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The Proximity-Concentration Trade-Off under Goods Price and Exchange Rate Uncertainty

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Abstract

The underlying model combines the proximity-concentration trade-off framework with the real option approach. In contrast to the latest trade models, uncertainty is introduced as a continuous phenomenon. Furthermore, the model contains the innovation of comparing two option values simultaneously. The implementation of goods price uncertainty turns out to reduce the probability of entering a new market as an exporter. FDI becomes the optimal entry mode with increasing uncertainty. Additionally, the model is extended by implementing exchange rate uncertainty in a period of appreciation.

Keywords: Export, FDI, Uncertainty, Real Option Approach
JEL classification: D81, D92, F17, F21, F23

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1. Introduction

The global economic integration has been increasingly influenced by international trade and foreign direct investments in the past two decades. According to the UNCTAD data (2006), since then domestic companies have steadily increased their exports and foreign plant shares (horizontal FDI) to access new markets. Besides this persistent growth, two additional striking developments can be identified in empirical data. Since the early 1980s the growth of FDI inflows has exceeded that of exports on average in every year until 2000. Within this period worldwide real GDP increased by 2.5% and global exports rose by 5.6% per year. In contrast global inflows of FDI increased by 17.7% (Navaretti and Venables, 2004). The major share of FDI originated in and were attracted by developed countries (Markusen, 2002). However, this last development has changed its nature since 2003, as global FDI inflows have maintained their growth only because developing countries have started to attract relatively more FDI inflows whereas developed countries have experienced a reduction in their inflow growth rates (UNCTAD-Statistics, 2006).

Given the increasing importance of exports and FDI, economic analyses focusing on these two elements of international economics have gained impetus. The first influential strand of explanation was the *Ownership*, *Location* and *Internalization Advantage* framework which was developed by John Dunning (1977, 1981). With the surge of FDI in the 1980s economists started to implement the OLI framework into models emphasizing different aspects of the three possible advantages. Among them were Horstman and Markusen (1987), Markusen and Venables (1998, 2000), Brainard (1993), Helpman (1984, 1985), Ethier and Markusen (1996) and Ehtier (1986). These models are either static general equilibrium or static partial equilibrium models. Common to the first four models is the assumption of different cost structures between export-oriented companies and multinational enterprieses (MNE) which have been considered as the driving force behind FDI. Brainard (1993) e.g. considers a two-country, two-sector model in which exporters are confronted with higher variable costs than foreign direct investors due to transport costs. However, the domestic production expansion for exports is associated with scale economies. Whether a company should serve a foreign market as an exporter or via FDI therefore depends on the trade-off between scale advantages in the domestic country and the proximity advantages in the foreign country. The author calls this hypothesis the *proximity-concentration trade-off*. In a cross-section analysis between the USA and 26 countries she provides empirical support for her hypothesis and concludes:

The proximity-concentration hypothesis predicts that firms should expand horizontally across borders whenever the advantage of access to the destination market outweigh the advantages from production scale economies. (Brainard, 1997)

The next influential strand of analytical models which explain export and FDI behavior, appeared under the umbrella of the so-called *New New Trade Theory*, referring to monopolistic competition models which include uncertainty over the productivity of firms that intend to enter new markets.[1] Based on the seminal contribution by Melitz (2003), Helpman et al.(2004) develop a model in which firms choose between an export and FDI mode to serve a foreign market, in the presence of the proximity-concentration trade-off. However, in contrast to earlier models, firms don't know their productivity performance until they incur the costs of investment (domestic, export and FDI). Once the companies are involved in one of the three possible investment strategies, they finally experience their productivity. The model predicts that the most productive firms will become foreign direct investors, less productive ones will export and the least productive ones will serve only the home market conditional on surviving. The authors analyze U.S. exports and affiliate sales data based on a Pareto distribution for ex ante uncertainty, covering 38 countries and 52 manufacturing sectors and they are able to prove the significance of their model.

The New New Trade Theory is a progress in international economics because it explains the involvement of multinational firms in export and FDI simultaneously. Additionally, the firm heterogeneity is combined with product uncertainty and sunk costs, elements which are observable in practice. Furthermore, as risk and sunk costs are crucial elements in investment decisions of investors, their implementation into the latest trade models is a major step forward. However, taking further empirical literature into account which deals with export and FDI decision associated with risk, it turns out that the specific type of risk involved is crucial for the expected outcome. Helpman et al. (2004) consider risk as a one time shock component. Once the companies enter the markets, uncertainty disappears. In contrast, investment models generally treat risk (a continuous phenomenon) by means

[1] The term *"New New Trade Theory"* has been popularized by Baldwin (2005). A concise literature overview of the latest developments in the New New Trade Theory is presented by Helpman (2006).

of a time-dependent variable, such as volatile prices in new markets or exchange rate volatility. A huge collection of empirical papers analyze the impact of exchange rate volatility on exports and FDI with different results. Égert and Morales-Zumaquero (2007) analyze the impact of exchange rate volatility on export developments in less developed countries and conclude, that an increase in exchange rate volatility appears to depress exports. Russ (2007) shows theoretically and empirically that FDI decisions can be influenced by exchange rate volatility significantly.

As empirical research is pointing out the importance of additional continuous variables for the analysis of export and FDI patterns besides productivity, the development of an appropriate model which takes both market entry modes simultaneously into account, might contribute to a better understanding of international economic developments. McDonald and Siegel (1986) provide a financial model which combines sunk costs, volatile variables and timing to determine the optimal investment decision of an investor through time. Their framework has become known as the *real option approach* which has been extended among others by Dixit and Pindyck (1994). Based on this dynamic approach, I develop a homogeneous partial equilibrium trade model with a stochastic process embedded into the proximity-concentration trade-off framework. In contrast to conventional models, investors are not only confronted with the choice between exporting and FDI, but they have also the possibility to postpone investment. The combination of the well-accepted assumptions of the proximity-concentration hypothesis and the real option theory provides additional equilibrium results which can be used to explain prevailing open questions. Neary (2006) reckons that foreign direct investments still grow faster than exports in the last years although transport costs are decreasing significantly. In the presented dynamic model such an equilibrium result is possible. Decreasing exports can appear in the presence of decreasing transport costs if prices are volatile.

2. Theoretical Framework

There is one risk neutral investor who intends to serve a new foreign market with her output y. The foreign country can either be served by exports or by a new foreign plant (horizontal FDI). The production function for both investment choices is given by the concave Cobb-Douglas function (1) with labor l as the only input factor.[2] Furthermore, irreversible fixed costs I arise for both market entry strategies. Therefore, the production technology provides increasing returns to scale. There is no labor supply constraint and y represents the output for each period t with an infinite investment horizon T.

$$y_t(l_t) = l_t^{\theta} \qquad \text{with} \quad 0 < \theta < 1. \tag{1}$$

Output prices p are given exogenously on the foreign market (price taker) and are assumed to be certain for the moment. Labor costs w are assumed to be equal and constant in both investment scenarios, but exports are subject to iceberg transport costs.[3] The output produced in the domestic country y_t^D shrinks by the constant factor $(\tau - 1)$ if it is transferred to the foreign market and therefore the amount y_t^E sold on the foreign market is given by

$$y_t^E = \frac{y_t^D}{\tau} \qquad \text{with} \quad \tau > 1. \tag{2}$$

For the export investment the corresponding profit flows (cash flows) in each period are derived from the maximization problem

$$\pi_t^E(p_t, w_t, \tau_t) = \max_{l_t} p_t \; y_t^E - w_t \; l_t \quad \text{s.t.} \quad y_t^E = \frac{y_t^D}{\tau_t} \quad \text{s.t.} \quad y_t^D = l_t{}^{\theta}. \tag{3}$$

The first order condition for labor input demand in period t is given by

$$l_t = \left(\frac{\theta \; p_t}{\tau_t w_t} \right)^{\frac{1}{1-\theta}} \tag{4}$$

and the instantaneous supply function by

$$y_t^D(l_t) = \left(\frac{\theta \; p_t}{\tau_t w_t} \right)^{\frac{\theta}{1-\theta}}. \tag{5}$$

[2] The amount of input factors can easily be extended by transforming l into a vector.
[3] The transport cost technology is given by $c(\tau) = \tau - 1$.

Clearly if transport costs increase, τ increases and as a result the optimal supply of the good decreases. Finally, the perpetual cash flow in the export scenario in each period t turns out to be

$$\pi_t^E(p_t, w_t, \tau_t) = (1-\theta)\left(\frac{\theta}{\tau^{\frac{1}{\theta}} w_t}\right)^{\frac{\theta}{1-\theta}} p_t^{\frac{1}{1-\theta}}. \tag{6}$$

It is possible to rewrite the cash flows in equation (6) with respect to total variable costs c^E and c^F. Since in the FDI scenario no transport costs arise ($\tau = 1$), total variable costs are equal to labor cost ($c^F = w$), whereas in the export scenario total variable costs are given by $c^E = w_t \tau_t^{\frac{1}{\theta}}$ and equation (6) can be restated as

$$\pi_t^i(p_t, c_t^i) = (1-\theta)\left(\frac{\theta}{c_t^i}\right)^{\frac{\theta}{1-\theta}} p_t^{\frac{1}{1-\theta}} \qquad \text{with} \quad i \in \{E, F\} \tag{7}$$

with the superscript F referring to FDI and E to exports. Equation (6) clearly demonstrates that transport costs have a negative impact on export cash flows, and one can conclude if labor costs are equal in both countries and transport costs arise only in the export scenario, then

$$\pi_t^E(p_t, c_t^E) < \pi_t^F(p_t, c_t^F). \tag{8}$$

The first part of the right-hand side in equation (6) consists only of constant values and can be summarized to

$$\pi^i(p_t, c^i) = Z_t^i p_t^\kappa \tag{9}$$

with

$$Z_t^i = (1-\theta)\left(\frac{\theta}{c^i}\right)^{\left(\frac{\theta}{1-\theta}\right)} \quad \text{and} \quad \kappa = \left(\frac{1}{1-\theta}\right) \quad \text{for} \quad i \in \{E, F\}.$$

The cash flows in equation (9) are convex in goods prices which is a standard result if the production function has a concave curvature.

The proximity-concentration trade-off under certainty

The two market access modes exhibit different relative costs. Exports are associated with fixed costs I^E which include the costs for a domestic production extension and a foreign distribution and service network. In the FDI mode, an investor is confronted with higher fixed costs I^F due to the necessity of a new production plant in the foreign country but experiences lower variable costs due to the lack of transport costs

$$I^E < I^F \quad \text{and} \quad c^E > c^F. \tag{10}$$

Given the cost structure of the two investment modes and the perpetual cash flows in equation (9), it is possible to calculate the value $v(p)$ of each investment mode if the opportunity cost is known. In the underlying model, δ_c is assumed to be the exogenous discount rate without a deeper specification and it is assumed to be constant over time ($\delta_c(t) = \delta_c$).[4] A switching strategy in form of becoming first a foreign direct investor and then an exporter is excluded. The value functions of the export and foreign direct investment mode are given by

$$v^E(p) - I^E = \frac{Z^E(p)^\kappa}{\delta_c} - I^E \tag{11}$$

and

$$v^F(p) - I^F = \frac{Z^F(p)^\kappa}{\delta_c} - I^F \tag{12}$$

and are depicted in figure (1) with respect to the price of the good. The crossing between the two functions determines the equilibrium cutoff price p_{Fc} at which the investor is indifferent between exporting and FDI. For prices below p_{Ec} none of the two investment strategies is worth to be started since the cash flows are not covering the fixed costs and the project values are both negative. For prices between the two cutoff points p_{Ec} and p_{Fc} clearly the export solution is dominating the FDI strategy. Due to the lower fixed costs I^E, the average costs are lower than in the FDI case and therefore the investor should serve the foreign market by exports. If the price of the good exceeds

[4] See Dixit and Pindyck (1994) for a detailed discussion.

p_{Fc}, the FDI mode becomes dominant because of the lower variable costs. Within this standard

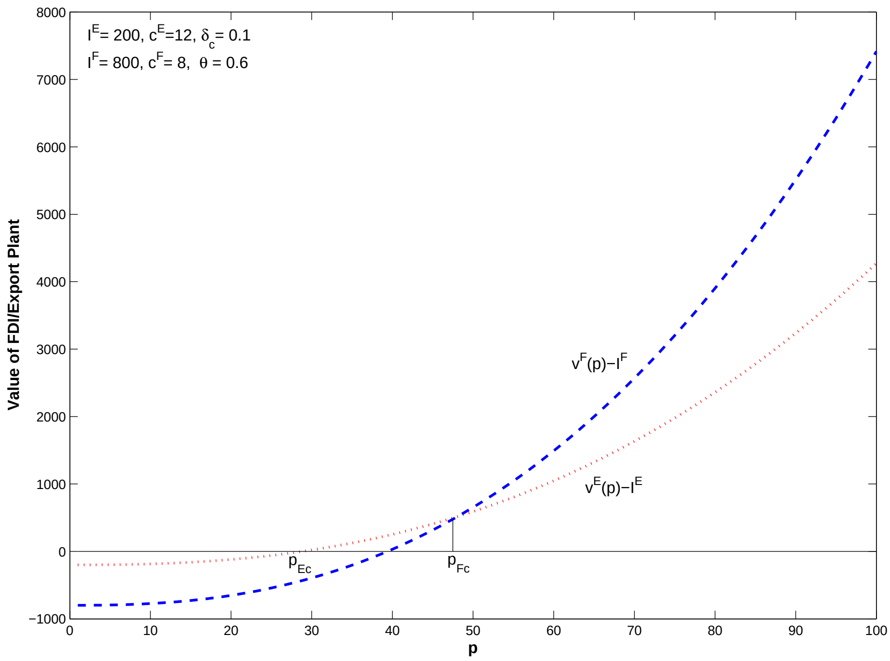


Figure 1: Proximity-Concentration Trade-Off (Certain Case)

framework under certainty, firms should expand horizontally across borders whenever the advantage of lower variable costs due to the lack of transport costs outweighs the advantage of lower fixed costs of a domestic production expansion. This result reflects the proximity-concentration hypothesis. The difference in the relative costs of the two market access modes is the source for a different degree of economies of scale and shapes the ultimate choice between the two alternative entry modes as in Helpman et al. (2004).

3. Investment Choice Under Uncertainty

Hitherto the proximity-concentration trade-off has been derived according to standard trade models as in Brainard (1993) or Markusen (2002). These types of models don't take into account that in the underlying problem investors are confronted not only with an optimization problem with respect to a different relative cost structure but also with different types of uncertainty over time, as e.g. goods price or exchange rate uncertainty. From the perspective of an investor, a crucial point is the type of uncertainty which arises within the investment problem. In the following, I extend the standard proximity-concentration trade-off framework by combining it with the real option theory. Two types of price uncertainty are analyzed whereas the term uncertainty is used in an interchangeable manner with the term risk.[5] In the first part, uncertainty is assumed in the price of the good which has different impacts on the export and FDI revenues. Afterwards the model is extended by introducing exchange rate uncertainty. These two types of price uncertainties deliver different equilibrium results.

3.1. Export or FDI under price uncertainty

In contrast to the standard proximity-concentration trade-off model, it is assumed that the price p of the good in the foreign country follows a geometric brownian motion.

$$dp = \alpha p dt + \sigma p dz \quad \text{with} \quad dz = \epsilon\sqrt{dt} \tag{13}$$

In equation (13), α is the expected growth rate of the price (e.g. due to macroeconomic developments). dz represents a Wiener process and is responsible for the uncertainty in the product price p. σ is the variance parameter which is responsible for the extent of uncertainty. ϵ is a randomly distributed variable with the mean of zero and a standard deviation of one (standard normal distribution). Therefore $E(dz) = 0$ and $E\left[(dz)^2\right] = dt$.[6]

Within these assumptions an investor is no longer confronted with a simple investment choice between exports and FDI, based on a traditional net present value (NPV) comparison. Additionally, the investor can postpone the investment decision by a certain period to gather further information about the development of the uncertain variable. Clearly, gathering information by waiting is associated with return losses since the investment is not taking place. Simultaneously, the waiting strategy offers the possibility to observe the behavior of the volatile variable and therefore the respective profit maximization can deliver a higher optimum. McDonald and Siegel (1986) call this additional value which can be achieved by waiting the *option value of an investment.* They derive

[5] In a concise way, risk is referring to a known probability distribution whereas uncertainty is referring to events in which the numerical probabilities cannot be specified.

[6] E refers to the expected value.

an investment rule which includes the option value of a project and it turns out that the fair value of an investment must be not only higher than its investment cost (Marshallian rule) but much higher. For the derivation of the optimal investment rule within the proximity-concentration trade-off, it is necessary to determine first the expected value of the real export investment $v_u^E(p)$ and the expected FDI value $v_u^F(p)$ under uncertainty. I refer to these values as the *risk-adjusted investment values* since the price uncertainty is incorporated.

The risk-adjusted investment value

An investor who holds the real investment associated with the risk in equation (13) over a period dt will expect a total return of $\mu = \delta + \alpha$ composed of

1. the expected appreciation of the price (α)
2. dividend (δ).

As risk is one major aspect within this valuation concept, it should be defined in a more rigorous manner. In the following, risk always refers to nondiversifiable risk, because with reference to the capital asset pricing model (CAPM) diversifiable risk can be eliminated by constructing appropriate portfolios. Given a market portfolio M and a riskless bond, it is possible to determine the appropriate return for any risk rate on the considered financial market.

Once the return for the market portfolio M's risk rate is known, it is possible to determine the risk premium for any firm's asset on the market, based on the covariance or correlation between the market portfolio M and the respective asset (Sharpe, 1964).

$$\mu_A = r + \Lambda\ \sigma_A \rho_{MA} \tag{14}$$

with

$$\Lambda = \frac{(\mu_M - r)}{\sigma_M} \tag{15}$$

Equation (14) states, given the correlation coefficient ρ between the market portfolio return and the considered investment return, and given Λ (the market price of risk), the expected total return rate of the considered asset is a sum of the riskless rate r and the respective risk premium. Consequently,

μ represents also the *risk-adjusted* discount rate for cash flows $\pi(p) = Zp$.[7] For such a simple cash flow structure the risk-adjusted value of an investment would be given by

$$v(p) = \int_0^\infty pe^{\alpha t}e^{-\mu t}dt = \frac{p}{\delta}. \tag{16}$$

However, in the underlying proximity-concentration trade-off framework the cash flows of the two investment modes $\pi^i(p) = Z^i p^\kappa$ are convex in p and therefore the risk-adjusted discount rate must be calculated by taking the convexity into account. Appendix A presents the derivation of the risk adjusted value of a real investment which is

$$v_u(p) = \frac{Zp^\kappa}{\delta'} \tag{17}$$

with the risk-adjusted discount rate

$$\delta' = r - \kappa(r - \delta) - \frac{1}{2}\sigma^2\kappa(\kappa - 1). \tag{18}$$

As it can be seen for a production technology with constant returns to scale ($\kappa = 1$), the risk-adjusted discount rate δ' is equal to the dividend payments δ of the investment. The fair value of the real investment with a convex cash flow structure ($\kappa > 1$) turns out to be risk-sensitive. Holding the dividend payments δ constant, as assumed, an increase in the volatility σ of prices decreases the risk adjusted discount rate δ' and therefore increases the expected value of the investment. Technically, this result is driven by the convexity of the underlying cash flows since its expected value will become higher according to Jensen's inequality. Therefore, I refer to this result as the *convexity-effect*. Given such a structure, an investor will have a higher incentive to execute an investment the higher the price volatility is if its option value is neglected.

[7] In the underlying framework, speculative bubbles are ruled out.

3.2. The fair value of an investment mode

Once the risk-adjusted investment values $v_u^i(p)$ of the two market entry modes are known, it is possible to derive their fair values including the option values $F^i(p)$ respectively. One possibility to calculate the fair value of an investment including the option value $F^i(p)$ is offered by the *contingent claims valuation*. This approach assumes that the final good of a project is traded on capital markets and $F^i(p)$ can be replicated by using the uncertain price of that final good. Of course not every good which is sold in foreign countries is traded on capital markets and therefore the replication method would only be applicable to a restricted set of investments. However, even if the final good of a real investment is not available on capital markets, the replication method can be applied to evaluate the fair value of the real project based on other assets or a portfolio of assets which comprise the same risk pattern as the real investment. Dixit and Pindyck (1994) refer to this approach as *asset spanning*. In the underlying problem, the value of the two projects (export and FDI) are risky because their value v depends on a stochastic variable p. Therefore, the diffusion process behind the value v could be derived from the volatile prices p by using the mentioned methods. As a result, the option value $F(v(p))$ of the two projects could be determined. However, this nested approach turns out to deliver very complicated results. Therefore, a third alternative is used here which results in the same investment rules as the replication and asset spanning method.
A riskless portfolio Θ is constructed by

1. holding one unit of the option $F(p)$
2. going short n units of an asset, which comprises the same risk return pattern as equation (13) $\rightarrow$ asset spanning: $n = F'(p)$[8]
3. the short position will require a payment of $\delta F'(p)p$ for each period dt.

A crucial assumption about the asset which is used to span the risk of the real investment is, that it pays no dividend. In other words, its expected return is given by μ and results only from its capital gain. Since this constructed portfolio Θ is riskless, its return must be equal to a riskless return

[8] See Dixit and Pindyck (1994, p. 118) for an analytical prove, that $n = F'(p)$ is the optimal short position.

$r[F(p) - F'(p)]dt$, with r as the relative return of a riskless treasury bond. This can be formulated as

$$dF(p) - F'(p)dp - \delta F'(p)pdt = r\left[F(p) - F'(p)\right]dt. \quad (19)$$

$dF(p)$ can be substituted by using Ito's lemma

$$dF = \frac{\partial F}{\partial t}dt + \frac{\partial F}{\partial p}dp + \frac{1}{2}\frac{\partial^2 F}{\partial p^2}\sigma^2 p^2 dt. \quad (20)$$

The result for the option value $F(p)$ is a second order differential equation which is linear in its dependent variable and its derivatives

$$\frac{1}{2}\sigma^2 p^2 F''(p) + (r - \delta)pF'(p) - rF(p) = 0.^9 \quad (21)$$

This homogeneous equation has a guess solution consisting of any two linearly independent solutions

$$F(p) = A_1 p^{\beta_1} + A_2 p^{\beta_2} \quad (22)$$

with

$$\beta_1 = \frac{1}{2} - \frac{r-\delta}{\sigma^2} + \sqrt{\left[\frac{r-\delta}{\sigma^2} - \frac{1}{2}\right]^2 + \frac{2r}{\sigma^2}} > 1 \quad (23)$$

$$\beta_2 = \frac{1}{2} - \frac{r-\delta}{\sigma^2} - \sqrt{\left[\frac{r-\delta}{\sigma^2} - \frac{1}{2}\right]^2 + \frac{2r}{\sigma^2}} < 0. \quad (24)$$

Appendix B presents the derivation of β and appendix C derives its properties. Based on equation (22), it is possible to formulate conditions to determine the constants A_1, A_2 and the threshold value $p*$, which triggers the real investment.

The first condition is given by

$$F(0) = 0. \quad (25)$$

It simply states that the option $F(p)$ should be worthless if the price of the underlying asset is equal to zero. Since β_2 is negative, condition (25) can only be true if $A_2 = 0$. As a result, the guess

[9] The effect of $F_t(p)$ is neglected since in the underlying continious case dt approaches zero.

solutions for equation (22) is reduced to

$$F(p) = Ap^{\beta}. \tag{26}$$

Two additional conditions are necessary to determine the trigger price p^* and the parameter A. These conditions are derived by considering the option value $F(p)$ at the threshold price p^*. First in equilibrium the value of the option $F(p^*)$ must be equal to the net value of the real investment $v(p^*)$.

$$F(p^*) = v_u(p^*) - I \tag{27}$$

Equation (27) is referred to as the *matching condition.* Additionally, for optimality the derivative of the option value must be equal to the derivative of the real investment value

$$F_p(p^*) = v_{up}(p^*). \tag{28}$$

Equation (28) is referred to as the *smooth-pasting* condition or *higher-order contact* (Dixit, 1993). If the two functions were not smooth at the trigger price p^*, a better maximum would be available. By using these conditions, it is possible to determine the cutoff prices and the three option parameters A^i and β for the underlying uncertain investment modes at which the option value of the project is equal to its real investment value.

$$p_i^* = \sqrt[\kappa]{\frac{\beta}{\beta-\kappa} I^i \frac{\delta'}{Z^i}} \tag{29}$$

with

$$\beta = \frac{1}{2} - \frac{r-\delta}{\sigma^2} + \sqrt{\left[\frac{r-\delta}{\sigma^2} - \frac{1}{2}\right]^2 + \frac{2r}{\sigma^2}} > 1 \tag{30}$$

and

$$A^i = Z^i \left(\left(\frac{\beta\, I^i \delta'}{(\beta-\kappa)\, Z^i}\right)^{\kappa^{-1}}\right)^{\kappa-\beta} \delta'^{-1} - I^i \left(\left(\frac{\beta\, I^i \delta'}{(\beta-\kappa)\, Z^i}\right)^{\kappa^{-1}}\right)^{-\beta} \tag{31}$$

for $i \in \{E, F\}$

Appendix D provides a concise derivation of the value p^*. The corresponding investment rule for the two market entry modes is therefore given by

$$v_u^i(p^*) = \frac{\beta}{\beta - \kappa} I^i. \tag{32}$$

If the volatility σ of the goods price p increases, the parameter β decreases ($\frac{\partial \beta}{\partial \sigma} < 0$).[10] Simultaneously, $\frac{\beta}{\beta-\kappa}$ increases and shifts up the threshold price p^*. The same effect drives up the expected investment value $v_u^i(p^*)$.

As it can be seen, the demanded real investment value is much higher than the investment costs I^i since the wedge $\frac{\beta}{\beta-\kappa}$ is bigger than one. In other words, an investor who includes the option value $F^i(p^*)$ in her assessment will demand a higher price of the good if its volatility σ increases. This is a standard result in the real option framework. Since the effect can be explained by observing σ, I refer to this result as the *uncertainty-effect*.

It is easier to interpret the economic intuition behind equation (32) if a numerical example is presented. Assume that the investment costs of a project are $I = 1$ with a volatility of the price $\sigma = 0.2$. The riskless interest rate is $r = 0.05$ and $\delta = 0.05$. The exponent of the production function is $\theta = 0.3$. With these parameter values $\beta = 2.16$, and the investment rule states $v_u^* = 2.96\ I$. Therefore, the underlying risky investment should be executed if its value is at least 2.96 times higher than the corresponding costs I, which is a huge difference to the Marshallian rule according to which an investment should be put into effect if the value of the project covers the investment costs I. Depicting the value function $v_u^E(p)$ of the export mode with its option value $F^E(p)$ allows to present the derived effects. In figure (2), the price level p_{Ec} represents the cutoff price under certainty which was derived in figure (1). Under certainty the investor should expand her domestic output for exports if prices are higher than p_{Ec}. The introduction of goods price uncertainty and the possibility of postponing an investment decision have two effects which are influencing the cutoff price, namely the convexity and uncertainty-effect. In figure (2) the continuous line represents the expected value of the export strategy. Due to the convexity-effect, the value function $v_u^E(p)$ is shifted up as the price of the good is volatile over time. In a scenario where the option value $F^E(p)$ of the

[10] Appendix C derives conditions which constrain β.

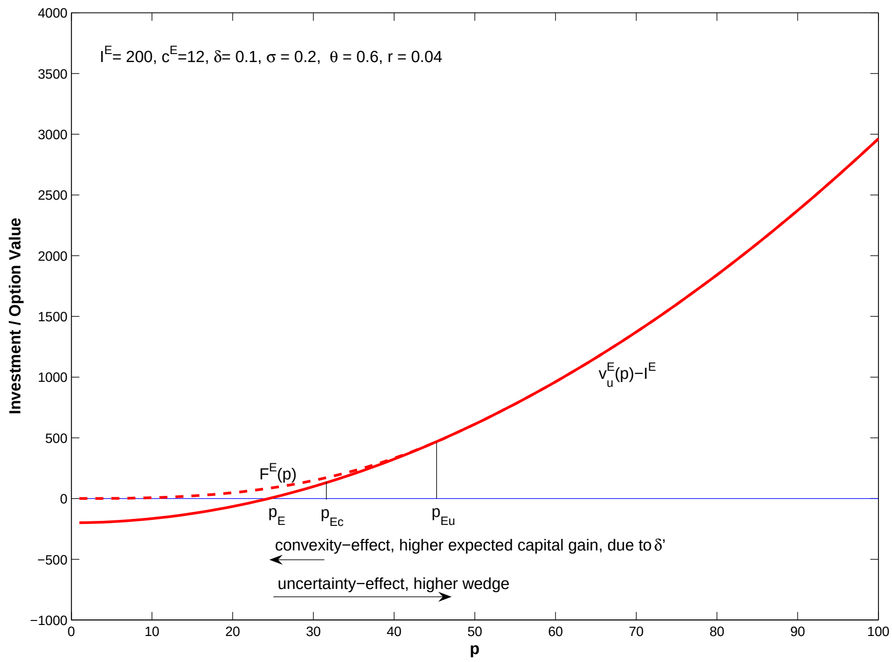


Figure 2: Cutoff price p_{Eu} for exports under uncertainty

investment is neglected, the investor would become an exporter if the prices are higher than p_E. The dashed line represents the option value of the export strategy and according to the optimality conditions the export cutoff price results if $F^E(p)$ is tangent to the expected investment value $v_u^E(p)$. This is the case at the price level p_{Eu}. The difference between the two functions $F^E(p)$ and $v_u^E(p)$ can be interpreted as the *value of waiting*. As long as the two functions don't coincide, there is a positive value of waiting and the investment should be postponed. Obviously, the uncertainty-effect shifts up the cutoff price of the export strategy and increases the value of waiting. Therefore, a risk-neutral investor will postpone the investment as long as the market price is lower than the cutoff price p_{Eu}. The crucial result in figure (2) is that the uncertainty-effect is bigger than the convexity-effect. Therefore, uncertainty leads to an investment which takes place at higher prices and implicitly later than under certainty. An independent presentation of the FDI mode in a graph is not necessary as the effects are the same as in figure (2). The only difference lies in the degree of convexity of the two value functions and the level of the equilibrium cutoff price which is higher due to the higher fixed costs.

Finally, it is possible to analyze the investment strategy of a risk-neutral investor who has to choose between the export and FDI modes including the option values of each strategy. Figure (3) depicts

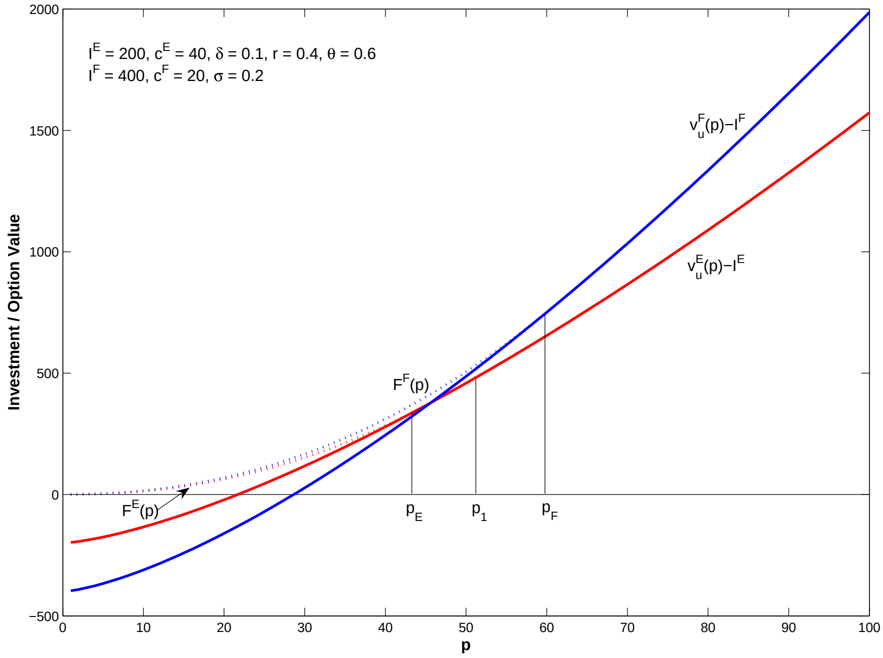


Figure 3: Export or FDI under uncertainty (A)

the value functions $v_u^i(p)$ of the two investment modes including the fixed costs for specific parameter values as continuous lines. The corresponding option values are represented by the dashed lines $F^E(p)$ and $F^F(p)$. Given the exemplary cost structure, the resulting cutoff prices provide the following investment plan. If the price p is smaller than p_E, the investor should wait and neither of the two investment strategies is executed, since the option values of both investments are higher than their expected values $v_u^i(p)$. For prices between p_E and p_F the investor would be willed to execute the export investment since its matching and smooth pasting conditions are fulfilled. However, within this price range the upper envelope in the graph is represented by the option value of the FDI mode. Therefore, the investor should wait and observe the price development. The economic intuition behind this price range is as follows. By waiting, the investor has the chance to observe the market and gather additional information concerning the FDI strategy. Given the price volatility in p, the FDI strategy offers a potentially higher return than the export strategy and therefore waiting is rational.[11]

[11] Such a strategy excludes strategic interaction between firms. It is assumed that there is no disadvantage if a firm enters a country later.

Generally spoken, the optimal strategy is derived by taking two conditions into account. First the investor has to compare the two cutoff prices and determine whether the prevailing market price is below or above them (necessary condition). However, such a comparison is not sufficient to identify the optimal strategy. Additionally the investor has to check the upper envelope of the value functions in figure (3) and control for, whether at the analyzed price level the upper envelope is given by an option value or not (sufficient condition). At a price level of p_1 in figure (3) e.g. exporting would be a profitable mode for an investor since $p_1 > p_E$. However, taking additionally the FDI mode into account it turns out that given the brownian motion (13) it is optimal to wait for the FDI mode as the upper envelope at p_1 is represented by its option value.

The resulting investment rule within the proximity-concentration trade-off framework differs from the standard option value models where the investment should be executed as soon as the price of an investment lies above the cutoff price. In the underlying framework, the investor has not only to control for the respective option value but also for the option value of the alternative investment. Figure (4) presents the value functions of the two investment modes with their option values at

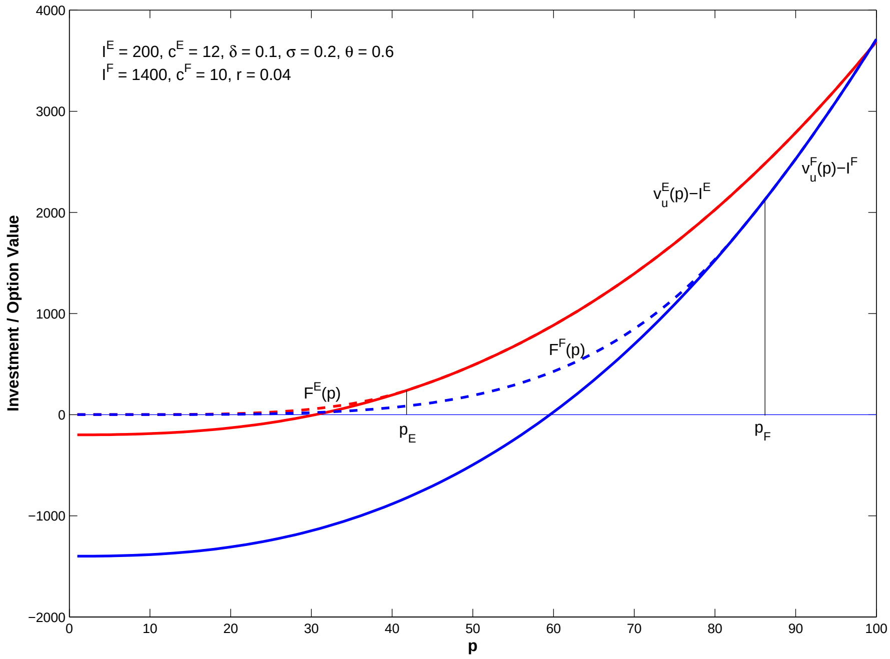


Figure 4: Export or FDI under uncertainty (B)

different relative fixed and variable costs, keeping the remaining parameters unchanged. The export

mode represents the dominant strategy for any price level since the upper envelope is either represented by the option value $F^E(p)$ or by the FDI value function $v_u^E(p)$.
Since the optimal strategy is derived from the upper envelope in figure (4) and since this envelope changes with different relative costs of the two investment modes, a crucial question is, whether it is possible to define unique optimal investment rules within the proximity-concentration trade-off framework with respect to the price volatility σ and the relative cost structure.

The necessary and sufficient condition of the optimal investment

As explained, the investor will choose the optimal investment mode by taking two conditions into account. In the first step, the equilibrium cutoff prices of the two modes are compared.

$$p_E = \sqrt[\kappa]{\frac{\beta}{\beta-\kappa} I^E \frac{\delta'}{Z^E}} \quad \text{and} \quad p_F = \sqrt[\kappa]{\frac{\beta}{\beta-\kappa} I^F \frac{\delta'}{Z^F}} \tag{33}$$

Under which condition the cutoff price p_E is bigger or smaller than p_F can be analyzed by considering the ratio of the two equilibrium prices.

$$\frac{p_E}{p_F} \gtrless 1 \quad \text{if} \quad \frac{I^E}{I^F} \gtrless \left(\frac{c^F}{c^E}\right)^{\frac{\theta}{1-\theta}} \qquad \textbf{(condition 1)} \tag{34}$$

Relation (34) shows that the cutoff price p_E will always be higher (lower) than p_F if the comparative advantage of the export strategy in fixed costs is lower (higher) than the comparative advantage of the FDI strategy in variable costs independently of the price volatility described by equation (13). This is a remarkable result because the proportional relationship between the two stochastic equilibrium cutoff prices is determined only by the deterministic relative costs although each of the equilibrium prices depends on the stochastic goods price. Figure (5) represents the relative fixed and variable costs of the two investment strategies within the proximity-concentration trade-off framework. Since the fixed costs for exporting are assumed to be lower than in the FDI mode, the upper level of the relative fixed costs is equal to unity. Given the assumed variable cost relationship between the two modes, the relative upper margin also equals to unity. The diagonal curve in figure (5) represents states in which the relative fixed costs are equal to the relative variable costs of the

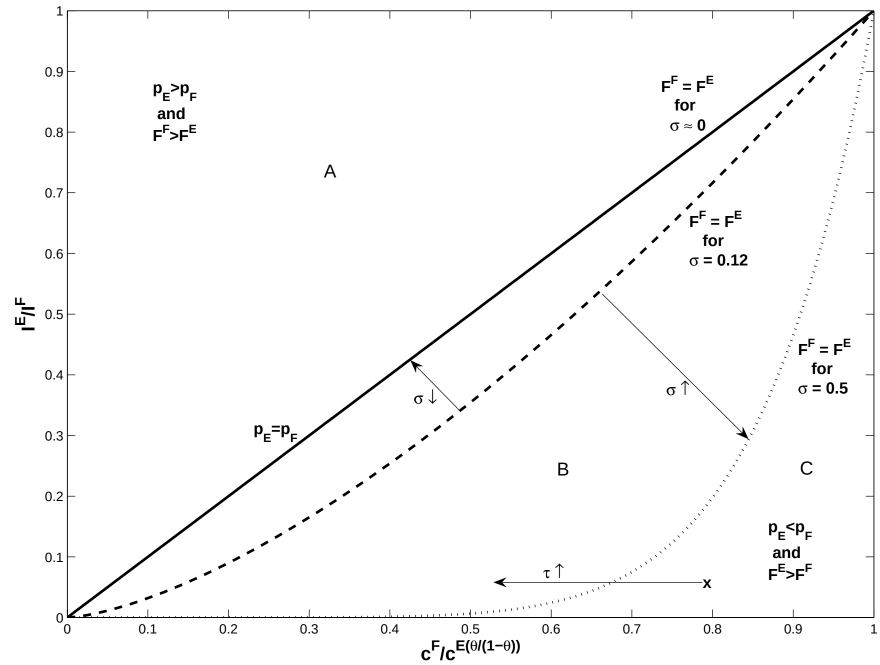


Figure 5: The optimal investment mode under uncertainty

two modes. According to condition 1, for such relative cost relationships the equilibrium cutoff prices are equal ($p_E = p_F$). Any relative cost structure above the diagonal curve provides a cutoff price for FDI which lies below the cutoff price of the export mode ($p_E > p_F$), independently of the degree of volatility σ of goods prices. The cutoff price for exports will be lower than the optimal FDI price for relative cost structures below the diagonal ($p_E < p_F$). The cross in figure (5) represents a specific relative cost structure for the two modes and an investor can immediately derive according to condition 1 that the export cutoff price will be lower than the FDI cutoff price. By comparing the prevailing market price with the two cutoff prices, it is possible to determine whether the value of waiting for each of the two investments is positive. If e.g. the market price is at a level of p_E as in figure (3), the investor would be willed to start exporting. However, the export mode would not be the optimal investment choice at that price level because the alternative mode's option value represents the upper envelope. By comparing the option values of the two investment modes, it is possible to determine the upper envelope in figure (3) for a price range in which both investment modes still possess a value of waiting.

$$F^E(p) = A^E\ p^\beta \lesseqgtr A^F\ p^\beta = F^F(p) \tag{35}$$

The option value of the export mode will be below (above) the option value of FDI if

$$\frac{A^E}{A^F} \lesseqgtr 1 \tag{36}$$

with

$$A^i = Z^i \left(\left(\frac{\beta\, I^i \delta'}{(\beta - \kappa)\, Z^i} \right)^{\kappa^{-1}} \right)^{\kappa-\beta} \delta'^{-1} - I^i \left(\left(\frac{\beta\, I^i \delta'}{(\beta - \kappa)\, Z^i} \right)^{\kappa^{-1}} \right)^{-\beta} \quad \text{for } i \in \{E, F\}. \tag{37}$$

Setting the two option values $F^F(p)$ and $F^E(p)$ relative to each other provides

$$\frac{A^E}{A^F} \lesseqgtr 1 \quad \text{if} \quad \left(\frac{c^F}{c^E} \right)^{\frac{\theta}{1-\theta}} \lesseqgtr \frac{I^E}{I^F} \left(\frac{I^E}{I^F} \right)^{(-\frac{\kappa}{\beta})} \quad \textbf{(condition 2)}. \tag{38}$$

Condition 2 is almost identical to condition 1 except the difference in the relative fixed costs on the right-hand side which turns out to be risk-sensitive since β includes the price volatility σ. For a price volatility equal to zero, the two conditions are identical because β approaches infinity. If the right-hand side in condition 2 equals the left hand-side, an investor is indifferent between the two modes, because the value of waiting in both modes is equal. An increase in the price volatility σ decreases β. As a result, a higher market price volatility increases the range in which the option value of exporting is smaller than the option value of FDI. The economic intuition of this causality is that a higher price volatility increases the value of waiting for FDI in a higher extent than for exporting. In such a case, the upper envelope in figure (3) is predominantly covered by the option function of the FDI mode.

The diagonal continuous curve in figure (5) represents states at which the relative variabel costs of the two modes are equal to their relative fixed cost ratio if the price volatility is close to zero ($\sigma \approx 0$). For relative cost structures which are above the continuous line, the option value of FDI will always be higher than the option value of the export mode. The investor should always wait for the FDI investment as it is representing the upper envelope in figure (3). For relative cost structures in the lower right corner, the option value of exporting will be higher than the option value of FDI. In such a case, the upper envelope in figure (3) will be represented by the export mode. The dashed line in figure (5) represents states at which the two option values are equal to each other with a price volatility $\sigma = 0.12$. Obviously, within the proximity-concentration trade-off framework an increase in the price volatility extends the range in which the option value of FDI is always higher than the option value of the export case. Although both investment modes are confronted with the same price volatility σ, the increase of the value of waiting for the FDI mode turns out to be always higher than the increase in the value of waiting for exporting. The area between the dashed and the continuous curves represents the difference in the value of waiting for the two modes if σ increases.

By taking into account condition 1 and 2 simultaneously it is possible to derive the optimal choice of an investor between the export mode and the FDI mode.

In figure (5) area A represents relative cost structures of the two modes which always result in a lower cutoff price p_F of the FDI mode and simultaneously a lower option value of the export mode. Therefore, in area A FDI will always be the optimal investment mode independently of the degree of price volatility as condition 1 and condition 2 reach their upper margin at the diagonal continuous curve.

For any relative cost structure in area C the export mode always turns out to be the optimal investment strategy because its cutoff price p_E will always be lower than the FDI cutoff price and simultaneously its option value at p_E will be always higher than the option value of FDI. This result only holds under the assumption that goods prices develop according to the geometric brownian motion (13) and the initial price at t_1 is below all cutoff prices.[12] As explained earlier, an increase in σ extends the region in which the option value of FDI becomes dominant and if $\sigma = 0$, the dashed line coincides with the diagonal continuous curve. This can be seen in figure (5). As σ increases, the dashed line becomes more convex and reduces area C.

The last area which needs to be analyzed is region B in which the cutoff price of the export mode will be always smaller than the cutoff price of the FDI mode. Simultaneously, the option value F^F will always dominate the option value of exporting. Figure (3) presents such a scenario in which $p_E < p_F$ and $F^F > F^E$. However, it is necessary to proof that the option value of FDI will be always

[12] One could imagine to draw prices from a distribution without a motion, but such an approach is not in accord with the real option approach and must be modelled in a different way.

the upper envelope between p_E and p_F if $F^F(p_E) > F^E(p_E)$. At a price level p_E, the investor is willed to execute the export mode but will wait, given the value of waiting for the alternative mode. The investor will always wait for the FDI mode if its option value F^F is not crossed by the value function v_u^E of the export mode between p_E and p_F as in figure (3).

If the slope of the FDI option value in p_E is higher than the slope of the export value function and if simultaneously $F_p^F(p_E)$ increases faster than $v_{up}^E(p_E)$, then the option value of the FDI mode will be always the upper envelope in figure (3). This causality holds due to the strict monotonicity (convexity) of the four functions. Therefore, area B in figure (5) would represent unambiguous cost structures for which FDI is the optimal investment mode.

Proof: We know that in p_E

$$F^E(p_E) = v_u^E(p_E) \quad \text{due to matching condition} \tag{39}$$

and

$$F_p^E(p_E) = v_{up}^E(p_E) \quad \text{smooth pasting condition.} \tag{40}$$

For any price $p > p_E$ the option value of FDI will be higher than the value function of exporting

$$\frac{F^F(p_E)}{v_u^E(p_E)} > 1 \tag{41}$$

if

$$\frac{F_p^F(p_E)}{v_{up}^E(p_E)} > 1 \quad \text{and} \quad \frac{F_{pp}^F(p_E)}{v_{upp}^E(p_E)} > 1 \tag{42}$$

with $F^F(p), F^E(p), v_u^E(p)$ and $v_u^F(p)$ strictly convex in p.

The two inequalities in (42) hold for

$$\left(\frac{c^F}{c^E}\right)^{\frac{\theta}{1-\theta}} > \frac{I^E}{I^F}\left(\frac{I^E}{I^F}\right)^{(-\frac{\kappa}{\beta})}. \tag{43}$$

Inequality (43) is equal to condition 2, for the case in which the option value of FDI is higher than the option value of exporting. It states that for relative cost structures which fulfill condition 2, the option value of FDI always will be the upper envelope in figure (3) and the value function of exporting will never cross the option value of FDI. This holds for any price p bigger than p_E. Therefore, area B in figure (5) turns out to represent relative cost structures which will always lead to FDI. This is a remarkable result because according to the analysis so far, it is sufficient to compare the two option values to determine the optimal investment strategy based on the relative cost structure and the prevailing price volatility σ.

Result

Assuming (10) *holds and the investor starts to observe the goods prices at a price level below the cut-off prices, then the export mode will be the optimal investment strategy if and only if $F^E(p) > F^F(p)$. The FDI mode will be the optimal strategy to serve the foreign market if and only if $F^E(p) < F^F(p)$ regardless of the relationship between p_E and p_F.*

Proof: See in the text □.

Introducing uncertainty in goods prices as a geometric brownian motion and giving an investor the possibility of postponing an investment changes the predictions of the proximity-concentration trade-off framework compared with a scenario under certainty. Indeed, an increase in the volatility of goods prices decreases the probability of executing both types of investment, because higher cut-off prices do result. This is a known result from the standard real-option theory. However, in the underlying framework an increase in the goods price volatility not only decreases the probability of executing both investments but furthermore, leads to a situation in which the share of FDI with respect to the export mode increases. Relative cost structures which favor the export mode under certainty, are leading to FDI under goods price uncertainty. Differently expressed, an increase in goods price volatility increases an investor's expected entry price. As the FDI mode delivers higher profits than the export mode at high prices, an increase in goods price volatility promotes predominantly the FDI mode to serve the foreign market.

Comparative statics

Transport costs τ, the goods price volatility σ and the exponent θ of the production function are of major interest within the proximity-concentration trade-off framework, since they are changing the equilibrium conditions.

		Export	FDI	$\frac{Export}{FDI}$
Transport Costs	$\tau \uparrow$	$\downarrow$	$\uparrow$	$\downarrow$
Goods Price Volatility	$\sigma \uparrow$	$\downarrow$	$\uparrow$	$\downarrow$
Degree of Concavity	$\theta \uparrow$	$\downarrow$	$\uparrow$	$\downarrow$
Relative Fixed Costs	$\frac{I^E}{I^F} \uparrow$	$\downarrow$	$\uparrow$	$\downarrow$
Relative Variable Costs	$(\frac{c^F}{c^E})^{\frac{\theta}{1-\theta}} \uparrow$	$\uparrow$	$\downarrow$	$\uparrow$

Table 1: Comparative Statics

Transport costs have been introduced as iceberg costs and are influencing only the variable costs of the export mode. An increase in τ is directly increasing the variable costs c^E. There is no impact on the variable cost of the FDI mode. In figure (5) and in condition (2), it can be seen that an increase of the transport costs is changing the relative cost ratio of the two modes in such a way, that exporting becomes less attractive for an investor.

An increase in θ (with $0 < \theta < 1$) reduces the concavity of the production function in l. Simultaneously, the cash-flow function $\pi(p)$ becomes more convex in p. Since the variable costs of the FDI mode are lower than in the export case, the convexity of the FDI cash-flows is higher, and a rise in θ increases the convexity of $\pi^F(p)$ more than for $\pi^E(p)$. Therefore, a rise in θ increases area B in figure (5) and promotes the FDI mode for a bigger relative cost range.

An increase in the price volatility σ increases the convexity of the indifference curve in figure (5). The range of relative costs which is favoring the FDI mode (area B) increases in σ. This result can also be derived from condition (2). As σ increases, β decreases and the relative cost range which favors FDI increases.

Besides these parameter effects, figure (5) also shows that for relative cost structures with relative high (low) variable costs in the foreign country and relative low (high) fixed costs in the home country, exporting (FDI) is the optimal investment strategy.

3.3. The timing of export and FDI under uncertainty

A convenient aspect of the real option approach is the fact that the cutoff prices are described by parameters without any reference to the probability distribution of the Wiener process in equation (13). However, this convenience appears with the cost of losing the time variable of the model. Based on the cutoff prices, it is possible to identify the optimal investment strategy but it is not possible to derive the timing of the corresponding investment given a specific geometric brownian motion. By running a Monte-Carlo simulation, it is possible to derive probability distributions for

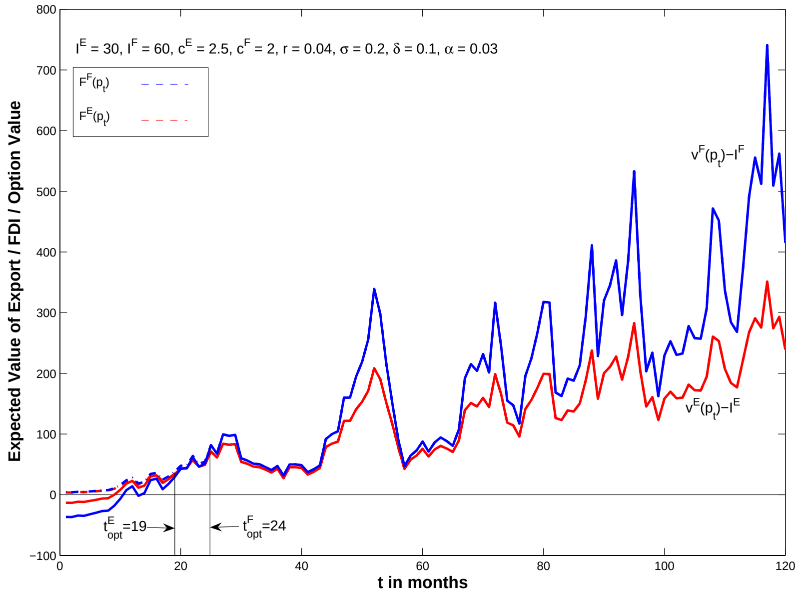


Figure 6: The timing of FDI (A)

the timing of the two investment modes for specific parameters. Figure (6) shows a sample path for the expected investment values of the export and FDI strategies $v^i(p) - I^i$ and the corresponding option values $F^i(p)$. The changes of the goods price over time are measured monthly and are given by the transformed geometric brownian motion

$$p_t = p_{t-1}\ (1 + \frac{\alpha}{12})\ dt + p_{t-1}\ 0.2\ \sqrt{\frac{1}{12}}\ \epsilon_t. \tag{44}$$

The chosen relative costs of the two investment modes are representing a relationship which is falling into area B in figure (5). Therefore, the FDI mode should always be the optimal strategy. Indeed, in figure (6) the upper function at any time t is either represented by the FDI option value $F^F(p)$ or the investment value function $v^F(p) - I^F$. After 19 months, an investor would be willed to serve the foreign market as an exporter, since there is no value of waiting for the export mode in that month. But taking into account the FDI mode, the investor should wait further 5 months and become a foreign direct investor. Clearly, the refusal of the export mode (gathering information) is associated with potential return losses between the 19^{th} and 24^{th} months. However, the waiting strategy turns out to be optimal as the FDI returns always are higher than the export returns and on the long run the initial losses are recovered. Figure (7) shows an alternative sample path for the

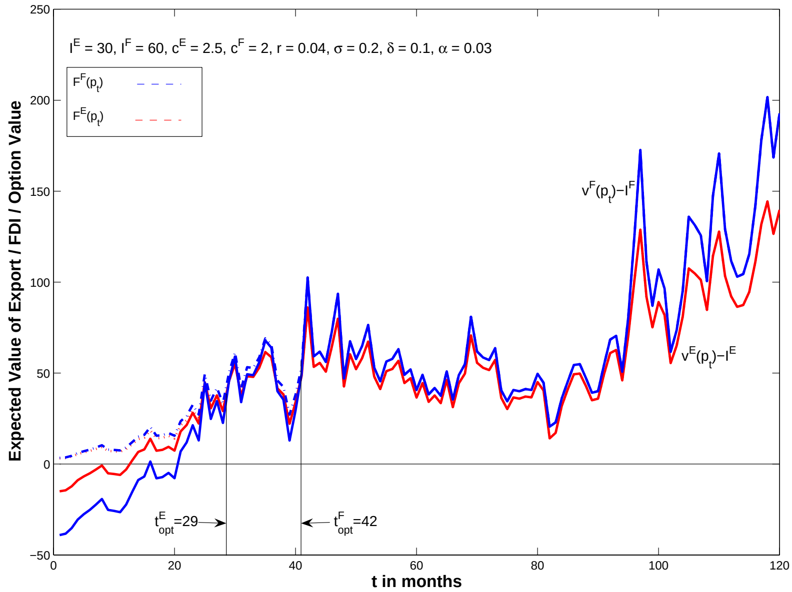


Figure 7: The timing of FDI (B)

different investment modes with the same parameters as in figure (6). As it can be seen, given the stochastic behavior of the goods price, the resulting sample paths can differ heavily from each other. Furthermore, the extent of waiting will be different in each sample path of the simulation.

3.4. Simulation results

The average time of waiting t^i_{opt} for the optimal investment mode is determined by a Monte-Carlo simulation for a specific price volatility σ_j. Based on the the geometric brownian motion (44), a price vector with $t = 120$ elements is generated (investment horizon of 10 years). In accordance with the presented theory, the cutoff prices are calculated and the optimal investment mode is determined. The process is repeated n=100000 times and finishes with the calculation of the average time of waiting for the optimal strategy. The whole procedure is nested into a second simulation which repeats the determination of the average value of waiting for the whole range of defined price volatilities σ, holding the remaining parameters constant (ceteris paribus). Figure (8) depicts the structure of the simulation.

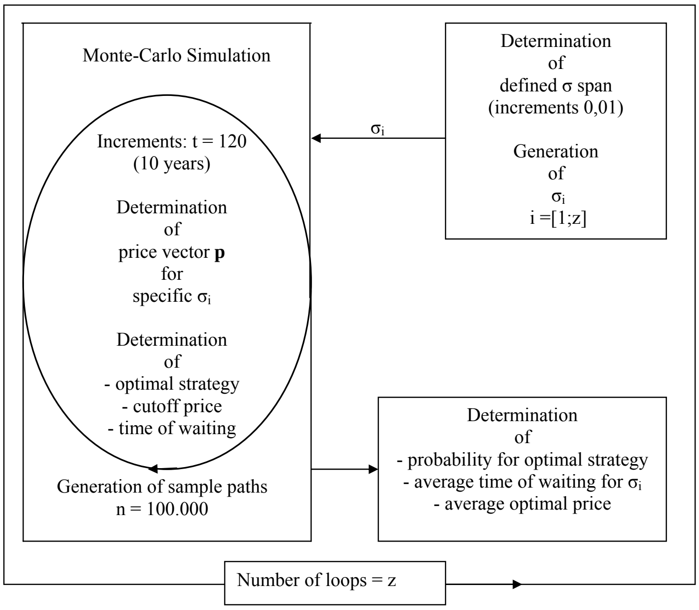
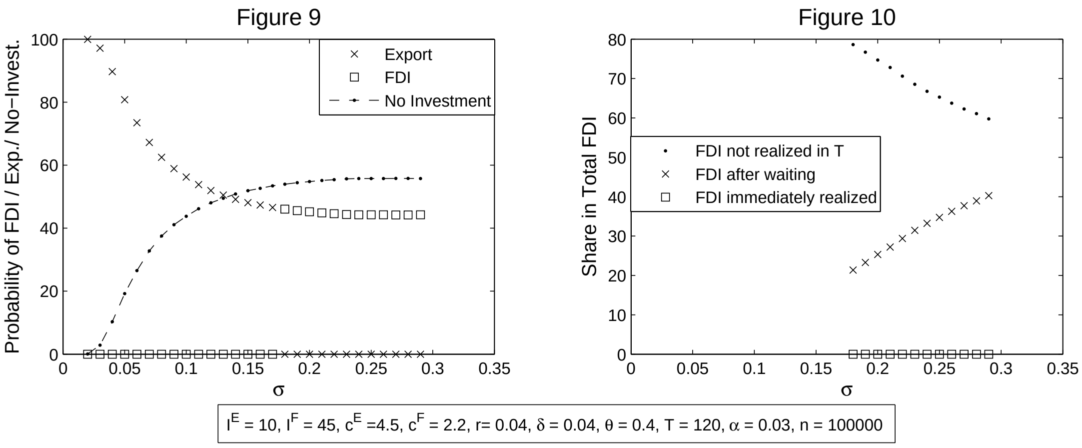


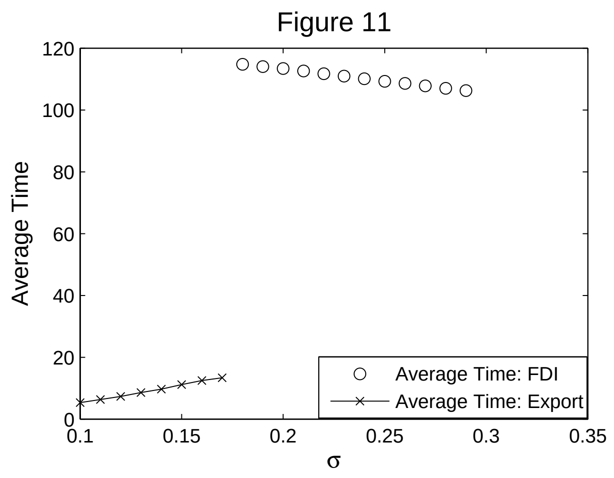
Figure 8: The structure of the simulation

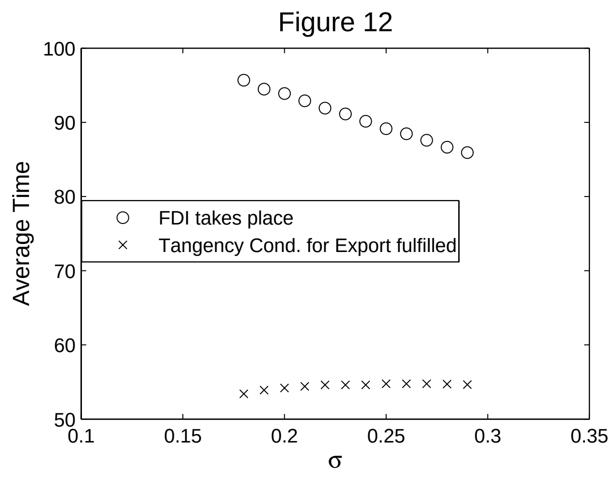
The chosen parameter values represent a relative cost relationship between the two investment modes which lies in area C in figure (5). Figure (9) presents the distribution of chosen strategies within the "n" different paths for each defined σ. As the price volatility increases, the probability of becoming an investor decreases and the probability of neglecting both investments monotonically increases. For the volatility range $0.02 < \sigma < 0.13$, the only investment mode which is taken into account turns out to be exporting. At a price volatility of $\sigma = 0.14$ suddenly FDI becomes the optimal investment mode and keeps on staying. This binary result is consistent with the previous theoretical results where the choice between the two investment modes has been identified unambiguously. The simulation provides the additional information that an increasing goods price volatility decreases the probability of starting any of the two investment modes. Figure (10) shows that FDI always takes place after a time, at which exporting would have been as such profitable. There is no case in which FDI takes place while exporting is not profitable (FDI immediately realized). This is also a predicted result of the model. Within the FDI mode samples, the majority of the investments would be executed after the considered investment horizon of 10 years. If the investment decision was constrained to this time range, the share of the FDI mode would be even lower.[13]



Furthermore, figure (10) shows that the share of the FDI mode which takes place within the time range of 10 years, increases with rising volatility. This phenomenon can be explained by the first passage time for geometric brownian motions (Song et al., 2002; Pattillo, 1998; Abel, 1983). As the price volatility increases, the state variable tends to reach its defined boundary faster. Figure (11) shows that the first passage time increases in the case of export, as the price volatility increases. However, for higher price volatilities the average waiting time turns out to decrease, which is the case in the FDI mode. In other words an increase in σ increases the value of waiting for exporting, as predicted implicitly by the theoretical model. For the FDI mode, a higher price volatility leads to a higher cutoff price but not necessarily to a higher average value of waiting. Figure (12) presents the average time of the FDI modes which appear only within the 10-year investment horizon. Clearly, the average time of the observed state variable decreases in σ.

[13] Although the simulation only considers a time range of t=120, it is possible to determine the optimal investment time for the corresponding FDI paths and include it into the analysis.





According to the simulation, an increase of the goods price volatility σ has 4 crucial effects:

1. The probability of any investment decreases (standard real option result)
2. The probability of serving the market by FDI (export) increases (decreases) (predicted by the model)
3. The value of waiting increases in the export mode (market is observed longer)
4. FDI takes place earlier (first passage time phenomenon).

4. The Choice under Exchange Rate Uncertainty

A crucial type of continuous risk besides goods price uncertainty is represented by exchange rate volatility. Multinational companies which intend to serve a new foreign market have to take into account the development of the exchange rate especially if the destination country has a floating currency system. Repatriated profits from foreign countries may be influenced delicately if a strong exchange rate depreciation appears. On the other hand, an appreciation may increase the value of repatriated profits although the foreign demand has not changed. From a theoretical point of view, the impact of the exchange rate volatility on revenues and therefore on the investment choice is not unambiguous. Empirical literature in international trade has tried to shed light on this theoretical puzzle. A large quantity of studies deliver conflicting results concerning the impact of exchange rate volatility either for FDI or for export behavior (Russ, 2007; Égert and Morales-Zumaquero, 2007). Theoretical models which take the standard real option approach into account generally suggest a negative effect of exchange rate volatility on FDI or export decisions (Campa, 1993). Based on this literature strand, empirical models also suggest a negative impact of exchange rate fluctuation on FDI (Bénassy-Quéré et al., 2001). However, the choice between exporting and FDI within the proximity-concentration trade-off framework under exchange rate uncertainty is not available so far. In the following, I extend the formulated model by combining it with exchange rate volatility. Given the lack of theoretical models which analyze the impact of a foreign exchange rate appreciation (Greenaway and Kneller, 2007), I present a case in which an investor needs to choose between exporting and FDI in a period of exchange rate appreciation.

4.1. The modified model structure under certainty

The assumptions of section 2 are modified in the following manner. The goods price on the foreign market is still exogenous but no longer uncertain. There is an exchange rate between the investor's home country and the new foreign market which is certain for the moment and given by

$$e = \frac{p}{p^*} \tag{45}$$

with p as the price in domestic currency and p^* as the price measured in foreign currency.[14] The profits in the foreign market are supposed to be repatriated, and therefore the investor chooses the market entry mode by comparing the FDI and export investment values, measured in the domestic currency. Since in the export mode output y is produced in the home country, only the revenues are influenced by the exchange rate, whereas in the FDI mode the variable costs (productions costs) are also affected. Therefore, the maximization problem of the investor is given by

$$\pi_t^E(e_t, p_t^*, w_t, \tau_t) = \max_{l_t} y_t^E \; p_t^* \; e_t - w_t \; l_t \quad \text{s.t.} \quad y_t^E = \frac{y_t^D}{\tau_t} \quad \text{s.t.} \quad y_t^D = {l_t}^\theta \tag{46}$$

$$\pi_t^F(e_t, p_t^*, w_t, \tau_t) = \max_{l_t} y_t \; p_t^* \; e_t - w_t \; e_t \; l_t \quad \text{s.t.} \quad y_t = {l_t}^\theta. \tag{47}$$

[14] The described scenario could be interpreted as a fixed exchange rate system.

The resulting periodical cash flows are given by

$$\pi_t^E(e_t, p_t^*, w_t, \tau_t) = (1-\theta)\left(\frac{\theta p_t^{*\frac{1}{\theta}}}{c^E}\right)^{\frac{\theta}{1-\theta}} e_t^{\frac{1}{1-\theta}} \quad \text{with} \quad c^E = \tau_t^{\frac{1}{\theta}} w_t \tag{48}$$

$$\pi_t^F(e_t, p_t^*, w_t, \tau_t) = (1-\theta)\left(\frac{\theta p_t^{*\frac{1}{\theta}}}{c^F}\right)^{\frac{\theta}{1-\theta}} e_t \quad \text{with} \quad c^F = w_t. \tag{49}$$

For a given constant discount rate δ_c and a relative cost structure according to the assumptions of the proximity-concentration trade-off (10), it is possible to determine the values of the two investment modes

$$v^E(e) - I^E = \frac{K^E e^\eta}{\delta_c} - I^E \tag{50}$$

$$v^F(e) - I^F = \frac{K^E e}{\delta_c} - I^F \tag{51}$$

$$\text{with} \quad K^i = (1-\theta)\left(\frac{\theta p^{*\frac{1}{\theta}}}{c^i}\right)^{\frac{\theta}{1-\theta}} \quad \text{and} \quad \eta = \frac{1}{1-\theta} \quad \text{for} \quad i \in \{E, F\} \tag{52}$$

The fixed costs I^i are paid una tantum and priced in the domestic currency (no exchange rate effect).[15] The two investment modes show different curvatures with respect to e. The value of the exporting firm is increasing exponentially in e whereas the FDI mode increases linearly as the exchange rate appreciates:

$$\frac{\partial v^E(e)}{\partial e} > 0 \quad \text{and} \quad \frac{\partial^2 v^E(e)}{\partial e^2} > 0, \tag{53}$$

$$\frac{\partial v^F(e)}{\partial e} > 0 \quad \text{and} \quad \frac{\partial^2 v^F(e)}{\partial e^2} = 0. \tag{54}$$

The over-proportional increase of the export firm value arises from the fact that an appreciation of the exchange rate has a positive impact on the revenue side but no impact on factor prices, since production costs incur in the home country. On the other hand, the FDI value increases with an appreciating exchange rate at a constant rate because besides the revenues production costs also

[15] In the FDI mode, one can imagine that the machineries are acquired in the home country or paid directly from the home country.

increase. Figure (13) depicts the value function for the export mode ($v^E(e)_1 - I_1^E$, $v^E(e)_2 - I_2^E$)

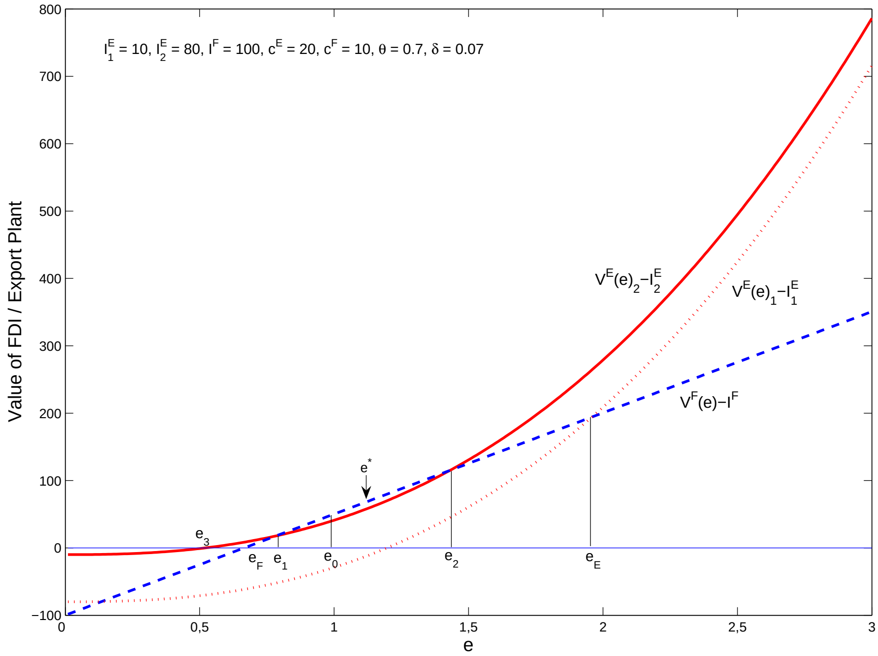


Figure 13: The impact of the exchange rate within the proximity-concentration trade-off framework

with two different fixed costs $I_1^E > I_2^E$ for specific exchange rates at a given goods price. The FDI value function is represented by the linear dashed line $v^F(e)$. Due to the different variable costs in the two entry modes, their slopes differ from each other whereas the level of their value functions is determined by the fixed costs. Comparing the export mode $v^E(e)_2 - I_2^E$ with the FDI mode at an exchange rate $e_0 = 1$ (no exchange rate effect) demonstrates that at the given relative cost structure, the FDI mode would be the optimal entry strategy, because it provides the highest profits.[16] Furthermore, it can be seen that the marginal profits $v^{E'}(e)$ for the export mode are increasing in e whereas in the FDI mode they are constant. A comparison of the two value functions $v^F(e) - I^F$ and $v^E(e)_1 - I_1^E$ provides a unique cutoff exchange rate e_E. For higher exchange rates, exporting is preferable. For any exchange rate below this cutoff value, FDI will be the optimal investment strategy between e_F and e_E. If the exchange rate is lower than e_F, none of the entry modes is executed. Comparing the export mode $v^E(e)_2 - I_2^E$ with the FDI mode results in the two cutoff exchange rates e_1 and e_2. The first cutoff exchange rate e_1 results from the very low fixed

[16] As $I_1^E > I_2^E$, FDI will dominate exporting in the other case, too.

costs in the export mode. Although the FDI mode exhibits an advantage in variable costs at that exchange rate level ($v^{F'}(e) > v^{E'}(e)$), the relatively low fixed costs are promoting the export mode for rates between e_1 and e_3 (fixed cost effect). At an exchange rate e_1, the FDI mode's variable cost advantage offsets the export strategy's fixed cost advantage and for exchange rates bigger than e_1 FDI becomes optimal. At an exchange rate e^* the marginal profits of the two modes are equal ($v^{E'}(e) = v^{F'}(e)$). For any exchange rate above e^* the variable cost advantage of the FDI mode is reduced by the exchange rate appreciation (exchange rate effect), since the factor costs of the export mode are not influenced. Finally, at a rate e_2, the exchange rate effect offsets the variable cost advantage of the FDI mode and exporting becomes optimal for $e > e_2$. For the given relative cost structure in figure (13), the FDI mode turns out to be optimal for exchange rates between e_1 and e_2 if it is compared with $v_2^E(e) - I_2^E$. For an appreciation of the exchange rate by more than 40%, exporting becomes the optimal entry mode.

Exchange rates turn out to influence the choice of an investor's entry mode differently than goods prices even without the introduction of uncertainty.

4.2. FDI or export under exchange rate uncertainty

In the following, the model is modified by assuming that the exchange rate follows a geometric brownian motion with

$$de = \alpha\ e\ dt + \sigma\ e\ dz \quad \text{with} \quad dz = \epsilon\sqrt{dt}. \tag{55}$$

The foreign country is assumed to be in a period of economic recovery, in which the exchange rate is appreciating. Therefore α represents the positive expected appreciation trend. dz represents a Wiener process and is responsible for the uncertainty. σ is the variance parameter which is responsible for the extent of uncertainty. ϵ is a randomly distributed variable with the mean of zero and a standard deviation of one (standard normal distribution). Therefore $E(dz) = 0$ and $E\left[(dz)^2\right] = dt$.[17] The investor will observe the market (waiting strategy) and choose between exporting and FDI, depending on the exchange rate. The exchange rate volatility σ has an impact on the expected export investment value, because of the convexity of the function (Jensen's inequality).

[17] E refers to the expected value.

The risk-adjusted investment value of exporting is given by

$$v_u(e) = \frac{K^E e^\eta}{\delta_e'} \quad \text{with} \quad \delta_e' = r - \eta(r-\delta) - \frac{1}{2}\sigma^2\eta(\eta-1). \tag{56}$$

An increase in the volatility reduces the risk-adjusted discount rate δ_e' and therefore increases the expected returns in the export mode. If the option value of the investment is neglected, an investor would enter a foreign market with high exchange rate volatility at low exchange rates (convexity effect). The expected value of the FDI mode under exchange rate uncertainty is not influenced as it is linear in e (no convexity effect). The corresponding option values $F^E(e)$ for the export mode and $F^F(e)$ for the FDI mode are derived on the basis of the boundary conditions as in section 3 and are given by

$$F^E(e) = Be^\gamma \tag{57}$$

with

$$B = K^E \left(\left(\frac{\gamma\, I^E \delta'}{(\gamma-\eta)\, K^E} \right)^{\eta^{-1}} \right)^{\eta-\gamma} \delta'^{-1} - I^E \left(\left(\frac{\gamma\, I^E \delta'}{(\gamma-\eta)\, K^E} \right)^{\eta^{-1}} \right)^{-\gamma} \tag{58}$$

and

$$F^F(e) = De^\gamma \tag{59}$$

with

$$D = (\gamma-1)^{-(1-\gamma)} \left(\frac{I^F}{K^F} \right)^{(1-\gamma)} \frac{K^F}{(\delta\gamma)^\gamma} \tag{60}$$

and

$$\gamma = \frac{1}{2} - \frac{r-\delta}{\sigma^2} + \sqrt{\left[\frac{r-\delta}{\sigma^2} - \frac{1}{2}\right]^2 + \frac{2r}{\sigma^2}} > 1. \tag{61}$$

Figure (14) depicts the investment value functions and the option value functions. At an exchange rate volatility of $\sigma = 0.1$, the expected export investment value becomes profitable for exchange rates higher than e_u^F. Due to the convexity effect, this rate is lower than the corresponding exchange

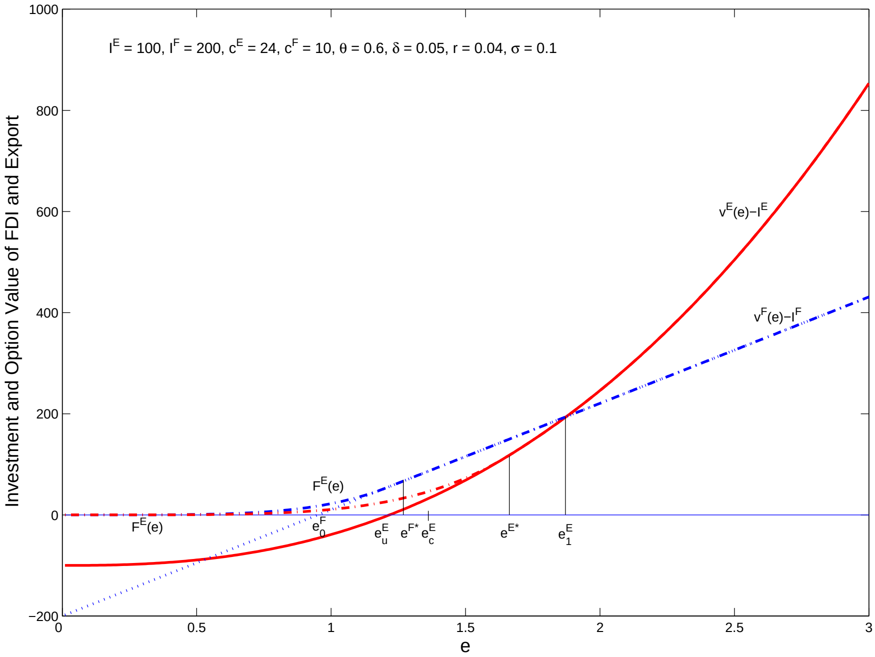


Figure 14: Export or FDI under exchange rate uncertainty (A)

rate e_c^F under certainty. However, if the export option value is taken into account, the investor will invest for rates higher than e^{E*} (uncertainty effect). For any exchange rate smaller than e^{E*}, there is a positive value of waiting and the export decision is postponed. In the FDI mode, there is no convexity effect and as in the certain case the investment turns out to be profitable for exchange rates bigger than e_0^F. However, taking the corresponding option value $F^F(e)$ into account, the investor will observe the market as long as the exchange rate is below e^{F*}, due to the uncertainty effect. The two decisive cutoff exchange rates are given by

$$e^{E*} = \sqrt[\gamma]{\frac{\gamma\ \delta_e'\ I^E}{(\gamma-\eta)K^E}} \quad \text{and} \quad e^{F*} = \frac{\gamma\ \delta\ I^F}{(\gamma-1)K^F}. \tag{62}$$

The optimal entry mode is determined by choosing the upper envelope in figure (14) at the prevailing exchange rates. In contrast to the price uncertainty case, there is not a unique upper envelope function. For exchange rates $e^{F*} < e < e_1^E$, the FDI mode represents the optimal entry mode whereas for exchange rates above e_1^E exporting becomes the optimal choice.

The economic intuition is as follows. For a low exchange rate volatility σ, FDI will be the optimal entry mode if the fixed costs in the export mode are not too low relative to the FDI fixed costs and simultaneously if the variable costs c^F are not too high relative to c^E. For low fixed cost in the export mode, the upper envelope in figure (14) is always represented by the export mode ($F^E(e)$ or $v^E(e) - I^E$). Figure (15) represents the value function for the two modes at the same parameter

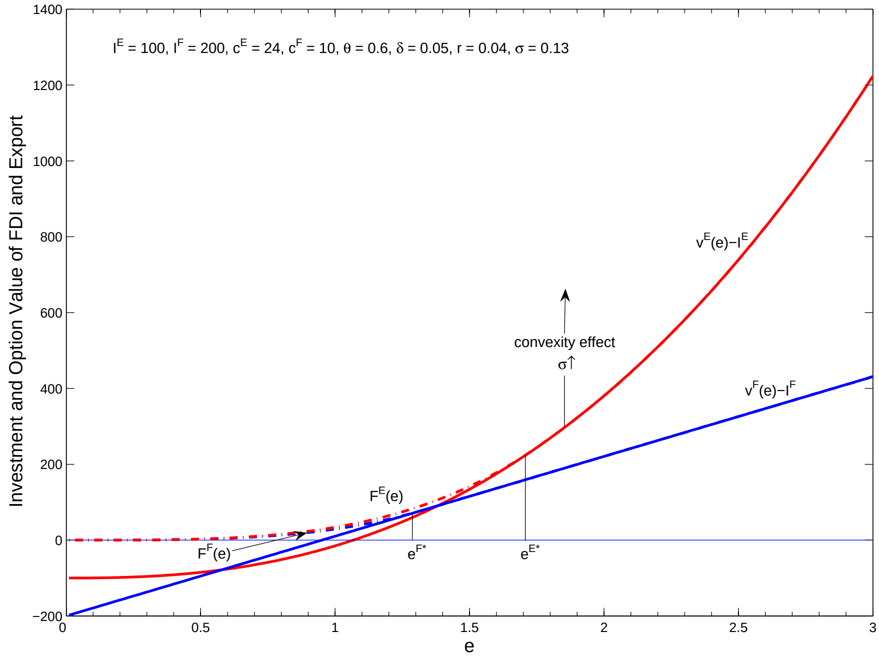


Figure 15: Export or FDI under exchange rate uncertainty (B)

values as in figure (14) except the exchange rate volatility σ. An increase in the exchange rate by only 3% simplifies the choices of the investor, since the upper envelope in the figure is merely represented by the export mode functions. Therefore exporting becomes the only optimal entry mode.

The increase in the exchange rate volatility has an asymmetric impact on the two entry modes. The convexity effect does not appear in the FDI strategy. As a consequence, the value of waiting in the export mode increases faster than in the FDI mode, and in figure (15) the range of exchange rates which promote exporting increases. In contrast to goods price uncertainty, within the proximity-concentration trade-off framework an increase in exchange rate uncertainty promotes exporting.[18]

[18] The result is referring to a scenario in which the foreign country experiences an exchange rate appreciation.

5. Conclusion

Multinational enterprises' choice between exporting and serving a foreign market through an affiliate plant has been explained predominantly by the proximity-concentration trade-off framework within international economics. Based on the assumption of asymmetric cost structures, the framework provides results which are empirically significant (Brainard, 1997; Helpman et al. 2004). Still, certain empirical patterns cannot be explained to a satisfying extent within this framework. Neary (2006) points out that transport costs have decreased steadily in the last years and contrary to the proximity concentration hypothesis the number of foreign affiliate firms has grown much faster than the number of exporting companies. One possible explanation for this ambiguous result is the negligence of continuous risk within the investment choice of multinational firms. Decision makers are confronted with goods price and exchange rate uncertainty and will anticipate and implement these aspects into their final choices. Real option models constitute a theoretical tool for this purpose. They are increasingly applied in corporations and should be therefore taken into account in prevailing models. Indeed, the underlying combination of the proximity concentration trade-off framework with commonly accepted real option assumptions turns out to revert the equilibrium results of standard trade models if goods price uncertainty is taken into account. The presence of volatile goods prices promotes FDI as the optimal strategy to serve a new foreign market. Price risk turns out to be a counteracting force e.g. to decreasing transport costs. Referring to Neary's empirical puzzle, one possible reason for increasing FDI in the presence of decreasing transport costs may lie in the presence of volatile goods prices. Given the increasing importance of emerging markets (UNCTAD-Statistics, 2006) which exhibit higher volatile environments and which simultaneously contribute to the increase in global FDI, the presented theoretical results contain a reasonable rational. Additionally, the extended model demonstrates that different types of price uncertainty have fundamentally different effects. In contrast to goods price uncertainty, exchange rate volatility turns out to promote primarily exporting. Furthermore, the presented model offers the possibility to determine the average timing of market entry in the presence of price risk. Given the lack of appropriate firm level data, the empirical verification is disposed to future research.

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Appendix

A. Risk-adjusted value of an investment

For an uncertain price:

$$dp = \alpha p dt + \sigma p dz \tag{63}$$

Cash flows in each period are given by:

$$\pi(p) = Z p^{\kappa} \tag{64}$$

Valuation by spanning a riskless portfolio:

- holding a unit of the investment project with value $v(p)$ over dt
- short position of $n = v'(p)$ units of output over dt

The value of this riskless portfolio is given by:

$$(\underbrace{\pi(p)}_{cash-flows} - \underbrace{n\delta p}_{dividendpayments})dt + (\underbrace{dv(p)}_{capitalgain} - \underbrace{v'(p)dp}_{costofshortposition}) \tag{65}$$

Using Ito's lemma, the capital gain overt dt can be reformulated as:

$$dv(p) = v'(p)dp + \frac{1}{2}v''(p)\sigma^2 p^2 dt \tag{66}$$

Substituting dp in equation (66) provides:

$$dv(p) = v'(p)\alpha p dt + \sigma p v'(p) dz + \frac{1}{2}v''(p)\sigma^2 p^2 dt \tag{67}$$

Rearranging equation (65) and substituting dv provides:

$$dv(p) - v'(p)dp = dv(p) - v'(p)dp \tag{68}$$

$$= \frac{1}{2}v''(p)\sigma^2 p^2 dt \tag{69}$$

Substituting (69) into equation (65) delivers:

$$(\pi(p) - v'(p)\delta p)dt + \frac{1}{2}v''(p)\sigma^2 p^2 dt \tag{70}$$

$$=(\pi(p) - v'(p)\delta p + \frac{1}{2}v''(p)\sigma^2 p^2)dt \tag{71}$$

As the spanned portfolio is riskless $\Rightarrow$ it must provide the riskless return r in each period dt:

$$r\left[v(p) - v'(p)p\right]dt \tag{72}$$

The result is the following condition:

$$(\pi(p) - v'(p)\delta p + \frac{1}{2}v''(p)\sigma^2 p^2)dt = r\left[v(p) - v'(p)p\right]dt \tag{73}$$

$$\Rightarrow \pi(p) + v'(p)(r-\delta)p + \frac{1}{2}v''(p)\sigma^2 p^2 - rv(p) = 0 \tag{74}$$

with

$$\pi(p) = Zp^{\kappa} \tag{75}$$

The risk-adjusted value of $v(p)$ in equation (74)

Since the second order differential equation (21) is linear in the dependent variable $v(p)$ and its derivatives

$\Rightarrow$ its general solution can be expressed as a linear combination of any two independent solutions:

$$v(p) = A_1 p^{\beta_1} + A_2 p^{\beta_2} \tag{76}$$

In the underlying case, we try the guess solution

$$v(p) = Z_1 p^{\kappa} \tag{77}$$

with

$$v'(p) = \kappa Z_1 p^{\kappa-1} \tag{78}$$

and

$$v''(p) = \kappa(\kappa-1) Z_1 p^{\kappa-2} \tag{79}$$

Substituting the guess function into equation (74) provides:

$$\frac{1}{2}\sigma^2 p^2 \kappa(\kappa-1) Z_1 p^{\kappa-2} + \kappa Z_1 p^{\kappa-1}(r-\delta)p - rZ_1 p^{\kappa} + Zp^{\kappa} = 0 \tag{80}$$

$$(\frac{1}{2}\sigma^2 \kappa(\kappa-1) + \kappa(r-\delta) - r) Z_1 p^{\kappa} + Zp^{\kappa} = 0 \tag{81}$$

$$Z_1 p^{\kappa} = \frac{Zp^{\kappa}}{r - \kappa(r-\delta) - \frac{1}{2}\sigma^2 \kappa(\kappa-1)} \tag{82}$$

The value function of equation (74) is therefore given by:

$$v(p) = \frac{Zp^{\kappa}}{\delta'} \tag{83}$$

with the risk-adjusted discount rate

$$\delta' = r - \kappa(r-\delta) - \frac{1}{2}\sigma^2 \kappa(\kappa-1) \tag{84}$$

B. Solution of a homogeneous differential equation

Given the second order differential equation (21)

$$\frac{1}{2}\sigma^2 p^2 F''(p) + (r-\delta)pF'(p) - rF(p) = 0$$

it is possible to state a general guess solution of the form

$$F(p) = Ap^{\beta} \tag{85}$$

since the differential equation is linear in the dependent variable F. Substituting the guess solution in equation (21) results in the quadratic equation

$$\frac{1}{2}\sigma^2\beta(\beta-1)Ap^{\beta} + (r-\delta)\beta Ap^{\beta} - rAp^{\beta} = 0 \tag{86}$$

$$\frac{1}{2}\sigma^2\beta(\beta-1) + (r-\delta)\beta - r = 0. \tag{87}$$

This quadratic equation is often called the fundamental quadratic equation and can be reformulated as

$$\Psi = \frac{1}{2}\beta^2 - \frac{1}{2}\beta + \frac{(r-\delta)}{\sigma^2}\beta - \frac{r}{\sigma^2} = 0. \tag{88}$$

The two solutions for equation (88) are given by

$$\beta_1 = \frac{1}{2} - \frac{r-\delta}{\sigma^2} + \sqrt{\left[\frac{r-\delta}{\sigma^2} - \frac{1}{2}\right]^2 + \frac{2r}{\sigma^2}} > 1 \tag{89}$$

$$\beta_2 = \frac{1}{2} - \frac{r-\delta}{\sigma^2} - \sqrt{\left[\frac{r-\delta}{\sigma^2} - \frac{1}{2}\right]^2 + \frac{2r}{\sigma^2}} < 0. \tag{90}$$

Therefore the proper shape of the guess solution is given by

$$F(p) = A_1 p^{\beta_1} + A_2 p^{\beta_2}. \tag{91}$$

However, due to the first optimality condition

$$F(0) = 0 \tag{92}$$

the second solution with $\beta < 0$ can be neglected. Otherwise the condition is not fulfilled.
The total differential of the fundamental quadratic equation Ψ delivers some important comparative

results.

As the volatility σ increases, β_1 will decrease. This has an important impact on the wedge in equation (32), since $\frac{\beta}{\beta-\kappa}$ will increase and therefore the expected trigger value of the investment will increase, too.

C. The fundamental quadratic equation

The fundamental quadratic equation (93) can be used to determine the parameter κ.

$$\Psi(\beta) = \frac{1}{2}\sigma^2\beta(\beta-1) + \beta(r-\delta) - r = 0 \tag{93}$$

For a general guess function $F(p) = A_1 p^{\beta_1} + A_2 p^{\beta_2}$, the corresponding graph looks as follows:

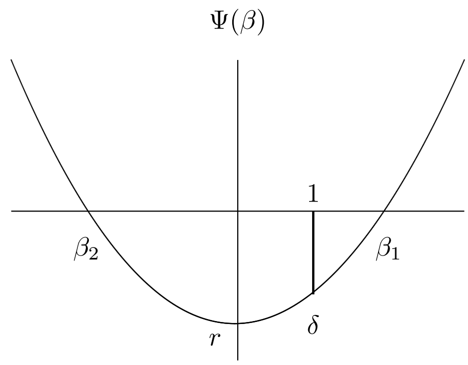

Figure 16: The fundamental quadratic equation

For the underlying value function $v(p)$, it is required that

$$\delta' > 0 \quad \text{with} \quad \delta' = r - \kappa(r-\delta) - \frac{1}{2}\sigma^2\kappa(\kappa-1) \tag{94}$$

otherwise $v(p)$ approaches infinity or is negative (economic intuition does not make sense).

As it can be seen, δ' is simply the negative of the fundamental quadratic Ψ. Therefore, the requirement $\delta' > 0$ is equivalent to the condition $\Psi(\beta) < 0$.

Figure (16) demonstrates, that the fundamental quadratic is negative for betas between β_1 and β_2. Therefore, it can be concluded that for $\delta' > 0$ it must be $\kappa < \beta_1$

As $\kappa = \frac{1}{1-\theta}$, it can be concluded

$$\theta < \frac{\beta_1 - 1}{\beta_1} \tag{95}$$

Furthermore, the production function is defined for

$$0 < \theta < 1. \tag{96}$$

And therefore $\kappa > 0$.

Under these conditions $\delta' > 0$ for $\beta_1 > \kappa > 0$.

D. The threshold price p^*

Given the optimality conditions

$$F(0) = 0 \tag{97}$$

$$Ap^{\beta} = \frac{Zp^{\kappa}}{\delta'} \tag{98}$$

$$\beta Ap^{(\beta-1)} = \frac{\kappa Zp^{\kappa-1}}{\delta'} \tag{99}$$

the cutoff price p^* which determines the investment threshold can be calculated as follows. Solving equation (98) for A provides

$$A = \frac{Zp^{\kappa-\beta}}{\delta'} - Ip^{-\beta}. \tag{100}$$

Substituting A in equation (99) provides

$$\beta p^{\beta-1}\left(\frac{Zp^{\kappa-\beta}}{\delta'} - Ip^{-\beta}\right) = \frac{\kappa Z p^{\kappa-1}}{\delta'} \tag{101}$$

$$\frac{\beta Z p^{\kappa-1}}{\delta'} - \beta I p^{-1} = \frac{\kappa Z p^{\kappa-1}}{\delta'} \tag{102}$$

$$(\beta-\kappa)\frac{Zp^{\kappa-1}}{\delta'} = \beta I p^{-1} \tag{103}$$

$$Zp^{\kappa} = \frac{\beta}{\beta-\kappa} I\delta' \tag{104}$$

$$p* = \sqrt[\kappa]{\frac{\beta}{\beta-\kappa}\frac{I}{Z}\delta'} \tag{105}$$

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