2012-2

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PhD Thesis

Routing and Scheduling in Liner Shipping

DEPARTMENT OF ECONOMICS AND BUSINESS
AARHUS UNIVERSITY • DENMARK

ROUTING
AND
SCHEDULING
IN
LINER SHIPPING

Ph.D. Dissertation

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DECEMBER 2011

Contents

Acknowledgements

It has been a pleasure to be part of CORAL and the former Department of Business Studies for the past 4 years. I would like to thank the administrative staff for helping me solve the administrative mysteries and for a large number of language revisions. I would like to thank the scientific staff for many interesting discussions on a large variety of topics and general inspiration. My fellow PhD students, both past and current, deserve a special thank you for creating a great atmosphere in the BS PhD corridor.

A special thank you is due to my advisor Professor Jens Lysgaard, who has been a steadying influence throughout the writing of this thesis. His constructive comments and guidance have been vital for the development of my thesis.

During my PhD study, I was fortunate to spend six months as a visiting scholar at both CIRRELT in Montreal, Canada, and at the School of Industrial and Systems Engineering at Georgia Tech in Atlanta, USA. I would like to thank Professor Gilbert Laporte for inviting me to visit CIRRELT and for introducing me to a number of interesting people. One of those people is Assistant Professor Oguz Solyali whom I would like to thank for many fruitful discussions on how to decompose my problem. I would also like to thank Associate Professor Ozlem Ergun for inviting me to Georgia Tech and for valuable comments and great discussions. The same thanks go to Associate Professor Alan Erera at Georgia Tech. In addition, I am grateful to Ferdinand Sallings Mindefond for financially supporting my two stays abroad as well as numerous conferences.

Finally, I would like to thank my family and friends for their encouragement and support throughout the past four years. A special thanks to Stig for always being there even when sailing the seven seas, for your insights into all things ship related, and for never letting me give up.

Sincerely,

Karina H. Kjeldsen
Aarhus, December 2011.

Summary

The thesis consists of three papers all related to routing and scheduling of ships and cargo in liner shipping.

Classification of ship routing and scheduling problems in liner shipping The first article provides a classification scheme for ship routing and scheduling problems in liner shipping in line with the current and future operational conditions of the liner shipping industry. Based on the classification, the literature consisting of twenty four articles is divided into four groups whose main characteristics are described. The literature within each group is reviewed, much of it for the first time.

A column generation based heuristic for routing ships and cargo in liner shipping The second article introduces a flow model for the routing of ships and cargo in liner shipping. The problem consists of establishing the routes to be sailed, which ship type to be used for each route, and the cargo routing. The weekly frequency predominant in the liner shipping industry is exploited to establish the number of ships to use per route and their speed. To find a solution to the flow model a Dantzig-Wolfe decomposition is applied. The master problem is solved by commercial software while a heuristic is developed for column generation.

Rescheduling ships and cargo in liner shipping in the event of disruptions The third paper introduces a new mathematical model for the simultaneous rescheduling of ships and cargo in liner shipping in the event of disruptions. The problem is modeled as a multicommodity flow problem with side constraints based on a time-space network. Given a list of disruptions, the planning period, and the ships and ports involved, the problem consists of constructing a set of ship schedules and cargo routings that allow the resumption of scheduled service at the end of the planning period while minimizing the operating cost. A Large Neighborhood Search (LNS) heuristic is developed to solve the problem. The LNS is tested on 20 test instances and the best objective value for all test instances is found within 3 minutes.

Resumé

Denne afhandling består af tre artikler om routing og scheduling af skibe og last inden for linjefart.

Classification of ship routing and scheduling problems in liner shipping Den første artikel introducerer et klassifikationssystem til routing og scheduling problemer for skibe i linjefart. Klassifikationssystemet er i overensstemmelse med både nuværende og fremtidige driftsmæssige betingelser for linjefart. Litteraturen, der består af 24 artikler, klassificeres i henhold til det udviklede klassifikationssystem. Baseret på klassificeringen kan litteraturen inddeles i fire grupper, hvis vigtigste egenskaber beskrives. Litteraturen inden for hver gruppe bliver gennemgået - meget af det for første gang.

A column generation based heuristic for routing ships and cargo in liner shipping Den anden artikel introducerer en flow model for routing af skibe og last i linjefart. Problemet består i at lave sejlruter, bestemme hvilken skibstype, der skal anvendes på hver sejlrute samt at fastlægge en rute for lasten. Den ugentlige frekvens, som er fremherskende inden for linjefart, udnyttes til at fastsætte såvel antallet af skibe, der anvendes per rute og deres hastighed. For at finde en løsning på flow modellen anvendes Dantzig-Wolfe dekomposition. Masterproblemet bliver løst ved hjælp af kommercielt software og subproblemet bliver løst ved hjælp af en heuristik.

Rescheduling ships and cargo in liner shipping in the event of disruptions Den tredje artikel introducerer en ny matematisk model, der sikrer den simultane planlægning af sejlplaner og fragtruter i tilfælde af disruptions. Problemet er modelleret som et multicommodity flow problem med sidebegrænsninger baseret på et time-space-network. Givet en liste af disruptions, en planlægningsperiode samt de involverede skibe og havne, består problemet i at konstruere nye sejlplaner for de involverede skibe og nye fragtruter, der sikrer at de oprindelige sejlplaner og fragtruter kan genoptages ved afslutningen af planlægningsperioden samtidig med, at driftsomkostningerne minimeres. En Large Neighborhood Search heuristik er udviklet til at løse problemet. Heuristikken testes på 20 testinstanser, og den bedste objektværdi for hver testinstans fremkommer inden for tre minutter.

Introduction

Introduction

Routing and scheduling is a large research area within communication and transportation. Within transportation there are four distinct modes which are truck, train, aircraft and ship. Shipping differs from trucking in that for example the draft of a ship is a function of how much weight is loaded on the ship whereby the ship-port compatibility is affected by the cargo loaded. Ships can also be diverted at sea and their voyages span days or weeks both of which are not true for trucks. The major differences between the transportation modes ship and train are that for trains the power unit is not an integral part of the transportation unit which it is for ships. In addition, by adding rail cars the transportation unit size for trains can be enlarged which is not possible for ships. Both planes and ships require large capital investment, they both pay port fees, and both operate internationally. However, airplanes come only in few varieties and do not operate around the clock whereas ships come in a large variety and do operate around the clock, making the two modes unlike as well. Because ships operate under different conditions than the other modes the ship routing and scheduling problems are also different (Christiansen et al., 2004).

Within sea transportation there is a distinction between three separate modes of operation: industrial, tramp and liner (Lawrence, 1972). In industrial shipping the cargo owner also owns the ship. Tramp ships are like taxis as they follow the available cargo and liner ships are operated on published schedule that affect the demand for their services and where each cargo only constitutes a small part of the ship capacity (Ronen, 1983). Continuing growth in the world population, increasing globalization, and the extensive depletion of local resources have resulted in increased world trade. Since there is already a heavy pressure on the road and air networks (Christiansen et al., 2004) and only little possibility of extending them, transportation by ship has experienced a rapid growth.

Today more than 80% of international trade in goods is carried by sea (UNCTAD, 2010b). The majority of these goods is presently transported by ships deployed in the industrial and tramp segments. However, 70% of the seaborne trade, in terms of value, is transported by container ships (WTO, 2008). The container ship fleet, which is exclusively deployed in liner shipping, constituted 13.3% of the world fleet's total deadweight tonnage in 2010. Since 1980 the share of containerized tonnage has increased eightfold, which is a reflection of the increased containerization of the trade in consumer and manufactured goods (UNCTAD, 2010a,c). Despite the growth in the liner shipping segment of the industry, there has been very little research to support it (Sigurd et al., 2007).

In 2005 the notoriously fragmented liner shipping industry saw several high profile acquisitions as Hapaq-Lloyd took over CP Ships, CMA CMG bought Delmas from the Bolloré Group, and A.P. Moller-Maersk acquired Royal P&O Nedlloyd. Despite this, the industry is still highly fragmented compared to other global industries and further mergers and acquisitions are expected (Leach, 2007). Prior to the mergers most scheduling was done manually, but the increased fleet sizes have made this increasingly difficult if an optimal fleet schedule is required (Christiansen et al., 2004).

The current financial crisis has had a tremendous impact on the liner shipping industry. As a consequence of the lower than expected demand for liner shipping transportation services there has been an excess of ship capacity resulting in very low freight rates. As a result, a large number of shipping lines are operating their services with losses. According to Alphaliner (2011), 14 out of 15 surveyed liner shipping companies posted operating losses ranging from -3% to -25% for the 3rd quarter of

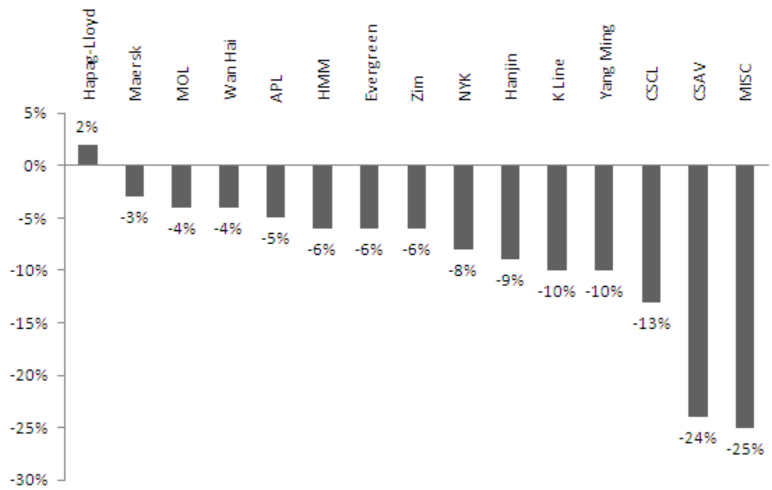

Figure 1: Operating margins for liner shipping companies surveyed by Alphaliner

2011. The decreasing profits have led to increased focus on improving asset utilization and minimizing the major operating costs.

The growth in the importance of the liner shipping industry, the increased fleet sizes operated by the liner shipping companies and increased focus on improving asset utilization and minimizing the major operating costs make it imperative that good solutions are found to the various routing and scheduling related problems in the industry.

Planning levels and terminology of liner shipping

Planning problems in liner shipping can be divided into three planning levels: strategic, tactical, and operational. Despite the heavy integration, especially between liner shipping companies and terminal operators, my focus will be on planning problems relating to the routing and scheduling of ships. Thus, I will not include problems such as terminal design, berth scheduling, or crew assignment. Before proceeding with the discussions of the three planning levels, it is worthwhile to clarify the terminology used both in the discussions and in the following articles.

Terminology

Throughout this introduction to the thesis and in the following articles the terminology below is used. Note that some words have a different meaning than their general usage when used in connection with liner shipping .

- The *shipper* is the owner of the transported cargo and contracts with a liner shipping company for the transportation.
- A *cargo* is a set of containers shipped from a single origin to a single destination. The volume of a cargo is given in TEU.
- *TEU* stands for Twenty-foot Equivalent Unit which is the most common size for a container. The measurement is often used to indicate volume of cargo and capacity of ships.
- *Routing* is the sequencing of port calls to be made by the available ships and the result is one or more routes. When a company involved in liner shipping creates routes, the company will often associate a ship type with each route in order to ensure adequate cargo capacity and compatibility between the ships and the ports that they visit.
- *Scheduling* is concerned with sequencing port calls and fixing the time of each port call for all ships involved.
- *Deployment* refers to the decisions on which of the available ships to use, which ships to use on a specific route and at what speed the ships should sail.
- *Fleet management* is concerned with laying up ships, chartering ships in or out of the fleet as well as the deployment of the ships.
- A *service* is the voyages provided by the collection of ships sailing on a specific route.

- A *voyage* is one traversal of a route starting at a port specified by the ship operator. It is usually one of the primary loading ports.
- A *move* is the act of loading or discharging one container from a ship.
- *Slow steaming* is a term used when the average speed of a voyage is less than the commercial speed. The term comes in three degrees: slow steaming, extra slow steaming, and super slow steaming.

Strategic planning

Strategic planning is concerned with decisions that are medium to long term and sets the stage for the tactical and operational planning. In liner shipping strategic decisions span 1–5 years with some decisions such as new building schemes and green field terminal projects extending 5–10 years into the future. Due to the length of the time horizon and the volatility of the liner shipping industry, knowledge about the future is limited and associated with a high degree of uncertainty. Therefore, strategic decisions are based on aggregated information.

Four of the most important strategic decisions for a liner shipping company are which trade lanes to participate in, the target market share, the long term fleet mix and size, and port choices. Initially, a company has to decide in which trade lanes to participate. Currently, only a handful of shipping lines operate globally and the majority of shipping lines has a regional base. Entry into new trade lanes can be very expensive since it takes time to obtain the volume needed to achieve the economies of scale enjoyed by the present operators. For each chosen trade lane the company has to decide how big a market share it wishes to obtain. Since shipping lines compete for cargo, such a decision influences the service level, e.g. the frequency that the shipping line must provide.

The trade lanes and market share, along with the shipping line's expectations to market growth influence the fleet size and mix problem. Together these aspects give aggregated volumes which can be used to estimate the required ship capacity while the choice of trade lanes decides the types of ships required. If the Far East–Europe trade lane is chosen, the shipping line will require very large ships to compete on economies of scale, and if the choice is the South America–Europe trade lane, ships with plenty of reefer plugs to transport refrigerated food are required. In addition, the shipping line has to decide the desired ratio between company owned and chartered ships. With regard to port choices, the main ports on each trade lane must be decided and so must the main transshipment ports that will connect the network. These decisions must be made in conjunction with the decisions on the fleet size and mix as they influence one another.

Additional strategic decisions for the shipping lines can be found in Christiansen et al. (2004), Christiansen et al. (2007) and Andersen (2010).

Tactical planning

Tactical planning focuses on decisions that are medium term which in liner shipping generally span anywhere from 2 months up to 1 year. The tactical decisions are based on the decisions made at the strategic level and will impact the operational planning decisions. Due to the shorter time horizon, the available information is more reliable, thus making it possible to make decisions based on the information directly rather than on the aggregation of information. In liner shipping, this means that rather than having volume information per trade lane, the volume information is on a port-to-port basis.

According to Christiansen et al. (2007), the main decision at the tactical planning level for liner shipping companies is fleet deployment. According to Andersen (2010) the main decisions at the tactical planning level also include the route design and the scheduling. The three decisions are interrelated and dependent on the strategic decisions made.

The route design or routing problem is the construction of a sequence of port calls that constitutes a route. For each trade lane a number of routes will often be constructed and some routes will cover more than one trade lane. When a liner shipping company creates routes, the company will often associate a ship type with each route in order to ensure adequate cargo capacity and compatibility between the ships and the ports that they visit. The allocation of ship types to routes constitutes part of the deployment decisions.

Scheduling, as opposed to routing, also includes the actual timing of the port calls and thus includes additional deployment decisions for all ships involved. The creation of a schedule includes the decision on which ship to deploy on the schedule and at what speed. All the schedules pertaining to one service are identical except for a time lag corresponding to the service frequency. This means that for each sea leg all the ships assigned to the schedules must maintain the same speed. The complete set of schedules for a liner shipping company constitutes the company's deployment decisions.

The schedules resulting from solving the scheduling problem are normally published for the coming 6 months. However, the published schedule may be changed later as a consequence of other tactical planning problems. Such tactical problems include planning for dry docking of ships in connection with the periodic surveys required for the ships to remain in class and the decision on whether to lay up ships or slow steam in case of excess ship capacity. Other tactical problems relevant for liner shipping companies can be found in Christiansen et al. (2004), Christiansen et al. (2007) and Andersen (2010).

Operational planning

Operational planning is concerned with the short term decisions which span anywhere from a few hours to a few months. The operational decisions are dependant upon the decisions taken at the strategic and tactical planning levels. Due to the short time horizon of this type of problems, information at this planning level is often based on a specific port, ship or cargo.

When a set of schedules from the tactical planning level has to be implemented, each ship in the fleet has to transition from its old schedule to its new schedule. When a chartered ship has to be returned to (received from) the owner, it has to be phased out of (in to) service. In both cases the transition that must be planned is an operational problem. Another type of transition problem, named disruption management, arises when the liner shipping company faces or will face a disruption. Disruption management consists of getting the ships and cargos back on the published schedule within a given recovery period with as little cost as possible.

Another operational problem is the problem of choosing a sailing route between two ports known as route planning or environmental routing (Christiansen et al., 2007). The navigator has to take water depths, currents, tides, waves, winds and company regulations into account. However, if the two ports are separated by a large body of water such as the Pacific or the Atlantic ocean, the navigator is aided by meteorologic services to adequately take into account prevalent weather and currents known as weather routing.

As a result of the trade imbalance large quantities of empty containers build up in ports with net imports. The empty container relocation problem consists of routing the empty containers on the existing network to ports with net exports. Additional operational planning problems appearing in liner shipping can be found in Christiansen et al. (2004), Christiansen et al. (2007) and Andersen (2010).

Articles

Based on the above, I have written three articles concerned with routing and scheduling problems in liner shipping. The first article: *Classification of ship routing and scheduling problems in liner shipping* covers all three planning levels as it surveys all articles on routing and scheduling in liner shipping. The second article: *A column generation based heuristic for routing ships and cargo in liner shipping* is concerned with the tactical problem of routing a given fleet in order to transport a set of known cargos. The last article: *Rescheduling ships and cargo in liner shipping in the event of disruptions* focuses on the operational problem of disruption management.

Classification of ship routing and scheduling problems in liner shipping

The first article presents a classification scheme for ship routing and scheduling problems in liner shipping and subsequently classifies existing literature on the subject according to the developed classification. The development of the classification scheme was based on the fact that there was no existing scheme exclusively concerned with liner shipping. The only prior classification scheme concerned with maritime transportation was done by Ronen (1983) but it encompasses all three operation modes in shipping. Since the three operation modes have significantly different characteristics, the newly developed classification scheme exclusively for liner shipping is different from the one developed by Ronen (1983). The literature on routing and scheduling in liner shipping is largely concerned with specific applications wherefore the problem formulations and solution methods are very diverse. This makes the development of a classification scheme important, as it may serve as a first step towards developing a general model or a group of models that cover the main problems within ship routing and scheduling in liner shipping.

The main problem was to determine which fields should be incorporated in the classification scheme and to establish why they are important. In addition, articles pertaining to routing and scheduling in liner shipping had to be classified according to the developed classification scheme. The purpose of the article is to give an overview of the models developed so far, their assumptions, constraints and solution methods. As such, the article presents the state-of-the-art within the area. This will enable a more informed choice of which model to use in a specific case. In addition, the article can be used for establishing which assumptions and constraints should be included in a generic model.

The method used is a review of existing taxonomies and classification schemes within the area of routing and scheduling combined with a review of the articles that define liner shipping as well as the current literature on routing and scheduling within liner shipping. Lawrence (1972), Ronen (1983), Ronen (1993), and Christiansen et al. (2004) are the main articles used for defining liner shipping. A number of taxonomies and classification schemes exist within the area of routing and scheduling. Bodin and Golden (1981), Bodin et al. (1983) and Desrochers et al. (1990) have all developed classification schemes for vehicle routing and scheduling problems. Assad (1988) also considers the routing and scheduling of the crew which is required to follow the vehicle. Based on the previously noted classifications, Ronen (1983) developed a scheme for use in connection with ship routing and scheduling problems in maritime transportation. Another application-specific scheme was developed later by Ronen (1988) with application to trucks.

The classification scheme has eighteen characteristics with anywhere from two to eight possible options. Twenty-four articles within ship routing and scheduling in liner shipping are classified and all but one option are used. The number of articles was rather small; however, this may be ascribed to the fact that articles concerning fleet management problems were only classified if they also consider routing or scheduling. By characterizing and considering the problems described in the articles, it is possible to divide the articles into groups according to the three problem types: Scheduling, routing and fleet management. Table 1 shows the number of classified articles that fall into each of the three problem types and the four possible combinations of the three problem types. There is a clear focus on problems involving fleet management aspects while scheduling without fleet management is rarely considered in the literature.

	Fleet management	
	No	Yes
Routing	6	8
Scheduling	2	8

Table 1: Classification of liner shipping articles according to problem types.

A column generation based heuristic for routing ships and cargo in liner shipping

The background for this article is the lack of models and solution methods that adequately reflect todays problems in the liner shipping industry. As mentioned above, one part of the literature deals with specific problems and another part of the literature only treats the most standard constraints. Furthermore, Rana and Vickson (1991) state that heuristics might be the only procedure that can solve the larger and more complex problems within liner shipping. This indicates heuristics are needed in future research.

The article has two objectives. The first objective is to develop a mathematical model for the routing of ships and cargo which considers various deployment issues, such as which ships to use on which routes and the speed of the ships. The focus on variable speed is based on the high correlation between speed and fuel consumption (Ronen, 1982) and the current fuel prices. Since the traversal time available for a route is fixed, the speed is influenced by the time spent in the various ports on the route. The most accurate estimate on the time spend in port is obtained if the ship, the port, and the volume of cargo are all taken into account as suggested by Lane et al. (1987). The resulting model is a flow model that incorporates all major aspects of the problem as experienced by the liner shipping companies including variable speed and variable time in port. In addition, the model exploits the weekly frequency generally used in the liner shipping industry to be able to compare the weekly cost of routes with varying time spans.

The second objective is to develop a heuristic which can handle the larger and more complex problems present in todays liner shipping problems. The developed heuristic is based on Dantzig-Wolfe decomposition and column generation. The decomposition splits the problem into a master problem which is a generalized set covering problem and a subproblem which generates promising columns for the master problem. The master problem is solved by commercial software while a heuristic is developed for column generation. The heuristic generates three types of columns a priori, namely dummy columns, transshipment columns, and route columns. Furthermore, the heuristic uses delayed column generation to generate route columns. In order to minimize the number of initial route columns, each column must adhere to five criteria relating to ports and regions when creating port sequences. Once the port sequences are established, four different methods are used for assigning cargo. All four procedures give preference to direct cargo over transshipment cargo.

To test the performance of the heuristic, 24 instance subsets, each containing between 2 and 12 test instances, are generated. The four methods for assigning cargo are tested and the Random method is considered the best. The effectiveness of the methods for restricting the set of allowable routes is tested as well. The methods can reduce the number of initial columns without excluding good solutions. The heuristic can solve problems that has the size of a feeder network and will do so reasonably fast. The longest time spent by the best cargo assigning method, Random, in reaching the final solution is less than an hour. Considering that the problem is tactical and only solved every six months, it is a reasonable time frame.

Rescheduling ships and cargo in liner shipping in the event of disruptions

According to Kohl et al. (2007) one of the objectives of disruption management in the airline industry is *to get back to the plan as soon as possible*. The motivation for this in the airline industry may be debatable (Kohl et al., 2007) but in liner shipping punctuality and reliability with regard to the published schedules have become a competitive requirement. This is underscored by several studies concerning service attributes such as Matear and Gray (1993), Suthiwartnarueput (1988) and Chiu (1996) who all name punctuality and reliability as one of the most important service attributes. Therefore the need to resolve disruptions and get the ships back on schedule as fast as possible in the least costly fashion are vital issues for the liner industry and as such an interesting area for research.

The purpose of this article is to introduce the problem of rescheduling ships and cargo in liner shipping when facing one or more past, current or future known disruptions and to present a method for solving the problem. Given the ships and cargo involved, a list of disruptions, and a recovery period, the problem consists of creating new schedules for the involved ships that adequately take into account the disruptions and new routings for the involved cargos that allow the network to return to its normal state of operation at the end of the recovery period at the minimum cost possible. When rescheduling the ships and rerouting cargos, there is a number of operational constraints that must be taken into account, including (but not limited to) ships' capacities, the ships' minimum and maximum service speed, and the productivity in the ports for the individual ship. The problem is modeled as a multicommodity flow problem with side constraints based on a time-space network.

To solve the problem, a Large Neighborhood Search (LNS) heuristic is developed. The heuristic, which alternates between a construction and a repair phase, iteratively destroys and repairs parts of the solution. The aim of the construction phase is to produce a set of feasible ship schedules given the disruptions. The construction phase may result in destroyed cargo routings. The repair phase attempts to repair the cargo routings destroyed by the disruptions and the construction phase. To diversify the search, randomness is included in both phases. The LNS is tested on 20 test instances and the best objective value for each test instance is found within 3 minutes. Considering that the problem is operational, the solution time is very satisfactory.

Contribution and future research possibilities

The contributions of this PhD project are a classification scheme and a classification of the existing literature on ship routing and scheduling in liner shipping as well as two generic models and solution methods, all of which add to the theory concerning routing and scheduling problems in liner shipping and aligns it with current industry standards and requirements. Furthermore, the developed models and methods can be used in decision support systems in the industry to ensure low cost delivery of the increasing global trade at the right time, at the right place thereby allowing global manufacturers to pursue production philosophies such as JIT and Lean.

A number of future research possibilities arise in connection with the performed research. Each article describes a number of future research possibilities related to the topic of the article. However, there is also a vast number of future research possibilities that are not directly related to the topics of the three articles in this project but which relate to the general topic of the project, namely routing and scheduling in liner shipping. One avenue for future research would be the simultaneous planning of operations at a dedicated terminal and the scheduling of ships calling the terminal. This would mean taking into account issues such as inventory cost and the impact of container dwell time on terminal productivity. Another possibility is to consider where ships should refuel when planning the ship routing. Both the cost of fuel and the availability of fuel vary greatly from port to port. Hence, considering refueling options may very well save money as well as ensure a steady supply of fuel when needed. A third area of possible future research is the simultaneous scheduling of multiple modes of transportation. It would be especially relevant to examine the interaction between ship and train schedules as terminals are often directly connected to the national train network.

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Chapter 1

Classification of ship routing and scheduling problems in liner shipping

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Abstract

This article provides a classification scheme for ship routing and scheduling problems in liner shipping in line with the current and future operational conditions of the liner shipping industry. Based on the classification, the literature is divided into groups whose main characteristics are described. The literature within each group is reviewed, much of it for the first time.

Key words: Transportation, routing, scheduling, classification scheme, maritime transportation

Published in INFOR, Vol. 49, No. 2, May 2011, pp. 139–152.

1.1 Introduction

The majority of international trade is transported by ships as the maritime shipping industry is the major provider of transportation of large volumes over great distances. The shipping industry is generally sub-categorized into three modes of operation namely industrial, tramp and liner (Lawrence, 1972). In industrial shipping, the owner of the cargo is also the owner of the ships. Tramp shipping resembles a taxi company in that the ships are sent to the port from which cargo is to be transported. Liner shipping resembles a bus network as it publishes schedules and competes for cargo based on the service provided (Ronen, 1983).

Within liner shipping, network design is deciding on which ports are to be used as hubs, considering factors such as operational constraints and costs, long-term fleet planning in terms of fleet size and mix as well as the routes and their frequency. The decision on which ports are established as hub ports also reflects the considerations concerning hub-and-spoke versus direct shipment for the various origin-destination pairs, which in turn reflects deliberations regarding transit times for the origin-destination pairs. The approximate transit times together with the frequency of the routes are major contributors in establishing the volume of cargo that the network will be able to garner from liner shipping customers (Christiansen et al., 2007; Reinhardt et al., 2007; Shintani et al., 2007; Agarwal and Ergun, 2008; Gelareh et al., 2010). For companies involved in liner shipping, network design is a strategic problem with a fairly long time horizon given that building new ships and container terminals typically has a completion time of two to five years. The routing problem appears both as part of the network design and as a separate problem with a shorter time horizon of generally six to eighteen months. In both cases, routing in liner shipping is the sequencing of port calls to be made by the available ships. An example of a route can be seen in Figure 1.1. When a company involved in liner shipping creates routes, the company will often

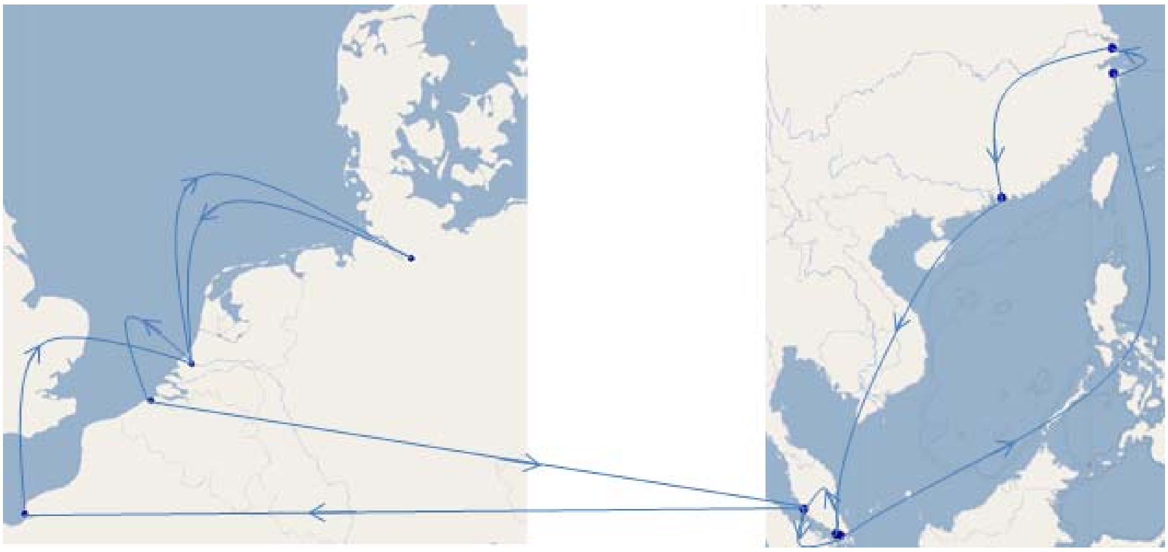

Figure 1.1: A ship route connecting Asia and Northern Europe

associate a ship type with each route in order to ensure adequate cargo capacity and compatibility between the ships and the ports that they visit.

While the routing problem is concerned with sequencing port calls, the scheduling problem is concerned with sequencing port calls and fixing the time of each port call for all ships involved. The two problem types are thus mutually exclusive. Should the sequence of the ports be given, then fixing the time of the port calls is known as a timetabling problem (French, 1982) which is a subproblem of a scheduling problem as the result is a schedule. Table 1.1 shows a schedule for a ship sailing between Asia and Northern Europe. Schedules, such as this, are published by the liner shipping companies with a time horizon of three to six months. When a company involved in liner shipping creates schedules, the company will most often associate a specific ship with each schedule in order to ensure that the published schedules are viable. Viability is required both at the start of the schedule period and on a continuous basis. Initially, the ship must be able to reach the phase-in port at the scheduled time considering the current position of the ship. On a continuous basis, the ship-port compatibility must be such that the ports can maintain a level of productivity on the ship that ensures that the port call can be finished within the time allotted in the schedule. In addition, the ship must be able to maintain the speed required to remain on schedule. Scheduling often involves a high number of deployment decisions which are decisions on which of the available ships to use, at what speed the ships should sail and which ship to use on a specific route. In liner shipping, deployment of the existing fleet is a part of fleet management which in connection with routing and scheduling in liner shipping is concerned with laying up ships, chartering ships in or out of the fleet as well as the deployment of the ships.

Port Name	Arrival Date	Departure Date
Singapore	25-11-2010 02:00	25-11-2010 22:00
Busan	01-12-2010 18:00	02-12-2010 10:00
Xingang	04-12-2010 03:30	04-12-2010 23:30
Dalian	05-12-2010 15:00	06-12-2010 04:00
Qingdao	07-12-2010 06:00	07-12-2010 16:00
Kwangyang	09-12-2010 16:00	10-12-2010 10:00
Shanghai	11-12-2010 20:00	12-12-2010 16:00
Suez Canal	27-12-2010 01:00	27-12-2010 17:00
Bremerhaven	04-01-2011 06:00	05-01-2011 14:00
Hamburg	06-01-2011 07:00	07-01-2011 03:00
Rotterdam	08-01-2011 11:00	09-01-2011 07:00
Felixstowe	10-01-2011 07:00	11-01-2011 15:00
Antwerp	12-01-2011 06:00	12-01-2011 22:00
Suez Canal	20-01-2011 19:00	21-01-2011 17:00
Singapore	03-02-2011 02:00	04-02-2011 15:00

Table 1.1: Schedule for Maersk Seville as found on Maerskline.com.

Ships deployed in liner shipping usually operate on closed routes consisting of a sequence of ports where each port may appear more than once in the sequence. Once a ship is assigned to a route, it will often perform multiple voyages where a voyage is one traversal of the route. Due to the closed routes and the fact that ships often load and discharge in each port of call, the ships are rarely if ever empty and it is therefore difficult to define the origin and destination of a voyage (Ronen, 1983). Furthermore, liner shipping is characterized by heavy containerization and the possibility of trans-shipping containers between routes. Based on these characteristics, and on the fact that, according to Alphaliner (2010), the 100 largest liner shipping companies account for 95% of the TEU capacity and have an average of 48 ships and a minimum of 4 ships, it is unlikely that a liner shipping routing or scheduling problem would contain only one ship.

Between 1990 and 2007, the volume of containers carried in liner shipping had an average annual growth rate of 9.8%. As a result, liner shipping expanded from transporting 5.1% in 1980 to 25.4% in 2008 of the world's dry cargo transported by sea (UNCTAD, 2008). The majority of cargo transported by liner ships is manufactured goods and high-value bulk commodities which have a higher value than the commodities transported in industrial and tramp shipping. Despite the growth in liner shipping and the increasing importance of the industry, research on routing and scheduling has been scarce until 2004, as reported by Christiansen et al. (2004). However, since then the volume of research in the area has been increasing steadily.

There are a number of classification schemes for routing and scheduling problems in transportation, but most are concerned with vehicle routing. Ronen (1983) is the only classification scheme that deals with maritime transportation. This scheme was created to cover all three operational modes of maritime transportation. The first characteristic of the classification scheme relates to the mode of operation but otherwise the scheme assumes that the same operational characteristics are relevant for routing and scheduling problems within the three modes of maritime transportation. Besides the classification scheme, Ronen (1983) also provides a review of ship routing and scheduling and closely related models in the literature. Similar reviews were published in Ronen (1993) and Christiansen et al. (2004), each covering the literature published in the decade between the reviews.

Since the publication of Ronen (1983), the liner shipping industry has developed considerably in terms of operational conditions, and liner shipping now differs significantly from the other two modes of maritime transportation with regard to routing and scheduling problems (Christiansen et al., 2004). Thus, the classification scheme developed by Ronen (1983) no longer reflects the operational conditions of the liner shipping industry. Along with the recent developments in the liner shipping industry, there has been an increase in the number of articles published on the subject of routing and scheduling related problems in liner shipping. This influx of articles has yet to be reviewed as the majority have been published after Christiansen et al. (2004).

The objective of this paper is twofold. First, we want to develop a classification scheme reflecting the operational conditions of ships in liner shipping today and aspects which will become important in the future. Secondly, we want to review the literature concerning routing and scheduling of ships in liner shipping. The articles are grouped according to the developed classification scheme.

The remainder of this paper is organized as follows. In section 2, a classification scheme is developed for routing and scheduling problems in liner shipping. Section 3 presents the classification scheme in a schematic overview and the literature within routing and scheduling in liner shipping is classified according to the developed scheme. In section 4, articles concerning routing and scheduling in liner shipping are grouped based on the classification scheme and subsequently reviewed. Finally, section 5 concludes the paper.

1.2 Classification scheme

The developed classification scheme is based on classification schemes from the vehicle routing and scheduling literature and on the literature concerned with liner shipping. The general classification schemes used can be found in Assad (1988), Bodin and Golden (1981), Bodin et al. (1983) and Desrochers et al. (1990). Ronen (1983) and Ronen (1988) are classification schemes for routing and scheduling of ships and trucks respectively which are also used. The most widely used literature concerning liner shipping is Christiansen et al. (2004) and Ronen (1983). In addition, several of the articles to be reviewed are used to exemplify some of the characteristics.

Though pure liner shipping routing and scheduling problems are substantially different from standard routing and scheduling problems, the classification schemes for the latter constitute a good base for the developed classification scheme. The reason is that the developed classification scheme is supposed to cover all kinds of liner shipping routing and scheduling problems, some of which contain elements that are not generally found in liner shipping. This is also why the articles to be reviewed are addressed while developing the classification scheme. An example is the characteristic *Number of starting points* found in section 1.2.1. According to the definition of liner shipping by Ronen (1983), there should be no such characteristic since liner shipping routes are closed loops without origin and destination, which means that there is no starting point. However, within the RoRo (Roll-on, Roll-of) segment of the liner shipping industry routes are not required to be closed loops and the ships do not necessarily operate on only one route, thus the ships will often have a starting point as will the routes. This is also the case in a large portion of the articles on liner shipping routing and scheduling problems to be reviewed. Hence, in order to cover the full spectrum of liner shipping routing and scheduling problems in the literature, this characteristic has to be included despite being irrelevant according to the definition of liner shipping. A number of the characteristics are based on a compromise between the three forms of literature that have gone into the development of the classification scheme.

1.2.1 Number of starting points

This characteristic is concerned with the number of starting points imposed on each ship or route. A starting point is a port or a position at sea which a route or ship is forced to call. Starting points come in various forms, but share the commonality that they require ships to call certain places or routes to include certain ports. Starting points can for example be the two extreme points between which a route has to be planned as in Rana and Vickson (1991). A starting point can also be the actual starting position at sea or in port for ships at the beginning of the planning horizon as in the RoRo application by Fagerholt et al. (2009). A starting point may also refer to a depot which all routes must contain as is the case in Fagerholt and Lindstad (2000). Within vehicle routing a starting point is a depot, and the possibilities are either one depot or multiple depots (Assad, 1988; Bodin and Golden, 1981; Bodin et al., 1983; Desrochers et al., 1990; Ronen, 1988). These are also the possibilities in Ronen (1983) where depots are termed origins. As in the cited classification schemes, the choices of one or multiple starting points are included which is further supported by the liner shipping articles mentioned earlier in this section. However, in line with the definition of liner shipping which states that a liner shipping voyage may not have an origin or destination as the voyages are closed loops (Ronen, 1983), the option of no starting points is also included. The characteristic thus becomes *Number of starting points* with the choices *None*, *One* and *Multiple*.

1.2.2 Type of operation

The demand experienced in liner shipping is characterized by an origin and destination pair. However, the type of operation varies. The various classification schemes (Assad, 1988; Bodin and Golden, 1981; Bodin et al., 1983; Desrochers et al., 1990; Ronen, 1983, 1988) have different alternatives for the type of operation such as delivery, pick-up and combinations of the two. In Ronen (1983) the alternatives emerge when answering *One* or *Multiple* to the two characteristics *The number of discharging ports per vessel voyage* and *The number of loading ports per vessel voyage*. The answer *Multiple* to both of these characteristics for example indicates a pick-up and delivery problem. However, it gives no indication of whether the actions take place simultaneously or not. If they take place simultaneously, then the problem has interwoven pick-ups and deliveries; otherwise pick-ups and deliveries are separated. The two types of problems differ wherefore it may be valuable to incorporate both possibilities in the classification scheme, as is done in Ronen (1983). However, it can be argued that if liner shipping follows the definition given by Ronen (1983) and consequently loading and discharging take place in each port of call, then an interwoven structure of deliveries and pick-ups should be the predominant alternative. Despite this, there are articles on routing and scheduling in liner shipping which describe a delivery operation (Fagerholt and Lindstad, 2000) while other articles treat a pick-up operation (Fagerholt, 1999). Therefore the following four choices will be available for the characteristic concerning the type of operation encountered in a problem: *Delivery*, *Pick-up*, *Pick-up and delivery separated* and *Pick-up and delivery interwoven*.

1.2.3 Nature of demand

The inclusion of the nature of demand as a characteristic is supported by Assad (1988), Bodin and Golden (1981), Bodin et al. (1983), Desrochers et al. (1990), Ronen (1983) and Ronen (1988). The choices are deterministic demand, stochastic demand and demand dependent on service. The first two choices are the standards in most of the cited classification schemes where demand is either predetermined or stochastic and hence described by a statistical distribution. If the demand is uncertain, a statistical distribution is most often assumed in order to more easily incorporate the demand into a model. The possibility of demand depending on the service provided is considered by Ronen (1983), and it is valid in liner shipping as the demand experienced by the individual liner company is expected to depend on the service that the company provides (Bendall and Stent, 2001; Boffey et al., 1979; Ronen, 1983, 1988). The characteristic *Nature of demand* thus includes the following three choices: *Deterministic*, *Stochastic* and *Demand dependent on service*.

1.2.4 Scheduling constraints at the ports

Issues regarding scheduling constraints at the ports are present in Assad (1988), Bodin and Golden (1981), Desrochers et al. (1990) and Ronen (1983), hence supporting the inclusion of a temporal aspect relating to ports in the classification scheme. Both Bodin and Golden (1981) and Ronen (1983) work with three possibilities concerning the time at which a port can be serviced. The first option is that the time to service a certain port is specified and fixed in advance. The second option is that time windows exist in which a port can be serviced. The last option is that there are no restrictions on when a port is to be serviced. In Desrochers et al. (1990), the choice concerning time windows is either one or multiple time windows. The number of time windows will not affect the nature of the problem; therefore the choices for this characteristic in the classification scheme are *Time of service fixed in advance*, *Time windows* and *No restrictions*.

1.2.5 Number of ships

A characteristic concerning the fleet size is included in Assad (1988), Bodin and Golden (1981), Bodin et al. (1983), Desrochers et al. (1990), Ronen (1983) and Ronen (1988), and the articles thereby support the inclusion of a similar characteristic in the scheme to be developed. Most of the classification schemes include the possibility of a fleet size of one. This is improbable in liner shipping as, according to Lawrence (1972), firms that operate ships in liner service “advertise a scheduled service between specified ports” and “employ a more extensive network of cargo solicitors and agents” which are both highly unlikely in the event that the firm only operates one ship. Assad (1988), Desrochers et al. (1990) and Ronen (1983) include the possibility that the size of the fleet is variable. A variable fleet size indicates that the problem involves fleet management aspects. The choices are therefore that either the fleet size and composition are fixed, or they can be changed. If the fleet is fixed, then the description of the problem will reveal the fleet size. If it can be changed, then the fleet size and composition are determined by the solution, and they are either constant or change over the scheduling period. The problems related to fixed fleet size are short-term problems, and the problems involving changeable fleets can change are medium to long-term problems (Ronen, 1983). The characteristic *Number of ships* is included with the following three choices: *Fixed*, *Changeable – constant over scheduling period* and *Changeable – changes over scheduling period*.

1.2.6 Fleet composition

The inclusion of the characteristic *Fleet composition* is supported by Assad (1988), Bodin and Golden (1981), Bodin et al. (1983), Ronen (1983) and Ronen (1988) as these include the same or a similar field. Either the fleet consists of ships that are identical in all important aspects, and hence the fleet is homogenous, or the ships differ from each other on some or all important aspects, and the fleet is considered heterogeneous. What constitutes important aspects will be defined from problem to problem; however, it is likely to include the aspects of capacity, speed and size in terms of draught, length and width. Since capacity is considered an important aspect and is therefore included in fleet composition, the inclusion of a characteristic for capacity restrictions, as done by Bodin and Golden (1981) and Bodin et al. (1983), is considered redundant. In Desrochers et al. (1990) the possibility of having no capacity restrictions is available. This is unlikely in liner shipping as there are few problems, if any, where it would be possible a priori to discern that no capacity constraints exist. Hence, the possibility of having no capacity constraints will not be included. The characteristic *Fleet composition* will therefore be included with the choices *Heterogeneous* and *Homogenous*.

1.2.7 Cruising speed

This characteristic is concerned with whether or not cruising speed is a decision variable, and the inclusion is supported by Ronen (1983). Ships are not restricted on speed in the same fashion as trucks which have to adhere to the prevalent speed limits as for example indicated by the inclusion of the characteristic Vehicle speed depends on type of road in Ronen (1988). Instead ships have a maximum speed, and they can sail at any speed up to this limit. However, the cost incurred while sailing is closely related to the chosen speed as the fuel consumption is strongly dependent on the speed (Perakis and Jeramillo, 1991). Consequently, the possibility of adjusting the speed is an option that can heavily influence the daily cost of a vessel (Ronen, 1983). Therefore the characteristic *Cruising speed* is included in the classification scheme with the choices

Yes or *No*.

1.2.8 Demand splitting

Since demand is characterized by an origin and destination pair, and it therefore cannot be satisfied from a different origin, demand splitting between ports is not relevant in liner shipping. However, splitting of demand between ships is relevant and this is the form that demand splitting takes in liner shipping. Demand splitting between ships can happen when a demand is to be transported from port A to port B and there are two or more services connecting port A and port B. The split takes place when part of the demand is shipped on one service and the remaining demand is shipped on one or more other services. The inclusion of demand splitting as a characteristic is supported by Bodin et al. (1983) which operates with either allowing or disallowing demand splitting. In Desrochers et al. (1990) it is important whether the splitting is allowed a priori or a posteriori. A posteriori splitting is only relevant when demand is stochastic. Since there are no articles on routing and scheduling in liner shipping which have both stochastic demand and allow splitting of demand, differentiating on the timing of the splitting is likely to be of little, if any, use in liner shipping. As differentiating on the timing of the splitting is not supported by other classification schemes either, it is not included in this scheme. In Assad (1988) the question is whether there are any rules for split deliveries, and Ronen (1988) looks at whether demand splitting is allowed between trucks or sources of origin, which in liner shipping would be ships and ports, respectively. Therefore, *Demand splitting* is included with the choices *Allowed* and *Not allowed*.

1.2.9 Partial satisfaction of demand

This characteristic is concerned with whether or not partial satisfaction of demand is allowed. The inclusion is supported by Bodin et al. (1983), Ronen (1983) and Ronen (1988) as these include the same or a similar characteristic. There can be several reasons for not wishing to load certain cargoes; however, the most prevalent is likely to be that the cost of transporting the cargo is higher than the income that it generates. The characteristic *Partial satisfaction of demand* is included with the choices *Allowed* and *Not allowed*.

1.2.10 Number of capacity types

In liner shipping the majority of the loaded cargo is either containerized or rolling stock, i.e. cars, trucks etc. The capacity for containers is most often measured in TEU (Twenty-foot Equivalent Unit) while the capacity for rolling stock is often measured in CEU (Car Equivalent Unit). In these cases, there is one capacity type. However, if for example container capacity is instead given as one capacity for twenty-foot containers and one capacity for forty-foot containers, as in Reinhardt et al. (2007), then there are multiple capacity types. This is also the case if the capacity is given as TEU capacity and reefer plug capacity. As multiple capacity types are very relevant for real-life problem instances within liner shipping and increase the complexity of the problem, the characteristic *Number of capacity types* is included with the choices *One* and *Multiple*.

1.2.11 Cargo transshipment

This characteristic is concerned with whether or not transshipment of cargo is allowed. The inclusion is supported by Desrochers et al. (1990), Ronen (1983) and Ronen (1988). The last article discusses transshipment between vehicles but in shipping transshipment has to include a port as transfer point. The reason is that the transfer of cargo from one ship to another at sea is likely to cause damage to either the ships or the cargo due to the risk of collision. The possibility of transshipment is important in liner shipping as it is imperative for hub-and-spoke networks which are used increasingly by the liner operators (Baird, 2006). Consequently, the characteristic is included in the classification scheme with the choices *Allowed* and *Not allowed*.

1.2.12 Number of routes per ship

One of the characteristics of liner shipping is that once a ship is assigned to a route, it will remain on the route performing multiple voyages of it during the planning period (Ronen, 1983). While this is true for the containerized liner shipping companies, the RoRo liner shipping companies seldom restrict their ships to one route only. Other applications also permit a ship to be assigned to several different routes during the planning period, to be sailed one after the other (Pesenti, 1995; Cho and Perakis, 1996). The inclusion of this characteristic is further supported by Assad (1988), Desrochers et al. (1990) and Ronen (1988). The last two of the cited articles include the choices *one* and *multiple*, which means that either a ship may only operate on one route during the planning period, or it may operate on several different routes during the planning period. The characteristic *Number of routes per ship* is included with the choices *One* and *Multiple*.

1.2.13 Planning horizon

This characteristic is concerned with the planning horizon, and whether or not it is defined as part of the problem description. If it is defined, then the choice is whether or not the routes must be completed within the planning horizon. The last premise is supported by Ronen (1988). The inclusion is important since the calculation of the objective function varies depending on whether all routes are completed within the given period or not. If they are not, then a rule must be established to take into account the partial routes and thereby the partial incomes and expenses that follow. The characteristic *Planning horizon* is included with the choices *Defined – ships must finish routes in the planning horizon*, *Defined – ships need not finish routes in the planning horizon* and *Undefined*.

1.2.14 Ships required to be empty

According to the definition of liner shipping given by Ronen (1983), ships may never be empty. Despite this, a large portion of the articles on the routing and scheduling of ships in liner shipping require that the ships have to be empty at some point. This characteristic is therefore concerned with whether the ships must be empty at some point or not. The choices are that either the ships are not required to be empty at any point on a route, or they are required to be empty at least once on a route. The distinction is important as the demand for the ships to be empty in one or more ports drastically reduces the level of complexity of the capacity calculations. This is due to the fact that if the ships are not required to be empty in one or more ports, then the capacity calculations have to take into account cargo carried over from previous routes and voyages. Furthermore, the requirement is found in case studies and often it is used as a way of making the problem more tractable. The characteristic thus warns readers of potential simplifications in the articles where the requirement is present.

In the articles where the ships are required to be empty, the requirement is either fulfilled at the starting point, as is the case in Bendall and Stent (2001) and Sigurd et al. (2007), or at the starting points for both the outbound and the inbound voyages, as in Rana and Vickson (1991), where the ship is required to be empty twice on a route. The characteristic *Ships required to be empty* is included with the choices *Yes* and *No*.

1.2.15 Port precedence requirement

A relationship between two ports exists when one has to be serviced before the other. This is a port precedence requirement, and it either exists or it does not. The inclusion of the characteristic is supported by Desrochers et al. (1990), Ronen (1983) and Ronen (1988). A precedence relationship can either be explicitly stated in a problem or it can be implied in the problem description. In liner shipping problems, when the type of demand is pick-up and delivery interwoven, then precedence is implied if the ship has to be empty at a given port, and transshipment is not allowed. The reason is that under these restrictions the only possibility of transporting cargo from the pick-up port (A) to the delivery port (B) is by loading it on a ship in A sailing towards B and reaching B without encountering a port with an empty requirement. The characteristic *Port precedence requirement* is included with the choices *Exists* and *None*.

1.2.16 Requirement for compatibility between ships and ports

This characteristic is concerned with whether or not there is one or more requirements for compatibility between ships and ports. Compatibility covers various issues – e.g. whether a ship can enter a port given the size of the ship (Ronen, 1983). With the increasing size of the liner ships this becomes more and more relevant as the largest ships can only call at a limited number of ports in the world due to their size (Baird, 2006). Only by taking this into account when constructing networks and routes, will these ships provide the optimal results in terms of carrying capacity and savings. Another compatibility issue is the availability of cranes. If a port is not equipped with cranes, it is paramount that the ships calling at the port have cranes. The inclusion of the characteristic is supported by Assad (1988), Desrochers et al. (1990) and Ronen (1988). *Requirement for compatibility between ships and ports* is included in the classification scheme with the choices *Exists* and *None*.

1.2.17 Cost types

The taxonomies by Bodin and Golden (1981), Bodin et al. (1983), Desrochers et al. (1990), Ronen (1983) and Ronen (1988) all include a characteristic concerning cost types; however, the content varies considerably. The most common cost types are variable or routing costs, fixed operating and capital costs and cost of unserviced demand. The last cost type arises for example when unserviced demand is expected to result in lost sales in which case the cost of unserviced demand is the cost of the lost sales. The overall division of costs will be based on the thus described tripartition. The cost types mentioned in Ronen (1983) and in the literature concerning routing and scheduling in liner shipping constitute the base for the cost types included in this taxonomy. The cost types are then as follows: the fixed costs include the fixed cost while in operation and while in lay-up, the variable costs include the steaming cost, the cost of entering a port, the cost of spending time in a port, the cost of cargo operation and the cost of transshipment. The cost of unserviced demand remains a separate category.

1.2.18 Objective

The last characteristic to be included in the classification scheme is the objective. The characteristic is considered in Bodin and Golden (1981), Bodin et al. (1983), Desrochers et al. (1990), Ronen (1983) and Ronen (1988). Historically, objectives have mainly focused on either monetary issues or maximizing utility. The definition of liner shipping states that the objective of liner operations is usually to maximize profits (Ronen, 1983). In case of lacking information minimizing cost can be utilized as the objective as it requires less information (Reinhardt et al., 2007). In light of the increasing focus on environmental issues it is relevant to include an objective concerned with minimizing the environmental impact. The impact could be CO_2 emission or the emission of other damaging substances. Hence the choices for the characteristic *Objective* become *Minimize cost*, *Maximize profit* and *Minimize environmental impact*.

1.3 Schematic overview

Based on the characteristics discussed in the previous sections, a schematic overview of the classification scheme for routing and scheduling problems for ships in liner shipping is developed.

In accordance with the objective of this paper, only articles concerned with routing or scheduling of ships involved in liner shipping are included in the overview. Timetabling of given liner shipping routes has also been included as this is considered a scheduling problem. Two types of closely related problems are excluded from the classification. The first is concerned with ships in liner shipping but not with the routing and scheduling of these. This includes problems related to pure fleet management and thus fleet deployment. Within liner shipping, ships are expected to remain on a closed route once assigned to it and to perform multiple voyages. Hence, from a deployment perspective the ships are assigned to one route and on the basis of this, other deployment decisions such as speed can be decided. However, if a problem does not match this expectation, a ship may be assigned to multiple routes, as is the case in Perakis and Jeramillo (1991) where the model determines how many voyages each ship has to make on each route. One could construe that new routes are being constructed; however, as per Perakis and Jeramillo (1991), this is a deployment problem and not a routing problem. The second type of related problems not included in the classification is concerned with routing and scheduling in liner shipping but not with the routing and scheduling of ships. There are a number of research areas that fall into this category, some of which are routing of empty and laden containers, routing on terminals of terminal equipment and scheduling of quay cranes for loading and discharging sequences. Obviously, these areas of research are outside the scope of this article. This leaves twenty-four articles relating to routing and scheduling of ships in liner shipping which are classified according to the developed classification scheme.

For the sake of convenience each article is assigned a number as shown in Table 1.2. In the schematic overview in Table 1.3 the number is used to indicate which characteristics are applicable for the individual article.

The list of problem characteristics covers the recurring characteristics of routing and scheduling problems in liner shipping. Most problems, however, have their individual idiosyncrasies, and these are not included in the classification scheme as they are particular to a specific problem and therefore not applicable to the problems in liner shipping in general. The references demonstrate that the current literature does not

1 – Kydland (1969)	13 – Fagerholt (2004)
2 – Olson et al. (1969)	14 – Sambracos et al. (2004)
3 – Boffey et al. (1979)	15 – Reinhardt et al. (2007)
4 – Lane et al. (1987)	16 – Shintani et al. (2007)
5 – Rana and Vickson (1991)	17 – Sigurd et al. (2007)
6 – Pesenti (1995)	18 – Agarwal and Ergun (2008)
7 – Cho and Perakis (1996)	19 – Alvarez (2009)
8 – Fagerholt (1999)	20 – Fagerholt et al. (2009)
9 – Fagerholt and Lindstad (2000)	21 – Imai et al. (2009)
10 –Bendall and Stent (2001)	22 – Karlaftis et al. (2009)
11 –Mourão et al. (2001)	23 – Yan et al. (2009)
12 –Ting and Tzeng (2003)	24 – Chuang et al. (2010)

Table 1.2: Numbers assigned to articles.

utilize all available choices. These choices are nevertheless included as the literature on routing and scheduling in liner shipping by no means is exhaustive and therefore cannot be expected to touch upon all facets of liner shipping. In addition, not all the articles are characterized according to all eighteen characteristics. This is due to omitted characteristics not being documented in the articles in question.

1.4 Classification of the liner shipping literature

The objectives of this section are first to divide the articles concerning routing and scheduling of ships in liner shipping into broad categories and second to review the articles based on the classification scheme developed. As routing and scheduling are mutually exclusive the three problem aspects, scheduling, routing and fleet management, can be combined into four different problem types as can be seen in Table 1.4. The problem types containing neither routing nor scheduling aspects are not treated as per section 1.3. The numbers in the body of Table 1.4 refer to the sections in which each problem type is treated. By characterizing the problems described in the articles according to three of the characteristics in the classification scheme while also considering the models presented in the articles, it is possible to categorize the articles in

No.	Characteristics	Options	Articles
1	Number of starting points	None	18-20, 23
		One	1, 6, 8–14, 16–17, 22, 24
		Multiple	3–5
2	Type of operation	Delivery	6, 9
		Pick-up	8, 13
		Pick-up and delivery separated	20, 24
		Pick-up and delivery interwoven	1–5, 7, 10–12, 14–19, 21–23
3	Nature of demand	Deterministic	2, 4–9, 11–23
		Stochastic	1, 24
		Dependent on service	3, 10
4	Scheduling constraints at the port	Time of service fixed in advance	2
		Time windows	4, 9, 11–12, 17–18, 20, 22–23
		No restrictions	1, 3, 5–8, 10, 13–16, 19, 21, 24
5	Number of ships	Fixed	3, 5, 12–13, 15, 21, 23–24
		Changeable – constant over scheduling period	1, 4, 6, 8–11, 14, 16–19, 22
		Changeable – changes over scheduling period	2, 7, 20
6	Fleet composition	Heterogenous	2, 4–9, 11–13, 15–20, 23
		Homogeneous	1, 3, 10, 14, 21–22, 24
7	Cruising speed	Yes	12, 16, 19
		No	1–11, 13–15, 17–18, 20–24
8	Demand splitting	Allowed	1–2, 5, 9–11, 15, 18–19, 23
		Not allowed	3–4, 6–8, 12–14, 16–17, 20–22, 24
9	Partial satisfaction of demand	Allowed	1, 3, 5–6, 10, 16, 18–19, 23–24
		Not allowed	2, 7–9, 11–15, 17, 20–22
10	Number of capacity types	One	3–8, 10–14, 16–17, 19–24
		Multiple	1–2, 9, 15, 18
11	Cargo transshipment	Allowed	11, 14–15, 18–19, 23
		Not allowed	1–10, 12–13, 16–17, 20–22, 24
12	Number of routes per ship	One	1, 4–5, 12, 15, 17–19, 21–24
		Multiple	2, 6–11, 13, 16, 20
13	Planning horizon	Defined – ships must finish routes	3, 5, 8–18, 20, 22
		Defined – ships need not finish routes	2, 4, 7, 19, 21, 23
		Undefined	1, 6, 24
14	Ships required to be empty	Yes	1, 5, 8–10, 13–14, 17, 20, 22, 24
		No	2–4, 6–7, 11–12, 15–16, 18–19, 21, 23
15	Port precedence requirement	Exists	1–2, 5, 10, 17, 24
		None	3–4, 6–9, 11–16, 18–22
16	Ship–port compatibility	Exists	2, 7, 11, 17, 19–20
		None	1, 3–6, 8–10, 12–16, 18, 21–24
17	Cost types	Fixed costs	
		– in operation	1, 6–8, 10–11, 16–17, 19, 21
		– in lay-up	2, 7, 19
		Variable costs	
		– steaming costs	1–2, 4–5, 7–8, 10–11, 13–20, 22–24
		– port entry charges	1, 4–5, 8, 10–11, 13–14, 16, 18, 23
		– time spent in port	4, 18, 23–24
		– cargo operation	1–2, 4, 10, 14, 19–20, 23
		– transshipment	15, 19, 23
		Cost of unserviced demand	19
18	Objective	Minimize costs	4, 7–9, 11, 13–15, 17, 19–20, 22
		Maximize profits	1–3, 5–7, 10, 16, 18, 23–24
		Minimizing environmental impact	

Table 1.3: Schematic overview of the classification scheme and the classified articles.

	Fleet management	
	No	Yes
Routing	1.4.1	1.4.3
Scheduling	1.4.2	1.4.4

Table 1.4: Defining the problem type.

accordance with the problem types mentioned in Table 1.4.

Characteristic (4) *Scheduling constraints at the ports* gives an indication of whether a problem is a routing or a scheduling problem. If the time of service is fixed in advance for one or more ports, it is highly likely that the problem is a scheduling problem. If there are time windows in the problem, it is likely that the problem is a scheduling problem but it depends on whether the width of the time windows is restrictive enough to fix a schedule. In both cases, the problem is only a scheduling problem if the result of the model is a schedule. If the problem is characterized by *No restrictions* in characteristic (4), then the problem is a routing problem.

When deciding whether a problem is a fleet management problem, there are three aspects to consider. First, there is the aspect of fleet size and mix. This is identified by characteristic (5) *Number of ships*. As described in section 1.2.5, the two changeable choices indicate a fleet management problem, while the value *Fixed* confirms that the fleet size and mix cannot be changed. Secondly, there is the aspect of speed as a decision variable in the model. This aspect is considered in characteristic (7) *Cruising speed*. If characteristic (7) takes the value *Yes*, then the problem is a fleet management problem since the deployment decision of setting the speed is considered. Should characteristic (7) take the value *No*, then the speed is given. The last aspect is the assignment of ships to routes. This aspect can only be ascertained by looking at the model. If the model assigns ships to routes, then the underlying problem is a fleet management problem. Only when the fleet size and mix cannot be changed, when the speed is given and the model does not assign ships to routes, is it possible to conclude that the problem is not a fleet management problem.

1.4.1 Routing without fleet management

The basic routing problem consists of a set of ports to be serviced by a number of ships. The problem is to construct a feasible set of routes while adhering to the dictates of the objective. A solution describes a route for each ship where the route is a sequence of ports which the ship has to visit along with an indication of the service to be provided at each location. Whatever temporal restrictions may exist have no impact on the result of the model. To ensure that there are no fleet management decisions in the problem, characteristic (5) must take the value *Fixed*, characteristic (7) must take the value *No* and the model must not assign ships to routes.

The article by Boffey et al. (1979) describes an interactive program and a heuristic method for solving the routing problem. The interactive program uses routes introduced by management; hence, only the problem described to demonstrate the heuristic method will be characterized in the following. All costs are assumed to be fixed in the short to medium term, and the objective is therefore to maximize revenue, which in this case is equivalent to maximizing profit. The demand is dependent on the service provided, and the critical service parameter is the transit time. The demand between two ports is fixed as long as the transit time remains below a stipulated critical limit. Once the transit time exceeds the limit, demand decreases at a fall-off rate which is dependent on the port pair in question.

In the article by Rana and Vickson (1991), the company has the possibility of not loading all available cargo, for example if it is not profitable to transport the cargo or if more profitable cargo is available in another port. Each ship is allowed one route which must be traversed an integer number of times in the planning horizon. However, as defined by Rana and Vickson (1991) a route can contain loops, and one route containing one or more loops can also be viewed as two or more connected routes. An example would be the route 2-4-5-6-4-3-2 where each number indicates a port. The voyage starts and ends in port 2, but it can also be viewed as the two routes 4-3-2-4 and 4-5-6-4 which are connected in port 4. In this case, the demand for one route per ship which is traversed an integer number of times is merely a demand for the routes to be connected and for the connected string of routes to be repeated an integer number of times.

The problem described by Fagerholt (2004) requires that each ship must sail at least one route. The problem is planned as a pick-up problem though in fact it involves interwoven pick-up and delivery. The reason for the simplification is that the empty containers delivered are not expected to influence the capacity constraints, and the problem has been simplified accordingly. The planning horizon is one week, and voyages consisting of multiple routes are expected to finish within this time horizon.

The article by Reinhardt et al. (2007) argues that in order for liner shipping problems to mirror the problems experienced in the industry, loops must be allowed in the developed routes. However, when the formulation is made linear, only simple routes are allowed. In both cases the formulation only permits one route per ship and this route is then repeated throughout the planning horizon. The problem considers multiple commodities because the different container types are considered as separate commodities also with regard to the capacity calculations as the ships have a separate capacity for each of the different container types. Transshipment is allowed and is associated with a cost. Since the cost varies from port to port, the solution to the problem shows the optimal ports for transshipment, the choice of which is part of designing a network.

The article by Imai et al. (2009) compares multi-port calling and hub-and-spoke networks by analyzing the container management cost for various scenarios. The routing problem is connected to the multi-port calling problem where a genetic algorithm is used in an attempt to minimize the distance of travel for the demands weighted by the volume. The problem characteristics reported in Table 1.3 are only related to the multi-port calling problem.

In the article by Chuang et al. (2010), a genetic algorithm is used to find one route for the ports in the problem. The genetic algorithm starts from an initial solution given by experts. The starting port is also the final port of the route and cannot be changed by the algorithm; the remaining ports can be excluded or repeated on the route as the algorithm sees fit. The fitness function is based on profit per time unit. The revenue is based on the cargo transported. Cargo is only transported from a load port if the next port on the route is the destination of the cargo. This means that there is never more than one cargo on board at a time. The cost is a combination of the cost of sailing from port to port and the cost of berthing in the ports based on the time spent at berth.

1.4.2 Scheduling without fleet management

The scheduling problem consists of constructing a feasible set of schedules while adhering to the dictates of the objective. The scheduling problem is characterized by either *Time windows* or *Time of service fixed in advance* in characteristic (4). However, to ensure that the problem is in fact a scheduling problem, the result of the model must be a schedule. As in the previous type of problem, characteristic (5) must take the value *Fixed*, characteristic (7) must take the value *No* and the model must not assign ships to routes.

The problem described in Ting and Tzeng (2003) involves several different types of time windows. There are hard time windows in busy ports and soft time windows in less busy ports where in fact the soft time windows are treated as a lack of time window constraints. In addition, either end of a hard time window may be soft. The goal of the applied dynamic programming algorithm is to minimize the total expected variation in time from available time windows to the estimated time windows. The algorithm starts with a preferred port sequence; however, if the time windows cannot be satisfied by adjusting cruising speed, buffer time and quay crane productivity, then the order of ports located on the same continent may be changed.

Yan et al. (2009) are concerned with the timetabling of two a priori planned routes and with planning the container shipments. The problem is modeled on a time-space network and as an integer multiple commodity network flow problem. The model works with an upper limit on the transit time for the cargo expected to be loaded during the planning horizon. The cost function of the model includes a holding cost which is accrued whenever a cargo is in a port other than the destination port. The capacity constraints of the model vary by voyage leg, which indicates that the model is able to incorporate the issue of draught restrictions.

1.4.3 Routing with fleet management

The routing problem with fleet management is a combination of a routing problem as described in section 1.4.1 and a fleet management problem. This means that besides constructing a feasible set of routes, the model also determines either the number and the types of ships to be used, the deployment of the ships, or both. For a problem of this type, at least one of the following three must be true: i) characteristic (5) takes one of the changeable characteristics, ii) characteristic (7) takes the value *Yes* or iii) the model assigns ships to routes.

Kydland (1969) was one of the first to address the problem of routing and fleet management in liner shipping. The model established by Kydland uses a large number of simulations to establish the optimal port rotation, the number of ships and the type of ships. There are several types of ships under consideration, but each simulation is done with a homogenous fleet. The model permits partial loading of cargo in a port. The cargo not loaded will then be available for the next departure, but if it is not loaded on that departure either, the cargo disappears. The model is described with stochastic demand, though it is also possible to use deterministic demand in the model. The ships are required to be empty at the starting point; however, this restriction can be relaxed in which case the method becomes an approximation, but the degree of approximation can be good if the port with the least amount of cargo in transit is chosen as the starting point.

Pesenti (1995) presents a hierarchical decision model for a heterogeneous fleet for both liner and tramp services. The possible routes are decided a priori by the strategic decision makers wherefore port precedence is not relevant for this article. Because the possible routes have already been established, the deterministic demand is given on a per route basis. This indicates that there is little, if any, interaction between the various routes, and consequently transshipment and splitting of demand between services are not allowed. The company is, however, allowed not to load all available cargo.

The article by Cho and Perakis (1996) presents a strategic routing problem with two optimization models for the problem. The two models differ as the first model has a given fleet size, and profit is maximized. The second model establishes the fleet size and minimizes the cost. Both models include the possibility of laying up ships and thereby changing the number of ships available over the scheduling period. There is no requirement for the ships to finish the planned routes within the planning horizon. This is a consequence of the fact that there is no integrality constraint for the variable that denotes the number of voyages that a ship has on a route during the planning horizon.

The problem presented in Fagerholt (1999) is a pick-up problem, and all the cargo moves from the ports to the starting point. In this problem the starting point is where the described feeder network connects to the main network. The fleet is heterogeneous only with respect to capacity as the speed and loading/unloading capacity are assumed to be identical for the different ship types. The routes are combined to finish within the planning horizon of one week. Thus, by repeating the obtained routings a weekly service of all ports is achieved. The required fleet size is then chartered for the expected duration of the plan.

The article by Sambracos et al. (2004) presents a much aggregated model and a solution that establishes the optimal fleet size and the routing of same. The model allows the possibility of transshipping cargo at all locations, and the solution shows that several transshipment centers are to be used. Since the model is built on the assumption that ports can accommodate any number and any type of ship, the solution provides a guideline as to where port expansions must be undertaken to support the optimal cargo transportation. The model is based on one commodity, namely containers, but the containers envisioned for use in the network are smaller than the standard sizes.

In the article by Shintani et al. (2007) focus is on empty containers, demonstrating that profits increase when repositioning of empty containers is considered if constructing routes. The problem description includes a requirement for weekly service, which means that when setting the planning horizon, the number of ships is fixed and vice versa. In the example the time horizon is fixed at twenty-one days wherefore the number of ships to be used is fixed at three as this is the number required to make weekly calls on a twenty-one-day roundtrip. The cruising speed is a decision variable, and the cost of proceeding at a given speed is included in the total shipping cost.

The article by Alvarez (2009) presents a joint routing and deployment problem where the possibility of rejecting cargo is available. This possibility ensures that the developed model always has a feasible solution. The rejection of cargo affects the objective function as there is a penalty cost associated with each TEU which is not loaded. There is no service requirement incorporated into the model, which means that the service provided to the ports is erratic and has varying transit times. The developed heuristic is fast and should be able to handle problem instances similar to the problems experienced by the major global liner shipping companies.

In the article by Karlaftis et al. (2009) one of the decisions to be made is which of the available ships to include. However, as there is no cost associated with including ships, the developed model focuses solely on minimizing the distance traveled. A genetic algorithm is used to generate a string of ports that can be divided into routes. To ensure that the typical operators in genetic algorithms can be used, the string does not include the main port. The string is divided into trips by a heuristic focusing on utilizing the ships to maximum capacity, thereby countering part of the effect of the fitness function which focuses on minimizing the total time spent. The problem includes a delivery deadline for all cargoes which is the upper limit of the soft time window which – when violated – gives rise to a penalty that is added to the fitness function.

1.4.4 Scheduling with fleet management

The scheduling problem with fleet management is a combination of a scheduling problem as described in section 1.4.2 and a fleet management problem. This means that besides constructing a feasible set of schedules, the model also determines either the number and the types of ships to be used, the deployment of the ships, or both. For a problem of this type, at least one of the following three must be true: i) characteristic (5) takes one of the changeable characteristics, ii) characteristic (7) takes the value *Yes* or iii) the model assigns ships to routes.

One of the first articles concerning with scheduling and fleet management problem in liner shipping was written by Olson et al. (1969). The authors developed a computer program to solve the problem under consideration. The procedure used by the program only remembers the last region visited by a ship and not the previous port, hence it cannot return a ship to its starting port. Therefore there are no starting points. The possibility of laying up a vessel at a given cost is used to control the number of available ships during the scheduling period. The procedure establishes a schedule based on the initial positions of the ships in the fleet. Once the first port for a ship has been decided, the initial positions together with the fixed speed ensure that the time of service is fixed for the first port.

In the article by Lane et al. (1987) the cargo becomes available for shipping at given times. Since there is a cost per unit of time that the cargo is waiting for loading, and a cost per unit of time that the ship is idle waiting for cargo, this gives rise to soft time windows. The ships must be taken from a pool of available ships, and the fleet size and types of ships to be used are part of the solution. Hence, the fleet can both increase and decrease within a given range. However, once the fleet has been established, it is fixed for the planning horizon, and within this period each ship is expected to perform its assigned route, though the ships are not required to finish the routes within the period.

The problem in Fagerholt and Lindstad (2000) is a delivery problem where cargo is loaded from the starting point which, in this case, also functions as the storage location for the cargo. There are three or four weekly calls to each location, and demand is estimated on a weekly basis, which indicates that splitting of demand is necessary. The nature of demand is stochastic; however, it is transformed to a deterministic demand by setting it at 150% of the average weekly demand. Time windows exist because the depot has fixed working hours and several of the installations are closed for operation at night.

In the article by Bendall and Stent (2001) demand depends on the service provided. The number of port calls in a given port during the planning period determines the size of the demand in that port. Port precedence exists because there is a 'natural predetermined pattern' (Bendall and Stent, 2001) for the spoke ports coupled with the fact that all routes also end at the point of starting, here named the hub port. The problem is solved in two stages. In the first stage the number of voyages to be made is found by a linear model, and in stage two these voyages are scheduled manually while taking into account a 24-hour non-operating period for maintenance and repair. The week-long schedule is subsequently repeated for successive time horizon periods.

In the article by Mourão et al. (2001), transshipment is a requirement rather than a possibility, the reason being that transshipment is the only method of getting cargo from the medium network to the feeder network. The two networks are connected in one starting point, namely the hub which is where the transshipment takes place. The ships deployed on one network cannot be used on the other network due to the demand for compatibility between ships and ports. The problem takes into account the ports' physical and logistical conditions and consequently operates with two ship types. The model chooses between different schedules named for the day of the week that the hub port is called.

The article presented by Sigurd et al. (2007) considers the compatibility between ports and ships. The ships have to finish the routes within the planning horizon, but there is no requirement that the ships commence at a specific starting point as long as they wrap around and call at the port from which they started one planning period later.

This ties in with the fact that the two-week plan developed by the model is repeated over a longer period. For some problem instances the time windows in connection with the demand for visit separation and lead time restrictions may lead to several ports having the time of service fixed in advance.

In the problem presented by Agarwal and Ergun (2008), demand is treated as a set of commodities. The demand is identified by a demand triplet, which is an origin port, a destination port and the day of the week when the demand becomes available. The last fact paired with the cost of storing cargo establishes soft time windows. Transshipment is used extensively, but no cost is associated with the transshipments.

In the article by Fagerholt et al. (2009), the problem consists of transporting a specified volume on given trade routes. Each trade route has to be covered by a predetermined number of voyages, each with a time window attached in which a ship will have to commence operation in the first port on the route. The duration of the voyage depends on which ship is assigned to the voyage. The model takes into account the starting position of all own ships and allows for chartering additional ships if this is the most cost efficient way of covering the required voyages. The model can be extended to include spot cargo, in which case the objective of the model changes to profit maximization.

1.5 Conclusion

In the first part of the article we develop a classification scheme for scheduling and routing problems in liner shipping, taking into account current operational conditions and aspects which will become important in the future. One of the main changes in the operational conditions is the increasing connectivity of the networks operated in the liner shipping industry which has prompted the addition of characteristic (8) – *Demand splitting* and the addition in characteristic (17) of the cost type *transshipment*. Furthermore, the industry is moving towards a setup more in line with the description of liner shipping found in Ronen (1983). This has led to the inclusion of several characteristics to support a similar development in the literature. An example is characteristic (12) *Number of routes* which reflects the tendency towards ships remaining on the same route throughout the planning horizon. As a consequence of this the ships may no longer be empty at any port and for that reason characteristic (14) *Ships required to be empty* has been included. With the ships remaining on one route and retaining cargo onboard at all times, it is relevant to include the choice *None* in characteristic (1) *Number of starting points* as no starting point is required for such an operation. Based on an increasing awareness of especially the carbon emissions in the liner shipping industry, the choice *Minimizing environmental impact* has been included as a possible objective. The choice has yet to be used in the literature; however, as the carbon footprint has become a selling point for the liner shipping companies, it is likely to appear in future literature.

In the second part of the article, twenty-four articles are reviewed, fourteen of which have not previously been included in any of the three previous reviews concerned with routing and scheduling in maritime transportation. It is interesting to note that the two important characteristics *Cargo transshipment* and *Cruising speed* have only been included in articles in the past ten years. It is evident from the reviews that the majority of the work within routing and scheduling in liner shipping contains an element of fleet management; this is the case with sixteen of the twenty-four articles reviewed in sections 1.4.3 and 1.4.4. It is worth noting that the articles are evenly split between routing with fleet management and scheduling with fleet management. On the other hand, there are only two articles on scheduling without fleet management, which is in line with the comment in section 1.1 stating that scheduling often involves a high number of deployment decisions and thereby includes fleet management.

1.6 Acknowledgement

The author wishes to thank several anonymous referees whose helpful comments led to an improved paper.

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Chapter 2

A column generation based heuristic for routing ships and cargo in liner shipping

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Abstract

The main problem in the liner shipping industry is designing the service network. The design process consists of deciding the frequency of service between port pairs, establishing the routes to be sailed and which ship type to be used for each route. Furthermore, the cargo-routing must be decided. The weekly frequency predominant in the liner shipping industry is exploited to establish the number of ships to use per route and their speed. A flow model representing the problem is developed. To find a solution to the flow model a Dantzig-Wolfe decomposition is

applied. The master problem is solved by commercial software, while a heuristic is developed for column generation.

Key words: Transportation, routing, network design, maritime transportation

2.1 Introduction

In the last decade a large number of mergers and acquisitions have taken place within the liner shipping industry. This has resulted in larger controlled fleets which in turn have made it impossible to create optimal fleet schedules by pen and paper as done traditionally (Christiansen et al., 2004). Furthermore, the shipping industry has experienced decreasing profits which have led to an increased focus on improving asset utilization and minimizing the major operating cost including fuel consumption. The combination of these factors is the motivation for seeking improved methods for routing and fleet deployment in liner shipping that take into account fuel consumption.

One of the major influences on fuel consumption is the speed at which the ships sail. The speed of a ship is in turn impacted by the time spent in the various ports on a route. This is due to the fact that a fixed amount of time is available for traversing the route and the more time the ship spends in port, the faster it will have to sail in order to make up for lost time. The most accurate estimate on time spent in port is obtained if the ship type, the port, and the volume of cargo are all taken into account as suggested by Lane et al. (1987). Within routing and fleet deployment in liner shipping only Shintani et al. (2007) and Alvarez (2009) consider the speed as a variable rather than a given input. However, in Alvarez (2009) the time in port is fixed for each port-ship combination while Shintani et al. (2007) considers the cargo volumes and the port. Hence neither of the two consider all three aspects influencing the time spent in port.

The model considered in this article includes the speed of the ships as a decision variable. Thus the speed for each ship must be decided. The chosen speed has a tremendous influence on fuel consumption. According to the description of the relationship between speed and fuel consumption in (Ronen, 1982), a ship with a maximum speed of 25 knots can save 36 % fuel by reducing the speed to 20 knots. In addition, the model takes into account the interplay between port, ship type, and cargo volume when estimating the time spent in port. When considering cargo volumes in a liner shipping perspective, it is important to take into account the additional volumes that will be added in some ports due to transshipment.

The objective of the project is twofold. First of all, the objective is to develop a mathematical flow model for the routing of ships and cargo in liner shipping while also considering various deployment issues such as which ships to use on which routes and what speed the ships should maintain. The model incorporates all major aspects of the problem as experienced by the liner shipping companies including the described aspects of variable speed, transshipment, and time in port. Secondly, a heuristic is developed based on Dantzig-Wolfe decomposition and column generation. As part of the heuristic, the performance of various methods for assigning cargo to routes are tested in terms of solution quality and the time required to obtain the solution. In addition, various methods for minimizing the set of allowable routes are tested as well.

In section 2.2 a literature review is introduced. In section 2.3 the problem and the assumptions as well as the notation and the mathematical flow model are described. In section 2.4 the heuristic is presented and in section 2.5 the computational results are discussed. Finally, in section 2.6 concluding remarks are presented.

2.2 Literature review

Despite the fact that maritime transportation is paramount for global trade, the volume of research on ship routing and scheduling is still rather low. However, it has been increasing steadily in the last 15 years. This is especially true for ship routing and scheduling within liner shipping where the number of articles have more than doubled within the last decade. The very first articles on ship routing and scheduling were reviewed by Ronen (1983) and included all three modes of maritime transportation. A decade later a review of the intervening work on ship routing and scheduling was done by Ronen (1993). This was followed by Christiansen et al. (2004) which reflected the increasing focus on maritime transportation. The review by Kjeldsen (2011) covered the entire period but focused solely on routing and scheduling problems as they appear in connection with liner shipping.

Within scheduling in liner shipping, the following four articles touch upon one or more of the aspects mentioned in section 2.1. Ting and Tzeng (2003) include the speed as a variable but the cost of changing the speed is not considered. Port productivity is considered when creating schedules hence the time spent in port depends on the volume and the port but not on the ship. Yan et al. (2009) allow transshipment and include the cost of same in the objective function. The two articles presented in Lane et al. (1987) and Agarwal and Ergun (2008) include the aspect of fleet deployment on top of the scheduling aspects. Lane et al. (1987) introduce the idea that the time in port is dependent on the ship, the port, and the cargo volume, but it is unclear how they use this dependency in the model. In Agarwal and Ergun (2008) the length of a port stay comes in multiples of days and is not dependent on neither cargo volume, port nor ship. The cost of transshipment is investigated during the computational studies in Agarwal and Ergun (2008), although the cost of transshipment is not considered in their initial model.

The following three articles are concerned with the routing problem in liner shipping and consider either the speed, transshipment, or the time spent in port. Reinhardt et al. (2007) impose a demand for weekly service. In addition, the authors allow transshipment of cargo and they include the cost of transshipment in the objective function. Shintani et al. (2007) include the deployment decisions as the speed is variable. The cost of a chosen speed is considered when making decisions. The time spent in port is dependent on the cargo volume and the port. In Alvarez (2009) the speed of the ships also has to be established and the cost of the chosen speeds constitutes a part of the total cost. Transshipment is allowed and has an associated cost. The time spent in port is dependent on the ship type and port but not on the cargo volume.

Set partitioning and column generation techniques are widely used for solving landbased transportation problems. Within maritime transportation the techniques have been applied less frequently, though there are a number of applications within tramp and industrial shipping. Appelgren (1969) applies the Dantzig-Wolfe decomposition to a shipping application with fixed and spot cargo. Kim and Lee (2005) is an example of a tramp application with fixed cargo sizes, whereas Brønmo et al. (2010) is an example of scheduling of tramp ships with flexible cargo sizes. Christiansen and Fagerholt (2002) and Bausch et al. (1998) are examples from industrial shipping. In addition, Christiansen and Nygreen (1998) and Persson and Göthe-Lundgreen (2005) use column generation to solve marine inventory routing problems. In liner shipping set partitioning and column generation has been applied in Lane et al. (1987) where a set of columns are generated a priori at the discretion of an analyst and the set partitioning problem chooses the least cost set of columns which satisfies all requirements. In Fagerholt (1999) the set partitioning problem has the same function and all viable columns are generated a priori based on combining single routes into multiple routes. Sigurd et al. (2007) use delayed column generation where columns are added in the branching process.

2.3 Problem Description

The liner shipping problem consists of creating routes for a heterogenous fleet of ships that ensure that all demands between the ports are delivered at the lowest possible cost. The pick-up and deliveries are interwoven and each demand is known by the port pair that stipulates the origin and destination. In liner shipping demand is considered to be dependent on the service that the liner shipping company provides (Bendall and Stent, 2001; Boffey et al., 1979; Ronen, 1983, 1988). One of the most important service parameters is the frequency of service offered for each port pair (Brooks, 1985). Hence by fixing the frequency it is possible to fix the demand to some extend. The major liner shipping companies such as Maersk Line (Maersk, 2011), MSC (MSC, 2011), CMA-CMG (CMA-CMG, 2011), Evergreen (Evergreen, 2011) and Hapag-Lloyd (Hapag-Lloyd, 2011) use weekly frequency for most of their services. Weekly frequency is also used in the model by (Reinhardt et al., 2007), (Agarwal and Ergun, 2008), and (Yan et al., 2009), wherefore this frequency has been chosen. When the frequency is fixed, the use of deterministic demand becomes less of an approximation.

Splitting demand on different routes is not allowed. This is mainly due to the fact that the effect of splitting demand is very small when the individual demands are small compared to the size of the ships (Dror and Trudeau, 1990) and when splitting demand would likely lead to an increase in port calls (Fagerholt, 2004) both of which is the case for a global liner shipping company. Partial satisfaction of demand is not allowed, because if it was allowed while the objective is to reduce the cost, then no cargo would be loaded, the ships would not be used, and the cost would be zero.

All demand is shipped in containers which come in several sizes. The two most prevailing sizes are TEUs (Twenty-foot Equivalent unit) and FEUs (Forty-foot Equivalent Unit). The number of TEUs is easily obtained as one FEU is equivalent to two TEUs. In the following, demand and ship capacities are measured in TEUs. When the liner shipping companies estimate the cost of transshipment and the time spent on discharges, loadings and transshipment, the measurement used is the number of moves. A move is the action of discharging or loading a container regardless of the container size. Thus there will not be the same number of moves as the number of TEUs when for example discharging at a port. However, the major shipping lines have a more or less fixed ratio between TEU and FEU for most trades. This ratio is used to correct the cost per transshipment and the time consumption per move so that the transshipment cost is per TEU and the time consumption is per TEU. The time consumption per TEU is dependent on the port and ship type; the port because each port has different ground handling capabilities and number of cranes, and the ship type because the size of the ship decides how many cranes can be operated on it simultaneously.

Transshipment is an intrinsic part of liner shipping. Since this is a tactical problem, port capacities and capabilities are fixed when it comes to transshipment. Therefore only a limited number of ports can function as transshipment hubs. This results in a limited number of transshipment options for each demand. The number of options will

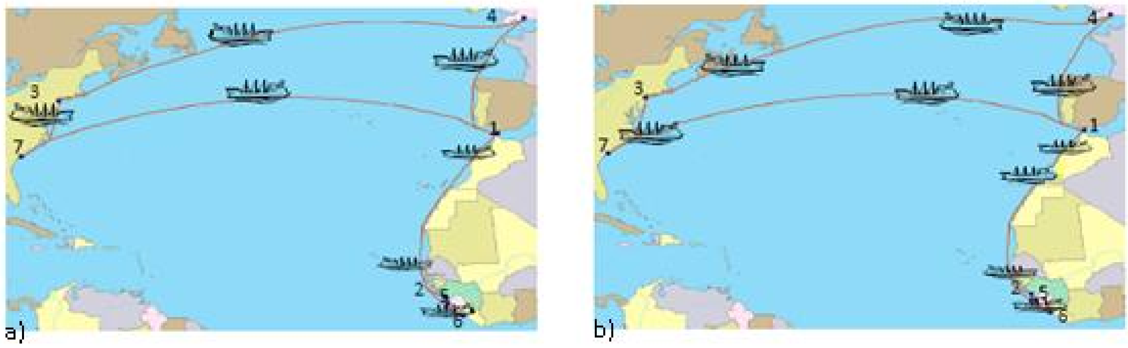

Figure 2.1: a) and b) show two variants of the routes 1-2-6-5 and 1-4-3-7.

depend on the number of ports in the network that has transshipment capabilities.

Ships involved in liner shipping are never expected to be empty (Ronen, 1983). Routes are cyclic and a ship is assigned to one route which it sails continuously. Therefore, all cargos loaded in a port can eventually be discharged in any other port on the route and thus there is no need for a port precedence requirement. Compatibility between ships and ports is required to ensure that ports can accommodate the ships calling and to ensure that cranes are available either in the ports or on the ships.

The fleet consists of a number of ships that can be divided into ship types. Ships of the same type will have similar physical characteristics such as speed capabilities, fuel consumption, draft and capacity. The major shipping lines have a number of ships of each type, and there seems to be a strong preference for similar ships on a route. Since all routes are sailed every week and the demand is identical week after week, this ensures that all cargo can be accommodated. This further guarantees that the number of ships required will be fixed. The speed of the ships is included as a decision variable because the time required to traverse each route greatly depends on the chosen speed as does the cost.

The weekly frequency on a route is obtained by having ships sail in a staggered pattern so that any two consecutive ships are exactly one week apart. This can be achieved by choosing a speed that ensures that the route takes an integer number of weeks to perform and then have the same number of ships sail the route exactly one week apart. That is, the number of ships sailing the route equals the route's duration measured in weeks.

This is exemplified in figure 2.1 where the routes in a) and b) have the same port sequences. However, they differ with respect to the durations and the number of ships sailing the routes. For example, in a) the upper route is a four week round trip sailed by four ships at a speed of 13.3 knots, whereas in b) the upper route is a five week round trip sailed by five ships at the lower speed of 10.5 knots. The idea of having the number of ships on a route equal the route's duration measured in weeks is also found in Agarwal and Ergun (2008). However, they present a model where each ship type has a fixed speed rather than a variable speed as is the case here.

Since the duration of a route in weeks is identical to the number of ships assigned to a route, the weekly cost of sailing a route (in total over all ships on the route) is equal to the cost of having one ship sail the entire route (in total over the duration of the route). Using this relation, the weekly cost of including any route is available irrespective of its duration, whereby it becomes possible to compare routes with varying time spans. As such, the length of the planning horizon is not required in the model. This is a central element in this paper's model construction. The corresponding mathematical aspects are explained further in subsection 2.3.1.

2.3.1 Notation

The mathematical model for the liner shipping problem is based on a flow formulation. Let N be the set of ports, and let A be the set of arcs (i, j) with $i, j \in N$ so that one or more ship types can feasibly sail from port i to port j if and only if (iff) $(i, j) \in A$. The distance from port i to port j, measured in nautical miles, is denoted d_{ij}. The set of ship types is denoted T. The port set $N_t \subseteq N$ is the set of feasible ports for ships of type t, and $A_t \subseteq A$ is the set of feasible arcs for ships of type t, i.e., a ship of type t can feasibly sail from port i to port j iff $(i, j) \in A_t$. Each ship type t has an upper and lower bound on the speed that the ships of that type can maintain over an extended period of time, these are denoted by S_t^- and S_t^+. In addition each ship type t has a given capacity denoted by Q_t. F_t is the fuel in tons per day consumed by a ship of type t when sailing at speed S_t^+. For each ship type t, V_t is the set of ships of type t indexed by v where $V_i \cap V_j = \emptyset$ for all $i \neq j$.

The set of cargos is denoted M. The volume of cargo m, measured in TEUs, is denoted G_m, and K_{mi} indicates whether there is a supply of cargo m available in port i or not. The origin and destination ports for cargo m are denoted O_m and D_m, respectively. The set of ports in which transshipment of cargo m is permitted is denoted $Z_m \subset N$. The time, measured in days, required in port i to either discharge one TEU from a ship or load one TEU onto a ship of type t is denoted E_{it}. Finally, B denotes the maximum number of times a port may be visited on one route.

There are four cost types in the model. The cost of one ton of fuel is denoted by C^F, C_t^U is the cost incurred when a ship of type t is used, C_i^T is the cost of loading or discharging one TEU at port i in connection with a transshipment, and C_{it}^P is the cost sustained when a ship of type t calls at port i.

The definition of decision variables in the model is somewhat untraditional for a flow model and should be viewed in close relation to the aspects related to the possibly different route durations. The general idea in the model is that any route is associated with a single ship as far as the mathematical representation is concerned, even if the duration of the route is more than one week. The model has a decision variable which represents the total duration in weeks of any route. In order to ensure that sufficiently many ships of each type are available for sailing each route, the model places an upper bound on the total duration of all routes that are assigned to ships of each type. This upper bound is equal to the number of available ships of that particular type.

The following decision variables are used and involve the above mentioned interpretation of the use of ships. The binary variable x_{ijv} equals 1 iff ship v sails directly from port i to port j, the binary variable y_{iv} equals 1 iff port i is called by ship v, and the binary variable u_{ijvm} equals 1 iff cargo m is transported from port i to port j on ship

v. In addition, variables δ and γ are introduced in order to identify whether a cargo is discharged (loaded) in other ports than the cargo's destination (origin). This takes place in the following way: the binary variable δ_{ivm} equals 1 iff cargo m is discharged in port $i \neq D_m$ from ship v, and γ_{ivm} equals 1 iff cargo m is loaded on ship v in port $i \neq O_m$. Moreover, β_v is the duration in weeks of the route assigned to ship v. Finally, S_v is the continuous variable denoting the speed for ship v (measured in nautical miles per day).

2.3.2 Mathematical Model

$$\min \quad \sum_{t\in T}\sum_{v\in V_t} C^F F_t(\frac{S_v}{S_t^+})^3 \sum_{(i,j)\in A_t} \frac{d_{ij}x_{ijv}}{S_v} + \sum_{t\in T}\sum_{v\in V_t} C_t^U \beta_v$$
$$+ \sum_{t\in T}\sum_{i\in N_t}\sum_{v\in V_t}\sum_{m\in M} C_i^T(\delta_{ivm}+\gamma_{ivm})G_m + \sum_{t\in T}\sum_{v\in V_t}\sum_{i\in N_t} C_{it}^P y_{iv} \tag{2.1}$$

$$\text{s.t.} \quad \sum_{j\in N_t} u_{ijvm} - \sum_{j\in N_t} u_{jivm} = 0,$$
$$\forall t \in T, v \in V_t, m \in M, i \in N_t \backslash (Z_m \cup \{O_m, D_m\}), \tag{2.2}$$
$$\sum_{j\in N_t} u_{jivm} - \sum_{j\in N_t} u_{ijvm} \leq \delta_{ivm}, \forall t \in T, v \in V_t, m \in M, i \in Z_m, \tag{2.3}$$
$$\sum_{j\in N_t} u_{ijvm} - \sum_{j\in N_t} u_{jivm} \leq \gamma_{ivm}, \forall t \in T, v \in V_t, m \in M, i \in Z_m, \tag{2.4}$$
$$\sum_{t\in T}\sum_{v\in V_t}\sum_{j\in N_t} u_{ijvm} - \sum_{t\in T}\sum_{v\in V_t}\sum_{j\in N_t} u_{jivm} = K_{mi},$$
$$\forall m \in M, i \in Z_m \cup \{O_m, D_m\}, \tag{2.5}$$
$$x_{ijv}Q_t \geq \sum_{m\in M} G_m u_{ijvm}, \forall t \in T, v \in V_t, (i,j) \in A_t, \tag{2.6}$$
$$S_t^- \leq S_v \leq S_t^+, \forall t \in T, v \in V_t, (i,j) \in A_t, \tag{2.7}$$

$$\frac{\sum_{(i,j)\in A_t} d_{ij}x_{ijv}}{S_v} + \sum_{m\in M}\sum_{i\in Z_m}\sum_{j\in N_t} G_m E_{it}(\delta_{ivm} + \gamma_{ivm})$$
$$+ \sum_{m\in M}\sum_{i\in\{O_m,D_m\}}\sum_{j\in N_t} G_m E_{it} x_{ijv} \leq 7\beta_v, \forall t \in T, v \in V_t, \tag{2.8}$$

$$\sum_{v\in V_t} \beta_v \leq |V_t|, \forall t \in T, \tag{2.9}$$

$$\sum_{j\in N_t} x_{ijv} - \sum_{j\in N_t} x_{jiv} = 0, \forall t \in T, v \in V_t, i \in N_t, \tag{2.10}$$

$$\sum_{i\in S}\sum_{j\in \bar{S}} x_{ijv} \geq y_{\tau v} + y_{\sigma v} - 1,$$
$$\forall S \subset N, \bar{S} = N\backslash S, \tau \in S, \sigma \in \bar{S}, t \in T, v \in V_t, \tag{2.11}$$

$$By_{iv} \geq \sum_{j\in N_t\backslash\{i\}} x_{ijv}, \forall t \in T, v \in V_t, i \in N_t, \tag{2.12}$$

$$x_{ijv} \in \{0,1\}, \forall t \in T, v \in V_t, (i,j) \in A_t, \tag{2.13}$$

$$y_{iv} \in \{0,1\}, \forall t \in T, v \in V_t, i \in N_t, \tag{2.14}$$

$$\delta_{ivm}, \gamma_{ivm} \in \{0,1\}, \forall t \in T, v \in V_t, m \in M, i \in Z_m, \tag{2.15}$$

$$u_{ijvm} \in \{0,1\}, \forall t \in T, v \in V_t, (i,j) \in A_t, m \in M, \tag{2.16}$$

$$\beta_v \geq 0 \text{ and integer}, \forall t \in T, v \in V_t, \tag{2.17}$$

where the value of K_{mi} in (2.5) is given as follows:

$$K_{mi} = \begin{cases} 1, & \text{when i} = O_m \\ -1, & \text{when i} = D_m \\ 0, & \text{otherwise.} \end{cases}$$

The objective (2.1) defines the total cost which is to be minimized. There are four terms in the objective function. The first term is the cost of fuel for sailing the routes. The second term is the cost of using the ships that must be deployed. The third term is the cost of transshipment. The last term is the cost of the calls to various ports.

Constraints (2.2) ensure that the cargo is not loaded or discharged in a port which is neither approved for transshipment of the cargo nor is it the origin or destination of the cargo. Together with constraints (2.5), constraints (2.2) also ensure that only the intended transshipment ports are used for transshipment purposes.

Constraints (2.3) together with constraints (2.15) ensure that if cargo m can be discharged in port i from ship v for transshipment, then either all or none of the cargo is discharged for transshipment. Constraints (2.4) together with constraints (2.15) ensure that the loading of transshipment cargo m in port i onto ship v is restricted in the same fashion. Constraints (2.3) and (2.4) are only enforced for transshipment ports as constraints (2.5) ensure that no other ports will have cargo for transshipment.

Constraints (2.5) together with the possible values of K_{mi} guarantee the proper flow of transshipment cargo. This is achieved by requiring that if cargo m has neither origin nor destination in port i ($K_{mi} = 0$) and it is loaded onto or discharged from ship v in port i, then the reverse action must also take place for cargo m in port i (this will then be done on another ship). Furthermore, the constraints ensure that cargo m is loaded at its origin and discharged at its destination.

Constraints (2.6) ensure that the capacity available on ship v sailing from i to j is sufficiently large to accommodate all the cargos loaded on ship v sailing from i to j. Constraints (2.7) ensure that each ship maintains a speed that is within the given lower and upper bounds.

The left-hand side of constraints (2.8) is the time (measured in days) required to perform the route planned for ship v. The first term is the time spent sailing the route. This is found by calculating the length of the route divided by the speed of the ship. The second term is the time spent in port by the ship in order to perform the transshipment moves. The last term is the time spent on loading cargos at their origin and and discharging them at their destinations. The right-hand side of (2.8) is the duration of the route measured in days, so that β_v is the duration of the route measured in weeks. As β_v is required to be integer (2.17), the duration of the route is a whole number of weeks.

Constraints (2.8) are modeled as inequalities rather than equalities as a consequence of the lower bound on ship speeds. In order to obtain an exact whole number of weeks as the duration of a route, it may be necessary to add idle time for the individual ship. Specifically, it is not safe to assume that the ship can just lower the speed until the left-hand side equals a given right-hand side since this may require a speed which is lower than the lower bound on the speed of the ship. Therefore, it may be necessary to include idle time on the route. This idle time is not included on the left-hand side of (2.8). Instead, idle time is given by the slack of the corresponding inequality.

As explained in subsection 2.3.1, the interpretation of the decision variables implies that whenever a route with a duration of β_v weeks is planned for a ship v of type t, then the solution should be interpreted so that β_v ships of type t are assigned to that particular route. Constraints (2.9) ensure that sufficiently many ships of each type are available by requiring that the total duration (measured in weeks) of all routes planned for ships of type t does not exceed the available number of ships of type t.

Constraints (2.10) ensure that the routes are cyclic by requiring that ship v departs from port i iff it arrives at port i. Constraints (2.11) and (2.12) ensure that the routes are connected. Furthermore, constraints (2.12) ensure adherence to the upper bound on the number of calls that one route may have to the same port.

Finally, constraints (2.13)-(2.16) restrict variables x, y, δ, γ, and u to be binary, and constraints (2.17) restrict the β variables to be nonnegative integers.

2.4 Heuristic solution method

The model presented in section 2.3.2 is solved by a heuristic based on Dantzig-Wolfe decomposition. The decomposition splits the problem into a master problem and a subproblem. The master problem is a generalized set covering problem and the subproblem is the problem of generating promising columns for the master problem.

The heuristic produces solutions for the case that $B = 1$, i.e., all routes in the heuristic solution are elementary.

In the following sections, the main components of the heuristic will be explained. The restricted master problem is presented in section 2.4.1. The model's transshipment columns are described in section 2.4.2, and the model's route columns are described in section 2.4.3. Finally, a brief overview of the heuristic as a whole is described in section 2.4.4.

2.4.1 The restricted master problem

The restricted master problem is a generalized set covering problem that incorporates the global constraints mentioned in the flow model. The formulation contains three types of columns, namely i) dummy columns, ii) transshipment columns, and iii) route columns. The nature of these columns are as follows.

For each cargo $m \in M$, the model contains one dummy column and an associated binary decision variable ω_m. The dummy columns serve only to ensure that the mathematical model has a feasible solution even if one or more cargos are not actually transported in the solution. Specifically, $\omega_m = 1$ iff cargo m is not transported in the solution. Accordingly, the cost (denoted by Ω^P) of a dummy column is set to a prohibitively large value.

The transshipment columns can be explained as follows. Each cargo m has a number of possible paths from its origin to its destination. One of these paths is the direct connection, i.e., a path consisting of only the single arc (O_m, D_m). Each of the other paths for cargo m is called a transshipment option and is represented by a corresponding transshipment column. The set of transshipment options for cargo m is denoted P_m. A binary decision variable θ_{mp} is associated with the p'th transshipment option of cargo m, so that $\theta_{mp} = 1$ iff transshipment option $p \in P_m$ is used. The cost of transshipment option p for cargo m is denoted by Ω^T_{mp} (the calculation of this cost is described in section 2.4.2).

A transshipment option is given by a sequence of two or more arcs. Let the set of arcs in transshipment option p for cargo m be denoted by Γ_{mp}. It is noted that the same arc (i, j) may be included in more than one transshipment option for cargo m. The set of all arcs which occur in one or more transshipment options for cargo m is denoted Ψ_m, i.e., $\Psi_m = \bigcup_{p \in P_m} \Gamma_{mp}$.

Each route column represents a route for one or more ships. The characteristics of a route, including cost calculations, are given in section 2.4.3.1. A binary decision variable λ_r is associated with each route column r, so that $\lambda_r = 1$ iff route r is used in the solution. The cost of route column r is denoted by Ω^R_r.

Let the coefficient $\Delta_{mr} = 1$ if cargo m is transported directly from O_m to D_m on route r, and $\Delta_{mr} = 0$ otherwise. Further, to represent the use of transshipment options, the coefficient $\Lambda_{mrij} = 1$ if cargo m is transported along $(i, j) \in \Psi_m$ on route r, and $\Lambda_{mrij} = 0$ otherwise. The coefficient Φ_{tr} represents the number of ships of type t assigned to route r.

In order to keep the model tractable, only a subset R of all feasible routes is considered. The routes in the model are constructed in two stages. The construction of initial route columns is described in section 2.4.3.1, and the construction of additional route columns is described in section 2.4.3.2.

This leads to the formulation given by (2.18)–(2.24).

$$\min \quad \sum_{m \in M} \Omega^P \omega_m + \sum_{m \in M} \sum_{p \in P_m} \Omega^T_{mp} \theta_{mp} + \sum_{r \in R} \Omega^R_r \lambda_r \tag{2.18}$$

$$\text{s.t.} \quad \omega_m + \sum_{p \in P_m} \theta_{mp} + \sum_{r \in R} \Delta_{mr} \lambda_r \geq 1, \forall m \in M, \tag{2.19}$$

$$- \sum_{p \in P_m | (i,j) \in \Gamma_{mp}} \theta_{mp} + \sum_{r \in R} \Lambda_{mrij} \lambda_r \geq 0, \forall m \in M, (i, j) \in \Psi_m, \tag{2.20}$$

$$\sum_{r \in R} \Phi_{tr} \lambda_r \leq |V_t|, \forall t \in T, \tag{2.21}$$

$$\omega_m \in \{0, 1\}, \forall m \in M, \tag{2.22}$$

$$\theta_{mp} \in \{0, 1\}, \forall m \in M, p \in P_m, \tag{2.23}$$

$$\lambda_r \in \{0, 1\}, \forall r \in R. \tag{2.24}$$

Equation (2.18) minimizes the cost of the selected columns. Constraints (2.19) ensure, for each cargo, that either the cargo is transported at least once—either as direct cargo or via one of the transshipment options—or the cargo is not transported as represented by selecting the cargo's associated dummy column.

Constraints (2.20) ensure that the selected routes will provide transportation on all arcs of that transshipment option if the demand is being transported via one of the transshipment options. Constraints (2.21) ensure that the number of ships used of each ship type on all the selected routes does not exceed the number of ships available of each ship type. The last constraints (2.22)–(2.24) ensure that the decision variables are either 0 or 1.

The restricted IP master is given by (2.18)–(2.24). The restricted LP master is a relaxation of the restricted IP master where the binary requirements in (2.22)–(2.24) are replaced by nonnegativity constraints.

Table 2.1 shows an example of a master problem and the columns required. For notational convenience in this example, cargos are named according to their origin and destination, e.g., 2-4 is a cargo with origin in port 2 and destination in port 4.

The first 14 columns are the dummy columns, the next 13 columns are the transshipment columns, and the last 5 columns are the route columns.

Cargo 2-4 must be transported either directly from 2 to 4 or by using the transshipment option along the path 2-1-4, i.e., with transshipment in port 1. The last transshipment column represents this transshipment option. If this option is chosen, then the cost is incurred of transshipping cargo 2-4 in port 1, and the model ensures that the two legs $(2, 1)$ and $(1, 4)$ are serviced by the chosen route columns, as represented by the last two constraints. Moreover, if the option is chosen, then the direct transportation of 2-4 is not required as constraint `C16`(2-4) is satisfied by the contribution from the transshipment column.

Regardless of whether it is the restricted LP master problem or the restricted IP master problem that is solved, the solution is found by using commercial software. In

		X1	X2	X3	X4	X5	X6	X7	X8	X9	X10	X11	X12	X13	X14	X15	X16	X17	X18	X19	X20	X21	X22	X23	X24	X25	X26	X27	X52	X53	X97	X98	X99	LHS		RHS
Ships	C1 (Type 1)																												4	3				4	≤	5
	C2 (Type 2)																														4	5	6	6	≤	7
Cargo	C3 (1-2)	1														1													1	1				1	=	1
	C4 (1-5)		1														1												1	1				1	=	1
	C5 (1-6)			1														1											1	1				1	=	1
	C6 (2-1)				1														1										1	1				1	=	1
	C7 (1-4)					1																									1	1	1	1	=	1
	C8 (1-3)						1													1											1	1	1	1	=	1
	C9 (5-1)							1													1								1	1				1	=	1
	C10 (1-7)								1													1									1	1	1	1	=	1
	C11 (4-7)									1													1								1	1	1	1	=	1
	C12 (4-3)										1													1							1	1	1	1	=	1
	C13 (6-1)											1													1				1	1				1	=	1
	C14 (7-2)												1													1								1	=	1
	C15 (3-2)													1													1							1	=	1
	C16 (2-4)														1													1						1	=	1
T/S	C17 (1-4)															-1																		0	=	0
	C18 (4-2)															-1																		0	=	0
	C19 (1-4)																-1																	0	=	0
	C20 (4-5)																-1																	0	=	0
	C21 (1-4)																	-1																0	=	0
	C22 (4-6)																	-1																0	=	0
	C23 (2-4)																		-1															0	=	0
	C24 (4-1)																		-1															0	=	0
	C25 (1-4)																			-1														0	=	0
	C26 (4-3)																			-1														0	=	0
	C27 (5-4)																				-1													0	=	0
	C28 (4-1)																				-1													0	=	0
	C29 (1-4)																					-1												0	=	0
	C30 (4-7)																					-1												0	=	0
	C31 (4-1)																						-1											0	=	0
	C32 (1-7)																						-1											0	=	0
	C33 (4-1)																							-1										0	=	0
	C34 (1-3)																							-1										0	=	0
	C35 (6-4)																								-1									0	=	0
	C36 (4-1)																								-1									0	=	0
	C37 (7-1)																									-1					1	1	1	0	=	0
	C38 (1-2)																									-1			1	1				0	=	0
	C39 (3-1)																										-1				1	1	1	0	=	0
	C40 (1-2)																										-1		1	1				0	=	0
	C41 (2-1)																											-1	1	1				0	=	0
	C42 (1-4)																											-1			1	1	1	0	=	0
	Cost (1000)	M	M	M	M	M	M	M	M	M	M	M	M	M	M	68	42	50	24	71	8	71	88	82	9	5	14	4	198	199	588	510	489	710		
	Choice																									1	1	1	1				1			

Table 2.1: The columns needed to solve the master problem for instance T1.5. A cost coefficient of 'M' represents a very large value.

order to speed up the solution process, cuts are added to the problem before the commercial solver is used. The cuts are the discrete dual-feasible function (DFF) presented in Clautiaux et al. (2010) which is the discrete equivalent of the DFF presented in Fekete and Schepers (2001). The cuts are added for constraints (2.21) and the heuristic produces one inequality for each pair of (t, k) with $t \in T$ and $1 \leq k \leq \min\{|V_t| - 2, 4\}$. The upper bound of 4 on the k-value is set to ensure that only a manageable number of inequalities are added. The inequalities are added before the commercial solver solves the restricted master problem and are removed again once the solution has been extracted.

2.4.2 Transshipment columns

Together with the other parts of the model, the transshipment columns serve the purpose of ensuring that the following aspects are dealt with in the model:

1. A cargo is either loaded directly or transshipped.
2. If a transshipment option is chosen, then all arcs of that transshipment option are covered by the selected routes.
3. The cost of transshipment is included in the total cost of the solution, as the objective coefficient of a transshipment column is the total cost of transshipping that particular cargo along the corresponding path from the cargo's origin to its destination.

Since each transshipment option for a cargo gives rise to a transshipment column, the number of transshipment options must be limited in order to keep the model at a manageable size. We have chosen to limit the transshipment options to one per cargo. For each cargo we have chosen the transshipment port which leads to the smallest distance deviation compared to traveling directly between origin and destination. If there are only two transshipment ports in a network, then cargo between these two ports will not have a transshipment option available.

The transshipment columns are generated initially in the solution process.

2.4.3 Route columns

Each route column represents a route which is given by the cargos transported, the ship type used, and the number of ships used. Furthermore, the column represents a choice of which ports to service, which order they are serviced in, and the speed of the ships. These aspects are contained implicitly in the cost of the column.

In the suggested heuristic, a large number of route columns are created a priori as described in section 2.4.3.1, whereas a smaller number of columns are created in a delayed refinement process as described in section 2.4.3.2.

2.4.3.1 Initial route columns

The heuristic commences by generating all possible sets of port combinations, subject to certain restrictions. The heuristic generates the individual set no more than once, rather than generating all permutations of the same set.

Set restrictions and port sequencing The following restrictions are introduced to ensure that the number of sets generated remains manageable. Restrictions iii) and iv) are based on the fact that ports are divided into regions. A region is a clearly defined set of ports that, with respect to the entire network, are comparatively close to each other as for example the Baltic Sea, the North American West Coast, or West Africa.

i) The number of ports in the sets must remain within an upper and lower bound. The idea of an upper bound stems from Agarwal and Ergun (2008) where the upper bound depends on the size of the area the routes need to cover. The routes published by global liner companies such as Maersk Line (Maersk, 2011) and CMA CMG (CMA-CMG, 2011) range between 3 and 20 ports. Based on this the lower bound has been fixed to 3, while the upper bound depends on the size of the network in question. Therefore, the upper bound is input to the heuristic.

ii) To ensure that all routes are connected to the rest of the network, any single route must contain at least one port with transshipment capabilities. Since the liner shipping companies have access to a limited number of ports with transshipment capabilities, the restriction diminishes the number of possible sets substantially.

iii) If a route visits a port in a region, then the route must visit at least two ports or a transshipment port in the region.

iv) The number of regions visited on a route cannot exceed a given maximum. The maximum is input to the heuristic. The number will largely depend on the size of the network and on how the regions are defined.

v) All ports on a route must be needed in order to transport the planned cargo. This restriction is added to ensure that we do not create routes to transport cargo which could just as well have been transported by a shorter route.

Both iii) and iv) are based on the current practices of the major shipping lines such as Maersk Line, MSC, and CMA CMG (Maersk, 2011; MSC, 2011; CMA-CMG, 2011). All restrictions ii)–v), but especially ii) and iii), will limit the number of possible ports on a route, and the limitation may be less restrictive than the upper bound in restriction i). Despite these limitations on the number of ports in a set, all port pairs will be represented in the generated columns several times.

The heuristic solves a traveling salesman problem (TSP) for each set. The solution to the TSP provides the sequence (a circuit) of the ports in the set. This implies that all routes in a solution are elementary, i.e., multiple calls to the same port on the individual route are not considered in this heuristic.

Cargo selection For any port sequence that has been established, a decision must be made as to which cargos should be transported on the route. For this purpose, the heuristic considers the ship types individually, and the ship types are considered in increasing order of capacity. Before assigning cargo to a route, the heuristic verifies that the ship-port compatibility is adhered to as well as whether there are sufficient ships to provide a weekly service on the route, given the upper bound on the speed.

For a given combination of route and ship type, four separate procedures have been established. All four procedures give preference to direct cargo over transshipment cargo. This preference is installed to mitigate the fact that customers rarely approve of seeing their cargo transshipped if there is a direct service. Before any of the procedures are commenced, the heuristic creates a list of possible cargos. The list of possible cargos consists of two segments. The first segment contains the direct cargos, i.e., cargos whose origin and destination ports are on the route. The second segment contains the transshipment cargos whose transshipment port and either origin or destination are on the route. Within each of the two segments, the order in which the cargos occur on the list is the same as the cargos' order in the input data. The first segment is placed at the top of the list and it has consequently higher priority than segment two whenever the cargos are considered from top to bottom on the list.

The first procedure is a greedy method for selecting cargos (Greedy). Given the list of possible cargos the greedy method goes through the list from top to bottom and simply adds the individual cargo if the capacity allows it. In any case, the procedure then proceeds to the next cargo on the list. How well this procedure performs is dependent on the order of the cargo in the input.

The second procedure incorporates an element of randomness (Random) in order to mitigate this weakness. Rather than picking the cargos in the listed order, this procedure picks them randomly within each of the two segments (still considering direct cargos before transshipped cargos). Otherwise, this procedure proceeds as the previous procedure.

The third procedure is somewhat more elaborate as the goal is to maximize the number of *c*argo-miles, i.e., cargo volume transported times the distance that the cargo is transported (MaxDist). The reason for looking at this rather than just the cargo volume transported is that cargos which have a long way to go are given priority as these cargos can be more difficult to transport. The procedure solves a mixed integer problem that maximizes the number of cargo-miles while adhering to the capacity constraint of the ship type. To give direct cargo priority over transshipment cargo, the objective coefficient of direct cargo is doubled. The mixed integer problem is solved via CPLEX.

The last procedure is based on the fact that in liner shipping routes often contain a transshipment port out of which the ship is transporting the maximum volume for the route (MaxVol). To emulate this, the procedure maximizes the cargo volume out of the route's smallest indexed transshipment port. This is done by solving a knapsack problem using dynamic programming. Once this is achieved, a similar knapsack problem is solved for each of the other ports on the route. Each such knapsack problem maximizes the volume on the ship when it leaves the given port, subject to the decisions made with respect to the cargos out of the already considered ports. At each stage, when choosing the next port to consider, the port with the smallest unused capacity on departure from the port is chosen.

In all four cases the procedure is carried out for both directions of the route. If the selected cargos differ for the two directions, then they are both added. Otherwise, the one with the smallest number of cargo-miles is added so that the cargo is onboard for as little time as possible.

Ship assignment For any circuit of ports and associated set of cargos, the number of ships—of the given type—to deploy on the route must be determined. For each route there may be several feasible numbers of ships to be deployed, each with its own cost. In order to establish which numbers of ships are feasible, the time spent in port must first be calculated. The time spent in a port is decided by the number of moves required to ensure complete loading and discharge of the planned cargo and by the port productivity for the given ship type. Once the time in port has been established, the heuristic considers the number of ships in increasing order and continues as long as an additional ship leads to a lower total cost. The number of available ships of the given ship type is an upper bound on the number of ships that can be considered on the route. In order for a number of ships to be feasible, the speed required to sail the route must be within the upper and lower bound on the speed for the given ship type.

Cost calculations For each of the possibly several feasible numbers of ships, the heuristic calculates the cost of sailing the route at the calculated speed and the cost of including the established number of ships of the chosen ship type. The heuristic also adds the port costs. The total cost calculated is the objective coefficient for the column.

2.4.3.2 Additional column generation

The restricted LP master and IP master may be solved several times. Irrespective of whether it is the LP master or the IP master that has been (re-)optimized, it is investigated whether one or more cargos are being transported more than once in the solution, represented by surplus in the set covering constraints (2.19) and (2.20). If so, an attempt is made to produce new columns based on existing columns, with the purpose of eliminating excess transportation of the individual cargos.

The general idea is that a new column is generated as a copy of an existing column except that the new column covers only a subset of those cargos covered by the original column.

In particular, if a cargo m is transported more than once on an arc (i, j), then there is potential for improving the solution simply by i) copying one of the columns on which cargo m is transported on arc (i, j), ii) removing the transport of cargo m on arc (i, j) in the created copy, and iii) adding the created copy to the restricted master problem. Provided that the new column has smaller cost than the column from which it was created, the addition of the column will result in a solution of lower cost.

In order to describe the procedure in detail, a bit of additional notation is introduced. Let y_r^* represent the value of y_r in the current solution to the restricted master problem. Moreover, let s_m^* represent the slack of constraint (2.19) corresponding to cargo m, and let s_{mij}^* represent the slack of constraint (2.20) corresponding to cargo m on arc (i, j).

The generation of new route columns is done by going through all route columns in the restricted master problem. In the following, a column from which other columns are produced is called a *source column*. The process involved in producing new columns is described in the following. Let α denote a new column produced from source column r.

i) Some of the coefficients of value 1 in source column r are given the value 0 in column α. Specifically, the change of coefficient from $\Gamma_{mrij} = 1$ to $\Gamma_{m\alpha ij} = 0$ is carried out for all combinations of m and (i, j) for which $y_r^* \leq s_{mij}^*$. The corresponding change is carried out in (2.19) from $\Delta_{mr} = 1$ to $\Delta_{m\alpha} = 0$ for any m for which $y_r^* \leq s_m^*$. As a consequence, a new column can replace its source column without affecting the rest of the solution.

ii) For the set of cargos resulting from i), the heuristic tries in turn a number of ship types, namely the current ship type for route r and those other ship types of smaller capacity whose capacity is sufficiently large to accommodate the newly identified cargos on the route. For each ship type considered, the heuristic goes through all possible values of the number of ships assigned to the route. All combinations of ship type and number of ships which lead to a feasible route are considered for producing new columns.

As such, a current route may lead to a number of new routes, all carrying the same set of cargos but with different ship types and a different number of ships.

The following restrictions and relaxations are introduced in relation to the generation of new route columns:

a) A column r is used as a source column only if $y_r^* > 0.1$. This limits the number of source columns with the intention of limiting the total computing time spent on additional column generation.

b) As a relaxation of the requirement in section 2.4.3.1 for the port sets, it is no longer required that the resulting route has either a transshipment port or at least two ports in each visited region. Furthermore the lower bound on the number of ports on a route is no longer imposed.

c) The removal of cargos when producing a new column from the source column may result in one or more ports becoming obsolete, i.e., there is no transport of cargos from/to the ports. Any obsolete ports are then removed from the route.

d) When removing cargos and ports from a column, the new speed will in most cases be smaller than the speed for the source column. However, the speed may not be lowered as much as indicated by the removal of cargos and ports, due to the lower bound on the speed. In such cases, the cost of the column is calculated as if the ships continuously sail at minimum speed. In practice the ships will be idle for the remaining time available.

The cost is calculated as for the initial columns. The cost of a new column can be higher than the cost of its source column because it is possible to create columns with different ship types. However, it is much more likely that the cost decreases as a consequence of the removal of cargos, for the following reasons:

1) Removal of cargos results in spending less time in port for loading and discharging thereby leaving more time for the sea passage. This allows for a reduction in speed which reduces the cost of sailing the route.

2) If the speed decreases sufficiently, then it may become possible to perform the route with a smaller number of ships thereby lowering the cost of including ships.

3) Lower volumes can result in a lower maximum volume which may lead to the possibility of using smaller ships which will decrease port costs, the inclusion costs, and possibly the sailing costs.

Since the cargos of the new column can always be transported in the same manner as in the source column, there will always be at least one new column with the same or lower cost as the source column. Therefore, the cost of the new solution will be the same as or lower than the cost of the previous solution.

As an example, table 2.2 illustrates the columns with positive y-values after the initial columns have been generated and the restricted master problem has been solved. The column furthest to the right shows the surplus for each set covering constraint, i.e., the amount by which the left-hand side (LHS) exceeds the right-hand side (RHS). Columns X40 and X43 will result in at least one new column each. In fact, the two columns result in the generation of 10 new columns. Column X43 results in the creation of the new column X99 which is shown in table 2.1. The number and type of ship remains the same, but the speed decreases and therefore so does the cost.

2.4.4 Overview of solution method

The heuristic starts with generating the support columns mentioned in section 2.4.2 and then proceeds to generate the route columns mentioned in section 2.4.3. The restricted LP master described in section 2.4.1 is then solved as described. Subsequently the solution is examined to locate any surplus in the set covering constraints. If surplus exists, then additional columns are generated as described in section 2.4.3.2. Once all the additional columns are generated, the restricted LP master is reoptimized as described in section 2.4.1 and the process continues.

If no new columns are added to the restricted LP master, or if the restricted LP master's objective value is unchanged after the latest LP iteration, then the integrality constraints are added and the restricted IP master is solved. The heuristic continues with the integrality constraints permanently added. Hence, if the solution to the restricted IP master contains surplus, then the heuristic proceeds by generating new columns on the same premisses as previously described. This process continues until no further surplus exists after which the solution is the final solution to the problem.

	X25	X26	X27	X40	X43	X53	X54	LHS	RHS	Surplus
C1 (Type 1)				5		3	2	5	5	
C2 (Type 2)					6			3	7	4
C3 (1-2)						1	1	1	1	
C4 (1-5)						1	1	1	1	
C5 (1-6)						1	1	1	1	
C6 (2-1)						1	1	1	1	
C7 (1-4)				1	1			1	1	
C8 (1-3)				1	1			1	1	
C9 (5-1)						1	1	1	1	
C10 (1-7)				1	1			1	1	
C11 (4-7)				1	1			1	1	
C12 (4-3)				1	1			1	1	
C13 (6-1)						1	1	1	1	
C14 (7-2)	1							1	1	
C15 (3-2)		1						1	1	
C16 (2-4)			1					1	1	
C17 (1-4)				1	1			1		1
C18 (4-2)										
C19 (1-4)					1			0.5		0.5
C20 (4-5)										
C21 (1-4)				1	1			1		1
C22 (4-6)										
C23 (2-4)										
C24 (4-1)				1	1			1		1
C25 (1-4)										
C26 (4-3)										
C27 (5-4)										
C28 (4-1)				1	1			1		1
C29 (1-4)										
C30 (4-7)										
C31 (4-1)										
C32 (1-7)										
C33 (4-1)										
C34 (1-3)										
C35 (6-4)										
C36 (4-1)				1	1			1		1
C37 (7-1)	-1			1	1					
C38 (1-2)	-1					1	1			
C39 (3-1)		-1		1	1					
C40 (1-2)		-1				1	1			
C41 (2-1)			-1			1	1			
C42 (1-4)			-1	1	1					
Cost (1000)	5	14	4	362	496	199	301	702		
Choice	1	1	1	0.5	0.5	0.5	0.5			

Table 2.2: The initial columns with positive y-values for instance `T1.4`. All empty entries contain zeros.

2.5 Computational results

In this section, the results of the computational study are presented after a description of how the test instances are generated. First, we establish the superiority of the random method over the three other methods, i.e., Greedy, MaxDist, and MaxVol, when it comes to assigning cargo to routes. Next, a deeper analysis of the random method is presented. Finally, some of the interesting characteristics of the solutions obtained by the heuristic are discussed. The heuristic is implemented in C++ in a Windows environment with extensive use of CPLEX 12.0. All computational experiments are performed on an HP EliteBook with an 2.53 GHz Intel Duo P8700 processor and 1.86 GB RAM. All times are reported in whole seconds.

2.5.1 Data Generation

A number of instances with varying characteristics are generated to test the robustness of the heuristic. The entire set of instances is divided into subsets, so that all instances in the same subset differ only wrt. the maximum number of ports allowed on a route. In the following, each subset of instances will be referred to as an *instance subset*. Table 2.3 shows the characteristics for each of the 24 instance subsets.

Each row in table 2.3 shows the number of the instance subset, the number of ports, ship types, ships, cargos, hubs, and regions, respectively, and the range for the maximum number of ports allowed on a route. The number of instances in the individual instance subset is given by the number of different values in the range for the maximum number of ports allowed on a route. In the following, an entire instance subset will be referred to using the notation `Ts`, where `s` is the instance subset number. The individual instances will be referred to using the notation `Ts.m`, where `m` is the maximum number of ports on a route. As such, instance subset `T1` contains the two instances `T1.4` and `T1.5`. The minimum number of ports on a route is 3 in all instances.

Instance subset	Ports	Ship types	Ships	Cargos	Hubs	Regions	Max. ports per route
1	7	2	12	14	2	3	4–5
2	7	2	12	21	2	3	4–5
3	7	2	16	14	2	3	4–5
4	7	2	16	21	2	3	4–5
5	7	1	12	21	2	3	4–5
6	7	1	16	21	2	3	4–5
7	7	2	16	14	7	3	4–5
8	7	2	16	14	2	1	4–7
9	15	3	21	30	3	3	4–12
10	15	3	21	45	3	3	4–12
11	15	3	27	30	3	3	4–12
12	15	3	27	45	3	3	4–12
13	15	1	21	45	3	3	4–12
14	15	1	27	45	3	3	4–12
15	15	3	27	30	15	3	4–12
16	15	3	27	30	3	1	4–15
17	25	5	35	50	4	4	4–6
18	25	5	35	75	4	4	4–6
19	25	5	45	50	4	4	4–6
20	25	5	45	75	4	4	4–6
21	25	1	35	75	4	4	4–6
22	25	1	45	75	4	4	4–6
23	25	5	45	50	25	4	4–6
24	25	5	45	50	4	1	4–6

Table 2.3: Characteristics of the instances.

As such, in `T1.4` all routes are required to visit either 3 or 4 ports. In `T1.5` all routes are required to visit 3, 4, or 5 ports.

The 24 instance subsets are divided into three groups (`T1`–`T8`, `T9`–`T16`, and `T17`–`T24`). The instances within the same group have the same ports. The ports are actual ports distributed across Africa, Europe, and North America. The sailing distances between ports are obtained from Netpas Distance 2.7 (Build 2715). The port cost and port productivity are both dependent on the port as well as the ship type. The port costs are estimated based on the web pages of various ports presented in appendix 2.C. The productivity for the port-ship type combinations is estimated based on information gathered from the ports (appendix 2.C) and from terminal operators and others with information on port and crane productivity (appendix 2.D).

The number of ship types for each group is based on the number of ships so as to achieve 6–9 ships per ship type which is within industry standards. For each group of instances there is a large and a small number of ships. The smaller of the numbers is based on the ratio between ships and ports as experienced by the liner shipping companies MSC and COSCO (MSC, 2011; COSCO, 2011; Alphaliner, 2011). The larger number is achieved by adding approximately 1/3 in terms of ships evenly to all ship types. The ship types used are based on ships found in the Maersk Line fleet (Maersk, 2011). Information such as capacity, maximum speed, fuel consumption and port compatibility comes from ship owners and operators (appendix 2.E) and port authorities (appendix 2.C). Ideally, minimum speed should be the speed resulting from the minimum safe operating load of the engine. However, this information is not available wherefore the minimum speed in the instances has been fixed to the minimum speed required to maneuver the ships. The cost of including the various ships is estimated based on Alphaliner's 07/2011 evolution of charter rates for cellular ships fall 2009 (Alphaliner, 2011). Each group includes two instance subsets with only one ship type which is constructed as the weighted average of the original ship types.

The number of cargos is the double or the triple of the number of ports. The origin-destination pair for each cargo is chosen based on general trade patterns. The demand quantities are randomly generated within intervals which vary dependent on the total ship capacity and sailing distances involved. The quantity interval is [5;400], [5;450], and [5;900], for the group of instances with 7, 15, and 25 ports, respectively. The generated volumes are assigned to port pairs based on existing trade patterns. Within each group of instances the cargos are the same. Hence, all instances in T9–T16 have 30 cargos in common, and all instances in T10, T12, T13, and T14 have 45 cargos in common.

A hub is a port with transshipment capabilities. The number of hubs chosen for each group is based on estimates of the required number of hubs given the number of ports. Within each region, the hubs are chosen randomly among the ports currently functioning as hubs for one or more of the liner shipping companies. The cost of a transshipment is estimated based on port information (appendix 2.C) and on terminal information (appendix 2.D).

For one instance subset in each of the three groups, namely T7, T15, and T23, all ports are hubs hence transshipment can take place in all ports. The cost of a transshipment is the same in all ports.

The number of regions is based on a natural geographic division of the ports. In T1–T8 the regions are Africa, Europe and North America, in T9–T16 the regions are Africa, Northern Europe and Southern Europe, and in T17–T24 the regions are Africa, Northern Europe, Southern Europe, and the Mediterranean. For all instances, the upper limit on the number of regions a route can visit is set to two.

The maximum number of ports on a route is within a range with four as the lower limit. The upper limit depends on the instance subset as follows. For T1–T7 and T9–T15, the restriction on at most two regions per route, together with the ports' distribution across regions, result in a maximum value of five ports per route for T1–T7 and a maximum value of 12 ports per route for T9–T15. For T8 and T16, where there is only one region, the maximum number of ports on a route varies in the range from four up to the total number of ports in the instance. The larger number of ports in T17–T24 has resulted in a more restrictive limitation on the number of ports on a route. The purpose of this is to avoid excessive computational effort, so that the maximum value is six ports per route for all these instances.

To summarize, in total there are 18 instances with 7 ports, 75 instances with 15 ports, and 24 instances with 25 ports.

2.5.2 Effectiveness of the cargo assigning algorithms

In this section the four methods for assigning cargo to routes are compared in terms of ability to solve the test instances, the objective value, and the time needed to reach the solution. The comparison will result in a choice of one of the methods to be used for the remainder of the article.

Each instance is solved by the three methods Greedy, MaxDist, and MaxVol. In addition, the method Random is run 10 times on each instance. The results reported are the average, the minimum, and the maximum of the 10 runs. Detailed results for all four methods can be found in appendix 2.A.

2.5.2.1 Ability to solve the test instances

All instances in T1–T8 can be solved by all four methods. Table 2.4 shows the performance of each of the four methods on T9–T16 and T17–T24.

Method	T9–T16	T17–T24	Total
MaxVol	75/75	19/24	94/99
MaxDist	75/75	20/24	95/99
Random	71/75	19/24	90/99
Greedy	67/75	16/24	83/99

Table 2.4: Number of solved instances.

A total of 96 out of 99 instances were solved by one or more methods. In total over the two groups of instances, the two more complex methods MaxVol and MaxDist are more likely to find a solution than Random and Greedy, with Greedy being the least likely of the four methods to solve all instances. When an instance cannot be solved, it is most often because it is too large for the MIP solver in CPLEX. Only in rare cases is the LP solver in CPLEX unable to solve the instance.

2.5.2.2 Objective values and computing times

For all instances in T1–T8, all four methods give the same results in terms of both objective value and computing time (the computing time is negligible, i.e., less than a second, for all four methods).

However, the methods are not equally good for the remaining two groups of instances. In the following the methods are compared wrt. three performance measures, namely frequency of best performance, average performance, and worst case performance.

Frequency of best performance It is observed which method performed best wrt. objective value and computing time, respectively, across the 96 instances solved by one or more methods. The results are summarized in table 2.5 which shows how often each of the four methods have had the best performance in terms of objective value and computing time, respectively. Table 2.5 shows that Random has the best objective value in 57 instances despite only solving 90 instances. With respect to computing time, Random has the best performance in 32 instances. This is only surpassed by Greedy which solves 44 instances in the shortest time.

Either MaxVol or MaxDist produces the best solution in terms of objective value for 15 instances, out of which 6 instances have not been solved by neither Greedy nor Random. Hence MaxVol or MaxDist is able to produce the best solution for only 10% of those instances which have also been solved by Greedy or by Random.

	Best objective			Best computing time		
Method	T9–T16	T17–T24	Total	T9–T16	T17–T24	Total
MaxVol	2	1	3	17	4	21
MaxDist	9	3	12	2	0	2
Random	45	12	57	27	5	32
Greedy	20	5	25	32	12	44

Table 2.5: Frequencies of best performance.

Average performance The average performance of a method is important as it gives insight into what can be expected of the method in the long run. For the two groups of instances with 15 and 25 ports, respectively, the average index for both the objective value and the time spent is shown in table 2.6. Each 'Total' column is obtained as a

	Average objective index			Average time index		
Method	T9–T16	T17–T24	Total	T9–T16	T17–T24	Total
MaxVol	113	137	118	542	538	541
MaxDist	111	117	112	956	1091	984
Random	102	100	102	246	378	274
Greedy	109	103	108	267	118	238

Table 2.6: Average performance indices. Best result is fixed to index 100.

weighted average based on the number of solved instances as reported in table 2.4.

For the instances T9–T16, the methods MaxVol, MaxDist, and Greedy have approximately the same performance in terms of objective value, while Random outperforms them with an index of 102. In terms of time consumption, Random and Greedy have the better performance. However, this can be ascribed to the appalling performance of the two other methods.

For the instances T17–T24 the performance is much more varied. MaxVol performs the worst with an index of 137 which indicates that on average the method produces results for this set of instances that are 37% above the best objective value found. Greedy and Random outperform the two other methods with 3% and 0% above the best solution, respectively. Regarding time, Greedy outperforms the other methods with an index of 118. MaxDist has the worst performance with an index of 1091, which means that on average solving the cargo assignment problem with this method is 11 times slower than the fastest solution.

Looking at the two groups of instances combined, Random has the best performance in terms of objective value with an average index value of 102. This number is obviously low partially due to the high number of best solutions produced by Random. However, if these instances are not taken into account, the value only increases to 104 which is still much lower than the second lowest index which is 111 produced by Greedy (see table 2.14). This means that when Random does not produce the best solution, on average the solution produced is only 4% above the best solution. The conclusion that Random has by far the best average performance in terms of objective value and that on average Greedy is the fastest at finding the solution is supported by tables 2.13 and 2.15 which show the average deviation from the best in terms of dollars and seconds.

Worst case performance While it is important for a method to be able to come up with the best solution and have a good average performance, it is also important that a method always produces fairly good solutions. Hence, the worst case performance of a method should rarely or better yet never be horrendous.

The worst performances of the methods on the instances T9–T16 and T17–T24 are presented in table 2.7 together with a specification of the instance on which the individual method produced that particular result. Instance T13.4 causes 3 out of 4 of the worst performances for T9–T16, while T21.4 causes 3 out of 4 of the worst performances for T17–T24. This is interesting as the two instances share the same characteristics as they both have one region, the smaller number of ships, and the larger number of cargos. This type of problem is obviously difficult to solve well and considering that it is two different methods (MaxDist and Random) that find the best solution, one could speculate that finding a good solution for this type of problem depends on chance rather than the design of the four methods. Instance set T21 causes poor performance for all other methods than Random. If removing the two worst instances for each method, the worst case performance of the four methods ranges from 101 to 129 with Random outperforming the other methods and producing the two lowest worst cases at 101 and 104.

MaxDist and MaxVol both solve more instances than Greedy and Random, but they perform poorly in terms of solution quality and time spent achieving the solution. Greedy is by far the best when it comes to time spent finding the solutions. Random is the method most likely to find the best solution, and even when it does not find the best solution, the solutions found are rarely really bad.

		Worst		2nd worst		3rd worst	
Group	Method	Index	Instance	Index	Instance	Index	Instance
T9–T16	MaxVol	373	T13.4	131	T16.15	129	T16.11
	MaxDist	413	T10.4	125	T16.15	123	T16.11
	Random	160	T13.4	135	T10.4	104	*
	Greedy	367	T13.4	117	T11.12	117	T11.11
T17–T24	MaxVol	558	T21.4	129	T21.5	124	T21.6
	MaxDist	331	T21.4	118	T21.6	116	T21.5
	Random	103	T20.5	102	T18.5	101	*
	Greedy	116	T21.4	106	T21.5	105	T17.4

Table 2.7: Empirical worst case performances. Best objective value is fixed to index 100. A * means that the result was obtained on multiple instances.

2.5.2.3 Choice of method

Based on the observed characteristics of the four cargo assigning methods, we have chosen to work with Random in the remainder of this article.

2.5.3 Analysis of the solutions

Using the Random method for assigning cargo to routes we will look at various problem aspects and their impact on the characteristics of the solutions. The first aspect to be discussed is the impact of having a heterogenous fleet of ships versus a homogenous fleet of ships. Secondly, we will discuss the impact of the number of cargos and ships on the number of columns generated, the LP iterations, and the total time required to solve the problem. Lastly, we will discuss the impact of the various actions taken to minimize the number of routes generated. This includes fixing the maximum number of ports per route, requiring a minimum of one hub per route, and regional restrictions.

The data and conclusions presented throughout section 2.5.3 are based on appendix 2.A.

2.5.3.1 Heterogenous versus homogenous ships

In order to empirically observe an effect of having heterogenous instead of homogenous ships, the number of initial columns produced has been recorded for certain pairs of instances. In particular, the number of initial columns has been recorded for each instance in four instance sets as shown in table 2.8.

Instance sets T10 and T13 both have 7 ports, 21 ships, and 45 cargos, but they differ wrt. the number of ship types. In T13 there is only one ship type while T10 has 3 ship types each with 7 ships. Column 4 shows the decrease from T10.m to T13.m, i.e., the increase in number of columns as an effect of heterogenous instead of homogenous ships.

Similarly, instance sets T12 and T14 also differ only wrt. heterogenous versus homogenous ships, but here there are 27 ships instead of the 21 ships in T10 and T13. The 27 ships in T14 are of one type only whereas T12 has 3 ship types each with 9 ships.

m	T10.m	T13.m	Diff.	T12.m	T14.m	Diff.
4	3567	2899	668	4759	2919	1840
5	9395	8920	475	13115	8995	4120
6	17416	18856	-1440	24998	19052	5946
7	24743	30215	-5472	36075	30467	5608
8	29306	38475	-9169	42967	38795	4172
9	31203	42403	-11200	46016	42843	3173
10	31693	43617	-11924	46819	44064	2755
11	31788	43897	-12109	46955	44361	2594
12	31821	43929	-12108	46952	44330	2622

Table 2.8: The number of initial columns for instances in different instance sets. The difference columns show the number of additional columns resulting from having heterogenous ships compared to homogenous ships.

In general, there may be two opposite effects of introducing different ship types for a fixed total number of ships. On one hand, different ship types may lead to more columns. This is simply due to the fact that for a given route one column can be created for every ship type that is capable of performing the route. This effect is present in particular when plenty of ships are available. On the other hand, different ship types may lead to fewer columns. Since the number of ships available is given, the number of ships available per ship type decreases as the number of ship types increases. In light of the requirement that the ships on a route must all be of the same ship type, fewer ships per ship type restrict the possibilities of creating columns for relatively long routes. This effect is particularly remarkable when all or almost all available ships are needed in a feasible solution.

Both effects can be seen from the results in table 2.8. Instance sets T12 and T14 have a relatively large number of ships available, and the effect of having multiple ship types is that more columns are generated. On the other hand, for instance sets T10 and T13 where the number of ships is relatively small, the effect is both positive and negative. For the smaller routes (with up to 5 ports per route) more columns are generated when there are multiple ship types. However, as the routes grow longer, having multiple ship types results in fewer columns.

Liner shipping companies generally work to utilize all available ships, wherefore a plenitude of ships will rarely occur. Considering this and the fact that many liner shipping networks have very long routes, the idea of working with different ship types in the model is likely to ensure that fewer columns are generated.

2.5.3.2 Number of cargos and ships

The number of cargos and the number of ships impact the search for a solution. If the number of cargos is increased, the number of columns generated increases. This can be ascertained by doing a pairwise comparison of instance subsets which differ only wrt. the number of cargos. For example, T1 is compared to T2. Similarly, T3 is compared to T4, T9 to T10, T11 to T12, T17 to T18, and T19 to T20. The percentage of port pairs between which there is cargo must increase as the number of cargos increases. With more connected port pairs more routes are likely to pass the requirement that all ports on a route must be needed in order to transport the planned cargos on the route (requirement v) in section 2.4.3.1), and thus more columns are generated.

Similarly, the effect of the number of ships can be ascertained by pairwise comparisons, this time between T1 and T3, T2 and T4, T9 and T11, T10 and T12, T17 and T19, and T18 and T20. If the number of ships is increased, the number of generated columns increases.

The size of the problem in terms of the number of ships and the number of cargos has a strong influence on the computing time. For the instances with 7 ports, the computing time is negligible. However, the computing times of the two groups of instances with 15 and 25 ports are sufficiently large to allow comparison. Accordingly, table 2.9 shows the computing times for different instance subsets.

Instance set	T9	T10	T11	T12	T17	T18	T19	T20
Number of cargos	30	45	30	45	50	75	50	75
Number of ships	21	21	27	27	35	35	45	45
Computing time (seconds)	3	35	5	53	68	612	93	839
Number of LP/MIP iterations	3.1	3.9	3.0	3.6	3.1	4.5	2.9	4.7

Table 2.9: Average computing times and number of iterations.

Table 2.9 shows that there is a sharp increase in the computing time as a result of increasing the number of cargos. However, increasing the number of ships leads to a much smaller increase in the computing time. As the time used to generate the initial columns is negligible, the vast majority of the time reported in table 2.9 is spent on generating additional columns and solving the various LPs and MIPs. While the number of LP and MIP iterations increases as the number of cargos increases, the number of iterations does not always increase when the number of ships increases. It is evident that the number of cargos will have a huge impact on the computing time as the problems grows larger.

2.5.3.3 Limiting the number of generated columns

We now look at the impact of the various actions implemented in the heuristic in an attempt to limit the number of columns generated.

Number of ports allowed per route This action affects both the solvability of the problem and the solution quality as measured in objective value. The impact on the solvability is apparent when looking at the group of instances with 25 ports. As the number of ports allowed on a route increases, the number of instances solved decreases.

We have observed the general pattern: increasing the number of allowed ports per route improves the objective value significantly as long as the number of allowed ports per route is below a certain value, above which there is a remarkable tailing off effect.

Specifically, for the instances with 7 ports it has no effect to increase the number of allowed ports per route from 4 to 5, with instance set T7 being the only exception.

For the instances with 15 ports, the effect of increasing the number of allowed ports per route tapers off above 9 ports per route. This is true for all the test instances within the group. The average objective values for all instances in T9–T16, except the three instances in T16 with more than 12 ports allowed on a route, are shown in table 2.10.

m	T9.m	T10.m	T11.m	T12.m	T13.m	T14.m	T15.m	T16.m
4	100	100	100	100	100	100	100	100
5	85	50	82	73	39	70	84	79
6	78	49	76	71	36	66	72	69
7	74	47	72	68	30	61	68	63
8	71	45	70	66	30	59	65	60
9	71	42	69	64	29	58	65	60
10	70	42	69	63	29	58	64	—
11	70	42	68	63	29	56	64	56
12	70	42	69	63	29	58	64	—

Table 2.10: Indexed objective values for the instances with 15 ports. The indices are calculated based on the objective value obtained by $m = 4$ within the same instance subset. A solution was not obtained for the instances T16.10 and T16.12.

For the instances with 25 ports, the effect of increasing the number of allowed ports per route has been significant for the number of ports that has been tested in the experiments. This indicates that allowing more than 6 ports per route could decrease the objective value even further.

To summarize, setting the number of ports allowed per route too low will result in increased cost. On the other hand, it is possible to limit the number of ports allowed per route thereby decreasing the number of columns without losing large reductions in cost. The challenge then becomes one of finding the value for the upper limit on the number of ports per route that has the effect of limiting the number of columns without too great an increase in cost.

Minimum one hub per route The effect of this requirement has not been analyzed directly, but removing this requirement has instead been approximated by giving all ports hub status. Unfortunately, this has the added effect of changing the transshipment patterns so any changes in the objective value may as well stem from these changes as from any changes in the route structure. The increase in the number of initial columns, however, predominantly arises from the change in the route structure as the change in transshipment patterns may decrease or increase the number of columns only slightly. The impact of removing the requirement can be seen by comparing the number of initial columns in test cases 3 and 7, 11 and 15, and 19 and 23 (see appendix 2.A).

m	T11.m	T15.m	Diff.	Diff. %
4	3314	6584	3270	99
5	9137	15664	6526	71
6	17803	28253	10450	59
7	26576	39906	13330	50
8	32505	47221	14716	45
9	35363	50282	14919	42
10	36203	51175	14972	41
11	36358	51312	14955	41
12	36331	51351	15019	41

Table 2.11: Average number of initial columns and the differences between T11 and T15.

Table 2.11 shows a comparison between the number of initial columns in T11 and T15. Just as the objective value, the percentage of initial columns saved by enforcing a minimum of one hub per route evens out at $m = 9$. This means that at the point where we would expect to find good solutions in terms of objective value, the number of initial columns will decrease with approximately 40% if the requirement is implemented. When looking at the solutions produced for T15.9, only one out of the 10 runs features a route that does not include a hub. It is therefore likely that if the transshipment pattern is not changed, then the impact of requiring a minimum of one hub per route only decreases the number of initial columns and has little or no effect on the quality of the solution in terms of objective value.

Regional restrictions There are two restrictions related to regional aspects. Firstly, there is an upper limit on the number of regions a route can visit. This limit is fixed to 2 regions for the three groups of instances. Secondly, in any visited region there must be at least 2 port visits or a hub visit. Again, instead of analyzing these restrictions directly, an approximation has been made. Rather than removing the restrictions, all ports have been assigned to the same region. This has been done in the construction of instance subsets T8, T16, and T24.

Among the instances with 7 ports, instance subsets T3 and T8 are the ones to consider as they only differ in the number of regions the ports are assigned to. The implementation of the regional restrictions ensures a decrease in the number of initial columns of 64% for $m = 4$ and 72% for $m = 5$, respectively, m being the maximum number of ports per route. Interestingly, the objective values are the same for the two instances. This means that the restrictions have not cut off the best solution. Hence the regional division and restrictions can be enforced without impacting the objective value of the solution.

Among the instances with 15 ports, T11 and T16 illustrate the consequences of imposing the regional restrictions. The percentage of initial columns that can be saved by implementing the restrictions increases as the number of ports per route increases. For $m = 4$ there is a saving of 66%, while the saving for $m = 11$ is 90%. These are significant reductions and they are most likely the reasons why T11 is solved in all 10 runs for all values of m while instances T16.10 and T16.12 cannot be solved and T16.11 is only solved in 3 of the 10 runs. However, the reduced number of columns and the increased solvability come at a cost. The cost of the solutions to T11 is 7% to 31% higher than the cost of the solution to T16. The cost difference increases as the number of ports per route increases. Hence, for this group of instances the regional requirement seriously hampers the efforts of finding a good solution, most likely because the division of ports into regions has been too restrictive.

Among the instances with 25 ports, the comparison is between T19 and T24. As in the previous comparisons, the regional restrictions result in a large decrease in the number of initial columns, which in turn results in greater solvability. In this case the increased solvability comes at a very low cost as the increase in the objective value is 1.4% for $m = 4$ and 0.2% for $m = 5$. This could indicate that the division of ports into regions implemented in this group of instances is a good fit.

Hence, the conclusion must be that the regional restrictions can greatly diminish the number of initial columns with no or little impact on the quality of the solution.

2.6 Concluding remarks and future research

In this paper we present a new mathematical model for the simultaneous routing of ships and cargo in liner shipping which also considers various deployment issues such as which ships to use on which routes and what speed the ships should maintain. The model also incorporates major aspects of the problem such as transshipment and that the time spent in port is dependent on the ship type, the port and the amount of cargo to be loaded and discharged. In addition, a heuristic is developed based on Dantzig-Wolfe decomposition and column generation. As part of the heuristic, several methods for assigning cargo to routes are tested, with Random being the better method in terms of solution quality. Various methods for restricting the set of allowable routes are tested as well. If employed correctly, the methods can aid in reducing the number of initial columns while not hindering good solutions. Lastly, in order to speed up the solution process, discrete dual-feasible function (DFF) cuts are added to the problem before CPLEX is used.

The developed heuristic can solve problems of the size of feeder networks and will do so reasonably fast. The best cargo assigning method is Random which never spends more than 47.5 minutes reaching a final solution. Considering that the problem is tactical and therefore only solved every 4–6 months, it is viable to spend a lot more time on solving the problem. This leaves computing time for more sophisticated features as discussed below. The discussed features are likely to increase the computing time, however, they are also likely to increase the size of the problems the heuristic can solve. The current heuristic can thus be regarded as a possible solution method for feeder networks and as a stepping stone for solving larger problems such as regional networks.

Next, we present some directions for future research. These fall into three distinct areas.

The first area of future research focuses on creating standards. Two standards are needed. First and foremost, a standard should be defined for the problem of routing ships and cargo in a liner shipping setting. Secondly, a benchmark suite which meshes with the agreed upon definition of the problem should be constructed. Together, the two standards will allow comparisons between results by different authors.

The second area is concerned with other methods for solving the problem presented in this paper. Currently the heuristic uses the MIP solver in CPLEX to find integer solutions. This prevents the heuristic from finding solutions to larger problems. Hence, finding a more efficient method for determining the integer solutions should allow the heuristic as a whole to solve larger problems. Another promising avenue for future research would be to base the column generation on pricing information. This would allow the heuristic to take into account the benefit of adding a certain port or cargo to a route. To get the full benefit of the pricing information the problem could be decomposed into a master problem and a pricing problem and solved by column generation and lagrangian relaxation. This method would follow the mathematical model more closely and could possibly allow non-elementary routes as well as more complex transshipment patterns.

The last area of future research is concerned with variations on the problem presented in this paper. There are numerous interesting variations and we have sought only to highlight a few. One variation would be to change the problem from minimizing the cost as is currently done to maximizing the profit. The change would include the distinction between required cargo that must be transported and optional cargo that may or may not be transported. Another variation is to include restrictions on when port calls can be made to busy ports. Such time slots would have capacity constraints with regards to the number of containers that can be loaded/discharged. A third possible variation springs from the fact that trade is not balanced which leads to a surplus or deficit of empty containers in many ports. Taking into consideration empty containers leads to a need for balancing flows in and out of ports over time.

2.A Detailed results

Table 2.12 shows detailed results obtained by using each solution method on each individual test instance. The columns show the test instance, the method used, the objective value of the solution, the number of cargos which are not transported, the time in seconds spent on finding the initial columns and on the entire method, respectively, the number of LP and MIP iterations, respectively, the number of initial columns, and finally the total number of columns generated. The entry '—' in the 'Obj.' column is used to indicate that the method in question is unable to solve that specific instance, in which case the remainder of the row is kept blank.

Each line for the methods MaxVol, MaxDist, and Greedy shows the eight performance measures corresponding to a single solution.

In contrast, Random produces 10 solutions for each instance. For each of the performance measures, the lines for Random Avg., Random Min., and Random Max. show the average, minimum, and maximum value of the 10 values of that performance measure. As such, the values for Random Min. and Random Max. may be taken from up to eight different solutions. The results for Random Min. and Random Max. are reported only when different from Random Avg.

Table 2.12: Performance of the four cargo assignment methods.

Instance	Method	Obj.	Not transp.	Time		Iterations		No. of columns	
				Initial	Total	LP	MIP	Initial	Total
T1.4	MaxVol	710406	0	0	0	2	1	89	99
T1.4	MaxDist	710406	0	0	0	2	1	89	99
T1.4	Greedy	710406	0	0	0	2	1	91	101
T1.4	Random Avg.	710406	0	0	0	2	1	91	101
T1.5	MaxVol	710406	0	0	0	2	1	92	102
T1.5	MaxDist	710406	0	0	0	2	1	92	102
T1.5	Greedy	710406	0	0	0	2	1	94	104
T1.5	Random Avg.	710406	0	0	0	2	1	94	104
T2.4	MaxVol	740486	0	0	0	2	1	108	114
T2.4	MaxDist	740486	0	0	0	2	1	102	108
T2.4	Greedy	740486	0	0	0	2	1	109	115
T2.4	Random Avg.	740486	0	0	0	2	1	110	116
T2.5	MaxVol	740486	0	0	0	2	1	113	119
T2.5	MaxDist	740486	0	0	0	2	1	108	114
T2.5	Greedy	740486	0	0	0	2	1	114	120
T2.5	Random Avg.	740486	0	0	0	2	1	115	121
T3.4	MaxVol	620015	0	0	0	2	1	115	127
T3.4	MaxDist	620015	0	0	0	2	1	115	127
T3.4	Greedy	620015	0	0	0	2	1	119	123
T3.4	Random Avg.	620015	0	0	0	2	1	119	123
T3.5	MaxVol	620015	0	0	0	2	1	120	132
T3.5	MaxDist	620015	0	0	0	2	1	120	132
T3.5	Greedy	620015	0	0	0	2	1	124	128
T3.5	Random Avg.	620015	0	0	0	2	1	124	128
T4.4	MaxVol	674750	0	0	0	2	1	141	151
T4.4	MaxDist	674750	0	0	0	2	1	131	138
T4.4	Greedy	674750	0	0	0	2	1	144	154
T4.4	Random Avg.	674750	0	0	0	2	1	145	156
T4.5	MaxVol	674750	0	0	0	2	1	150	160
T4.5	MaxDist	674750	0	0	0	2	1	141	148
T4.5	Greedy	674750	0	0	0	2	1	153	163
T4.5	Random Avg.	674750	0	0	0	2	1	154	165
T5.4	MaxVol	783647	0	0	0	2	1	113	117
T5.4	MaxDist	783647	0	0	0	2	1	113	117
T5.4	Greedy	783647	0	0	0	2	1	113	117
T5.4	Random Avg.	783647	0	0	0	2	1	113	117
T5.5	MaxVol	783647	0	0	0	2	1	119	123
T5.5	MaxDist	783647	0	0	0	2	1	119	123
T5.5	Greedy	783647	0	0	0	2	1	119	123
T5.5	Random Avg.	783647	0	0	0	2	1	119	123

continued

Table 2.12: Performance of the four cargo assignment methods, continued.

Instance	Method	Obj.	Not transp.	Time Initial	Time Total	Iterations LP	Iterations MIP	No. of columns Initial	No. of columns Total
T6.4	MaxVol	778619	0	0	0	2	1	113	117
T6.4	MaxDist	778619	0	0	0	2	1	113	117
T6.4	Greedy	778619	0	0	0	2	1	113	117
T6.4	Random Avg.	778619	0	0	0	2	1	113	117
T6.5	MaxVol	778619	0	0	0	2	1	119	123
T6.5	MaxDist	778619	0	0	0	2	1	119	123
T6.5	Greedy	778619	0	0	0	2	1	119	123
T6.5	Random Avg.	778619	0	0	0	2	1	119	123
T7.4	MaxVol	1298333	0	0	0	2	1	193	207
T7.4	MaxDist	1298333	0	0	0	2	1	193	207
T7.4	Greedy	1298333	0	0	0	2	1	193	207
T7.4	Random Avg.	1298333	0	0	0	2	1	193	207
T7.5	MaxVol	1164219	0	0	0	2	1	201	215
T7.5	MaxDist	1164219	0	0	0	2	1	201	215
T7.5	Greedy	1164219	0	0	0	2	1	201	215
T7.5	Random Avg.	1164219	0	0	0	2	1	201	214
T8.4	MaxVol	620015	0	0	0	2	1	295	307
T8.4	MaxDist	620015	0	0	0	2	1	321	333
T8.4	Greedy	620015	0	0	0	2	1	326	330
T8.4	Random Avg.	620015	0	0	0	2	1	334	338
T8.5	MaxVol	620015	0	0	0	2	1	376	388
T8.5	MaxDist	620015	0	0	0	2	1	411	423
T8.5	Greedy	620015	0	0	0	2	1	428	432
T8.5	Random Avg.	620015	0	0	0	2	1	440	444
T8.6	MaxVol	620015	0	0	0	2	1	404	416
T8.6	MaxDist	620015	0	0	0	2	1	438	450
T8.6	Greedy	620015	0	0	0	2	1	466	470
T8.6	Random Avg.	620015	0	0	0	2	1	481	485
T8.7	MaxVol	620015	0	0	0	2	1	407	419
T8.7	MaxDist	620015	0	0	0	2	1	438	450
T8.7	Greedy	620015	0	0	0	2	1	472	476
T8.7	Random Avg.	620015	0	0	0	2	1	489	493
T9.4	MaxVol	1800442	0	1	2	3	2	2981	3039
T9.4	MaxDist	1901239	0	3	5	3	2	2205	2316
T9.4	Greedy	1803325	0	0	0	3	2	2510	2557
T9.4	Random Avg.	1780996	0	0	0	3	2	2524	2583
T9.4	Random Min.	1761507	0	0	0	3	1	2499	2541
T9.4	Random Max.	1803325	0	0	1	4	2	2552	2638
T9.5	MaxVol	1503865	0	4	6	5	2	7198	7288
T9.5	MaxDist	1518044	0	12	13	3	2	5100	5156

continued

Table 2.12: Performance of the four cargo assignment methods, continued.

Instance	Method	Obj.	Not transp.	Time Initial	Time Total	Iterations LP	Iterations MIP	No. of columns Initial	No. of columns Total
T9.5	Greedy	1503865	0	0	2	4	2	6621	6721
T9.5	Random Avg.	1505533	0	0	1	3	2	6600	6665
T9.5	Random Min.	1480780	0	0	1	3	2	6548	6606
T9.5	Random Max.	1543630	0	0	2	4	2	6656	6718
T9.6	MaxVol	1415185	0	11	15	3	3	11433	11504
T9.6	MaxDist	1347519	0	30	31	3	1	7879	7899
T9.6	Greedy	1391944	0	0	0	3	1	12656	12710
T9.6	Random Avg.	1396993	0	0	3	3	2	12524	12596
T9.6	Random Min.	1381622	0	0	2	3	2	12471	12549
T9.6	Random Max.	1447049	0	0	5	5	3	12557	12631
T9.7	MaxVol	1382230	0	21	23	3	2	13836	13874
T9.7	MaxDist	1328247	0	52	54	2	2	9486	9524
T9.7	Greedy	1391944	0	0	6	3	3	18616	18701
T9.7	Random Avg.	1319518	0	0	4	3	2	18376	18416
T9.7	Random Min.	1278223	0	0	1	2	1	18326	18362
T9.7	Random Max.	1350093	0	0	7	4	3	18431	18476
T9.8	MaxVol	1315924	0	27	30	2	2	14621	14656
T9.8	MaxDist	1282544	0	69	69	2	1	10095	10104
T9.8	Greedy	1391944	0	0	8	3	3	22591	22694
T9.8	Random Avg.	1260238	0	0	3	3	2	22325	22368
T9.8	Random Min.	1235444	0	0	1	2	1	22233	22274
T9.8	Random Max.	1284533	0	1	5	3	2	22412	22472
T9.9	MaxVol	1315924	0	30	32	2	2	14792	14828
T9.9	MaxDist	1282544	0	64	64	2	1	10239	10248
T9.9	Greedy	1391944	0	1	9	3	3	24415	24518
T9.9	Random Avg.	1262821	0	1	5	3	2	24164	24217
T9.9	Random Min.	1235444	0	1	2	2	1	24052	24121
T9.9	Random Max.	1282544	0	3	15	8	2	24256	24316
T9.10	MaxVol	1315924	0	32	34	2	2	14812	14851
T9.10	MaxDist	1282544	0	67	67	2	1	10251	10260
T9.10	Greedy	1391944	0	1	9	3	3	24961	25064
T9.10	Random Avg.	1254541	0	1	5	3	2	24694	24756
T9.10	Random Min.	1232033	0	1	1	2	1	24581	24652
T9.10	Random Max.	1268141	0	3	12	5	2	24773	24849
T9.11	MaxVol	1315924	0	31	34	2	2	14815	14854
T9.11	MaxDist	1282544	0	67	67	2	1	10251	10260
T9.11	Greedy	1391944	0	1	9	3	3	25052	25158
T9.11	Random Avg.	1245071	0	1	5	3	2	24785	24832
T9.11	Random Min.	1220384	0	1	2	2	1	24707	24731
T9.11	Random Max.	1275933	0	3	10	4	3	24905	24923

continued

Table 2.12: Performance of the four cargo assignment methods, continued.

Instance	Method	Obj.	Not transp.	Time Initial	Time Total	Iterations LP	Iterations MIP	No. of columns Initial	No. of columns Total
T9.12	MaxVol	1315924	0	32	34	2	2	14815	14854
T9.12	MaxDist	1282544	0	67	68	2	1	10251	10260
T9.12	Greedy	1391944	0	1	9	3	3	25059	25162
T9.12	Random Avg.	1242708	0	1	5	3	2	24808	24860
T9.12	Random Min.	1229219	0	1	2	2	1	24677	24688
T9.12	Random Max.	1261834	0	3	16	5	3	24887	24968
T10.4	MaxVol	3209736	0	2	12	5	3	3754	3915
T10.4	MaxDist	12696168	1	6	52	3	3	2911	3029
T10.4	Greedy	3076185	0	0	12	4	3	3548	3672
T10.4	Random Avg.	4160453	0	0	20	5	3	3567	3705
T10.4	Random Min.	3037434	0	0	7	3	2	3547	3655
T10.4	Random Max.	12761672	1	0	64	7	4	3590	3744
T10.5	MaxVol	2414792	0	7	61	4	3	9043	9188
T10.5	MaxDist	2172496	0	20	24	3	2	7084	7175
T10.5	Greedy	2077207	0	0	5	3	3	9286	9386
T10.5	Random Avg.	2092411	0	0	4	5	2	9395	9520
T10.5	Random Min.	2048201	0	0	2	3	2	9371	9485
T10.5	Random Max.	2135535	0	0	8	7	3	9421	9561
T10.6	MaxVol	2241432	0	19	74	5	3	14966	15080
T10.6	MaxDist	2082450	0	45	56	3	3	11480	11604
T10.6	Greedy	2061278	0	0	31	6	4	17231	17343
T10.6	Random Avg.	2018685	0	0	46	4	3	17416	17515
T10.6	Random Min.	1924445	0	0	18	3	2	17357	17453
T10.6	Random Max.	2105893	0	0	69	5	4	17462	17565
T10.7	MaxVol	2081114	0	30	99	5	3	18920	19065
T10.7	MaxDist	2086604	0	76	125	4	3	14717	14782
T10.7	Greedy	1919437	0	0	51	4	3	24497	24576
T10.7	Random Avg.	1943696	0	0	96	4	3	24743	24832
T10.7	Random Min.	1862857	0	0	14	3	2	24684	24770
T10.7	Random Max.	1988348	0	2	319	5	4	24843	24916
T10.8	MaxVol	2026612	0	39	64	5	2	20566	20677
T10.8	MaxDist	1987600	0	104	149	5	3	16550	16673
T10.8	Greedy	1801287	0	1	15	6	2	29057	29148
T10.8	Random Avg.	1857396	0	1	62	4	2	29306	29389
T10.8	Random Min.	1801049	0	0	22	3	2	29228	29317
T10.8	Random Max.	1930331	0	3	173	6	4	29385	29464
T10.9	MaxVol	2125245	0	43	111	4	2	21074	21175
T10.9	MaxDist	1868885	0	93	111	4	3	17275	17415
T10.9	Greedy	1718279	0	1	10	4	2	31002	31066
T10.9	Random Avg.	1740013	0	1	21	4	2	31203	31264

continued

Table 2.12: Performance of the four cargo assignment methods, continued.

			Not	Time		Iterations		No. of columns	
Instance	Method	Obj.	transp.	Initial	Total	LP	MIP	Initial	Total
T10.9	Random Min.	1718279	0	1	8	3	2	31103	31162
T10.9	Random Max.	1786439	0	3	46	5	2	31291	31349
T10.10	MaxVol	2008780	0	44	84	4	2	21199	21304
T10.10	MaxDist	1914921	0	96	101	4	1	17479	17555
T10.10	Greedy	1721556	0	1	18	3	2	31524	31573
T10.10	Random Avg.	1727040	0	1	21	3	2	31693	31756
T10.10	Random Min.	1704001	0	1	8	2	2	31650	31696
T10.10	Random Max.	1770631	0	3	66	5	3	31759	31814
T10.11	MaxVol	2008780	0	45	80	4	2	21222	21327
T10.11	MaxDist	1905260	0	96	107	5	2	17506	17588
T10.11	Greedy	1721556	0	1	21	4	2	31609	31666
T10.11	Random Avg.	1735150	0	1	21	3	2	31788	31843
T10.11	Random Min.	1721440	0	1	7	2	2	31691	31735
T10.11	Random Max.	1786118	0	3	38	5	2	31886	31934
T10.12	MaxVol	2008780	0	44	80	4	2	21222	21327
T10.12	MaxDist	1905185	0	98	108	4	1	17509	17581
T10.12	Greedy	1721556	0	1	18	4	2	31616	31673
T10.12	Random Avg.	1726830	0	1	22	4	2	31821	31885
T10.12	Random Min.	1704001	0	1	7	3	2	31731	31794
T10.12	Random Max.	1781231	0	3	58	7	2	31899	31950
T11.4	MaxVol	1627248	0	1	2	3	2	3953	3998
T11.4	MaxDist	1652337	0	4	4	3	2	2883	2934
T11.4	Greedy	1652337	0	0	0	3	2	3298	3362
T11.4	Random Avg.	1640711	0	0	1	3	2	3314	3367
T11.4	Random Min.	1632323	0	0	0	3	2	3293	3329
T11.4	Random Max.	1652621	0	0	3	5	3	3346	3397
T11.5	MaxVol	1379493	0	4	9	4	2	9979	10060
T11.5	MaxDist	1375244	0	14	18	3	2	6967	7030
T11.5	Greedy	1365445	0	0	31	3	3	9145	9202
T11.5	Random Avg.	1349672	0	0	9	4	2	9137	9204
T11.5	Random Min.	1332532	0	0	1	3	2	9062	9148
T11.5	Random Max.	1379493	0	0	28	5	2	9170	9240
T11.6	MaxVol	1290220	0	12	15	3	2	16199	16262
T11.6	MaxDist	1232539	0	32	33	3	1	11000	11016
T11.6	Greedy	1347262	0	0	22	4	3	18029	18118
T11.6	Random Avg.	1244637	0	0	4	3	3	17803	17852
T11.6	Random Min.	1226876	0	0	0	2	1	17726	17760
T11.6	Random Max.	1285195	0	0	7	4	3	17915	17973
T11.7	MaxVol	1245871	0	21	31	6	2	19772	19811
T11.7	MaxDist	1212797	0	57	59	3	2	13385	13416

continued

Table 2.12: Performance of the four cargo assignment methods, continued.

			Not	Time		Iterations		No. of columns	
Instance	Method	Obj.	transp.	Initial	Total	LP	MIP	Initial	Total
T11.7	Greedy	1324909	0	0	34	4	3	26957	27038
T11.7	Random Avg.	1179237	0	0	5	3	2	26576	26611
T11.7	Random Min.	1157533	0	0	1	2	1	26488	26523
T11.7	Random Max.	1198748	0	0	8	3	2	26689	26728
T11.8	MaxVol	1226827	0	27	34	4	2	20959	21010
T11.8	MaxDist	1194058	0	76	76	2	1	14308	14327
T11.8	Greedy	1316230	0	0	46	3	3	32964	33020
T11.8	Random Avg.	1152987	0	0	5	3	2	32505	32542
T11.8	Random Min.	1108372	0	0	2	2	1	32450	32487
T11.8	Random Max.	1188756	0	1	11	4	2	32598	32654
T11.9	MaxVol	1226827	0	31	37	4	2	21222	21273
T11.9	MaxDist	1194058	0	67	68	2	1	14532	14553
T11.9	Greedy	1316230	0	1	44	3	3	35738	35794
T11.9	Random Avg.	1132590	0	1	6	3	2	35363	35395
T11.9	Random Min.	1108372	0	1	1	2	1	35206	35239
T11.9	Random Max.	1167493	0	3	14	3	2	35525	35540
T11.10	MaxVol	1226827	0	32	39	4	2	21254	21305
T11.10	MaxDist	1194058	0	70	70	2	1	14522	14573
T11.10	Greedy	1316230	0	1	42	3	3	36576	36632
T11.10	Random Avg.	1136385	0	1	6	3	1	36203	36238
T11.10	Random Min.	1108372	0	1	2	2	1	36133	36162
T11.10	Random Max.	1167472	0	3	10	4	2	36288	36322
T11.11	MaxVol	1226827	0	32	38	4	2	21259	21310
T11.11	MaxDist	1194058	0	70	71	2	1	14522	14573
T11.11	Greedy	1316230	0	1	32	3	3	36719	36775
T11.11	Random Avg.	1121296	0	1	5	3	2	36358	36396
T11.11	Random Min.	1108372	0	1	2	2	1	36168	36233
T11.11	Random Max.	1164562	0	3	9	4	2	36520	36531
T11.12	MaxVol	1226827	0	32	38	4	2	21259	21310
T11.12	MaxDist	1194058	0	70	70	2	1	14522	14573
T11.12	Greedy	1316230	0	1	53	3	3	36730	36786
T11.12	Random Avg.	1124908	0	1	7	3	2	36331	36369
T11.12	Random Min.	1108372	0	1	2	2	1	36158	36185
T11.12	Random Max.	1166989	0	3	23	4	2	36581	36635
T12.4	MaxVol	2379565	0	2	8	4	3	5053	5171
T12.4	MaxDist	2534906	0	7	21	5	3	3854	3996
T12.4	Greedy	2345580	0	0	8	4	3	4724	4857
T12.4	Random Avg.	2431030	0	0	11	4	2	4759	4889
T12.4	Random Min.	2370437	0	0	4	3	2	4742	4852
T12.4	Random Max.	2535339	0	0	30	6	3	4787	4934

continued

Table 2.12: Performance of the four cargo assignment methods, continued.

Instance	Method	Obj.	Not transp.	Time Initial	Time Total	Iterations LP	Iterations MIP	No. of columns Initial	No. of columns Total
T12.5	MaxVol	2018513	0	8	77	4	3	12701	12838
T12.5	MaxDist	1849891	0	21	29	3	2	9828	9942
T12.5	Greedy	1797528	0	0	4	5	2	12952	13110
T12.5	Random Avg.	1785113	0	0	4	4	2	13115	13224
T12.5	Random Min.	1766972	0	0	2	3	2	13027	13110
T12.5	Random Max.	1804436	0	0	7	5	3	13170	13273
T12.6	MaxVol	1886749	0	18	92	5	3	21510	21655
T12.6	MaxDist	1780941	0	49	61	3	4	16308	16382
T12.6	Greedy	1774392	0	0	25	3	3	24678	24776
T12.6	Random Avg.	1717179	0	0	50	4	3	24998	25090
T12.6	Random Min.	1636679	0	0	8	3	2	24880	24968
T12.6	Random Max.	1762671	0	0	74	5	3	25085	25176
T12.7	MaxVol	1814525	0	31	89	4	2	27508	27627
T12.7	MaxDist	1766636	0	83	112	3	3	21211	21307
T12.7	Greedy	1728141	0	0	101	4	3	35608	35697
T12.7	Random Avg.	1661353	0	0	120	4	3	36075	36161
T12.7	Random Min.	1616142	0	0	32	3	2	35964	36038
T12.7	Random Max.	1693808	0	0	303	4	4	36220	36318
T12.8	MaxVol	1753603	0	43	71	5	2	30052	30156
T12.8	MaxDist	1708220	0	107	133	4	3	24026	24112
T12.8	Greedy	1592956	0	1	43	2	3	42576	42629
T12.8	Random Avg.	1603193	0	1	97	4	3	42967	43040
T12.8	Random Min.	1574743	0	0	58	3	2	42892	42951
T12.8	Random Max.	1618851	0	1	197	4	3	43133	43199
T12.9	MaxVol	1761120	0	49	101	4	2	30850	30948
T12.9	MaxDist	1637728	0	95	125	6	3	25153	25287
T12.9	Greedy	1548192	0	1	16	3	2	45587	45648
T12.9	Random Avg.	1561487	0	1	80	4	3	46016	46087
T12.9	Random Min.	1552218	0	1	33	3	2	45895	45970
T12.9	Random Max.	1578389	0	3	225	5	3	46069	46154
T12.10	MaxVol	1734663	0	52	108	4	2	31047	31141
T12.10	MaxDist	1630830	0	99	113	5	2	25475	25548
T12.10	Greedy	1532624	0	1	10	3	1	46413	46439
T12.10	Random Avg.	1534478	0	1	57	3	2	46819	46876
T12.10	Random Min.	1532624	0	1	9	3	1	46724	46774
T12.10	Random Max.	1542906	0	3	166	4	3	46951	47023
T12.11	MaxVol	1734663	0	53	102	4	2	31084	31178
T12.11	MaxDist	1630830	0	100	121	5	2	25518	25592
T12.11	Greedy	1520635	0	1	11	3	1	46550	46591
T12.11	Random Avg.	1524721	0	1	35	4	2	46955	47007

continued

Table 2.12: Performance of the four cargo assignment methods, continued.

Instance	Method	Obj.	Not transp.	Time Initial	Time Total	Iterations LP	Iterations MIP	No. of columns Initial	No. of columns Total
T12.11	Random Min.	1517040	0	1	12	2	1	46827	46856
T12.11	Random Max.	1532981	0	3	123	5	3	47135	47173
T12.12	MaxVol	1734663	0	53	102	4	2	31084	31178
T12.12	MaxDist	1630830	0	100	115	3	3	25523	25581
T12.12	Greedy	1520635	0	1	12	3	1	46561	46602
T12.12	Random Avg.	1523492	0	1	26	3	2	46952	46991
T12.12	Random Min.	1516681	0	1	13	3	1	46803	46867
T12.12	Random Max.	1532936	0	1	62	4	2	47071	47116
T13.4	MaxVol	12991755	1	3	44	5	3	3306	3364
T13.4	MaxDist	3478884	0	2	6	5	3	2360	2429
T13.4	Greedy	12770784	1	0	5	3	3	2827	2877
T13.4	Random Avg.	5571512	0	0	6	4	3	2899	2959
T13.4	Random Min.	3218154	0	0	2	3	2	2855	2879
T13.4	Random Max.	13100864	1	0	12	5	3	2933	3031
T13.5	MaxVol	2367267	0	4	20	4	3	9717	9809
T13.5	MaxDist	2271270	0	10	15	4	2	7091	7183
T13.5	Greedy	2143632	0	0	4	3	2	8800	8884
T13.5	Random Avg.	2168435	0	0	7	4	3	8920	9009
T13.5	Random Min.	2060779	0	0	3	3	2	8876	8957
T13.5	Random Max.	2248529	0	0	13	5	3	8970	9062
T13.6	MaxVol	2207996	0	12	177	5	3	19314	19429
T13.6	MaxDist	2131098	0	19	119	3	3	14062	14129
T13.6	Greedy	2039818	0	0	63	4	3	18665	18735
T13.6	Random Avg.	1985492	0	0	169	4	3	18856	18953
T13.6	Random Min.	1824869	0	0	51	3	2	18752	18856
T13.6	Random Max.	2082356	0	0	460	6	3	18919	19078
T13.7	MaxVol	1846236	0	26	47	5	2	28283	28396
T13.7	MaxDist	1948767	0	35	268	5	3	20704	20795
T13.7	Greedy	1852577	0	0	118	3	3	29731	29814
T13.7	Random Avg.	1694827	0	0	167	4	3	30215	30308
T13.7	Random Min.	1653465	0	0	19	3	2	30122	30244
T13.7	Random Max.	1747016	0	0	425	5	4	30305	30397
T13.8	MaxVol	1891886	0	42	69	4	2	33590	33683
T13.8	MaxDist	1781874	0	49	181	4	3	25021	25091
T13.8	Greedy	1878316	0	1	1240	4	3	37844	37915
T13.8	Random Avg.	1681098	0	1	453	4	3	38475	38561
T13.8	Random Min.	1598844	0	0	35	3	2	38367	38457
T13.8	Random Max.	1727287	0	1	1741	6	3	38577	38655
T13.9	MaxVol	1870829	0	49	99	5	3	35670	35768
T13.9	MaxDist	1780573	0	125	560	4	3	26993	27093

continued

Table 2.12: Performance of the four cargo assignment methods, continued.

Instance	Method	Obj.	Not transp.	Time Initial	Time Total	Iterations LP	Iterations MIP	No. of columns Initial	No. of columns Total
T13.9	Greedy	1870502	0	1	367	4	3	41778	41846
T13.9	Random Avg.	1607225	0	1	746	3	3	42403	42492
T13.9	Random Min.	1524281	0	1	25	3	2	42228	42339
T13.9	Random Max.	1643356	0	1	2466	4	4	42479	42585
T13.10	MaxVol	1873919	0	52	139	4	3	36240	36317
T13.10	MaxDist	1780014	0	129	1184	4	4	27619	27693
T13.10	Greedy	1818605	0	1	248	2	3	43024	43069
T13.10	Random Avg.	1617621	0	1	1418	4	3	43617	43696
T13.10	Random Min.	1565456	0	1	154	3	2	43557	43640
T13.10	Random Max.	1667535	0	1	3562	4	4	43688	43752
T13.11	MaxVol	1867976	0	52	120	3	3	36290	36369
T13.11	MaxDist	1780014	0	131	3832	3	4	27749	27842
T13.11	Greedy	1806034	0	1	507	4	3	43260	43326
T13.11	Random Avg.	1611920	0	1	769	3	3	43897	43976
T13.11	Random Min.	1547716	0	1	80	3	2	43756	43842
T13.11	Random Max.	1665478	0	3	1516	4	4	44108	44185
T13.12	MaxVol	1867976	0	54	148	3	3	36300	36379
T13.12	MaxDist	1780014	0	49	1212	3	4	27769	27862
T13.12	Greedy	1806034	0	1	972	3	3	43280	43340
T13.12	Random Avg.	1598561	0	1	788	4	3	43929	44014
T13.12	Random Min.	1526360	0	1	39	3	2	43725	43811
T13.12	Random Max.	1651156	0	1	1758	6	4	44020	44103
T14.4	MaxVol	2663310	0	1	5	4	2	3329	3401
T14.4	MaxDist	2541509	0	2	3	5	2	2375	2467
T14.4	Greedy	2623755	0	0	6	3	3	2849	2926
T14.4	Random Avg.	2642846	0	0	7	4	3	2919	2995
T14.4	Random Min.	2529893	0	0	1	3	2	2904	2977
T14.4	Random Max.	2793781	0	0	21	5	3	2937	3009
T14.5	MaxVol	1956800	0	4	12	3	3	9797	9892
T14.5	MaxDist	1879024	0	8	11	4	2	7147	7233
T14.5	Greedy	1833530	0	0	3	4	3	8879	8981
T14.5	Random Avg.	1839340	0	0	4	4	3	8995	9073
T14.5	Random Min.	1832401	0	0	2	3	2	8948	9026
T14.5	Random Max.	1853970	0	0	9	5	3	9033	9139
T14.6	MaxVol	1914675	0	11	107	3	3	19491	19583
T14.6	MaxDist	1752307	0	20	43	3	3	14190	14266
T14.6	Greedy	1745341	0	0	56	4	3	18834	18922
T14.6	Random Avg.	1750870	0	0	353	4	3	19052	19146
T14.6	Random Min.	1715323	0	0	64	3	3	18991	19072
T14.6	Random Max.	1777496	0	0	943	4	4	19142	19237

continued

Table 2.12: Performance of the four cargo assignment methods, continued.

			Not	Time		Iterations		No. of columns	
Instance	Method	Obj.	transp.	Initial	Total	LP	MIP	Initial	Total
T14.7	MaxVol	1701238	0	25	54	3	3	28573	28650
T14.7	MaxDist	1692725	0	37	177	6	3	20910	21032
T14.7	Greedy	1711907	0	0	135	3	3	29999	30078
T14.7	Random Avg.	1608047	0	0	622	4	3	30467	30550
T14.7	Random Min.	1570196	0	0	93	3	3	30360	30443
T14.7	Random Max.	1643957	0	0	1758	5	4	30601	30672
T14.8	MaxVol	1709563	0	43	105	5	3	33945	34049
T14.8	MaxDist	1658438	0	51	396	5	4	25278	25392
T14.8	Greedy	1718848	0	0	1987	4	3	38176	38280
T14.8	Random Avg.	1556708	0	1	1149	3	3	38795	38872
T14.8	Random Min.	1532033	0	0	315	2	2	38734	38801
T14.8	Random Max.	1591913	0	1	2739	5	3	38856	38947
T14.9	MaxVol	1709628	0	50	144	4	3	36049	36135
T14.9	MaxDist	1658438	0	46	446	4	4	27275	27374
T14.9	Greedy	1641466	0	1	589	4	3	42145	42236
T14.9	Random Avg.	1525451	0	1	1033	4	3	42843	42929
T14.9	Random Min.	1487207	0	1	32	3	2	42751	42809
T14.9	Random Max.	1579106	0	3	2041	5	4	42949	43025
T14.10	MaxVol	1704028	0	52	439	3	3	36623	36720
T14.10	MaxDist	1658438	0	49	2351	4	4	27907	27982
T14.10	Greedy	1632366	0	1	430	3	3	43398	43472
T14.10	Random Avg.	1536683	0	1	1546	3	3	44064	44142
T14.10	Random Min.	1449890	0	1	39	3	2	43980	44034
T14.10	Random Max.	1586149	0	3	3586	4	4	44137	44235
T14.11	MaxVol	1705083	0	52	949	4	2	36673	36757
T14.11	MaxDist	1716102	0	49	2202	5	2	28037	28109
T14.11	Greedy	1623600	0	1	633	3	3	43634	43679
T14.11	Random Avg.	1492417	0	1	804	3	3	44361	44450
T14.11	Random Min.	1445145	0	1	18	3	2	44259	44330
T14.11	Random Max.	1548275	0	3	3792	4	3	44514	44605
T14.12	MaxVol	1705083	0	52	845	4	2	36683	36767
T14.12	MaxDist	1716102	0	49	2749	4	2	28057	28129
T14.12	Greedy	1623600	0	1	385	3	3	43654	43699
T14.12	Random Avg.	1524318	0	1	1334	3	3	44330	44407
T14.12	Random Min.	1481545	0	1	277	3	2	44207	44266
T14.12	Random Max.	1562101	0	3	3005	4	3	44465	44549
T15.4	MaxVol	1833524	0	1	4	3	2	8474	8542
T15.4	MaxDist	1818722	0	7	8	3	2	6087	6159
T15.4	Greedy	1821409	0	0	4	3	2	6631	6732
T15.4	Random Avg.	1781277	0	0	2	3	2	6584	6664

continued

Table 2.12: Performance of the four cargo assignment methods, continued.

			Not	Time		Iterations		No. of columns	
Instance	Method	Obj.	transp.	Initial	Total	LP	MIP	Initial	Total
T15.4	Random Min.	1754726	0	0	1	2	2	6557	6638
T15.4	Random Max.	1822873	0	0	3	5	2	6610	6693
T15.5	MaxVol	1534761	0	7	18	4	2	18067	18168
T15.5	MaxDist	1530544	0	24	27	4	2	12892	12969
T15.5	Greedy	1485244	0	0	6	3	2	15821	15915
T15.5	Random Avg.	1494257	0	0	9	4	2	15664	15752
T15.5	Random Min.	1486672	0	0	4	3	2	15581	15654
T15.5	Random Max.	1505116	0	0	18	4	3	15702	15796
T15.6	MaxVol	1359628	0	17	21	3	2	27242	27328
T15.6	MaxDist	1345374	0	53	56	3	2	19548	19592
T15.6	Greedy	1315299	0	0	7	3	2	28528	28607
T15.6	Random Avg.	1281442	0	0	8	3	2	28253	28317
T15.6	Random Min.	1253185	0	0	2	2	1	28164	28257
T15.6	Random Max.	1317142	0	0	24	6	3	28307	28421
T15.7	MaxVol	1355924	0	28	45	3	2	32772	32837
T15.7	MaxDist	1323649	0	86	93	5	2	24205	24273
T15.7	Greedy	1259949	0	1	13	3	2	40263	40336
T15.7	Random Avg.	1211475	0	1	10	3	2	39906	39960
T15.7	Random Min.	1189528	0	1	7	3	2	39818	39880
T15.7	Random Max.	1226132	0	1	18	5	2	40022	40084
T15.8	MaxVol	1376735	0	37	64	3	2	34572	34640
T15.8	MaxDist	1303215	0	110	118	4	2	26246	26299
T15.8	Greedy	1214811	0	1	8	4	2	47612	47691
T15.8	Random Avg.	1165401	0	1	11	3	2	47221	47269
T15.8	Random Min.	1126261	0	1	3	2	1	47158	47205
T15.8	Random Max.	1197154	0	1	20	4	3	47339	47352
T15.9	MaxVol	1214779	0	41	42	3	1	34923	34946
T15.9	MaxDist	1303215	0	97	105	3	2	26702	26768
T15.9	Greedy	1214282	0	1	14	3	3	50724	50780
T15.9	Random Avg.	1150767	0	1	12	3	2	50282	50325
T15.9	Random Min.	1114428	0	1	4	2	1	50180	50243
T15.9	Random Max.	1168311	0	4	21	4	3	50384	50467
T15.10	MaxVol	1214779	0	42	45	3	1	34987	35010
T15.10	MaxDist	1303215	0	101	119	3	2	26762	26823
T15.10	Greedy	1214811	0	1	10	3	2	51591	51656
T15.10	Random Avg.	1132516	0	1	13	2	1	51175	51197
T15.10	Random Min.	1115951	0	1	2	2	1	51052	51060
T15.10	Random Max.	1167472	0	4	68	3	2	51272	51326
T15.11	MaxVol	1214779	0	42	44	3	1	34992	35015
T15.11	MaxDist	1303215	0	102	120	3	2	26762	26823

continued

Table 2.12: Performance of the four cargo assignment methods, continued.

Instance	Method	Obj.	Not transp.	Time Initial	Time Total	Iterations LP	Iterations MIP	No. of columns Initial	No. of columns Total
T15.11	Greedy	1214282	0	1	17	4	3	51734	51801
T15.11	Random Avg.	1139347	0	1	11	3	2	51312	51358
T15.11	Random Min.	1112938	0	1	3	2	1	51233	51296
T15.11	Random Max.	1166581	0	4	24	6	3	51387	51408
T15.12	MaxVol	1214779	0	43	45	3	1	34992	35015
T15.12	MaxDist	1303215	0	101	119	3	2	26762	26823
T15.12	Greedy	1214811	0	1	22	3	2	51745	51821
T15.12	Random Avg.	1137763	0	1	11	3	2	51351	51393
T15.12	Random Min.	1108631	0	1	2	2	1	51263	51326
T15.12	Random Max.	1177800	0	4	28	5	3	51408	51475
T16.4	MaxVol	1602078	0	3	8	5	3	11967	12079
T16.4	MaxDist	1600342	0	11	14	4	2	8334	8443
T16.4	Greedy	1513940	0	0	2	3	3	9797	9915
T16.4	Random Avg.	1535466	0	0	3	4	2	9813	9915
T16.4	Random Min.	1524874	0	0	1	3	2	9768	9863
T16.4	Random Max.	1579906	0	0	8	6	2	9847	9947
T16.5	MaxVol	1323121	0	20	46	6	3	35969	36098
T16.5	MaxDist	1217904	0	67	75	4	3	24236	24323
T16.5	Greedy	1225250	0	0	19	4	2	33221	33311
T16.5	Random Avg.	1220348	0	0	15	4	2	33186	33258
T16.5	Random Min.	1176709	0	0	7	3	2	33066	33144
T16.5	Random Max.	1262351	0	1	30	6	3	33270	33356
T16.6	MaxVol	1179915	0	93	229	3	3	73727	73794
T16.6	MaxDist	1112884	0	259	354	4	3	47551	47639
T16.6	Greedy	1066777	0	2	47	4	2	82777	82828
T16.6	Random Avg.	1063436	0	2	161	4	3	82436	82499
T16.6	Random Min.	1033076	0	1	53	3	2	82361	82440
T16.6	Random Max.	1089574	0	6	540	5	4	82512	82578
T16.7	MaxVol	1135193	0	293	692	3	3	111399	111471
T16.7	MaxDist	1088281	0	710	1133	4	3	72469	72541
T16.7	Greedy	1059616	0	4	791	5	4	157285	157354
T16.7	Random Avg.	972181	0	5	520	4	2	156230	156279
T16.7	Random Min.	915735	0	4	16	3	1	156067	156128
T16.7	Random Max.	1049909	0	14	1494	5	3	156431	156502
T16.8	MaxVol	1092037	0	599	846	3	2	136150	136203
T16.8	MaxDist	1054797	0	1436	1838	3	3	89105	89170
T16.8	Greedy	—							
T16.8	Random Avg.	918777	0	8	192	4	2	235853	235905
T16.8	Random Min.	907303	0	8	70	3	1	235568	235612
T16.8	Random Max.	937717	0	9	426	4	3	236223	236281

continued

Table 2.12: Performance of the four cargo assignment methods, continued.

Instance	Method	Obj.	Not transp.	Time Initial	Time Total	Iterations LP	Iterations MIP	No. of columns Initial	No. of columns Total
T16.9	MaxVol	1091137	0	809	2236	5	3	147146	147227
T16.9	MaxDist	1054797	0	1965	3476	2	3	97053	97103
T16.9	Greedy	—							
T16.9	Random Avg.	915507	0	14	368	4	2	298268	299129
T16.9	Random Min.	904055	0	11	151	2	2	290939	298790
T16.9	Random Max.	926031	0	37	627	5	3	299314	299360
T16.10	MaxVol	1091679	0	2349	3641	3	2	150496	150566
T16.10	MaxDist	1054797	0	2460	4127	4	3	99301	99372
T16.10	Greedy	—							
T16.10	Random Avg.	—							
T16.11	MaxVol	1103686	0	1012	2150	3	3	150996	151055
T16.11	MaxDist	1054797	0	2676	3477	3	3	99716	99768
T16.11	Greedy	—							
T16.11	Random Avg.	857606	0	15	41	3	1	353724	353742
T16.11	Random Min.	850855	0	15	39	2	1	353687	353718
T16.11	Random Max.	862013	0	15	43	3	2	353757	353769
T16.12	MaxVol	1103686	0	1033	2046	3	3	151051	151110
T16.12	MaxDist	1054797	0	2770	3536	3	3	99752	99804
T16.12	Greedy	—							
T16.12	Random Avg.	—							
T16.13	MaxVol	1103686	0	1035	2198	3	3	151056	151115
T16.13	MaxDist	1054797	0	2775	4605	4	3	99757	99813
T16.13	Greedy	—							
T16.13	Random Avg.	—							
T16.14	MaxVol	1103686	0	1018	2183	3	3	151056	151115
T16.14	MaxDist	1054797	0	2787	4612	4	3	99757	99813
T16.14	Greedy	—							
T16.14	Random Avg.	—							
T16.15	MaxVol	1103686	0	1045	2209	3	3	151056	151115
T16.15	MaxDist	1054797	0	2797	4626	4	3	99757	99813
T16.15	Greedy	—							
T16.15	Random Avg.	842043	0	26	78	3	1	360614	360626
T16.15	Random Min.	825405	0	15	40	2	1	360497	360512
T16.15	Random Max.	851578	0	47	135	3	1	360687	360695
T17.4	MaxVol	3129766	0	13	42	3	2	15061	15173
T17.4	MaxDist	2664726	0	51	59	2	2	10451	10498
T17.4	Greedy	2810333	0	0	13	3	2	14341	14437
T17.4	Random Avg.	2685780	0	0	16	3	2	14035	14112
T17.4	Random Min.	2642918	0	0	8	3	2	13987	14080
T17.4	Random Max.	2771832	0	1	28	3	2	14105	14176

continued

Table 2.12: Performance of the four cargo assignment methods, continued.

			Not	Time		Iterations		No. of columns	
Instance	Method	Obj.	transp.	Initial	Total	LP	MIP	Initial	Total
T17.5	MaxVol	2625167	0	133	245	4	2	59089	59246
T17.5	MaxDist	2328937	0	610	630	5	1	42053	42158
T17.5	Greedy	2305085	0	2	85	3	3	75442	75510
T17.5	Random Avg.	2279416	0	4	119	3	2	73754	73811
T17.5	Random Min.	2244493	0	2	75	2	1	73447	73511
T17.5	Random Max.	2321079	0	7	238	5	2	73886	73945
T17.6	MaxVol	2275093	0	881	996	3	2	137562	137647
T17.6	MaxDist	2172440	0	5638	6243	4	2	104613	104703
T17.6	Greedy	—							
T17.6	Random Avg.	2114016	0	19	1677	3	2	254803	254883
T17.6	Random Min.	2075823	0	10	988	2	2	254532	254613
T17.6	Random Max.	2137340	0	36	2850	4	3	255113	255190
T18.4	MaxVol	5046950	0	16	61	4	3	17722	17973
T18.4	MaxDist	4606837	0	111	142	4	2	14074	14256
T18.4	Greedy	4126127	0	0	6	3	2	18645	18770
T18.4	Random Avg.	4167236	0	0	59	5	3	18342	18525
T18.4	Random Min.	3965338	0	0	15	4	2	18273	18488
T18.4	Random Max.	4289558	0	0	119	7	4	18396	18579
T18.5	MaxVol	3610530	0	168	461	4	3	76546	76803
T18.5	MaxDist	3378763	0	927	1087	4	3	63485	63703
T18.5	Greedy	3227985	0	2	102	6	2	99652	99872
T18.5	Random Avg.	3280457	0	2	1165	4	3	97936	98133
T18.5	Random Min.	3165727	0	2	276	4	2	97631	97828
T18.5	Random Max.	3383703	0	2	2341	5	4	98128	98356
T18.6	MaxVol	3376023	0	1136	3281	6	3	201014	201256
T18.6	MaxDist	3161563	0	7216	8944	5	2	182572	182743
T18.6	Greedy	—							
T18.6	Random Avg.	—							
T19.4	MaxVol	2795836	0	13	59	3	2	20288	20411
T19.4	MaxDist	2386947	0	44	51	3	2	13760	13824
T19.4	Greedy	2486331	0	0	12	4	2	19448	19542
T19.4	Random Avg.	2405431	0	0	15	3	3	18927	18994
T19.4	Random Min.	2386871	0	0	8	2	2	18864	18915
T19.4	Random Max.	2478117	0	0	22	5	3	19008	19081
T19.5	MaxVol	2380218	0	144	782	3	3	83715	83859
T19.5	MaxDist	2178707	0	536	630	3	2	58626	58725
T19.5	Greedy	2149022	0	2	125	4	3	108966	109041
T19.5	Random Avg.	2105245	0	2	172	3	2	105956	106017
T19.5	Random Min.	2091354	0	2	72	2	2	105756	105810
T19.5	Random Max.	2123052	0	2	359	5	3	106286	106344

continued

Table 2.12: Performance of the four cargo assignment methods, continued.

Instance	Method	Obj.	Not transp.	Time Initial	Time Total	Iterations LP	Iterations MIP	No. of columns Initial	No. of columns Total
T19.6	MaxVol	2102222	0	728	869	3	2	201412	201500
T19.6	MaxDist	—							
T19.6	Greedy	—							
T19.6	Random Avg.	—							
T20.4	MaxVol	3886624	0	17	64	5	3	24066	24355
T20.4	MaxDist	3710582	0	105	143	4	3	18663	18917
T20.4	Greedy	3620488	0	0	20	4	2	25382	25630
T20.4	Random Avg.	3558735	0	0	126	5	2	24870	25065
T20.4	Random Min.	3507829	0	0	26	3	2	24785	24962
T20.4	Random Max.	3634296	0	0	299	8	3	24963	25140
T20.5	MaxVol	—							
T20.5	MaxDist	3073044	0	862	1149	3	2	89691	89809
T20.5	Greedy	2857930	0	2	81	4	2	144790	144966
T20.5	Random Avg.	2951327	0	2	1552	4	3	141721	141893
T20.5	Random Min.	2878453	0	2	431	3	2	141445	141620
T20.5	Random Max.	3010187	0	2	2770	6	3	142083	142239
T20.6	MaxVol	—							
T20.6	MaxDist	—							
T20.6	Greedy	—							
T20.6	Random Avg.	—							
T21.4	MaxVol	25188035	2	6	33	3	3	13905	14011
T21.4	MaxDist	14926024	1	81	296	4	3	11241	11341
T21.4	Greedy	5212985	0	0	13	4	3	12737	12811
T21.4	Random Avg.	4510016	0	0	10	4	2	12590	12655
T21.4	Random Min.	4354774	0	0	3	3	2	12527	12603
T21.4	Random Max.	4740523	0	1	28	5	3	12667	12726
T21.5	MaxVol	4195467	0	110	481	6	3	81162	81310
T21.5	MaxDist	3782680	0	925	1141	5	2	64027	64162
T21.5	Greedy	3453252	0	2	758	4	3	83508	83598
T21.5	Random Avg.	3264296	0	3	311	4	3	81907	81986
T21.5	Random Min.	3205497	0	2	111	3	2	81794	81879
T21.5	Random Max.	3316705	0	7	584	5	4	82042	82131
T21.6	MaxVol	3276176	0	924	2471	4	3	291908	292031
T21.6	MaxDist	3134226	0	4211	8619	5	3	233040	233172
T21.6	Greedy	—							
T21.6	Random Avg.	2651878	0	12	795	5	3	346940	347032
T22.4	MaxVol	4073250	0	6	36	4	2	13785	13912
T22.4	MaxDist	3989482	0	29	39	4	2	11201	11322
T22.4	Greedy	3549795	0	0	2	3	2	12758	12816
T22.4	Random Avg.	3571863	0	0	9	4	3	12607	12667

continued

Table 2.12: Performance of the four cargo assignment methods, continued.

			Not	Time		Iterations		No. of columns	
Instance	Method	Obj.	transp.	Initial	Total	LP	MIP	Initial	Total
T22.4	Random Min.	3530139	0	0	3	3	2	12551	12617
T22.4	Random Max.	3691902	0	1	15	5	3	12656	12730
T22.5	MaxVol	3323595	0	96	163	4	2	79947	80074
T22.5	MaxDist	3136901	0	322	476	5	3	63966	64081
T22.5	Greedy	3020085	0	2	186	4	3	83725	83826
T22.5	Random Avg.	2919723	0	4	249	4	3	82032	82121
T22.5	Random Min.	2883138	0	2	82	3	2	81854	81940
T22.5	Random Max.	2969634	0	7	546	5	3	82216	82308
T22.6	MaxVol	—							
T22.6	MaxDist	2823662	0	2506	3563	5	3	233428	233552
T22.6	Greedy	—							
T22.6	Random Avg.	2573780	0	18	310	4	3	347684	347765
T22.6	Random Min.	2541485	0	11	216	3	3	347329	347394
T22.6	Random Max.	2595488	0	41	371	4	3	348013	348091
T23.4	MaxVol	3652984	0	16	100	3	2	43083	43278
T23.4	MaxDist	3425154	0	129	164	4	2	23646	23834
T23.4	Greedy	3343538	0	0	15	3	2	26705	26867
T23.4	Random Avg.	3340229	0	1	39	3	2	35850	36013
T23.4	Random Min.	2869075	0	0	13	3	2	26468	26646
T23.4	Random Max.	3434256	0	4	150	5	3	119798	119936
T23.5	MaxVol	2976100	0	214	576	2	2	150357	150471
T23.5	MaxDist	2880174	0	1314	1475	4	3	87326	87468
T23.5	Greedy	2872843	0	4	104	4	2	120827	120970
T23.5	Random Avg.	2881607	0	4	217	4	2	119599	119737
T23.5	Random Min.	2867923	0	4	142	3	2	119373	119501
T23.5	Random Max.	2894694	0	4	297	6	2	119867	119998
T23.6	MaxVol	—							
T23.6	MaxDist	—							
T23.6	Greedy	—							
T23.6	Random Avg.	—							
T24.4	MaxVol	2773803	0	46	380	3	2	68112	68253
T24.4	MaxDist	2602661	0	258	258	4	2	45233	45346
T24.4	Greedy	2396613	0	1	19	3	2	58714	58809
T24.4	Random Avg.	2372841	0	1	47	3	2	57767	57815
T24.4	Random Min.	2363724	0	1	26	2	2	57682	57720
T24.4	Random Max.	2394223	0	1	83	3	2	57892	57926
T24.5	MaxVol	2295209	0	1111	1678	4	2	300714	300839
T24.5	MaxDist	2169276	0	4684	5084	3	2	219541	219583
T24.5	Greedy	2126846	0	8	695	5	3	348448	348559
T24.5	Random Avg.	2100975	0	8	363	3	2	341610	341683

continued

Table 2.12: Performance of the four cargo assignment methods, continued.

Instance	Method	Obj.	Not transp.	Time Initial	Time Total	Iterations LP	Iterations MIP	No. of columns Initial	No. of columns Total
T24.5	Random Min.	2096402	0	7	257	3	2	341442	341517
T24.5	Random Max.	2105547	0	8	469	3	2	341778	341849
T24.6	MaxVol	—							
T24.6	MaxDist	—							
T24.6	Greedy	—							
T24.6	Random Avg.	—							

2.B Average performances

Tables 2.13, 2.14, and 2.15 show average performance measures for the four methods for the two groups of instances T9–T16 and T17–T24. Moreover, a performance measure over the two groups combined has been calculated as a weighted average using as weights the number of solved test instances as shown in table 2.4. This performance measure for the two groups combined is shown under the heading 'All' in all three tables.

In table 2.13 the average is taken over all solved test instances.

	Cost ($)			Time (seconds)		
Method	T9–T16	T17–T24	All	T9–T16	T17–T24	All
MaxVol	268625	1456499	508727	131	289	163
MaxDist	222345	694686	321785	488	1669	737
Random	51880	13545	43787	132	196	146
Greedy	211224	88839	187632	87	50	80

Table 2.13: The average additional cost and time compared to the best solution.

In tables 2.14 and 2.15 the average is taken over all those solved test instances where the solution is not the best solution.

In relation to the index values in table 2.14, the best solutions have index 100. The numbers reported as 'Average cost index' are the average index values for all observations that do not contain the best objective value. The numbers reported in columns 'Average time index' are the average index values of the time used to reach a solution for all observations that do not contain the best time.

	Average cost index ($)			Average time index (seconds)		
Method	T9–T16	T17–T24	All	T9–T16	T17–T24	All
MaxVol	114	139	119	672	654	668
MaxDist	112	120	114	980	1091	1003
Random	105	101	104	330	477	361
Greedy	113	104	111	419	172	371

Table 2.14: The average index compared to the best solution but excluding best solutions.

	Cost ($)			Time (seconds)		
Method	T9–T16	T17–T24	All	T9–T16	T17–T24	All
MaxVol	278893	1537415	533275	172	366	211
MaxDist	252665	817277	371531	501	1669	747
Random	136426	36764	115386	209	266	221
Greedy	301107	129221	267972	167	202	174

Table 2.15: The average additional cost and time compared to the best solution but excluding best solutions.

2.C Port information

`http://www.portgot.se/prod/hamnen/ghab/dalis2b.nsf`

`http://www.aarhushavn.dk/en/welcome.htm`

`http://www.portoffelixstowe.co.uk/`

`http://webserver.apba.es/portal/page?_pageid=388,171476&_dad=portal&_schema=PORTAL`

`http://www.hafen-hamburg.de/en/overview/Container%20Terminals`

`http://www.bremenports.de/623_2`

http://www.portofrotterdam.com/en/Pages/default.aspx

http://www.dunkerque-port.fr/en/

http://www.havre-port.net/pahweb.html

http://www.portdebarcelona.es/en/home_apb

http://www.valenciaport.com/en-US/paginas/default_en_us.aspx

http://www.zeebruggeport.be/en

http://www.marseille-port.fr/v_anglaise/index.htm

http://www.nycterminal.com/t3/index.php?id=106

http://www.port-of-charleston.com/

http://www.scspa.com/charleston/default.asp

http://www.nationalportauthorityliberia.org/index.html

http://www.paa-ci.org/

http://ghanaports.gov.gh/GPHA/index.php

http://www.togoport.net/togoport/

http://www.portdakar.sn/?lang=en

2.D Productivity information

http://www.apmterminals.com

http://www.eurogate.de/live/eg_site_en/show.php3?id=17&nodeid=17&_language=en

http://www.t-p-o.fr/index-gb.asp

http://www.internationalpsa.com/factsheet/map.html

http://www.setv.ci/content/view/5/6/lang,en/

http://www.hph.com/globalbusiness/ports.aspx

http://www.otal.com

http://www.unctad.org/en/docs/rmt2010ch5_en.pdf

http://www.beckettrankine.com/downloads/BCT.PDF

2.E Ship information

http://www.hanse-bereederung.de/fleet/containerships.php

http://www.harperpetersen.com/ships/18.html

http://www.rickmers.com/index.php?id=488&no_cache=1

http://www.wehrship.de/index.php?id=22&L=1

http://www.cma-cgm.com/ProductsServices/ContainerShipping/VesselFleet/Default.aspx

http://www.cosco.com/en/fleet/BoatList.jsp?parCatName=Container%20ship&leftnav=/7/1

http://www.evergreen-line.com/static/jsp/vessel.jsp

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Chapter 3

Rescheduling ships and cargo in liner shipping in the event of disruptions

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Abstract

This paper introduces a new mathematical model for the simultaneous rescheduling of ships and cargo in liner shipping in the event of disruptions. The problem is modeled as a multicommodity flow problem with side constraints based on a time-space network. Given a list of disruptions, the planning period, and the ships and ports involved, the problem consists of constructing a set of ship schedules and cargo routings that allows the resumption of scheduled service at the end of the planning period while minimizing the operating cost. To solve the problem, a Large Neighborhood Search (LNS) heuristic is developed. The heuristic, which alternates between a construction and a repair phase, iteratively destroys and repairs parts of the solution. The aim of the construction phase is to produce a set of feasible ship schedules given the disruptions while the repair phase attempts to repair the cargo routings destroyed by the construction phase. To diversify the search, randomness is included in both phases. The LNS is tested on 20 test instances and the best objective value for each test instances is found within 3 minutes.

Key words: Disruption management, maritime transportation, liner shipping, transportation

3.1 Introduction

In liner shipping, ships operate on closed routes consisting of a sequence of ports where each port may appear more than once in the sequence. Once a ship is assigned to a route, it will perform multiple voyages on the route. In liner shipping, the cargo is mostly containerized and each cargo is characterized by an origin port, a destination port, a volume, a time at which the cargo becomes available for loading at the origin port, the planned delivery time at the destination port, and the cost of delivering the cargo later than planned. The ships are assigned to trade lanes based on ship and port characteristics. Once the ships are assigned to their respective trade lanes, the liner shipping companies create schedules based on the available ships and the forecasted cargos. While ships and cargo are the main components of the schedule, additional considerations must be made. These include berth availability in the various ports, transit time requirements, frequency of service between main hubs, and the level of buffer to be incorporated in the schedule to maintain schedule reliability. The buffer can be included in one of two ways. The buffer can be included as an extension of the port call where the scheduled port call is longer than what would normally be required. This is done for ports with high performance variability both in terms of productivity and congestion and when the volume is uncertain. The buffer can also be included by increasing the time for sailing thereby allowing the ships to not sail at maximum service speed. It is generally included to secure schedule reliability. However, in recent years lowering the speed has also been used as a way of absorbing surplus ship capacity. According to Salles (2011) more than 80% of services connecting Europe and the Far East are Extra Slow Steaming (ESS) which means that ships with a service speed of 24–25 knots are sailing 18 knots +/- 1 knot. This provides a great deal of buffer for the schedule and should help increase schedule reliability.

Despite this careful planning of the schedules, the normal operation is often disrupted by unforeseen events. In fact, Christiansen et al. (2004) state that maritime operations have large operational uncertainty. Reasons for the uncertainty in liner shipping include port congestion, lower than expected port productivity, bad weather at sea, labor strikes, mechanical failures, and tidal windows. According to Notteboom (2006), port congestion and lower than expected port productivity account for 86% of schedule unreliability on the East Asia-Europe routes in the fourth quarter of 2004. Given the nature of liner shipping networks, once a disruption happens it will cascade through the network and influence other ports and ships (Notteboom and Rodrigue, 2008).

The economic impact of disruptions in liner shipping is substantial and has an effect on two fronts. There is the cost to the shipping line which includes, amongst others, increased bunker cost, increased port costs both from additional port calls and from stevedores who have to be paid even if they are not used, charter cost for additional ships to transport the cargo, and intangible costs such as loss of goodwill and loss of customers. An increase in service speed by just a couple of knots results in a dramatic increase of fuel consumption. For example, increasing service speed from 18 to 24 knots for an 6,600 TEU container vessel increases its fuel consumption by as much as 130 tons per day. With the current bunker prices of about USD 650 per ton, this translates into an increase in cost of USD 84.500 for each day the higher speed has to be maintained. In addition to the cost to the liner shipping company, there is the cost to the customers who have cargo onboard the affected ships. Notteboom (2006) estimates that a one day delay with a ship carrying 4,000 full TEU's from the Far East to Belgium leads to extra costs for the customers of at least EUR 57,000.

Despite the high likelihood of disruptions in liner shipping, the cascading effect of unopposed disruptions, and the high cost of disruptions, very little research has been done in the area. To our knowledge, only chapter 6 in Andersen (2010) touches upon the problem and then only in a paragraph stating that the developed solution method is also capable of addressing the network recovery/rescheduling problem. However, this is only true if the disruption has already happened when the rescheduling problem is solved. If we want to start the rescheduling while a disruption is taking place or in preparation for a known future disruption, then the developed solution method cannot be used.

In contrast to the liner shipping industry, the airline industry has seen a substantial amount of research on disruption management. The research was pioneered by Teodorović and Guberinić (1984) who presented a problem where one or more aircraft are unavailable and the objective is to minimize the total passenger delay. The general survey on disruption management within airline scheduling done by Clausen et al. (2010) divides the literature into three main areas. The first area focuses on aircraft recovery, the second on crew recovery, and the third on integrated and passenger recovery where passenger recovery is considered an integrated recovery task. The integrated recovery of passengers and flights resembles the problem of rescheduling ships and cargo in liner shipping as both problems are concerned with the means of transportation (plane versus ship) as well as the transported (passengers versus cargo).

According to Kohl et al. (2007), passengers are generally given low priority in the disruption management literature. However, the next three papers focus on the passenger recovery. Bratu and Barnhart (2006) present two models that develop recovery plans for aircraft, crews, and passengers simultaneously by determining which flight leg departures to postpone and which to cancel. The objective is to minimize the airline operating costs, estimated passenger delay, and disruption costs. The models are evaluated by simulation with data provided by a major US airline. In Zhang and Hansen (2008) ground transportation is introduced as an alternative to flights when the capacity of a hub airport is temporarily reduced. The inserted ground transportation supports the passenger recovery. An integer model is presented to assist in deciding if and how to delay or cancel flights and when to substitute flights with buses. The focus of the model is to minimize passenger costs arising due to the passenger recovery and the operating cost of the transportation. The work, on which Bisaillon et al. (2011) is based, was initiated in the context of the 2009 ROADEF Challenge. The challenge consisted of the airline recovery problem combining fleet assignment, aircraft routing, and passenger assignment. To solve the problem, the authors propose a large neighborhood search heuristic with three phases where randomness in the first phase ensures a larger search space.

The purpose of this paper is to introduce the problem of rescheduling ships and cargo in liner shipping when facing one or more past, current or future known disruptions and to provide a solution method. Given the ships and cargo involved, a list of disruptions, and a recovery period, the problem consists of creating new schedules for the ships that adequately take into account the disruptions and a new routing for each cargo that allows the network to return to its normal state of operation at the end of the recovery period at the minimum cost possible. When rescheduling the ships and rerouting cargos, a number of operational constraints must be taken into account, including (but not limited to) ships' capacities, the ships' minimum and maximum service speed, and the productivity in the ports given each ship.

The remainder of this paper is organized as follows. Section 3.2 introduces the problem in more detail and section 3.3 presents the model. The solution method is introduced in section 3.4 and section 3.5 contains the computational results. This is followed by the conclusion in section 3.6.

3.2 Problem Description

Liner shipping companies that need rescheduling of ships and cargo in the event of disruptions will most likely have a large set of published schedules. These schedules consist of a sequence of port calls where the time of each port call is fixed as illustrated by table 3.1. In addition, the network will be such that transshipping cargo between different services is an integrated part of operating the network. The vast majority of transshipments takes place at designated hub ports which are always frequented by two or more routes on a regular basis. Non-hub ports can have transshipment capabilities, but, these ports are only used for smaller volumes or in the case of emergencies.

	Arrival		Departure	
Port Name	Date	Time	Date	Time
Algeciras	05-02-2012	14:00	06-02-2012	20:00
Suez Canal	11-02-2012	19:00	12-02-2012	17:00
Tanjung Pelepas	24-02-2012	23:00	26-02-2011	02:00
Vung Tau	28-02-2012	01:00	28-02-2012	17:00
Nansha New Port	02-03-2012	12:00	03-03-2012	03:00
Yantian	03-03-2012	13:00	04-03-2012	12:00
Hong Kong	04-03-2012	18:00	05-03-2012	06:00
Los Angeles	18-03-2012	18:00	22-03-2012	03:00

Table 3.1: Schedule for Grete Maersk sailing between Europe and the US West coast as found on Maerskline.com.

When disruptions happen in a liner shipping network, their effect on the network needs to be contained both in time and space. The containment in time leads to a decision on the end time of the planning period by which time the ships and cargos must be back on schedule. The containment in space leads to a decision on which ships and ports are to be used in getting the network back on schedule. All ports and ships that are directly involved in the disruptions are included but other ships and ports can also be include depending on their proximity to the disruptions and their perceived ability to alleviate the effects of the disruptions.

The challenge is to find new ship schedules and cargo routings that minimize the operational cost given the capacity constraints and port productivity constraints. The operational cost consists of the cost of fuel for sailing, all port related costs, the cost of transshipments, and the delay cost. The latter cost is the cost of delivering cargo after their planned time of delivery. The cost can be an actual cost stipulated in the contract between the shipper and the liner shipping company or it can be an estimated cost covering intangibles such as lost future sales.

Disruptions in liner shipping have many causes. As mentioned in section 3.1, Notteboom (2006) estimates that port congestion and lower than expected port productivity account for the majority of schedule unreliability on the East Asia–Europe routes. Other common sources of disruption are poor weather which can both delay ships on route and close ports, mechanical problems onboard ships, unexpected waiting time in connection with pilot, towage, bunkering, and tidal windows in ports and access channels. Other less common sources of disruption are strikes, civil unrest, search and rescue missions, ship arrests, and waiting for cargo. More severe disruptions exist and they will likely result in the ship being out of the network for the foreseeable future. Such disruptions include fire incidents, ship collisions, ship grounding, and piracy. Due to the expected length of such disruption these types of incidents are not considered in this article.

When a liner shipping company decides to react to disruptions by rescheduling their ships and cargo, the following information concerning the disruptions must be available. The starting time and ending time of each disruption must be known. A disruption may already have started or it can be starting in the near future. The ending time of the disruptions must also be within the foreseeable future. In addition, the ships and ports involved in each disruption must be known.

Liner shipping companies can use a number of actions to try to recover from disruptions. Notteboom (2006) cites four common actions taken to recover schedules: rearranging the port calls, increasing the speed, omitting port calls, and 'cut and run' which means that the ship departs before all load moves and in some cases even all discharge moves have been performed. In addition, the author mentions the possibility of using ships that are not yet in service for recovering the schedules. For major disruptions this notion can be extended and shipping lines will then charter a ship merely for the purpose of recovering from a disruption. Finally, the author mentions the possibility of increasing port productivity as a way to recover from disruptions. However, this action can only be used to recover small amounts of time and only in a limited number of ports around the world. Another method for schedule recovery is discharging cargo in another port than the final port and then routing the cargo via another mode of transportation to the final port. Trains and trucks are used for this purpose quite often in Europe due to the relatively small distances. However, the method was also used during the 2002 US West Coast lockout of longshoremen where cargo was discharged in Mexico for intermodal transport to the US.

3.3 Modeling and notation

We formulate the simultaneous ship and cargo rescheduling in the event of disruptions problem as a multicommodity flow problem with side constraints on a time-space network given by a multigraph $G = (V, A)$ with vertex set V and arc set A. The elements V and A are described in the following.

3.3.1 Vertices

Each vertex in $V = V_P \cup V_O \cup V_D$ represents a specific physical location at a specific time, where V_P represents ports, V_O represents origins of ships, and V_D represents destinations of ships. The three sets are mutually exclusive. V_O contains a distinct vertex for each ship, where O^h denotes the origin vertex of ship h. V_D contains a set of distinct vertices for each ship, where D^h denotes the possible destination vertices of ship h. For any h, all vertices in D^h represent the same location and cover a time interval $[\Theta_-^h; \Theta_+^h]$, and we consider the rescheduling of ship h to be completed once the ship reaches one of the vertices in D^h.

We use the notation v_{pt} to denote the vertex representing location $p \in V$ at time t. Moreover, when convenient, we use a single letter without subscripts to denote a vertex in V; for any such $i \in V$ we let $location(i)$ and $time(i)$ denote the location and time, respectively, of vertex i. As such, $i = v_{pt}$ is equivalent to $p = location(i)$ and $t = time(i)$.

The planning period starts at $t = 1$ and ends at $t = T$. All origin vertices represent the location of ships at time 0, i.e., $time(i) = 0 \; \forall i \in V_O$. All destination vertices are placed in time after the end of the planning period, i.e., $time(i) > T \; \forall i \in V_D$.

3.3.2 Arcs and associated variables

The arc set A is composed of a number of different types of arcs, all representing flows of ships and cargos. Any arc $(i,j) \in A$ is directed from $i = v_{pt}$ to $j = v_{p't'}$ where $t' > t$, i.e., any arc points forward in time.

In general, for any ordered pair of vertices $(i,j) \in A$, there may be zero, one, or multiple (i.e., parallel) arcs from i to j. The possibility of parallel arcs implies that it is convenient to introduce several subsets of A, where each subset represents a distinct type of flow on ships or cargos.

We introduce an extended notation in order to be able to specify the particular type of an arc. For the ships, we have the following three types of arcs:

Voyage arcs $(v_{pt}, v_{p't'})^h_{\mathrm{VOY}}$ represents that ship h sails from v_{pt} to $v_{p't'}$, where $p \neq p'$. We let the binary variable $x(v_{pt}, v_{p't'})^h_{\mathrm{VOY}}$ take the value 1 if and only if (iff) the corresponding arc is used. Moreover, we let $x^h(i+)$ denote the sum of x-variables associated with voyage arcs for ship h out of vertex $i \in V$ and we let $x^h(-i)$ denote the sum of x-variables associated with voyage arcs for ship h into vertex $i \in V$. In addition, X^h is the set of all voyage arcs for ship h.

Berthing arcs $(v_{pt}, v_{p,t+1})^h_{\mathrm{B}}$ represents that ship h is berthed and under operation in port p from time t to $t+1$. We let the binary variable $y^h(i+)$ take the value 1 iff the berthing arc out of $i = v_{pt}$ is used. Similarly, we let the binary variable $y^h(-i)$ take the value 1 iff the berthing arc into $i = v_{pt}$ is used.

Waiting arcs $(v_{pt}, v_{p,t+1})^h_{\mathrm{W}}$ represents that ship h waits in port p from time t to $t+1$. We let the binary variable $w^h(i+)$ take the value 1 iff the waiting arc out of $i = v_{pt}$ is used. Similarly, we let the binary variable $w^h(-i)$ take the value 1 iff the waiting arc into $i = v_{pt}$ is used.

In addition, we have the following four types of arcs for the cargos:

Onboard arcs $(v_{pt}, v_{p't'})^{mh}_{\mathrm{O}}$ represents the transport of cargo m onboard ship h from v_{pt} to $v_{p't'}$, including any situation where a cargo is onboard a ship while the ship is at port (in such a situation, the corresponding arc has $p = p'$ and $t' = t+1$). We let the continuous variable $u(v_{pt}, v_{p't'})^{mh}_{\mathrm{O}}$ denote the fraction of cargo m transported on ship h from v_{pt} to $v_{p't'}$. Moreover, we let $u^{mh}(i+)$ denote the sum of u-variables associated with onboard arcs for cargo m on ship h out of vertex $i \in V$ and we let $u^{mh}(-i)$ denote the sum of u-variables associated with onboard arcs for cargo m on ship h into vertex $i \in V$. In addition, U^{mh} is the set of all onboard arcs for cargo m on ship h.

Port arcs $(v_{pt}, v_{p,t+1})^m_{\mathrm{P}}$ represents that cargo m is waiting in port p (for initial loading or for transshipment purposes) from time t to $t+1$. We let the continuous variable $z^m(i+)$, where $i = v_{pt}$, represent the fraction of cargo m being held in port p from time t to $t+1$. Similarly, we let the continuous variable $z^m(-i)$, where $i = v_{pt}$, represent the fraction of cargo m being held in port p from time $t-1$ to t.

Loading arcs $(v_{pt}, v_{p,t+1})^{mh}_{\mathrm{L}}$ represents that cargo m is being loaded on ship h in port p from time t to $t+1$. We let the continuous variable $s^{mh}(i+)$, where $i = v_{pt}$, denote the fraction of cargo m being loaded on ship h in port p from time t to $t+1$. Similarly, we let the continuous variable $s^{mh}(-i)$, where $i = v_{pt}$, denote the fraction of cargo m being loaded on ship h in port p from time $t-1$ to t.

Discharge arcs $(v_{pt}, v_{p,t+1})^{mh}_{\mathrm{D}}$ represents that cargo m is being discharged from ship h in port p from time t to $t+1$. We let the continuous variable $r^{mh}(i+)$, where $i = v_{pt}$, denote the fraction of cargo m being discharged from ship h in port p from time t to $t+1$. Similarly, we let the continuous variable $r^{mh}(-i)$, where $i = v_{pt}$, denote the fraction of cargo m being discharged from ship h in port p from time $t-1$ to t.

3.3.3 Ship flows

Each ship h has a capacity, CAP^h, which is the maximum number of TEUs the ship can carry at any given time.

Voyage arcs The time span $t'-t$ of a voyage arc $(v_{pt}, v_{p't'})^h_{\mathrm{VOY}}$ from location p to p' is determined by the chosen speed. Since the speed is variable, there can be several voyage arcs from a given $v_{pt} \in V$ arriving at port p' at different times.

Two costs are associated with each voyage arc $(v_{pt}, v_{p't'})^h_{\mathrm{VOY}}$. One is the fuel cost SC^h_{ij}, where $i = v_{pt}$ and $j = v_{p't'}$, of sailing ship h from location p to p' at the speed corresponding to the time span $t'-t$. The second cost is the cost of calling the destination port p' of the voyage arc, PC^h_j, which includes all one-off costs incurred when a ship calls a port such as pilots and certain harbor dues.

Berthing arcs When a ship is in port and under operation, it is represented by a berthing arc. The cost BC^h_i of a berth arc is the cost of having ship h berthed at p for one time unit. This cost encompasses all costs connected with a port stay that are paid as a function of the time spent at berth. If a port is closed due to a disruption, the flow on the associated berthing arcs is fixed to 0.

Waiting arcs When a ship is waiting, it is denoted by a waiting arc. There is no cost associated with the waiting arcs. A waiting arc can appear in one of two places: either it appears between two berthing arcs or it follows a voyage arc. In the latter case, the speed of the voyage arc must be the minimum possible speed for that arc, otherwise it would be cheaper to lower the speed and deleting the waiting arc.

The operational constraint, ship-port compatibility, is incorporated directly in the time-space network, i.e., compatibility is ensured when creating the individual arcs. Ship-port compatibility can be lacking for a number of reasons. First of all, the lack of compatibility can be permanent as is the case if the ship is too large to enter the port or if neither the port nor the ship has cranes, as this means that there is no possibility of loading or discharging cargo. Secondly, the lack of compatibility can be temporary as is the case if the port is closed at certain times, for example at night, or if tides reduce the draft so significantly that ships are unable to safely enter the port.

A ship can enter the planning period in one of three ways. Firstly, the ship can be berthed at the start of the planning period. Secondly, the ship can be at a port waiting. Lastly, the ship can be at sea, in which case there will be a limited number of possible first ports in the planning period; this limit arises due to the fact that the possibilities will most likely have to be calculated manually by the ship's crew as they are the only ones who know the ship's current position. To ensure that exactly one arc is used for each ship for entering the planning period, constraints (3.2) are added.

Once the planning period ends, ship h must finish at one of its destination vertices. This gives rise to constraints (3.3).

3.3.4 Cargo flows

The set of cargos is denoted M and indexed by m. Each cargo is designated by a volume G^m, an origin vertex $O_c^m = v_{pt}$ where p is the origin port of cargo m and t is the time at which cargo m becomes available for loading, and a destination vertex $D_c^m = v_{p't'}$ where p' is the destination port of cargo m and t' is the planned delivery time of cargo m at the destination.

If the planned delivery time for a cargo is later than the end of the planning period, we require that the cargo be onboard a ship at the end of the planning period. That is, we do not permit the cargo to be delivered during the planning period.

However, if a cargo is planned to be delivered during the planning period, the cargo must still be delivered during the planning period after disruptions have been taken into account.

In any case, we let B^m_- denote the set of vertices representing the destination of cargo m at the various times in the planning period at which cargo m may be delivered at the destination. That is, $B^m_- = \emptyset$ if $time(D^m_c) > T$, and $B^m_- = \{v_{pt} | p = location(D^m_c) \wedge time(O^m_c) + 1 \leq t \leq T\}$ if $time(D^m_c) \leq T$. Similarly, we let B^m_+ denote the set of vertices representing the destination of cargo m at the various times after the planning period at which cargo m may be delivered at the destination. That is, $B^m_+ = \{v_{pt} | p = location(D^m_c) \wedge T < t \leq time(D^m_c)$ if $time(D^m_c) > T$, and $B^m_+ = \emptyset$ if $time(D^m_c) \leq T$.

Each cargo also has a unique delay cost, FC^m_i, which is the cost of delivering cargo m at $time(i)$. As long as the cargo is not late, i.e. $time(i) \leq time(D^m_c)$, the delay cost will be 0. When the cargo is delivered after the planned delivery time, the cost of the delay will be a non-decreasing function of the delay.

Onboard arcs The transport of a cargo from one vertex to another onboard ship h is modeled by an onboard arc. An onboard arc must mirror a voyage arc, a berth arc or a waiting arc while also adhering to the capacity constraints on the ships. This gives rise to constraints (3.9) and (3.10).

Port arcs When a cargo is waiting in port either for the initial loading or for transshipment purposes, it is modeled by a port arc.

Loading arcs When a cargo is being loaded onboard a ship, it is modeled by a loading arc.

Discharge arcs Once a cargo has been loaded it must also be discharged, this is modeled by a discharge arc.

Since cargo can only be loaded on or discharged from ship h if ship h is in port, the cargo loading arcs and cargo discharge arcs must mirror a berth arc for ship h. When loading and discharging cargo, the productivity $PROD_j^h$ of a port p for a given ship h must be taken into consideration. These two limitations give rise to constraint (3.8). The costs of loading cargo in $location(O_c^m)$ and discharging cargo in $location(D_c^m)$ are not considered as the costs cannot be avoided given that all cargo should be transported. However, the cost of transshipment changes with changing routings and must be captured. The transshipment cost per TEU, TC_j, in port j is incurred for any cargo m which requires a load on another ship in order for the cargo to arrive at $location(D_c^m)$, i.e., the cost is incurred for all cargos except those for which port j is the destination $location(D_c^m)$. In case a port is unable to handle transshipments, the transshipment cost, TC_j, is fixed at a prohibitively high rate. This will ensure that the port is not used for transshipment.

There are three possible ways in which a cargo can become available for onwards shipping. The first two are related to the way the ships operate when entering the planning horizon. Firstly, the cargo can be onboard a ship which is at sea at the start of the planning horizon. Secondly, the cargo can be onboard a ship which is in port at the start of the planning horizon. The third possibility is that the cargo becomes available in a port during the planning period. Constraints (3.8), (3.9), and (3.10) ensure that in case a cargo enters the planning horizon in the first or second way, then there will be a ship carrying the cargo at sea or in port. Regardless of how a cargo becomes available for onward shipping, it can only become available once. This is ensured by constraint (3.5).

By the end of the planning period all cargos must either be delivered at their individual destination ports or be en route for timely delivery. This gives rise to constraint (3.6) which ensures that either a ship carries the cargo towards the destination or the cargo has already been unloaded at the destination port before the end of the planning horizon.

A number of constraints are required to ensure that the cargos move correctly between ports and ships. Constraints (3.11) are required to ensure that before loading a cargo in p at time t the cargo must have either been discharged from a ship in p at time $t-1$ or it must have been waiting in p at time $t-1$. Constraints (3.12) are included to ensure that cargos are onboard ship h before they are discharged from the ship. Constraints (3.13) are required to guarantee that a cargo is only transported on ship h from port p at time t if the cargo was loaded onboard ship h in port p at time $t-1$ or was already onboard ship h in port p at time $t-1$.

3.3.5 Mathematical Model

Given the timespace network as well as the operating constraints, the model can be formulated as a mixed integer program. The objective of the model is to reschedule the ships and the cargos at minimum cost. The model is formulated as follows:

$$\begin{aligned}
\min \quad & \sum_{h\in H}\sum_{(i,j)\in X^h} x(i,j)^h_{\text{VOY}}(SC^h_{ij} + PC^h_j) + \sum_{h\in H}\sum_{i\in V} y^h(i+)BC^h_i \\
& + \sum_{h\in H}\sum_{m\in M}\sum_{i\in B^m} FC^m_i r^{mh}(-i) + \sum_{h\in H}\sum_{m\in M}\sum_{i\in V\setminus B^m} r^{mh}(-i)G^m TC_i \qquad (3.1)
\end{aligned}$$

$$\begin{aligned}
\text{s.t.} \quad & y^h(O^h+) + w^h(O^h+) + x^h(O^h+) = 1, \quad \forall h \in H \qquad (3.2) \\
& \sum_{i\in D^h} x^h(-i) = 1, \quad \forall h \in H \qquad (3.3) \\
& y^h(i+) + w^h(i+) + x^h(i+) \\
& - y^h(-i) - w^h(-i) - x^h(-i) = 0, \quad \forall h \in H, \forall i \in V_P \qquad (3.4) \\
& \sum_{h\in H} \big(s^{mh}(O^m_c+) + u^{mh}(O^m_c+)\big) + z^m(O^m_c+) = 1, \quad \forall m \in M \qquad (3.5) \\
& \sum_{h\in H}\left(\sum_{i\in B^m_-} r^{mh}(-i) + \sum_{i\in B^m_+} u^{mh}(-i)\right) = 1, \quad \forall m \in M \qquad (3.6) \\
& \sum_{h\in H} \big(s^{mh}(i+) + r^{mh}(i+) + u^{mh}(i+)\big) + z^m(i+)
\end{aligned}$$

$$- \sum_{h \in H} \left(s^{mh}(-i) + r^{mh}(-i) + u^{mh}(-i) \right) - z^m(-i) = 0,$$

$$\forall m \in M, i \in V_P \setminus \{B_-^m \cup O_c^m\} \tag{3.7}$$

$$\sum_{m \in M} \left(s^{mh}(i+) + r^{mh}(i+) \right) G^m \leq y^h(i+) PROD^h(i+),$$

$$\forall h \in H, i \in V \setminus V_D \tag{3.8}$$

$$\sum_{m \in M} u(i,j)_{\mathrm{O}}^{mh} G^m \leq x(i,j)_{\mathrm{VOY}}^h CAP^h, \forall h \in H, (i,j) \in X^h \tag{3.9}$$

$$\sum_{m \in M} u(i,j)_{\mathrm{O}}^{mh} G^m \leq \left(y^h(i+) + w^h(i+) \right) CAP^h,$$

$$\forall h \in H, (i,j) \in U^{mh}, port(i) = port(j) \tag{3.10}$$

$$\sum_{c \in H} r^{mc}(-i) + z^m(-i) = s^{mh}(i+) + z^m(i+),$$

$$\forall h \in H, m \in M, i \in V_P \setminus \{B_-^m \cup O_c^m\} \tag{3.11}$$

$$u^{mh}(-i) \geq r^{mh}(i+), \quad \forall h \in H, m \in M, i \in V_P \tag{3.12}$$

$$s^{mh}(-i) + u^{mh}(-i) \geq u^{mh}(i+), \quad \forall h \in H, m \in M, i \in V_P \setminus \{O_c^m\} \tag{3.13}$$

$$x^h(i+) = \sum_{j | (i,j) \in X^h} x(i,j)_{\mathrm{VOY}}^h, \quad \forall h \in H, i \in V \tag{3.14}$$

$$x^h(-i) = \sum_{j | (j,i) \in X^h} x(j,i)_{\mathrm{VOY}}^h, \quad \forall h \in H, i \in V \tag{3.15}$$

$$y^h(v_{pt}+) = y^h(-v_{p,t+1}), \quad \forall h \in H, p \in V_O \cup V_P, t \in \{0, \ldots, T-1\} \tag{3.16}$$

$$w^h(v_{pt}+) = w^h(-v_{p,t+1}), \quad \forall h \in H, p \in V_O \cup V_P, t \in \{0, \ldots, T-1\} \tag{3.17}$$

$$u^{mh}(i+) = \sum_{j | (i,j) \in U^{mh}} u(i,j)_{\mathrm{O}}^{mh}, \quad \forall m \in M, h \in H, i \in V \tag{3.18}$$

$$u^{mh}(-i) = \sum_{j | (j,i) \in U^{mh}} u(j,i)_{\mathrm{O}}^{mh}, \quad \forall m \in M, h \in H, i \in V \tag{3.19}$$

$$z^m(v_{pt}+) = z^m(-v_{p,t+1}), \quad \forall m \in M, p \in V_P, t \in \{1, \ldots, T-1\} \tag{3.20}$$

$$s^{mh}(v_{pt}+) = s^{mh}(-v_{p,t+1}),$$

$$\forall m \in M, h \in H, p \in V_P, t \in \{1, \ldots, T-1\} \tag{3.21}$$

$$r^{mh}(v_{pt}+) = r^{mh}(-v_{p,t+1}),$$

$$\forall m \in M, h \in H, p \in V_O \cup V_P, t \in \{0, \ldots, T-1\} \quad (3.22)$$

$$x(i,j)^h_{\text{VOY}} \in \{0,1\}, \quad \forall h \in H, (i,j) \in X^h \quad (3.23)$$

$$x^h(i+), y^h(i+), w^h(i+) \in \{0,1\}, \quad \forall h \in H, i \in V \quad (3.24)$$

$$0 \leq u(i,j)^{mh}_{\text{O}} \leq 1, \quad \forall m \in M, h \in H, (i,j) \in U^{mh} \quad (3.25)$$

$$0 \leq u^{mh}(i+) \leq 1, \quad \forall h \in H, m \in M, i \in V \quad (3.26)$$

$$0 \leq z^m(i+) \leq 1, \quad \forall m \in M, i \in V \quad (3.27)$$

$$0 \leq s^{mh}(i+) \leq 1, \quad \forall h \in H, m \in M, i \in V \quad (3.28)$$

$$0 \leq r^{mh}(i+) \leq 1, \quad \forall h \in H, m \in M, i \in V \quad (3.29)$$

Equation (3.1) defines the objective which is to be minimized. There are four terms in the objective. The first term is the fuel cost of sailing the chosen routes and the cost associated with calling the ports. The second term is the cost of the ships staying at the berths. The third term is the cost of delaying the cargo past the original delivery time. The last term is the cost incurred as a result of transshipping cargo.

Constraints (3.2) guarantee that each ship enters the planning period exactly once while constraints (3.3) ensure that each ship sails to its destination port in time to ensure that the ship is capable to commencing the previously scheduled port call on time. Constraints (3.4) ensure a constant, connected flow for each ship from its origin O^h to its destination D^h. Together the three constraints (3.2), (3.3), and (3.4) ensure that each ship performs at most one activity at a time.

Constraints (3.5)–(3.7) have similar effects for the cargo as had the previous 3 constraint types described for the ships. Constraints (3.5) ensure that cargo only becomes available once, while constraints (3.6) ensure the cargo is delivered to the destination port, $location(D^m_c)$. Constraints (3.7) guarantee that the cargo flow is connected and that cargo arriving at a port will also depart the port unless the port is the destination port. Together the constraints ensure that, at any vertex in the time-space network, the fraction of cargo flowing is less than or equal to one.

Constraints (3.8) ensure that cargo is only loaded on (discharged from) a ship in a port if the ship is actually in the port when the loading (discharge) is attempted. Furthermore, the constraints ensure that the volume of cargo planned for loading (discharge) in a given port can be handled productivity-wise given the port and ship.

Constraints (3.9) guarantee that cargo m is only onboard ship h from i to j, where $location(i) \neq location(j)$, if ship h is sailing from i to j. Constraints (3.10) guarantee that cargo m is only onboard ship h from i to j, where $location(i) = location(j)$, if ship h is either berthed or waiting at $location(i)$ from time $time(i)$ to $time(j)$. In addition, (3.9) and (3.10) ensure that at any given time the total volume of cargo onboard a ship is less than or equal to the capacity of the ship.

Constraints (3.11)–(3.13) ensure that cargo movements happen in the correct order. Constraints (3.11) ensure that before a cargo is eligible for loading or for waiting in port it must have been discharged from a ship or already be waiting in port. Constraints (3.12) guarantee that a cargo is onboard the ship in the period prior to any attempt of discharging the cargo from the same ship. Constraints (3.13) ensure that if a cargo is onboard a ship in the current period, then the cargo was either onboard the same ship in the previous period or the cargo was loaded onto the ship in the previous period.

Constraints (3.14) and (3.15) ensure that the relationship between the summary variable $x^h(i+)$ ($x^h(i-)$) and the variables $x(i,j)^h_{\text{VOY}}$ ($x(j,i)^h_{\text{VOY}}$) described in section 3.3.2 is maintained. Constraints (3.16) and (3.17) ensure that the model recognizes the head and tail of a berthing (waiting) arc as pertaining to the same arc. Constraints (3.18) and (3.19) ensure that the relationship between the summary variable $u^{mh}(i+)$ ($u^{mh}(i-)$) and the variables $u(i,j)^{mh}_{\text{O}}$ ($u(j,i)^{mh}_{\text{O}}$) described in section 3.3.2 is maintained. Constraints (3.20), (3.21), and (3.22) ensure that the model recognizes the head and tail of a port (loading or discharge) arc as pertaining to the same arc.

Constraints (3.23) and (3.24) ensure that the variables pertaining to the ships are binary and constraints (3.25)–(3.29) ensure that the variables pertaining to the cargo are continuous and remain between zero and one.

3.4 Solution method

The model presented in the previous section can be solved by the MIP-solver in a commercial solver such as CPLEX. However, it takes CPLEX 90 minutes to solve a problem with 42 time periods, 3 ships, 6 ports, and 11 cargos. Most of the rescheduling problems in the industry will be much larger than this example. In addition, the time available for deciding on a suitable solution will be in the range 60 to 120 minutes. Hence, given the size of the real rescheduling problems and the time available for solving them, we develop a heuristic as the most viable approach to solving the problems.

The developed heuristic is a Large Neighborhood Search (LNS) heuristic. LNS is a general heuristic search paradigm that was originally proposed by Shaw (1998). It closely resembles the Ruin and Recreate heuristic presented by Schrimpf et al. (2000). The basic idea behind LNS is to improve the initial solution by repeatedly destroying and repairing parts of the solution. Rather than generating an initial solution, the original schedule is used as the initial solution. This approach is used due to the complexity of generating a feasible initial solution and because the original schedule was a good solution at the time of its creation. Though the original schedule is no longer operable due to the disruptions, good alternative solutions closely related to the original schedule are likely to exist, which makes the original schedule a good starting point for the search for good solutions.

In the context of rescheduling in liner shipping the disruptions make the current ships' schedules inoperable and the purpose of the LNS is to create a new feasible schedule for the ships and new routings for the cargos. Our method contains two phases, construction and repair, which are repeated until a given computing time is reached. The aim of the construction phase is to construct a feasible schedule for all ships involved while taking into account the disruptions. In most cases this will lead to the destruction of cargo routings. The aim of the repair phase is to repair the cargo routings by changing the schedules which must retain feasibility. In order to diversify the search, randomness is included in both the construction and repair phase. For each iteration of the construction phase there will be multiple iterations of the repair phase.

For the purpose of the LNS, each disruption can only be concerned with one ship or one port. Since an actual disruption may affect several ships and ports, disruptions are split into one disruption for each ship or port affected. A nationwide strike in South Africa will for example lead to the closure of multiple ports which will be handled as one disruption for each affected port. In addition, there may be ships caught in the closed ports that are unable to depart which will be handled as a separate disruption for each ship. Due to this split of the actual disruptions, the LNS views each disruption as affecting either a ship or a port.

3.4.1 Construction phase

In the construction phase, the focus is on constructing a feasible schedule for the ships taking into account the disruptions. We start by randomly sorting the disruptions in order to obtain a different ordering each time the construction phase is performed. Then, starting with the original ship schedule, we try to construct a feasible schedule for each ship by advancing, delaying, and canceling port calls. When canceling a port call, the new schedule is achieved by connecting the previous port call with the subsequent port call. We consider one ship at the time and we check whether the schedule is feasible after taking the disruptions into account. During the construction phase we do not allow the amount of time set aside for a port call to be below two time periods. In addition, port calls not affected by the disruptions cannot be changed.

Each disruption either pertains to a ship or to a port. If a disruption pertains to a ship, there are four relevant scenarios for the placement of port calls vis-a-vis the disruption.

i) The ship is disrupted for the entire time that the port call should have taken place. We remove the port call from the schedule and create a leg connecting the previous port call with the subsequent port call.

ii) The port call is before and during the disruption. We attempt to advance the port call by taking the following actions: moving waiting time, increasing the speed, decreasing the length of the port call. If these actions fail to advance the port call sufficiently for it to take place before the disruption, then the port call is canceled and we create a leg connecting the previous port call with the subsequent port call.

iii) The disruption happens between two port calls. The speed is increased to make up for the time lost due to the disruption. Once the speed reaches the maximum speed and if the schedule is still infeasible, the length of the forward port call is diminished. If the schedule is still infeasible after the speed has been increased to the maximum and the length of the port call has been diminished to two time periods, then the port call is canceled and we create a leg connecting the previous port call with the subsequent port call.

iv) The port call is planned to take place during and after the disruption. We attempt to delay the arrival at the port to after the disruption is over. This is done by decreasing the speed before the port call and increasing the speed after the port call. In addition, the length of the port call can be diminished. If the schedule is still infeasible, the port call is canceled and a leg is created connecting the previous port call with the subsequent port call.

If a disruption pertains to a port, we consider the inbound and outbound legs connected to the disrupted port.

i) We begin by advancing the port call in an effort to finish it before the disruption commences. We start by decreasing the waiting time for the current port call. We proceed by increasing the speed of the ship on the inbound leg, the speed is increased until feasibility is obtained or until the speed reaches the upper limit for the ship's speed.

ii) If advancing the port call does not ensure feasibility, we attempt to delay the port call so that it begins after the disruption is over. We decrease the speed on the inbound leg until either feasibility is obtained or the speed reaches the lower limit for the ship's speed. Feasibility is not obtained unless the speed on the outbound leg does not surpass the upper limit for ship's speed while arriving at the subsequent port on time. If feasibility is not obtained, we maintain the low speed and further delay the arrival at the port by increasing the waiting time. This is done as long as the subsequent port can be reached on time using the maximum speed for the ship.

iii) If the schedule is still infeasible, we remove the port call and connect the previous port and the subsequent port.

Before proceeding to the next phase of the LNS a new solution is found by using the LP solver in CPLEX. We are using the LP solver rather than the MIP solver because the ship-related arcs are already established, and hence the LP solver is used solely for the cargo routings.

3.4.2 Repair phase

In the second phase, the repair phase, the focus is on making the solution feasible with respect to the cargos that cannot be accommodated after the construction phase. In order for the solution to remain feasible with respect to the ships, none of the existing port calls are moved or deleted. Instead the repair phase uses four procedures in an attempt to increase the length of the existing port calls and the possibility of adding new port calls between existing ones. The port calls are extended as this provides additional time for loading (discharging) cargo.

1) The procedure advances the start of the port call by diminishing the waiting time before the port call.

2) The procedure delays the departure of the port call by diminishing the waiting time before the next port call.

3) The procedure delays the departure of the port call by increasing the outbound speed.

4) The procedure advances the start of the port call by increasing the inbound speed.

These four procedures are used in different order depending on the problem at hand. However, in all cases the procedures related to waiting times are used first as these are not associated with any cost, whereas increasing the speed adds to the cost of sailing. The existing port calls can be extended with as little as one time period. Likewise, new port calls can be created for cargo volumes requiring as little as one time period for the port call.

The repair phase starts by randomly sorting both cargos and ships in order to treat the cargos and the ships in a different order each time the repair phase is performed. The cargos not yet accommodated are either onboard a ship at the beginning of the planning period or they become available during the planning period. Since there is no origin port for the onboard cargo the two types of cargo are dealt with separately.

Onboard cargo For the onboard cargo, there are three ways of changing the schedules to accommodating more cargo.

1) If the ship on which the cargo is onboard calls the destination port of the cargo, an attempt is made to lengthen the port call.

2) The cargo is sought transshipped. This is done by discharging the cargo from the current ship in a port (the transshipment port) and then having another ship transport it from that transshipment port to the destination port. In order for a transshipment to take place, the current ship must depart the transshipment port before the second ship arrives. The three port calls are lengthened as needed and possible.

3) If part of the cargo is still not accommodated, then an attempt is made for the current ship to make an inducement call to the destination port.

Cargo that becomes available For the cargo that becomes available during the planning period there are four ways of changing the schedule to accommodate more cargo.

1) If a ship calls both the origin and the destination port of the cargo, then an attempt is made to lengthen the two port calls to accommodate more cargo.

2) An attempt is made to transship the cargo. For this purpose two ships are needed. The first ship must call the origin of the cargo after the cargo becomes available but before calling the transshipment port. The second ship must call the established transshipment port after the first ship has departed and before calling the destination port of the cargo. All four port calls are lengthened as needed.

3) For all ships that call the origin port of the cargo after the cargo becomes available, an attempt is made to insert a port call to the destination port after the port call at the origin port. The port call is extended to the degree needed.

4) For all ships that call the destination port of the cargo after the cargo becomes available, an attempt is made to insert a port call to the origin port of the cargo prior to the port call at the destination port. The inserted port call has to commence after the cargo becomes available.

Based on the schedule achieved through the previous steps a final attempt is made to accommodate the remaining cargo by extending the port calls to the origin and/or destination ports. This is done to take full advantage of the newly added port calls. The solution for the current iteration is found by employing the LP solver in CPLEX.

3.5 Computational experiments

In this section, we describe how the test instances are generated and we present the results of the computational study. The heuristic is implemented in C++ in a Windows environment with extensive use of CPLEX 12.0. All computational experiments are performed on a HP EliteBook with an 2.53 GHz Intel Duo P8700 processor and 1.86 GB RAM. All reported times are rounded to full seconds of CPU time.

For the computational experiments, we have used a maximum computing time of 1,800 seconds. Within the computing time, the construction phase has a maximum number of iterations which is calculated as the number of disruptions squared. Likewise, the number of iterations in the repair phase is limited to twice the number of cargos. However, the repair phase will terminate after 100 iterations without improvement to the solution either in terms of objective value or amount of cargo not transported.

3.5.1 Data

The computational experiments are conducted on generated data rather than actual data. The test instances span from small instances with 50 cargos, up to large instances with more than 400 cargos.

The generation of data starts with one or more randomly generated disruptions. Each disruption pertains either to a ship or a cargo. In addition, each disruption has a starting time and finishing time. When the disruption is known, the length of the planning period must be fixed. The planning period is the time after which ships should be back on schedule at the next port of call and cargos with planned delivery time before the end of the planning period should be delivered and cargos with planned delivery after the end of the planning period must be onboard the planned ship. Since most services in the liner shipping industry are weekly, we have chosen to use 10 and 17 days as the length of the planning period, as this will ensure that 1 (10 days) or 2 (17 days) ships are available for rescheduling on most of the involved services.

With a planning period of 10 or 17 days, the size of the time-space network becomes very large if the time period of the time-space network is one hour. Therefore, we have chosen to fix the length of a time period to 4 hours. By extending the length of each time period to 4 hours we decrease the size of the time-space network while at the same time ensuring that the rescheduling of ships and cargo is done in a timely manner. As a result, a planning period of 10 days equals 60 time periods while a planning period of 17 days gives 102 time periods.

Once the disruptions and number of time periods have been established, the ships and ports to be included in the heuristic can be established. The ships and ports may be involved in the heuristic in three ways. For the test instances all the ships and their schedules are taken from the publicly available schedules for the major shipping lines.

i) *Affected* - ships are *affected* if they are disrupted or if they are scheduled to call a port while the port is disrupted. Ports are *affected* if they are disrupted or if a ship's disruption is expected to directly affect the ship's call to the port.

ii) *Included* - ships are *included* when they have a scheduled call to an *affected* port during the planning period. All ports to which *affected* or *included* ships have scheduled port calls during the planning period are considered *included* as are origins and destinations of the *affected* and *included* ships.

iii) *Possible* - ships are termed *possible* when they are candidates for inducement calls to affected ports. The term *possible* is used for ports to which *possible* ships have scheduled port calls during the planning period as well as for origins and destinations of *possible* ships.

Given the information on *affected*, *included*, and *possible* ports and ships, the cargo is randomly generated while taking into account the schedules. The generated volume of each cargo lies within the interval 15 to 200 TEU while the generated daily delay cost per TEU lies in the interval 100 to 2,000 USD. The cost of not transporting cargo is 1,000,000 USD per TEU which should be significant enough to ensure that as much cargo as possible is transported. All test instances are generated such that all cargo can be transported without incurring any delay cost.

To test the heuristic, 20 test instances have been generated. Each test instance is characterized by the disruptions, the length of the planning period, the number of ships, the number of ports and the cargos as shown in table 3.2. The test instances are based on 5 disruptions. The first two disruptions, A and B, each lead to 4 test instances; 2 test instances with 60 time periods and 2 test instances with 102 time periods. Given the number of time periods, the first test instance contains the *affected* and *included* ships, whereas the larger instance also includes the *possible* ships. The same is the case for the ports.

Test		Time	No. of		
instance	Disruption	periods	ships	ports	cargos
1	A	60	2	4	40
2	A	60	3	5	46
3	A	102	4	6	100
4	A	102	7	8	200
5	B	60	3	4	50
6	B	60	6	7	102
7	B	102	4	6	103
8	B	102	7	8	200
9	C	60	5	10	193
10	C	60	9	12	389
11	C	102	8	16	449
12	D	60	5	10	193
13	D	60	9	12	389
14	D	102	8	16	449
15	E	60	5	10	193
16	E	60	9	12	389
17	E	102	8	16	449
18	D and E	60	5	10	193
19	D and E	60	9	12	389
20	D and E	102	8	16	449

Table 3.2: The test instances generated for testing the performance of the heuristic.

The three disruptions C, D, and E are similar in that the *affected*, *included*, and *possible* ships and ports are identical. Each disruption leads to 3 test instances; 1 test instance with 60 time periods with the *affected* and *included* ships and ports, 1 test instance with 60 time periods and the *affected*, *included*, and *possible* ships and ports, and 1 test instance with 102 time periods with the *affected* and *included* ships and ports. The last 3 test instances with the 2 disruptions D and E, have the same characteristics.

3.5.2 Performance of the heuristic

The performance of the heuristic on the generated test instances is summarized in table 3.3. As expected, the heuristic's computing time increases as the size of the test instances increases and as the number of disruptions increases, with several test instances pushing the heuristic to terminate based on the 1,800 seconds upper time limit. However, it is worth noting that despite the occasional long run time, the best objective value for all test instances was found within 152 seconds.

The cost of disruption is reported in 1,000 USD and it is the difference between the operational cost with and without the disruption. The operational cost consists of the sailing cost, the transshipment cost, the port and berth costs as well as the delay cost. When the cost of disruption is negative, it is most often due to a decrease in speed and omitted or shortened port calls.

Since all *affected* and *included* ships and ports are included in the smallest test instance created for a disruption, and given that all cargo can be transported when there are no disruptions, it should be impossible for the volume of cargo not transported to increase when the *possible* ships and ports are added to the test instances. Therefore, the amount of cargo not transported in test instance 4 represents an anomaly. Test instance 4 can be separated into two parts; test instance 3 and the *possible* ships and ports and the cargo associated with the aforementioned. Given disruption A, the *possible* ships and ports and the associated cargo have been tested by the heuristic and the ships are able to transport all cargo without any delay cost. Hence test instance 4 should have the same volume of not transported cargo as test instance 3 at the most. We are unable to establish the cause of this anomaly.

Given a number of time periods, if the volume of cargo not transported remains the same, then the cost of disruption will remain the same or decrease as the possible ships and ports are added. In addition, increasing the number of time periods will result in the same or a lower volume of cargo not transported.

Test instance	Time (secs.)		Disruption cost (1,000 USD)	Not transported cargo (TEU)
	Total	Best obj.		
1	17	1	-1,279	354
2	35	13	-1,302	354
3	117	12	214	269
4	411	9	175	284
5	33	10	3,785	1,290
6	205	6	3,785	1,290
7	109	2	997	1,040
8	451	59	996	1,040
9	214	5	-2,144	554
10	789	16	-2,144	554
11	1,800	37	61	420
12	213	5	-1,015	554
13	792	15	-1,015	554
14	1,800	99	-106	420
15	220	5	1,255	2
16	912	17	1,253	2
17	1,800	152	1,473	0
18	896	5	243	556
19	1,800	33	240	556
20	1,800	110	1,120	420

Table 3.3: A summary of the results obtained from running the heuristic on the generated test instances.

The results clearly show that it is more effective to extend the planning period than to include the possible ships and ports in an attempt to decrease the volume of cargo not transported. This may partially be due to the fact that the heuristic only allows inducement calls to the origin or destination port of the cargo, thereby excluding the possibility of inducement calls for transhipment purposes or to both the origin and the destination ports.

3.6 Conclusion

In this paper, we present a new dedicated mathematical model for the simultaneous rescheduling of ships and cargo in liner shipping in the event of disruptions. The model is a multicommodity flow problem with side constraints based on a time-space network. The model is able to handle past, present and future disruptions.

We develop a Large Neighborhood Search (LNS) heuristic to solve the problem. The LNS consists of a construction phase, which constructs a feasible schedule for all ships involved while taking into account the disruptions, and a repair phase, which attempts to repair the cargo routings by changing the schedules while maintaining schedule feasibility. The LNS is tested on 20 test instances and the best objective value for all test instances is found within 3 minutes.

A number of possible extensions of the model warrant further research. As mentioned in section 3.5.2, it may be beneficial to add a procedure in the repair phase that considers inducement calls for transshipment purposes and inducement calls to both origin and destination of a cargo. Both procedures should increase the effectiveness of including the possible ships and ports in a test case, thereby decreasing the volume of cargo not transported. Another addition to the LNS that may improve both the cost and the volume of cargo not transported would be a local search implemented after the repair phase to search the immediate area around the best solutions found.

Another possible extension of the model is the inclusion of intermodal transport. As mentioned in section 3.2, intermodal transportation is widely used as a way of recovering from a disruption and therefore it would be interesting to include the possibility in the model. Given the time-space network, the intermodal transportation could be included as arcs with given time requirement and cost that connect two ports but do not require the presence of a ship. Another method for recovering from a disruption mentioned in section 3.2, namely using additional ships chartered for the specific purpose, would also be an interesting extension.

Including time windows would be another interesting extension of the problem. Within liner shipping time windows can be qualified in a number of ways. They can be given as a fixed time period within which any one ship can be operated and the port productivity will be the standard rate for that ship. However, a time window can also be given as the number of moves the port is able to provide within a given time period, in which case the normal productivity rate for the port can no longer be used. Hard time windows could be incorporated into the time-space network by not creating the berthing edges outside of the time window. This has the advantage of greatly pruning the possible decision tree for each ship. The inclusion of time windows would have the additional effect of making the ships interdependent and not independent as in the present model.

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ISBN 978-8778826-16-9