Proceedings of the IASS 2022 Symposium affiliated with APCS 2022 conference
Innovation·Sustainability·Legacy
19 – 22 September 2022, Beijing, China

Why Did Ancient Hellenic Temples Collapse?
The Case Study: Athena Pronaia Tholos at Delphi

Attilio PIZZIGONI*, Vittorio PARIS[a], Valentina BEATINI[b]

[*]Architect, former Associate Professor, University of Bergamo - Italy
attilio.pizzigoni@gmail.com

[a] University of Bergamo, Italy. e-mail: vittorio.paris@unibg.it
[b] Aarhus University, Denmark. e-mail: valentina.beatini@cae.au.dk

Abstract

The hypothesis that the stone roof of Delphic Tholos has been vaulted has been already stated in a previous article by the same authors, *The Stone Roof,* cited in the references. Such thesis is coherent with the finds of the stone tiles preserved in the Archaeological Museum of Delphi. Within this document, the structural state has been examined at two different scales (local equilibrium and global equilibrium). The relevance of the stereotomy in achieving the balance is highlighted by studying the aggregations of a few tiles (local equilibrium) using a Non-Smooth Contact Dynamic Formulation. The study shows that any kinematics mechanism is avoided due to the tiles' geometry. The overall state of the Delphic Tholos has been estimated (global equilibrium) through a Graphic Static method, the Graphic static tool of the Modified Thrust Line Method. The analyses show how the masses of the acroteria and entablature (geisons, or cornice, frieze and architrave) provide the contrast needed to achieve the balanced state. In particular, the remotion of acroteria, connected with the frieze by the cornice, could be the reason for the collapse of the entire structure.

Keywords: stone roof, interlocking blocks, reciprocal self-sustainable equilibrium, Greek temples, Delphi.

1. Introduction

The art of stone cutting to cover vaults of the sizeable structural span was neither the exclusive prerogative of Philibert de l'Orme's France [1] nor of the great medieval masters of the Gothic cathedrals in Northern Europe [2]. Such a technique flourished in the Hellenic cities of the V and IV centuries BCE, as demonstrated, among the many examples, by the ruins of the Temple of Apollo in Bassai [3] by the great architect Ictino. Current knowledge of the ancient technique relies only on archaeological finds: the treatises of the Roman imperial era did not discuss it, perhaps because they focused on the development of the opus caementicium. To understand if the advanced technology was meant for purely technical purposes or also structural ones, the authors consider as a case study the roof of the tholos of Delphi in the archaeological site of Marmarià, the hypothesis of the existence of the vaulted stone roof of the Delphic Tholos has been already stated in [4]. The structure, also known as the temple of Athena Pronaia, is a circular building with an outer diameter of 800 dactyls (about 15 m). From the archaeological excavation campaigns [5] [6] [7] [8], several fragments of its stone tiles have emerged: they have complex shapes and are finely cut; however, they have not yet allowed a unique reconstruction hypothesis of the roofing structure. Archaeologists have proposed different roofs with wooden substructures, but all the hypotheses assume the presence of additional elements or details of which there is no evidence. The presented study aims to verify whether it would have been possible to create a self-supporting roof structure consisting of such tiles only. In the previous study by the same authors [4], the possible shape of the tiles was reconstructed from the archaeological fragments of the tiles preserved in the Archaeological Museum of Delphi, see Figure 1. Though traditionally called tiles, the blocks have weight, grooves and recesses, it has been assumed they could absolve a structural role. The possible rules of their reciprocal spatial combination have been investigated through physical and digital models. The tiles have two layers, and the supposed reconstruction of the lower one highlights the possibility of combining the tiles in couples to create staggered squared patterns, Figure 2a. The upper layer, whose reconstruction presents more minor uncertainties, is symmetrical, Figure 2b. Eventually, based on the finding and the size of the temple, a vaulted structure was hypothesised, Figure 2c. In the derived hypothesis, the tiles create partially interlocking blocks: due to their asymmetry and their arrangement, they can become topologically interlocking [9] along specific planes of contact.

Now, a two-fold analysis is carried out to further evaluate if and to what extent the stereotomy of the stone had been conceived with a structural intention. Firstly, the local equilibrium of the tiles in an arch jack, plate-bande [10], is investigated through a Non-Smooth Contact Dynamic method. Although the model does not reproduce the actual orientation and boundary conditions of the tiles, it allows us to isolate the specific contribution of the geometry of the contact surface from the behaviour of the whole dome. Eventually, it is verified through graphic statics whether the shape of the vault alone or with the help of complementary elements like the tabulation and the acroteria could guarantee the stability of the structure under its weight.

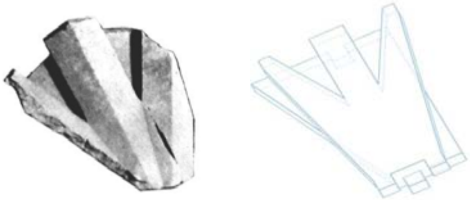

Figure 1 The damaged blocks from the archaeological excavation and the superimposed tiles proposed in the paper. Axonometry

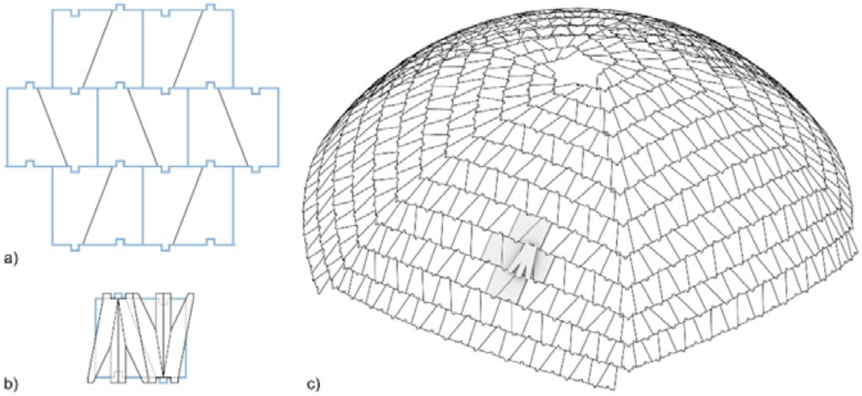

Figure 2 The dome (c) tiling is supposed to be composed of staggered modules made of two tiles: a lower asymmetrical layer (a), and an upper symmetrical layer (b).

2. Local stability

The contribution to the stability provided by the interlocking blocks is correlated to many geometrical parameters: the flow of forces within the dome to the supports, the bond pattern, and the blocks' geometry. To distinguish the latter, a vertical jack arch has been modelled composed of the same tiles hypothesised for the dome reconstruction. The arch has an average span of 2.25m, an average thickness of 0.11m and a structural depth of 0.74m. Eventually, the blocks were subjected to static self-weight and analysed within a Non-Smooth Contact Dynamic (NSCD) approach. The NSCD formulations belong to the Discrete Elements Method (DEM), that are aimed at dynamically analysing distinct bodies represented as discrete rigid elements. Like in the classical Heyman's assumption, masonry is considered to have infinite compressive strength and no tensile strength, but here it can also account for shear failure and any movement caused by the interaction of the blocks at their frictional contact surfaces. Assuming perfectly rigid contacts, the method does not require any material parameter except the density, here set as g=1800Kg/m3, the friction coefficient, set as m=0.4, and the compliance here set as 1/k = 1-10. The analyses have been performed in an ad-hoc NSCD masonry simulator within the open-source Project Chrono C++ platform [11] integrated by a parametric design environment for the pre-and post-processing of the resultant forces. The time step is set to 0.001s. The reader is referred to [12] for a detailed discussion of the simulator and to [13] or [14] for examples of its implementation, focusing on the geometry of the whole structure and the bond pattern, respectively. In the following cases, two sets of images have been produced. The first is a technical drawing illustrating the undeformed model and, in red, the superimposed thrust line that has been obtained composing the contact forces resulting from the analyses at a simulation time equal to 1sec. In the figures, the hatched blocks are the fixed support. Eventually, progressive simulation stages have been reported, produced by the NSCD tool.

The case in Figure 3 analyses the lower layer of the tiles. Some of the contact surfaces between the tiles have clumsy orientations because they cannot be perpendicular to any potential thrust action forces that could sustain the jack arch. Moreover, the jack arch has no side supports that could resist the thrust. As depicted in the simulation pictures in Figure 4, the structure fails in sliding.

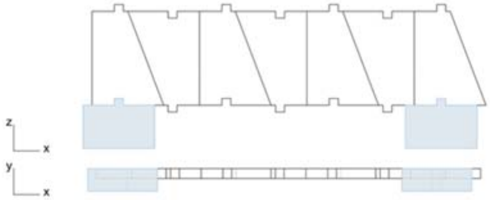

Figure 3: Blocks modelled as the lower layer of the target tiles and supported by end columns. No thrust action can be developed.

Figure 4: Displacement of the interlocking blocks of Fig.3 at simulation time *t* equal to 0s, 0.5s and 1s

In Figure 5, the blocks reproduce the upper layer of the target tiles. Again, the contact surfaces are not reasonably set. However, the supports are perfectly rigid end tiles, which may represent the case of well-compressed side supports. They activate a friction force that avoids the sliding failure previously illustrated. The simulations, Figure 6, show no visible movement.

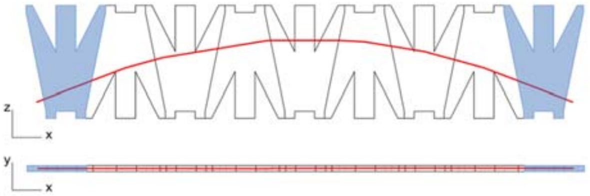

Figure 5: Blocks modelled as the upper layer of the target tiles and supported by fixed blocks at the side ends. No sliding occurs, and a thrust action can be developed.

Figure 6: Displacement of the interlocking blocks of Fig.5 at simulation time *t* equal to 0.5s, 2s and 3s

Figures 7 and 8 show the model and analyses of the whole tiles. Notice that the third and fifth tiles cannot develop any interlocking effect on their contact surfaces to the left. This initiates a local sliding failure, which activates a global torsional mechanism. The latest is also read by the plot of the thrust line, which shows a sudden change in the inclination of the curve to the right end due to the tile before the last touching the support. For the same reasons, the thrust line also pushes to the left and forward, moving out of the horizontal projection of the supports and depicting the failure mechanism. Contrary to the case of Figure 3, the partially interlocking tiles are thus able to develop a global resistance mechanism in case of local displacements.

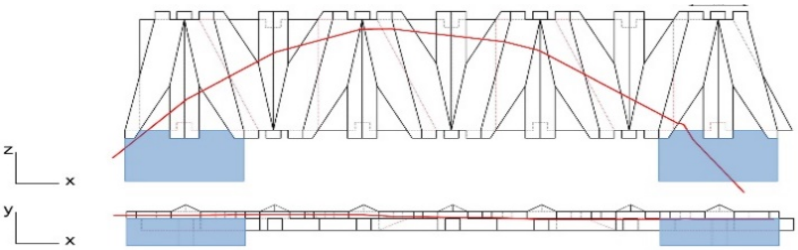

Figure 7: Blocks modelled as the target tiles and supported by fixed blocks at the bottom ends. No sliding occurs, and a thrust action can be developed.

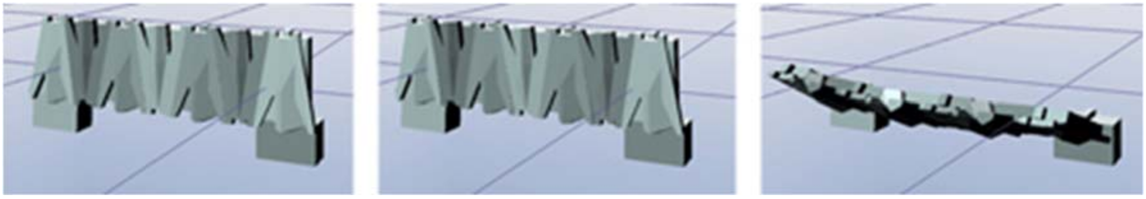

Figure 8: Displacement of the interlocking blocks of Fig.7 at simulation time *t* equal to 0.5s, 2s and 3s

Figures 9 and 10 depict the previous case with thicker supports so that the entire horizontal projection of the end tiles lies within them. The thrust line is still non symmetrical, Figure 9, and slightly pushing to the left. Similarly, it can still be noted in the elevation of the jump in the tangent toward the right support. However, the end tiles oppose the thrust here, and the structure does not fail. The partially interlocking blocks can thus avoid the propagation of a local failure, and ensure global stability, assuming enough resistance can be developed at the supports. The arch jack can stand despite the lack of lateral supports.

It shall be remarked that the resistance mechanism is still an arch action: the tiles are not topologically interlocking, a mechanism would be possible, and the resistance is not totally due to the assumed infinite compression strength of the material. Figure 11 depicts the simulation results from the same case considering frictionless contacts. The structure fails in sliding.

Another interesting aspect is the lack of symmetry of the thrust line. The thrust line pushing to the right implies a higher support force to the same side, which agrees with the supposed bond pattern depicted in Figure 1. Indeed, as detailed in [4], the tiles have been positioned along spirals lying on the surface of the dome in such a way that each tile can partially rest on the ones to its left.

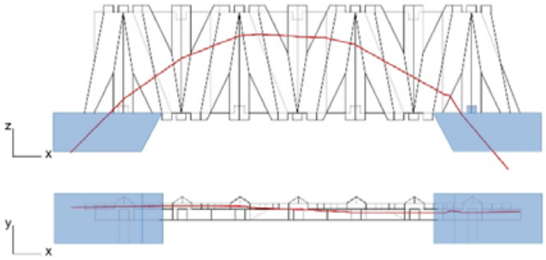

Figure 9: Blocks modelled as the target tiles and supported by large, fixed blocks at the bottom ends. No sliding occurs, and a thrust action can be developed.

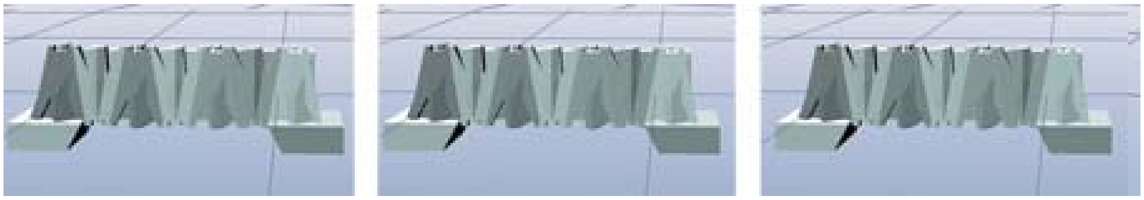

Figure 10: Displacement of the interlocking blocks of Fig.9 at simulation time *t* equal to 0.5s, 2s and 3s

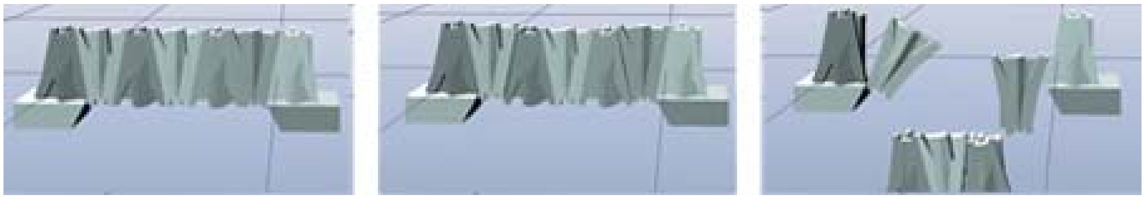

Figure 11: Displacement of the interlocking blocks of Fig.7, assuming frictionless contacts, at simulation time *t* equal 0s, 0.1s and 0.5s.

3. Global stability

The overall stability of the dome has been analysed within the frame of Heyman theory [15], i.e. investigations have been pursued respecting three assumptions: (i) masonry has no tensile strength, (ii) compression stresses are so low that masonry has effectively an unlimited compressive strength and (iii) sliding failure does not occur. According to this approach, structural stability in masonry structures is mainly related to the geometry of structures and the way their masses act.

Analyses have been performed using a Graphics Static (GS) tool: the Modified Thrust Line Method (MTLM) [16] formulated by Eddy in 1878 [17]. Its digital implementation is described in [18] [16] and applicated in [19] [20]. The MTLM allows considering the action of the hoop forces in axial symmetrical shells, i.e., it can graphically describe membrane behaviour by determining a distribution of hoop forces [20].

The primary purpose of the explorations carried out in this section is to understand the role of the acroteria as additional but needed loads to achieve the stability of the stone roof. Two different scenarios have been examined, the first in which the presence of acroteria was assumed. Due to the structural symmetry, the analyses were carried out regarding a sector of 18°, i.e., considering one of the twenty columns and the relative portion of entablature and vault. Dimensions and the geometries of the Delphic Tholos elements are modeled as described in [4]; the thickness of the vaulted stone roof traced is 21cm, and about 80cm is the diameter of the columns. A specific weight of $26kN/m^3$ has been assumed for all elements. Two different scenarios have been examined, the first in which the presence of acroteria is ignored, see Figure 12. In this scenario, the structural state is balanced, and at the base of the column, the Geometrical Safety Factor (GSF) is about 1.12 [21]. The GSF denotes the safe grade of a structure, and its value can change between one and infinite. For a given section, the GFS is estimated by the ratio of the section's half-thickness and the distance of the thrust line concerning the middle point of the section itself. High GSF values imply safe structures; however, they should be related to structural elements, e.g., for buttresses, a safe value is about 2 [22], while, for arches, even lower values are considered safe.

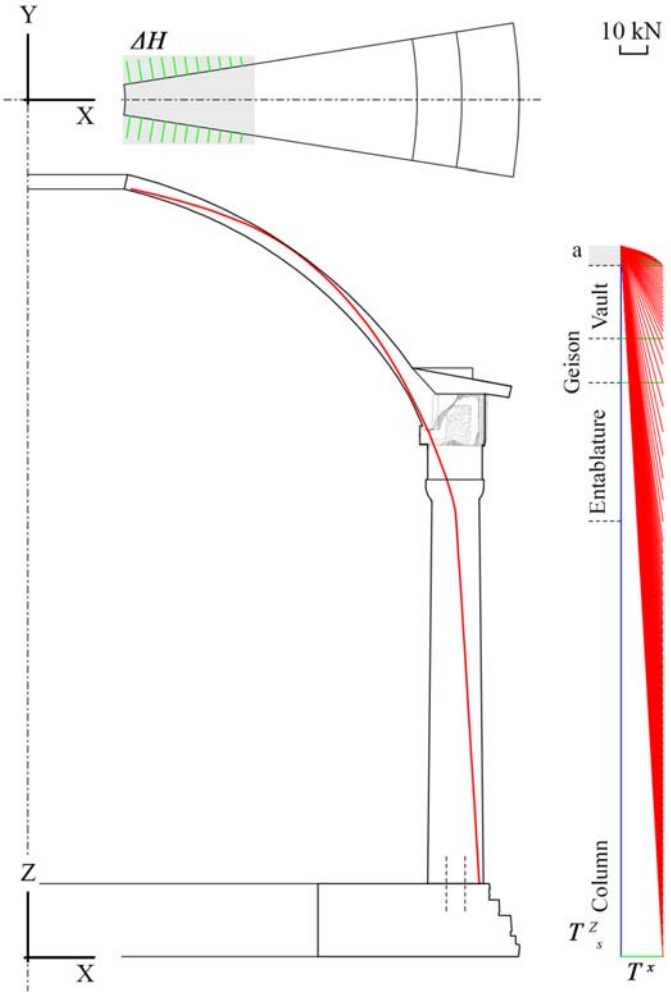

Figure 12: Delphic Tholos Equilibrium. The additional load of acroteria is neglected. The structure finds a balanced state with a safety coefficient of 1.12 [17].

Thus, GSF equal to 1.12 is a low value and suggests a grade of safety not enough to presume a practical building application. Furthermore, as shown in Figure 12, the GSF has been estimated regarding a configuration close to the minimal thrust ($T^x = 16.43$kN); hence the value of GFS is close to the higher admissible for the structure analysed. In the second scenario, the presence of acroteria is acknowledged by placing one statue in correspondence with each column, see Figure 13. Thus, the acroterium ($w_a = 39$kN) weight was supposed to be applied to stabilise the horizontal trust ($T^x = 16.43$kN). With such weight, the variation of the GSF is significant, with a value of 1.73.

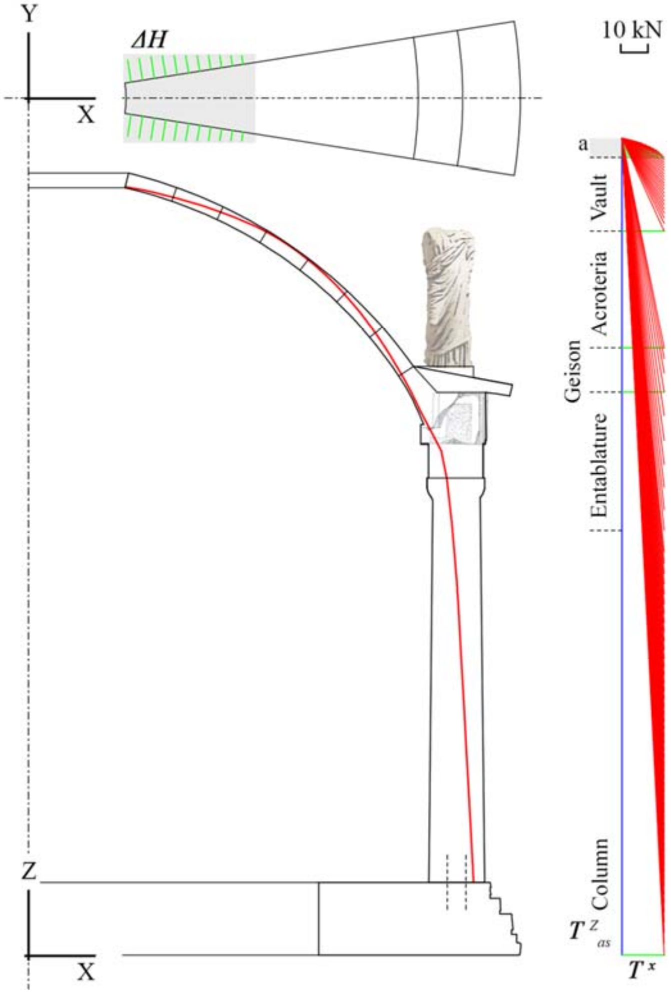

Figure 13: Delphic Tholos Equilibrium. The additional load of acroteria is considered. The structure finds a balanced state with a safety coefficient of 1.78.

The considerable variation of GSF is understandable by estimating the ratio between the column's weight and the acroteria's weight, which is about 2.33, value is exceptionally high if compared to the one related to gothic cathedrals. Unlike gothic structures, where the gargoyle statues play a role only to avoid failures at the top of buttresses, here, the acroteria have a primary role in overall equilibrium. The findings also confirm the hypothesis: the friezes often present fractures, see Figure 14, compatibly with the distribution of forces traced in Figure 13. The analyses show that the line of thrust is affected by the presence of acroteria, but also the presence of geisons, placed just below, plays a structural role. In fact, the geometry and mass (12.5kN) of geisons suggest that they contribute to contrasting the horizontal thrust; their remotion and removal of acroteria could undermine global stability.

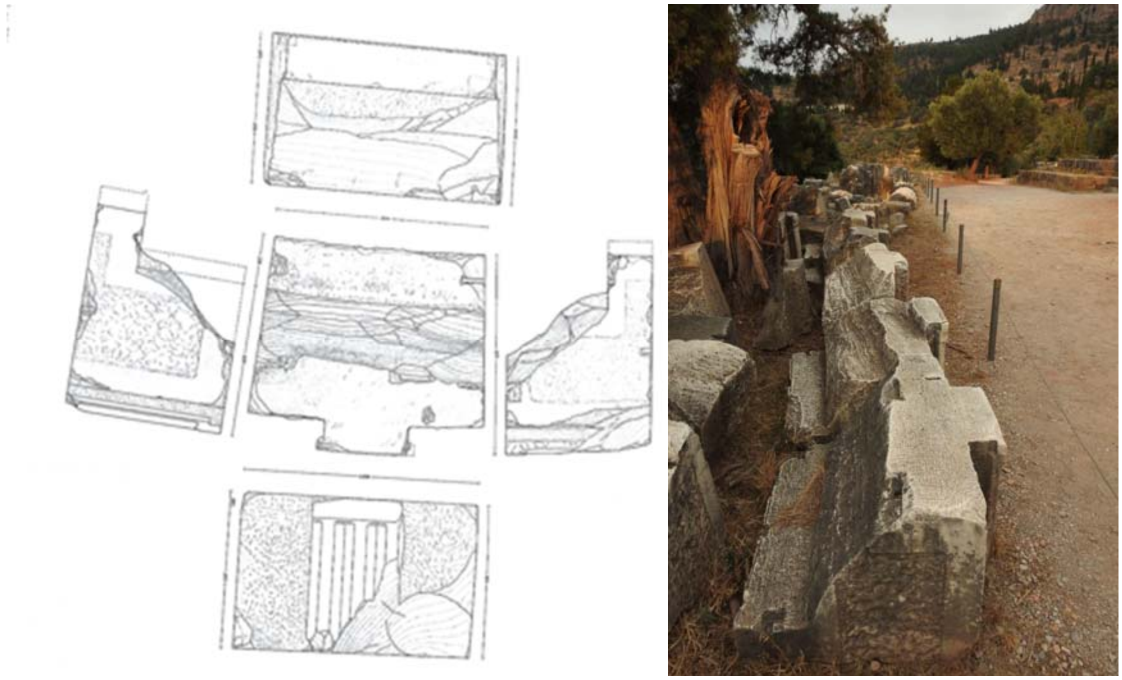

Figure 14 - (Left) Survey of block no. 076, Drawing by Joko Ito [8]. (Right) In the numerous Triglyphical blocks of the Marmarià site, all stone blocks show similar fractures imputable to the action of the thrust of the roof vault. As shown in the global analyses, removing acroterae, geisons and metopes masses could endangered the equilibrium of the structures leading to action (shear?) responsible for the fractures visible in the finds. The rights on the monument depicted, under the jurisdiction of the Ephorate of Antiquities of Phocis, belong to the Hellenic Ministry of Culture and Sport. Photo AP: ©Hellenic Ministry of Culture and Sport (Fund for the Archaeological Resources)

4. Conclusion

Constructing over the geometrical reconstruction of the Tholos of Delphi presented in [4], the study discussed the stability of the dome, the possible structural role of the composing tiles, and the possible cause of failure.

The NSCD analyses discussed in Section 2 demonstrate the local equilibrium provided by the partially interlocking tiles to ensure local equilibrium when friction contacts are modelled, and the possibility offered by the stereotomy to transmit a non-symmetrical thrust action to the supports. Whereas the exact shape of the tiles may be subjected to further refinements, with consequences on their ability to resist by form, the authors believe that such carefully modelled heavy stones, and similar tiles, shall deserve attention. This, together with the well-acknowledged geometrical knowledge of the ancient Greeks may open new interpretations and hypotheses on an area of archaeology that has been overshadowed in the structural field. Indeed, the lack of ruins and treatises shall not be considered an indication of poor construction technology, as many factors now and even more in ancient times contributed to the fortune of technologies [23].

The global equilibrium of the stone roof under self-weight has been discussed in Section 3 through a 3D graphic static approach. The analyses aimed to ensure the stability of the proposed structural surface and discuss the reasons for the temple's collapse. Accordingly, it has been found that the global equilibrium is a function of the masses of the entablature and acroteria.

Having retraced the reasons for the stability of the temple allows us to assert that among the possible causes for its collapse is to be found in the loss of the masses resting over the entablature. This seems to be confirmed by the section drawings of the relief of the friezes produced by the Japanese archaeological School directed by Joko Ito: the figures illustrate frieze elements all fractured along the same inclination located orthogonal to the thrust line obtained from the graphic static analyses. The reader may further refer to Figure 18 in [4] for more photographic details of the ruins. Accordingly, it is reasonable to suppose that the temple collapsed due to increased shear stress, as it would have happened if the weight of the acroteria, metopes and geisons had been removed.

From the above, the ancient temples may have collapsed when these elements for any reason ceased, and with it, the ability of the structure to achieve equilibrium, avoiding overturning.

Acknowledgements

It is hereby recognised that the above presentation is based on the specific contributions of the authors who, due to their different expertise, have contributed in a complementary way to this research. The first author collected information on archaeological finds, also researching the possible tiling considering the geometrical knowledge and technical skills available in ancient Greece. The other authors contributed respectively on the global and local stability.

The authors thank Professor Hugo Mulder for sparring on the analytical model; the French School of Athens and the Archaeological Museum of Delphi for their availability and, in particular, the Hellenic Ministry of Culture and Sports/ Archaeological Resources Fund for granting the reproduction of the images of the archaeological finds of the Tholos of Athena Pronaia for scientific use. The rights on the monument depicted, which falls under the jurisdiction of the Ephorate of Antiquities of Fokida, belong to the Ministry of Culture and Sports (Law 4858/2021).

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