# Essays on Warehouse Operations

by

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# Summary

The thesis consists of three papers related to the warehousing industry.

**A dynamic programming algorithm for the space allocation and aisle positioning problem.** The first paper considers a material handling system with gravity flow racks and addresses the problem of minimizing the total number of replenishments over a period subject to practical constraints related to the need for aisles granting safe and easy access to storage locations. In this paper, an exact dynamic programming algorithm is proposed for the problem. The computational study shows that the exact algorithm can be used to find optimal solutions for numerous SAAPP instances of moderate size.

**Scheduling in a cross-dock environment to minimize mean completion time.** The second paper studies the problem of minimizing the mean completion time of outbound trucks in a cross-dock environment with a single inbound door and a single outbound door. Minimizing mean completion time improves the cross-dock's responsiveness and is known to lead to stable or balanced utilization of resources, and reduction of in-process inventory. The problem is addressed from a flow shop scheduling perspective and its similarities with the classical $F2||\sum C_j$ problem are exploited. Indeed, the problem is a generalization of $F2||\sum C_j$, which is known to be $\mathcal{NP}$-hard. A branch-and-bound algorithm is proposed to find an optimal solution to the problem. Computational results show that the branch-and-bound algorithm

can optimally solve problem instances of moderate size within a reasonable amount of time.

**Scheduling trucks in a cross-dock with mixed service mode dock doors.** The problem considered in the third paper is to schedule inbound and outbound trucks subject to time windows at a multi-door cross-dock in which an intermixed sequence of inbound and outbound trucks can be processed at the dock doors. The focus is on operational costs by considering the cost of handling temporarily stored products, as well as the cost of tardiness due to processing outbound trucks after their respective due dates. A mathematical model for optimal solution is derived and an adaptive large neighborhood search heuristic is proposed to compute near-optimal solutions of real size instances. Computational experiments show that the proposed heuristic algorithm can obtain high quality solutions within short computation times.

# Preface

This thesis was prepared at the School of Business and Social Sciences, Aarhus University in partial fulfillment of the requirements for acquiring the Ph.D. degree in Economics and Business. The work was carried out at the Department of Economics and Business.

The thesis extends the literature by proposing an exact algorithm for a known warehousing problem and introducing two new research problems within the area of cross-dock operations. The thesis considers exact and heuristic solution methods to solve the problems.

Three research papers are included in the thesis, of which one has been accepted for publication. In addition to the three papers, there is a supplementary report, which contains further background and details for the application environment.

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Sincerely,

Peter Bodnar
Aarhus, July 2013.

# Contents

# Chapter 1

# Introduction

Warehouses are essential components of any supply chain. In a warehouse items are handled in order to level out the variability and imbalances of the material flow caused by factors such as seasonality in demand, production scheduling, transportation, and consolidation of items (Gu et al., 2007). Inventories in warehouses are capital intensive assets that require storage areas, handling equipment, and information systems. In addition, warehouse operations are repetitive, labor intensive activities. The capital and operating costs of warehouses represent about 20-25% of the logistics costs (Frazelle, 2002; Baker and Canessa, 2009). Therefore, improvements in the planning and control of warehousing systems can contribute to the success of any supply chain.

A warehouse is typically divided into functional areas that are designed to facilitate the material flow (Tompkins et al., 2010). The main warehouse areas are outlined in the following: receiving area, reserve and forward storage area, and shipping area. Operations in the receiving area include the processing (i.e., unloading) of carriers, item identification, and quantity and quality inspection. Received items are then moved to a storage area or directly to the shipping area. The storage area is often divided into a reserve and a forward storage area. The reserve storage area covers typically distant and heavily accessible locations, e.g., the uppermost part of a rack, and is used to ensure the replenishment for the forward storage area. Customer demand is primarily satisfied from the forward storage area, where the items are typically stored in convenient size and the storage locations are easily accessible. In the shipping area, items are sorted, consolidated and loaded on the carriers. While this is a general material flow in a warehouse, the actual material flow depends mainly on the role of the particular warehouse in the supply chain.

Specialized warehouses are established to fulfill the different requirements, e.g., production warehouse, distribution warehouse, and cross-dock. The main function of a production warehouse is buffering and storage, it supplies raw or semi-finished material for production and may prepare finished items for shipment; the typical objective is the minimization of operation and investment costs given the storage capacity and response time (Rouwenhorst et al., 2000).

Distribution warehouse (or distribution center) handles, in addition, the distribution of items. In this case, the general objective is to achieve high throughput at minimum operational and investment costs. In a cross-dock (or transshipment center), storage is scarcely presented, incoming items are immediately sorted and new customized shipments are created (De Koster et al., 2007).

In a typical warehouse 65% of the operating expenses are consumed by order picking (Ruben and Jacobs, 1999), which includes the item picking to replenish the forward area and to fulfill customer order. Therefore, this thesis focuses on those decision problems and warehouse systems that aim to reduce or even eliminate the order picking costs. Chapter 2 addresses a problem of minimizing the number of items picked to replenish the forward storage area during the planning horizon. The problem has been introduced by Anken et al. (2011), and a heuristic algorithm has been proposed to obtain near-optimal solutions. The study presented in chapter 2 extends the existing literature and proposes an exact algorithm.

However, order picking costs can be further reduced by directly moving items from the receiving to the shipping area, that is, by cross-docking. Two studies presented in chapter 3 and 4 consider cross-dock environments and address the problem of scheduling inbound and outbound trucks. The truck scheduling problem emerges when the number of docks is less than the number of trucks that are available to be processed. In chapter 3, an exact algorithm is proposed to the case when the performance criterion is the mean completion time of outbound trucks. In chapter 4, the objective is to minimize the operation costs which consist of the following: the storage and retrieval costs and the tardiness costs.

In the remaining part of this chapter, further background information and details are provided for the application environment. Many studies propose a framework to classify the existing literature on warehousing. These frameworks are based on either application areas, methodology, levels of decision, or a combination of these. One of the first literature reviews is presented by Matson and White (1982); their framework is based on application areas such as transfer lines, flexible manufacturing, and order picking systems. van den Berg (1999) focuses

also on application areas and reviews the research on storage and order picking. A study by De Koster et al. (2007) considers low-level, picker-to-parts order-picking systems in a framework that classify the literature based on the different levels of decisions. Ashayeri and Gelders (1985) propose a framework based on the solution approach and distinguish exact, approximate, and simulation methods. Wäscher (2004) classify the literature based on both application area and solution approach. Cormier and Gunn (1992) presents a comprehensive study that clusters literature based on the level of decisions. Rouwenhorst et al. (2000) propose a framework based on decision levels, material flow, and organization aspects. Recently, Gu et al. (2007, 2010a) present two studies, which cover warehouse operations and design, respectively, with an outlook to performance evaluation. In this chapter, the considered warehousing decisions are structured in a framework that first clusters the problems based on the levels of decisions, and then (where appropriate) the subproblems are addressed in a sequence that follows the typical material flow.

The structure of this chapter is the following. Section 1.1 presents high-level warehouse design decisions, which establish the overall structure and define the functional areas. Section 1.2 elaborates on the operation decisions that define the details of the material and information flow. These decisions are at a tactical level for one organization and at an operational level for another depending on the particular characteristics of the organization. The problems addressed in this thesis are connected to operations decisions. Section 1.3 presents the motivation of the research and provides an outline for the following chapters.

## 1.1 Warehouse design

Warehouse design is often tackled by a stepwise approach with interrelated phases and frequent reiteration (Baker and Canessa, 2009). The overall structure of a warehouse design is determined by the role of the given warehouse in the supply chain. This role defines the business requirements and key focus areas, as well as the constraints on the design. Based on the assem-

bled and analyzed data, operation principles and stock keeping units (SKUs) are defined which aid managers reducing the range of feasible configurations. An SKU serves as an alphanumeric product number to identify and keep track of the stored items; each item is assigned an SKU number based on its characteristic (Tompkins et al., 2010). According to Rouwenhorst et al. (2000) the next phase is to develop functional specifications and operating procedures. The priorities in this phase are established by taking into account the performance evaluation, which is traditionally production and cost oriented, e.g., maximize throughput and minimize total discounted cost. Ashayeri et al. (1985) present a model that seeks to minimize the sum of investment and operation costs in automated warehousing systems subject to constraints on throughput, warehouse size, and crane capacity. According to Goetschalckx and Ashayeri (1989), customer oriented objectives are increasingly pursued, e.g., minimize mean processing time (De Koster and Van der Poort, 1998; Jarvis and McDowell, 1991), or minimize tardiness (Chan and Kumar, 2009). The problem of finding a trade-off between the objectives emerges often in the presence of both production and customer oriented objectives. For instance, Kovács (2011) shows the contradiction between minimizing the maximum of response time (also called order cycle time) and minimizing the average response time (also called average picking time).

Warehouse design decisions address also equipment selection and internal arrangement decisions, which cover the allocation of functional areas that exhibit various configurations in terms of size, layout, and procedures (Pliskin and Dori, 1982; Heragu et al., 2005). Ashayeri and Gelders (1985) provide a literature review on various methods that can assist with the evaluation of equipment types. Size and dimension related decisions are influenced by expected inventory levels, which establish the maximum capacity and define the resource requirements (Francis and White, 1992). The warehouse sizing problem has been addressed by Goh et al. (2001). They model a warehousing cost with a piecewise linear function assuming a multi-SKU system and provide an exact algorithm for the case of separable inventory costs and an approximation algorithm for the case of joint inventory costs.

A storage area is often divided into a reserve and a forward area (Tompkins et al., 2010).

The problem of sizing these areas has been addressed in the papers by Bhaskaran and Malmborg (1990) and Malmborg (1996a) with a stochastic cost-savings model and an integrated evaluation model, respectively. These models can be used to evaluate different scenarios. The paper of Hackman et al. (1990) is among the earliest studies on modeling the product allocation problem (also called forward-reserve allocation problem), that is, the problem of determining which product to place in the forward storage area and in what quantities. A greedy heuristic is proposed to solve the problem based on a prior ranking of the products. Hackman and Platzman (1990) provide a mixed integer linear programming formulation to address more general instances of the above problem. van den Berg et al. (1998) provide a knapsack based heuristic assuming unit-load replenishment. For the base problem, Gu et al. (2010b) provide a branch-and-bound algorithm.

In a storage area, products are allocated to *storage locations*, which are organized in *single-block* or *multi-block* layout where longitudinal and cross *aisles* separate the blocks. That is, a block is a group of compartments, e.g., conventional fix racks or bin cabinets. Recently, mobile and flexible systems are also available, e.g., gravity flow rack and rail-embedded motorized rack. Bassan et al. (1980) present some optimal design parameters for determining the internal layout of storage bays in a rectangular-shaped warehouse. Rosenblatt and Roll (1984) extend the work of Bassan et al. (1980) and propose a simulation based procedure that integrates the warehouse sizing problem, and the internal layout problem. *Input/output (I/O) point* (also referred as pickup and drop-off point) is the location, where goods arrive and leave the storage area. The distance between the I/O point and a storage location has an essential effect on the time required to store or retrieve an item assigned to that particular location. The traditional layout is commonly referred to as rectangle-in-time, which refer to the shape of the isolines that are computed to identify locations at identical distances from the nearest I/O point. Recently, Gue and Meller (2009) and Gue et al. (2012) present novel aisle designs, e.g., 'flying-V' and 'fishbone', with the aim to reduce the travel time by relaxing the conventional requirement of parallel picking aisles and orthogonal cross aisles. The results show that the total travel

time can be reduced by enabling non-orthogonal cross aisles and orienting the picking aisles in different directions. However, the reduction decreases as the number of I/O points increases.

Proposed warehouse design solutions are finally evaluated and assessed. Scenarios are often constructed to consider various situations and configurations; decisions in this phase can be facilitated with simulation models (Baker and Canessa, 2009). These models may also include operation decisions, which are presented in the following section.

## 1.2 Operations

Operation decisions define the material flow in a warehouse, which is elaborated in the following. First, items arrive and received in the facility. Then the item is either forwarded to the shipping area or allocated in a storage location. The best storage location is close to a dock so that the cost of (i) unloading the items at a dock and transporting them to the storage area and (ii) accessing items and transferring them to a dock for loading is minimized. As a result, items compete for storage locations that are closest to the docks (Ahuja et al., 1993). When an order is received, the picker retrieves the ordered items. Note that the retrieval can be triggered either by an order for item replenishment in the forward storage area or by a customer order. To ensure efficiency, the picker should follow a route that minimizes the cost of the retrieval. Order consolidation can be considered to facilitate an efficient picking. The sorting is necessary if items have to be clustered by customer order after the completion of the picking (De Koster et al., 2007). Finally, items are loaded on the carriers and shipped. The main stages in this process are: receiving, storage, order picking, and shipping.

### 1.2.1 Receiving and shipping

*Receiving* and *shipping* are warehouse operations, which represent two extreme connection points of the warehouse procedures. Receiving includes typically carrier processing (i.e., unloading), item identification, recording the goods receipt, quantity and quality inspection, un-

packing, and sorting activities; whereas, shipping includes finishing, batching, packing, and loading operations.

The *truck-to-dock assignment* problem emerges in multi-door warehouses, where a set of docks are available to process a set of carriers and the problem is to assign the docks to the carriers so that some performance criteria are met. The paper of Tsui and Chang (1990) is among the first studies on the truck–to–dock assignment problem with the objective to minimize total travel time inside the facility. The authors present a bilinear objective formulation and provide a local search algorithm. For the same formulation, Tsui and Chang (1992) describe a branch-and-bound algorithm. The paper of Oh et al. (2006) presents the problem in the context of a mail distribution center and proposes a genetic algorithm heuristic. Miao et al. (2009) consider the problem with operational time constraints assuming that trucks are pre-scheduled with hard time windows during which a dock is fully occupied; tabu search and a genetic algorithm are proposed in the paper.

When several trucks are to be processed at a facility with limited number of docks, the *truck scheduling* policy determines the order of trucks. That is, the truck scheduling problem is the problem of defining the start and completion times for the processing of each truck so that some performance criteria are met. In this problem, the time dimension is taken into account; hence the objective function also tends to be time-related, such as the minimization of makespan, defined as the completion time of the last outbound vehicle. (Boysen and Fliedner, 2010). For excellent reviews on truck scheduling the reader is referred to Agustina et al. (2010), Boysen and Fliedner (2010) and Van Belle et al. (2012).

### 1.2.2 Storage

*Storage* is the process of allocating items in the warehouse. Since warehouse storage locations and pickers are generally scarce resources, therefore high allocation efficiency is required in terms of utilization of both picker effort and storage capacity. Storage includes the following interrelated activities: sequencing and consolidation, storage location assignment, and shuffling.

*Item sequencing* determines the order, according to which items are sorted to be processed, e.g., allocated or shuffled. Item sequencing is typically based on a first come, first served or on an earliest due date order. However, items can be consolidated (or clustered) according to decisions and restrictions determining whether different items can be placed in the same compartment. A dedicated compartment accommodates only one item. While in a mixed compartment, more items can be allocated. e.g., a stored pallet may consist of several different products, or a rack location may cover several slots and a product can be assigned to each slot. Item consolidation may yield both improved storage utilization and increased complexity (Anken et al., 2011). Steudel (1979) presents a heuristic for the pallet loading problem, where units of a single product are placed on a pallet forming a layout that minimizes the unused area. The problem is formulated as a two-dimensional cutting stock problem, where the original area is partitioned into sub-areas. Tsai et al. (1988) present stylized model with an LP formulation for the two-dimensional palletizing problem with products of different size. However, item consolidation is frequently limited by compliance restrictions among products or between product and location, e.g., product-to-product restriction is common in industries where products can pollute or damage each other (Heskett, 1964).

The *storage location assignment* policy determines where a given item is stored. The main difference between the product allocation and the storage location assignment problems is that the former considers the reserve and forward storage areas and product volumes, while latter focuses typically on multiple locations and individual items. Storage location assignment policies can be classified into two main groups, namely, *dedicated* and *shared* policies. Dedicated storage location assignment commits compartments to products during the planning horizon. This policy requires a high storage capacity to store the maximum inventory of each product (Tompkins et al., 2010). However, once the products and the locations are matched, only the quantity has to be updated at every transaction, therefore this policy improves the transparency and the picker's familiarity with the locations of the different products. Shared storage location assignment allows the compartment to accommodate different products; therefore the location

may be potential for any product upon necessary capacity and product-location compliance. This policy can be computationally intensive. However, the main advantage of it is that the storage capacity required must only fulfill the peak inventory level of all products during the planning horizon (Tompkins et al., 2010). That is, the storage capacity requirement with the shared storage location assignment is lower.

Dedicated storage location assignment is commonly used to reduce the picking effort, since related assignment methods, in general, place popular items to easily accessible locations; thereby the average picking time is shortened. Shared storage location assignment is usually applied in order to scale down the storage capacity requirement. The two assignments can be compared based on picking effort and utilized storage space. Malmborg (1996b) shows that for a given $\alpha/\beta$ 'ABC-curve', where $\alpha\%$ of the stored items are responsible for $\beta\%$ of the retrieval transactions, a skewness factor $s = \log_\alpha \beta$ indicates which one of the two policies may provide superior performance. As the value of the $s \in (0, 1)$ increases the advantage of the shared policy decreases. When $s = 1$, all items have the same level of demand retrievals.

The storage location assignment influences essentially the expected total storage and order picking time, which consists of travel time, stowing and retrieving time, and administration related time. The travel time may take up to 50 % of the total time spent on storing and picking an item (Francis and White, 1992). Therefore, several studies have addressed storage location assignment problem with the objective to minimize the travel time. The storage location assignment policies frequently considered in the literature are: random, closest-open-location, popularity based, turnover based, class based, and duration-of-stay based (DOS) rule.

*Random storage location assignment* policy allocates items to locations with equal likelihood resulting in leveled storage space utilization. When comparing the performances of different methods, the random assignment is usually applied as a benchmark. In practice, items are commonly allocated to the available compartment closest to the I/O point; this method is called the *closest-open-location policy*. However, the performance of the closest-open-location policy is often approximated by applying the random policy (De Koster et al., 2007).

Product characteristics can be incorporated in the decision to improve the performance. Information about product popularity, which is the average number of retrieval orders during the planning period, can be used to reduce expected storage and retrieve time. The most frequently ordered product is assigned to the location closest to the I/O point, the second most frequently ordered product to the second closest location, etc. Therefore high-runner items are placed at the easiest accessible locations, while low-demand items are allocated farther, this facilitates that the expected storage/retrieval time is low. However, such a policy may increase aisle congestion and create unbalanced utilization.

Hausman et al. (1976) provide a seminal study on the *full turnover-based assignment rule*, in which the highest-turnover item is assigned to the location closest to the I/O point, and show that this approach offers superior performance with respect to the expected total travel time compared to the closest-open-location rule. Turnover-based assignment can also be adopted in a facility using a throughput-to-storage ratio (Liu, 2004), or a density-turnover index (Chuang et al., 2012). Lee and Elsayed (2005) propose an iterative search procedure to determine the space requirements for warehouse systems operating with a full turnover-based storage rule. The inverse of the turnover rate of a product is called the *cube-per-order index* (COI), which was first presented by Heskett (1963, 1964). The COI based policy is found to be optimal under certain circumstances (Francis and White, 1992). Malmborg and Bhaskaran (1990) show the optimality of the COI in a system where one storage and one retrieval is executed within a picking cycle. Moreover, they consider the cases when the COI rule provides alternative layouts and propose a heuristic to select one. Recently, Yu and De Koster (2013) present that when full turnover-based storage is coupled with dedicated storage, then this policy does not always outperform class-based and random storage assignment rule in terms of the expected travel times.

*Class-based storage assignment rule* is a shared assignment policy which clusters products into classes based on product characteristics, such as popularity or turnover. The item classes are assigned to a group of storage locations. The class with the highest average popularity or

turnover is assigned to locations closest to the I/O group, and within the class the items are conventionally assigned to random storage locations (Kulturel et al., 1999). The main procedures, i.e., establishing the boundaries of the product classes and assigning storage locations to these classes are done either simultaneously or in a cyclical manner. The selected item attributes determine the development of distinguished classes. Rosenblatt and Eynan (1989) show the decrease of marginal cost benefit when increasing the number of classes. Muppani and Adil (2008a,b) present a branch-and-bound and a simulated annealing approach to form storage classes considering storage-space cost and handling cost. To the case when demand is stochastic, Ang et al. (2012) propose a robust optimization based approach to solve the multi-period storage assignment problem.

Administrative cost can be reduced when items from the same shipment are grouped and handled together (Roll and Rosenblatt, 1983). Frazelle and Sharp (1989) introduce the *correlated assignment policy* (also referred as family grouping) which can reduce both the time required to locate the item and the interleaving time, which is the travel time between two adjacent retrievals within the same picking cycle. The *contact based correlation* is based on the frequency by which the items are picked sequentially (Garfinkel, 2005). Whereas, the *complementary based correlation* measures the strength of joint demand (Mantel et al., 2007). Chuang et al. (2012) propose a model for the two-stage clustering-assignment problem assuming dedicated storage assignment policy and orders with multiple items. In the first stage, items are clustered into groups based on the correlation between them. In the second stage, groups are assigned to storage locations.

Goetschalckx and Ratliff (1990) present the DOS rule, which incorporates information about each individual item and determines the storage location based on its duration-of-stay, which is the expected time interval between the arrival (or storage) and the departure (or retrieval) times of a given item. Kulturel et al. (1999) present a comparison between the class-based full-turnover rule and the class-based DOS rule assuming three classes and stochastic demand. The results show that the class-based full-turnover rule tends to provide superior results with

respect to total travel time.

However, it is possible to relax the traditional requirement that items are committed to the locations, where they were once assigned to, until the retrieval of the last unit. *Shuffling* can improve storage utilization of a warehouse by replacing the items. That is, a low-demand item allocated close to the I/O point can be swapped with a high-runner item allocated further from the I/O point, or similar items can be grouped in a new location. Jaikumar and Solomon (1990) present a heuristic for the shuffling (or reallocation) problem, their approach assumes sufficient slack time to perform the shuffling. The paper emphasizes the hierarchy of the problems. The first problem is the long-term storage location assignment based on *ex ante* data, followed by an optional reallocation problem, and finally a short-term storage location assignment problem based on *ex post* data. In the paper of Muralidharan et al. (1995), the shuffling problem is formulated as a precedence-constrained asymmetric traveling salesman problem, and it is addressed by two heuristic methods. Sadiq et al. (1996) present an improvement heuristic for dynamic warehouse environments. The heuristic aims to improve order picking and shuffling time by incorporating expected demand and correlation among products.

### 1.2.3 Order picking

An order consists of a set of order lines, which indicate the product code and the required quantity. Orders may be clustered to increase the efficiency of the retrieval; this is referred to as *order consolidation*. A *cycle* is a route from the I/O point to the requested storage location(s) and back to the I/O point. Single-command cycle policy (or single-address system) allows only either one storage or one retrieval activity at a cycle. A single storage and a single retrieval activity are combined in a dual-command cycle policy (or dual-address system). Finally, multi-command cycle policy allows several storage and several retrieval activities within one cycle. When multiple orders request the same product, the items can be picked in batches. In this case sorting is required before delivering the items, i.e., the picked products are divided into smaller quantities corresponding to the orders. Sorting can pursue a sequential or a simultaneous

approach (Roodbergen and Vis, 2006). The former is called *pick-and-sort*, i.e., sorting is done after items have been accumulated during picking. The latter is called *sort-while-pick*, i.e., sorting is done during picking.

Order picking can be manual, mechanical or automatic. The configuration of picking systems, including the level of mechanization, may vary among different departments and product groups. The main types of picking systems are the following: *picker-less*, *picker-to-product*, and *product-to-picker*. A picker-less system is a completely automatic system, e.g., items are loaded on a conveyor from an A-frame. In a product-to-picker system the item is transported to the location of the picker, e.g., by using mini-load or carousel. In a picker-to-product system a picker goes to the location of the product, retrieves it, and delivers it to an I/O point or he may place the item on a conveyor belt, the latter case is called a pick-to-belt system (Anken et al., 2011).

Picker-to-product systems can be divided into sequential and parallel picking processes. In a sequential picking process only one picker works on an order. It can be further divided into discrete (or single order) picking and batch picking depending on the number of orders handled simultaneously by the picker. On the other hand, in a parallel picking process several pickers may work simultaneously on the same order but on different order lines. To increase the familiarity of the picker with the locations of the products, pickers can be assigned to zones, which are groups of storage locations. In general, zones have determined boundaries; therefore the order can be split based on the location of the ordered products. Jane and Laih (2005) present a clustering model to construct a set of zones assuming a parallel picking process. The problem is $\mathcal{NP}$-hard, hence a heuristic approach is proposed for real size instances. Parikh and Meller (2008) show that the workload-imbalance is greater in zone picking systems when compared to the case that orders are first consolidated and then picked sequentially. Bartholdi III et al. (2001) presents the advantages of a flexible setup based on sequential picking, called the *bucket-brigade policy*, where pickers work in a line and as soon as one becomes idle he moves up the line and takes over the order from an adjacent picker. Simulation results show

that if walking time is insignificant, then the bucket-brigade policy can increase production rate and balance the workload among the pickers.

Picker-to-product systems are traditionally connected to pick-to-paper systems, where the orders are printed, and the picker follows the instructions on the hard copy. However, pick-to-light and pick-to-voice (or pick-by-voice) systems are also available, in these cases the items are indicated by light or the picker is informed via radio, respectively. In a manual or semi-mechanical picker-to-product system the picker is typically a warehouse employee with a picking cart or a vehicle, whereas in a heavily-mechanical or automated picker-to-product system the picker may be a crane. There is an extensive literature on semi-automatic and fully automatic storage systems. Sarker and Babu (1995) provide a literature review on automated storage and retrieval systems. Johnson and Brandeau (1996) extend the review by including automated guided vehicle systems. Recently, a comprehensive literature review is presented by Roodbergen and Vis (2009).

The *order-picking problem* (or retrieval sequencing problem) is to find the sequence of retrievals for a set of orders in a multi-command system which minimizes the travel time. The sequencing and routing of pickers can be established by individual or standardized policies (Petersen II and Schmenner, 1999). Individual routing policy determines the sequence of visited locations for each storage/retrieve cycle regardless of previous routes. Ratliff and Rosenthal (1983) show that the order-picking problem is a variant of the traveling salesman problem, and propose a network flow model in which the vertices represent the ordered items locations, the aisle ends, and the I/O points. Due to the complexity of establishing individual routes, the sequence of visited locations are often developed based on a selected heuristic algorithm (Petersen II, 1999; Roodbergen, 2001), e.g., transversal, largest gap, and composite. When the transversal (or S-shape) method is applied the picker enters the aisle containing a pick and traverses it. In the largest gap method, the 'gap' is defined as the distance between any two adjacent items, or between a cross-aisle and the nearest item. The picker enters each aisle, which contains an ordered product, up to the largest gap and leaves the same direction as

she entered. A composite method combines the previous two methods. These methods are frequently used in practice (Hall, 1993). Roodbergen and Vis (2006) consider the transversal and largest gap methods and shows that their performance with respect to total travel distance depends on the layout of the order picking area.

## 1.3 Outline of the thesis

This thesis focuses on warehouse operations. Order picking is known to be the most labor intensive and costly activities in a warehouse (De Koster et al., 2007). Order picking arises when items are picked to satisfy customer demand or to replenish the forward area. In a typical warehouse environment, 65% of the total operating expenses are attributed to order picking functions (Ruben and Jacobs, 1999). Therefore, improvements that facilitate the reduction of order picking costs are of great interests.

In chapter 2, a study presented by Anken et al. (2011) is extended. The study considers a distribution center with a reserve and a forward storage area, and focuses on reducing the number of replenishments to ensure product availability over a period. The problem is called the space allocation and aisle positioning problem (SAAPP). It addresses simultaneously two important and interrelated problems, namely, the product allocation problem and the storage location assignment problem. Anken et al. (2011) propose a two-phase heuristic algorithm to obtain near-optimal solutions and a mathematical formulation that can be used with a commercial software to obtain optimal solutions for small size instances. The complexity of the SAAPP is due to practical requirements related to safe and easy access to the items in the considered warehouse system, which is a picker-to-product pick-to-belt system with gravity flow racks that composed of multiple locations and multiple slots per location. Chapter 2 focuses on an exact approach for the SAAPP and provides an algorithm that can obtain optimal solutions for instances of moderate size. The key contributions presented in this chapter are (1) a graph representation of the problem, and (2) an exact dynamic programming algorithm.

However, order picking costs can be minimized or even eliminated in cross-docking facilities. In a cross-dock, items arrive typically from several suppliers, unloaded from inbound trucks and sorted so that items of one supplier can be consolidated with other supplier items for common final delivery destinations. Then, the items are loaded on outbound trucks. During this process, items are either moved directly from inbound to outbound trucks or stored in a temporary storage (or buffer) area for a short time (Bartholdi III and Gue, 2004). The benefit of cross-docking is that it can reduce order picking and inventory holding costs compared to traditional warehousing, it can also reduce transportation costs and increase the capacity utilization of the trucks compared to direct transportation between suppliers and customers. However, cross-docking requires the planning of the docking activities and the scheduling of the trucks. The truck scheduling problem emerges repeatedly during the daily cross-dock operations and has a key role in a rapid transshipment process (Boysen and Fliedner, 2010). In this thesis, chapters 3 and 4 consider truck scheduling problems in a cross-docking environment.

In chapter 3 the performance criteria is to minimize the mean completion time. This objective is known to improve responsiveness and leads to stable or balanced utilization of resources (Rajendran and Ziegler, 1997; Framinan and Leisten, 2003). Moreover, it can facilitate a high turnover of items and the timely completion of trucks processed at the cross-dock. Some properties of the problem are described and an exact algorithm is proposed to obtain solutions for instances of moderate size. The key contributions in this chapter are (1) the identification of some optimality properties, (2) the valid bounding procedures and dominance criteria, and (3) a branch-and-bound algorithm that is able to obtain optimal solution for instances of moderate size.

Finally, in chapter 4 the investigated truck scheduling problem considers explicitly both the order picking costs and the timely completion of trucks. The performance criterion of the problem is to minimize the operational costs, consisting of storage, retrieval, and tardiness costs. The cost of storage and retrieval relates to those items that are placed in the temporary storage area instead of being directly loaded on the outbound trucks. The problem is shown to be $\mathcal{NP}$-

hard therefore for large size instances a heuristic algorithm is proposed. In this chapter, the key contributions are (1) a mathematical model that can be used to solve instances of moderate size, (2) a proof of problem complexity, and (3) a heuristic algorithm that can be used to obtain near-optimal solutions for large instances.

# Chapter 2

# A Dynamic Programming Algorithm for the Space Allocation and Aisle Positioning Problem

# A Dynamic Programming Algorithm for the Space Allocation and Aisle Positioning Problem

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**Abstract**

The space allocation and aisle positioning problem (SAAPP) in a material handling system with gravity flow racks is the problem of minimizing the total number of replenishments over a period subject to practical constraints related to the need for aisles granting safe and easy access to storage locations. In this paper, we develop an exact dynamic programming algorithm for the SAAPP. The computational study shows that our exact algorithm can be used to find optimal solutions for numerous SAAPP instances of moderate size.

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## 2.1 Introduction

Storage locations are often grouped into a *forward area* and a *reserve area*; customer demand is primarily satisfied from the forward area, where the products are typically stored in convenient size and the storage locations are easily accessible. The reserve area covers typically distant and heavily accessible locations, e.g., an external warehouse or the uppermost part of a rack, and is used to ensure the replenishment for the forward area (Tompkins et al., 2010).

In the forward area, a *picker-to-product*, *pick-to-belt* system may be utilized, which enables the order picker to walk down the line removing cases from pallet storage locations and placing them on a take-away belt or roller conveyor; such a system provides high picking productivity and less travel distance between picks (Frazelle, 2002). In order to further increase efficiency by decreasing travel time/distance, products can be stored on a *gravity flow rack* (Gu et al., 2010a), which is a storage rack with deep locations that utilizes rollers to 'flow' the products from the back of the location to the front, thereby making the products more accessible for small-quantity order picking. This is a first-in, first-out (FIFO) storage system that provides high throughput pallet storage and retrieval as well as efficient space utilization (Frazelle, 2002).

In a gravity flow system, a *dedicated location* accommodates only a single product, and it requires a single access point from the front, thus a dedicated location is both simple and space efficient. A location with multiple commodities is referred as a *mixed location*; the maximum number of different products stored at a mixed location is equal to the number of slots (i.e., the depth of the location). A mixed location can increase the number of different products stored on a gravity flow rack; however, it requires an access point from the side so that the picker can reach any of the products. Gravity flow systems are capital intensive, hence efficient utilization is essential.

In such a warehouse system, the space allocation and aisle positioning problem (SAAPP) is defined by Anken et al. (2011) as the problem of minimizing the total number of replenishments over a period subject to some intuitive and practical constraints related to the need for aisles granting safe and easy access to storage locations. The defined problem includes an advance

replenishment period, in which the forward area can be replenished while no product is retrieved. The SAAPP considers two interrelated problems and seeks to find an optimal solution for them simultaneously. The first is the determination of the number of units per product allocated to the forward area. The second problem is the storage location assignment problem. Separately, these problems have been extensively investigated. For a literature review we refer to Rouwenhorst et al. (2000), Wäscher (2004) and Gu et al. (2007).

However, previous research in the field of material handling seems to underestimate the effect of interrelations between the managerial decisions on warehouse performance. It is thus of interest to learn how material handling decisions are interconnected, and develop integrated material handling systems. Need for research on integration of problems has been expressed earlier in Matson and White (1982) and De Koster et al. (2007).

In this paper, we define the SAAPP as a shortest path problem with resource constraints on an appropriately defined graph and introduce a dynamic programming algorithm for its solution. Similarly to Anken et al. (2011), we take the approach of addressing the SAAPP with the equivalent objective of maximizing the number of allocated product units during the advance replenishment period, instead of minimizing the total number of replenishments. The correspondence of these two objectives is valid, since for each product the number of allocated units is at most its periodic demand and any unit that is not allocated in advance must be replenished during the period.

The main contribution of this research is our exact algorithm which is able to optimally solve SAAPP instances of moderate size in reasonable time. Indeed, the instances that we are able to solve to proven optimality are considerably larger than those solved in Anken et al. (2011).

In the following, we first briefly describe the problem along with an illustrative example. Thereafter we present our modelling of the problem, in particular we introduce a graph representation of the SAAPP where any feasible solution is given as a directed path between a specified pair of vertices in the graph. Further, we present our dynamic programming algorithm

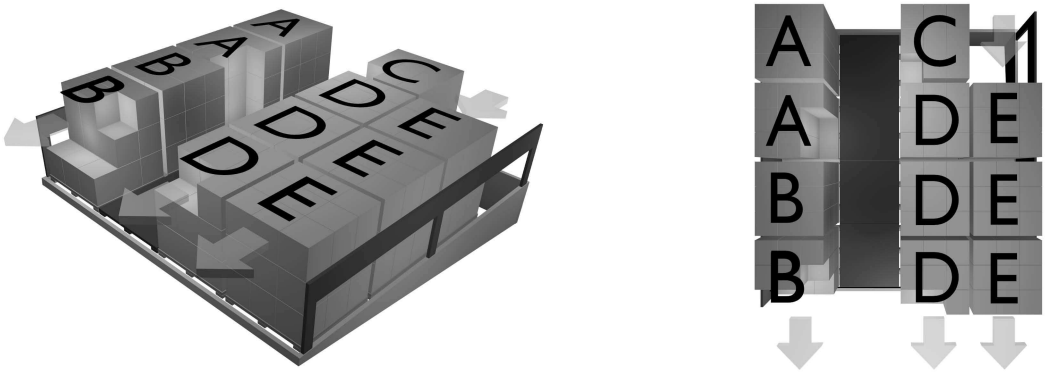
for finding an optimal path in this graph. Then, the results of the computational experiments are reported. Finally, the conclusion is presented.

## 2.2 Problem description

The SAAPP is to maximize the number of allocated product units (or, equivalently, minimizing the total number of replenishments over a period), given a unit-load, gravity flow storage system with $n$ locations, $m$ slots per location, $p$ different products with a periodic demand $d_j$ $(j = 1, \ldots, p)$ and the following requirements according to Anken et al. (2011): (i) each product receives at least one slot, (ii) each mixed location is adjacent to an empty location (i.e., an empty aisle), (iii) all allocated units of a product are located in the minimum number of consecutive locations and (iv) all units of a given product are located on only one side of an empty location. Moreover, it is given in the problem formulation by Anken et al. (2011) that the solution focuses on the advance replenishment, which is distinguished from replenishment during the period as in van den Berg et al. (1998), and that the periodic demand is stated in unit-loads.

Figure 2.1 illustrates a gravity flow system with pallet loads, case picking, 4 locations and 4 slots per location.

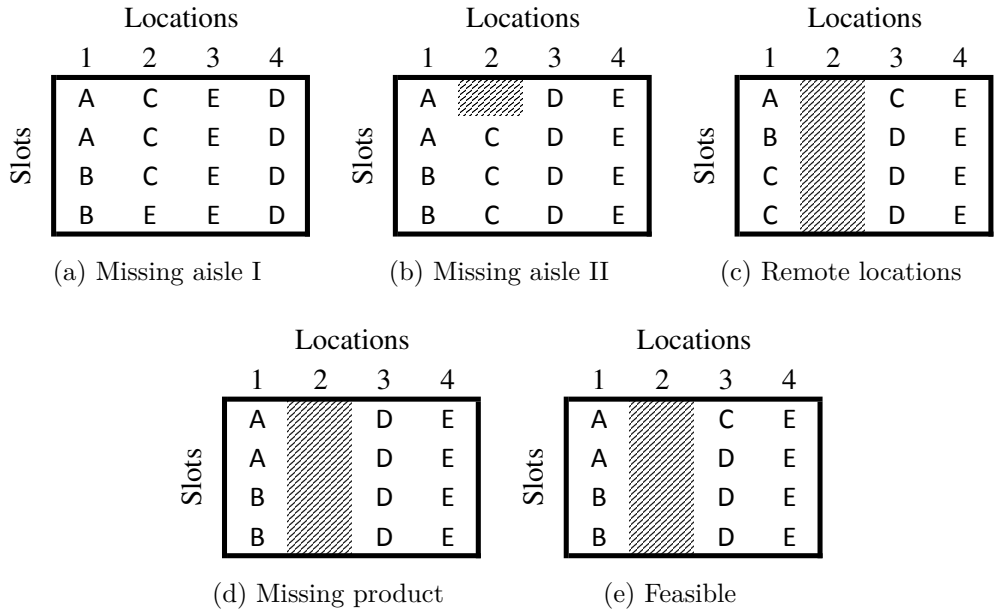
As an example, we may consider a system with $n = 4$, $m = 4$, and $p = 5$. The products are named A, B, C, D, and E with respective demand values 2, 2, 3, 4, and 5. We shall present five solutions in Figure 2.2, out of which four are infeasible due to constraint violation and only one is feasible. We depict a simple storage system viewed from above, as in Figure 2.1b. The empty storage slots are indicated with diagonal stripe pattern. Figure 2.2a depicts an infeasible solution since it contains two mixed locations (1 and 2) without an empty aisle. In Figure 2.2b, location 1 is mixed without an adjacent empty aisle. In Figure 2.2c, the solution is infeasible because product C is stored in two non-consecutive locations (1 and 3) on both sides of an empty location. In Figure 2.2d, product C is missing which makes the solution infeasible.




(a) View from an angle (b) View from above

Figure 2.1: An industrial gravity flow storage system

Finally, Figure 2.2e presents a feasible solution which is also optimal for this example.




(a) Missing aisle I (b) Missing aisle II (c) Remote locations

(d) Missing product (e) Feasible

Figure 2.2: Four infeasible solutions and one feasible solution for the example problem instance.

## 2.3 Modelling

In the following, we introduce our graph representation of the SAAPP. Then we eliminate certain solution symmetries by clustering products. Further, we define a location's status according to whether the location is empty, mixed, or dedicated. Finally we present the solution space and labels used in our dynamic programming algorithm.

### 2.3.1 Graph representation

Given the $p$ products with demands $d_1, \ldots, d_p$, a feasible solution may be characterized as a vector of allocated quantities $a_1, \ldots, a_p$, where $a_j$ is the allocated quantity of product $j$, satisfying $1 \leq a_j \leq d_j$ for $j = 1, \ldots, p$, together with information on where to place the $a_j$ units for each product $j = 1, \ldots, p$.

In order to represent the placement of the units for each product, we use a directed graph $G = (V, A)$ with vertex set $V$ and arc set $A$. The vertex set $V$ contains a vertex $v_{ij}$ for each position in the storage system defined by the combination of location $i$ and slot $j$, so we have $v_{ij} \in V$ for $i = 1, \ldots, n$ and $j = 1, \ldots, m$. In addition, we introduce vertex $v_{0m}$ as a source vertex, so we have $|V| = nm + 1$ vertices in $G$. It is noted that the location of the source vertex is an artificial location.

Any arc in $A$ is directed from some vertex $v_{ij}$ to another vertex $v_{kl}$ so that either $(i < k)$ or $((i = k)$ and $(j < l))$. The direction of arcs implies that the graph is acyclic, so the vertices in $V$ can be arranged in topological order (Ahuja et al., 1993). In our particular case, the topological order is $(v_{0m}, v_{11}, \ldots, v_{1m}, v_{21}, \ldots, v_{2m}, \ldots, v_{nm})$. Accordingly, we let an *order* vector describe the position of each vertex in the topological order, specifically we define $\text{order}(v_{ij}) = (i-1)m + j$ for all $v_{ij} \in V$. It is noted that $\text{order}(v_{0m}) = 0$.

We define $V[v_{ij}; v_{kl}] = \{v_{ab} \in V : \text{order}(v_{ij}) < \text{order}(v_{ab}) \leq \text{order}(v_{kl})\}$, that is, the string of vertices in the topological order starting at the successor of $v_{ij}$ and ending at $v_{kl}$. The length of any arc $(v_{ij}, v_{kl}) \in A$ is defined as $|V[v_{ij}; v_{kl}]| = \text{order}(v_{kl}) - \text{order}(v_{ij}) = (k-i)m + l - j$.

We say that an arc $(v_{ij}, v_{kl}) \in A$ *covers* the vertex set $V[v_{ij}; v_{kl}]$. As such, the arcs along any path in $G$ from $v_{0m}$ to $v_{nm}$ collectively cover all vertices in $V \setminus \{v_{0m}\}$ and so that each $v_{ij} \in V \setminus \{v_{0m}\}$ is covered by exactly one arc.

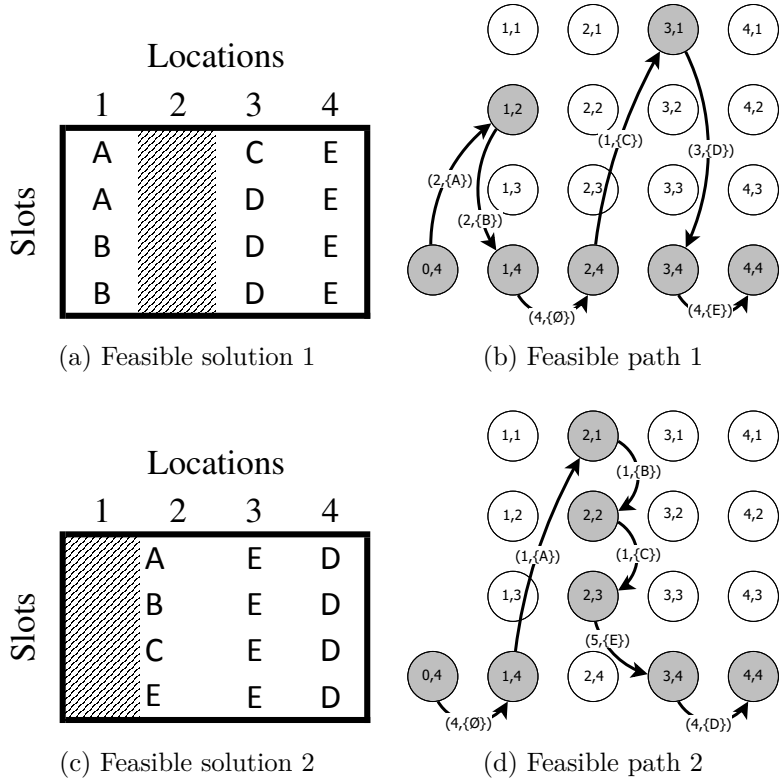
A feasible solution to the SAAPP amounts to determining, for each $v_{ij} \in V \setminus \{v_{0m}\}$, which product to place in that position, or to leave the position empty. The $a_j$ positions that product $j$ is assigned to are represented by consecutive vertices in their topological order. It follows that we can describe any feasible solution as a path from $v_{0m}$ to $v_{nm}$ in $G$, where the characteristics of the individual arc describe which product, if any, to assign to the positions covered by the arc. Each of the $p$ products is represented by exactly one arc along the path, and any additional arcs along the path represent assignment of empty space.

Figure 2.3 depicts the correspondence between feasible solutions and feasible paths, using the example in the previous section with $n = 4$, $m = 4$, $p = 5$, and where the respective demand values for products A, B, C, D, and E are 2, 2, 3, 4, and 5. The location of $v_{0m}$ cannot be used as an empty aisle, so if the first location is mixed, then the second location must be empty. Similarly, if the last location is mixed, then the location before the last must be empty.

In Figures 2.3b and 2.3d, each vertex is labelled with a pair of values, where the first value indicates the location, and the second indicates the slot. The source vertex is labelled $(0, 4)$. The arcs are also labelled with a pair of values, the first indicates the number of allocated product units, the second indicates the allocated product. Product type $\emptyset$ represents a dummy product, which takes up empty positions, thus 4 units of product $\emptyset$ in a location with 4 slots produces an empty aisle.

We note that there are several ways to extend a path, and that any path in the graph will be elementary, as a consequence of the way that the graph is constructed. Figure 2.4 presents ways to extend paths from certain positions.

The arcs in Figure 2.4 are labelled with a pair of values, where the first value is the number of units assigned, and the second value is a set of product types that can be allocated. Arcs connecting shaded vertices indicate an already decided partial path. Dashed arcs entering non-



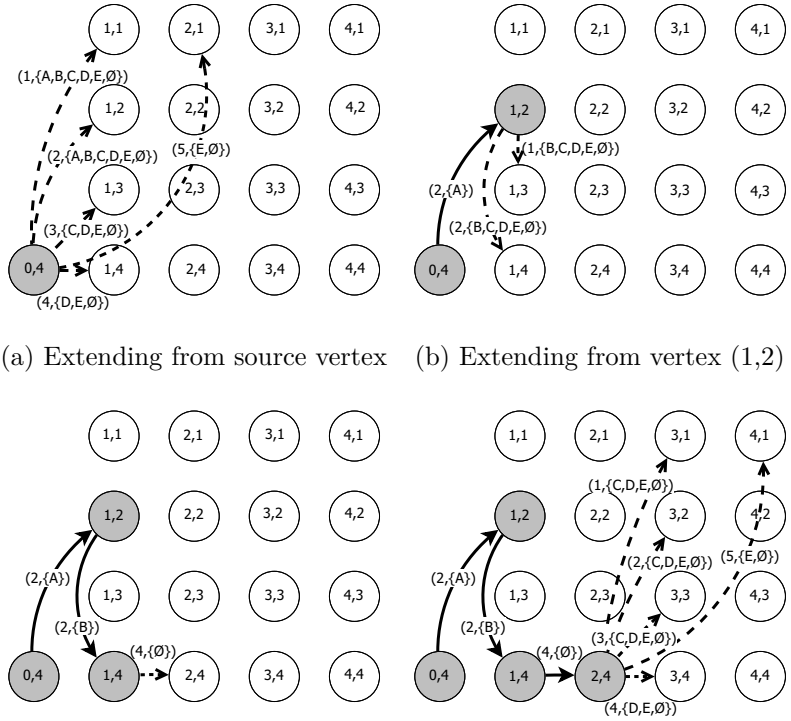

(a) Feasible solution 1 (b) Feasible path 1

(c) Feasible solution 2 (d) Feasible path 2

Figure 2.3: Feasible solutions and the corresponding paths in the graph.

shaded vertices indicate potential extensions of a path. In any potential extension, the dummy product $\emptyset$ is always contained in the set of product types.

Figure 2.4a depicts the situation of starting a path by creating an arc from the source vertex. We may choose one of the products in the set. For instance, if we start with the arc labelled $(4, \{D, E, \emptyset\})$, we assign 4 units of one of the products in the set; product A, B, and C are not contained in the set since their demand is strictly less than 4. Similarly, in Figure 2.4b the path extensions from vertex $(1, 2)$ are presented. In Figure 2.4c, the only extension from vertex $(1, 4)$ is to create an empty aisle given that the first location is mixed, hence we have a set with the only element being $\emptyset$. Finally, Figure 2.4d depicts potential path extensions from vertex $(2, 4)$ in the situation where products A and B are allocated in location 1 and location 2 is an empty aisle.




(a) Extending from source vertex (b) Extending from vertex (1,2)

(c) Extending from vertex (1,4) (d) Extending from vertex (2,4)

Figure 2.4: Extending a path.

We note that some path extensions do not lead to a feasible solution. In Figure 2.4d, for instance, the extension from vertex $(2, 4)$ by assigning 4 units of product D means that the remaining products C and E must be assigned to the last location, which would then become mixed. This, however, would be infeasible due to the absence of an empty aisle adjacent to the last location.

### 2.3.2 Product clusters

Two allocated products with identical demands may be swapped in any feasible solution; such an operation just amounts to choosing another permutation of products with equal demands. In order to eliminate symmetries with respect to such permutations, we express the prod-

ucts and their demands in a compact way. In particular, we divide the $p$ products into $d^* = \max_{j=1,\ldots,p}\{d_j\}$ clusters, where cluster $i$ contains the products with demand equal to $i$, for $i = 1, \ldots, d^*$. As such, a characterization of the products is given by the vector $\Gamma_0 = (\gamma_0^1, \gamma_0^2, \ldots, \gamma_0^{d^*})$, where $\gamma_0^i = |\{j : d_j = i\}|$ for $i = 1, 2, \ldots, d^*$. Using this representation, we do not distinguish between products in the same cluster. For example, the five products in the example of Figure 2.2 are characterized by the vector (0,2,1,1,1) without distinguishing between products A and B.

### 2.3.3 Empty/Mixed/Dedicated combinations

For any path ending at any vertex in location $i$, we keep information on the status of locations $i-1$ and $i$ with respect to the possibility of mixing location $i$ and whether or not mixing location $i$ implies a requirement for allocating an empty aisle at location $i+1$. We define a location's EMD-status as (E), (M), or (D), according to whether the location is empty, mixed, or dedicated, respectively, and we let $\text{EMD}_i$ denote the EMD-status of any location $i$. While we can combine the three possibilities for locations $i$ and $i-1$ to produce $3 \times 3$ combinations, it is in fact sufficient to distinguish between only five cases, for the following reason: if location $i$ is empty, then $\text{EMD}_{i-1}$ makes no difference with respect to possible extensions of the path; if, however, location $i$ is not empty, then we only need to keep track of whether or not location $i-1$ is empty, as this determines whether or not a mixed location $i$ implies that location $i+1$ needs to be empty. The resulting five cases are described below as part of a path label.

### 2.3.4 Path labels

For presentational convenience is it assumed in the following that, unless otherwise stated, any path considered starts at vertex $v_{0m}$.

With any path we associate a label which keeps the information needed to identify the path, and we let $\mathbf{v}_{ij}$ denote the set of labels associated with paths ending at vertex $v_{ij}$. Each label $\lambda \in \mathbf{v}_{ij}$, associated with the path $P_\lambda$, contains the following information:

1. the total number of allocated product units along $P_\lambda$; this is the objective value of the path and equals the total length of all those arcs along $P_\lambda$ that represent allocation of product units (i.e., excluding arcs representing empty positions);

2. $\Gamma_\lambda = (\gamma_\lambda^1, \gamma_\lambda^2, \ldots, \gamma_\lambda^{d^*})$ shows the number of yet unallocated products. As such, along $P_\lambda$ we have allocated $\gamma_0^i - \gamma_\lambda^i$ products from cluster $i$, for $i = 1, \ldots, d^*$;

3. $\Theta_\lambda$ is the combination of $\text{EMD}_i$ and $\text{EMD}_{i-1}$ for $P_\lambda$ and captures the aforementioned five cases:

   a) $\Theta_\lambda =$ (E) if location $i$ is empty. In this case the EMD-status for location $i-1$ makes no difference;

   b) $\Theta_\lambda =$ (M,MD): Location $i$ is mixed, and location $i-1$ is mixed or dedicated;

   c) $\Theta_\lambda =$ (D,MD): Location $i$ is dedicated, and location $i-1$ is mixed or dedicated;

   d) $\Theta_\lambda =$ (M,E): Location $i$ is mixed, and location $i-1$ is empty;

   e) $\Theta_\lambda =$ (D,E): Location $i$ is dedicated, and location $i-1$ is empty.

   We note that if $\lambda \in \mathbf{v}_{im}$ and the corresponding location is dedicated, then cases (c) and (e) could be merged, since the information about the previous location plays no role in the continuation of the path. In the following we refer to $\Theta_\lambda$ as the EMD-value of $P_\lambda$.

## 2.4 A dynamic programming algorithm

Given that $G$ is acyclic, all paths in $G$ can be constructed by taking in topological order all vertices in $V \setminus \{v_{nm}\}$ as source vertex $v_{ij}$ and extend each path ending at $v_{ij}$ by adding arcs $(v_{ij}, v_{kl})$ in all the ways that are relevant according to the problem characteristics. When all vertices in $V \setminus \{v_{nm}\}$ have been used as source vertex, an optimal solution is provided by one of the paths ending at $v_{nm}$.

In combination with this, we have found it useful also to introduce a threshold value $T$ as an estimate of the optimal number of allocated units. Indeed, in the construction of paths as sketched above we are concerned only with solutions having at least $T$ allocated units.

Initially, $T$ is set equal to an upper bound. In a single iteration of the path construction there may or may not exist a solution with at least $T$ allocated units. If one or more such solutions exist, then the best found solution is indeed optimal and we terminate the procedure, otherwise we decrease $T$ and perform another iteration.

A stepwise description of the procedure is as follows.

Step 1: Create at vertex $v_{0m}$ a dummy label $\lambda$ which represents an empty path (i.e., with no arcs) with the following information: $\Gamma_\lambda = \Gamma_0$ and the total number of allocated product units is 0. The EMD-value for this empty path is artificially set equal to $\Theta_\lambda$ = (D,E), so that if location 1 is mixed, then location 2 must be empty. This dummy label is the only label in $\mathbf{v}_{0m}$.

Step 2: Perform steps 3–7 for all source vertices $v_{ij}$ satisfying $0 \leq \text{order}(v_{ij}) \leq nm - 1$, where the vertices are selected in increasing order of order$(v_{ij})$. Finally, go to step 8.

Step 3: For each pair of labels $\lambda$, $\lambda' \in \mathbf{v}_{ij}$, detect whether $\lambda$ dominates $\lambda'$ using the dominance rule. Remove from $\mathbf{v}_{ij}$ any label as soon as it is found to be dominated.

Step 4: Select labels $\lambda \in \mathbf{v}_{ij}$ in non-increasing order of the total number of allocated product units. For each label $\lambda$ perform steps 5–7.

Step 5: Perform a feasibility verification of the given $\lambda$. If infeasibility is detected, then return to step 4.

Step 6: Compute an upper bound (UB) for the given $\lambda$. If UB $< T$, then return to step 4.

Step 7: Create a set of successors $S(\lambda)$ of $\lambda$ as described hereinafter.

Step 8: If there exists at least one label in $\mathbf{v}_{nm}$, then identify in $\mathbf{v}_{nm}$ a label with the maximum value of allocated product units; this provides us with an optimal solution. Otherwise, if there are no labels in $\mathbf{v}_{nm}$, then we can conclude that the optimal solution contains less than $T$ allocated units, hence we reduce $T$ by one and return to step 2.

### 2.4.1 Label elimination

We have developed a number of procedures which allow us to eliminate certain labels as our algorithm proceeds. These procedures are described in the following.

**A dominance rule.** We have derived the following set of conditions for detecting, for a given label set $\mathbf{v}_{ij}$ and a given pair of labels $\lambda$, $\lambda' \in \mathbf{v}_{ij}$ whether $\lambda$ dominates $\lambda'$:

1. The total number of allocated product units of $\lambda$ must be greater than or equal to that of $\lambda'$.

2. The number of remaining products must be the same for the two labels, i.e.,

$$\sum_{i=1}^{d^*} \gamma_\lambda^i = \sum_{i=1}^{d^*} \gamma_{\lambda'}^i$$

3. Assume that condition 2 is satisfied, and let $r = \sum_{i=1}^{d^*} \gamma_\lambda^i$ denote the number of remaining products. Moreover, let $(d_\lambda^{[1]}, \ldots, d_\lambda^{[r]})$ be the $r$ demands in $\Gamma_\lambda$ sorted in non-increasing order, and similarly let $(d_{\lambda'}^{[1]}, \ldots, d_{\lambda'}^{[r]})$ be the $r$ demands in $\Gamma_{\lambda'}$ sorted in non-increasing order. Then condition 3 is the following: $d_\lambda^{[i]} \geq d_{\lambda'}^{[i]}$ for $i = 1, \ldots, r$. The reason is that for any choice of quantities in the completion of $\lambda'$ we can choose the same quantities in the completion of $\lambda$ by choosing the product with demand $d_\lambda^{[i]}$ in the extension of $P_\lambda$ as a replacement of the product with demand $d_{\lambda'}^{[i]}$ in the extension of $P_{\lambda'}$. Furthermore, if $d_\lambda^{[i]} > d_{\lambda'}^{[i]}$ for one or more values of $i$, then the completion of $\lambda$ may lead to a strictly better solution.

4. The combination of $\Theta_\lambda$ and $\Theta_{\lambda'}$ must be one of those marked with a checkmark '✓' in Table 2.1. The two checkmarks in parentheses in Table 2.1 apply only when the two labels are in the last slot of a dedicated location $i$, since in that case location $i+1$ is not required to be empty, no matter whether or not location $i-1$ is empty. However, if a label is in one of the $m-1$ first slots of location $i$, then an additional product might be allocated in location $i$ with a label representing that location $i$ is mixed.

Table 2.1: Dominance relations between EMD-values.

| | $\Theta_{\lambda'}$ | | | | |
|---|---|---|---|---|---|
| $\Theta_\lambda$ | (E) | (D,E) | (M,E) | (D,MD) | (M,MD) |
| (E) | ✓ | ✓ | ✓ | ✓ | ✓ |
| (D,E) | | ✓ | ✓ | ✓ | ✓ |
| (M,E) | | ✓ | ✓ | ✓ | ✓ |
| (D,MD) | | (✓) | (✓) | ✓ | ✓ |
| (M,MD) | | | | | ✓ |

The general idea underlying this dominance rule is that if, for any completion (until $v_{nm}$) of $P_{\lambda'}$ there is a corresponding completion of $P_\lambda$ which results in a better or equally good solution, then $\lambda$ dominates $\lambda'$. If all the above conditions are satisfied, then $\lambda$ dominates $\lambda'$.

**Feasibility verification.** In Anken et al. (2011, pp. 39–40) it is shown how to calculate the maximum number of possible products that can be placed in the $nm$ positions, taking into account the problem requirements with respect to empty aisles. Obviously, this maximum is determined by allocating only one unit of each product, and a feasible solution exists only if the actual number of products does not exceed the maximum number of products that can be allocated, as it is required to allocate at least one unit of each product.

In our algorithm, we have generalized these calculations in order to determine, for any path currently ending at some vertex $v_{ij}$, whether the remaining space from $v_{ij}$ to $v_{nm}$ is sufficient for allocating all the yet unallocated products.

For any label $\lambda$ associated with the path $P_\lambda$, we let $\bar{p}(\lambda)$ denote the maximum number of products which can be allocated in the remaining positions until $v_{nm}$, starting from the current

end vertex $v_{ij}$ of the path $P_\lambda$. Any label $\lambda$ along a feasible path must satisfy $\sum_{i=1}^{d^*} \gamma_\lambda^i \leq \bar{p}(\lambda)$, so otherwise label $\lambda$ can be discarded. Our computations of $\bar{p}(\lambda)$ follow the general idea in Anken et al. (2011).

**Upper bounds.** For any path $P_\lambda$ to $v_{ij}$ we compute an upper bound UB on the final number of allocations that may be obtained at $v_{nm}$ by extending $P_\lambda$. This bound is the sum of two terms, namely the number of allocations made along the path until $v_{ij}$, and an upper bound $\bar{a}_\lambda$ on the remaining allocations that can be made from $v_{ij}$ to $v_{nm}$. The following are involved in the computation of $\bar{a}_\lambda$:

1. The remaining number of available positions $nm - \text{order}(v_{ij})$ is surely a valid value of $\bar{a}_\lambda$.

2. The number of yet unallocated product units ($\sum_{i=1}^{d^*} i\gamma_\lambda^i$) is also a valid value of $\bar{a}_\lambda$.

3. If $\Theta_\lambda = (\text{M,MD}) \vee \Theta_\lambda = (\text{D,MD})$, then any extension of $P_\lambda$ will involve that either the following location must be an empty aisle (i.e., if we assign further products to the current location which then will be mixed), producing $m$ empty positions, or the remaining $m - j$ positions in the current location are left empty. Together, this implies at least $m - j$ empty positions on the path from $v_{ij}$ to $v_{nm}$.

4. The set of products with demands less than $m$ implies a certain amount of empty positions to be allocated. A product $j$ with $d_j < m$ will produce $m - d_j$ empty positions if it is assigned to a dedicated location, and otherwise it must be assigned to a mixed location which will generate a requirement for an empty aisle. We consider collectively all the $\sum_{i=1}^{m-1} \gamma_\lambda^i$ yet unassigned products with demands smaller than $m$. For a fixed number of empty aisles we can determine a lower bound on the implied number of empty positions by assigning up to $2m$ products on the two sides of each aisle—where we choose the products in nondecreasing order of demand—and assign the remaining products to dedicated locations; this produces $m$ empty positions for each empty aisle plus the sum of the $m - d_j$ values over all products $j$ assigned to dedicated locations; we recall that only

products with demand smaller than $m$ are considered. We obtain a sequence of lower bounds by considering the case with $0, 1, 2, \ldots$ etc. empty aisles, and finally we choose the lower bound with the minimum number of empty positions.

Further details are omitted for the sake of brevity. If the computed upper bound UB $< T$, then we can discard label $\lambda$.

**Iterative probing.** When determining the value of the threshold level $T$, we first compute an upper bound UB on the final number of allocations based on the dummy label $\lambda$ at vertex $v_{0m}$ and set $T =$ UB. As described earlier, if one or more solutions exist with at least $T$ allocated units, then we terminate the procedure, otherwise we decrease $T$ and perform another iteration. There are several alternative methods for updating $T$. We have chosen to use a simple method of decreasing $T$ by a constant amount after each iteration until an optimal solution is found. In fact, we have chosen to decrease $T$ by only one unit after each iteration, as we have experienced that a $T$ value below the optimal number of allocated units can significantly increase the total computation time.

### 2.4.2 Extending a path

Given a label $\lambda$ representing a path from $v_{0m}$ to $v_{ij}$, we produce the set of successor labels $S(\lambda)$ of $\lambda$ by adding arcs $(v_{ij}, v_{kl}) \in A$.

For each $q = 1, \ldots, d^*$ we consider the possibility of extending the path by adding an arc $(v_{ij}, v_{kl})$ which represents an allocation of the quantity $q$. A straightforward implementation would be to extend the path from $v_{ij}$ with an arc for each cluster $i \geq q$ for which $\gamma_\lambda^i > 0$. However, we observe that for any choice of $q$ it suffices to consider only one cluster by choosing the smallest among sufficient demands. Formally, we let $\rho(q, \lambda)$ denote the cluster from which we choose the product to allocate, given that $q$ units must be allocated starting from label $\lambda$, and we determine the cluster by setting $\rho(q, \lambda) = \min\{i | i \geq q \wedge \gamma_\lambda^i > 0\}$. Any other choice of

cluster for the given value of $q$ would lead to a dominated label according to condition 3 of the dominance rule.

In addition to allocating units of a product, we also have the possibility of allocating empty positions. We consider these two alternatives further in the following, where we also describe the other ingredients involved in extending a path.

**Allocating empty positions.** Without loss of generality we use the convention that empty positions within a location are placed at the highest-indexed slots in the location. As such, any arc representing empty positions ends at slot $m$. If the arc completes the current location, then it is of the form $(v_{ij}, v_{im})$, otherwise it is used for creating an empty aisle and is of the form $(v_{im}, v_{i+1,m})$.

Allocation of $q \in [m+1; 2m-1]$ uses two arcs by first completing the current location and then creating an empty aisle. Allocation of $2m$ or more empty positions in one step does not occur; however, adjacent empty aisles can be produced by repeatedly creating single aisles.

**Allocating product units.** Given a product and a quantity $q$ to be allocated, we allocate the product to the $q$ positions in $V[v_{ij}; v_{kl}]$. So, in general, the arc representing the allocation of $q$ units is directed from $v_{ij}$ to $v_{kl}$ so that $q = (k-i)m + l - j$. With respect to feasibility, it is required to use the smallest possible number of locations for storing the $q$ units. In terms of our allocation procedure this requirement amounts to ensuring that if $i < k$, then the total quantity of the product which is allocated to locations $i$ and $k$ exceeds $m$, otherwise the quantity at location $i$ could be moved to location $k$, resulting in one less location for storing the product. This can be expressed by the restriction that $l > j$ whenever $(i < k) \wedge (j < m)$. However, there is no restriction when $j = m$, as in that case we do not allocate any units of the product in location $i$.

**Ensuring feasibility with respect to mixed locations and aisles.** Any mixed location must be adjacent to an empty aisle. For satisfying this requirement we make use of the EMD-value for the label $\lambda$ that we are extending by the arc $(v_{ij}, v_{kl})$, resulting in label $\mu$ at vertex

$v_{kl}$.

In the following we distinguish between whether or not we extend from the last slot (i.e., $j = m$) in a location. It is noted that these two cases have quite different possibilities with respect to feasibility. The details are as follows.

1. Suppose that $j = m$, i.e., we extend from the last slot in location $i$. Note that only in this case is it possible to have $\Theta_\lambda = (\text{E})$. We distinguish between two situations:

   a) $\Theta_\lambda = (\text{M,MD})$: Location $i$ is mixed and must be adjacent to an empty aisle. Since the previous location is not an empty aisle, there is only one feasible extension, namely to allocate an empty aisle at location $i+1$. The resulting label has $\Theta_\mu = (\text{E})$.

   b) $\Theta_\lambda \neq (\text{M,MD})$: There are no restrictions on the extensions in any of these four cases, so we consider all values $q = 1, \ldots, d^*$ as the quantity for the next allocation. For each value of $q$, the extension produces the first allocated product in the resulting location $k$, which then becomes dedicated. The resulting label has $\Theta_\mu = (\text{D,E})$ if $(\Theta_\lambda = (\text{E})) \wedge (k = i+1)$, otherwise $\Theta_\mu = (\text{D,MD})$. Note that this also covers all possibilities where $q > m$ resulting in $k > i+1$. Finally, we also consider the possibility of creating an empty aisle at location $i+1$, producing $\Theta_\mu = (\text{E})$.

2. Suppose that $j < m$. This means that we have already assigned at least one product to location $i$, so $\Theta_\lambda \neq (\text{E})$. For any of the four possible values of $\Theta_\lambda$ we may fill the current location by adding $m - j$ empty positions, which will produce the label $\Theta_\mu = \Theta_\lambda$ at vertex $v_{im}$. Alternatively, if we extend the path by allocating product units, then location $i$ will be mixed in the new label, and we distinguish between the following two situations:

   a) $\Theta_\lambda = (\text{M,MD}) \vee \Theta_\lambda = (\text{D,MD})$: The previous location is not empty, so the fact that the current location is mixed after this allocation means that the next location must be empty. Therefore, the quantity that we allocate must not exceed the remaining

space in the current location, i.e., we consider only quantities $q = 1, \ldots, \min\{m - j, d^*\}$ to be allocated. The resulting label has $\Theta_\mu$ = (M,MD).

b) $\Theta_\lambda$ = (M,E) $\vee$ $\Theta_\lambda$ = (D,E): The previous location is empty, so the current location can be mixed without requiring the next location to be empty. Consequently, we consider all quantities $q = 1, \ldots, d^*$ to be allocated, subject to the restriction that if $k > i$, i.e., if we allocate more than $m - j$ units, then it must hold that $l > j$ in order to use the smallest possible number of locations for the $q$ units. Concerning the resulting label, we note that for $q \leq m - j$, we obtain $\Theta_\mu$ = (M,E), as the allocation is kept within the current location. For $q > m - j$ (implying $k > i$) the resulting label is $\Theta_\mu$ = (D,MD).

## 2.5 Computational experiments

Computational experiments were carried out on a personal computer P8700 (2.53 GHz) with 4 GB of RAM running Windows 7. Algorithms were implemented in C with Visual Studio 10.0.

We have implemented and analyzed the following approaches: our exact algorithm Saapp, the Cplex 12.4 implementation (Anken et al., 2011, the model of), and in addition we also analyzed the Saapp algorithm with an initial threshold value $T$ set to the optimal value, so the algorithm finds an optimal solution in a single iteration. The latter setup is denoted as Saapp/si and shows the computation time spent in the last iteration, i.e., the time needed to prove optimality of a solution.

We have tested the performance of our algorithm by generating SAAPP instances as proposed by Anken et al. (2011, p. 9). Accordingly, the number of products is in the range $[0.9n, \ldots, 1.3n]$. A CPU time limit of one hour was set for both the Saapp algorithm and the Cplex implementation.

Table 2.2 presents our computational results. Each line represents average results for problems with $p = 0.9n, 1.0n, 1.1n$, and $1.3n$. For each value of $p$ a total of 10 problem instances

were tested.

The two columns under the 'No. of replenishments' heading show the average number of replenishments of the instances that are solved by CPLEX and SAAPP, respectively, within the CPU time limit. The number of replenishments can be automatically translated into wasted time in the picking area (Anken et al., 2011) and is computed by taking the total demand over the period and deducting from it the total number of allocated product units during the advanced replenishment. The actual number of solved instances is presented under the 'No. of solved instances' heading. The three columns under the 'CPU time (sec)' heading show the time required by CPLEX, SAAPP and SAAPP/SI to provide an optimal solution, averaged over the solved instances. To facilitate the comparison, SAAPP/SI is applied only to those instances that are solved by SAAPP within the CPU time limit.

Clearly, with respect to computing time SAAPP outperforms CPLEX when the number of locations is greater than 15. In addition, the results show that SAAPP can solve problems of moderate size. Moreover, the parameter with the highest impact on the computation time is the number of slots.

The difference in computing time between SAAPP and SAAPP/SI shows the time spent on iterations before we reach the $T$ value that equals the optimal objective value. We consider this difference as the potential saving in computation time that might be obtained by more advanced schemes for initializing and updating the $T$ value. We consider the time difference between SAAPP and SAAPP/SI to be relatively modest, hence the iterative probing seems to be an appropriate approach for the SAAPP.

Finally, Table 2.3 presents the average number of allocated units in the optimal solution obtained with our SAAPP algorithm. Each value is the average result over 10 instances with the specified $n$, $m$ and $p$ configuration. If we consider the effect, in terms of the number of allocated units, of increasing either $n$ or $m$ by one, then the parameter with the highest impact per unit change seems to be the number of slots, within the problem sizes that have been investigated. Indeed, a unit increase of $m$ increases the capacity of the flow rack by $n$ slots. Hence, if more

Table 2.2: Average results for our test instances

| | | No. of replenishments | | No. of solved instances | | CPU time (sec) | | |
|---|---|---|---|---|---|---|---|---|
| $n$ | $m$ | CPLEX | SAAPP | CPLEX | SAAPP | CPLEX | SAAPP | SAAPP/SI |
| 10 | 2 | 3.83 | 3.83 | 40 | 40 | 0.14 | 0.11 | 0.00 |
| | 3 | 4.33 | 4.33 | 40 | 40 | 0.46 | 0.12 | 0.00 |
| | 4 | 5.15 | 5.15 | 40 | 40 | 0.77 | 0.11 | 0.00 |
| 15 | 2 | 5.08 | 5.08 | 40 | 40 | 0.78 | 0.11 | 0.00 |
| | 3 | 5.98 | 5.98 | 40 | 40 | 13.21 | 0.12 | 0.01 |
| | 4 | 7.40 | 7.40 | 40 | 40 | 30.63 | 0.14 | 0.02 |
| 20 | 2 | 7.30 | 7.30 | 40 | 40 | 2.75 | 0.12 | 0.00 |
| | 3 | 8.35 | 8.35 | 40 | 40 | 174.08 | 0.15 | 0.02 |
| | 4 | 9.70 | 9.70 | 40 | 40 | 803.98 | 0.34 | 0.14 |
| 25 | 2 | 8.63 | 8.63 | 40 | 40 | 39.40 | 0.13 | 0.01 |
| | 3 | 9.88 | 10.10 | 33 | 40 | 331.78 | 0.25 | 0.09 |
| | 4 | 11.77 | 12.33 | 26 | 40 | 1546.75 | 1.79 | 1.04 |
| 30 | 2 | 10.93 | 10.93 | 40 | 40 | 177.43 | 0.15 | 0.01 |
| | 3 | 10.46 | 12.25 | 13 | 40 | 1800.17 | 0.75 | 0.41 |
| | 4 | | 14.93 | | 40 | | 17.06 | 10.97 |
| 35 | 2 | | 12.15 | | 40 | | 0.16 | 0.03 |
| | 3 | | 13.45 | | 40 | | 1.77 | 1.14 |
| | 4 | | 17.03 | | 40 | | 150.20 | 104.18 |
| 40 | 2 | | 14.38 | | 40 | | 0.18 | 0.03 |
| | 3 | | 16.55 | | 40 | | 9.54 | 6.13 |
| | 4 | | 19.49 | | 35 | | 811.31 | 590.94 |
| 45 | 2 | | 16.13 | | 40 | | 0.23 | 0.05 |
| | 3 | | 18.50 | | 40 | | 41.26 | 26.03 |
| | 4 | | 22.44 | | 18 | | 1248.16 | 943.70 |

products must be allocated in a gravity flow rack system of moderate size, then one should investigate the possibility of increasing the number of slots per location, which would imply deeper flow racks in the storage system.

Moreover, Table 2.3 shows that for a fixed number of locations and slots, more products result in fewer allocated units. This is due to the requirement of allocating at least one unit

per product. Specifically, more products tend to produce more mixed locations which in turn increase the number of empty aisles, leaving less space for allocation of products.

Table 2.3: Average number of allocated units with SAAPP

(a) $p = 0.8n$

| | n | | | | | |
|---|---|---|---|---|---|---|
| $m$ | 15 | 20 | 25 | 30 | 35 | avg |
| 2 | 26.80 | 35.40 | 44.50 | 53.40 | 62.90 | 44.60 |
| 3 | 40.30 | 53.30 | 67.00 | 80.10 | 94.50 | 67.04 |
| 4 | 53.90 | 71.90 | 89.90 | 107.80 | 125.30 | 89.76 |
| avg | 40.33 | 53.53 | 67.13 | 80.43 | 94.23 | |

(b) $p = 1.0n$

| | $n$ | | | | | |
|---|---|---|---|---|---|---|
| $m$ | 15 | 20 | 25 | 30 | 35 | avg |
| 2 | 25.90 | 34.00 | 43.10 | 51.20 | 60.40 | 42.92 |
| 3 | 39.10 | 52.20 | 65.50 | 79.10 | 92.30 | 65.64 |
| 4 | 53.10 | 71.00 | 88.70 | 105.70 | 124.00 | 88.50 |
| avg | 39.37 | 52.40 | 65.77 | 78.67 | 92.23 | |

(c) $p = 1.1n$

| | $n$ | | | | | |
|---|---|---|---|---|---|---|
| $m$ | 15 | 20 | 25 | 30 | 35 | avg |
| 2 | 25.00 | 33.40 | 41.90 | 49.70 | 58.10 | 41.62 |
| 3 | 38.90 | 51.00 | 64.70 | 76.70 | 91.30 | 64.52 |
| 4 | 52.60 | 70.40 | 87.70 | 104.80 | 122.40 | 87.58 |
| avg | 38.83 | 51.60 | 64.77 | 77.07 | 90.60 | |

(d) $p = 1.3n$

| | $n$ | | | | | |
|---|---|---|---|---|---|---|
| $m$ | 15 | 20 | 25 | 30 | 35 | avg |
| 2 | 22.00 | 28.00 | 36.00 | 42.00 | 50.00 | 35.60 |
| 3 | 37.80 | 50.10 | 62.40 | 75.10 | 88.10 | 62.70 |
| 4 | 50.80 | 67.90 | 84.40 | 102.00 | 120.20 | 85.06 |
| avg | 36.87 | 48.67 | 60.93 | 73.03 | 86.10 | |

Based on the results with the set of generated problem instances, we conclude that the

exact SAAPP algorithm can be applied to obtain optimal solutions of moderate size instances.

## 2.6 Conclusions

In this paper, we have presented an exact dynamic programming algorithm for the space allocation and aisle positioning problem (SAAPP) which deals with two important and interrelated problems, namely, the product allocation problem and the storage location assignment problem.

To the best of our knowledge, this is the first exact algorithm for the SAAPP. We have introduced an appropriately defined graph representation of the SAAPP, which explores the characteristics of the underlying problem structure and enables the development of the dynamic programming algorithm.

Indeed, our computational study performed on numerous data shows that our exact algorithm can be used to find optimal solutions to SAAPP instances of moderate size.

# Chapter 3

# Scheduling in a Cross-Dock Environment to Minimize Mean Completion Time

# Scheduling in a Cross-Dock Environment to Minimize Mean Completion Time

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**Abstract**

This paper studies the problem of minimizing the mean completion time of outbound trucks in a cross-dock environment with a single inbound door and a single outbound door. Minimizing mean completion time improves the cross-dock's responsiveness and is known to lead to stable or balanced utilization of resources, rapid turn-around of vehicles and reduction of in-process inventory. We address the problem from a flow shop scheduling perspective and exploit its similarities with the classical two-machine flow shop problem, i.e., $F2||\sum C_j$. Indeed, our problem is a generalization of $F2||\sum C_j$, which is known to be $\mathcal{NP}$-hard. We propose a branch-and-bound algorithm to find an optimal solution to the problem. Computational results show that our branch-and-bound algorithm can optimally solve problem instances of moderate size within a reasonable amount of time.

## 3.1 Introduction

Cross-docks are facilities in which inbound vehicles deliver products that are rapidly distributed, consolidated, and transferred to outbound vehicles. Cross-docks aim to minimize or even elim-

inate the cost of storing and retrieving products. Products are either transferred from inbound to outbound vehicles or placed in a buffer area for a short period, typically less than 24 hours. Moreover, cross-docking can reduce transportation costs by enabling higher capacity utilization for inbound and outbound vehicles. Cross-docking has been successfully implemented in several industries such as at retail chains (Stalk et al., 1992), mailing companies (Forger, 1995), automobile manufacturers (Witt, 1998), and less-than-truckload logistics providers (Gue, 1999).

The literature on cross-docking is mainly focused on truck scheduling and truck-to-dock assignment problems. For recent comprehensive literature reviews we refer to Agustina et al. (2010), Boysen and Fliedner (2010), and Van Belle et al. (2012). Truck-to-dock assignment is the problem of assigning inbound and outbound trucks to docks (Tsui and Chang, 1990, 1992; Oh et al., 2006; Miao et al., 2009). Note that the truck-to-dock assignment problem is different from the truck scheduling problem, which takes time dimension into account while time is not considered in the dock door assignment problem (Konur and Golias, 2013).

In the existing literature on truck scheduling problems in cross-docking, optimization criteria are typically based on completion times of the vehicles. The most common and widely studied performance measure of cross-dock operations is the makespan, defined as the completion time of the last outbound vehicle (Yu and Egbelu, 2008; Chen and Song, 2009; Chen and Lee, 2009; Boysen et al., 2010). This objective is appropriate when the cost of a schedule depends on how long the processing system is devoted to the entire set of vehicles. Moreover, the makespan is known to be directly related to the maximization of the throughput and the usage of the resources, e.g., dock doors.

In this research we focus on individual outbound trucks and minimize the mean completion time, which is equivalent to minimizing the total completion time defined as the sum of the completion times of the outbound trucks. The objective is also referred to as the total flow time if all trucks are available from the beginning of the planning horizon. This performance measure improves the cross-dock's responsiveness and leads to stable or balanced utilization of resources, rapid turn-around of vehicles, and reduction of in-process inventory (Rajendran

and Ziegler, 1997; Framinan and Leisten, 2003). Therefore, it is considered to be relevant and meaningful for today's dynamic production environment (Liu and Reeves, 2001).

This objective criterion has been recently addressed by Briskorn et al. (2013) and Soltani and Sadjadi (2010). Briskorn et al. (2013) consider the problem of minimizing the total weighted completion time in a single dock cross-docking facility with inventory constraints. The authors show that the problem is $\mathcal{NP}$-hard and propose a branch-and-bound and a dynamic programming algorithm. Soltani and Sadjadi (2010) assume a single inbound and a single outbound dock system with no temporary storage and develop two robust hybrid meta-heuristic methods.

We extend the literature by focusing on minimizing the total completion time of outbound trucks in a single inbound and single outbound door cross-dock environment with temporary storage and propose an exact branch-and-bound approach.

The structure of this paper is the following. In Section 3.2 we describe the problem and provide an illustrative example. The properties of the problem are presented in Section 3.3. In Section 3.4 we describe lower bound procedures which can be embedded in a branch-and-bound framework as presented in Section 3.5. Results of the computational experiments are reported in Section 3.6. Finally, conclusions are presented in Section 3.7.

## 3.2 Problem description

We consider a cross-dock with a single inbound and a single outbound dock door. We assume complete information about the inbound and outbound vehicles. Each inbound truck may carry several products to a set of outbound trucks, and each outbound truck may receive products from several inbound trucks. We assume a pre-distribution system, which is applied frequently in practice (Yan and Tang, 2009), i.e., each incoming product is assigned to a single outbound truck prior to the product's arrival at the cross-dock, hence each product can be considered as a unique product type and a transshipment matrix can completely describe the predecessor relations between trucks.

Moreover, each truck is available from the beginning of the planning horizon and associated with a known processing time. Processing (i.e., loading) of an outbound truck cannot be initiated before all its associated products have been unloaded from their respective inbound trucks and the outbound dock has become idle. We assume that the transshipment time of products in the cross-dock is negligible (i.e., we assume that it is zero) when searching for a solution. Finally, storage capacity is assumed to be unlimited and sufficient workforce is assumed to be available in the cross-dock to process the trucks.

Cross-dock scheduling is closely related to traditional machine scheduling (Chen and Lee, 2009; Briskorn et al., 2013). However, cross-dock scheduling involves some peculiarities, for which classical machine scheduling approaches or even notation cannot be used directly, e.g., in a cross-dock inbound vehicles might contain multiple product units, which are not preassigned to a specific vehicle but may satisfy the demand of multiple outbound vehicles for the respective product (Boysen and Fliedner, 2010).

We formulate the problem as a two-machine flow shop problem and consider two sets of jobs with precedence constraints. The job sets correspond to inbound and outbound vehicles while the two machines correspond to a single inbound and a single outbound dock, respectively.

For notational convenience, when referring to a problem we follow the 3-field notation $[\alpha|\beta|\gamma]$ for cross-docking scheduling problems introduced by Boysen and Fliedner (2010), where $\alpha$ is door environment, $\beta$ is the operational characteristics, and $\gamma$ is the objective. The notation uses square brackets in order to clearly distinguish the considered cross-docking problem from problems in a classical scheduling environment. Accordingly, our problem can be denoted as $[E2|t_\lambda{=}0|\sum C_j]$ and stated as the following.

$[E2|t_\lambda{=}0|\sum C_j]$ Problem: There are two machines $M_1$ and $M_2$ and two set of jobs $\mathcal{I} = \{i_1, i_2, \ldots, i_n\}$ and $\mathcal{O} = \{o_1, o_2, \ldots, o_m\}$. Each job $j \in \mathcal{I}$ must be processed on $M_1$ and requires a processing time of $a(j)$. Each job $j \in \mathcal{O}$ must be processed on $M_2$ and requires a processing time of $b(j)$. For each job $j \in \mathcal{O}$, there is a corresponding predecessor set $P_j$ of jobs in $\mathcal{I}$, such that all jobs in $P_j$ must be completed on $M_1$ before $j$ can be processed on $M_2$. We aim to schedule jobs in $\mathcal{I}$ and $\mathcal{O}$, such that the total completion time of jobs in $\mathcal{O}$, i.e., the sum of the $m$ completion times on $M_2$, is minimized.

The problem addressed in this paper can be considered as a generalized two-machine flow shop problem, which is a common problem in production environments where the two machines represent two processing stages that products must visit in order. In particular, a special case of our problem, when a single inbound truck serves a single outbound truck, is equivalent to the two-machine minimum total completion time flow shop problem. We follow the notation by Graham et al. (1979) and use $F2||\sum C_j$ to denote this classical problem. We note that in $F2||\sum C_j$ the transshipment time of the jobs between the first and second machine is also ignored, i.e., assumed to be zero. The $F2||\sum C_j$ is known to be $\mathcal{NP}$-hard in the strong sense (Gonzalez and Sahni, 1978; Framinan and Leisten, 2003), therefore, $[E2|t_\lambda{=}0|\sum C_j]$ is also $\mathcal{NP}$-hard.

Furthermore, we use the following notation throughout the paper:

| | |
|---|---|
| $S_i$ | is the successor set of job $i \in \mathcal{I}$, $S_i \subseteq \mathcal{O}$, i.e., $o \in S_i$ if and only if $i \in P_o$ |
| $\pi$ | is a sequence of jobs in $\mathcal{I}$ processed on $M_1$, |
| | $\pi = (\pi_1, \pi_2, \ldots, \pi_l)$ with $l \leq n$, i.e., $\pi_k$ is the $k$th element of $\pi$ |
| $\sigma$ | is a sequence of jobs in $\mathcal{O}$ processed on $M_2$, |
| | $\sigma = (\sigma_1, \sigma_2, \ldots, \sigma_l)$ with $l \leq m$, i.e., $\sigma_k$ is the $k$th element of $\sigma$, |
| | $\sigma$ is a complete sequence of jobs in $\mathcal{I}$ if $l = m$, |
| | otherwise it is a partial sequence |
| $\bar{\sigma}$ | is the complement set of a sequence $\sigma$, i.e., $\bar{\sigma} = \mathcal{O} \setminus \sigma$ |

| | |
|---|---|
| $\Omega$ | is a schedule that corresponds to a tuple $(\pi, \sigma)$ |
| $C(j)$ | is the completion time of job $j \in \{\mathcal{I}, \mathcal{O}\}$ |
| $C^i(\Omega)$ | is the completion time of the last job on $M_i$ according to $\Omega$ |

$[E2|t_\lambda{=}0|\sum C_j]$ can be formulated as follows:

$$\min \sum_{o \in \mathcal{O}} C(o) \tag{3.1}$$

subject to

$$C(i) - a(i) \geq 0 \qquad \forall i \in \mathcal{I} \tag{3.2}$$

$$C(o) - b(o) \geq C(i) \qquad \forall o \in \mathcal{O}, i \in P_o \tag{3.3}$$

$$C(i) - a(i) \geq C(i') \vee \; C(i') - a(i') \geq C(i) \qquad \forall i, i' \in \mathcal{I}, i \neq i' \tag{3.4}$$

$$C(o) - b(o) \geq C(o') \vee \; C(o') - b(o') \geq C(o) \qquad \forall o, o' \in \mathcal{O}, o \neq o' \tag{3.5}$$

The objective function (3.1) aims to minimize the total completion time. Constraints (3.2) ensure that the processing of any job on $M_1$ does not start before the beginning of the planning horizon. Constraints (3.3) ensure that the processing of any job on $M_2$ can commence only after all its predecessors are completed on $M_1$. Constraints (3.4)–(3.5) ensure that each machine can process at most one job at a time.

Consider an example with 6 inbound and 4 outbound trucks, i.e., $\mathcal{I} = \{i_1, i_2, \ldots, i_6\}$ and $\mathcal{O} = \{o_1, o_2, o_3, o_4\}$. The predecessor relations and processing times are given in Table 3.2. A feasible schedule is presented in Figure 3.1, for this instance this is also an optimal schedule with a total completion time of 110, i.e., a mean completion time of 27.5. That is, an optimal solution to this problem is the following: $\pi = (i_2, i_4, i_5, i_3, i_6, i_1)$ and $\sigma = (o_1, o_2, o_3, o_4)$.

Table 3.2: Example

(a) Jobs in $\mathcal{I}$

| Job | $i_1$ | $i_2$ | $i_3$ | $i_4$ | $i_5$ | $i_6$ |
|---|---|---|---|---|---|---|
| $a(j)$ | 6 | 5 | 8 | 4 | 4 | 6 |
| $S_j$ | $\{o_4\}$ | $\{o_1, o_4\}$ | $\{o_3, o_4\}$ | $\{o_1, o_2\}$ | $\{o_2, o_4\}$ | $\{o_3\}$ |

(b) Jobs in $\mathcal{O}$

| Job | $o_1$ | $o_2$ | $o_3$ | $o_4$ |
|---|---|---|---|---|
| $b(j)$ | 5 | 10 | 4 | 8 |
| $P_j$ | $\{i_2, i_4\}$ | $\{i_4, i_5\}$ | $\{i_2, i_3, i_5, i_6\}$ | $\{i_1, i_2, i_3\}$ |

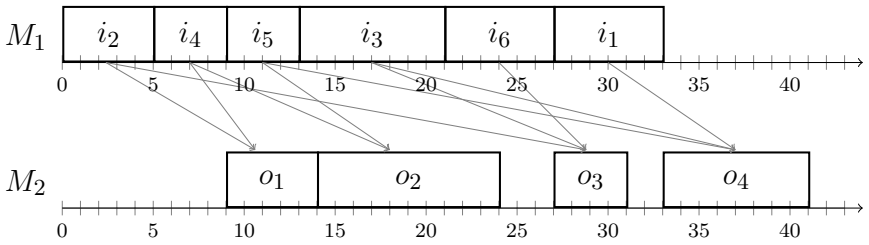



Figure 3.1: Representation of a schedule (arrows represent predecessor relations)

## 3.3 Optimality properties

Jobs in $\mathcal{I}$ can be scheduled consecutively on $M_1$ without slack (or idle time) between any two jobs, since no job in $\mathcal{I}$ has a predecessor. Therefore, a sequence of jobs in $\mathcal{I}$ and their respective processing times provide complete information about the schedule on $M_1$, namely starting and completion times of each job in $\mathcal{I}$.

There is a feasible solution for any given sequence of jobs in $\mathcal{I}$, since one can always schedule jobs in $\mathcal{O}$ in an arbitrary order after the completion time of the last job on $M_1$.

The following property is useful for sequencing jobs in $\mathcal{I}$ without successors.

**Property 1.** *There exists an optimal schedule such that job $i \in \mathcal{I}$ with $S_i = \emptyset$ is sequenced after any $i' \in \mathcal{I}$ with $S_{i'} \neq \emptyset$.*

*Proof.* Assume that some job $i \in \mathcal{I}$ with $S_i = \emptyset$ is sequenced immediately before some other

job $i' \in \mathcal{I}$ with $S_{i'} \neq \emptyset$. When exchanging these two jobs, $i, i'$ on $M_1$, no job is delayed on $M_2$. Therefore, applying such exchanges iteratively, any job $i \in \mathcal{I}$ with $S_i = \emptyset$ can be sequenced after any $i' \in \mathcal{I}$ with $S_{i'} \neq \emptyset$ without delaying the completion time of any job on $M_2$. □

Moreover, any job $i \in \mathcal{I}$ with $S_i = \emptyset$ that is sequenced last on $M_1$ can be removed from a solution, since the quality of the solution is measured only by considering completion times on $M_2$. Therefore, from now on, we assume that $S_i \neq \emptyset$ for all $i \in \mathcal{I}$.

Just as there exists a feasible solution for any given sequence of jobs in $\mathcal{I}$, there exists also a feasible solution for any given sequence of jobs in $\mathcal{O}$ (cf. Chen and Lee, 2009, on the makespan minimization problem). In particular, for a given sequence $\sigma$ of jobs in $\mathcal{O}$, one can create a sequence of jobs on $M_1$ according to the order in which their successors are processed on $M_2$, i.e., by first sequencing jobs in $P_{\sigma_1}$, then jobs in $P_{\sigma_2} \setminus P_{\sigma_1}$, then iteratively sequencing jobs in $P_{\sigma_j} \setminus \left(\cup_{k=1}^{j-1} P_{\sigma_k}\right)$ for $j = 3, \ldots, m$. In each iteration, the jobs that are just added to the schedule can be sequenced arbitrarily and the processing of the current $j \in \sigma$ begins as soon as all jobs in $P_j$ are completed and $M_2$ is idle. Let the *stepwise-extended order* of inbound jobs for a given sequence $\sigma \in \mathcal{O}$ be defined as $P_{\sigma_1}, P_{\sigma_2} \setminus P_{\sigma_1}, P_{\sigma_3} \setminus (P_{\sigma_1} \cup P_{\sigma_2}), \ldots, P_{\sigma_m} \setminus \left(\cup_{k=1}^{m-1} P_{\sigma_k}\right)$.

**Property 2.** *There exists an optimal schedule such that if jobs in $\mathcal{O}$ are processed on $M_2$ according to the sequence $\sigma$, then jobs in $\mathcal{I}$ are processed on $M_1$ in the stepwise-extended order.*

*Proof.* Assume that a schedule is given and consider two jobs $j, j' \in \sigma$, where $\sigma = (o_1, o_2, \ldots, o_m)$ (after re-indexing) such that $j$ is sequenced immediately before $j'$ on $M_2$ and some $i' \in P_{j'} \setminus P_j$, is sequenced before some $i \in P_j$ on $M_1$. The claim is that exchanging $i$ and $i'$ does not increase the total completion time of jobs in $\sigma$. Without loss of generality, assume the following partial sequence on $M_1$: $(i', \alpha, i)$, where $\alpha$ is a partial sequence of jobs in $\pi \setminus \{i, i'\}$. We note that the processing of $j$ can start only after the completion of $i$. Therefore, exchanging $i$ and $i'$, which produces the partial sequence $(i, \alpha, i')$, does not delay the completion time of $j$. Moreover, such exchanging procedure does not delay the completion time of $j'$ either since $\sigma$ is unchanged and defines that job $j'$ is processed after $j$. After exchanging $i$ and $i'$ the new solution is

still feasible. Repeating this exchange process, jobs on $M_1$ will eventually be sequenced in the stepwise-extended order as claimed. □

In the following, we assume that whenever a $\sigma$ is given, then jobs in $\cup_{j\in\sigma}P_j$ are contained in $\pi$ and sequenced on $M_1$ in the stepwise-extended order.

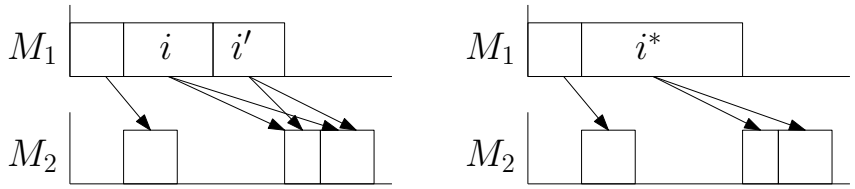
Furthermore, we can establish that a set of jobs in $\mathcal{I}$ with each element having the same successor set can be considered as one block. The following property states this observation formally.

**Property 3.** *If for some jobs $i, i' \in \mathcal{I}$, $i \neq i'$ we have that $S_i = S_{i'}$, then there exists an optimal schedule such that $i$ and $i'$ are processed consecutively on $M_1$.*

*Proof.* Assume that two jobs $i, i' \in \mathcal{I}$ with $S_i = S_{i'}$ are not processed consecutively on $M_1$ and let $\alpha$ be the partial sequence of jobs processed between $i$ and $i'$, i.e., the partial sequence of these jobs is $(i, \alpha, i')$. Note that no job $j \in S_i$ can start being processed after having completed $i$ on $M_1$ since any such job $j$ also has $i'$ as predecessor, thus the processing of $j$ can be started only after $i'$ is completed on $M_1$. Hence, exchanging $i$ and any job in $\alpha$ does not delay any job $j \in S_i$, and the new solution is still feasible. As a conclusion, $i$ can be exchanged with all jobs in $\beta$ resulting in the partial sequence $(\beta, i, i')$, which is at least as good as $(i, \beta, i')$ with respect to the total completion time. □

Consider two jobs $i, i' \in \mathcal{I}$, with $S_i = S_{i'}$; Property 3 implies that $i$ and $i'$ together can be considered as one block when scheduling the inbound trucks. Therefore, whenever two such jobs are identified, we can reduce the size of the problem by replacing the two jobs with a newly constructed job that represents the block, i.e., the new job represents both original jobs (see Figure 3.2). In particular, let *job clustering* of two jobs $i, i' \in \mathcal{I}$, with $S_i = S_{i'}$ be defined as constructing a new job $i^*$, where $S_{i^*} := S_i$ and $a(i^*) := a(i) + a(i')$, and setting $\mathcal{I} := \mathcal{I} \setminus \{i, i'\} \cup \{i^*\}$ and $P_o := P_o \setminus \{i, i'\} \cup \{i^*\}, \forall o \in S_{i^*}$.

**Corollary 1.** *Job clustering of two jobs $i, i' \in \mathcal{I}$, with $S_i = S_{i'}$ does not change the total completion time of jobs on $M_2$.*



(a) Original instance (b) New instance

Figure 3.2: Illustration of a job clustering (arrows represent predecessor relations)

Now, we present a set of conditions based on a lower bound procedure presented by Ignall and Schrage (1965) for the $F2||\sum C_j$; a complete schedule is optimal if it satisfies all conditions in the set.

**Property 4.** *A complete schedule $\Omega = (\pi, \sigma)$ is optimal if all of the following conditions hold:*

- *jobs in $\mathcal{O}$ are sequenced according to the* shortest processing time *(SPT) rule;*
- *the first job in $\sigma$ has the minimum total predecessor processing time, i.e.,* $\sum_{i \in P_{\sigma_1}} a(i) = \min_{o \in \mathcal{O}} \sum_{k \in P_o} a(k)$*;*
- *$M_2$ is not idle between processing any two jobs in $\sigma$.*

*Proof.* Consider a relaxation of the $[E2|t_\lambda{=}0|\sum C_j]$ problem, the single machine problem with the set $\mathcal{O}$ of jobs and processing times $b(j)$ $\forall j \in \mathcal{O}$. The optimal solution for this relaxation can be found by sequencing jobs in $\mathcal{O}$ according to the SPT rule. Let $\sigma$ denote such a complete sequence of jobs in $\mathcal{O}$. We note that in this schedule, the machine is not idle between any two jobs.

In our cross-dock environment the earliest time that the processing of any job in $\mathcal{O}$ can start on $M_2$ is $\min_{o \in \mathcal{O}} \sum_{k \in P_o} a_k$. Let $r$ denote this minimum total predecessor processing time. Hence, moving all jobs on $M_2$ to the right by $r$ time units, i.e., increasing the total completion time with $m \cdot r$, is a valid lower bound to our problem.

If we adopt $\sigma$ for the $[E2|t_\lambda{=}0|\sum C_j]$ problem and in the resulting schedule $M_2$ is not idle between any two jobs while considering the original predecessor relations, then the objective

value is equal to the provided lower bound so $\sigma$ is optimal also for the $[E2|t_\lambda{=}0|\sum C_j]$ problem. □

With the help of this property, we can identify the following special case of our problem which can be solved in $O(m \log m)$ time.

**Corollary 2.** *If $P_j = P_{j'}$ for all $j, j' \in \mathcal{O}$, the problem is polynomially solvable by sequencing jobs in $\mathcal{O}$ according to the SPT rule.*

In the general case, there are $n!m!$ feasible sequences in total. However, due to Property 2 the search space reduces to consider only outbound job sequences, i.e., $O(m!)$ sequences. Clearly complete enumeration is limited to the smallest instances.

## 3.4 Lower bounds

In this section, we present procedures that provide valid lower bounds. These procedures are useful to evaluate the performance of a heuristic method and they can be embedded in exact approaches such as branch-and-bound.

### 3.4.1 Predecessor relaxation based lower bounds

First, we build on the ideas presented by Ignall and Schrage (1965) for the $F2||\sum C_j$ in their seminal paper. The key element is to process jobs on $M_2$ without idle times by allowing either that $M_2$ can process multiple jobs simultaneously or by relaxing the predecessor processing constraints on $M_1$. We denote the lower bounds obtained with these approaches as $LB_1$ and $LB_2$, respectively.

To apply the lower bound procedure $LB_1$, we first construct an instance for the $F2||\sum C_j$ problem based on the instance in question for our $[E2|t_\lambda{=}0|\sum C_j]$. For this, we consider an instance $I$ of $[E2|t_\lambda{=}0|\sum C_j]$, in which there exists at least one job in $\mathcal{I}$ such that it has more than one successor job in $\mathcal{O}$. We aim to relax the successor relations and obtain an instance

in which one inbound truck serves one outbound truck, i.e., $|S_i| = 1\ \forall i \in \mathcal{I}$. This is achieved with an iterative procedure, as follows. In each iteration, a job $i \in \mathcal{I}$ is divided into a set of newly constructed parts where the total processing time of the parts is $a(i)$ and each part $k$ has a successor set $S_k \subseteq S_i$. We then replace job $i$ with the set of newly constructed parts.

In particular, let *job splitting* of a job $i \in \mathcal{I}$ be defined as a three phase procedure that operates as follows. First, we construct a set $\mathcal{D}(i)$ of new jobs, where $\sum_{k\in\mathcal{D}(i)} a(k) = a(i)$ and $\cup_{k\in\mathcal{D}(i)} S_k \subseteq S_i$. Second, we set $\mathcal{I} := \mathcal{I} \setminus \{i\} \cup \mathcal{D}(i)$ and remove $i$ from the predecessor sets, i.e., $P_j := P_j \setminus \{i\}\ \forall j \in \mathcal{O}$. Finally, we update the predecessor sets, so that they contain the elements of $\mathcal{D}(i)$, i.e., for all $k \in \mathcal{D}(i)$ we set $P_j := P_j \cup \{k\}\ \forall j \in S_k$.

Then a new instance $I'$ is constructed by modifying $I$ such that we apply job splitting on a job $i \in \mathcal{I}$, with $|S_i| > 1$. We call the new instance obtained *reversible* if $S_k = S_i$ for all $k \in \mathcal{D}(i)$; otherwise it is called *non-reversible*. The following observation is used to establish a valid lower bound.

**Proposition 1.** *The total completion time of an optimal schedule for $I'$ is a valid lower bound of the total completion time for $I$.*

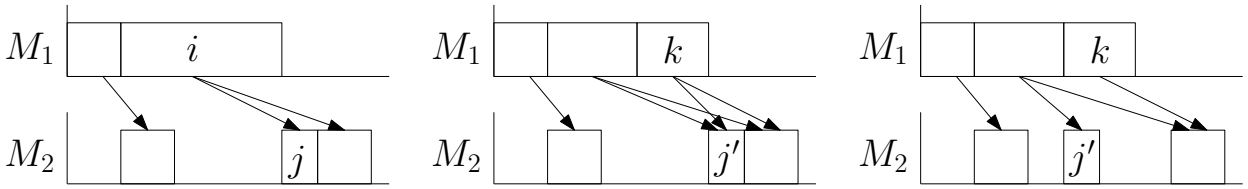
*Proof.* We can distinguish two cases based on whether $I'$ is reversible or not. We will prove that the claim holds in both cases.

First, consider the case that $I'$ is reversible. That is, $S_k = S_{k'}$ for all $k, k' \in \mathcal{D}(i)$, which implies that jobs in $\mathcal{D}(i)$ can be processed consecutively on $M_1$ (see Property 3) and any two jobs in $\mathcal{D}(i)$ can be clustered. Hence, the optimal total completion times for $I$ and $I'$ are identical.

Second, consider the case that $I'$ is non-reversible. That is, there exists some $k \in \mathcal{D}(i)$ such that $S_k \subset S_i$. Let $\mathcal{I}'$ and $\mathcal{O}'$ denote the set of jobs in the instance $I'$ that are processed on $M_1$ and $M_2$, respectively. Since the job splitting procedure operates on the predecessor sets but does not modify the set of jobs on $M_2$, we have that each job in $\mathcal{O}'$ corresponds to a job in $\mathcal{O}$, where $\mathcal{O}$ is the set of jobs in the instance $I$ that are processed on $M_2$. Without loss of generality, assume that $j' \in \mathcal{O}'$ corresponds to $j \in \mathcal{O}$ and $i \in P_j$ while $k \notin P_{j'}$, where $k \in \mathcal{D}(i)$.

This implies that the processing of job $j' \in \mathcal{O}'$ can begin on $M_2$ before job $k \in \mathcal{I}'$ is completed on $M_1$, i.e., the total processing time required to complete the jobs in $P_{j'}$ is less that of $P_j$. $\square$

As a consequence, the job splitting procedure (see Figure 3.3) is not the reverse of creating compound jobs (see Figure 3.2).




(a) Original instance (b) New instance (reversible) (c) New instance (non-reversible)

Figure 3.3: Illustration of a job splitting, where $k \in \mathcal{D}(i)$ (arrows represent predecessor relations)

Applying the job splitting procedure, we can construct an $F2||\sum C_j$ instance based on our $[E2|t_\lambda{=}0|\sum C_j]$ instance. $F2||\sum C_j$ can be stated as follows.

> $F2||\sum C_j$ Problem: There is a set of jobs $J$ and two machines, $M_1$ and $M_2$. Each job $j \in J$ is processed on $M_1$ and then processed on $M_2$ with the processing time $A(j)$ and $B(j)$, respectively. A job is completed when its processing is finished on $M_2$. The objective is to minimize the total completion time of jobs in $J$.

We let $J := \mathcal{O}$, i.e., each job $j \in J$ corresponds to a job $j' \in \mathcal{O}$. For each job $j \in J$, we create the processing times $A(j)$ and $B(j)$ on $M_1$ and $M_2$, respectively, as follows. First, we set $B(j) := b(j')$ for all $j \in J$. Then, we compute the value of $A(j)$ using the job splitting procedure. While there are many ways to split jobs, we have chosen a simple rule. We split each job $i \in \mathcal{I}$ such that $\mathcal{D}(i)$ is a singleton and the job $k \in \mathcal{D}(i)$ is a predecessor of a single job.

In particular, we initially set the processing time on $M_1$ to zero for each job $j \in J$, i.e., $A(j) = 0\ \forall j \in J$. Then, we iterate through the jobs in $\mathcal{I}$ and apply the job splitting procedure to each job $i \in \mathcal{I}$ in two steps. In the first step, we construct a single job $k$ with $a(k) = a(i)$

and set $\mathcal{D}(i) := \{k\}$. In the second step, the successor set $S_k \subseteq S_i$ is constructed such that it contains a single element. For this, let $Q(i)$ denote a set of jobs in $J$, such that each job $j \in Q(i)$ corresponds to a job $j' \in S_i$. We note that $S_i \subseteq \mathcal{O}$ and $Q(i) \subseteq J$. We identify a job in $Q(i)$ with the minimum processing time on $M_1$, let $j^*$ denote this job, i.e., $j^* = \arg\min_{j \in Q(i)}\{A(j)\}$. Then, we set the single job in $S_k$ so that it is the job in $\mathcal{O}$ that corresponds to $j^*$. Finally, we increase the processing time of $j^*$ on $M_1$ by $a(i)$, i.e., $A(j^*) := A(j^*) + a(i)$, and the procedure iteratively continues with the following job in $\mathcal{I}$.

We note that the value of $A(j)$ $\forall j \in J$ can be obtained, even if some elements of the previously described procedure are omitted. Indeed, for each $i \in \mathcal{I}$, it is sufficient to find $j^* \in J$ and update the processing time of $j^*$ on $M_1$. The pseudo code of the procedure is presented in Algorithm 1. It enables us to consider a starting schedule $\Omega = (\pi, \sigma)$, where $\sigma$ is a sequence of $r$ jobs. For this, we introduce the following notation. For each $i \in \mathcal{I}$, let $\bar{Q}(i)$ denote a set of jobs in $J$, where each job $j \in \bar{Q}(i)$ corresponds to a job $j' \in S_i \setminus \sigma$.

**Algorithm 1** Computing $A(j)$ $\forall j \in J$

```
1  A(j) = 0   ∀j ∈ J
2  for each i ∈ I do
3     if i ∉ P_j for all j ∈ σ then
4           j* = arg min_{j∈Q̄(i)}{A(j)} (break ties arbitrary)
5           A(j*) := A(j*) + a(i)
6     end if
7  loop
```

Once $A(j)$ and $B(j)$ are available for all $j \in J$ we can compute a lower bound as described by Ignall and Schrage (1965), that is

$$LB_1 = \sum_{j=1}^{r} C(\sigma_j) + \sum_{p=1}^{m-r} \Big[C^1(\Omega) + (n - r - p + 1)\ A(j_p) + B(j_p)\Big] \tag{3.6}$$

where $j_1, j_2, \ldots, j_{n-r}$ is a permutation of the unscheduled jobs in $J$ such that $A(j_1) \leq A(j_2) \leq \cdots \leq A(j_{n-r})$. That is, the processing of each job can start on $M_2$ immediately after being

processed on $M_1$, irrespective of whether $M_2$ is idle or not. The optimal solution of this relaxed problem is given by sequencing the jobs according to the SPT rule on $M_1$. The complexity of computing $LB_1$ for a given schedule is $O(m \log m)$ at the root node and $O(m)$ for all other nodes (Della Croce et al., 1996).

We incorporate a further improvement to this lower bound based on a relaxation of the problem as proposed by Della Croce et al. (1996). In particular, consider two adjacent jobs $j$ and $j'$ in $J$ such that $j$ is sequenced immediately before $j'$. In $F2||\sum C_j$ we have that $j'$ cannot be processed on $M_2$ before time $\max\{C^2(j), C^1(j')\}$, where $C^i(j)$ is the completion time of job $j$ on $M_i$. This constraint is omitted in a relaxed problem, called $F2_{rel}||\sum C_j$, which has that $j'$ cannot be processed on $M_2$ before time $\max\{C^1(j) + B(j), C^1(j')\}$.

It is shown in Della Croce et al. (1996) that a lower bound to $F2_{rel}||\sum C_j$ is given by $LB_{DNT_1} = LB_1 + \Gamma$, where $\Gamma$ is the minimum solution value over all permutations of unscheduled jobs in $J$ for the expression

$$\sum_{p=1}^{m-r} \Big[ \max\Big\{0, B(j_p) - A(j_{p+1})\Big\} + \max\Big\{0, A(j_p) - A(j_{p+1})\Big\}\Big] \tag{3.7}$$

To find $\Gamma$ an asymmetric traveling salesman problem based model is proposed in the paper and a lower bound of $\Gamma$ is found by a bipartite weighted matching relaxation. The complexity of computing $LB_{DNT_1}$ is $O(m^3)$.

The second lower bound procedure is based on a relaxation that the processing of jobs on $M_2$ can start immediately after the minimum total predecessor time is completed on $M_1$ and $M_2$ is idle. That is, the earliest processing time is identical for each unscheduled job in $\mathcal{O}$. This approach is equivalent to relaxing constraint (3.3) in the formulation and introducing

$$C(o) - b(o) \geq \min_{i \in \mathcal{I}}\{C(i)\} \quad \forall o \in \mathcal{O} \tag{3.8}$$

The optimal solution of this relaxed problem is given by sequencing the jobs according to the SPT rule on $M_2$. We note that this approach is not restricted to the $F2||\sum C_j$ formulation, therefore we may utilize the information from the $[E2|t_\lambda{=}0|\sum C_j]$ problem. In particular, we

use the information about the predecessor sets when selecting $k$ in $\bar{\sigma}$ and call the resulting formulation $LB_2$. Let $\Omega = (\pi, \sigma)$ denote a starting partial schedule, we thus have that

$$LB_2 = \sum_{j=1}^{r} C(\sigma_j) + \sum_{p=1}^{m-r} \left[ \max \left\{ C^2(\Omega) \ , \ C^1(\Omega) + \min_{k \in \bar{\sigma}} \{ \sum_{i \in P_k \setminus \pi} a(i) \} \right\} + (n - r - p + 1) \cdot b(o_p) \right] \tag{3.9}$$

where $o_1, o_2, \ldots, o_{n-r}$ is a permutation of jobs in $\bar{\sigma}$ such that $b(o_1) \leq b(o_2) \leq \cdots \leq b(o_{n-r})$.

The complexity of computing $LB_2$ for a given $\sigma$ is $O(nm + m \log m)$ at the root node and $O(m)$ for all other nodes.

### 3.4.2 A preemption based lower bound

Here, we follow the idea presented by Ahmadi and Bagchi (1990) for the $F2||\sum C_j$ and adapt it to the cross-docking context. We therefore construct, an $m$-job single machine problem instance with unequal release dates, in which the set of jobs is $J := \mathcal{O}$, i.e., each job $j \in J$ corresponds to a job $j' \in \mathcal{O}$. Each job $j \in J$ is associated with a processing time $B(j) := b(j')$ and a release date

$$r_j = \max \left\{ C^2(\Omega), C^1(\Omega) + \sum_{i \in P_j \setminus \pi} a(i) \right\} \tag{3.10}$$

where $\Omega = (\pi, \sigma)$ is a starting partial schedule. The preemptive relaxation of this problem, i.e., $1|r_j, pmtn|\sum C_j$ is efficiently solvable by applying the *shortest remaining processing time* (SRPT) rule (Ahmadi and Bagchi, 1990; Chu, 1992). For this, the release dates are computed first. Second, when a job is to be selected for processing among the available ones, the job with the smallest remaining processing time is chosen. An arriving job preempts the job being processed if the processing time of the new arrival is strictly smaller than the remaining processing time of the job in process.

Let $LB_3$ denote the solution value of an optimal solution to the $1|r_j, pmtn|\sum C_j$. The lower bound $LB_2$ is dominated by $LB_3$. However, for a given schedule, the complexity of computing $LB_3$ is $O(nm + m \log m)$ at the root node and $O(m \log m)$ for all other nodes.

## 3.5 Branch-and-bound

Among the exact methods, branch-and-bound algorithms have been widely applied to minimization problems in which a solution is a permutation of a set of objects, particularly for machine scheduling problems like the $F2||\sum C_j$. A branch-and-bound algorithm is generally characterized by a branching rule, a search strategy, node elimination rules, lower-bound cost functions, upper-bound cost functions, and node dominance relations. We describe the details of these elements in the following.

### 3.5.1 Branching rule

We adopt the branching rule presented in the paper of Ignall and Schrage (1965), in which the authors address the $F2||\sum C_j$ problem by a branch-and-bound approach with branching on the job to be appended to a partial sequence. The rationale for this approach is that for every explored node corresponding to a partial sequence $\sigma$ the completion times on both machines are already computed and hence a new partial sequence constructed by appending a job to the partial sequence can be evaluated in a few, simple steps (T'kindt et al., 2004).

A diagram representing the branch-and-bound process is called a search tree. In this tree, each node represents a sub-problem with a given fixed partial sequence of jobs, that is, we branch on the successive job $j \in \mathcal{O}$ to be inserted in the sequence. In particular, the $k$th level of the search tree corresponds to a partial sequence with $k$ elements.

### 3.5.2 Search strategy

To decide which node to branch from in the search tree, we consider the best first, the breadth first and the depth first search strategies. It is known that the first two mentioned strategies may require significantly more storage space when compared to last one. When the number of generated nodes is a key element, as in our problem, depth first is generally considered the most efficient search strategy (T'kindt et al., 2004). We therefore apply this strategy and select

the active node at the lowest level in the tree, namely the node with the largest number of sequenced jobs, i.e., we explore along each branch of the search tree as far as possible before backtracking.

### 3.5.3 Elimination rules

For each sub-problem of the search tree a lower bound is computed. In order to adopt the tightest lower bound, we choose the maximum of the computed values as the lower bound to be used for pruning branches:

$$LB = \max\{LB_{DNT_1}, LB_2, LB_3\} \tag{3.11}$$

If the lower bound of a given node is greater than or equal to the current upper bound (UB) in our minimization problem, then the node is pruned.

### 3.5.4 Upper bounds

At the beginning of the search we establish an initial UB by applying heuristic procedures. Moreover, we can improve on this UB during the search by evaluating complete sequences of jobs in $\mathcal{O}$, which are created during the execution of the lower bound procedures.

#### Initial upper bound

Several heuristic procedures have been proposed for the $F2||\sum C_j$ problem. These often follow a general framework composed of three phases, namely job indexing, solution construction, and solution improvement (Framinan et al., 2005; Pan and Ruiz, 2013). Framinan et al. (2005) cluster the existing heuristics into two groups: simple and composite methods. A heuristic is called composite if it employs another heuristic for one or more of the three phases. Otherwise the procedure can be categorized as a simple heuristic.

The solution to the problem can be described by the job schedule on $M_2$ alone since it defines the job schedule on $M_1$ as well. Therefore, the heuristic procedures are focused on a sequence of jobs on $M_2$.

We have implemented a simple heuristic, the procedure of Laha and Sarin (2009), called FL-LS, which is considered the best performer among simple heuristics (Pan and Ruiz, 2013) and runs in $O(m^4)$ time, where $m$ is the number of jobs in $\mathcal{O}$. The procedure is outlined in the following.

Step 1: Compute $T_j$, the total processing time on both machines for each job $j \in \mathcal{O}$, i.e.,

$$T_j = \sum_{i \in P_j} a(i) + b(j) \quad \forall j \in \mathcal{O} \tag{3.12}$$

Step 2: Sort jobs in $\mathcal{O}$ in non-decreasing order of their total processing times.

Step 3: Set $k = 1$ and construct a partial sequence with the first job from the sorted list.

Step 4: Update $k := k + 1$. Select the $k$th job from the list and insert it into the best of the $k$ possible positions in the partial sequence. Then each job of this sequence (except the last inserted) is placed into all possible positions and the best sequence is selected among those generated. This becomes the next current sequence.

Step 5: If $k = m$ then terminate, otherwise go to step 4.

We have also implemented a composite heuristic, called C2, which is proposed by Framinan et al. (2005) and reported to provide the best results among the tested procedures. The procedure comprises the following three stages:

In the first stage, jobs are sorted based on the index function proposed by Liu and Reeves (2001), which takes the sum of two terms, namely, a weighted total machine idle time, and an artificial total completion time. Then, an initial solution is constructed following an idea

described by Framinan and Leisten (2003). That is, jobs are inserted into a starting partial sequence one by one according the job indices. In particular, the $k$th job is inserted into each of the $k$ possible slots of the best partial solution. Out of these $k$ sequences the best is kept. Then a pairwise interchange is performed to search for improvements. If a better result is obtained then the new partial solution is retained as the best partial sequence.

Once an initial sequence of jobs in $\mathcal{O}$ is obtained, the procedure continues with the second stage in which a solution is constructed according to the idea presented by Rajendran and Ziegler (1997). That is, in this stage each job in $\mathcal{O}$ is inserted into each of the possible slots.

A neighborhood strategy, called forward insertion exchange - restart (FIE-R) is used in the third stage. This strategy exchanges each job with any other job following it in the sequence. When the exchange is actually made because the new solution improves the current one, the process starts from the first job in the new sequence.

**Sequence based upper bound**

The complete sequences of jobs in $\mathcal{O}$ that are constructed by the lower bound procedures can be used at each node of the search tree to update a current $UB$. For the procedure described in Section 3.4.1 two complete sequences are constructed when computing $LB_1$ and $LB_2$. Moreover, another sequence is created during the procedure for computing $LB_3$. We note that while the schedule constructed in $LB_3$ allows preemption, we can construct a feasible schedule without preemption for the $[E2|t_\lambda{=}0|\sum C_j]$ by sequencing the outbound jobs based on the beginning or completion of their processing on $M_2$.

We apply the original predecessor relations when evaluating a sequence and hence obtain a feasible schedule for the $[E2|t_\lambda{=}0|\sum C_j]$. Therefore, such an obtained solution value is a valid upper bound for the problem. Taking the minimum of the solution values of these schedules, we obtain an upper bound. If it is strictly less than the current $UB$, then the latter is updated.

### 3.5.5 Dominance relations

A common way to shorten the enumeration is to consider dominance conditions while branching from a node. Dominance conditions can be derived by comparing two nodes of the search tree, that is, two partial solutions of the given problem. These conditions are commonly used for machine scheduling problems and typically involve precedence conditions among pairs of jobs. Therefore, whenever we are considering to branch from a node by adding an unscheduled job to the end of a partial sequence, we apply the dominance rules described hereafter.

First, we may use the following dominance criterion which is based on the idea presented by Della Croce et al. (1996) for the $F2||\sum C_j$.

**Dominance 1.** *Given a starting partial schedule $\Omega = (\pi, \sigma)$ and a pair of unscheduled jobs $j$ and $j'$ in $\mathcal{O}$, $j \neq j'$. If*

$$P_j \setminus \pi \subseteq P_{j'} \setminus \pi \tag{3.13}$$

$$b(j) \geq b(j') \tag{3.14}$$

*and*

$$\max\left\{C^1(\Omega) + \sum_{i \in P_j \setminus \pi} a(i)\ ,\ C^2(\Omega)\right\} + b(j) \leq \max\left\{C^1(\Omega) + \sum_{i' \in P_{j'} \setminus \pi} a(i')\ ,\ C^2(\Omega)\right\} + b(j') \tag{3.15}$$

*then the partial sequence $(\sigma, j)$ dominates $(\sigma, j')$.*

*Proof.* Consider two complete sequences on $M_2$, namely, $(\sigma, j, \alpha, j', \beta)$ and $(\sigma, j', \alpha, j, \beta)$, where $\sigma$ is a starting partial sequence, $\alpha$ and $\beta$ are partial sequences, and no job is sequenced twice. We will prove that $(\sigma, j, \alpha, j', \beta)$ dominates $(\sigma, j', \alpha, j, \beta)$ by verifying that the total completion time of $(\sigma, j, \alpha, j', \beta)$ is not greater than that of $(\sigma, j', \alpha, j, \beta)$.

The proof can be divided into five sequential phases, where each phase considers a starting partial sequence in the following order: $(\sigma)$, $(\sigma, j)$, $(\sigma, j, \alpha)$, $(\sigma, j, \alpha, j')$, and $(\sigma, j, \alpha, j', \beta)$. Let $C^i(\gamma)$ denote the completion time of the last job on $M_i$ according to the sequence $\gamma$ of jobs in $\mathcal{O}$.

Phase 1: The completion times of jobs in $\sigma$ are identical for both sequences, i.e., for both $(\sigma, j, \alpha, j', \beta)$ and $(\sigma, j', \alpha, j, \beta)$.

Phase 2: Due to (3.13) and (3.15), we have that $C^1(\sigma, j) \leq C^1(\sigma, j')$ and $C^2(\sigma, j) \leq C^2(\sigma, j')$.

Phase 3: Consider a starting partial sequence $(\sigma, j, j', \alpha)$. Together with (3.13) we can establish that

$$C^1(\sigma, j, \alpha) \leq C^1(\sigma, j, j', \alpha) = C^1(\sigma, j', \alpha) \tag{3.16}$$

Due to (3.15) and (3.16), we have that

$$C^2(\sigma, j, \alpha) \;\; \leq \;\; C^2(\sigma, j', \alpha) \tag{3.17}$$

Phase 4: The completion time of the last job on $M_1$ is identical in the two sequences $(\sigma, j, \alpha, j')$ and $(\sigma, j', \alpha, j)$, i.e.,

$$C^1(\sigma, j, \alpha, j') \;\; = \;\; C^1(\sigma, j', \alpha, j) \tag{3.18}$$

Due to (3.14), (3.17), and (3.18), we have that

$$\max\Big\{C^1(\sigma, j, \alpha, j'), C^2(\sigma, j, \alpha)\Big\} + b(j') \leq \max\Big\{C^1(\sigma, j', \alpha, j), C^2(\sigma, j', \alpha)\Big\} + b(j) \tag{3.19}$$

That means that

$$C^2(\sigma, j, \alpha, j') \;\; \leq \;\; C^2(\sigma, j', \alpha, j) \tag{3.20}$$

Phase 5: When all predecessor jobs are completed of the starting partial sequences $(\sigma, j, \alpha, j')$ and $(\sigma, j', \alpha, j)$, due to (3.18) we have that the completion time of each remaining job in $\mathcal{I}$ is the same for both complete sequences, i.e., for both $(\sigma, j, \alpha, j', \beta)$ and $(\sigma, j', \alpha, j, \beta)$. Thus together with (3.20) we have that

$$C^2(\sigma, j, \alpha, j', \beta) \;\; \leq \;\; C^2(\sigma, j', \alpha, j, \beta) \tag{3.21}$$

As a conclusion $(\sigma, j, \alpha, j', \beta)$ will result in a total completion time that is not greater than that of $(\sigma, j', \alpha, j, \beta)$. □

The following dominance criterion is based on the study by Della Croce et al. (2002) on the $F2||\sum C_j$ problem. The key idea is that under certain conditions we can immediately identify the next job to sequence.

**Dominance 2.** *Given a starting sub-schedule $\Omega = (\pi, \sigma)$ of an optimal solution, if there exists a job $j \in \bar{\sigma}$ such that*

$$b(j) = \min_{j' \in \bar{\sigma}} \left\{ b(j') \right\} \tag{3.22}$$

$$\sum_{i \in P_j \setminus \pi} a(i) = \min_{j' \in \bar{\sigma}} \left\{ \sum_{i' \in P_{j'} \setminus \pi} a(i') \right\} \tag{3.23}$$

*and*

$$\sum_{i \in P_j \setminus \pi} a(i) \leq b(j) \tag{3.24}$$

*then $(\sigma, j)$ is a starting subsequence of an optimal solution.*

A proof sketch is provided in the following. Suppose that in an optimal solution there is a partial sequence $\alpha$ of $r$ jobs in $\mathcal{O} \setminus \{\sigma, j\}$ immediately after $\sigma$ and before $j$, that is, we consider the sequence $(\sigma, \alpha, j)$. If we reinsert $j$ before $\alpha$ and rearrange jobs on $M_1$ according to Property 2, then all $r$ jobs in $\alpha$ are moved to the right by at most $b(j)$ time units. At the same time, the completion time of job $j$ is reduced by at least $r \cdot b(j)$ time units. Hence, the total completion time of $(\sigma, j, \alpha)$ is not worse than that of $(\sigma, \alpha, j)$.

If more than one such job exists that satisfy the criteria in Dominance 2, then we may arbitrarily choose the job with the lowest index.

Moreover, whenever a new job $j \in \mathcal{O}$ is appended at the end of a partial sequence resulting in the current subsequence $\sigma$, we call an operator that changes the order of the scheduled jobs and as a result we obtain a new sequence $\sigma^*$, which is compared with the current one. The condition applied to our problem is based on the well-known dynamic programming dominance

property presented by Potts and Van Wassenhove (1983). When evaluating whether one sequence dominates the other we apply a generalization of the dynamic programming dominance criterion presented by Della Croce et al. (2002).

We outline the key idea in the following. Suppose that $\sigma^*$, which is a permutation of the jobs in $\sigma$, has a smaller total completion time and a greater makespan than that of $\sigma$. The greater makespan of $\sigma^*$ implies that the total completion time of the remaining (unscheduled) jobs that extend $\sigma^*$ is greater than or equal to that of $\sigma$. However, if the difference between the two total completion times of the already scheduled jobs offsets the difference in the total completion times related to the remaining jobs, then $\sigma^*$ dominates $\sigma$.

**Dominance 3.** *(Della Croce et al., 2002) Given two partial schedules $\Omega = (\pi, \sigma)$ and $\Omega^* = (\pi^*, \sigma^*)$, such that $\sigma^*$ is a permutation of the jobs in $\sigma$, $\sigma \neq \sigma^*$. If*

$$\sum_{j^* \in \sigma^*} C(j^*) \leq \sum_{j \in \sigma} C(j) \tag{3.25}$$

*and*

$$|\bar{\sigma}| \cdot \Big[ C^2(\Omega^*) - C^2(\Omega) \Big] \leq \sum_{j \in \sigma} C(j) - \sum_{j^* \in \sigma^*} C(j^*) \tag{3.26}$$

*then $\sigma^*$ dominates $\sigma$.*

A proof sketch is provided in the following. Assume that the jobs in $\bar{\sigma}$ are sequenced arbitrarily. We show the total completion time of the complete sequence $(\sigma^*, \bar{\sigma})$ is not greater than that of the complete sequence $(\sigma, \bar{\sigma})$. Let $C^i(\gamma)$ denote the completion time of the last job on $M_i$ according to the sequence $\gamma$ of jobs in $\mathcal{O}$. We note that $C^1(\sigma^*, \alpha) = C^1(\sigma, \alpha)$ for any partial sequence $\alpha$, such that $\alpha$ is a starting subsequence of $\bar{\sigma}$. We can distinguish two cases based on the value of $\Delta := C^2(\Omega^*) - C^2(\Omega)$. First, the case that $\Delta \leq 0$ is trivial. We consider the case that $\Delta > 0$ from now on. For all $j \in \bar{\sigma}$, we have that

$$\begin{aligned} C^2(\sigma^*, \alpha, j) &= \max \Big\{ C^1(\sigma^*, \alpha, j) \ , \ C^2(\sigma^*, \alpha) \Big\} + b(j) && (3.27) \\ &\leq \max \Big\{ C^1(\sigma, \alpha, j) \ , \ C^2(\sigma, \alpha) + \Delta \Big\} + b(j) \leq C^2(\sigma, \alpha, j) + \Delta && (3.28) \end{aligned}$$

Now, consider the total of $|\bar{\sigma}|$ remaining jobs. Let $\sum_{j\in\bar{\sigma}} C_\sigma(j)$ denote the total completion time of jobs in the partial sequence $\bar{\sigma}$, where the complete sequence is $(\sigma, \bar{\sigma})$, i.e., jobs in $\bar{\sigma}$ are sequenced after the starting sequence $\sigma$. Due to (3.28), we have that

$$\sum_{j\in\bar{\sigma}} C_{\sigma^*}(j) \leq \sum_{j\in\bar{\sigma}} \Big(C_\sigma(j) + \Delta\Big) = \sum_{j\in\bar{\sigma}} C_\sigma(j) + |\bar{\sigma}| \cdot \Big[C^2(\Omega^*) - C^2(\Omega)\Big] \tag{3.29}$$

Together with (3.26) we have that

$$\sum_{j\in\bar{\sigma}} C_{\sigma^*}(j) - \sum_{j\in\bar{\sigma}} C_\sigma(j) \leq \sum_{j\in\sigma} C(j) - \sum_{j^*\in\sigma^*} C(j^*) \tag{3.30}$$

After rearranging, we obtain $\sum_{j^*\in\sigma^*} C(j^*) + \sum_{j\in\bar{\sigma}} C_{\sigma^*}(j) \leq \sum_{j\in\sigma} C(j) + \sum_{j\in\bar{\sigma}} C_\sigma(j)$ and hence $\sigma^*$ dominates $\sigma$.

In case both conditions, namely, (3.25) and (3.26), happen to be equalities in Dominance 3, then we choose arbitrarily one of the sequences.

Dominance 3 is used as follows. Let $j$ denote the last job in $\sigma$. Following Della Croce et al. (2002), we apply the following operators to obtain a permutation of jobs in $\sigma$ for all jobs $j' \in \sigma \setminus \{j\}$: (i) $j$ is exchanged with $j'$, (ii) $j$ is re-inserted immediately before $j'$, and (iii) $j'$ is re-inserted after $j$. Then each pair consisting of the new starting sequence $\sigma^*$ and the original $\sigma$ is compared using Dominance 3.

## 3.6 Computational experiments

The branch-and-bound procedure was coded in C++. The instances were run on a P8700, 2.53 GHz computer with 4GB of RAM.

The number of jobs is chosen such that $m \in \{15, 20, 25, 30\}$ and $n \in \{10, 15, 20, 25, 30\}$. For each instance, a transshipment matrix is generated randomly with a preset probability $p$ that a truck $i \in \mathcal{I}$ is a predecessor of some $j \in \mathcal{O}$. We set $p$ identical for all inbound and outbound truck pairs within one instance, i.e., $i \in P_j$ with probability $p$ for all $i \in \mathcal{I}, o \in \mathcal{O}$. Should the generated instance contain a truck with empty predecessor or successor set, the instance is discarded and another is generated.

In the literature about the mean completion time flow shop problems, the processing times for each job are typically taken from the uniform distribution $U[1, 10]$ (cf. van de Velde, 1990; Della Croce et al., 2002; Akkan and Karabatı, 2004) or $U[1, 99]$ (cf. Taillard, 1993; Framinan et al., 2003; Pan and Ruiz, 2013). The latter distribution is known to produce difficult problem instances (Framinan et al., 2005). We follow the convention and generate the instances applying these two distributions. For each problem size, 20 instances were randomly generated.

Table 3.3 presents results for instances with processing times in $U[1, 10]$. The CPU times are given in seconds. Under 'No. of nodes', we report the number of nodes in the search tree. As expected, the number of nodes and the CPU time are strongly correlated. Therefore, if one aims to reduce the computation time, it is appropriate to consider also on the reduction of the number of nodes, which is facilitated with the selected depth first search strategy in our approach.

The results show that our procedure provides an optimal solution in a reasonable time for instances with up to 25 outbound trucks if processing times are in the range $U[1, 10]$.

Table 3.4 presents results for instances with processing times in $U[1, 99]$. The computational time is greater compared to the case with processing times in $U[1, 10]$.

We note that due to the capacity of the trucks carrying pallets or cases in a cross-dock environment, the number of outbound trucks served per inbound truck is often bounded in practice. Therefore, as the number of trucks increases (and exceeds a threshold level corresponding to the capacity of the trucks), the value of $p$ tends to 0. In the literature, there is only a limited amount of information about the density of the transshipment matrix. In the study of Gue (1999), the inbound trucks at a considered test site contain products for an average of 6.4 destinations. In case of 25 inbound and 25 outbound trucks this yields that $p = 0.26$ on average, assuming no correlation between the loads. The results show that instances of this moderate size can be solved optimally by our algorithm within a reasonable time.

Among the problem parameters, the value of $p$ seems to have the biggest impact on the computation time. Figure 3.4 illustrates the change in CPU times and number of nodes when

Table 3.3: Average results for instances with processing times in $U[1, 10]$

| | | $p = 0.25$ | | | | $p = 0.5$ | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | CPU (sec) | | No. of nodes (×1000) | | CPU (sec) | | No. of nodes (×1000) | |
| $m$ | $n$ | avg | max | avg | max | avg | max | avg | max |
| 15 | | | | | | | | | |
| | 10 | 0.0 | 0.0 | 0.2 | 1.1 | 0.0 | 0.0 | 0.0 | 0.1 |
| | 15 | 0.2 | 2.0 | 3.6 | 23.7 | 0.0 | 0.0 | 0.2 | 0.4 |
| | 20 | 0.2 | 1.0 | 3.6 | 11.1 | 0.0 | 0.0 | 0.2 | 0.8 |
| | 25 | 0.7 | 7.0 | 8.0 | 55.1 | 0.0 | 0.0 | 0.3 | 0.7 |
| | 30 | 1.6 | 7.0 | 13.8 | 50.7 | 0.0 | 0.0 | 0.6 | 1.5 |
| 20 | | | | | | | | | |
| | 10 | 0.0 | 0.0 | 0.5 | 3.5 | 0.0 | 0.0 | 0.1 | 0.5 |
| | 15 | 0.2 | 2.0 | 1.9 | 13.7 | 0.0 | 0.0 | 0.3 | 1.1 |
| | 20 | 2.1 | 15.0 | 14.1 | 92.9 | 0.0 | 0.0 | 0.8 | 3.2 |
| | 25 | 8.5 | 28.0 | 43.0 | 138.5 | 0.0 | 0.0 | 1.1 | 2.7 |
| | 30 | 37.9 | 290.0 | 168.3 | 1226.8 | 0.1 | 1.0 | 1.5 | 5.3 |
| 25 | | | | | | | | | |
| | 10 | 0.0 | 0.0 | 0.7 | 2.6 | 0.0 | 0.0 | 0.1 | 0.6 |
| | 15 | 0.7 | 5.0 | 5.0 | 25.5 | 0.0 | 0.0 | 0.4 | 1.0 |
| | 20 | 6.3 | 28.0 | 27.1 | 112.2 | 0.0 | 0.0 | 0.9 | 2.7 |
| | 25 | 24.8 | 112.0 | 89.9 | 402.6 | 0.0 | 0.0 | 1.1 | 2.4 |
| | 30 | 147.9 | 595.0 | 461.6 | 1963.8 | 0.3 | 2.0 | 2.5 | 7.9 |
| 30 | | | | | | | | | |
| | 10 | 0.0 | 0.0 | 0.7 | 1.6 | 0.0 | 0.0 | 0.2 | 1.2 |
| | 15 | 6.9 | 60.0 | 26.2 | 200.2 | 0.0 | 0.0 | 0.6 | 2.1 |
| | 20 | 24.9 | 121.0 | 64.6 | 283.0 | 0.1 | 1.0 | 1.1 | 3.6 |
| | 25 | 114.0 | 317.0 | 293.1 | 845.9 | 0.6 | 3.0 | 2.6 | 8.3 |
| | 30 | 504.7 | 3154.0 | 1202.5 | 8406.0 | 0.9 | 4.0 | 3.0 | 10.1 |

0.0 CPU time indicates that the computation time is $< 0.05$ sec.

$p$ is between 0.1 and 1. The computation time seems to increase exponentially as $p$ decreases. We note that as $p$ decreases, the difference reduces between the optimal solution value for an instance $I$ for the $[E2|t_\lambda{=}0|\sum C_j]$ and the one for an instance $I'$ for the $F2||\sum C_j$, which is generated based on $I$ (see Section 3.4). This is due to the decrease in the number of predecessor relations. On the other hand, when $p = 1$, an optimal solution is obtainable in polynomial

Table 3.4: Average results for instances with processing times $U[1, 99]$

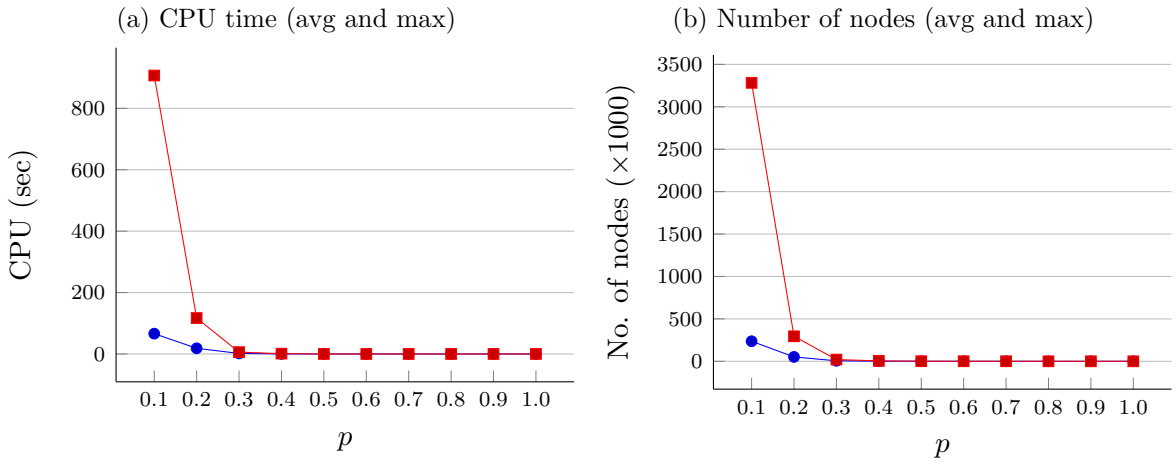
| | | $p = 0.25$ | | | | $p = 0.5$ | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | CPU (sec) | | No. of nodes (×1000) | | CPU (sec) | | No. of nodes (×1000) | |
| $m$ | $n$ | avg | max | avg | max | avg | max | avg | max |
| 15 | | | | | | | | | |
| | 10 | 0.0 | 0.0 | 0.5 | 3.2 | 0.0 | 0.0 | 0.1 | 0.2 |
| | 15 | 0.4 | 4.0 | 5.5 | 37.8 | 0.0 | 0.0 | 0.2 | 0.5 |
| | 20 | 0.3 | 1.0 | 4.8 | 12.7 | 0.0 | 0.0 | 0.4 | 1.1 |
| | 25 | 0.7 | 8.0 | 9.6 | 69.5 | 0.0 | 0.0 | 0.5 | 1.3 |
| | 30 | 1.9 | 7.0 | 16.2 | 52.2 | 0.0 | 0.0 | 0.8 | 2.4 |
| 20 | | | | | | | | | |
| | 10 | 0.0 | 0.0 | 1.2 | 6.4 | 0.0 | 0.0 | 0.1 | 1.1 |
| | 15 | 0.4 | 5.0 | 4.6 | 36.8 | 0.0 | 0.0 | 0.6 | 1.8 |
| | 20 | 5.2 | 31.0 | 30.4 | 179.9 | 0.1 | 1.0 | 1.4 | 6.5 |
| | 25 | 17.2 | 84.0 | 92.1 | 427.4 | 0.2 | 1.0 | 2.3 | 7.7 |
| | 30 | 59.1 | 443.0 | 273.6 | 1966.4 | 0.4 | 3.0 | 2.9 | 17.3 |
| 25 | | | | | | | | | |
| | 10 | 0.1 | 1.0 | 2.0 | 8.7 | 0.0 | 0.0 | 0.4 | 2.4 |
| | 15 | 4.6 | 39.0 | 21.6 | 168.5 | 0.0 | 0.0 | 0.9 | 3.2 |
| | 20 | 23.7 | 146.0 | 96.2 | 559.9 | 0.3 | 1.0 | 2.5 | 6.9 |
| | 25 | 75.8 | 396.0 | 273.9 | 1379.9 | 0.3 | 1.0 | 2.4 | 6.7 |
| | 30 | 330.1 | 946.0 | 1081.6 | 3313.8 | 1.2 | 4.0 | 5.2 | 17.7 |
| 30 | | | | | | | | | |
| | 10 | 0.3 | 1.0 | 2.8 | 5.7 | 0.1 | 1.0 | 0.7 | 3.4 |
| | 15 | 61.2 | 417.0 | 195.4 | 1342.3 | 0.2 | 1.0 | 1.5 | 5.0 |
| | 20 | 133.6 | 996.0 | 349.8 | 2476.7 | 0.6 | 6.0 | 3.0 | 20.4 |
| | 25 | 423.1 | 944.0 | 1163.5 | 2509.6 | 1.8 | 9.0 | 6.2 | 23.6 |
| | 30 | - | - | - | - | 2.3 | 11.0 | 6.4 | 31.3 |

0.0 CPU time indicates that the computation time is $< 0.05$ sec.
- indicates that the values were not computed

time (see Section 3.3).

We have tested the performance of the proposed algorithm on randomly generated $F2||\sum C_j$ problems. The results are presented in Table 3.5. For each configuration, 20 problem instances were generated with processing times in $U[1, 10]$. The results show that an optimal solution is

Figure 3.4: Results with $m = n = 20$ and processing times in $U[1, 10]$




obtainable for instances with up to 20 jobs. These results are inferior to the ones obtainable with the state-of-the-art algorithms for the $F2||\sum C_j$ problem, which are able to obtain an optimal solutions for instances with up to 40 jobs in a reasonable time (cf. Della Croce et al., 2002; Hoogeveen et al., 2006). The $F2||\sum C_j$ has attracted a lot of attention because of its notorious intractability (Hoogeveen et al., 2006). The difference in the performance between our algorithm and the specialized exact algorithms for the $F2||\sum C_j$ is caused by the additional complexity of the overlapping predecessor sets of the trucks in a cross-dock environment. While significant overlapping helps to reduce the computational time (see Figure 3.3), the structures of the two problems are not identical and several of the efficient lower bound procedures and dominance criteria applied for the $F2||\sum C_j$ problem become invalid for our problem.

Table 3.5: Average results for $F2||\sum C_j$ instances

| | CPU (sec) | | No. of nodes (×1000) | |
|---|---|---|---|---|
| No. of jobs | avg | max | avg | max |
| 15 | 0.1 | 1.0 | 1.7 | 10.5 |
| 20 | 26.6 | 404.0 | 144.8 | 2194.1 |
| 25 | 2236.4 | 21199.0 | 8172.8 | 80507.8 |

Finally, we have tested the performance of the lower and upper bound procedures called

at the beginning of the search. Table 3.6 presents the results, where each value is an average over 20 instances with processing times in $U[1, 10]$. $LB^*$ ($UB^*$) denotes the result of the best performing lower (upper) bound procedure at the root node of the search tree. The relative difference $rd(\alpha, \beta)$ between the solution value obtained with a bounding procedure and the optimal solution value is computed as $rd(\alpha, \beta) = (\alpha/\beta - 1) \cdot 100$. The results show that the relative difference between an optimal solution and the $LB^*$ is 32.9% on average, while the relative difference between the $UB^*$ and an optimal solution is only 2% on average.

Table 3.6: Average performance of lower and upper bound procedures

| | | $p = 0.25$ | | $p = 0.5$ | |
|---|---|---|---|---|---|
| $m$ | $n$ | $rd(opt, LB^*)$ | $rd(UB^*, opt)$ | $rd(opt, LB^*)$ | $rd(UB^*, opt)$ |
| 15 | | | | | |
| | 10 | 15.0 | 1.5 | 14.8 | 1.1 |
| | 15 | 33.8 | 1.7 | 29.1 | 2.7 |
| | 20 | 41.8 | 1.3 | 38.6 | 2.3 |
| | 25 | 41.0 | 1.1 | 43.2 | 0.9 |
| | 30 | 39.4 | 0.6 | 49.0 | 0.6 |
| 20 | | | | | |
| | 10 | 11.0 | 1.2 | 13.5 | 0.6 |
| | 15 | 19.0 | 2.1 | 26.1 | 2.0 |
| | 20 | 40.1 | 2.8 | 38.4 | 2.1 |
| | 25 | 51.0 | 1.5 | 44.7 | 2.0 |
| | 30 | 53.9 | 2.1 | 49.1 | 1.5 |
| 25 | | | | | |
| | 10 | 8.4 | 1.3 | 11.2 | 1.0 |
| | 15 | 17.1 | 1.5 | 22.8 | 2.1 |
| | 20 | 39.0 | 2.6 | 31.8 | 3.1 |
| | 25 | 47.3 | 3.3 | 38.9 | 3.1 |
| | 30 | 61.6 | 2.5 | 46.9 | 2.2 |
| 30 | | | | | |
| | 10 | 7.0 | 0.8 | 8.4 | 1.0 |
| | 15 | 14.7 | 2.7 | 19.5 | 3.2 |
| | 20 | 32.4 | 4.3 | 29.4 | 3.1 |
| | 25 | 49.0 | 2.4 | 38.3 | 3.0 |
| | 30 | 56.9 | 3.2 | 45.2 | 2.0 |

To summarize, for this complex problem in a cross-dock environment the results show that our exact approach is able to solve instances of moderate size in a reasonable time.

## 3.7 Conclusions

This paper studies the problem of scheduling inbound and outbound vehicles with the objective of minimizing the total completion time in a single inbound and single outbound dock cross-docking environment with temporary storage. The performance measure improves the cross-dock's responsiveness and is known to lead to stable or balanced utilization of resources, rapid turn-around of vehicles, and reduction of in-process inventory (Rajendran and Ziegler, 1997; Framinan and Leisten, 2003).

Our solution approach is based on the relation of the problem with machine scheduling. Indeed, the problem is a generalization of a well-studied $NP$-hard problem, the two-machine flow shop problem, i.e., $F2||\sum C_j$. We present optimality properties, three lower bound procedures, and propose an exact branch-and-bound algorithm, in which we branch on the new outbound trucks that are appended to a partial outbound truck sequence.

To the best of our knowledge, this is the first exact approach to the problem. The computational experiments show that when the transshipment matrix is dense, the proposed branch-and-bound algorithm can provide proven optimal solutions within seconds for problems with up to 60 outbound trucks. However, if the number of trucks increases and/or the transshipment matrix becomes sparse, the computation time increases rapidly. The results show that the procedure is able to find optimal solutions for moderate size instances of 40-50 trucks within a reasonable time.

While cross-dock facilities with a single inbound and a single outbound dock are rare in practice, the solution approach can be used as a building block for solving more complex problems. In particular, for problems where the truck-to-dock assignment is done prior to truck sequencing. Furthermore, it seems that the upper bound algorithms implemented at the

beginning of the search obtain good solutions. It is thus of interest to study other complex truck-scheduling problems in a cross-dock environment and search for heuristic approaches that are able to provide near-optimal solutions.

# Chapter 4

# Scheduling Trucks in a Cross-Dock with Mixed Service Mode Dock Doors

# Scheduling Trucks in a Cross-Dock with Mixed Service Mode Dock Doors

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**Abstract**

The problem considered in this paper is to schedule inbound and outbound trucks subject to time windows at a multi-door cross-dock in which an intermixed sequence of inbound and outbound trucks can be processed at the dock doors. We focus on operational costs by considering the cost of handling temporarily stored products, as well as the cost of tardiness due to processing outbound trucks after their respective due dates. A mathematical model for optimal solution is derived and an adaptive large neighborhood search heuristic is proposed to compute near-optimal solutions of real size instances. Computational experiments show that the proposed heuristic algorithm can obtain high quality solutions within short computation times.

## 4.1 Introduction

Cross-docking is a process that aims to minimize the inventory-holding function of a warehouse by rapidly consolidating shipments from diverse sources with known destinations and transferring them to outbound trucks such that in-between storing is minimized. Nevertheless, between unloading and loading, products might be stored temporarily (e.g., for a few hours) to wait for a truck to arrive (Vis and Roodbergen, 2008). In a recent survey with 219 respondents, only 20.6% of those currently cross-docking practice *pure* cross-docking, i.e., cross-docking without temporary storage; most respondents first temporarily store products in the warehouse (Saddle Creek Corporation, 2011). Shipments typically spend less than 24 hours at the facility, sometimes less than an hour (Bartholdi III and Gue, 2004).

One may divide cross-docking operations by the handling units (Bartholdi III et al., 2008): in *case-pick cross-docking*, the distributor receives pallets of product and ships cases, or even individual items; while *unit-load cross-docking* is exclusively about processing unit size products. In this paper, we investigate cross-dock systems that are devoted entirely to unit-load handling.

According to Bartholdi III and Gue (2004) the first design decision is the total number of docks. Two types of docks are considered: inbound docks (also called receiving doors), where trucks dock to be unloaded, and outbound docks (also called shipping doors), where trucks dock to collect freight for a specific destination. Studies on cross-docking tend to make the assumption that facilities designed for cross-docking are long, narrow rectangles (I-shape), in which the docks are positioned on two opposite sides of the facility such that one side of the cross-dock is the 'inbound side' where inbound trucks are processed, and another side is the 'outbound side' where outbound trucks are processed, that is, products are moved across the facility (cf. Bartholdi III and Gue, 2000; Van Belle et al., 2012; Bellanger et al., 2013).

In the aforementioned survey, however, only 53.7% of the respondents who cross-dock today use facilities that are specifically designed or set up for cross-docking and just 27.3% of those planning to cross-dock in the near future expect to use such a facility (Saddle Creek Corpo-

ration, 2011). This implies that the typical material flow may not necessary be characterized with an 'I' shape, in which products are moved from dedicated inbound to dedicated outbound docks. Moreover, not distinguishing between dock types is reasonable, since docks are primarily designed for generic trucks, i.e., docks are 'flexible' by design and can be operated in a mixed service mode (cf. Shakeri et al., 2012). A typical dock can be used either as an inbound or as an outbound dock at any time.

However, utilizing dock flexibility may impose complexity from a managerial point of view. Bartholdi III and Gue (2000) report that most terminals conduct inbound and outbound operations, so named because of the type of freight handled. Unloading and loading procedures may involve different equipment and handling requirements may differ substantially. Moreover, requested employee competencies may also differ in the two procedures, a fact that must be taken into consideration especially if the organization opts for flexible or temporary employees. Therefore, when mixed service mode docks are used, the complexity of managerial decisions is likely to increase. On the other hand, flexibility should be designed into every function and operation of the warehouse (Brockmann and Godin, 1997). Hence, it is of interest to investigate the potential of utilizing the inherent flexibility to increase the efficiency and reduce costs.

In this paper, we focus on truck scheduling with the objective to minimize the operational costs, consisting of storage, retrieval and tardiness costs. The cost of storage and retrieval relates to those loads that are placed in the temporary storage area instead of being directly loaded on outbound trucks. The cost of tardiness is considered due to an obvious priority of any cross-dock, that is, to respect the truck due times agreed with the customers.

The contribution of this research is twofold. First, we derive a mathematical model that can be used to obtain optimal solution for instances of moderate size. We show that the problem is $\mathcal{NP}$-hard. Therefore, for large instances, we propose an adaptive large neighborhood search based approach, which is shown to provide good quality solutions. Second, we take into consideration the potential of mixed service mode docks in a cross-docking environment. In particular, we investigate the effect of utilizing a subset of docks in a mixed service mode and

evaluate their impact with respect to the operational costs.

In the next section, we present the related literature. In Section 4.3, we describe the problem. Our model is presented in Section 4.4. Then, in Section 4.5, the proposed heuristic procedure is described. Results of the computational experiments are reported in Section 4.6. Finally, conclusions are presented in Section 4.7.

## 4.2 Literature review

Several papers study cross-docking. For recent comprehensive literature reviews we refer to Agustina et al. (2010), Boysen and Fliedner (2010) and Van Belle et al. (2012). Cross-dock decisions include truck scheduling, assigning trucks to docks and products to locations. In this section, we present literature related to these three areas.

The product-to-location assignment problem emerges in a cross-dock environment since each arriving item has to be assigned to a storage location unless it is immediately transferred to another truck. Several models assume that the products can be kept at the doors with an infinite buffer capacity (Yu and Egbelu, 2008; Vahdani and Zandieh, 2010; Bellanger et al., 2013). This may be a reasonable assumption when the ratio of trucks per door is low. However, in case of a high ratio, products are stored in a separate temporary storage area to facilitate the operations at the doors. This additional handling increases the operational costs. The handling cost has been addressed by Vis and Roodbergen (2008) and Sadykov (2012). Vis and Roodbergen (2008) consider the problem of determining the short-term storage locations such that the total travel distance in the entire facility is minimized with the assumption of prior truck scheduling and truck-to-dock assignment. Sadykov (2012) analyzes a single inbound and single outbound door cross-dock with a temporary storage location and shows that the problem of truck scheduling with the objective to minimize the storage usage is $\mathcal{NP}$-hard in the strong sense even for the case in which each inbound truck carries at most two product types, each outbound truck carries at most one product type, all storage costs are unitary, and the storage

capacity is unlimited.

Truck-to-dock assignment is the problem of assigning inbound and outbound trucks to docks. Note that the truck-to-dock assignment problem is different from the truck scheduling problem, which takes the time dimension into account, while time is not considered in the dock door assignment problem (Konur and Golias, 2013). The paper of Tsui and Chang (1990) is among the first studies on the truck-to-dock assignment problem. The authors present a bilinear objective formulation and provide a local search algorithm. For the same formulation, Tsui and Chang (1992) describe a branch-and-bound solution approach. The paper of Oh et al. (2006) presents the problem in the context of a mail distribution center and proposes a genetic algorithm heuristic. Miao et al. (2009) consider the problem with operational time constraints assuming that trucks are pre-scheduled with hard time windows during which a dock is fully occupied. The investigated problem is $\mathcal{NP}$-hard, hence tabu search and a genetic algorithm are proposed in the paper.

Recently, several studies have addressed the problem from a scheduling point-of-view. The objective function tends to be time-related, such as the minimization of makespan (Boysen and Fliedner, 2010). Many studies focus on cross-docks with a single inbound and a single outbound dock (Yu and Egbelu, 2008; Chen and Lee, 2009; Boysen et al., 2010; Soltani and Sadjadi, 2010; Larbi et al., 2011).

For this configuration, Yu and Egbelu (2008) provide a mixed integer linear programming formulation, assuming unlimited storage capacity in the cross-dock and propose a heuristic method to minimize the makespan. For the same formulation, Vahdani and Zandieh (2010), Boloori Arabani et al. (2011), and Liao et al. (2012) propose metaheuristic approaches. This configuration has been extended by Boloori Arabani et al. (2010) and Boloori Arabani et al. (2012); these studies include time windows and aim to minimize the earliness and tardiness, for which metaheuristic methods are proposed.

Chen and Lee (2009) formulate the minimum makespan truck scheduling problem as a two-machine flow-shop problem, where an operation on the first machine is to unload the inbound

products, while collecting those products and loading them into an outbound truck is considered as an operation on the second machine. The authors assume that processing a truck on the second machine cannot be started unless all the corresponding loads have been unloaded from the trucks that are processed on the first machine. With reduction from the classical two-machine flow-shop problem, Chen and Lee (2009) show that the problem is strongly $\mathcal{NP}$-hard. Moreover, the problem remains strongly $\mathcal{NP}$-hard even if all processing times are unitary.

For the case when temporary storage is forbidden, Soltani and Sadjadi (2010) propose two hybrid metaheuristics, i.e., hybrid simulated annealing and hybrid variable neighborhood search, to solve the truck scheduling problem such that the makespan is minimized.

Boysen et al. (2010) present a novel formulation for the truck scheduling problem in a single inbound, single outbound door cross-dock by assuming that the planning horizon is split into time intervals. They show that the problem is $\mathcal{NP}$-hard even for a restricted case, in which all handling operations per truck are completed within a single time interval, internal transportation time is ignored, and the intermediate buffer is not limited in size.

The above mentioned studies provide insights and may be used as building blocks in an approach to tackle a more general problem. However, cross-docks with a single inbound and single outbound door are rare, thus studies on multi-door systems are necessary (Liao et al., 2013).

The study of Chen and Song (2009) extends the work of Chen and Lee (2009) by considering multiple machines corresponding to doors, and develops a heuristic solution approach. With the objective to minimize makespan, Shakeri et al. (2012) consider a limited number of forklifts inside the multi-door warehouse and address the truck scheduling problem with a heuristic algorithm. A hybrid flow-shop model with the same objective function has been developed for this problem context by Bellanger et al. (2013). The authors present a branch-and-bound method for small and a heuristic for real size instances.

For the case when time windows are considered, finding schedules to minimize earliness and tardiness has been addressed by Li et al. (2004). The authors provide a two-phase parallel

machine formulation and develop two genetic algorithm based methods, one of which uses integer programming to solve an embedded subproblem. An alternative approach is presented for the same problem by Álvarez Pérez et al. (2009) based on tabu search.

Boysen (2010) analyzes the case when no storage is available in the multi-door cross-dock. The proposed bounded dynamic programming and simulated annealing algorithms aim to minimize three objectives, namely flow time, processing time, and tardiness, assuming that the service times over all trucks do not deviate significantly. The time is discretized into fixed-length periods, which are sufficiently long to process an inbound truck. Due to the zero-inventory policy the outbound truck schedule can be easily derived given an inbound truck schedule.

Alpan et al. (2011) and Alpan et al. (2011) analyzed the truck scheduling problem with a focus on operational costs. In particular, the focus is on minimizing the sum of handling cost and outbound truck preemption cost. The authors present a bounded dynamic programming and a heuristic approach, while assuming discretized time.

In this study, we address the truck scheduling problem and extend the paper of Boysen et al. (2010) with the main difference being that we take a direct focus on operational costs and consider a multi-door cross-dock environment. We show that the problem is $\mathcal{NP}$-hard and present a heuristic approach that follows the adaptive large neighborhood search (ALNS) principles (Ropke and Pisinger, 2006a). ALNS has been successfully applied in complex scheduling problems, including vehicle routing problems (Ropke and Pisinger, 2006b; Pisinger and Ropke, 2007; Stenger et al., 2013) and crane scheduling in container terminals (Gharehgozli et al., 2013).

Our study differs from most of the previous studies on truck scheduling in a multi-door cross-dock in the following aspects: (i) the objective is to minimize operation costs, namely, the sum of handling and tardiness costs, and (ii) a set of doors can function in a mixed service mode.

## 4.3 Problem description

The considered cross-dock has multiple docks, of which some are inbound, some are outbound, and some can operate in a mixed service mode. Let $D_I$, $D_O$, and $D_F$ denote the number of docks of these three types, respectively. Particular origins and destinations are not pre-assigned to docks; therefore peak loads can be absorbed by using multiple docks and trucks for any of the origins and destinations.

We consider two sets of trucks, namely a set $\mathcal{I}$ of inbound and a set $\mathcal{O}$ of outbound trucks, both carrying unit size loads (e.g., pallets or roll cages). We assume that for each truck complete information is available about the loads, and that the loads have been pre-sorted (also called pre-distributed, cf. Yan and Tang, 2009), i.e., each incoming unit load is assigned to a single outbound truck prior to arrival. This implies that the relations between the trucks can be expressed with a $|\mathcal{I}| \times |\mathcal{O}|$ transshipment matrix, where element $f_{ij}$ corresponds to the number of loads to be moved from truck $i \in \mathcal{I}$ to truck $j \in \mathcal{O}$.

When an inbound truck is docked, all its products are unloaded. If a product cannot be loaded directly on the assigned outbound truck, then it must be stored in a temporary storage area and a given handling cost, $\beta$ occurs per unit load. We assume that the capacity of the temporary storage area is unlimited and $\beta$ is identical for all loads. Such a handling cost includes both administrative as well as internal transportation costs. Administrative costs may cover the cost of assigning a temporary storage location to a product and recording it. Transportation cost may cover the cost of transporting a product to an assigned temporary storage location, stowing it, then retrieving the product and transporting it to a dock. In cross-docking environments, a closest–open–location storage policy is often followed, where the first empty location that is encountered by the employee is used to store the product. In this case, the transportation cost associated with the random storage policy can be used as an appropriate approximation (De Koster et al., 2007).

A time window is given in terms of arrival time, $r_i$ and due time, $d_i$ for each truck $i \in \{\mathcal{I}, \mathcal{O}\}$. Since with continuous time there are infinite possibilities of when to assign a truck to a dock,

the time will be divided into time intervals. Let $\mathcal{T}$ denote the set of time intervals. The determination of the time interval length is driven by the expected length of loading and unloading operations (cf. Boysen et al., 2010; Boysen, 2010). We assume that for each truck the processing takes the same amount of time, and all scheduled transshipment operations are completed in one time interval with sufficient workforce. This can be seen as a reasonable approximation of reality, whenever the number of loads per vehicle does not vary strongly (Boysen, 2010).

Preemption of a docked truck is not allowed. That is, once the processing of a truck begins the truck is kept at the dock until its completion. Trucks can be processed in a single time interval, hence when an inbound truck is docked it is completed within the same time interval. However, the completion time of an outbound truck depends on the load arrivals. In particular, the processing of an outbound truck can be completed only after each load for that truck has been unloaded from a set of inbound trucks. Whenever the loading of an outbound truck is completed after its due time, the total cost increases due to the tardiness. We assume that the cost $\gamma$ of tardiness per time interval is known. Such cost depends on the agreed due time of an outbound truck, which may be determined internally, driven by the working hours and internal schedule of the cross-dock, or externally, e.g., by the customer. Moreover, the cost per time unit may include other factors, e.g., overtime hours, an agreement with specified indemnity, or an estimated loss in value for the customer. Tardy completion of an outbound truck may lead to tardy delivery to the customer, i.e., poor service performance, which may result in loss of goodwill for the cross-dock. In this study, we assume that $\gamma$ is the same for all trucks. However, including different values is easily achievable in the proposed model.

Our purpose is to find a schedule both for the inbound and outbound trucks so that the sum of handling and tardiness costs is minimized. The objective function is a trade-off between the two considered costs. We have the following decisions to make at each time interval: (i) select some outbound trucks to be processed, (ii) select some inbound trucks to be processed.

Figure 4.1 presents a cross-dock example with six docks, three of which are currently used

by inbound trucks and three are used by outbound trucks. The temporary storage area is presented as a shaded rectangle. The transportation flow is indicated with dotted lines.

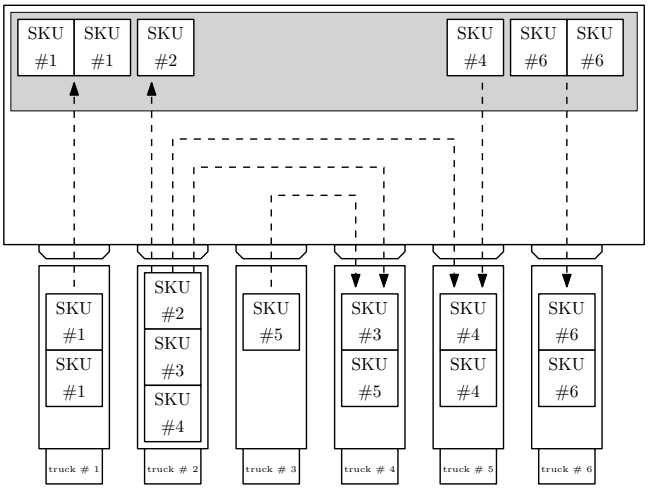



Figure 4.1: A representation of material flow in a cross-dock with temporary storage area

When docks are mainly used for outbound flows, then the cost of handling is reduced since unit loads are likely to be transported directly to the outbound trucks. However, in this case only a few docks are available for inbound trucks and this may result in a bottleneck for the inbound flow. As a consequence, outbound trucks may be completed late and thus the total cost of tardiness may increase.

When docks are mainly used for inbound flows, then the unit loads may have to be stored temporarily, and thus handled twice. That is, the total cost of handling increases. Nevertheless, products 'flow' into the cross-dock rapidly, which may provide increased flexibility for the scheduling of the outbound trucks which can result in lower total tardiness cost.

## 4.4 Model formulation

A mathematical programming model can be formulated to the problem, which is referred to as the full problem (FP). The following notations are used:

$\mathcal{I}$ set of inbound trucks

$\mathcal{O}$ set of outbound trucks

$\mathcal{T}$ set of time intervals

$D_I$ number of dedicated inbound docks

$D_O$ number of dedicated outbound docks

$D_F$ number of flexible docks

$f_{ij}$ number of unit loads on $i \in \mathcal{I}$ associated with $j \in \mathcal{O}$

$r_i$ arrival time of truck $i \in \{\mathcal{I}, \mathcal{O}\}$

$d_i$ due time of truck $i \in \{\mathcal{I}, \mathcal{O}\}$

$\beta$ handling cost per unit load

$\gamma$ tardiness cost per time interval for outbound trucks

Additionally, the following notations are used to simplify the formulation:

$\delta_j^t$ is computed as $\delta_j^t = \max\{0, t - d_j\},\ \forall j \in \mathcal{O}, t \in \mathcal{T}$,

$\mathcal{T}_i^E$ set of time intervals before truck $i$ arrives, i.e., $\mathcal{T}_i^E = \{t | t \in \mathcal{T}, t < r_i\},\ \forall i \in \{\mathcal{I}, \mathcal{O}\}$

$\mathcal{T}_i^L$ set of time intervals after the due time of truck $i$, i.e., $\mathcal{T}_i^L = \{t | t \in \mathcal{T}, d_i < t\},\ \forall i \in \{\mathcal{I}, \mathcal{O}\}$

Variables:

$x_i^t$ is 1 if inbound truck $i \in \mathcal{I}$ is processed at time interval $t \in \mathcal{T}$ and 0 otherwise

$y_j^t$ is 1 if outbound truck $j \in \mathcal{O}$ is processed at time interval $t \in \mathcal{T}$ and 0 otherwise

$\hat{y}_j^t$ is 1 if the loading of outbound truck $j \in \mathcal{O}$ is completed at $t \in \mathcal{T}$ and 0 otherwise

$s_j^t$ total number of units in the storage area for $j \in \mathcal{O}$ at (the end of) $t \in \mathcal{T}$

$l_j^t$ total number of units loaded on $j \in \mathcal{O}$ at (the end of) $t \in \mathcal{T}$

$\hat{s}_i^t$ increment in the number of units stored in the storage area for $i \in \mathcal{O}$ at $t \in \mathcal{T}$, when compared to the previous time interval

The objective of the presented problem is to minimize the total cost, which is defined as

the sum of handling and tardiness costs. The formulation of the FP is the following:

$$\min \sum_{t\in\mathcal{T}} \sum_{j\in\mathcal{O}} \left(\beta \hat{s}_j^t + \gamma \delta_j^t \hat{y}_j^t\right) \tag{4.1}$$

subject to

$$\sum_{i\in\mathcal{I}} x_i^t + \sum_{j\in\mathcal{O}} y_j^t \leq D_I + D_F + D_O \qquad \forall t \in \mathcal{T} \tag{4.2}$$

$$\sum_{i\in\mathcal{I}} x_i^t \leq D_I + D_F \qquad \forall t \in \mathcal{T} \tag{4.3}$$

$$\sum_{j\in\mathcal{O}} y_j^t \leq D_F + D_O \qquad \forall t \in \mathcal{T} \tag{4.4}$$

$$\sum_{t=r_i}^{d_i} x_i^t = 1 \qquad \forall i \in \mathcal{I} \tag{4.5}$$

$$\sum_{t\in\mathcal{T}} \hat{y}_j^t = 1 \qquad \forall j \in \mathcal{O} \tag{4.6}$$

$$s_j^{t-1} + l_j^{t-1} + \sum_{i\in\mathcal{I}} x_i^{t-1} f_{ij} = s_j^t + l_j^t \qquad \forall j \in \mathcal{O}, \quad t \in \mathcal{T} \tag{4.7}$$

$$l_j^t - l_j^{t-1} \leq \sum_{i\in\mathcal{I}} f_{ij} y_j^t \qquad \forall j \in \mathcal{O}, \quad t \in \mathcal{T} \tag{4.8}$$

$$l_j^{t-1} \leq l_j^t \qquad \forall j \in \mathcal{O}, \quad t \in \mathcal{T} \tag{4.9}$$

$$y_j^t \leq \hat{y}_j^t + y_j^{t+1} \qquad \forall j \in \mathcal{O}, \quad t \in \mathcal{T} \tag{4.10}$$

$$\sum_{i\in\mathcal{I}} f_{ij} \hat{y}_j^t \leq l_j^t \qquad \forall j \in \mathcal{O}, \quad t \in \mathcal{T} \tag{4.11}$$

$$s_j^t - s_j^{t-1} \leq \hat{s}_j^t \qquad \forall j \in \mathcal{O}, \quad t \in \mathcal{T} \tag{4.12}$$

$$x_i^t = 0 \qquad \forall i \in \mathcal{I}, \quad t \in \{\mathcal{T}_i^E, \mathcal{T}_i^L\} \tag{4.13}$$

$$y_j^t = 0 \qquad \forall j \in \mathcal{O}, \quad t \in \mathcal{T}_i^E \tag{4.14}$$

$$x_i^t \in \{0,1\} \qquad \forall i \in \mathcal{I}, \quad t \in \mathcal{T} \tag{4.15}$$

$$y_j^t, \hat{y}_j^t \in \{0,1\} \qquad \forall j \in \mathcal{O}, \quad t \in \mathcal{T} \tag{4.16}$$

$$\hat{s}_j^t, s_j^t, l_j^t \in \mathbb{Z}^+ \qquad \forall j \in \mathcal{O}, \quad t \in \mathcal{T} \tag{4.17}$$

Constraints (4.2) ensure that the maximum number of docked trucks at any time interval is at most the number of docks. Constraints (4.3) ensure that the number of inbound trucks docked at any time interval is at most the number of docks that can be used to process inbound trucks. Similarly, constraints (4.4) ensure that the maximum number of outbound trucks docked at a given time interval $t$ is at most the number of docks that can be used to process outbound trucks. Constraints (4.5) ensure that each inbound truck is unloaded exactly once. This also implies that inbound trucks cannot be used as temporary storage locations. Constraints (4.6) ensure that all outbound trucks must be completed eventually. Constraints (4.7) ensure that items that are unloaded from an inbound truck are either stored at the temporary storage area or loaded on the associated outbound truck. We note that if $\beta > 0$ then at most one of $s_j^t$ and $l_j^t$ is non-zero due to the objective function. Constraints (4.8) ensure that whenever an outbound truck is loaded it must be docked. Constraints (4.9) ensure that the number of unit loads on any outbound truck is non-decreasing. Constraints (4.10) ensure that the outbound trucks are kept at the outbound docks until they are completed. Constraints (4.11) ensure that if an outbound truck is completed, then all the unit loads for that truck are loaded. Constraints (4.12) ensure that the increment of units for outbound truck $j$ in the temporary storage is computed as the (non-negative) difference of quantities stored in the temporary storage area. Constraints (4.13) and (4.14) ensure that no truck can be docked before its arrival time, and no inbound truck can be docked after its due time. Finally, constraints (4.15)–(4.17) specify domains of decision and auxiliary variables.

The formulation can be tightened by ensuring that once an outbound truck is completed it is not docked again:

$$\sum_{t'=1}^{t-1} \hat{y}_j^{t'} + y_j^t \leq 1 \qquad \forall j \in \mathcal{O}, \quad t \in \mathcal{T} \tag{4.18}$$

We note that one can easily extend the model to express different tardiness costs per trucks, for example, by replacing the expression $\gamma\delta_j^t$ in (4.1) with arbitrary $\gamma_j^t$, where $\gamma_j^t$ is the tardiness cost of truck $j \in \mathcal{O}$ when it is completed at time $t$.

Boysen and Fliedner (2010) have introduced a 3-field notation $[a|b|c]$ for cross-docking scheduling problems, where $a$ is door environment, $b$ is the operational characteristics, and $c$ is the objective. The notation uses square brackets in order to clearly distinguish the considered cross-docking problem from problems in a classical scheduling environment. Accordingly, our problem can also be denoted as $[\mathrm{EM}|r_i, p_i = 1, d_i, t_p = 0|\sum T_i + \sum S_p]$ , where EM indicates that a subset of docks is operated in an exclusive (or dedicated) service mode and another subset in a mixed service mode, $r_i$ is the arrival time, $p_i$ is the processing time of, and $d_i$ is the due time of truck $i \in \{\mathcal{I}, \mathcal{O}\}$. Furthermore, $t_p$ denotes the transshipment time inside the cross-dock, $\sum T_i$ is the total tardiness, and $\sum S_p$ is the total number of loads placed in the temporary storage.

**Proposition 2.** $[\mathrm{EM}|r_i, p_i = 1, d_i, t_p = 0|\sum T_i + \sum S_p]$ *is strongly $\mathcal{NP}$-hard.*

*Proof.* We transform instances of $F2|CD, p_{ik} = 1|C_{max}$, which is known to be strongly $\mathcal{NP}$-hard (Chen and Lee, 2009), to $[\mathrm{EM}|r_i, p_i = 1, d_i, t_p = 0|\sum T_i + \sum S_p]$ . We restate $F2|CD, p_{ik} = 1|C_{max}$ in the following.

> $F2|CD, p_{ik} = 1|C_{max}$ Problem: There are two machines, $M_1$ and $M_2$, and two sets of jobs, $J^1 = \{J_{11}, J_{21}, \ldots, J_{n1}\}$ and $J^2 = \{J_{12}, J_{22}, \ldots, J_{m2}\}$, where each $J_{i1}, i = 1, \ldots, n$ must be processed on $M_1$, and requires a processing time of $p_{i1} = 1$, and each $J_{j2}, j = 1, \ldots, m$, must be processed on $M_2$ and requires a processing time of $p_{j2} = 1$. That is, $p_{i1} = p_{j2} = 1$ for all $i$ and $j$. For each $J_{j2}$ in $J^2$, there is a corresponding subset of $J^1$, call it $S_j$, such that $J_{j2}$ can be processed on $M_2$ only after all jobs in $S_j$ have been completed on $M_1$. We search for a sequence of jobs in $J^1$ on $M_1$ and of jobs in $J^2$ on $M_2$ such that the makespan, i.e., the time between the start time of processing the first job on $M^1$ and the completion time of the last job on $M^2$, is minimized.
>
> Given an instance of $F2|CD, p_{ik} = 1|C_{max}$ and a positive integer $M$, the question we ask is "can we find a feasible schedule $F2|CD, p_{ik} = 1|C_{max}$ with a makespan no larger than $M$?"

Now, we construct an instance of $[\text{EM}|r_i, p_i = 1, d_i, t_p = 0|\sum T_i + \sum S_p]$ as follows. First, we assume a single inbound and single outbound door cross-dock facility, i.e., $D_I = D_O = 1$ and $D_F = 0$. Moreover, $\beta = 0$ and $r_i = 0\ \forall i \in \{\mathcal{I}, \mathcal{O}\}$.

Corresponding to each job $J_{i1}$ in $F2|CD, p_{ik} = 1|C_{max}$ we construct a job $i$ in $\mathcal{I}$, thus $|\mathcal{I}| = n$. Corresponding to each job $J_{j2}$ in $F2|CD, p_{ik} = 1|C_{max}$ we construct a job $j$ in $\mathcal{O}$, thus $|\mathcal{I}| = m$. For each pair of jobs $i = 1, \ldots, n$ and $j = 1, \ldots, m$, we have that $f_{ij} = 1$ if $J_{i1} \in S_j$ and 0 otherwise.

Furthermore, let $\gamma = 1$, $|\mathcal{T}| \geq \max\{M, |J^1| + |J^2|\}$, $d_i = |\mathcal{T}|\ \forall i \in \mathcal{I}$ and $d_j = M\ \forall j \in \mathcal{O}$. That is, the time window of any inbound truck $i \in \mathcal{I}$ spans the whole planning horizon, and each outbound truck has the same due date at time $M$.

The question we ask is "can we find a feasible schedule to our problem $[\text{EM}|r_i, p_i = 1, d_i, t_p = 0|\sum T_i + \sum S_p]$ with a total cost not greater than 0?"

As the length of such an $[\text{EM}|r_i, p_i = 1, d_i, t_p = 0|\sum T_i + \sum S_p]$ instance is polynomially bounded in $|J^1| \cdot |J^2|$, it can be readily generated on the basis of any instance of $F2|CD, p_{ik} = 1|C_{max}$ in polynomial time.

$\rightarrow$ A solution of any YES-instance of $F2|CD, p_{ik} = 1|C_{max}$, i.e., for which a feasible schedule exists with makespan no greater than $M$, is also feasible for $[\text{EM}|r_i, p_i = 1, d_i, t_p = 0|\sum T_i + \sum S_p]$ with total cost not greater than 0, since each truck is completed at time $M$ at the latest.

$\leftarrow$ Assume that there exists a feasible YES-instance of $[\text{EM}|r_i, p_i = 1, d_i, t_p = 0|\sum T_i + \sum S_p]$ (generated as described before), i.e., the total cost is not greater than 0. In such a schedule, some outbound trucks can be docked for more than one time interval; let $\hat{\mathcal{O}}$ denote the set of such trucks, $\hat{\mathcal{O}} \subseteq \mathcal{O}$. Since $\beta = 0$, the total cost depends only on the completion time of the trucks; therefore, we can delay the start time of docking any $j \in \hat{\mathcal{O}}$ until the time interval when $j$ is completed without changing the total cost. Using this delay operation on all the trucks in $\hat{\mathcal{O}}$ we obtain a schedule, in which each truck is processed only in one time interval. The new schedule is still feasible for $[\text{EM}|r_i, p_i = 1, d_i, t_p = 0|\sum T_i + \sum S_p]$ without changing

the total cost. Recall that $d_j = M,\ \forall j \in \mathcal{O}$. The obtained schedule leads to a corresponding $F2|CD, p_{ik} = 1|C_{max}$, with a makespan no greater than $M$.

We can conclude that a solution with a total cost not greater than 0 exists if and only if the corresponding instance of $[\text{EM}|r_i, p_i = 1, d_i, t_p = 0|\sum T_i + \sum S_p]$ is a YES-instance. $\mathcal{NP}$-hardness of our problem immediately follows. □

The $\mathcal{NP}$-hardness of our problem implies that the time requirement of an exact approach to obtain an optimal solution increases exponentially as the size of the problem increases (unless $\mathcal{P} = \mathcal{NP}$), hence exact algorithms are likely to be inefficient when the number of trucks is large. Since we are interested in real-size instances, it is appropriate to develop algorithms which give near-optimal solutions. Therefore, we propose a heuristic approach, which is explained in the following section.

## 4.5 Metaheuristic approach

In order to find a good feasible solution for real-size instances, a heuristic has been developed that follows the principles of Adaptive Large Neighborhood Search (Ropke and Pisinger, 2006a,b; Pisinger and Ropke, 2007). As in Ropke and Pisinger (2006b), the master level search framework is based on simulated annealing (SA). In the following, we first outline the framework of the proposed approach. Second, we present a heuristic method to construct initial feasible solutions. Then we define a schedule representation, which is used in the iterations of an improvement heuristic. Finally, we describe the neighborhood operators, which are used to modify the schedule in order to obtain improvements, along with a reduced problem formulation to obtain a complete candidate solution, the acceptance of which is based on the SA principles. The pseudo code of the heuristic is presented below in Algorithm 2.

The heuristic operates as follows. First, a set of feasible solutions, $X$ is created. A solution consists of an inbound truck schedule $\pi$ and an outbound truck schedule $\sigma$. Let $z(x)$ denote the objective value of solution $x$. From $X$, we select a solution with the lowest objective value

and record it as the best found solution, $x^*$. Then, the heuristic iterates until the terminating criterion is met.

In each iteration, the algorithm tries to find a solution with a lower objective value by first changing a part of the solution, namely, $\pi$ for which a set $N$ of neighborhood operators has been developed. In each iteration, a neighborhood operator $n(\cdot) \in N$ is selected in a probabilistic manner, such that the better performance a neighborhood operator had previously, the more likely it is to be selected in the successive iterations. This is achieved by using a *roulette wheel selection* principle, where each $n \in N$ has a probability corresponding to its weight $\mu_n$. In particular, $n \in N$ is selected with probability $\mu_n / \sum_{i=1}^{|N|} \mu_i$. Then, an inbound truck schedule $\pi'$ is selected from the neighborhood $n(\pi)$.

Once $\pi'$ is given, the problem to obtain a new feasible solution $x'$ reduces to decide when each outbound truck should be first docked since preemption is not allowed. We call the resulting model a restricted problem, RP. The main difference between RP and FP, is that in RP we search for an $\sigma$ that minimizes the total cost given a $\pi$.

The current solution is updated following the SA principles, which have two key parameters, namely, a temperature $T$ and a cooling ratio $r$. A new feasible solution $x'$ is always accepted if its objective value is better than that of $x$. Otherwise, $x'$ replaces $x$ with a probability of $e^{-(z(x')-z(x^*))/T}$. Following Ropke and Pisinger (2006a), the temperature is initialized as $T := -w \cdot z(x^*) / \ln 0.5$ where $w$ is an arbitrary parameter and $x^*$ is the best found solution, i.e., a solution that is $w\%$ worse than $z(x^*)$ is accepted with probability 0.5 (cf. Kovacs et al., 2012). Then $T$ is updated using the recursion $T := rT$ with $0 < r < 1$. Furthermore, if within the last $n_T$ consecutive iterations the current solution has not been improved, then we apply a diversification strategy to direct the search into other regions of the solution space (cf. Stenger et al., 2013). In particular, we reset the temperature $T$ such that its new value is computed the same way as the initial temperature. Finally, we set the terminating criterion such that a total of 5,000 iterations are executed.

**Algorithm 2** Pseudo code of the implemented heuristic

Construct a set of feasible initial solutions $X$
Choose a $x^* \in X$ such that $z(x^*) \leq z(x_0)\ \forall x_0 \in X$
Set the current solution $x \leftarrow x^*$
Construct an inbound truck schedule $\pi$ corresponding to $x$

**until** stop criterion is met **do**
    Choose a neighborhood operator, $n() \in N$ based on adaptive weights
    Choose an inbound truck schedule, $\pi' \in n(\pi)$
    Apply RP to $\pi'$, resulting in $x'$
    **if** $z(x') < z(x^*)$ **then**
        Set $x^* \leftarrow x'$
    **end if**
    **if** $z(x') \leq z(x)$ **then**
        Set $\pi \leftarrow \pi'$, $x \leftarrow x'$
    **else if** Uniform$[0; 1) \leq e^{-(z(x')-z(x^*))/T}$ **then**
        Set $\pi \leftarrow \pi'$, $x \leftarrow x'$
    **end if**
    Update $T$
    Update the score of $n()$
**repeat**

**return** $x^*$

### 4.5.1 Constructing an initial feasible solution

In this section we propose a heuristic algorithm that follows the principles presented in Yu and Egbelu (2008) and can be used to create feasible solutions. The same set of assumptions holds in this algorithm as in the FP.

The procedure iterates through the time intervals in $\mathcal{T}$. In each time interval $t \in \mathcal{T}$ a set of processed inbound trucks $\mathcal{I}^t$ and a set of docked outbound trucks $\mathcal{O}^t$ is created, $\mathcal{I}^t \subseteq \mathcal{I}$ and $\mathcal{O}^t \subseteq \mathcal{O}$. At the beginning of each iteration, we first initialize $\mathcal{O}^t := \{j \mid j \in \mathcal{O}^{t-1}, j \text{ is not completed}\}$. Second, we define $\mathcal{I}^t$, and finally, update $\mathcal{O}^t$.

Defining $\mathcal{I}^t$ is done in two steps. In the first step, a set of candidate inbound trucks, $\mathcal{C}(\mathcal{I})^t$

is created. In particular,

$$\mathcal{C}(\mathcal{I})^t := \{i \mid i \in \mathcal{I}, r_i \leq t \leq d_i, i \notin \cup_{t'=0}^{t'=t-1} \mathcal{I}^{t'}\} \tag{4.19}$$

In the second step, we define $\mathcal{I}^t \subseteq \mathcal{C}(\mathcal{I})^t$ based on decision rules that arise from the following selection strategies: (i) select the truck with the earliest arrival time (FCFS), (ii) select the truck with the earliest due time (EDD), (iii) select the truck with the largest number of units transferred directly to outbound trucks (cf. Yu and Egbelu, 2008), and (iv) select the truck that, when processed, completes the largest number of outbound trucks.

In the third strategy, we assign a weight, $w_i$ for each $i \in \mathcal{C}(\mathcal{I})^t$ to denote the number of units transferred directly to outbound trucks, and set $w_i := \sum_{j \in \mathcal{O}^t} f_{ij} + \Delta_i$, where the first part takes in consideration the already docked outbound trucks, while $\Delta_i$ is based on the set $\mathcal{C}(\mathcal{O})^t$ of available but not yet docked outbound trucks, such that

$$\mathcal{C}(\mathcal{O})^t := \{j \mid j \in \mathcal{O}, r_j \leq t, j \notin \cup_{t'=1}^{t'=t-1} \mathcal{O}^{t'}\} \tag{4.20}$$

Let $S_i^t$ denote the set of the first $D_O + D_F - |\mathcal{O}^t|$ outbound trucks when the elements in $\mathcal{C}(\mathcal{O})^t$ are sorted in non-increasing order of load units received from $i$. We then define $\Delta_i := \sum_{j \in S_i^t} f_{ij}$.

In the fourth strategy, the computation is similar to the previous one, expect that in this case, we are interested in the number of outbound trucks that can be completed when an inbound truck is processed.

Based on these four strategies we apply four decision rules to define $\mathcal{I}^t$ by selecting sequentially, at most $D_I + D_F$ trucks from $\mathcal{C}(\mathcal{I})^t$. The first three decision rules follow the idea of the first three strategies. However, the last one combines the third and fourth strategy with weights $\beta$ and $\gamma$, respectively.

Finally we update $\mathcal{O}^t$. As for the set of inbound trucks, we have also four decision rules to select trucks from $\mathcal{C}(\mathcal{O})^t$. The first two decision rules for outbound trucks are identical to the first two described for the inbound trucks. Similarly, the third rule aims to reduce handling cost by selecting the outbound truck that minimizes the handling cost at time interval $t$ given

$\mathcal{I}^t$. The fourth selection rule is to choose the outbound truck which minimizes the sum of handling and tardiness costs at time $t$, with weights $\beta$ and $\gamma$, respectively.

The number of outbound trucks selected in each time interval is determined such that inbound trucks can be processed before their due time. For this, at the start of the constructive heuristic we let $\pi^t := \{i \mid i \in \mathcal{I}, d_i = t\},\ \forall t \in \mathcal{T}$. Whenever an inbound truck $i$ is processed, we update $\pi^{d_i} := \pi^{d_i} \setminus \{i\}$. Thus, we can limit the maximum number of outbound trucks docked at time $t$ as follows:

$$|\mathcal{O}^t| \leq D_O + D_F - \max\left\{0, \max_{t' \in \mathcal{T}, t \leq t'} \left\{| \pi^{t'} |\right\} - D_I\right\} \qquad \forall t \in \mathcal{T} \tag{4.21}$$

The constructive heuristic algorithm returns the best feasible solution found when applying the total of 16 combinations of the decision rules.

### 4.5.2 Representation of the inbound truck schedule

Since the constructive heuristic may yield a suboptimal solution, another heuristic algorithm is used to improve on the solution quality. The time requirement of the search for an improved solution depends heavily on the representation of the inbound truck schedule $\pi$, which is modified in each iteration, hence, a compact representation is proposed.

The representation of $\pi$ follows the idea presented by Fu et al. (2005), i.e., the sequence of trucks is represented by a tuple where the sets of trucks processed in a time interval are separated by a symbol. In our case, this symbol represents the end of a time interval and is **0**.

Figure 4.2 shows an example for six inbound trucks and five time intervals. The sequence represents a $\pi$ according to which inbound truck 1 is processed in the first time interval, inbound trucks 2 and 3 in the second time interval, and inbound trucks 4, 5, and 6 are processed in the fourth time interval.

| 1 | **0** | 2 | 3 | **0** | **0** | 4 | 5 | 6 | **0** | **0** |
|---|---|---|---|---|---|---|---|---|---|---|

Figure 4.2: Inbound truck schedule representation

Note that if no inbound truck is docked in a time interval, then two **0** values are consecutive in the sequence. For example, in Figure 4.2 we see that during time intervals 3 and 5 no inbound truck is processed.

### 4.5.3 Neighborhood operators

The performance of the algorithm depends on the efficiency of the neighborhood operators, which are designed to introduce moves from one solution to another. Position-based neighborhoods are commonly used for permutations that represent scheduling problems. Therefore, several position-based neighborhood operators were selected for this problem.

Following the principles of the ALNS, the neighborhood operators consist of destroy and repair (also called ruin and recreate) operators, so called because operators in the first set create an infeasible solution, e.g., some inbound trucks are removed from their time interval, while the repair operators take the infeasible solution and rectify it, i.e., the removed truck is reinserted in a time interval.

The following destroy procedures are used:

- rRnd: A randomly selected truck is removed from the schedule. All trucks in the schedule have an equal probability of being selected. This is a standard procedure in a variable neighborhood search.

- rMax: We identify a time interval with the highest number of trucks processed simultaneously (call it *maxtime*) and randomly select a truck in it, which is then removed from the schedule. This procedure aims to reduce the bottlenecks in the material flow by reducing the maximum number inbound trucks processed simultaneously at any time interval, so that some additional doors may become available to process outbound trucks.

- rCrt: A truck is selected based on a *critical ratio*, that is, a value is assigned to each truck and applying the roulette wheel selection principle one truck is selected to be rescheduled in either an earlier or a later time interval. In particular, assuming that the truck is

to be rescheduled earlier, the critical ratio $cr_i^{\leftarrow}$ of truck $i \in \mathcal{I}$, is computed as follows, $cr_i^{\leftarrow} = \sum_{j \in S_i} \max\{0, \lambda_i - d_j\}$, where $\lambda_i$ is the time interval that truck $i$ is currently scheduled in, and $S_i$ is the set of outbound trucks that receive products from truck $i$, i.e., $S_i := \{j \mid j \in \mathcal{O}, f_{ij} > 0\}$. That is, the probability of rescheduling earlier inbound truck $i$ increases with each outbound truck $j$ that is served by $i$ and $d_j < \lambda_i$, where the increase depends on the difference between $d_j$ and $\lambda_i$. When an inbound truck is to be scheduled later, the critical ratio is $cr_i^{\rightarrow} = \sum_{j \in S_i} \max\{0, d_j - \lambda_i\}$ for each $i \in \mathcal{I}$.

We use the following repair procedures:

- iBck: A truck is reinserted in an earlier time interval, i.e., backward reinsertion.
- iFrw: A truck is reinserted in a later time interval, i.e., forward reinsertion.
- iSwp: Swap two trucks.

Instead of assigning a weight to each of the aforementioned procedures, we assign the weight to pairs, hence a neighborhood operator $n \in N$ is a combination of a removal procedure and an insertion procedure. This approach allows us to develop complex neighborhood operators.

We can use the information about the current solution, which consists of an inbound truck schedule $\pi$ and an outbound truck schedule $\sigma$. The inbound truck schedule can be changed so as to facilitate the direct loading of outbound trucks. In particular, we have developed a neighborhood operator, referred to as riI2O, that can be applied when there exists a time interval $t$ in which a (non-empty) set $\mathcal{I}^t$ of inbound trucks is scheduled, while no outbound truck is docked in the next time interval, i.e., each unit load arriving at $t$ must be stored temporarily. In this case, we select an inbound truck $i \in \mathcal{I}^t$ and reschedule it to another time interval $t'$, such that in the current solution at time $t' + 1$ there is at least one docked outbound truck $j$, with $f_{ij} > 0$.

Finally, we develop a neighborhood operator structure that works iteratively. It starts by removing an inbound truck $i$ and trying to reinsert it to a different time interval, either earlier

(riChnBck) or later (riChnFrw). Assume for now that a removed truck $i$ is reinserted only in a later time interval. If that time interval $t = d_i$, then truck $i$ is reinserted into $t$. Otherwise, if at $t$ another $i' \in \mathcal{I}, i \neq i'$ is scheduled and $i'$ can be rescheduled later, then truck $i$ is reinserted into $t$, and truck $i'$ is removed from the schedule; we may then reindex $i'$ to $i$ and the procedure continues recursively. Else, $t + 1$ is investigated to decide whether truck $i$ should be reinserted in it. We note that this neighborhood operator can provide a new inbound truck schedule that is also obtainable with multiple random removal and reinsertion pairs. However, this operator is able to both diversify the search and provide solutions, which are unlikely to be reached with simple neighborhood operators.

Consider an example with a total of three docks available to process inbound trucks and suppose that the current inbound truck schedule is the one presented in Figure 4.2. Assume that inbound truck 3, with $d_3 = 5$ is selected randomly and removed from the second time interval. Then, another time interval $t = 4$ is selected, in which truck 4 with $d_4 = 5$ is currently scheduled. We reinsert truck 3 at $t = 4$ and remove truck 4, hence the total number of inbound trucks processed in that time interval is three, which is the maximum number of inbound trucks that can be processed simultaneously. Then, another time interval $t = 5$ is selected, in which truck 4 is rescheduled. The starting and the new inbound truck schedule are presented on Figure 4.3 with arrows indicating the changes.

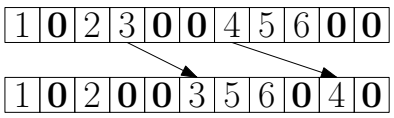



Figure 4.3: Example of neighborhood operator

Table 4.4 presents the total of 11 neighborhood operators that are applied in the algorithm. Should one of the neighborhood operators create an infeasible inbound truck schedule, then the move is forbidden and a new operator is selected.

At the beginning of the procedure, we weight all neighborhood operators equally; in particular, the initial weight is 1 for each operator (cf. Ropke and Pisinger, 2006a). During the

Table 4.4: Neighborhood operators

| Operator | Short description |
|---|---|
| rRnd & iBck | random removal and backward insertion |
| rRnd & iFrw | random removal and forward insertion |
| rRnd & iSwp | two random trucks are exchanged |
| rMax & iBck | random removal from a maxtime and backward insertion |
| rMax & iFrw | random removal from a maxtime and forward insertion |
| rCrt & iBck | removal based on critical ratio and backward insertion |
| rCrt & iFrw | removal based on critical ratio and forward insertion |
| rCrt & iSwp | two trucks are selected based on critical ratio and then exchanged |
| riI2O | an inbound truck is reinserted to facilitate direct loading |
| riChnBck | iterative removal and backward insertion |
| riChnFrw | iterative removal and forward insertion |

procedure, the weights $\mu_n$ are updated based on their respective performances, for all $n \in N$. Let $\xi_n$ be the score assigned to $n \in N$ with an initial value of 0. After a neighborhood operator $n$ modified the current solution, its score is updated as follows (cf. Stenger et al., 2013):

$$\xi_n := \begin{cases} \xi_n + \delta^1 & \text{if } n \in N \text{ yielded a solution that improved the global best solution} \\ \xi_n + \delta^2 & \text{if } n \in N \text{ yielded a solution that improved the current solution,} \\ & \text{but not the global} \\ \xi_n + \delta^3 & \text{otherwise,} \end{cases} \tag{4.22}$$

where $\delta^1$, $\delta^2$, and $\delta^3$ are user defined parameters. Following Ropke and Pisinger (2006a), the weights are updated periodically, i.e., every 100 iterations. Let $\theta_n$ denote the number of times $n \in N$ modified the current solution in the last 100 iterations, and $\rho \in (0, 1)$ denote the parameter called the *reaction factor*. The weights are updated as follows, if $\theta_n = 0$ then $\mu_n$ is unchanged, otherwise $\mu_n := (1 - \rho)\mu_n + \rho(\xi_n/\theta_n)$. Then, $\xi_n$ and $\theta_n$ are reset to 0, $\forall n \in N$.

### 4.5.4 The reduced problem

In the reduced problem (RP), we consider the problem of determining $\sigma$ given $\pi$, that is, the arrival time of each load is given, as well as the number of docks $D_O^t$ available for outbound

truck processing for all $t \in \mathcal{T}$, where $D_O^t := D_O + D_F - \max\{0, \sum_{i\in\mathcal{I}} x_i^t - D_I\}, \forall t \in \mathcal{T}$.

Let $\mathcal{S}_j$ denote the set of time intervals when a unit load becomes available for truck $j \in \mathcal{O}$, clearly $\mathcal{S}_j \subseteq \mathcal{T}$. Furthermore, let $\alpha_j = \min_{t\in\mathcal{S}_j}\{t\}$ and $\omega_j = \max_{t\in\mathcal{S}_j}\{t\}$, i.e., $\alpha_j/\omega_j$ is the first/last time interval when a unit load becomes available for $j \in \mathcal{O}$.

We may split the planning time horizon into three parts for each truck $j \in \mathcal{O}$: (i) between $t = 0$ and $\alpha_j - 1$ there is no need to start the processing of $j \in \mathcal{O}$, since no product is available to be loaded on it, (ii) if the docking of $j \in \mathcal{O}$ starts between time $\alpha_j$ and $\omega_j - 1$ then it must stay at the door until time $\omega_j$, and (iii) between $\omega_j$ and $|\mathcal{T}|$ the completion of $j \in \mathcal{O}$ requires only one time interval. Hence, we may focus on the time interval when an outbound truck $j$ is first docked. Thus, let $\tilde{y}_j^t$ be 1 if outbound truck $j \in \mathcal{O}$ is first docked at $t \in \mathcal{T}$ and 0 otherwise.

To express whether an outbound truck $j$ is still docked at time $t$ given that it is first docked at time $t'$, we use the aforementioned observation and define $f_j^{t',t}$ for each $j \in \mathcal{O}$ and $t', t \in \mathcal{T}$. In particular, we set

$$f_j^{t',t} = \begin{cases} 1 & \text{if } t' = t \text{ or } t' < t \leq \omega_j \\ 0 & \text{otherwise} \end{cases} \tag{4.23}$$

When computing the cost, $c_j^t$, incurred when $j \in \mathcal{O}$ is first docked at $t \in \mathcal{T}$, we take in consideration the accumulated cost of tardiness until $j$ is completed and the accumulated cost of handling before $j$ is first docked. Thus, we have that

$$c_j^t = \gamma\delta_j^{\max\{t,\omega_j\}} + \beta \sum_{t'=1}^{t-2} \sum_{i\in\mathcal{I}} x_i^{t'} f_{ij} \quad \forall j \in \mathcal{O}, t \in \mathcal{T} \tag{4.24}$$

Clearly, $c_j^t \leq c_j^{t+1}$, $\forall j \in \mathcal{O}, t \in \mathcal{T}$. The first part of the equation (4.24) defines the tardiness cost, while the second part is based on the observation that if outbound truck $j$ is first docked at time $t$, then each load for $j$ that arrives at time $t-1$ (or later) can be directly loaded, while loads for $j$ that arrive before $t-1$ must be stored temporarily.

A mathematical programming model can be formulated to the RP:

$$\min \sum_{j\in\mathcal{O}} \sum_{t\in\mathcal{T}} c_j^t \tilde{y}_j^t \tag{4.25}$$

subject to

$$\sum_{t\in\mathcal{T}} \tilde{y}_j^t = 1 \qquad \forall j \in \mathcal{O} \tag{4.26}$$

$$\sum_{t'\in\mathcal{T}} \sum_{j\in\mathcal{O}} \tilde{y}_j^{t'} f_j^{t',t} \leq D_O^t \qquad \forall t \in \mathcal{T} \tag{4.27}$$

$$\tilde{y}_j^t \in \{0,1\} \qquad \forall j \in \mathcal{O}, \quad t \in \mathcal{T} \tag{4.28}$$

$$\tilde{y}_j^t = 0 \qquad \forall j \in \mathcal{O}, \quad t \in \mathcal{T}_j^E \tag{4.29}$$

The objective function (4.25) seeks to minimize the total cost. Constraints (4.26) ensure that the processing of each outbound truck is started exactly once. Constraints (4.27) ensure that the number of docked outbound trucks at time $t$ is at most the number of available dock doors for outbound trucks at time $t$. The domain of the decision variables is indicated by (4.28)–(4.29).

Moreover, we tighten the formulation based on the observation that docking $j \in \mathcal{O}$ first in time interval $t < \omega_j$ where $t \notin \mathcal{S}_j$ can be excluded from consideration, since such a solution has the same quality with respect to total costs as the one in which truck $j$ is first docked in time interval $t + 1$. We thus have that

$$\tilde{y}_j^t = 0 \qquad \forall j \in \mathcal{O}, t \in \{t' | t' \in \mathcal{T}, t' < \omega_j \text{ and } t' \notin \mathcal{S}_j\} \tag{4.30}$$

Hence, we ensure that the processing of outbound truck $j$ does not start before $\omega_j$ in a time interval when no product becomes available for $j$.

When solving the reduced problem, we follow the approach presented in the paper of Li et al. (2004), in which the authors propose a metaheuristic method for truck scheduling in a cross-docking environment, which uses integer programming to solve an embedded subproblem. Similarly, we have implemented RP using a commercial solver. Experiments show that a solver such as Gurobi can be used to obtain optimal solution for real size instances in a reasonable time.

## 4.6 Computational experiments

The FP was implemented in Gurobi 5.01, and the ALNS was coded in C++ calling Gurobi 5.01 to solve the RP. The instances were run on a P8700, 2.53 GHz computer with 4GB of RAM. In order to facilitate the comparison, a single thread was used by Gurobi 5.01 in all cases.

With regards to the ALNS, while we aim to keep the user defined parameters as low as possible, the values for the following parameters have to be established: $w$, $r$, $n_T$, $\delta^1$, $\delta^2$, $\delta^3$, and $\rho$. Following Stenger et al. (2013), we use the parameter vector: $(r, n_T, \delta^1, \delta^2, \delta^3, \rho) = (0.95, 200, 9, 3, 1, 0.3)$. Finally, we choose $w = 0.2$. No claim is made that this choice of parameter values is the best possible; however, the setting used generally worked well on our test problems.

### 4.6.1 Generating instances

The time windows were generated following a procedure proposed by Konur and Golias (2013), i.e., a 12-hour shift operation is considered. In each generated instance, a time interval represents a 45-minute interval; therefore the 12 hours are converted to 16 time intervals. First, a random value in the uniform distribution $U[1, 16]$ is generated for each truck and is used as the mid-time window, i.e., $(d_i - r_i)/2$, then the time window is established applying a given time window length (in case $r_i < 1$ or $d_i > 16$ for some truck $i$, then the time window is adjusted such that it falls in $[1, 16]$). Since an outbound truck might be completed after its due time, the total planning horizon considers a longer period. In our experiments we consider a total of 32 time intervals, which is equivalent to 24 hours.

Following Gue (1999), the average number of different destinations per truck has been determined to be within the range $[1, 7]$. In particular, each $|\mathcal{I}| \times |\mathcal{O}|$ transshipment matrix is generated such that the number of destinations for each $i \in \mathcal{I}$ and the number of inbound trucks that deliver products for each $j \in \mathcal{O}$ is taken from one of the two considered uniform distributions, namely, $U[3, 5]$ and $U[5, 7]$. That is, the average number of different destinations

per inbound truck is 4 and 6, respectively, where the actual outbound trucks are selected randomly. For all generated instances, the number of total unit loads per truck is set to 24, which corresponds to the standard trailer load in Western Europe measured in EUR pallets.

In order to facilitate the comparison of different configurations, each generated instance has equal number of inbound and outbound docks, as well as equal numbers of inbound and outbound trucks. In all cases $\gamma = 10$. Moreover, the transshipment matrices and the time windows are generated independently; this enables that (i) the $|\mathcal{I}| \times |\mathcal{O}|$ transshipment matrix is identical for instances with a given combination of number of trucks and average number of shipments per destination, and (ii) the set of time windows is identical for instances with a given combination of a number of trucks and time window length.

### 4.6.2 Numerical results

In this part, we test moderate and large size instances, and provide computational results when they are solved by the commercial software Gurobi and our proposed heuristic algorithm. The results aim to illustrate the sensitivity of solution quality and computation time to the following input characteristics: number of docks, ratio of mixed service mode docks ($D_F$) compared to the total number of docks, number of trucks, average number of shipments per truck, and time windows length (abbreviated as t.w. length). The results are presented in Tables 4.5-4.6 for a total of 192 instances.

A commonly used evaluation metric, the relative difference ($rd$), is adopted for the experiments (cf. Shakeri et al., 2012). Given two objective function values $z_A(x)$ and $z_B(x)$ obtained by applying algorithms A and B, respectively, to solve a problem instance $x$, the $rd$ of A is computed as follows, $rd\ (\%) = (z_A(x)/z_B(x) - 1) \cdot 100$.

Table 4.5 presents the results for instances with only 10 doors in total. Gurobi has found the optimal solution for most of the instances within the time limit of 30 minutes. Our ALNS is always within a 5.36% gap from the best available solution, and within 1% on average, while the average CPU time is less than 30 seconds.

Table 4.5: Numerical results with 10 doors

| $\beta/\gamma$ | 0.1 | | | | | | | | 0.5 | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| t.w. length | 2 | | | | 3 | | | | 2 | | | | 3 | | | |
| | Gurobi | | ALNS | | Gurobi | | ALNS | | Gurobi | | ALNS | | Gurobi | | ALNS | |
| $D_F$ (%) | sol. | CPU | *rd* (%) | CPU | sol. | CPU | *rd* (%) | CPU | sol. | CPU | *rd* (%) | CPU | sol. | CPU | *rd* (%) | CPU |
| *(a) no. of trucks = 30, avg no. of destinations per truck = 4* | | | | | | | | | | | | | | | | |
| 0 | 1010 | 6 | 0.10 | 14 | 397 | 1 | 0.00 | 14 | 1550 | 17 | 0.00 | 14 | 1185 | 1 | 2.11 | 13 |
| 20 | 977 | 4 | 0.00 | 13 | 394 | 1 | 0.00 | 13 | 1440 | 5 | 0.00 | 14 | 1170 | 1 | 1.28 | 13 |
| 40 | 951 | 3 | 0.00 | 13 | 391 | 1 | 0.77 | 12 | 1375 | 5 | 3.64 | 13 | 1155 | 1 | 0.00 | 13 |
| 60 | 925 | 2 | 0.00 | 12 | 391 | 1 | 0.00 | 12 | 1335 | 5 | 0.75 | 13 | 1150 | 1 | 0.43 | 12 |
| 80 | 925 | 2 | 0.00 | 13 | 391 | 1 | 0.00 | 12 | 1310 | 3 | 2.67 | 13 | 1150 | 1 | 0.43 | 12 |
| 100 | 925 | 2 | 0.00 | 12 | 391 | 1 | 0.00 | 12 | 1310 | 3 | 2.67 | 13 | 1150 | 0 | 0.43 | 12 |
| *(b) no. of trucks = 30, avg no. of destinations per truck = 6* | | | | | | | | | | | | | | | | |
| 0 | 1126 | 15 | 0.53 | 14 | 498 | 5 | 0.00 | 13 | 1850 | 35 | 0.54 | 14 | 1355 | 41 | 0.00 | 14 |
| 20 | 1099 | 16 | 0.18 | 15 | 486 | 5 | 0.62 | 13 | 1735 | 45 | 0.00 | 15 | 1310 | 19 | 0.00 | 13 |
| 40 | 1065 | 16 | 0.19 | 14 | 474 | 4 | 0.00 | 13 | 1640 | 46 | 0.00 | 14 | 1255 | 11 | 0.00 | 13 |
| 60 | 1032 | 9 | 0.19 | 13 | 466 | 4 | 0.00 | 13 | 1545 | 29 | 0.65 | 14 | 1210 | 4 | 0.00 | 13 |
| 80 | 1032 | 14 | 0.19 | 13 | 466 | 5 | 0.64 | 13 | 1515 | 30 | 0.66 | 14 | 1210 | 4 | 0.00 | 12 |
| 100 | 1032 | 16 | 0.19 | 14 | 466 | 5 | 0.64 | 12 | 1515 | 63 | 0.66 | 13 | 1210 | 6 | 0.00 | 13 |
| *(c) no. of trucks = 60, avg no. of destinations per truck = 4* | | | | | | | | | | | | | | | | |
| 0 | 1666 | 226 | 0.42 | 40 | 1276 | 134 | 0.47 | 30 | 3200 | 732 | 4.06 | 43 | 2880 | t.l. | 2.08 | 26 |
| 20 | 1597 | 58 | 1.44 | 32 | 1217 | 192 | 0.66 | 24 | 3015 | 739 | 3.81 | 46 | 2650 | 1645 | 2.45 | 27 |
| 40 | 1561 | 163 | 1.28 | 28 | 1201 | 362 | 1.08 | 22 | 2880 | 1157 | 1.39 | 36 | 2530 | 1217 | 0.59 | 28 |
| 60 | 1554 | 99 | 1.03 | 22 | 1191 | 138 | 0.00 | 23 | 2800 | 465 | 5.36 | 26 | 2475 | t.l. | 1.62 | 25 |
| 80 | 1554 | 109 | 0.97 | 22 | 1191 | 139 | 0.00 | 22 | 2800 | 533 | 5.36 | 24 | 2475 | t.l. | 1.62 | 24 |
| 100 | 1554 | 76 | 0.97 | 22 | 1191 | 235 | 0.00 | 22 | 2800 | 452 | 5.36 | 24 | 2475 | t.l. | 1.62 | 25 |
| *(d) no. of trucks = 60, avg no. of destinations per truck = 6* | | | | | | | | | | | | | | | | |
| 0 | 2134 | 1610 | 0.66 | 25 | 1343 | 503 | 1.19 | 27 | 4075 | t.l. | 1.72 | 28 | 3055 | t.l. | 1.31 | 28 |
| 20 | 2026 | 1058 | 0.69 | 25 | 1291 | 917 | 0.70 | 24 | 3870 | t.l. | 0.52 | 33 | 2865 | t.l. | 1.57 | 27 |
| 40 | 1955 | 1294 | 0.15 | 29 | 1257 | 1095 | 0.80 | 24 | 3700 | t.l. | 1.08 | 34 | 2715 | t.l. | 0.92 | 24 |
| 60 | 1943 | t.l. | 0.05 | 23 | 1234 | 804 | 1.13 | 23 | 3600 | t.l. | 2.36 | 27 | 2720 | t.l. | 0.92 | 25 |
| 80 | 1943 | 1798 | 0.15 | 23 | 1234 | 138 | 0.73 | 23 | 3590 | t.l. | 1.95 | 25 | 2725 | t.l. | -0.73 | 23 |
| 100 | 1943 | 970 | 0.10 | 23 | 1234 | 359 | 0.73 | 23 | 3605 | t.l. | 2.22 | 25 | 2710 | t.l. | -0.18 | 23 |

rd (%) refers to the relative difference when the best found solution is compared to the one provided by Gurobi; CPU time is in seconds; t.l. indicates that the search has been terminated due to time limit (i.e., 1800 sec), in which case the reported solution is the best found integer solution

Table 4.6: Numerical results with 20 doors

| $\beta/\gamma$ | 0.1 | | | | | | | | 0.5 | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| t.w. length | 2 | | | | 3 | | | | 2 | | | | 3 | | | |
| | Gurobi | | ALNS | | Gurobi | | ALNS | | Gurobi | | ALNS | | Gurobi | | ALNS | |
| $D_F$ (%) | sol. | CPU | *rd* (%) | CPU | sol. | CPU | *rd* (%) | CPU | sol. | CPU | *rd* (%) | CPU | sol. | CPU | *rd* (%) | CPU |
| *(a) no. of trucks = 60, avg no. of destinations per truck = 4* | | | | | | | | | | | | | | | | |
| 0 | 1452 | 16 | 1.31 | 23 | 1086 | 31 | 1.47 | 21 | 2520 | 47 | 3.57 | 24 | 2130 | 152 | 1.17 | 21 |
| 20 | 1391 | 8 | 0.72 | 20 | 1036 | 11 | 1.54 | 20 | 2365 | 31 | 2.75 | 22 | 1980 | 455 | 6.06 | 20 |
| 40 | 1359 | 4 | 1.25 | 20 | 1003 | 12 | 1.89 | 20 | 2250 | 5 | 2.44 | 20 | 1840 | 176 | 3.80 | 20 |
| 60 | 1356 | 4 | 1.11 | 20 | 987 | 6 | 1.82 | 20 | 2235 | 5 | 3.36 | 20 | 1720 | 53 | 3.49 | 20 |
| 80 | 1355 | 4 | 0.89 | 20 | 987 | 6 | 2.13 | 20 | 2230 | 4 | 3.59 | 20 | 1720 | 73 | 3.20 | 20 |
| 100 | 1355 | 4 | 0.89 | 20 | 987 | 9 | 2.13 | 20 | 2230 | 4 | 3.59 | 20 | 1720 | 51 | 3.20 | 20 |
| *(b) no. of trucks = 60, avg no. of destinations per truck = 6* | | | | | | | | | | | | | | | | |
| 0 | 1808 | 448 | 0.00 | 26 | 1154 | 50 | 1.39 | 21 | 3275 | 1476 | 1.07 | 29 | 2380 | 1381 | 1.89 | 23 |
| 20 | 1757 | 391 | 0.23 | 22 | 1113 | 44 | 0.81 | 21 | 3020 | 1313 | 0.83 | 23 | 2230 | 1466 | 0.90 | 21 |
| 40 | 1697 | 268 | 0.18 | 21 | 1084 | 15 | 0.00 | 21 | 2805 | 579 | 0.71 | 22 | 2095 | 1065 | 0.48 | 21 |
| 60 | 1653 | 189 | 0.42 | 21 | 1064 | 11 | 0.94 | 20 | 2645 | 220 | 1.13 | 22 | 1995 | 138 | 0.50 | 20 |
| 80 | 1646 | 119 | 0.30 | 21 | 1063 | 11 | 1.03 | 20 | 2585 | 104 | 1.55 | 22 | 1965 | 228 | 0.51 | 20 |
| 100 | 1646 | 184 | 0.30 | 21 | 1063 | 11 | 1.03 | 20 | 2585 | 334 | 1.55 | 22 | 1965 | 318 | 0.51 | 21 |
| *(c) no. of trucks = 120, avg no. of destinations per truck = 4* | | | | | | | | | | | | | | | | |
| 0 | 3005 | 94 | 0.77 | 65 | 2555 | t.l. | 0.98 | 53 | 5830 | t.l. | 2.23 | 66 | 5300 | t.l. | 2.36 | 52 |
| 20 | 2905 | 125 | 0.45 | 77 | 2467 | 613 | 0.57 | 45 | 5560 | t.l. | 1.71 | 70 | 4980 | t.l. | 1.71 | 49 |
| 40 | 2810 | 73 | 0.75 | 60 | 2404 | 774 | 1.25 | 41 | 5290 | t.l. | 0.76 | 70 | 4725 | t.l. | 2.01 | 41 |
| 60 | 2768 | 140 | 0.90 | 44 | 2393 | 543 | 1.04 | 40 | 5100 | t.l. | 1.18 | 48 | 4570 | t.l. | 3.83 | 44 |
| 80 | 2768 | 109 | 1.05 | 44 | 2393 | 285 | 0.71 | 40 | 5090 | 1545 | 2.16 | 47 | 4585 | t.l. | 2.73 | 42 |
| 100 | 2768 | 61 | 1.05 | 45 | 2393 | 1471 | 0.71 | 40 | 5090 | 703 | 2.16 | 48 | 4570 | t.l. | 3.06 | 43 |
| *(d) no. of trucks = 120, avg no. of destinations per truck = 6* | | | | | | | | | | | | | | | | |
| 0 | 3869 | t.l. | 0.88 | 44 | 2761 | t.l. | 0.62 | 42 | 7550 | t.l. | 1.46 | 46 | 6005 | t.l. | -0.83 | 47 |
| 20 | 3685 | 1497 | 0.54 | 42 | 2664 | t.l. | 0.19 | 43 | 7165 | t.l. | 1.54 | 45 | 5575 | t.l. | 0.63 | 46 |
| 40 | 3583 | t.l. | 0.31 | 43 | 2574 | t.l. | 0.54 | 42 | 6895 | t.l. | 0.36 | 44 | 5400 | t.l. | -1.30 | 44 |
| 60 | 3488 | t.l. | 0.54 | 43 | 2546 | t.l. | 0.75 | 40 | 6630 | t.l. | 1.36 | 45 | 5240 | t.l. | -0.57 | 44 |
| 80 | 3475 | 1146 | 0.52 | 44 | 2544 | t.l. | 0.83 | 40 | 6625 | t.l. | 0.98 | 46 | 5260 | t.l. | -0.95 | 44 |
| 100 | 3475 | t.l. | 0.26 | 44 | 2548 | t.l. | 0.67 | 41 | 6635 | t.l. | 0.60 | 46 | 5295 | t.l. | -1.61 | 44 |

rd (%) refers to the relative difference when the best found solution is compared to the one provided by Gurobi; CPU time is in seconds; t.l. indicates that the search has been terminated due to time limit (i.e., 1800 sec), in which case the reported solution is the best found integer solution

Table 4.6 presents the results for instances with 20 doors. The results show that the commercial software fails to provide an optimal solution rapidly as the size of the problem increases. In particular, Gurobi finds the optimal solution for only 79 out of the 96 instances within the time limit. Our ALNS is always within a 6.06% gap from the best known solution, and considering only the instances for which the optimal solution value is known, the average gap of ALNS is 1.43%.

The results show that the computation time to obtain an optimal solution with Gurobi depends on the truck to dock ratio and the number of destinations per truck. Indeed, these parameters can be used to characterize the complexity of the operations in a cross-dock.

We have tested only instances with a feasible solution; however, there exist over-constrained instances without feasible solutions with respect to the scheduling, e.g., when the number of inbound trucks with identical time windows $(a, b)$ is more than the maximum number of trucks that can be processed at the docks during the time intervals $(a, b)$. Such infeasibility can be easily detected either when solving FP or when attempting to construct a starting feasible solution in the ALNS algorithm.

In addition, Tables 4.5–4.6 show that the proposed ALNS algorithm is insensitive to the number of destinations per truck both in quality and calculation time. However, the number of trucks determines the size of the RP and influences its computation time requirement. Indeed, as the number of trucks doubles, the computation of the ALNS tends to require twice as much time.

Finally, the results do not indicate clear correlation between the length of the time windows and the computation time for Gurobi. The CPU time for the ALNS heuristic is not affected by this instance characteristic. We note that, in practice, it is uncommon that trucks have wide time windows. Hence, the considered short time windows used in these instances are reasonable to approximate real instances.

Table 4.7 presents information on the frequency of using each neighborhood operator within the ALNS algorithm. Each row represents an average result over 16 instances that are presented

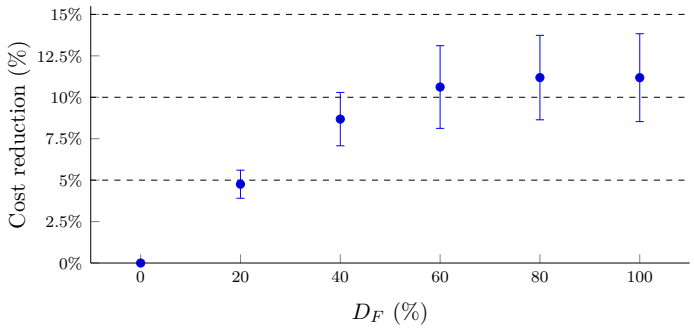
Table 4.7: Frequency of using the neighborhood operators as a percentage of the total number of iterations

| $D_F$ (%) | rRnd & iBck | rRnd & iFrw | rRnd & iSwp | rMax & iBck | rMax & iFrw | rCrt & iBck | rCrt & iFrw | rCrt & iSwp | irI2O | riChnBck | riChnFrw |
|---|---|---|---|---|---|---|---|---|---|---|---|
| *(a) no. of docks = 10* | | | | | | | | | | | |
| 0 | 9.53 | 10.54 | 11.03 | 7.31 | 10.48 | 8.48 | 6.10 | 8.67 | 7.31 | 9.81 | 10.74 |
| 20 | 9.46 | 10.42 | 11.23 | 7.14 | 10.85 | 8.72 | 6.14 | 8.40 | 7.24 | 9.73 | 10.69 |
| 40 | 9.11 | 10.73 | 11.00 | 6.94 | 10.93 | 8.85 | 6.46 | 8.35 | 7.32 | 9.42 | 10.89 |
| 60 | 9.54 | 10.49 | 11.01 | 7.20 | 10.50 | 9.08 | 6.37 | 8.57 | 7.29 | 9.59 | 10.36 |
| 80 | 9.38 | 10.27 | 10.96 | 7.38 | 10.48 | 9.00 | 6.38 | 8.56 | 7.37 | 9.71 | 10.53 |
| 100 | 9.35 | 10.23 | 10.92 | 7.45 | 10.61 | 8.96 | 6.40 | 8.51 | 7.31 | 9.76 | 10.51 |
| *(b) no. of docks = 20* | | | | | | | | | | | |
| 0 | 9.31 | 10.29 | 10.91 | 7.06 | 11.13 | 8.17 | 6.00 | 9.00 | 9.09 | 9.07 | 9.99 |
| 20 | 9.15 | 10.27 | 10.95 | 7.03 | 11.09 | 8.14 | 6.04 | 8.91 | 9.10 | 9.35 | 9.98 |
| 40 | 9.13 | 10.39 | 11.06 | 7.41 | 11.04 | 8.29 | 6.05 | 8.79 | 8.61 | 9.26 | 9.99 |
| 60 | 9.16 | 10.02 | 11.02 | 7.63 | 10.97 | 8.69 | 5.82 | 9.12 | 8.66 | 9.13 | 9.79 |
| 80 | 9.29 | 9.77 | 11.04 | 7.71 | 11.12 | 8.57 | 5.98 | 8.89 | 8.81 | 9.02 | 9.82 |
| 100 | 9.24 | 9.86 | 11.05 | 7.78 | 11.13 | 8.50 | 5.97 | 8.91 | 8.79 | 8.98 | 9.79 |

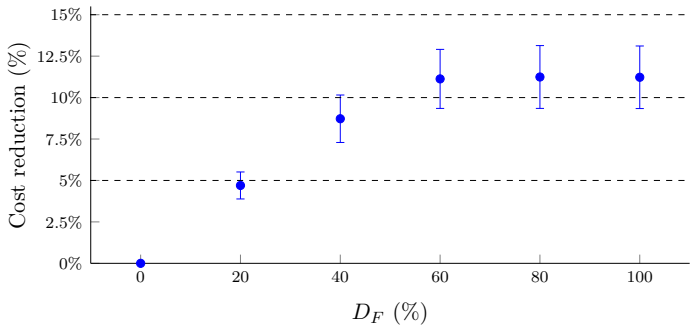
in Table 4.5 and 4.6. The results indicate that the frequency of using the different neighborhood operators do not vary significantly from one another.

Figure 4.4a shows the average cost reduction and the 95% confidence intervals when the solution is compared to the case with 0 docks operated in a multi-purpose mode. The figure is based on the results obtained with Gurobi and presented in Tables 4.5–4.6; only proven optimal solutions are considered. The results indicate that an average of 8-10% cost decrease is obtainable if 40-60% of the total number of docks are operated in mixed service mode. Intuitively, this is due to pooling the truck queues. These flexible docks operate similarly to multi-purpose servers that can handle different types of arriving entities. Therefore, even when all of the servers dedicated to the arriving entity type are busy, there is an option to serve the arriving entity with a multi-purpose server. However, pooling is not always beneficial (cf. van Dijk and van der Sluis, 2008; Joustra et al., 2010). In our case, the potential improvement in costs decreases when the ratio of the flexible docks increases compared to the total number of doors. In particular, the results indicate that when this ratio is around 60% an additional mixed service mode dock does not improve the solution significantly.

Finally, we present the computational results for large size instances when they are solved by our proposed ALNS algorithm. For this, a total of 96 instances are generated. The results are presented in Figure 4.4b, where each interval is based on the results of 16 instances with the following characteristics: the number of docks is $\{40, 50\}$, the average number of destinations per truck is $\{4, 5\}$, $\beta/\gamma$ is $\{0.1, 0.5\}$, and the length of time window is $\{2, 3\}$. The number of trucks is 250 in all instances.




(a) Results obtained with Gurobi for instances with $\{10, 20\}$ doors and $\{30, 60, 120\}$ trucks




(b) Results obtained with ALNS for instances with $\{40, 50\}$ doors and 250 trucks

Figure 4.4: Mean and confidence interval plot for cost reduction

The results show that the proposed ALNS provides similar improvement opportunities as the commercial solver regardless of the size of the problem. Table 4.8 shows that the computation time requirement scales well as the problem size increases. In particular, the average CPU time is within 3 minutes, while the maximum time is within 4 minutes in all cases.

Table 4.8: CPU time (sec) requirement of ALNS for large size instances with 250 trucks

| no. of doors | 40 | | 50 | |
|---|---|---|---|---|
| $D_F$ (%) | avg | max | avg | max |
| 0 | 128 | 174 | 135 | 234 |
| 20 | 139 | 200 | 117 | 141 |
| 40 | 165 | 212 | 110 | 129 |
| 60 | 169 | 211 | 103 | 121 |
| 80 | 113 | 131 | 99 | 111 |
| 100 | 115 | 135 | 101 | 112 |

We note that the computation time tends to be shorter as the number of multi-purpose docks increases. This is due to that as more docks are available to process inbound trucks, a neighborhood operator is more likely to provide immediately a feasible inbound truck schedule. Therefore, the time spent to move from one solution to another is reduced.

As a conclusion, the proposed ALNS algorithm is able to solve real size instances within a reasonable time, providing solutions with a consistent gap when compared to the optimal solution.

## 4.7 Conclusions

To the best of our knowledge, this is the first study on truck scheduling in a cross-docking environment that also shows the effects of operating a set of doors in a mixed mode.

We have presented a model which minimizes the operation costs, i.e., the sum of handling and tardiness costs. It can be used to find optimal truck schedules for moderate size instances. We have implemented the model with a commercial solver, Gurobi, and tested the sensitivity of the solution value to several problem characteristics. The results show that the number of trucks per dock and the number of destinations per truck are the main factors of the computation time when searching for an optimal solution.

For larger instances Gurobi cannot provide a solution in a reasonable time, therefore we propose a heuristic algorithm that follows the principles of the Adaptive Large Neighborhood

Search and show that it provides a consistent gap when compared to the best known solution obtained by the commercial solver. Results show that the computation time requirement of the proposed heuristic scales well with the problem size.

Our computational results confirm that a few mixed service mode docks can improve the performance of the cross-dock, as they decrease the total costs. However, the improvement decreases as the number of mixed service mode docks increases.

# Bibliography


Agustina, D., C. K. M. Lee, and R. Piplani (2010). A review: Mathematical modles for cross docking planning. *International Journal of Engineering Business Management 2*(2), 47–54.

Ahmadi, R. H. and U. Bagchi (1990). Improved lower bounds for minimizing the sum of completion times of $n$ jobs over $m$ machines in a flow shop. *European Journal of Operational Research 44*(15), 331–336.

Ahuja, R. K., T. L. Magnanti, and J. B. Orlin (1993). *Network Flows: Theory, Algorithms, and Applications* (1 ed.). Prentice Hall: New Jersey.

Akkan, C. and S. Karabatı (2004). The two-machine flowshop total completion time problem: Improved lower bounds and a branch-and-bound algorithm. *European Journal of Operational Research 159*(2), 420–429.

Alpan, G., A.-L. Ladier, R. Larbi, and B. Penz (2011). Heuristic solutions for transshipment problems in a multiple door cross docking warehouse. *Computers & Industrial Engineering 61*(2), 402–408.

Alpan, G., R. Larbi, and B. Penz (2011). A bounded dynamic programming approach to schedule operations in a cross docking platform. *Computers & Industrial Engineering 60*(3), 385–396.

Álvarez Pérez, G. A., J. L. González-Velarde, and J. W. Fowler (2009). Crossdocking – Just in time scheduling: An alternative solution approach. *Journal of the Operational Research Society 60*(60), 554–564.

Ang, M., Y. F. Lim, and M. Sim (2012). Robust storage assignment in unit-load warehouses. *Management Science 58*(11), 2114–2130.

Anken, N., J.-P. Gagliardi, J. Renaud, and A. Ruiz (2011). Space allocation and aisle positioning for an industrial pick-to-belt system. *Journal of the Operational Research Society 62*(1), 38–49.

Ashayeri, J., L. Gelders, and L. N. Van Wassenhove (1985). A microcomputer-based optimization model for the design of automated warehouses. *International Journal of Production Research 23*(4), 825–839.

Ashayeri, J. and L. F. Gelders (1985). Warehouse design optimization. *European Journal of Operational Research 21*(3), 285–294.

Baker, P. and M. Canessa (2009). Warehouse design: A structured approach. *European Journal of Operational Research 193*(2), 425–436.

Bartholdi III, J. J., D. D. Eisenstein, and R. D. Foley (2001). Performance of bucket brigades when work is stochastic. *Operations Research 49*(5), 710–719.

Bartholdi III, J. J. and K. R. Gue (2000). Reducing labor costs in an LTL crossdocking terminal. *Operations Research 48*(6), 823–832.

Bartholdi III, J. J. and K. R. Gue (2004). The best shape for a crossdock. *Transportation Science 38*(2), 235–244.

Bartholdi III, J. J., K. R. Gue, and K. Kang (2008). Staging protocols for unit-load crossdocking. In M. Lahmar (Ed.), *Facility Logistics*, pp. 153–172. Auerbach Publications.

Bassan, Y., Y. Roll, and M. J. Rosenblatt (1980). Internal layout design of a warehouse. *AIIE Transactions 12*(4), 317–322.

Bellanger, A., S. Hanafi, and C. Wilbaut (2013). Three-stage hybrid-flowshop model for cross-docking. *Computers & Operations Research 40*(4), 1109–1121.

Bhaskaran, K. and C. J. Malmborg (1990). Economic tradeoffs in sizing warehouse reserve storage area. *Applied Mathematical Modelling 14*(7), 381–385.

Boloori Arabani, A. R., S. M. T. Fatemi Ghomi, and M. Zandieh (2010). A multi-criteria cross-docking scheduling with just-in-time approach. *The International Journal of Advanced Manufacturing Technology 49*, 741–756.

Boloori Arabani, A. R., S. M. T. Fatemi Ghomi, and M. Zandieh (2011). Meta-heuristics implementation for scheduling of trucks in a cross-docking system with temporary storage. *Expert Systems with Applications 38*(3), 1964–1979.

Boloori Arabani, A. R., M. Zandieh, and S. M. T. Fatemi Ghomi (2012). A cross-docking scheduling problem with sub-population multi-objective algorithms. *The International Journal of Advanced Manufacturing Technology 58*(5-8), 741–761.

Boysen, N. (2010). Truck scheduling at zero-inventory cross docking terminals. *Computers & Operations Research 37*(1), 32–41.

Boysen, N. and M. Fliedner (2010). Cross dock scheduling: Classification, literature review and research agenda. *Omega 38*(6), 413–422.

Boysen, N., M. Fliedner, and A. Scholl (2010). Scheduling inbound and outbound trucks at cross docking terminals. *OR Spectrum 32*(1), 135–161.

Briskorn, D., F. Jaehn, and E. Pesch (2013). Exact algorithms for inventory constrained scheduling on a single machine. *Journal of Scheduling 16*(1), 105–115.

Brockmann, T. and P. Godin (1997). Flexibility for the future in warehouse design. *IIE Solutions 27*(7), 22–25.

Chan, F. T. S. and V. Kumar (2009). Hybrid TSSA algorithm-based approach to solve warehouse-scheduling problems. *International Journal of Production Research 47*(4), 919–940.

Chen, F. and C.-Y. Lee (2009). Minimizing the makespan in a two-machine cross-docking flow shop problem. *European Journal of Operational Research 193*(1), 59–72.

Chen, F. and K. Song (2009). Minimizing makespan in two-stage hybrid cross docking scheduling problem. *Computers & Operations Research 36*(6), 2066–2073.

Chu, C. (1992). A branch-and-bound algorithm to minimize total flow time with unequal release dates. *Naval Research Logistics 39*(6), 859–875.

Chuang, Y.-F., H.-T. Lee, and Y.-C. Lai (2012). Item-associated cluster assignment model on storage allocation problems. *Computers & Industrial Engineering 63*(4), 1171–1177.

Cormier, G. and E. A. Gunn (1992). A review of warehouse models. *European Journal of Operational Research 58*(1), 3–13.

De Koster, R. B. M., T. Le-Duc, and K. J. Roodbergen (2007). Design and control of warehouse order picking: A literature review. *European Journal of Operational Research 182*(2), 481–501.

De Koster, R. B. M. and E. S. Van der Poort (1998). Routing orderpickers in a warehouse: A comparison between optimal and heuristic solutions. *IIE Transactions 30*(5), 469–480.

Della Croce, F., M. Ghirardi, and R. Tadei (2002). An improved branch-and-bound algorithm for the two machine total completion time flow shop problem. *European Journal of Operational Research 139*(2), 293–301.

Della Croce, F., V. Narayan, and R. Tadei (1996). The two-machine total completion time flow shop problem. *European Journal of Operational Research 90*(2), 227–237.

Forger, G. (1995). UPS starts world's premiere cross-docking operation. *Modern Materials Handling 50*(13), 36–38.

Framinan, J. M. and R. Leisten (2003). An efficient constructive heuristic for flowtime minimisation in permutation flow shops. *Omega 31*(4), 311–317.

Framinan, J. M., R. Leisten, and C. Rajendran (2003). Different initial sequences for the heuristic of Nawaz, Enscore and Ham to minimize makespan, idletime or flowtime in the static permutation flowshop sequencing problem. *International Journal of Production Research 41*(1), 141–148.

Framinan, J. M., R. Leisten, and R. Ruiz-Usano (2005). Comparison of heuristics for flowtime minimisation in permutation flowshops. *Computers & Operations Research 5*(5), 1237–1254.

Francis, R. L. and J. A. White (1992). *Facility Layout and Location: An Analytical Approach* (2 ed.). Prentice-Hall, Englewood Cliffs, NJ.

Frazelle, E. H. (2002). *World-class warehousing and material handling*. McGraw-Hill: New York.

Frazelle, E. H. and G. P. Sharp (1989). Correlated assignment strategy can improve order-picking operation. *Industrial Engineering 4*, 33–37.

Fu, Z., R. Eglese, and L. Y. O. Li (2005). A new tabu search heuristic for the open vehicle routing problem. *Journal of the Operational Research Society 56*(3), 267–274.

Garfinkel, M. (2005). *Minimizing multi-zone orders in the correlated storage assignment problem*. Ph.d. thesis, Georgia Institute of Technology.

Gharehgozli, A. H., G. Laporte, T. Yu, and R. B. M. De Koster (2013). Scheduling two yard cranes handling requests with precedences in a container block with multiple open locations. working paper.

Goetschalckx, M. and J. Ashayeri (1989). Classification and design of order picking systems. *Logistics Information Management 2*(2), 99–106.

Goetschalckx, M. and H. D. Ratliff (1990). Storage policies based on the duration stay of unit loads. *Management Science 36*(9), 1120–1132.

Goh, M., O. Jihong, and T. Chung-Piaw (2001). Warehouse sizing to minimize inventory and storage costs. *Naval Research Logistics 48*(4), 299–312.

Gonzalez, T. and S. Sahni (1978). Flowshop and jobshop schedules: complexity and approximation. *Operations Research 26*(1), 36–52.

Graham, R. L., E. L. Lawler, J. K. Lenstra, and A. H. Rinnooy Kan (1979). Optimization and approximation in deterministic sequencing and scheduling: a survey. *Annals of Discrete Mathematics 5*, 287–326.

Gu, J., M. Goetschalckx, and L. F. McGinnis (2007). Research on warehouse operation: A comprehensive review. *European Journal of Operational Research 177*(1), 1–21.

Gu, J., M. Goetschalckx, and L. F. McGinnis (2010a). Research on warehouse design and performance evaluation: A comprehensive review. *European Journal of Operational Research 203*(3), 539–579.

Gu, J., M. Goetschalckx, and L. F. McGinnis (2010b). Solving the forward-reserve allocation problem in warehouse order picking systems. *Journal of the Operational Research Society 61*(6), 1013–1021.

Gue, K. R. (1999). The effects of trailer scheduling on the layout of freight terminals. *Transportation Science 33*(4), 419–428.

Gue, K. R., G. Ivanovic, and R. D. Meller (2012). A unit-load warehouse with multiple pickup and deposit points and non-traditional aisles. *Transportation Research Part E: Logistics and Transportation Review 48*(4), 795–806.

Gue, K. R. and R. D. Meller (2009). Aisle configurations for unit-load warehouses. *IIE Transactions 41*(3), 171–182.

Hackman, S. T. and L. K. Platzman (1990). Near-optimal solution of generalized resource allocation problems with large capacities. *Operations Research 38*(5), 902–910.

Hackman, S. T., M. J. Rosenblatt, and J. M. Olin (1990). Allocating items to an automated storage and retrieval system. *IIE Transactions 22*(1), 7–14.

Hall, R. W. (1993). Distance approximations for routing manual pickers in a warehouse. *IIE Transactions 25*(4), 76–87.

Hausman, W. H., L. B. Schwarz, and S. C. Graves (1976). Optimal storage assignment in automatic warehousing systems. *Management Science 22*(6), 629–638.

Heragu, S. S., L. Du, R. J. Mantel, and P. C. Schuur (2005). Mathematical model for warehouse design and product allocation. *International Journal of Production Research 43*(2), 327–338.

Heskett, J. (1963). Cube-per-order index – A key to warehouse stock location. *Transportation & Distribution Management 3*(4), 27–31.

Heskett, J. (1964). Putting the cube-per-order index to work in warehouse layout. *Transportation & Distribution Management 4*, 23–30.

Hoogeveen, H., L. van Norden, and S. van de Velde (2006). Lower bounds for minimizing total completion time in a two-machine flow shop. *Journal of Scheduling 9*(6), 559–568.

Ignall, E. and L. Schrage (1965). Application of the branch and bound technique to some flowshop scheduling problems. *Operations Research 13*(3), 400–412.

Jaikumar, R. and M. Solomon (1990). Dynamic operational policies in an automated warehouse. *IIE Transactions 22*(4), 370–376.

Jane, C.-C. and Y.-W. Laih (2005). A clustering algorithm for item assignment in a synchronized zone order picking system. *European Journal of Operational Research 166*(2), 498–496.

Jarvis, J. M. and E. D. McDowell (1991). Optimal product layout in an order picking warehouse. *IIE Transactions 23*, 93–102.

Johnson, M. E. and M. L. Brandeau (1996). Stochastic modeling for automated material handling system design and control. *Transportation Science 30*(4), 330–350.

Joustra, P., E. van der Sluis, and N. M. van Dijk (2010). To pool or not to pool in hospitals: a theoretical and practical comparison for a radiotherapy outpatient department. *Annals of Operations Research 178*(1), 77–89.

Konur, D. and M. M. Golias (2013). Cost-stable truck scheduling at a cross-dock facility with unknown truck arrivals: A meta-heuristic approach. *Transportation Research Part E: Logistics and Transportation Review 49*(1), 71–91.

Kovács, A. (2011). Optimizing the storage assignment in a warehouse served by milkrun logistics. *International Journal of Production Economics 133*(1), 312–318.

Kovacs, A. A., S. N. Parragh, K. F. Doerner, and R. F. Hartl (2012). Adaptive large neighborhood search for service technician routing and scheduling problems. *Journal of Scheduling 15*(5), 579–600.

Kulturel, S., N. E. Ozemirel, C. Sepil, and Z. Bozkurt (1999). Experimental investigation of shared storage assignment policies in automated storage/retrieval systems. *IIE Transactions 31*(8), 739–749.

Laha, D. and S. C. Sarin (2009). A heuristic to minimize total flow time in permutation flow shop. *Omega 37*(3), 734–739.

Larbi, R., G. Alpan, P. Baptiste, and B. Penz (2011). Scheduling cross docking operations under full, partial and no information on inbound arrivals. *Computers & Operations Research 38*, 889–900.

Lee, M.-K. and E. A. Elsayed (2005). Optimization of warehouse storage capacity under a dedicated storage policy. *International Journal of Production Research 43*(9), 1785–1805.

Li, Y., A. Lim, and B. Rodrigues (2004). Crossdocking - JIT scheduling with time windows. *Journal of the Operational Research Society 55*(12), 1342–1351.

Liao, T. W., P. J. Egbelu, and P. C. Chang (2012). Two hybrid differential evolution algorithms for optimal inbound and outbound truck sequencing in cross docking operations. *Applied Soft Computing 12*(11), 3683–3697.

Liao, T. W., P. J. Egbelu, and P. C. Chang (2013). Simultaneous dock assignment and sequencing of inbound trucks under a fixed outbound truck schedule in multi-door cross docking operations. *International Journal of Production Economics 141*(1), 212–229.

Liu, C.-M. (2004). Optimal storage layout and order picking for warehousing. *International Journal of Operations Research 1*(1), 37–46.

Liu, J. and C. R. Reeves (2001). Constructive and composite heuristic solutions to the $P//\sum C_i$ scheduling problem. *European Journal of Operational Research 132*(2), 439–452.

Malmborg, C. J. (1996a). An integrated storage system evaluation model. *Applied Mathematical Modelling 20*(5), 359–370.

Malmborg, C. J. (1996b). Storage assignment policy tradeoffs. *International Journal of Production Research 34*(2), 363–378.

Malmborg, C. J. and K. Bhaskaran (1990). A revised proof of optimality for the cube-per-order index rule for stored item location. *Applied Mathematical Modelling 14*(2), 87–95.

Mantel, R. J., P. C. Schuur, and S. S. Heragu (2007). Order oriented slotting: a new assignment strategy for warehouses. *European Journal of Industrial Engineering 1*(3), 301–316.

Matson, J. O. and J. A. White (1982). Operational research and material handling. *European Journal of Operational Research 11*(4), 309–318.

Miao, Z., A. Lim, and H. Ma (2009). Truck dock assignment problem with operational time constraint within crossdocks. *European Journal of Operational Research 192*(1), 105–115.

Muppani, M. V. R. and G. K. Adil (2008a). A branch and bound algorithm for class based storage location assignment. *European Journal of Operational Research 189*(2), 492–507.

Muppani, M. V. R. and G. K. Adil (2008b). Efficient formation of storage classes for warehouse storage location assignment: A simulated annealing approach. *Omega 36*(4), 609–618.

Muralidharan, B., R. J. Linn, and R. Pandit (1995). Shuffling heuristics for the storage location assignment in an AS/RS. *International Journal of Production Research 33*(6), 1661–1673.

Oh, Y., H. Hwang, C. N. Chab, and S. Leec (2006). A dock-door assignment problem for the Korean mail distribution center. *Computers & Industrial Engineering 51*(2), 288–296.

Pan, Q.-K. and R. Ruiz (2013). A comprehensive review and evaluation of permutation flowshop heuristics to minimize flowtime. *Computers & Operations Research 40*(1), 117–128.

Parikh, P. J. and R. D. Meller (2008). Selecting between batch and zone order picking strategies in a distribution center. *Transportation Research Part E: Logistics and Transportation Review 44*(5), 696–719.

Petersen II, C. G. (1999). The impact of routing and storage policies on warehouse efficiency. *International Journal of Operations & Production Management 19*(10), 1053–1064.

Petersen II, C. G. and R. W. Schmenner (1999). An evaluation of routing and volume-based storage policies in an order picking operation. *Decision Sciences 30*(2), 481–501.

Pisinger, D. and S. Ropke (2007). A general heuristic for vehicle routing problems. *Computers & Operations Research 34*(8), 2403–2435.

Pliskin, J. S. and D. Dori (1982). Ranking alternative warehouse area assignments: A multi-attribute approach. *IIE Transactions 14*(1), 19–26.

Potts, C. N. and L. N. Van Wassenhove (1983). An algorithm for single machine sequencing with deadlines to minimize total weighted completion time. *European Journal of Operational Research 12*(12), 379–387.

Rajendran, C. and H. Ziegler (1997). An efficient heuristic for scheduling in a flowshop to minimize total weighted flowtime of jobs. *European Journal of Operational Research 103*(1), 129–138.

Ratliff, H. D. and A. S. Rosenthal (1983). Order-picking in a rectangular warehouse: a solvable case of the traveling salesman problem. *Operations Research 31*(3), 507–521.

Roll, Y. and M. J. Rosenblatt (1983). Random versus grouped storage policies and their effect on warehouse capacity. *Material Flow 1*(3), 199–2005.

Roodbergen, K. J. (2001). *Layout and routing method for warehouses*. Ph. D. thesis, RSM Erasmus University, The Netherlands.

Roodbergen, K. J. and I. F. A. Vis (2006). A model for warehouse layout. *IIE Transactions 38*(10), 799–811.

Roodbergen, K. J. and I. F. A. Vis (2009). A survey of literature on automated storage and retrieval systems. *European Journal of Operational Research 194*(2), 343–362.

Ropke, S. and D. Pisinger (2006a). An adaptive large neighborhood search heuristic for the pickup and delivery problem with time windows. *Transportation Science 40*(4), 455–472.

Ropke, S. and D. Pisinger (2006b). A unified heuristic for a large class of vehicle routing problems with backhauls. *European Journal of Operational Research 171*(3), 750–775.

Rosenblatt, M. J. and A. Eynan (1989). Deriving the optimal boundaries for class-based automatic storage/retrieval systems. *Management Science 35*(12), 1519–1524.

Rosenblatt, M. J. and Y. Roll (1984). Warehouse design with storage policy considerations. *International Journal of Production Research 22*(5), 809–821.

Rouwenhorst, B., B. Reuter, V. Stockrahm, G. J. van Houtum, R. J. Mantel, and W. H. M. Zijm (2000). Warehouse design and control: Framework and literature review. *European Journal of Operational Research 122*(3), 515–533.

Ruben, R. A. and F. R. Jacobs (1999). Batch construction heuristics and storage assignment strategies for walk/ride and pick systems. *Management Science 45*(4), 575–596.

Saddle Creek Corporation (2011). 2011 cross-docking trends report. http://www.sclogistics.com/component/content/article/9-uncategorised/150-white-papers. accessed on 15 November 2012.

Sadiq, M., T. L. Landers, and G. D. Taylor (1996). An assignment algorithm for dynamic picking systems. *IIE Transactions 28*(8), 607–616.

Sadykov, R. (2012). Scheduling incoming and outgoing trucks at cross docking terminals to minimize the storage cost. *Annals of Operations Research 201*(1), 423–440.

Sarker, B. R. and P. S. Babu (1995). Travel time models in automated storage/retrieval systems: A critical review. *International Journal of Production Economics 40*(2–3), 173–185.

Shakeri, M., M. Y. H. Low, S. J. Turner, and E. W. Lee (2012). A robust two-phase heuristic algorithm for the truck scheduling problem in a resource-constrained crossdock. *Computers & Operations Research 39*(11), 2564–2577.

Soltani, R. and S. J. Sadjadi (2010). Scheduling trucks in cross-docking systems: A robust meta-heuristics approach. *Transportation Research Part E: Logistics and Transportation Review 46*(5), 650–666.

Stalk, G., P. Evans, and S. L. E. (1992). Competing on capabilities: The new role of corporate strategy. *Harvard Business Review 70*(2), 57–69.

Stenger, A., D. Vigo, S. Enz, and M. Schwind (2013). An adaptive variable neighborhood search algorithm for a vehicle routing problem arising in small package shipping. *Transportation Science 47*(1), 64–80.

Steudel, H. J. (1979). Generating pallet loading patterns: A special case of the two-dimensional cutting stock problem. *Management Science 25*(10), 997–1004.

Taillard, E. (1993). Benchmarks for basic scheduling problems. *European Journal of Operational Research 64*(2), 287–285.

T'kindt, V., F. Della Croce, and C. Esswein (2004). Revisiting branch and bound search strategies for machine scheduling problems. *Journal of Scheduling 7*(6), 429–440.

Tompkins, J. A., J. A. White, Y. A. Bozer, E. H. Frazelle, and J. M. A. Tanchoco (2010). *Facilities Planning* (4th ed.). John Wiley & Sons.

Tsai, R. D., E. M. Malstrom, and H. D. Meeks (1988). A two-dimensional palletizing procedure for warehouse loading operations. *IIE Transactions 20*(4), 418–425.

Tsui, L. Y. and C.-H. Chang (1990). A microcomputer based decision support tool for assigning dock doors in freight yards. *Computers & Industrial Engineering 19*(1-4), 309–312.

Tsui, L. Y. and C.-H. Chang (1992). An optimal solution to a dock door assignment problem. *Computers & Industrial Engineering 23*(1-4), 283–286.

Vahdani, B. and M. Zandieh (2010). Scheduling trucks in cross-docking systems: Robust meta-heuristics. *Computers & Industrial Engineering 58*(1), 12–24.

Van Belle, J., P. Valckenaers, and D. Cattrysse (2012). Cross-docking: State of the art. *Omega 40*(6), 827–846.

van de Velde, S. L. (1990). Minimizing the sum of the job completion times in the two-machine flow shop by lagrangian relaxation. *Annals of Operations Research 26*(1–4), 257–268.

van den Berg, J. P. (1999). A literature survey on planning and control of warehousing systems. *IIE Transactions 31*(8), 751–762.

van den Berg, J. P., G. P. Sharp, A. J. R. M. (Noud) Gademann, and Y. Pochet (1998). Forward-reserve allocation in a warehouse with unit-load replenishments. *European Journal of Operational Research 111*(1), 98–113.

van Dijk, N. M. and E. van der Sluis (2008). To pool or not to pool in call centers. *Production and Operations Management 17*(3), 296–305.

Vis, I. F. A. and K. J. Roodbergen (2008). Positioning of goods in a cross-docking environment. *Computers & Industrial Engineering 54*(3), 677–689.

Wäscher, G. (2004). Order picking: A survey of planning problems and methods. In H. Dyckhoff, R. Lackes, and J. Reese (Eds.), *Supply Chain Management and Reverse Logistics* (1 ed.)., pp. 323–347. Springer.

Witt, C. E. (1998). Crossdocking: Concepts demand choice. *Material Handling Engineering 53*(7), 44–49.

Yan, H. and S.-l. Tang (2009). Pre-distribution and post-distribution cross-docking operations. *Transportation Research Part E: Logistics and Transportation Review 45*(6), 843–859.

Yu, W. and P. J. Egbelu (2008). Scheduling of inbound and outbound trucks in cross docking systems with temporary storage. *European Journal of Operational Research 184*(1), 377–396.

Yu, Y. and R. B. M. De Koster (2013). On the suboptimality of full turnover-based storage. *International Journal of Production Research 51*(6), 1635–1647.