On the evaluation of commonality strategy in product line design: The effect of valuation change and distribution channel structure

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Abstract

This paper investigates a manufacturer's optimal decisions in relation to the adoption of the commonality strategy in a decentralized channel as opposed to a centralized channel. Our model, through valuation premium and discount, captures the possible changes in the perceived quality made by customers as a result of the use of common design in the high-quality and low-quality products. We show that commonality always help reduce the extent of quality distortion encountered by the low-valuation segment, regardless of the channel structure. We also show that the adoption of commonality strategy in a decentralized channel is able to reduce channel efficiency loss, which may lead to improvements in the channel profits even when there is no cost saving associated with the use of common components. The valuation premium and discount are influential on several important measure including optimal quality levels, channel profits, and consumer surplus. Furthermore, we point out an important issue on the coordination between the manufacturer and the retailer that must be addressed if the commonality strategy is to be implemented in a decentralized channel.

Key words: component commonality; product line; marketing-manufacturing interface; channel coordination;

1. Introduction

It is commonly recognized that product line proliferation presents a major operational challenge for many firms today despite its importance for attracting different consumer segments. A proliferation of products negatively affects operational performance due to higher forecast errors, higher manufacturing costs, and higher overhead and administrative costs (Lee and Billington, 1994). The use of component commonality or standardization has been widely regarded as a cost-effective way for firms to mitigate the negative effects of product line proliferation. The major benefits of using commonality include a reduction in production costs due to economies of scale in production and a reduction in inventory holding costs due to risk pooling. Examples of commonality are abundant and can be found in many industries including automotive, computers, and electronics; see Ulrich (1995), Fisher et al. (1999), and Desai et al. (2001). But the use of commonality is not without negative effects as the customers may not perceive the products as substantially different. This may particularly affect the customers' valuation of the high-end product in a negative way, resulting in reduced sales and profits. Manufacturing and marketing must therefore consider the above-described trade-off when deciding whether a component should be made common or not. More precisely, the decision must be based on an evaluation of the overall profits that result from both the cost-related and revenue-related effects.

The use of commonality in product line design is indeed an area that has been well studied in the literature (Kim and Chhajed 2000; Ramdas 2003; Heese and Swaminathan 2006). Most existing papers study the effects of commonality in product line design by considering the decision faced by a single player in the supply chain, e.g. the manufacturer. Such a decision problem captures situations in which the manufacturer sells directly to consumers, but it neglects other situations in which the manufacturer must sell the products through intermediate parties such as distributors or retailers. When the manufacturer sells directly to consumers, i.e. uses a centralized distribution channel, it has full control of the ultimate targeting of the products to the different consumer segments through its pricing and quality level decisions. In a decentralized distribution channel, however, the manufacturer depends on intermediate parties such as retailers, to sell its products to consumers. As different parties along a distribution channel may not share the same interest, the distribution channel is often not fully coordinated. Several operational and marketing decisions such as price and product quality may be affected by the channel structure. Moreover, when designing a product line, the manufacturer must be aware that the ultimate targeting is made by the retailer and could therefore be different from the manufacturer's intended targeting strategies.

This article aims to uncover the extent to which the existence of a retailer in a distribution channel influences the manufacturer's component commonality strategy. We focus on situations in which the manufacturer has motivation to target the different segments and is dominant regarding product line design decisions such as quality and possible use of commonality, but it has to rely on the retailer who has the option of not offering the full product line as intended by the manufacturer. In this case, the manufacturer must design a product line and determine wholesale prices such that the retailer is willing to carry the full product line. This issue is relevant due to the increasing power of retailers (Kumar 1999; Corsten and Kumar 2005) and the intensifying focus of retailers on SKU rationalization (Hamstra 2009). The problem addressed in this paper can be exemplified by electronic products such as TVs or PCs sold at retail stores. Consider for example, Samsung's range of smart TVs of different screen sizes that share the same widgets or Lenovo's X series that share the same keyboard or battery. It is evident that some retailers are not motivated to sell the complete product line offered by the manufacturer.

From a marketing perspective, it would also be interesting to examine the extent to which the effects of commonality on customers' valuation of products should be taken into account when the manufacturer considers adopting the commonality strategy. Consider the example of Apple offering iPhone SE and iPhone 6 that share many features but differ in screen size. Because of the common design, buyers may perceive iPhone SE similar to iPhone 6. The buyers of iPhone SE may be willing to pay a higher price for the product. On the other hand, the similarity of the two products may also lower the customers' willingness to pay for iPhone 6. The effects of commonality on customers' valuation of products have been studied previously (see e.g. Kim and Chhajed 2000), but the results are only known for the situation in which manufacturers operate in a centralized channel. Our paper will extend the results to the decentralized channel.

In this paper, we study the effects of product commonality on a manufacturing firm selling its products through a retailer in a market where customers' valuation of products may change due to the use of common design. We contrast the results with those obtained in past literature that predominantly consider only a single firm. More specifically, we aim to answer the following questions:

(i) *How does the use of component commonality affect product line design strategy in a decentralized channel? How does the effect differ from that of the centralized channel?*
(ii) *How will the effect of commonality in a decentralized channel change when customers' valuation of products may change due to product commonality?*

The remainder of this article is organized as follows. In section 2, we review the extant literature. In section 3, we present the model and the result for the product line design problem in a decentralized channel. We contrast this result with the result in the centralized channel, and emphasize the existence of channel inefficiency loss in the decentralized channel. In section 4, we examine the role of commonality in product line design in the decentralized channel differentiated by whether or not the valuation change factors are present. We also discuss some coordination issues between the manufacturer and the retailer that are pertinent to the implementation of the commonality strategy in the decentralized channel. In section 5, we conclude the article and discuss some possible extensions of the study.

2. Literature review

This paper is related to three streams of literature. The first one is the literature examining the costs and benefits of using commonality in product line design. We refer the reader to Ramdas (2003) and Labro (2004) for reviews of this literature. The benefits of commonality have been identified by several authors. Rutenberg (1969) recognizes the economies of scale in production as a result of using a common component in different products. The benefits of commonality in the form of reduction in inventory costs have been studied by Baker et al. (1986) and Gerchak et al. (1988). Cooper (1994) and Pisano and Rossi (1994) recognize the reduction in overall investments in production equipment. Meixell (2005) suggests that designing component commonality into products has a stabilizing effect on production schedules.

Several other authors also identify the costs of using component commonality due to potential demand cannibalization across different products and over-design of components. Like we do in this paper, these authors consider the marketing-operations trade-off in their models. Kim and Chhajed (2000) consider a model to examine when commonality should be introduced and how much commonality should be offered. In particular, they examine the effects of commonality on customers' valuation of products when it is used to design a product line, which we also consider in this paper. Krishnan and Gupta (2001) study the appropriateness of using commonality in product development and its impact on product planning decisions. Their results suggest that commonality is not expedient when market diversity is too extreme or when there are high levels of economies of scale associated with unique components. The two papers above only consider products with a single attribute. Models considering two attributes like ours are presented in the following three papers. Desai et al. (2001) analyze costs and benefits of commonality by considering two attributes (components) in their model. Their analysis also includes the decision on which component to be made common. They make a comparison between the strategy of not using commonality and two different commonality strategies that are differentiated by whether the high-quality or low-quality component is made common. Heese and Swaminathan (2006) consider a model similar to that of Desai et al. (2001). They explicitly consider the interactions between decisions regarding commonality, production cost, quality levels, and cost-reduction effort. They challenge the conventional result that suggests that reduced product differentiation due to commonality leads to less attractive product lines. They show that in the presence of quality and cost-reduction effort decisions, commonality might eventually result in more attractive product lines. Most recently, Kim et al. (2013) consider the case of a non-dominating preference structure in a two-segment market where each market segment has an attribute it values more than the other segment. They show that in this non-dominating preference structure, commonality can actually relieve cannibalization. But all the above papers consider the commonality strategy from a manufacturer's point of view, i.e. in cases where a manufacturer sells its product directly to consumers. We contribute to this stream of literature by incorporating distribution channel considerations into the classic component commonality decision.

Our paper is also related to the marketing and operations literature that examines the effects of distribution channel structure on product line design strategies. Villas-Boas (1998) considers a product line design problem in a distribution channel that consists of a manufacturer selling a product line through a retailer. The paper considers a vertically differentiated market, composed of two segments that are different in their relative preference for quality. To study the effects of channel structure on the optimal design of a product line, they make comparisons between the decentralized channel and the centralized channel. They find that a manufacturer offers products that are more differentiated in a decentralized channel than in a centralized channel. Liu and Cui (2010) study a product line design problem in a distribution channel where consumers are only horizontally differentiated. They focus on the optimal number of products in a product line and consider both channel profits and social welfare in their analysis. They show that a decentralized distribution channel can be an efficient structure by providing a product line length that is optimal from a social welfare perspective. Shi et al. (2013) consider a market where consumers can be horizontally or vertically differentiated. In contrast to Liu and Cui (2010) and Shi et al. (2013) who focus on the case where the manufacturer is more powerful than the retailer such that the manufacturer is able to make a take-it-or-leave-it offer of products to the retailer, Villas-Boas (1998) considers the case where the retailer may decide whether to carry the full product line offered by the manufacturer or only part of it. Kolay (2015) extends the model in Villas-Boas (1998) by considering add-on services in addition to the main attribute of the products. She examines the conditions under which these add-on services should be provided by the manufacturer or by the retailer. In our paper, we consider a product line design problem in a distribution channel similar to that of Villas-Boas (1998) and Kolay (2015). We believe that the problem is relevant as we see many real life examples of retailers with the option of choosing the products they wish to carry from the full product line designed by the manufacturer. In comparison to Villas-Boas (1998) and Kolay (2015), what is new in our paper is the option of using commonality in a distribution channel. Our study provides new insights into the attractiveness of the commonality strategy as we show that the strategy can actually be used to alleviate the quality distortions pertinent in product line design.

Although many studies have analyzed the effects of the distribution channel structure on product-line strategies, there is, however, relatively scant literature focusing on coordination issues in the channel. Many authors have studied channel coordination in contexts other than product-line design (Jeuland and Shugan 1983, Moorthy 1987, Ingene and Parry 1995, and Raju and Zhang 2005, Cachon and Lariviere 2005). Channel coordination in the context of product line strategies have been studied by Hua et al. (2011). They consider the product line design problem in a vertically differentiated market and design a revenue-sharing contract between the manufacturer and retailer in order to improve the performance of the channel. Shao et al. (2013) study a supply chain consisting of a manufacturer who distributes a product line through two competing retailers and consider demand uncertainty. They show that retail price floors or inventory buybacks can achieve coordination. In this paper, though we do not develop specific coordination mechanisms, we discuss coordination issues between the manufacturer and the retailer that must be resolved prior to the implementation of the commonality strategy in a decentralized channel, a topic that has never been studied previously.

3. Product line design in a decentralized channel

We consider the product line design problem in a supply chain that consists of a manufacturer and a retailer in a monopolistic setting. The manufacturer produces and sells products to the retailer who then sells the products to end consumers. We consider a vertically differentiated market where consumers belong to one of two segments. We label them as the high-valuation (*H*) and low-valuation (*L*) segments. The size of the market is normalized to 1, and the fraction of segment *s, s* = *L*, *H* is denoted by α_s, where $\alpha_L + \alpha_H = 1$.

The manufacturer designs a product line that consists of two products, one targeted at each segment. The product is characterized by a bundle of two vertical attributes for which more is always better for all consumers. An attribute may represent a design subsystem, a product module, or a component of the product. The index for each of the attributes is denoted by k =1, 2. By definition, each of the two products is intended to be sold to each market segment and the index s'=*L*, *H* will also be used to distinguish the two products. The quality level of attribute *k* = 1, 2 offered in product s'=*L*, *H* is denoted by $q_{s'k}$ (> 0).

Consumers in a segment are homogeneous in their quality valuation for attributes. Consumers in segment *s* give quality valuation (part worth) u_{sk} for attribute *k*. We assume the presence of a dominating preference structure. That is, the quality valuation from consumers in segment *H* is always greater than the quality valuation from consumers in segment *L* and this applies to both attributes. This means that $u_{H1} > u_{L1}$ and $u_{H2} > u_{L2}$. Following the established literature on product line design (see e.g. Mussa and Rosen 1978, Kohli and Krishnamurthi 1987, Moorthy and Png 1992), we assume that the total valuation of a consumer (consumer utility) in segment *s* for product s' is denoted by $u_{s1}q_{s'1} + u_{s2}q_{s'2}$. The way the consumer's overall utility is derived from the quality of the attributes is consistent with the spirit of the multi-attribute utility models widely employed in the marketing literature (see e.g. Kotler and Keller 2012). In addition, a similar approach is also used in conjoint analysis, which is one of the widely used marketing research techniques in real-world applications (see Cattin and Wittink 1982 for a survey of conjoint analysis applications). We assume that the cost of providing a unit of quality increases at an increasing rate as the level of quality increases, and this applies to both attributes. For mathematical tractability, we assume a quadratic function $c_k q_{s'k}^2$, where c_k is a cost coefficient for attribute k. Consistent with the notion of additive separability of attribute part worths, we assume that the overall quality cost of the product is the sum of individual quality costs from different attributes. This assumption has been made by e.g. Kim et al. (2013) and can be justified for products where the cost of providing a unit of quality in one attribute is independent of the cost of providing a unit of quality in the other attributes. Further, we view the cost of providing a unit of quality as the marginal cost of production. This assumption has been made by many papers in the quality-based segmentation literature (see e.g. Mussa and Rosen 1978 and Moorthy 1984). Introducing a marginal cost of production that depends on the quality of the product can be justified in the settings where producing a higher quality product requires, for example, more expensive materials or more man hours.

Now consider the interactions between the manufacturer and retailer. The retailer can decide on the ultimate targeting of the products to the different market segments. It is certainly in the manufacturer's interest to design the product line in such a way that the retailer is willing to carry the full line and to target each product to the segment intended by the manufacturer. In a decentralized distribution channel, the manufacturer decides on the quality levels of the two attributes in the two products $q_{s'k}, s' = L, H$ and $k = 1,2$ and sets the wholesale prices charged to the retailer, denoted by w_L and w_H. The retailer then decides on the retail prices, denoted by p_L and p_H. Our model assumes that the market size is fixed and the same for both the centralized and decentralized channels, which allows us to focus on the effect of channel structure on product line decisions. This assumption is common in the literature on product line design (e.g. Moorthy 1984; Villas-Boas 1998; Liu and Cui 2010).

Our model considers a monopolistic setting at both the manufacturer and the retailer levels, ignoring the effects of competition on the commonality strategy. Furthermore, we also exclude other channel structures (e.g. dual channel). This allows us to focus on examining how the effectiveness of the commonality strategy in the centralized channel, as widely reported in the literature, is affected when the manufacturer now depends on the retailer to sell its products to consumers. Further, we also need the following assumption in order to ensure that the manufacturer is willing to offer two products in the product line:

$$\alpha_H < Max\left\{\frac{u_{L1}}{u_{H1}(1+\alpha_L)}; \frac{u_{L2}}{u_{H2}(1+\alpha_L)}\right\} \quad \text{(A1)}$$

Assumption (A1) suggests that the size of the high-valuation segment cannot be too large in order to ensure that the optimal two-product solution exists. As will be explained in the next section, this assumption is required to ensure that at least one attribute has positive optimal quality level in the low-end product in the decentralized channel. Should this assumption be violated, the manufacturer will only sell one product targeted at the high-valuation segment. We provide a more detailed explanation of the relation between Assumption (A1) and the optimal quality levels of the low-end product in the decentralized channel in the Appendix. The motivation behind the assumption is because we want to focus on the situations where there are potential misalignments between the manufacturer's and retailer's targeting strategies, and such situations occur when the manufacturer's intention is to design and sell complete product line that consists of the low-end and high-end products.

In order to show more explicitly the extent of quality distortion in each of the two product line design problems, we introduce the notion of efficient quality. The efficient quality for a segment is the quality that maximizes the difference between a customer's valuation and the firm's marginal cost of quality (Moorthy and Png 1992). In other words, the efficient quality levels are the optimal quality levels achieved under perfect market information. In such a setting, the firm can observe consumer types and can tailor its product offering and price to each individual consumer without worrying about potential cannibalization. The efficient quality of attribute k (k = 1 and 2) targeted at segment s ($s = L$ and H) maximizes $u_{sk}q_{sk} - c_k {q_{sk}}^2$, and is equal to $q_{sk}^{*e} = \frac{u_{sk}}{2c_k}$. Further, following Moorthy and Png (1992), we introduce $R_k = \frac{\alpha_H}{\alpha_L}\left(\frac{u_{Hk}}{u_{Lk}} - 1\right)$, which is a measure of potential cannibalization in attribute k (k = 1 and 2).

We use the following notation: (i) C and D, to differentiate the centralized channel from the decentralized channel; and (ii) NC and C, to differentiate the no-commonality strategy from the commonality strategy.

The decentralized channel – no commonality strategy (*D-NC*)

We extend the model of Villas-Boas (1998) by considering two product attributes instead of a single attribute. To obtain the equilibrium decisions for both the retailer and manufacturer in the decentralized channel, we consider the situation in which it is of the manufacturer's interest to induce the retailer to carry the full product line. We first consider the retailer's optimal pricing and targeting strategies, based on which the manufacturer must design a product line such that the retailer is willing to carry the full product line.

The retailer's problem

If the retailer is to carry the full product line consisting of two products, the problem can be formulated as:

Problem *R D-NC:*

$$Max_{p_H,p_L}\, \pi_{D-NC}^{R} = \alpha_H(p_H - w_H) + \alpha_L(p_L - w_L) \quad (1)$$

Subject to

$$u_{L1}q_{L1} + u_{L2}q_{L2} \geq p_L \quad (2)$$

$$u_{H1}q_{H1} + u_{H2}q_{H2} \geq p_H \quad (3)$$

$$u_{H1}q_{H1} + u_{H2}q_{H2} - p_H \geq u_{H1}q_{L1} + u_{H2}q_{L2} - p_L \quad (4)$$

$$u_{L1}q_{L1} + u_{L2}q_{L2} - p_L \geq u_{L1}q_{H1} + u_{L2}q_{H2} - p_H \quad (5)$$

In the above formulation, constraints (2) and (3) are participation constraints for the low-valuation segment and high-valuation segment, respectively, and constraints (4) and (5) are self-selection or incentive compatibility constraints. In line with the standard solution used in the marketing literature (e.g. Moorthy 1984), the retailer's optimal pricing decision is obtained by making the participation constraint for the low-valuation segment, i.e. (2) and the incentive compatibility constraint for the high-valuation segment, i.e. (4) binding. Substituting p_L with $u_{L1}q_{L1} + u_{L2}q_{L2}$, and p_H with $u_{H1}q_{H1} + u_{H2}q_{H2} - u_{H1}q_{L1} - u_{H2}q_{L2} + p_L$ yields

$$\pi_{D-NC}^{R} = \alpha_H(u_{H1}q_{H1} + u_{H2}q_{H2} - u_{H1}q_{L1} - u_{H2}q_{L2} + u_{L1}q_{L1} + u_{L2}q_{L2} - w_H) + \alpha_L(u_{L1}q_{L1} + u_{L2}q_{L2} - w_L) \quad (6)$$

Instead of carrying both products, the retailer may also choose to carry only the low-quality product and sell it to both segments or to carry only the high quality product and sell it to the high-valuation segment.

If the retailer carries only the low quality product and sells it to both segments, it gets a profit equal to $u_{L1}q_{L1} + u_{L2}q_{L2} - w_L$. The following condition must hold to make sure that the retailer is willing to carry both products instead of carrying just the low quality product:

$$\alpha_H(u_{H1}q_{H1} + u_{H2}q_{H2} - u_{H1}q_{L1} - u_{H2}q_{L2} + u_{L1}q_{L1} + u_{L2}q_{L2} - w_H) + \alpha_L(u_{L1}q_{L1} + u_{L2}q_{L2} - w_L) \geq u_{L1}q_{L1} + u_{L2}q_{L2} - w_L$$

$$\Leftrightarrow w_H - w_L \leq u_{H1}q_{H1} + u_{H2}q_{H2} - u_{H1}q_{L1} - u_{H2}q_{L2} \quad (7)$$

Condition (7) suggests that the two wholesale prices cannot be too different.

Similarly, if the retailer decides to carry only the high quality product and sells it only to the high-valuation segment, it can get a profit equal to $\alpha_H(u_{H1}q_{H1} + u_{H2}q_{H2} - w_H)$. The condition to make sure that the retailer prefers to carry both products to carrying just the high quality product can be written as:

$$\alpha_H(u_{H1}q_{H1} + u_{H2}q_{H2} - u_{H1}q_{L1} - u_{H2}q_{L2} + u_{L1}q_{L1} + u_{L2}q_{L2} - w_H) + \alpha_L(u_{L1}q_{L1} + u_{L2}q_{L2} - w_L) \geq \alpha_H(u_{H1}q_{H1} + u_{H2}q_{H2} - w_H)$$

$$\Leftrightarrow w_L \leq \frac{u_{L1}q_{L1} + u_{L2}q_{L2} - \alpha_H(u_{H1}q_{L1} + u_{H2}q_{L2})}{\alpha_L} \quad (8)$$

Condition (8) suggests that the wholesale price of the low quality product must not be too high.

The manufacturer's problem

If the manufacturer wishes to offer two products, then it has to make sure that the retailer is motivated to carry both products, i.e. (7) and (8) are satisfied. The manufacturer's problem can then be formulated as:

Problem *M D-NC*:

$$Max_{w_L,w_H,q_{L1},q_{L2},q_{H1},q_{H2}} \quad \pi^M_{D-NC} = \alpha_H(w_H - c_1 q_{H1}^2 - c_2 q_{H2}^2) + \alpha_L(w_L - c_1 q_{L1}^2 - c_2 q_{L2}^2) \tag{9}$$

subject to (7) and (8).

In line with Villas-Boas (1998), we show that the manufacturer's profit is maximized by making the two constraints (7) and (8) binding (the proof is given in the appendix). The optimal solution is $q^*_{H1\,D-NC} = q^{*e}_{H1}$; $q^*_{H2\,D-NC} = q^{*e}_{H2}$; $q^*_{L1\,D-NC} = \frac{u_{L1}-\alpha_H u_{H1}(1+\alpha_L)}{2c_1\alpha_L^2} = q^{*e}_{L1}\left(1 - R_1 - \frac{1}{\alpha_L}R_1\right)$; $q^*_{L2\,D-NC} = \frac{u_{L2}-\alpha_H u_{H2}(1+\alpha_L)}{2c_2\alpha_L^2} = q^{*e}_{L2}\left(1 - R_2 - \frac{1}{\alpha_L}R_2\right)$. To ensure that at least one attribute is offered in the low-end product, i.e., $q^*_{L1\,D-NC} > 0$ or $q^*_{L2} > 0$, we need $R_1 + \frac{1}{\alpha_L}R_1 < 1$ or $R_2 + \frac{1}{\alpha_L}R_2 < 1$, which is equivalent to Assumption (A1).

While our focus is on the decentralized channel involving a manufacturer and a retailer, we also consider the centralized distribution channel as a benchmark. The model for the problem in the centralized channel (*C-NC*) is presented in the appendix. The optimal solution for the centralized channel is $q^*_{H1\,C-NC} = q^{*e}_{H1}$; $q^*_{H2\,C-NC} = q^{*e}_{H2}$; $q^*_{L1\,C-NC} = \frac{u_{L1}-\alpha_H u_{H1}}{2c_1\alpha_L} = q^{*e}_{L1}(1 - R_1)$; $q^*_{L2\,C-NC} = \frac{u_{L2}-\alpha_H u_{H2}}{2c_2\alpha_L} = q^{*e}_{L2}(1 - R_2)$.

The result shows that while the high-valuation segment gets the efficient quality for the two attributes, the low-valuation segment gets the quality that is lower than the efficient quality. In the decentralized channel, the quality of the low-end product is distorted from the efficient quality by a factor of $R_k + \frac{1}{\alpha_L}R_k$ in attribute *k*. This extent of quality distortion in the decentralized channel is more severe than that of the centralized channel since the quality of the low-end product in the centralized channel is only distorted from the efficient quality by a factor of R_k.

The quality distortion in the centralized channel occurs because the manufacturer is unable to observe the preferences of its customers such that the price of the high quality product must be reduced to ensure that the low-end product does not cannibalize the high-end product. More severe quality distortion in the decentralized channel occurs due to the potential misalignment of the manufacturer's and the retailer's targeting strategies, resulting in channel efficiency loss, defined as the difference of profit in the decentralized channel from the profit in the centralized channel. The fact that the manufacturer must keep the wholesale prices not too high, as reflected in (7) and (8), also shows the existence of double marginalization, defined as the difference between the wholesale and retail prices. In (8), the wholesale price of the low-end product needs to be discounted from the retail price to prevent the retailer from selling only the high-end product. The other condition in (7), reflects the manufacturer's action to prevent the retailer from selling only the low-end product targeted at the two segments, i.e., to prevent cannibalization of the high-end product by the low-end product.

Should this potential misalignment not exist, e.g. in the case of the manufacturer selling only a single product, the double marginalization will not be present. Consequently, in equilibrium, the wholesale price will be the same as the retail price, and the manufacturer will take all the profit.

4. The role commonality in a decentralized channel

4.1 Model

In this section, our primary focus is on examining how commonality may influence the product line decisions in the decentralized channel. The manufacturer now has the choice of adopting the commonality strategy in which its two products share a common component or platform. We assume that attribute 1 is common for the two products whereas attribute 2 is unique. Similar to Kim and Chhajed (2000), we consider the effects of commonality on customers' valuation of products. Although, by assumption, the quality of the common attribute is the same for both the low-end and high-end products, there is a valuation change due to product similarity. The common attribute in the low-end product will provide a valuation premium β_p for its buyers in segment *s* such that the valuation for the common attribute becomes $u_{s1}(1+\beta_p)q_1$. In contrast, there is a valuation discount β_d for the common attribute used in the high-end product and its valuation becomes $u_{s1}(1-\beta_d)q_1$. Note that these valuation changes are associated with the presence of common design in the products regardless the segment customers belong to. In other words, the valuation of customers in both segments on the common attribute will undergo a valuation premium (discount) when considering buying the low (high) end product. Note that while our model considers two independent product attributes, Kim and Chhajed (2000) use the notion of overall quality comprised of a modular design and a custom design.

If the manufacturer decides to adopt the commonality strategy, there is a fixed cost denoted by F, incurred by the manufacturer to develop a common platform from scratch. For the common attribute (attribute 1), we assume that the production cost is reduced by a factor of θ $(0 \leq \theta < 1)$, which can be due to scale economies, pooling effects, as well as simplifications in manufacturing and inventory management processes. Our assumption on cost saving is consistent with past literature (Kim and Chhajed 2000; Kim et al. 2013). The exogenously given cost saving factor in our model allows us to maintain analytical tractability and to clearly focus on the effect of commonality on product line design decisions.

The decentralized channel – commonality strategy (*D-C*)

We now present the product line design problem with commonality in the decentralized channel.

The retailer's problem

The retailer's problem is formulated as:

Problem *R D-C*:

$$Max_{p_H,p_L}\,\pi^R_{D-C} = \alpha_H(p_H - w_H) + \alpha_L(p_L - w_L) = u_{L1}(1+\beta_p)q_1 + u_{L2}q_{L2} + \alpha_H\big(u_{H2}(q_{H2} - q_{L2}) - u_{H1}q_1(\beta_d+\beta_p) - w_H\big) - \alpha_L w_L \quad (10)$$

Subject to

$$u_{L1}(1+\beta_p)q_1 + u_{L2}q_{L2} \geq p_L \quad (11)$$

$$u_{H1}(1-\beta_d)q_1 + u_{H2}q_{H2} \geq p_H \quad (12)$$

$$u_{H1}(1-\beta_d)q_1 + u_{H2}q_{H2} - p_H \geq u_{H1}\left(1+\beta_p\right)q_1 + u_{H2}q_{L2} - p_L \quad (13)$$

$$u_{L1}\left(1+\beta_p\right)q_1 + u_{L2}q_{L2} - p_L \geq u_{L1}(1-\beta_d)q_1 + u_{L2}q_{H2} - p_H \quad (14)$$

Following Kim and Chhajed (2000), the retailer's optimal pricing decision is obtained by making (11) and (13) binding. Hence, the retailer's prices are $p_L = u_{L1}\left(1+\beta_p\right)q_1 + u_{L2}q_{L2}$ and $p_H = u_{L1}\left(1+\beta_p\right)q_1 + u_{L2}q_{L2} - u_{H1}q_1\left(\beta_d+\beta_p\right) + u_{H2}\big(q_{H2} - q_{L2}\big)$. By making (11) and (13) binding, it can be shown that (12) is always satisfied. In order to ensure that (14) is always satisfied, we need to assume that the high-valuation segment's perceived quality gap between the high and low-end products is greater than that of the low-valuation segment. Technically, we assume:

$$(u_{H2} - u_{L2})(q_{H2} - q_{L2}) - (u_{H1} - u_{L1})q_1(\beta_d+\beta_p) > 0. \quad \text{(A2)}$$

More detailed explanation of the necessity of Assumption (A2) is provided in the Appendix.

Following the same method used in problem *D-NC*, the manufacturer needs to meet the following constraints in order to induce the retailer to buy the two products.

$$w_H - w_L \leq u_{H2}(q_{H2} - q_{L2}) - u_{H1}q_1(\beta_d+\beta_p), \text{ and} \quad (15)$$

$$w_L \leq \frac{u_{L1}(1+\beta_p)q_1 + u_{L2}q_{L2} - \alpha_H u_{H1}q_1(1+\beta_p) - \alpha_H u_{H2}q_{L2}}{\alpha_L} \quad (16)$$

The manufacturer's problem

If the manufacturer wishes to offer two products, it has to make sure that the retailer wants to carry both products, i.e. that the constraints (15) and (16) above are satisfied.

Problem *M D-C*:

$$Max_{w_L,w_H,q_1,q_{L2},q_{H2}} \ \pi_{D-C}^{M} = \alpha_H(w_H - c_1(1-\theta)q_1^2 - c_2 q_{H2}^2) + \alpha_L(w_L - c_1(1-\theta)q_1^2 - c_2 q_{L2}^2) - F \tag{17}$$

subject to (15) and (16).

The optimal quality levels are as follows: $q_{1\ D-C}^{*} = \frac{(1+\beta_p)(u_{L1}-\alpha_H u_{H1})-\alpha_L\alpha_H u_{H1}(\beta_d+\beta_p)}{2c_1(1-\theta)\alpha_L}$;

$q_{L2\ D-C}^{*} = \frac{u_{L2}-\alpha_H u_{H2}(1+\alpha_L)}{2c_2\alpha_L^2} = q_{L2}^{*e}\left(1 - R_2 - \frac{1}{\alpha_L}R_2\right)$; $q_{H2\ D-C}^{*} = q_{H2}^{*e}$.

The model for the benchmark problem in the centralized channel (*C-C*) are presented in the Appendix. The optimal solution for problem C-C is $q_{1\ C-C}^{*} = \frac{u_{L1}(1+\beta_p)-\alpha_H u_{H1}(\beta_d+\beta_p)}{2c_1(1-\theta)}$; $q_{L2\ C-C}^{*} = \frac{u_{L2}-\alpha_H u_{H2}}{2c_2\alpha_L} = q_{L2}^{*e}(1-R_2)$; $q_{H2\ C-C}^{*} = q_{H2}^{*e}$.

4.2 Discussion

The solution shows that the commonality strategy applied to the first attribute does not affect the quality of the second attribute. Hence, throughout our analysis, we focus only on the first attribute. In the special case of no valuation change, i.e., when $\beta_p = \beta_d = 0$, it can be shown that when the cost saving factor is zero, i.e., $\theta = 0$, the optimal quality of the first attribute in the commonality strategy and decentralized chanel is equal to $q_{1\ D-C}^{*} = \frac{u_{L1}-\alpha_H u_{H1}}{2c_1\alpha_L} = q_{L1}^{*e}(1-R_1)$. The quality of the common attribute is improved by a factor of $\frac{1}{\alpha_L}R_1$ so that it now becomes equal to the optimal quality the low-end product in the no-commonality strategy and centralized channel, $q_{1\ C-NC}^{*}$. We summarize the above finding in the following proposition.

Proposition 1

In the special case of no valuation change, i.e., when $\beta_p = \beta_d = 0$, commonality reduces the quality distortion in the low-quality product that typically arises in the decentralized channel.

In the commonality strategy and decentralized channel and in the case of no cost saving, the low-valuation segment now gets the same quality level as in the no commonality strategy and centralized channel, which shows that commonality in the decentralized channel is able to ease the quality distortion effect encountered by the low-valuation segment. But this is at the expense for customers in the high-valuation segment who now get a lower quality level. The optimal quality level of the common attribute can also be written as the weighted average of the first attribute's low quality level and the first attribute's high quality level in the no-commonality strategy, where the weighting factors are the sizes of the low-valuation segment and the high-valuation segment, respectively, i.e., $q^*_{1\,D-C} = \alpha_H q^*_{H1\,D-NC} + \alpha_L q^*_{L1\,D-NC} = \alpha_H q^{*e}_{H1} + \alpha_L q^{*e}_{L1}\left(1 - R_1 - \frac{1}{\alpha_L} R_1\right)$. This reflects the fact that the optimal design of commonality will be decided from the balance between the loss of profit from the high-valuation segment and the increase of profit from the low-valuation segment. As a comparison, commonality in the centralized channel improves the quality of the low-end product by a factor of R_1 so that the low-valuation segment gets the efficient quality. In other words, the extent of quality improvement in the decentralized channel is higher than that of the centralized channel. This finding is new since none of the existing studies in the literature has examined the role of commonality in a decentralized channel.

By examining the upper bounds for the wholesale prices in (15) and (16), we can see that, in comparison to the no commonality strategy, commonality reduces the difference between the wholesale prices of the high-end and low-end products. In other words, commonality reduces the retailer's motivation for selling only the low-end product rather than selling the two products. As a result, the manufacturer has a motivation to increase the quality level of the first attribute compared to the no commonality strategy. Our study shows an important finding that by adopting commonality in the decentralized channel the firm is able to reduce the effect caused by the potential misalignment of the manufacturer's and retailer's targeting strategies. Interestingly, this not only reduces the extent of quality distortion encountered by the low-valuation segment, but may also increase the channel profit. This finding is summarized in the following proposition.

Proposition 2

Suppose the fixed cost is zero and there is no cost saving and no valuation change. The adoption of commonality strategy in the decentralized channel may increase the channel profit. This is, however, *only possible when the size of the high-valuation segment is larger than the size of the low-valuation segment, i.e.,* $\alpha_H > 0.5$.

In Proposition 2, we show that there is a region where the adoption of commonality strategy in the decentralized channel may increase the channel profit even though there is no cost saving. To be more precise, the region stated in Proposition 2 includes two possible situations. In the first situation where $\alpha_H < \frac{u_{L1}}{u_{H1}(1+\alpha_L)}$, i.e., the optimal quality level of the first attribute in the low-end product is positive, commonality strategy in the decentralized channel increases the channel profit when the size of the high-valuation segment is larger than the size of the low-valuation segment, regardless of whether or not the second attribute is offered in the low-end product. In the second situation where $\alpha_H > \frac{u_{L1}}{u_{H1}(1+\alpha_L)}$, we have a special case in which the first attribute is not offered in the low-end product in the no commonality stratey since its calculated optimal quality level is negative. In the proof of Proposition 2, we show that the claim made in Proposition 2 remains valid in this situation as long as $0.5 < \alpha_H < \frac{{u_{L1}}^2\,(1-{R_1}^2)}{{u_{H1}}^2}$. For the ease of exposition and to focus on our main results, in the rest of the analysis, we will only consider the solution space with positive quality levels in the two attributes.

The finding in Proposition 2 is interesting since the existing literature on commonality in the centralized channel shows that the cost saving is the only possible source of motivation for adopting the commonality strategy. Commonality in the decentralized channel is able to increase the quality level for the low-valuation segment to the extent that is higher than that of the centralized channel. This increase in quality is more significant when the measure of potential cannibalization in the first attribute is larger. Interestingly, when the two segments are of equal size, the increase of profit from the low-valuation segment due to commonality is exactly the same as the loss of profit from the high-valuation segment.

Increasing the size of the high-valuation segment, α_H, has two effects. First, it will increase the potential cannibalization, which also means that the extent of quality improvement due to commonality will be higher, resulting in a larger profit gain from the low-valuation segment. Second, it will increase the profit loss from the high-valuation segment. When α_H is sufficiently high, the quality improvement would be large enough so that the profit gain from the low-valuation segment outweighs the profit loss from the high-valuation segment. On the contrary, when the size of the low-valuation segment is larger, the potential cannibalization becomes less severe such that the profit gain from the low-valuation segment would not be sufficient to offset the profit loss from the high-valuation segment. This particular finding is counter intuitive and has so far been not discussed in the existing literature studying commonality or distribution channel in product line design.

The effect of valuation change factors appears to be important. Higher values of β_p and β_d intensify product cannibalization, and will therefore force the manufacturer (the retailer) to reduce the wholesale price (retail price) of the high-end product. On the other hand, higher β_p allows the manufacturer (the retailer) to increase the wholesale (retail) price of the low-end product. It can be shown that the resultant of the two opposing forces of increased β_p is always in favor of quality and profit improvement.

In the following proposition we provide the minimum cost saving needed to optimally adopt commonality.

Proposition 3

(a) *In the decentralized channel, it is beneficial to adopt commonality if $\theta \geq \theta_D^{min}$, where $\theta_D^{min} = 1 - \frac{\left\{(1+\beta_p)^2 (u_{L1} - \alpha_H u_{H1})^2 - \alpha_L^2 \alpha_H^2 u_{H1}^2 (\beta_d + \beta_p)^2\right\}}{\alpha_L \{\alpha_L \alpha_H \, u_{H1}{}^2 + \alpha_L^2 \, u_{L1}{}^2 (1-R_1)^2 - u_{L1}{}^2 R_1^2 + 4 \, c_1 \alpha_L F\}}$; The sign of θ_D^{min} can be negative or positive in the cases with and without valuation changes.*

(b) The minimum cost saving in the decentralized channel is lower than that in the centralized channel, i.e. $\theta_D^{min} < \theta_C^{min}$.

In Proposition 3(a), we derive the minimum cost saving factor, θ_D^{min}, that will make the commonality strategy the preferred option as opposed to the no-commonality strategy. In the special case where $\beta_p = \beta_d = 0$, and $\alpha_H = 0.5$, we have $\theta_D^{min} = 0$, that corresponds to the result in Proposition 1. Proposition 2(b) reveals that, under the same parameter values, the profit improvement resulting from the adoption of commonality strategy in the decentralized channel is always higher than that of the centralized channel. This is directly associated with the fact that in the decentralized channel, commonality reduces the quality distortion effect in the low-end product at a higher level than in the centralized channel. This finding provides new insights into the importance of commonality strategy in the decentralized channel. While the existing literature pinpoints motivations for considering commonality in product line design in a centralized channel, our study reveals that the motivations are even stronger when firms operate in a decentralized channel.

We present Figures 1 to 3 to provide more insights regarding the above results. In Figure 1, we depict how the values of the minimum cost saving rate are influenced by the proportion of the high-valuation segment and also by the valuation changes. The figure is plotted for the following parameters: $F = 0$; $u_{L1} = 4$; $u_{H1} = 4.5$; $u_{L2} = 5.5$; $u_{H2} = 6$; $c_1 = 1$; $c_2 = 1$. Although the figure is plotted for this specific set of problem parameters, the insight discussed here is not parameter specific. The figure also shows that when β_p is positive, both θ_C^{min} and θ_D^{min} are lower. The figure shows that the minimum cost saving is increasing in α_H in the centralized channel, but it is initially increasing and then decreasing in the decentralized channel. The increasing part occurs in the region where α_H is small, i.e., where the measure of cannibalization is low so that the profit loss from the high-valuation segment dominates the profit gain from the low-valuation segment. In this region, increasing α_H will result in larger loss of profit from the high-valuation segment, and hence, increase the minimum cost saving. There is, however, a value of α_H that represents a turning point at which the quality improvement is significant enough such that the profit gain from the low-valuation segment starts to dominate the profit loss from the high-valuation segment.

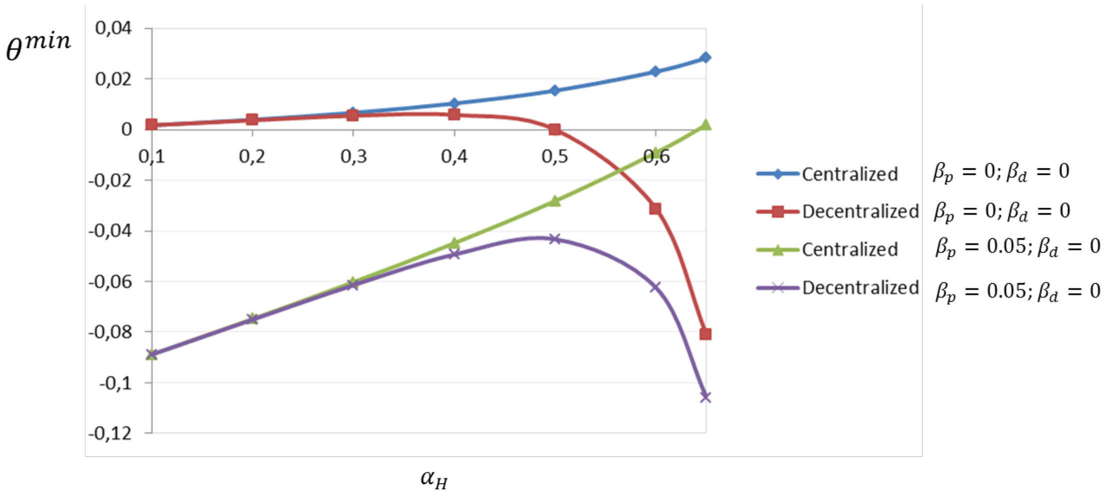

Figure 1. The minimum cost saving as a function of α_H and how it is affected by valuation changes

In Figure 2, we show how the the channel profits are influenced by the proportion of the high-valuation segment. We use the same parameter values as for Figure 1 and plot the channel profits for the decentralized channel under both the no commonality and commonality strategies, for the case with zero valuation changes ($\beta_p = \beta_d = 0$) and zero cost saving ($\theta = 0$). The figure illustrates our claim in Proposition 2, i.e., the commonality strategy increases the channel profit when $\alpha_H > 0.5$ even when there is no cost saving.

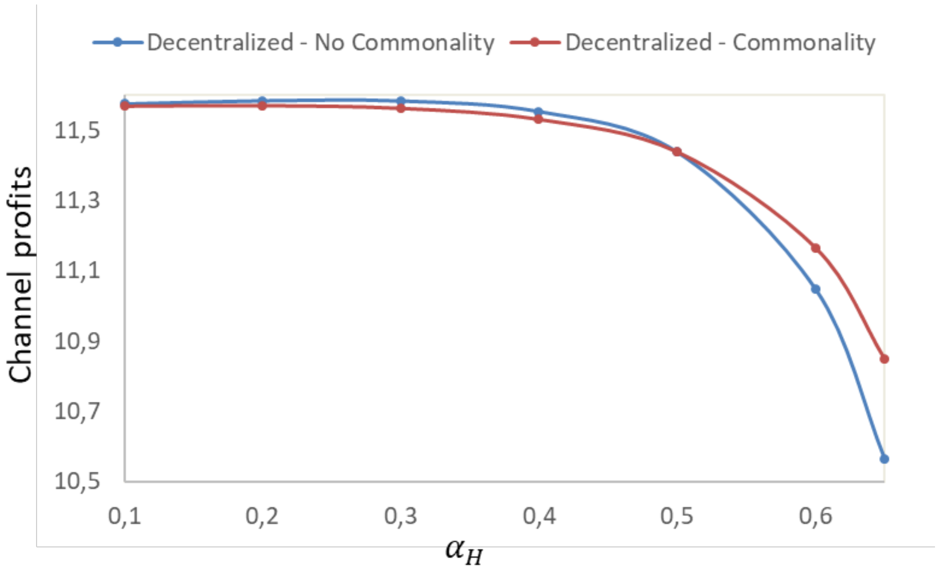

Figure 2. The channel profit comparison (no-commonality vs. commonality) as a function of α_H for the case with zero valuation changes ($\beta_p = \beta_d = 0$) and zero cost saving ($\theta = 0$)

As the measure of the potential cannibalization is not only affected by the size of each segment, but also by the ratio of quality valuations $\frac{u_{H1}}{u_{L1}}$, in Figure 3 we show how the ratio $\frac{u_{H1}}{u_{L1}}$ affects the minimum cost saving in the case of no valuation changes. We use the same parameter values as in Figure 1 and maintain the value of u_{L1} at 4.0, but include two additional values of u_{H1}: $u_{H1} = 5.0$ and $u_{H1} = 4.3$. The figure shows that the effect of the potential cannibalization is differentiated by whether it is the high-valuation segment or the low-valuation segment that is larger. When the size of the low-valuation segment is larger, a higher value of $\frac{u_{H1}}{u_{L1}}$ will require a larger cost saving to make the commonality strategy attractive. However, when the size of the high-valuation segment is larger, a higher value of $\frac{u_{H1}}{u_{L1}}$ will result in more significant profit improvement. As for comparison, a higher value of $\frac{u_{H1}}{u_{L1}}$ will always increase the minimum cost saving in the centralized channel, regardless of the relative size of each segment. Our study complements the existing literature by pinpointing the importance of the size of each segment in evaluating commonality in a decentralized channel.

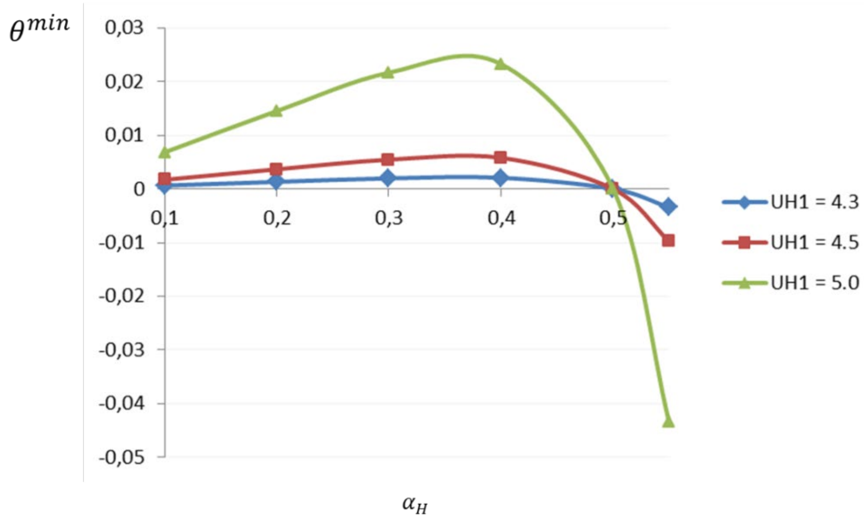

Figure 3. The minimum cost saving as a function of α_H and how it is affected by the ratio $\frac{u_{H1}}{u_{L1}}$

From a marketing perspective, it would also be interesting to understand how the manufacturer's component commonality strategy and the presence of valuation changes will affect consumer surplus. Recall that the standard solution approach in the product line design literature, which we also use in this paper, determines the optimal quality levels by making the participation constraint of the low-valuation segment and the self-selection constraint of the high-valuation segment binding. Accordingly, consumers in the low-valuation segment will have zero surplus whereas consumers in the high-valuation segment will have positive surplus.

Proposition 4

The use of commonality strategy in a decentralized channel will give consumers in the high-valuation segment higher surplus compared to the no-commonality strategy.

The use of commonality requires the manufacturer to reduce the price of the high-end product in the extent that is higher than that of the no-commonality strategy, which results in higher consumer surplus, as stated in Proposition 4. Finally, we relate our results to the concept of social welfare by integrating the results on the channel profits and consumer surplus. Our study shows that it is possible to use the commonality strategy to enhance the channel profits and consumer surplus at the same time, thereby improving the social welfare. This articulates the role of commonality in removing major causes of quality distortion prevalent in product line design. The discussion on social welfare will be incomplete without paying attention to how the profits in the channel are distributed between the manufacturer and the retailer, which is the main topic discussed in the following section.

4.3 Coordination

In Section 4.1, we have discussed the effects of commonality on the channel's profitability in the decentralized channel. What has not been discussed is how the adoption of the commonality strategy may change the profits gained by the manufacturer and the retailer. This topic is relevant especially when discussing the implementation of the commonality strategy since there could be misalignments between the manufacturer's and channel's objectives. Our main focus here is more on putting forward this misalignment issue rather than presenting a specific supply chain contract. We summarize the main finding in the following proposition.

Proposition 5

Let $\theta_{D(M)}^{min}$ ($\theta_{D(SC)}^{min} = \theta_{D}^{min}$) denote the minimum cost saving rate required so that the commonality strategy gives the manufacturer (channel) a larger profit compared to the no-commonality strategy. The inequality $\theta_{D(SC)}^{min} < \theta_{D(M)}^{min}$ always holds.

In Proposition 5, we show there are situations in which the channel (and retailer) obtains larger profits due to commonality, but the manufacturer suffers from reduced profits. These situations occur when $\theta_{D(SC)}^{min} < \theta < \theta_{D(M)}^{min}$. Figure 4 below is presented to illustrate the situation. The parameter values used in the figure are: $F = 0$; $u_{L1} = 4$; $u_{H1} = 5$; $u_{L2} = 5.5$; $u_{H2} = 7$; $c_1 = 1$; $c_2 = 1$; $\alpha_H = \alpha_L = \frac{1}{2}$, and $\beta_p = \beta_p = 0$. The figure shows the profits for both the retailer and the manufacturer in the no-commonality and commonality strategies as the functions of the cost saving rate. In this example, $\theta_{D(SC)}^{min} = 0$, and $\theta_{D(M)}^{min} = 0.3$. Suppose the manufacturer has the possibility to develop common design which gives potential cost saving of $\theta < 0.3$. Without coordination between the manufacturer and the retailer, the channel will fail to exploit an opportunity to enhance its profitability.

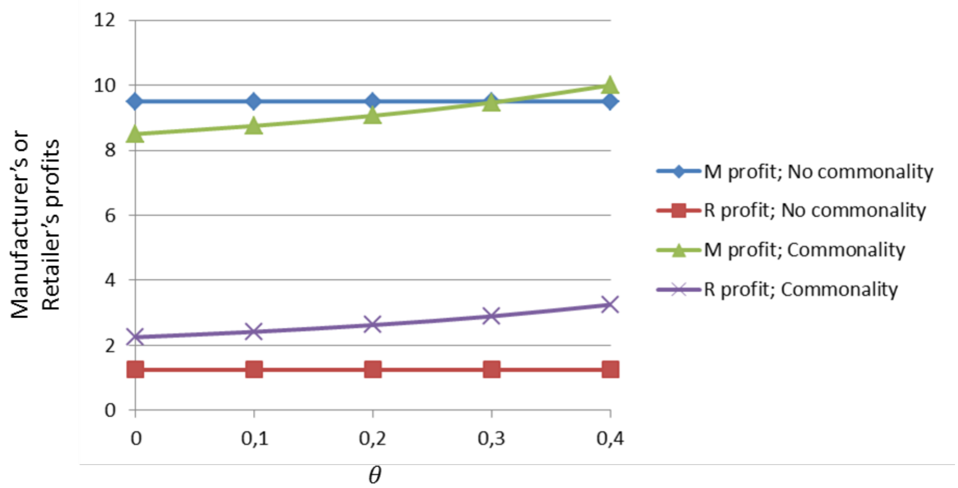

Figure 4. The manufacturer's and retailer's profits in the no-commonality and commonality strategies

The main aspect we want to point out here is that there should be some coordination mechanism in place with respect to the implementation of the commonality strategy in a decentralized channel, ensuring that both the manufacturer and the retailer are made better-off, or at least, not made worse-off. In principle, several vertical contracts between the manufacturer and the retailer could solve the problems discussed above. If the manufacturer is in the position as a first mover, it could, for example, charge the retailer a fixed fee up to the amount that equals the increase of the channel profits. Similarly, knowing that commonality could increase its profits, the retailer can also take the initiative to induce the manufacturer to incorporate commonality in its product line design and transfer a fixed fee such that the manufacturer is not made worse-off. Other types of contract such as revenue sharing or cost sharing could also be applied.

5. Conclusions

In this article, we extend the classic component commonality decision to consider the decentralized distribution channel in which a manufacturer must sell its products through a retailer. Our benchmark is the centralized distribution channel in which the manufacturer makes commonality decisions without the presence of the retailer's strategic behavior. We particularly consider a decentralized distribution channel where the manufacturer's intention to sell the full product line is not always in line with the retailer's profit maximizing strategies. Hence, regardless of whether or not component commonality is adopted, the manufacturer must ensure that the retailer is willing to carry the full product line and to target each product to the segment intended by the manufacturer. In our model, we also consider the possible presence of customers' valuation changes due to the use of common design in the two products offered in the product line. Our analysis yields a number of interesting managerial insights pertaining the effect of distribution channel structure and valuation changes.

Regardless the channel structure, our results show that the use of commonality in product line design may help firms in restoring the quality levels that have been distorted due to potential cannibalization in the centralized channel and due to potential misalignment of the manufacturer's and retailer's product line strategies. This, in turn, may lead to improvements for the channel's profitability. Our study reveals an important finding, which suggests that commonality in product line design may increase the profits even when there are no cost savings associated with the use of common components. Amid the extensive literature and widely practice of commonality, this finding provides a new insight into the benefits of commonality beyond the cost saving motive.

How customers perceive the quality of the products due to the use of common design appears to be influential, and this is consistent across all the important measures including quality levels, channel profits and consumer surplus. The valuation premium has positive influences whereas the valuation discount has the opposite effect. The valuation premium is indeed a factor that could allow the commonality strategy to generate larger profits even when there are no cost savings. However, we have also shown that even with a zero valuation premium, commonality may increase the profits in a decentralized channel. This finding, in particular, extends the literature on product line design in a distribution channel and on commonality by showing that commonality can actually be used as a means to reduce channel efficiency loss, leading to larger profits, even though not supported by cost saving. The existing supply chain management literature points out the importance of coordination, which has motivated us to examine how the channel's profits are allocated between the manufacturer and the retailer. We have analytically proved that while commonality may increase profits for the channel and the retailer, the manufacturer may actually suffer from a reduction in profits. Our analytical result highlights the importance of vertical contract designs that must be developed prior to the implementation of the commonality strategy in a decentralized channel.

We acknowledge some limitations of this paper and would like to suggest several topics for future research. First, our model assumes that the two attributes are independent such that the total consumer valuation (quality cost) of a product is represented as the sum of consumer valuations (quality costs) of the two individual attributes. The extent to which our results will change if other utility and cost functions are used is still unclear.

Second, our model considers a monopolistic setting at both the manufacturer and the retailer levels, ignoring the effects of competition on the commonality strategy. For example, when there are competing retailers, each may prefer to carry different products from the product lines offered by the manufacturers. In relation to this, another line of research relevant is to examine how the results in this paper may change when a manufacturer introduces a competitive element with the retailer by adopting a dual channel strategy where the manufacturer, in addition to selling the products through a retailer, also sells the products through its own on-line stores.

Third, our model setup does not capture the interplay between risk pooling and the market segmentation effects that are inherent when demand uncertainty is present. Rong et al. (2015) has shown that the presence of demand uncertainty can reduce the benefit of market segmentation, i.e., the quality difference between products becomes less sensitive to consumer heterogeneity. On the contrary, as shown in this paper, when a decentralized channel is adopted, the product line chosen by the manufacturer should be more distorted, leading to a greater difference between the products in the line. It would be interesting to examine how the attractiveness of the commonality strategy in a decentralized channel will be affected when the risk pooling effect is modeled endogenously in the presence of demand uncertainty.

References

Baker, K.R., Magazine, M.J., Nuttle, H.L.W., The effect of commonality on safety stocks in a simple inventory model. *Management Science*, 1986, 32(8), 982-988.

Cachon, G.P. and Lariviere, M.A., Supply chain coordination with revenue-sharing contracts: strengths and limitations. *Management Science*, 2005, 51(1), 30-44.

Cattin, P. and Wittink, R., Commercial uses of conjoint analysis. *Journal of marketing*, 1982, 46(3), 44-53.

Cooper, R., Nissan Motor Company Ltd.: Target Costing System. *Harvard Business School Case*, 1994, Boston, MA.

Corsten, D. and Kumar, N., Do suppliers benefit from collaborative relationships with large retailers? An empirical investigation of efficient consumer response adoption. *Journal of Marketing*, 2005, 69(3), 80-94.

Desai, P., Kekre, S., Radhakrishnan, S. and Srinivasan, K., Product differentiation and commonality in design: balancing revenue and cost drivers. *Management Science*, 2001, 47(1), 37-51.

Fisher, L. M., Ramdas, K. and Ulrich, K. T., Component sharing in management of product variety: A study of automotive braking systems. *Management Science*, 1999, 45(3), 297-315.

Gerchak, Y., Magazine, M.J. and Gamble, A.B., Component commonality with service level requirements. *Management Science*, 1988, 34(6), 753-760.

Hamstra, M., Retailers increase focus on SKU rationalization. *Supermarket News*, 2009, 57 (41), 28-30.

Heese, H.S. and Swaminathan, J., Product line design with component commonality and cost reduction effort. *Manufacturing and Service Operations Management*, 2006, 8(2), 206-219.

Hua, Z., Zhang, X. and Xu, X., Product design strategies in a manufacturer-retailer distribution channel. *Omega*, 2011, 39, 23-32.

Ingene, C.A. and Parry, M.E., Channel coordination when retailers compete. *Marketing Science*, 1995, 14(4), 360-377.

Jeuland, A.P. and Shugan, S.M., Managing channel profits. *Marketing Science*, 1983, 2(3), 239-272.

Kim, K. and Chhajed, D., Commonality in product design: cost saving, valuation change, and cannibalization. *European Journal of Operational Research*, 2000, 125, 602-621.

Kim, K., Chhajed, D. and Liu, Y., Can commonality relieve cannibalization in product line design? *Marketing Science*, 2013, 32 (3), 510-521.

Kohli, R. and Krishnamurthy, R., A heuristic approach to product design. *Management Science*, 1987, 33(12), 1523-1533.

Kolay, S., Manufacturer-provided services vs. Retailer-provided services: Effect on product quality, channel profits and consumer welfare. *International Journal of Research in Marketing*, 2015, 32, 124-154.

Kotler, P. and Keller, K., *Marketing Management*, 2012, 14th ed., Prentice Hall Inc.

Krishnan, V. and Gupta, S., Appropriateness and impact of platform-based product development. *Management Science*, 2001, 47(1), 52-68.

Kumar, N., The power of trust in manufacturer-retailer relationship. *Harvard Business Review*, 1999, 74, 92-106.

Labro, E., The cost effects of component commonality: a literature review through a management-accounting lens. *Manufacturing and Service Operations Management*, 2004, 6(4), 358-367.

Lee, H.L., and Billington, C., *Designing products and processes for postponement. S. Dasu and C. Eastman, eds. Management of Design: Engineering and Management Perspectives* (Kluwer Academic Publishers, Boston), 1994, 105-122.

Liu, Y. and Cui, T.H., The length of product line in distribution channels. *Marketing Science*, 2010, 29 (3), 474-482.

Meixell, M.J., The impact of setup costs, commonality and capacity on schedule stability: An exploratory study. *International Journal of Production Economics*, 2005, 95(1), 95-107.

Moorthy, K.S., Market segmentation, self selection and product line design. *Marketing Science*, 1984, 3 (4), 288-307.

Moorthy, K.S., Managing channel profits: comment. *Marketing Science*, 1987, 6(4), 375-379.

Moorthy, K.S. and Png, I.P.L., Market segmentation, cannibalization, and the timing of product introductions. *Management Science*, 1992, 38(3), 346-359.

Mussa, M. and Rosen, S., Monopoly and product quality. *Journal of Economic Theory*, 1978, 18, 301-317.

Pisano, G. and Rossi, S., ITT Automotive Global Manufacturing Strategy. *Harvard Business School Case,* 1994, Boston, MA.

Raju, J. and Zhang, Z.J., Channel coordination in the presence of a dominant retailer. *Marketing Science*, 2005, 24(2), 254-262.

Ramdas, K., Managing product variety: An integrated review and research directions. *Production and Operations Management*, 2003, 12, 79-101.

Rong, Y., Chen, Y.J., Shen, Z.J.M., The impact of demand uncertainty on product line design under endogenous substitution. *Naval Research Logistics*, 2015, 62, 143-157.

Rutenberg, D.P., Design commonality to reduce multi-item inventory: optimal depth of a product line. *Operations Research*, 1969, 19(2), 491-509.

Shao, J., Krishnan, H., and McCormick, S.T., Distributing a product line in a decentralized supply chain. *Production and Operations Management*, 2013, 22(1), 151-163.

Shi, H., Liu, Y., and Petruzzi, N.C., Consumer heterogeneity, product quality and distribution channels. *Management Science*, 2013, 59 (5), 1162-1176.

Ulrich, K. L., The role of product architecture in the manufacturing firm. *Research Policy*, 1995, 24(3), 419-440.

Villas-Boas, J.M., Product line design for a distribution channel. *Marketing Science*, 1998, 17 (2), 156-169.

Appendix

Proof of solution to Problem *M D-NC*

By making the two constraints (7) and (8) binding, we have :

$$w_H = w_L + u_{H1}q_{H1} + u_{H2}q_{H2} - u_{H1}q_{L1} - u_{H2}q_{L2}$$

$$w_L = \frac{u_{L1}q_{L1} + u_{L2}q_{L2} - \alpha_H(u_{H1}q_{L1} + u_{H2}q_{L2})}{\alpha_L}$$

The profit function for Problem *M D-NC* can be written as:

$\pi^M_{D-NC} = \alpha_H\left(\frac{u_{L1}q_{L1}+u_{L2}q_{L2}-\alpha_H(u_{H1}q_{L1}+u_{H2}q_{L2})}{\alpha_L} + u_{H1}q_{H1} + u_{H2}q_{H2} - u_{H1}q_{L1} - u_{H2}q_{L2} - c_1q_{H1}^2 - c_2q_{H2}^2\right) + \alpha_L\left(\frac{u_{L1}q_{L1}+u_{L2}q_{L2}-\alpha_H(u_{H1}q_{L1}+u_{H2}q_{L2})}{\alpha_L} - c_1q_{L1}^2 - c_2q_{L2}^2\right)$.

Taking the first-order partial derivatives of the profit with respect to q_{H1}, q_{H2}, q_{L1}, q_{L2} and equaling them to zero give: $\frac{\partial\pi^M_{D-NC}}{\partial q_{H1}} = \alpha_H(u_{H1} - 2c_1q_{H1}) = 0 \rightarrow q_{H1}^* = \frac{u_{H1}}{2c_1} = q_{H1}^{*e}$; $\frac{\partial\pi^M_{D-NC}}{\partial q_{H2}} = \alpha_H(u_{H2} - 2c_2q_{H2}) = 0 \rightarrow q_{H2}^* = \frac{u_{H2}}{2c_2} = q_{H2}^{*e}$; $\frac{\partial\pi^M_{D-NC}}{\partial q_{L1}} = \alpha_H\left(\frac{u_{L1}-\alpha_H u_{H1}}{\alpha_L} - u_{H1}\right) + \alpha_L\left(\frac{u_{L1}-\alpha_H u_{H1}}{\alpha_L} - 2c_1q_{L1}\right) = 0 \rightarrow q_{L1}^* = \frac{u_{L1}-\alpha_H u_{H1}(1+\alpha_L)}{2c_1\alpha_L^2} = \frac{u_{L1}}{2c_1}\left(\frac{1}{\alpha_L^2} - \frac{\alpha_H u_{H1}(1+\alpha_L)}{\alpha_L^2 u_{L1}}\right) = \frac{q_{L1}^{*e}}{\alpha_L^2}\left(1 - \frac{\alpha_H u_{H1}(1+\alpha_L)}{u_{L1}}\right) = \frac{q_{L1}^{*e}}{\alpha_L^2}\left(1 - \frac{\alpha_H u_{H1}}{u_{L1}} - \frac{\alpha_L\alpha_H u_{H1}}{u_{L1}}\right) = \frac{q_{L1}^{*e}}{\alpha_L}\left[\frac{1}{\alpha_L}\left(1 - \frac{\alpha_H u_{H1}}{u_{L1}}\right) - \frac{\alpha_H u_{H1}}{u_{L1}}\right] = \frac{q_{L1}^{*e}}{\alpha_L}\left[1 - R_1 - \left(1 - \alpha_L(1 - R_1)\right)\right] = q_{L1}^{*e}\left(1 - R_1 - \frac{1}{\alpha_L}R_1\right)$;

$\frac{\partial\pi^M_{D-NC}}{\partial q_{L2}} = \alpha_H\left(\frac{u_{L2}-\alpha_H u_{H2}}{\alpha_L} - u_{H2}\right) + \alpha_L\left(\frac{u_{L2}-\alpha_H u_{H2}}{\alpha_L} - 2c_2q_{L2}\right) = 0 \rightarrow q_{L2}^* = \frac{u_{L2}-\alpha_H u_{H2}(1+\alpha_L)}{2c_2\alpha_L^2} = q_{L2}^{*e}\left(1 - R_2 - \frac{1}{\alpha_L}R_2\right)$, where $q_{sk}^{*e} = \frac{u_{sk}}{2c_k}$.

Motivation of Assumption (A1)

To ensure that the optimal quality level of attribute *k* in the low-quality product is positive, i.e. $q_{Lk}^* > 0$, we need $u_{Lk} - \alpha_H u_{Hk}(1 + \alpha_L) > 0 \Leftrightarrow \alpha_H < \frac{u_{Lk}}{u_{Hk}(1+\alpha_L)}$. Since the profit function is concave in quality, it can be shown that in the case where $q_{Lk}^* \leq 0$, it will be optimal to remove attribute *k* in the low quality product, i.e., $q_{Lk}^* = 0$. This implies that in the case where both $q_{L1}^* < 0$ and $q_{L2}^* < 0$, i.e., $u_{L1} - \alpha_H u_{H1}(1 + \alpha_L) < 0$ and $u_{L2} - \alpha_H u_{H2}(1 + \alpha_L) < 0$, the manufacturer will only offer the high-quality product in the product line. Hence, the manufacturer will only offer two products if at least one attribute has positive optimal quality level in the low-quality product, which can be met by the following condition: $\alpha_H < Max\left\{\frac{u_{L1}}{u_{H1}(1+\alpha_L)}, \frac{u_{L2}}{u_{H2}(1+\alpha_L)}\right\}$.

Motivation of Assumption (A2)

By making (11) and (13) binding, $p_L = u_{L1}(1+\beta_p)q_1 + u_{L2}q_{L2}$;

$$p_H = u_{H1}(1-\beta_d)q_1 + u_{H2}q_{H2} - u_{H1}(1+\beta_p)q_1 - u_{H2}q_{L2} + p_L$$
$$= u_{L1}(1+\beta_p)q_1 + u_{L2}q_{L2} - u_{H1}q_1(\beta_d+\beta_p) + u_{H2}(q_{H2}-q_{L2})$$

By substituting p_H, (12) can be written as:

$$u_{H1}(1-\beta_d)q_1 + u_{H2}q_{H2} \geq u_{L1}(1+\beta_p)q_1 + u_{L2}q_{L2} - u_{H1}q_1(\beta_d+\beta_p) + u_{H2}(q_{H2}-q_{L2})$$

$$\Leftrightarrow u_{H1}q_1 \geq u_{L1}(1+\beta_p)q_1 + u_{L2}q_{L2} - u_{H1}q_1\beta_p - u_{H2}q_{L2}$$

$$\Leftrightarrow (u_{H1}-u_{L1})q_1 \geq -q_1\beta_p(u_{H1}-u_{L1}) - q_{L2}(u_{H2}-u_{L2}),$$

which is always true.

From (14), we have: $u_{L1}(1-\beta_d)q_1 + u_{L2}q_{H2} - p_H \leq 0 \leftrightarrow p_H \geq u_{L1}(1-\beta_d)q_1 + u_{L2}q_{H2}$. By substituting p_H:

$$u_{L1}(1+\beta_p)q_1 + u_{L2}q_{L2} - u_{H1}q_1(\beta_d+\beta_p) + u_{H2}(q_{H2}-q_{L2}) \geq u_{L1}(1-\beta_d)q_1 + u_{L2}q_{H2}$$

$$\Leftrightarrow u_{L1}\beta_p q_1 - u_{L2}(q_{H2}-q_{L2}) + u_{H2}(q_{H2}-q_{L2}) - u_{H1}q_1(\beta_d+\beta_p) + u_{L1}\beta_d q_1 \geq 0$$

$$\Leftrightarrow u_{L1}q_1(\beta_d+\beta_p) + (u_{H2}-u_{L2})(q_{H2}-q_{L2}) - u_{H1}q_1(\beta_d+\beta_p) \geq 0$$

The above inequality can be rearranged as

$$u_{H2}q_{H2} + u_{H1}(1-\beta_d)q_1 - u_{H2}q_{L2} - u_{H1}(1+\beta_p)q_1$$
$$\geq u_{L2}q_{H2} + u_{L1}(1-\beta_d)q_1 - u_{L2}q_{L2} - u_{L1}(1+\beta_p)q_1$$

where the left term is the high-valuation segment's perceived quality gap between the high and low-end products, and the right term is the low-valuation segment's perceived quality gap between the high and low-end products.

Model formulation and solution to Problem *C-NC*

Problem *C-NC*:

$Max_{p_H,p_L,q_{H1},q_{H2},q_{L1},q_{L2}}\ \ \pi_{C-NC} = \alpha_H(p_H - c_1q_{H1}^2 - c_2q_{H2}^2) + \alpha_L(p_L - c_1q_{L1}^2 - c_2q_{L2}^2)$

Subject to (2) – (5).

By making (2) and (4) binding, we have:

$$p_L = u_{L1}q_{L1} + u_{L2}q_{L2};\ p_H = p_L + u_{H1}(u_{H1} - q_{L1}) + u_{H2}(q_{H2} - q_{L2})$$

By substituting p_L and p_H to the profit function in (1) we have:

$$\pi_{C-NC} = \alpha_H(u_{H1}q_{H1} + u_{H2}q_{H2} - u_{H1}q_{L1} - u_{H2}q_{L2} + u_{L1}q_{L1} + u_{L2}q_{L2} - c_1q_{H1}^2 - c_2q_{H2}^2) + \alpha_L(u_{L1}q_{L1} + u_{L2}q_{L2} - c_1q_{L1}^2 - c_2q_{L2}^2)$$

Taking the first-order partial derivatives of the profit with respect to q_{H1}, q_{H2}, q_{L1}, q_{L2} and equaling them to zero give: $\frac{\partial\pi_{C-NC}}{\partial q_{H1}} = \alpha_H(u_{H1} - 2c_1q_{H1}) = 0 \rightarrow q_{H1}^* = \frac{u_{H1}}{2c_1} = q_{H1}^{*e}$; $\frac{\partial\pi_{C-NC}}{\partial q_{H2}} = \alpha_H(u_{H2} - 2c_2q_{H2}) = 0 \rightarrow q_{H2}^* = \frac{u_{H2}}{2c_2} = q_{H2}^{*e}$; $\frac{\partial\pi_{C-NC}}{\partial q_{L1}} = \alpha_H(-u_{H1} + u_{L1}) + \alpha_L(u_{L1} - 2c_1q_{L1}) = 0 \rightarrow q_{L1}^* = \frac{u_{L1} - \alpha_H u_{H1}}{2c_1\alpha_L} = \frac{u_{L1}}{2c_1}\left[\frac{1}{\alpha_L}\left(1 - \frac{\alpha_H u_{H1}}{u_{L1}}\right)\right] = \frac{u_{L1}}{2c_1}\left[\frac{1}{\alpha_L}\left(\alpha_L - \alpha_H\left(\frac{u_{H1}}{u_{L1}} - 1\right)\right)\right] = \frac{u_{L1}}{2c_1}\left[1 - \frac{\alpha_H}{\alpha_L}\left(\frac{u_{H1}}{u_{L1}} - 1\right)\right] = q_{L1}^{*e}(1 - R_1)$;

$\frac{\partial\pi_{C-NC}}{\partial q_{L2}} = \alpha_H(-u_{H2} + u_{L2}) + \alpha_L(u_{L2} - 2c_2q_{L2}) = 0 \rightarrow q_{L2}^* = \frac{u_{L2} - \alpha_H u_{H2}}{2c_2\alpha_L} = q_{L2}^{*e}(1 - R_2)$.

Proof of solution to Problem *M D-C*

By making constraints (15) and (16) binding, we have:

$$w_H = w_L + u_{H2}(q_{H2} - q_{L2}) - u_{H1}q_1(\beta_d + \beta_p)$$

$$w_L = \frac{u_{L1}\left(1 + \beta_p\right)q_1 + u_{L2}q_{L2} - \alpha_H u_{H1}q_1\left(1 + \beta_p\right) - \alpha_H u_{H2}q_{L2}}{\alpha_L}$$

The profit function for Problem *M D-C* can be written as:

$\pi_{D-C}^M = \alpha_H\left(\frac{u_{L1}(1+\beta_p)q_1 + u_{L2}q_{L2} - \alpha_H u_{H1}q_1(1+\beta_p) - \alpha_H u_{H2}q_{L2}}{\alpha_L} + u_{H2}q_{H2} - u_{H2}q_{L2} - c_1(1-\theta)q_1^2 - c_2q_{H2}^2\right) + \alpha_L\left(\frac{u_{L1}(1+\beta_p)q_1 + u_{L2}q_{L2} - \alpha_H u_{H1}q_1(1+\beta_p) - \alpha_H u_{H2}q_{L2}}{\alpha_L} - c_1(1-\theta)q_1^2 - c_2q_{L2}^2\right)$.

Taking the first-order partial derivatives of the profit with respect to q_1, q_{H2}, q_{L2} and equaling them to zero give: $\frac{\partial\pi_{D-C}^M}{\partial q_1} = \frac{u_{L1}(1+\beta_p) - \alpha_H u_{H1}(1+\beta_p)}{\alpha_L} - 2c_1(1-\theta)q_1 - \alpha_H u_{H1}(\beta_d + \beta_p) = 0 \rightarrow q_1^* = \frac{(1+\beta_p)(u_{L1} - \alpha_H u_{H1}) - \alpha_L\alpha_H u_{H1}(\beta_d + \beta_p)}{2c_1(1-\theta)\alpha_L}$; $\frac{\partial\pi_{D-C}^M}{\partial q_{H2}} = \alpha_H u_{H2} - 2\alpha_H c_2 q_{H2} = 0 \rightarrow q_{H2}^* = \frac{u_{H2}}{2c_2} = q_{H2}^{*e}$; $\frac{\partial\pi_{D-C}^M}{\partial q_{L2}} = \frac{u_{L2} - \alpha_H u_{H2}}{\alpha_L} - \alpha_H u_{H2} - 2\alpha_L c_2 q_{L2} = 0 \rightarrow q_{L2}^* = \frac{u_{L2} - \alpha_H u_{H2}(1+\alpha_L)}{2c_2\alpha_L^2} = q_{L2}^{*e}\left(1 - R_2 - \frac{1}{\alpha_L}R_2\right)$

For the case with zero valuation changes ($\beta_p = \beta_d = 0$) and zero cost saving ($\theta = 0$), $q_1^* = \frac{u_{L1}-\alpha_H u_{H1}}{2c_1\alpha_L}$, which can also be written as $\frac{u_{L1}-\alpha_H u_{H1}(1+\alpha_L)}{2c_1\alpha_L} + \frac{\alpha_L\alpha_H u_{H1}}{2c_1\alpha_L} = \alpha_L q_{L1}^* + \alpha_H q_{H1}^*$

Model formulation and solution to Problem *C-C*

Problem *C-C*:

$$Max_{p_H,p_L,q_1,q_{H2},q_{L2}} \ \ \pi_{C-C} = \alpha_H(p_H - c_1(1-\theta)q_1^2 - c_2q_{H2}^2) + \alpha_L(p_L - c_1(1-\theta)q_1^2 - c_2q_{L2}^2) - F$$

Subject to (11) - (14).

The optimal solution to Problem *C-C* is obtained by making constraints (11) and (13) binding, which yields $p_L = u_{L1}(1+\beta_p)q_1 + u_{L2}q_{L2}$ and $p_H = p_L - u_{H1}q_1(\beta_d+\beta_p) + u_{H2}(q_{H2}-q_{L2})$. Substituting p_L and p_H in the profit function with these equations and taking the first-order partial derivatives of the profit with respect to q_1, q_{H2}, q_{L2} yield: $\frac{\partial\pi_{C-C}}{\partial q_1} = u_{L1}(1+\beta_p) - 2c_1(1-\theta)q_1 - \alpha_H u_{H1}(\beta_d+\beta_p) = 0 \rightarrow q_1^* = \frac{u_{L1}(1+\beta_p)-\alpha_H u_{H1}(\beta_d+\beta_p)}{2c_1(1-\theta)}$; $\frac{\partial\pi_{C-C}}{\partial q_{H2}} = \alpha_H u_{H2} - 2\alpha_H c_2 q_{H2} = 0 \rightarrow q_{H2}^* = \frac{u_{H2}}{2c_2} = q_{H2}^{*e}$; $\frac{\partial\pi_{C-C}}{\partial q_{L2}} = u_{L2} - \alpha_H u_{H2} - 2\alpha_L c_2 q_{L2} = 0 \rightarrow q_{L2}^* = \frac{u_{L2}-\alpha_H u_{H2}}{2c_2\alpha_L} = q_{L2}^{*e}(1-R_2)$.

Appendix B

Proof of Proposition 1

From direct comparison, when the cost saving and valuation changes are set to zero, i.e., $\theta = 0$ and $\beta_d = \beta_p = 0$, $q_{1\,D-C}^* = \frac{u_{L1}-\alpha_H u_{H1}}{2c_1\alpha_L} > q_{L1\,D-NC}^* = \frac{u_{L1}-\alpha_H u_{H1}(1+\alpha_L)}{2c_1\alpha_L^2}$.

Proof of Proposition 2

When the cost saving and valuation changes are set to zero, i.e., $\theta = 0$ and $\beta_d = \beta_p = 0$, and when $\alpha_H < \frac{u_{L1}}{u_{H1}(1+\alpha_L)}$, the difference in channel profits between the no-commonality strategy and the commonality strategy in the decentralized setting can be written as:

$\pi_{D-C}^* - \pi_{D-NC}^* = \frac{u_{L1}-\alpha_H u_{H1}(1+\alpha_L)}{2c_1\alpha_L^2}\left(u_{L1} - \alpha_H u_{H1} - \frac{u_{L1}-\alpha_H u_{H1}(1+\alpha_L)}{2\alpha_L}\right) + \frac{\alpha_H u_{H1}}{2c_1}\left(u_{H1} - \frac{u_{H1}}{2}\right) - \frac{u_{L1}-\alpha_H u_{H1}}{2c_1\alpha_L}\left(u_{L1} - \frac{u_{L1}-\alpha_H u_{H1}}{2\alpha_L}\right) = \frac{\alpha_H\,(u_{H1}-u_{L1})^2(1-2\alpha_L)}{4\alpha_L{}^3\,c_1}$. When $\alpha_H = \alpha_L = 0.5$, $\pi_{D-C}^* = \pi_{D-NC}^*$. When $\alpha_H > \alpha_L$, $\pi_{D-C}^* > \pi_{D-NC}^*$.

When $\alpha_H > \frac{u_{L1}}{u_{H1}(1+\alpha_L)}$, we can set $q_{L1}^* = 0$.

$$\pi^*_{D-C} - \pi^*_{D-NC} = u_{L1}q_1 - c_1 q_1^2 - \alpha_H q_{H1}(u_{H1} - c_1 q_{H1})$$
$$= \frac{u_{L1} - \alpha_H u_{H1}}{2c_1\alpha_L}\left(u_{L1} - \frac{u_{L1} - \alpha_H u_{H1}}{2\alpha_L}\right) - \alpha_H \frac{u_{H1}^2}{4c_1} = \frac{u_{L1}^2(1 - {R_1}^2)}{4c_1} - \alpha_H \frac{u_{H1}^2}{4c_1}$$

It can be shown that when $0.5 < \alpha_H < \frac{u_{L1}^2(1-{R_1}^2)}{u_{H1}^2}$, $\pi^*_{D-C} - \pi^*_{D-NC} > 0$.

Proof of Proposition 3

(a) The difference in channel profits in the decentralized channel between the no-commonality strategy and the commonality strategy can be written as:

$$\pi^*_{D-C} - \pi^*_{D-NC}$$
$$= \{((2\alpha_L - 1)u_{L1} + \alpha_H u_{H1})(1 + \beta_p) - \alpha_L\alpha_H u_{H1}(\beta_d + \beta_p)\}$$
$$- \frac{(u_{L1} - \alpha_H u_{H1}(1 + \alpha_L))((2\alpha_L - 1)u_L + \alpha_H u_{H1}(1 + \alpha_L)) + \alpha_L\alpha_H u_{H1}(u_{H1}(1 + \alpha_H(1 + \alpha_L)) - 2u_{L1}) + 4Fc_1\alpha_L^3}{4c_1\alpha_L^3}$$

The profits for the two strategies are equal when $\theta = \theta_D^{min} = 1 - \frac{\alpha_L\{(1+\beta_p)(u_{L1}-\alpha_H u_{H1}) - \alpha_L\alpha_H u_{H1}(\beta_d+\beta_p)\}\{((2\alpha_L-1)u_{L1}+\alpha_H u_{H1})(1+\beta_p) - \alpha_L\alpha_H u_{H1}(\beta_d+\beta_p)\}}{(u_{L1}-\alpha_H u_{H1}(1+\alpha_L))((2\alpha_L-1)u_L+\alpha_H u_{H1}(1+\alpha_L)) + \alpha_L\alpha_H u_{H1}(u_{H1}(1+\alpha_H(1+\alpha_L)) - 2u_{L1}) + 4Fc_1\alpha_L^3}$, which can be simplified as $\theta_D^{min} = 1 - \frac{\{(1+\beta_p)^2(u_{L1}-\alpha_H u_{H1})^2 - \alpha_L^2\alpha_H^2 u_{H1}^2(\beta_d+\beta_p)^2\}}{\alpha_L\{\alpha_L\alpha_H\, {u_{H1}}^2 + \alpha_L^2\, {u_{L1}}^2(1-R_1)^2 - {u_{L1}}^2 R_1^2 + 4\, c_1\alpha_L F\}}$.

The sign of θ_D^{min} can be positive or negative whether there are valuation changes or not.

(b) First, we derive the minimum cost saving in the centralized channel. The difference in channel profits in the centralized channel between the no-commonality strategy and the commonality strategy can be written as: $\pi^*_{C-C} - \pi^*_{C-NC} = \frac{\alpha_L\{u_{L1}(1+\beta_p) - \alpha_H u_{H1}(\beta_d+\beta_p)\}^2}{4c_1(1-\theta)\alpha_L} - \frac{(1-\theta)\{\alpha_H u_{H1}^2 - 2\alpha_H^2 u_{L1}u_{H1} + (2\alpha_H-1)u_{L1}^2 + 2\alpha_L(u_{L1}-\alpha_H u_{H1})\}}{4c_1(1-\theta)\alpha_L} - F$. The profits for the two strategies are equal when $\theta = \theta_C^{min} = 1 - \frac{\alpha_L\{u_{L1}(1+\beta_p) - \alpha_H u_{H1}(\beta_d+\beta_p)\}^2}{\{\alpha_H u_{H1}^2 - 2\alpha_H^2 u_{L1}u_{H1} + (2\alpha_H-1)u_{L1}^2 + 2\alpha_L(u_{L1}-\alpha_H u_{H1}) + 4Fc_1\alpha_L\}} = 1 - \frac{\{\alpha_L u_{L1}(1+\beta_p)(1-R_1) + \alpha_H u_{H1}(1-\beta_d)\}^2}{\alpha_H\, {u_{H1}}^2 + \alpha_L\, {u_{L1}}^2(1-R_1)^2 + 4\, c_1 F}$.

The cost savings in the centralized and decentralized channels are given by:

$$\pi^{SC}_{C-C} - \pi^{SC}_{C-NC} = \{u_{L1}(1 + \beta_p) - \alpha_H u_{H1}(\beta_d + \beta_p)\}q_{1C} - c_1(1 - \theta)q_{1C}^2 - (u_{L1} - \alpha_H u_{H1})q_{L1C}$$
$$- \alpha_H(u_{H1}q_{H1} - c_1 q_{H1}^2) + \alpha_L c_1 q_{L1C}^2 - F$$

$$\pi^{SC}_{D-C} - \pi^{SC}_{D-NC} = \{u_{L1}(1+\beta_p) - \alpha_H u_{H1}(\beta_d + \beta_p)\}q_{1D} - c_1(1-\theta)q_{1D}^2 - (u_{L1} - \alpha_H u_{H1})q_{L1D} - \alpha_H(u_{H1}q_{H1} - c_1 q_{H1}^2) + \alpha_L c_1 q_{L1D}^2 - F$$

The differences between the cost savings in the decentralized and centralized channels are given by:

$$(\pi^{SC}_{D-C} - \pi^{SC}_{D-NC}) - (\pi^{SC}_{C-C} - \pi^{SC}_{C-NC}) = \{u_{L1}(1+\beta_p) - \alpha_H u_{H1}(\beta_d + \beta_p)\}(q_{1D} - q_{1C}) - c_1(1-\theta)(q_{1D}^2 - q_{1C}^2) - (u_{L1} - \alpha_H u_{H1})(q_{L1D} - q_{L1C}) + \alpha_L c_1(q_{L1D}^2 - q_{L1C}^2)$$

Since $\pi^{SC}_{D-C} - \pi^{SC}_{D-NC} > 0, \pi^{SC}_{C-C} - \pi^{SC}_{C-NC} > 0$ and $q_{1C} > q_{1D}, q_{L1C} > q_{L1D}$, we can conclude that $(\pi^{SC}_{D-C} - \pi^{SC}_{D-NC}) - (\pi^{SC}_{C-C} - \pi^{SC}_{C-NC}) < 0$, which also means $\theta_D^{min} < \theta_C^{min}$.

Proof of Proposition 4

In the decentralized channel, consumer surplus (CS) for the no-commonality and commonality are

$$CS_{D-NC} = (u_{H1} - u_{L1})q_{L1(D-NC)} + (u_{H2} - u_{L2})q_{L2(D-NC)}$$

$$CS_{D-C} = (u_{H1} - u_{L1})(1+\beta_p)q_{1(D-C)} + (u_{H2} - u_{L2})q_{L2(D-C)}$$

Since we have $q_{1(D)} > q_{L1(D)}$, we can see from the expressions above that $CS_{D-C} > CS_{D-NC}$.

Proof of Proposition 5

Recall that

$$\theta^{min}_{D(SC)} = \pi^{SC}_{D-C} - \pi^{SC}_{D-NC} = u_{L1}(1+\beta_p)q_1 - \alpha_H u_{H1}(\beta_d + \beta_p)q_1 - c_1(1-\theta)q_1^2 - F - u_{L1}q_{L1}^* - \alpha_H(u_{H1}(q_{H1}^* - q_{L1}^*) - c_1 q_{H1}^2) + \alpha_L c_1 q_{L1}^2$$

and

$$\theta^{min}_{D(M)} = \pi^{M}_{D-C} - \pi^{M}_{D-NC} = \frac{(1+\beta_p)q_1(u_{L1} - \alpha_H u_{H1}) - q_{L1}^*(u_{L1} - \alpha_H u_{H1})}{\alpha_L} - \alpha_H u_{H1} q_1(\beta_d + \beta_p) - \alpha_H u_{H1}(q_{H1}^* - q_{L1}^*) - c_1(1-\theta)q_1^2 + \alpha_H c_1 q_{H1}^2 + \alpha_L c_1 q_{L1}^2 - F$$

After some algebraic manipulation, we have:

$$\theta_{D(SC)}^{min} - \theta_{D(M)}^{min} = \frac{\alpha_H\{q_{L1}^* - (1+\beta_p)q_1\}\{u_{H1} - u_{L1}\}}{\alpha_L}$$

Since $q_1 > q_{L1}^*$, $q_{L1}^* - (1+\beta_p)q_1$ is always negative, i.e. $\theta_{D(SC)}^{min} < \theta_{D(M)}^{min}$.