals has 2 as their only member, or in other words although the number 2 itself is no open interval.

If two subsets have no common elements they are said to be *disjunct*. In this case their intersection is the empty set Ø. As special cases we have that

$A \cap P \backslash A \equiv \emptyset$

$A \cup P \backslash A \equiv P$

Topological spaces

Topological spaces are structures defined on the power set of a set, i.e. on the set of subsets of a given set *P*, the so-called point set. Topological spaces are tailored to infinite point sets (as for instance the set of real numbers or geometrical objects) which will be presupposed in this introduction but could in principle be defined on finite sets. The latter is a so-called "degenerate" case with an interpretation diverging radically from the "normative" infinite cases. I shall comment on that later.

The infinite point set need not to be "points" in geometry or in any other sense, they can be elements or "objects" without any further specification. Because of the above mentioned *correspondence* between logical operations and set algebraic operations a topological space is first of all a special *logical structure* which is basic for the understanding of how topological spaces are *unique* compared with the structure in power sets defined from *unlimited* use of set algebraic operations and corresponding logical operations.

A little metaphorically you can say that a topological space is a structure which at the same time has an intensional side (the logic) and an extensional side (the sets) or even more metaphorically perhaps that a topological space is not only an ontological structure describing sets "in themselves" but also an epistemological structure describing the "access" to the sets or their "openness" towards acts of definition or selection. This has to do with what I in the first pre-reading paper called "Topology with a subject" and illustrated with the Mandelbrot Set as an example.

What sometimes makes it difficult to catch the "plot" in topological spaces and to understand their impressing analytic power is that on one hand their uniqueness and raison is derived from the logical (intensional or epistemological) side, but on the other hand that their defining axioms are expressed exclusively in extensional or set algebraic terms, in this way “hiding” their essence, or to go even further, their dialectics.

In a topological space defined on the power set of some set *P* some of the subsets are denoted *open sets* and others not. As a special case all or none of the subsets can be declared open, but this is another “degenerate” case. The distinction between sets being open or not is the “heart” of topology, without which we are left with ordinary set theoretical algebra.

The *logic* behind the axiomatic definition (to which I shall return) of a topological space is an *interpretation* of the concept “*open set*” as a set which is open to definition or selection in the sense that if a point *x* is a member of an open set, it can also be *decided* that it is a member, but not necessarily that it for any member of the complement of the open set can be decided that it is *not* a member. Here a basic *asymmetry* between membership and not-membership is introduced or in other words a basic asymmetry between an open set and its complement, an asymmetry which is not present in set algebra itself and is basic in topology.

The concept of decision is undefined or rather only defined implicitly through a presupposition that *an infinity of independent decisions is impossible*. Here some subjectivity or reference to what is *practicable* is sneaking into pure mathematics, but as with the Axiom of Choice (discussed in my first pre-reading introduction) it is implicit and disguised in extensional statements of “existence”. For the use of Mathematics as a tool in Psychology it is, however, indispensable to catch these layers of deeper logic behind the axiomatic dress.

Now to *the structure of open sets* in a topological space! The first axiom simply says

Ax. A: *The intersection of two open sets is an open set.*

If we shall decide if a point *x* is member of the intersection of the open sets *A* and *B* we have to decide that it is member of *A and* is a member of *B.* This is possible if they are both open and if we can make the two decisions. It is possible to make two decisions and combine them with the logical and-operation. Therefore the intersection is also open. And by *finite induction* this should be possible as long as the set of open sets in the intersection is finite. So “two” in the axiom could be interpreted as meaning “finite”. However there are reasons not to use a term as “finite” in basic axioms, because it does not belong to so-called *first-order-language*. It should be noted that Ax. A does not say that intersections of an infinity of open sets cannot be open in some cases. If it is the case in *all* cases, however, we have one of the degenerate and uninteresting cases.

The second axiom says

Ax. B: *Any union of open sets is an open set.*

If we shall decide if a point *x* is a member of the union of a set of open sets it is sufficient to decide that it is member of one of them irrespectively of whether the set of open sets is finite or infinite.

This is possible if we can find just one of the sets, which is assumed to be possible. Therefore the union is also open.

At last two necessary axioms:

Ax. C: *The empty set Ø is an open set.*

Ax. D: *The point set P itself is an open set.*

If a subset of the power set of *P*, or in other words a set of subsets in *P* is organized following the Axioms Ax. A, B, C and D, they are open sets in a *topological space*. Or the other way round: The set of open sets in a topological space satisfies the axioms Ax. A, B, C, D which defines the concept of topological space. *Nothing more is needed!*

You can interpret a topological space as a structure in a power set or you can interpret it as the logical structure of decidability.

The unique quality of the structure is the *asymmetry* expressed in Ax. A and Ax. B using the terms "two" and "any" respectively and the corresponding asymmetry between intersection and union of sets. It is also obvious that in a topological space on a finite point set this asymmetry disappears as both terms "two" and "any" in this case can be translated to "any finite number of".

One simple *example* of a topological space can be defined on the point set of all real numbers (the real axis). Here the open sets could be any union of open intervals, including the whole axis itself and the empty set. "Union" is here used in a general sense including a single open interval. The intersection of two unions of open intervals will itself be a union of open intervals or the empty set, and the union of any unions of open intervals will itself be a union of open intervals. So all four axioms are satisfied. This is also called *the standard topology on the real axis*. The *asymmetry* in the example is revealed by the fact that there exist intersections of unions of open intervals which is neither itself a union of open intervals or the empty set. An example was already given above with the intersection of all open intervals including the number 2 as a point. This intersection is just the number 2 which is no open interval. So in Ax. A "two" could not be replaced with "any" as in Ax. B.

The standard topology on the real axis is just an example. The logical structure defining a topological space is not tied to any specific choice of the point set, although some central applications of course have played a role in the history[1] of the concept, e.g. some geometrical applications to which I shall return later.

Open sets, boundaries and closed sets

Now I shall try to derive some concepts from the four axioms defining a topological space. Let us look at some arbitrary subset *X* in the point set *P*. We can now define *the interior of X* as the union

[1] Topology as a general logical and set theoretical structure not necessarily tied to pictorial representations or geometry was developed mainly by Polish and German mathematicians since around 1920.

of all open sets in *X*. *The interior is an open set* according to Ax. B, and it is easily seen that *an open set is its own interior*. In the same way shall we define *the exterior of X* as the union of all open sets in *X*'s complement, i.e. in *P\X*. Also *the exterior is an open set*, according to Ax. B. The set of points in *P* which are neither in *X*'s interior or in *X*'s exterior are defined as *the boundary of X*. It is easily seen from the definition that *the boundary of X is also the boundary of X's complement in P*. I also follows from the definition that *the boundary of X is the empty set Ø if and only if both X and its complement in P are open*. Or in other words, *if either X or P\X is not open then they have a common not-empty boundary*.

These were definitions which could be applied to any subset *X* in *P*. Now let us focus on the special case when *X* is an open set. If the complement of *X* is also open their common boundary was as mentioned above the empty set Ø. So let us instead look at the situation when *X* is open and its complement is not open. In this case *X* and its complement have a common non-empty boundary as proven above. None of the points in the boundary can, however, belong to *X*. If they did they would also belong to *X*'s interior which is *X* itself because *X* is open, and then they could by definition not belong to the boundary. So we have that *no points in the boundary to an open set belongs to the open set*. In other words we have that *all points in the boundary of open set belongs to its complement*. A subset in *P* which *contains all points of its boundary* is called a *closed set*.

From this it follows that the *complement of an open set is a closed set*, and vice versa. If two open sets are each others' complement their common boundary was the empty set Ø, so they also both include their boundary because the empty set is included in every subset. It also follows that *a subset of P is both open and closed if and only if its complement is also open*. This applies also to the open sets *P* and Ø which are each others' complements, cf. Ax. C and D.

The boundary to a set (open, closed or neither) is by definition the complement to the union of the set's interior and its exterior which according to Ax. B is open because the interior and the exterior are both open. So being the complement to an open set *the boundary to any subset in P is a closed set*. Further can we can prove that *the boundary to an open set or a closed set has no interior*. In other words does a boundary to an open or a closed set not contain any open set in *P*. If openness is interpreted in relation to decidability of membership of sets as in the above reasons given for the four axioms we must conclude that the *boundary* between an open set and its complement in this sense is *undecidable*.

Boundaries to sets that are neither open nor closed are more complicated and can in fact have interiors, but this is not of any relevance in the present context.

As mentioned are open and closed sets each others' complements, also in the special case when a set is both open and closed. From the above *duality rules* the four axioms could as well take their departure in the concept of closed sets, although the interpretation in relation to decidability would be more indirect.

Ax. A': *The union of two closed sets is a closed set.*

Ax. B': *Any intersection of closed sets is a closed set.*

Ax. C’: *The point set P itself is a closed set.*

Ax. D’: *The empty set Ø is a closed set.*

I is noteworthy that a concept of *boundary* which seems to match our intuitive notion of an undecidable border between decidable manifolds or domains can be derived alone from the simple logical structure in the four axioms defining a topological space without any appeal to visual or pictorial representation. As we shall see below the logical concept of topological space is, however, extremely shaped to describe just this, including important general properties of geometry. But the applications are much wider because of the high degree of generality in the simple basic axioms, as we shall see.

Some important cases of a topological space

Although the set of four axioms defining a topological space is already a strong analytic tool in logic and mathematics some axioms have to be added to make it even stronger and more precise in relation to many domains of application. As a point of departure we could have a look at the above mentioned standard topology of the real axis. It has to be stressed, however, that this is only an *example*. The axioms to be presented have a much wider application than just this simple case.

I the standard topology on the real axis R the open sets were all unions of open intervals including the empty set Ø. An example of an open set X will be the union of the two open intervals $x < 1$ and $2 < x$, where x is a real number. The complement R\\X of X is the closed interval $1 \leq x \leq 2$, and as the complement to an open set it is also a closed set in the standard topology on R. The common boundary of X and its complement is the set {1; 2} containing the two points 1 and 2 and being closed as well. An example of a set which is neither open nor closed could be the set of all rational numbers q for which $1 \leq q \leq 2$.

The real axis R is *connected* in the sense, that it has no “holes” or “fractures” which could split it in two. I is not possible to divide it in two complementary open non-empty sets. The complement of an open set will always be a closed set which is not open, unless the open set is R itself or the empty set Ø. If we divide R in two non-empty complementary sets there will always be a non-empty boundary. Or in other words, the only subsets in R which are both open and closed are R itself and the empty set Ø. This could be used in an axiom defining a *connected topological space*.

Ax. E (connectedness): *No other subsets of the point set P are both open and closed than P itself and the empty set Ø.*

Again we see that we are able to define a fundamental property of a point set, being connected, with very few and simple logical means.

In the standard topology any point in the point set is obviously member of some open interval, and even stronger, for any two points there are two disjunct open intervals each with one of the two points as a member. There are always two disjunct open intervals which “separate” the two points from each other. This is a general property of a very broad family of topological spaces used in mathematics and especially in geometry. It is also called the Hausdorff-property after the German mathematician Felix Hausdorff (1868-1942). A topological space having this property is called a *Hausdorff-space* and is defined by adding the so-called *Hausdorff-Axiom*:

Ax. F (Hausdorff): *For any two points in P there are two disjunct open sets so that one point is in the one and the other point in the other open set.*

A topological space satisfying the six axioms is called *a connected Hausdorff-space*.

If we exclude the extremely degenerate case where the point set has only one point or is the empty set with a supplementary axiom

Ax. G (existence): *P has more than one point,*

then Ax. C can be derived from Ax. A, F and G, and Ax. D can be derived from Ax. B, F and G, so we are in fact left with *only five independent axioms Ax. A, B, E, F and G,* and axioms Ax. C and D now become derived statements or *theorems*.

These *five axioms* taken together defining a *connected Hausdorff-space* have very strong implications and are defining what we call a *continuum* as we know it more intuitively from geometrical objects as lines and planes with an infinity of closely connected points. This simple logical structure is also what defines the structure of point sets in *geometrical topology*, the well-known "rubber sheet geometry".[2]

I shall now present a few derived consequences of the five axioms defining a connected Hausdorff-space, i.e. derived statements or *theorems*.

Th. 1: *In a connected Hausdorff-space is any finite set of points a closed set.*

Proof: Let us chose a point *x* in *P*. According to Ax. F and G for any other point *y* in *P* there exists an open set containing *y* and not *x*. The union of all such open sets for all points *y* different from *x* is itself open according to Ax. B, and it is the complement of *x*. Being the complement of an open set *x* is therefore a closed set. This will apply to all the single points in the finite set. The finite set is their union and must be closed according to Ax. A' (the closed set version of Ax. A). The empty set

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[2] Of special interest are the so-called compact continua or *compact, connected Hausdorff-spaces*. Sometimes the term *continuum* is used in this more narrow sense, and *geometrical topology* can then be defined as the classification of continua, i.e. compact, connected Hausdorff-spaces.

Compact topological spaces are "bounded" like e.g. a circle line or the surface of a sphere or a torus, while non-compact topological spaces are "open-ended" like the real axis or the 2-dimensional plane. Compactness can also be defined with a simple axiom, but this is of less importance in the present context where we will discuss connected Hausdorff-spaces in general.

Ø is by definition also finite and is closed according to Ax C (now being a theorem). This proves Th. 1.

Th. 2: *In a connected Hausdorff-space is any open set infinite or the empty set*.

Proof: Let us hypothetically assume that *P* is finite. According to Ax. G it can then be divided in two complementary non-empty finite sets. They are both closed according to Th. 1 and therefore also both open. But according to Ax. E no other subsets in *P* can be both open and closed than *P* itself and Ø. Therefore *P* cannot be finite. This implies that any finite subset of *P* has a non-empty complement. The finite subset is closed according to Th. 1 but as it has a non-empty complement it cannot also be open according to Ax. E. So no finite subset in P except the empty set is open. This proves Th. 2.

When combining Th. 1 and 2 we see that any finite set, and as a special case any single point, in a connected Hausdorff-space is closed and has no interior (strictly speaking it has an empty interior), i.e. no decidable proper parts.

It is remarkable that it in this way is proven that any connected Hausdorff-space defined from the very simple set of five axioms, with no reference to infinity, must be *infinite*. There are other examples of the "hidden" force of this "innocent" looking logical structure, defining an infinite logic.

Topological logic and decidability

The above mentioned Standard Topology on the real axis was one example of a connected Hausdorff-space. I shall now use this example to illustrate some aspects of the concept of *decidability*, still remembering that the logical structure of a connected Hausdorff-space is much more general and not tied to any example.

In the example chosen here the point set will not be the whole real axis but only its positive part "R+", i.e. the part with real numbers x for which $0 < x$. Let us define three disjunct subsets in R+, where X ∪ Y ∪ Z = R+.

X: $x < 1$

Y: {1}

Z: $1 < x$

In the standard topology X and Z are open, and Y is closed.

The complement of X is $Y \cup Z$ or $1 \leq x$, which is closed.

The complement of Z is $X \cup Y$ or $x \leq 1$, which is closed

One way of interpreting openness is, as said, to tie it to *decidability*. I the example we can imagine some sort of *measurement* as a criterion for deciding membership of subsets. If we think of measurement as an act of *comparison* of a given object x with some *norm*, or point of reference, it will be close to practical reality to say that a *difference* can always be detected within finite time, however small, but that *identity* is undecidable in the sense that we can never know if an undetected difference is due to identity or due to that we have not yet reached a level of precision or a step in the comparison procedure where a difference would be detected. There is an *asymmetry* between detecting difference and identity. Difference can be detected with certainty within finite time, identity can never. This is parallel to the example with the Mandelbrot Set given in my first pre-reading introduction.

If $x \in X$, i.e. $x < 1$, it can in principle be decided by comparing x with 1. If $x \in Z$, i.e. $1 < x$, the same is the case. If, however, $x \in Y$, i.e. $x = 1$, this can't be decided with certainty by comparing x with 1. Membership of open sets as X and Z can be decided. Membership of not-open isolated points as Y is indecidable. We shall also say that membership of non-open sets as $X \cup Y$, i.e. $x \leq 1$, and $Y \cup Z$, i.e. $1 \leq x$, are undecidable because they contain at least one point, i.e. 1, for which the membership is not decidable.

To move a little outside the mathematical world itself we can think of the weight of objects being decided by comparing their weight with the Standard Kilo in Paris.[3] We can imagine that we sort objects by comparison with the Standard Kilo and place them left or right to 1 if they weight less or more, respectively. If we have an object where we can't make the decision we will not declare that it weights exactly 1 kilo. Because in that case we could use it as a new Standard Kilo, and that would of course not be accepted by the authorities as long as we have the old one intact. That would break the internationally agreed procedures for prototypes. So we must instead declare it as undecidable. As a consequence we can never fill in the point 1 with one of the objects we compare with the Standard Kilo if decidability is based on measurement. Between the two open sets left and right to 1 there is a "hole" which can't be filled in this way.

But that "hole" is not empty after all, because it is inhabited in advance with the Standard Kilo. The weight of the Standard Kilo is not decided by comparison with the Standard Kilo, which was the defining criterion for membership of the open sets X and Z. But this does not mean that the weight of the Standard Kilo or its identity is undecidable in a wider sense. It has just *another status* as a "*point of reference*" for the measurements or comparisons. The Standard Kilo has been *chosen* or *selected* by other criteria than all the objects being compared with it. If we want to know if a given object is the Standard Kilo it will not suffice to measure it. We have to *locate* it. We have to ask the authorities of the address in Paris and hope to be let in after the security procedure and be allowed to watch it with reverence (and some heart-beating affection, I guess).

So objects which are undecidable or unidentifiable by *one* kind of decision can be decidable by *another*. If they are not defined by their properties or *universal* qualities, as e.g. weight, they might

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[3] The kilogram is the only unit in international standards for measurement that is still defined by reference to an artifact prototype kept safe and stable as far as possible. The other units for e.g. time and distance are defined by reference to universal physical phenomena in principle reproducible irrespectively of place and time.

be defined by being chosen or selected as *particulars*, more or less irrespectively of their properties. This is like defining a finite set by listing its individual members as was referred to in the introduction to “Formal Logic and Theory of Sets”.

And these chosen objects might, as the Standard Kilogram, function as single points of reference *framing* the logical organization or topology of other objects in topological spaces.

Points of reference are as isolated points and closed sets in a topology organized in another way than the open sets in the topology. They form a more “*discrete*” structure, but the two kinds of decisions and corresponding sets are also interacting in a way which can be expressed in axioms and theorems. Together they form a “*duality topology*” with great analytic force, in fact.

In my first pre-reading paper to the Remich workshop I presented such a *dual topology*. The open sets were called *sense categories* and some sets of isolated points *choice categories*. I refer to the interpretations given there.

The topology of *sense categories* is, however, a little more *general* than the topology of connected Hausdorff-spaces which in relation to that is a *special case*. I chose, however, this more special case in the above presentation because it is the well-known *standard topology* behind *geometric topology* (“rubber sheet topology”) to which is referred frequently in the other pre-readings to the Remich workshop, in some cases just referred to as “*ordinary point set topology*” or equivalent terms. I hope this choice will contribute to make the different pre-readings and the presentations at the workshop more *comparable*, which may also mean make the *differences* more transparent.

The topology with sense categories as open sets and the standard topology of connected Hausdorff-spaces are identical except that Ax. E (connectedness) is replaced with another axiom

Ax. H (perfectness): *No sense category contains just one point*[4]

A topological space satisfying the set of the five axioms Ax. A, B, F, G, and H is called a *perfect Hausdorff-space* or often just a perfect topological space, sometimes even just a *perfect topology*. It follows from Th. 2 above that Ax. H is already fulfilled in all connected Hausdorff-spaces. A perfect topology is a more *general* structure than the connected Hausdorff-spaces in the sense that *any connected Hausdorff-space is a perfect topology, but not all perfect topologies are connected Hausdorff-spaces*.

However, both Th. 1 and Th. 2 can as well be proven from the more general set of axioms. A perfect topology is also necessarily *infinite*.

One advantage perfect topologies have compared with the more special connected Hausdorff-spaces (not to be proven here) is that *in a perfect topology is any not-empty open set (sense category) itself a point set (a so-called subspace) organized with the same topology*. The logical structure of

[4] In the presentation of the axioms in my first pre-reading paper (Ax. 5, p. 5) points were called “objects” as it is also done in other contexts. Open sets were accordingly called sense categories and the point set was denoted the “universe of objects” Ù.

perfectness is "*hereditary*" in mathematical terminology or is a "*global*" structure repeating itself infinitely in all its detail in the same way as in some fractal structures. This makes the structure independent of explicit definition of the point set or the "universe" to which it is applied, which again has great interpretative advances when applied in e.g. psychology. It so to say "frees" the logical structure from ties to specific domains of objects.

In my first pre-reading paper (p. 5) the five axioms defining *sense categories* as open sets in a perfect topology (there called Ax. 1, 2, 3, 4 and 5) were supplemented with four axioms describing the logical structure of *choice categories* (Ax. 7, 8, 9 and 10) and with two axioms describing their *interaction* (Ax. 6 and 11).

From this set of 11 axioms several *theorems* can be derived which can be interpreted in a *psychological context* and reveal some of the conditions that frame our practical and conceptual interaction with the world, and our affections as well. But that must wait for the workshop.

This should only be an introduction to help establishing some *common baseline* of mathematical topology to save time and prevent misunderstandings in relation to the important issues we shall discuss at the workshop. But I must confess that I could not resist a temptation to present a little introduction also to my own presentation at the workshop hoping in this way to make that less technical and more intuitive and visual.

REFERENCE:

Mammen, J. "Topology with a subject". Pre-reading introduction to Remich workshop, distributed April 24, 2015, from Jaan Valsiner as

Mammen J (2015) SHORT INTRODUCTION TO TOPOLOGIES (for Remich Workshop).pdf