Lecture notes on

Electrical theory behind the measurement of body fluids with bioimpedance spectroscopy (BIS)

with applications to the measurement device
4200 HYDRA ECF/ICF Bio-Impedance Spectrum Analyzer
from Xitron Technologies

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Preface

Bioimpedance spectroscopy (BIS) is a painless and relatively simple measurement method. The measured data can be used to determine the amount of fluids in the body as well as estimate body composition.

The aims of these notes are

- to give an understanding of the electric theory underlying BIS measurements
- to give an account of the equations and calculations used in the device *4200 HYDRA ECF/ICF Bio-Impedance Spectrum Analyzer* from Xitron Technologies. For simplicity, the device will be called Xitron Hydra 4200.
- to create a basis for research applications of the device and underlying equations

To fulfil the aims, the notes must be both readable and usable. As the underlying theory is quite complex, this is not an easy task: A very readable text may be too shallow to be really useful, while a very thorough text may become too long and too complex to be readable. In an attempt to solve this dilemma, the writing has been based on the following considerations:

1. A person with little mathematical training and no special knowledge of electric theory should be able to understand the overall picture and follow the main flow of the notes.
2. On the other hand, the treatment of the subject must not be shallow, and more mathematically oriented readers should not be cheated from seeing equations and derivations.

In the resulting text, concepts are explained in words before being presented as equations, and long derivations are kept in separate paragraphs. The separate paragraphs can be skipped by readers who are satisfied with seeing the main formulas and their assumptions. This is, however, not a text where any equation can be skipped.

Aalborg, July 7th 2008

Revised edition

This constitutes a slightly revised edition of the original notes, with the main differences being that "fluid" is now consistently abbreviated F for fluid (ECF, ICF, and TBF) instead of W for water (ECW, ICW, and TBW), and some figures have been redrawn.

Aalborg, March 17th 2010

Note: The author of these notes has no affiliation with Xitron Technologies.

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Overview of chapters

Chapter 1: A brief, basic course on resistance, capacitance, and impedance: The first chapter introduces the basic concepts from the theory of electricity used in these notes. In principle, the chapter can be read with no background knowledge of electricity, but basic knowledge on resistance and Ohm's law will come in handy.

Chapter 2: Bioimpedance spectroscopy – fundamentals: How electric currents flow in tissue, and why impedance depends on frequency.

Chapter 3: Electrical model for BIS: An introduction to the electric model that is the basis of calculations in much of the literature on BIS. The chapter is to some extent a repetition of chapter 2, but with a different perspective.

Chapter 4: Resistivity: Introduction to the concept of resistivity (which is not the same as resistance, although the two are connected). Connection between resistivity and (body) volume is described. Finally, the chapter gives an introduction to the so-called Hanai mixture theory, from which a few formulas are used in later chapters.

Chapter 5: Calculation of extra-cellular fluid, ECF: This chapter describes the formulas used by the Xitron Hydra 4200 device for calculation of the volume of extra-cellular fluid, abbreviated ECF (or ECW for extra-cellular water). The formulas are presented in the beginning of the chapter, (16) and (17). Then a derivation of the formulas is given. The reader may skip the derivation, as long as he or she is aware of the assumptions.

Chapter 6: Calculation of total body fluid, TBF, and intra-cellular fluid, ICF: This chapter describes the formulas used by the Xitron Hydra 4200 device for calculation of the volume of total body fluid (abbreviated TBF or TBW) and the volume of intra-cellular fluid (abbreviated ICF or ICW). As in the previous chapter, the formulas are first presented, (32) - (36), and then derived. And again, the derivation may be skipped by the non-mathematical reader.

Chapter 7: The impact of deviations in ρ_{ECF} and ρ_{ICF}: In the previous chapters, extra- and intra-cellular resistivities were treated as basic constants. This chapter analyses how deviations in these "constants" influence the calculated fluid volumes, V_{ECF}, V_{TBF}, and V_{ICF}. The chapter may be skipped at first reading. Readers tempted to skip the chapter at *all* readings are advised to take a look at the example and read the summary.

Chapter 8: Calculation of BCM and FFM: A short presentation of the formulas used by the Xitron Hydra 4200 for calculation of Body Cell Mass (BCM) and Fat-Free Mass (FFM).

Chapter 9. Practical considerations on measurements: Some words and explanations on how BIS measurements should be made, and the implications of doing it wrong.

Chapter 10. Summary of formulas: Summary of the formulas presented in these notes for calculating fluid volumes. The chapter further presents formulas (without derivation) for the reverse calculation, i.e., how the basic constants can be calculated if the fluid volumes and measured resistances are known.

Chapter 1: A brief, basic course on resistance, capacitance, and impedance

Ohm's law

Electric current in a wire can be compared to running water in a stream of water. The water runs when there is a fall in the stream. The steeper the fall, the more pressure in the stream. In a mountain river the running water may exert a large pressure – utilized for instance in Norway for driving turbines to produce power. However, steepness alone is not a measure of the might of a river. The Mississippi River may not have a steep fall, but it is wide and during a day a very large amount of water flows through the river.

In a wire, the ***voltage drop*** is a measure of the pressure on the electric charges (electrons) in the wire. The ***current intensity*** tells how much electric charge flows through the wire pr. second. If the voltage drop increases then the current intensity will also increase.

The connection between voltage drop and current intensity is given by the ***electrical resistance***. For a given voltage drop, a high resistance will result in a low current intensity. Imagine a water pipe: A narrow pipe (high resistance) will hinder the flow of water, while a wide pipe (low resistance) will allow a large flow of water.

This connection is described by Ohm's law:

$$\text{current} = \frac{\text{voltage drop}}{\text{resistance}}$$

$$I = \frac{U}{R}$$

which may also be written:

$$U = R \cdot I \tag{1}$$

The letters stand for:

U is voltage drop (measured in Volt, V)

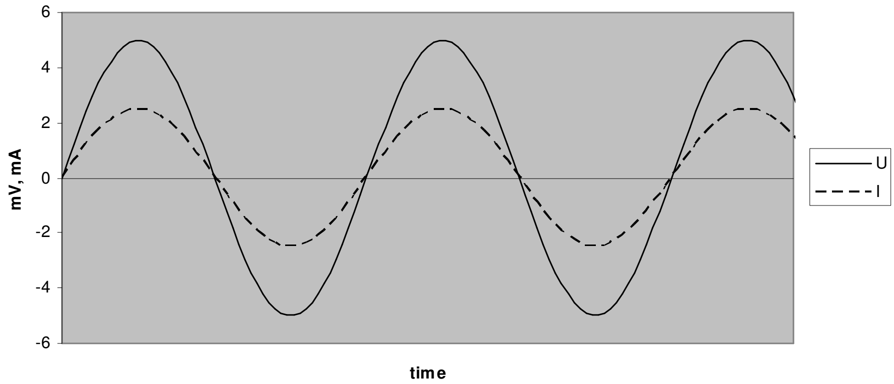
R is resistance (measured in Ohm, Ω)

I is current intensity (measured in Ampere, A)

Alternating current

The current coming out of the socket in wall is alternating current. On average, the voltage drop is 230 V (in Europe), but the direction constantly changes, so the current alternates between running in one direction and the opposite direction.

The voltage drop and the current intensity are in constant change, but the wires with the resistance remain the same. From Ohm's law we see that when the voltage drop (U) increases then the current intensity (I) also increases, and when the voltage drop decreases then the current intensity also decreases. So, the voltage drop and the current intensity changes in the same rhythm. We say that voltage and current are ***in phase***.

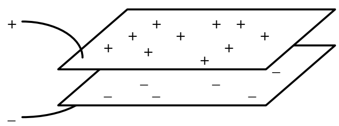


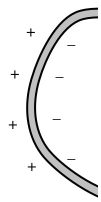
The illustration above shows an example where the resistance is R = 2 Ω: At all times, the numerical value of U is twice the numerical value of I.

In the case of household sockets, the current makes 50 back-and-forth cycles every second, i.e., the frequency is 50 Hertz (Hz). In the case of bioimpedance spectroscopy, a whole range of frequencies is used for measurements. For instance, the Xitron Hydra 4200 measures at frequencies ranging from 5,000 to 1,000,000 Hz (from 5 to 1000 kHz).

Capacitance

Resistance is a measure of a wire's (lack of) ability to conduct an electric current. ***Capacitance*** is a measure of the ability to accumulate electric charges for a shorter or longer time. Capacitance arises where two surfaces of electrically conductive materials are close, but have no direct contact. A technical example can be two metal sheets that do not touch. A biological example can be a cell in the body – the fluid outside the cell and the fluid inside the cell are able to conduct electricity, but the cell membrane is not, so there is no direct contact.





When voltage is applied to such a system, a current will run at first. This creates a surplus of negative electric charge on the one surface (at the minus side), and a surplus of positive electric charge on the other surface (at the plus side). But like charges repel each other (+ repel +, and – repel –). When so much charge has been accumulated that the repulsive force equals the force from the voltage, then the current cease flowing. If the current is direct (non-alternating), nothing more will happen. A direct current cannot pass a capacitance.

What happens if the current is alternating? At first, the same as above: Negative charge will accumulate on one surface, positive charge on the other, making it more and more difficult for the current to run. But then the voltage changes and the charges begin to run the opposite way. First the accumulations of charge will be emptied, then new accumulation begin, only opposite to before. This goes on for a short while, then the voltage changes again, etc. etc.

In this way, alternating current can run even though there is a capacitance. No individual charges pass the capacitance, but as a collective phenomenon the alternating electric current can "pass" the capacitance.

This is true even if one side is surrounded by insulating material, as in the case of a cell where the inside fluid is surrounded by the insulating cell membrane. This is because charges on the outside of the cell can attract or repel charges on the inside of the cell, just as a magnet can attract or repel another magnet without touching it.

Impedance and phase difference

When alternating current pass a capacitance, the peak value of the voltage drop will be related to the peak value of the current intensity. This is at first sight similar to Ohm's law:

$$U_{max} = Z \cdot I_{max} \qquad (2)$$

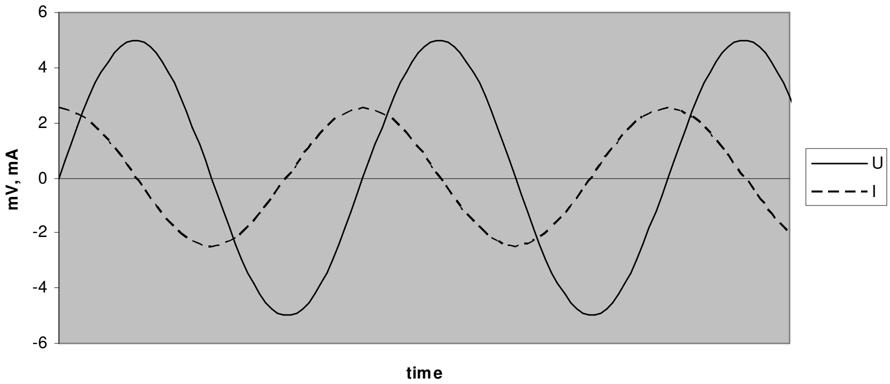
where Z is the so-called ***impedance***. Impedance and resistance have the same unit: Ohm or **Ω**. But capacitance has a twist we did not see in resistance: The rhythm of the current intensity is not the same as the rhythm of the voltage drop.

To understand this, imagine the capacitance being a dam, and the electric current being water flowing towards the dam. In time, a lake will accumulate behind the dam. If the level of the lake becomes as high as the level from which the water originates, then the water will stop running. But the accumulated water has built up a height difference. There is now a large drop in height from water level before the dam to water level after the dam.

Likewise with the capacitance: At first current flows easily. Over time, electric charge accumulates, and the voltage drop from one side to the other increases. As the voltage drop increases, current intensity decreases. When the voltage drop is at its peak value, the current intensity has dropped to zero.

For the dam, the story stops here. For the electric current, something new happens. Since we are talking about alternating current, the electric voltage changes, and current begins to flow in the opposite direction. First the capacitance is emptied of the accumulated charge. At the point where all charge has been emptied, the voltage drop over the capacitance is zero, while current intensity is at its negative peak value (negative current intensity = current flowing backwards). From this point the whole story starts over, just with opposite sign. And so on.

The key point here is that current intensity and voltage drop have different rhythms. *First* the current intensity is high, *then* the voltage drop becomes high. As a result, the current intensity (I) is ¼ cycle ahead of the voltage drop (U). Voltage and current are ***out of phase***. The difference between the phases is called the ***phase difference***.



An example is given in the above illustration, corresponding to impedance being Z = 2 Ω.

Except for the phase, these curves correspond to those for a resistance. You may say that impedance is "alternating-current-resistance". Both impedance and resistance has the unit Ohm or Ω.

Impedance and frequency

The impedance of a capacitance depends on two things:

- The "size" of the capacitance: The larger the capacitance, the lower the impedance. The reason is that a large capacitance can accumulate a large amount of electric charge before it becomes hard for the current to flow.
- The frequency of the alternating current: The higher the frequency, the lower the impedance. If frequency is high then the direction of the current will change before much charge has been accumulated. So a high-frequency current experiences only situations where current runs relatively unhindered.

For a given capacitance we have too extreme cases of frequency (f):

f = 0: Current with frequency 0 (i.e., no change of direction = direct current) cannot pass a capacitance. Impedance Z is infinitely large.

f = ∞: Current with infinite frequency will experience no resistance at all, i.e. impedance Z is 0.

In the middle range, $0 < f < \infty$, impedance Z has a finite value and there is a phase difference.

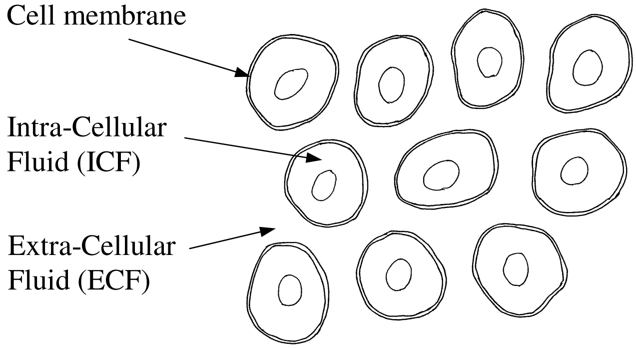
At this point we are ready to turn to our main subject: The study of impedance in biological materials at various frequencies, or *bioimpedance spectroscopy*.

Chapter 2: Bioimpedance spectroscopy – fundamentals

Cells and cell fluids

Each cell in the body is surrounded by a cell membrane made of fats. The cells contain intra-cellular fluid, abbreviated ICF (also called ICW for intra-cellular water). The cells are surrounded by extra-cellular fluid, ECF (also called ECW).

Electric currents can run in the intra- and the extra-cellular fluids, whereas the cell membrane is electrically insulating.



(Illustration based on De Lorenzo *et al* 1997)

The idea of bioimpedance spectroscopy (BIS) is to measure the impedance of biological tissue at a series of frequencies. This is done by sending weak alternating currents through the tissue, while measuring both the impedance and the phase difference for each frequency. The "tissue" can be quite large, for instance a whole person.

Currents with very low or very high frequency

Alternating current with *very low* frequency will behave as direct current: The cell membrane is electrically insulating, meaning that no current can get to the inside of the cells. So the current will run in the extra-cellular fluid only.

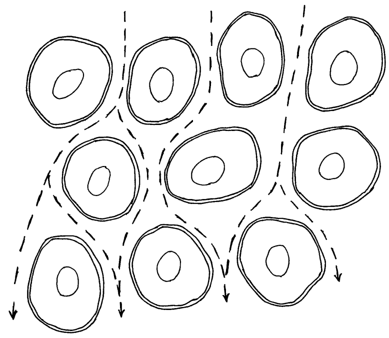
If current intensity (I) and voltage drop (U) are measured, the resistance (R) can be calculated with Ohm's law. The resistance at zero frequency is called R_0.

At frequencies above 0, the cell membrane will work as a capacitance, as described in the previous chapter. The impedance of a capacitance depends on the frequency of the alternating current, but at infinitely high frequency the impedance will be zero.

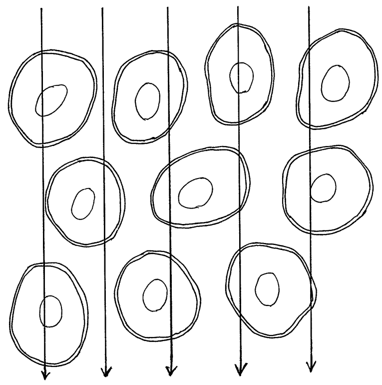
This means that at *very high* frequency the current can pass the cell membranes almost unhindered. The current does not have to run around the cells but can take a shortcut through the intra-cellular fluid.

Resistance at infinite frequency is called R_∞. Since the high-frequency current can take a more direct route than the low-frequency current, $R_\infty < R_0$.

Low frequency current



High-frequency current

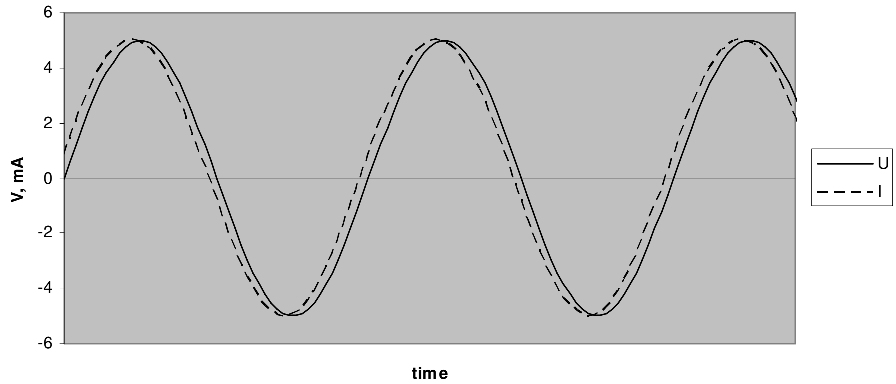


Medium size frequencies and phase difference

At both very low and very high frequency we could ignore capacitance and phase difference. For frequency f = 0 the cell membranes were 100% effective barriers, which the current had to run around. For frequency f = ∞ the current could run as if there had been no cell membranes at all. But in the medium range the cell membranes act as capacitances, and phase difference comes into play.

Phase difference can be measured in degrees. A whole cycle of the alternating current is 360º, a full circle. In the case of a capacitance, we have seen that the phase difference is ¼ cycle, i.e. 90º.[1] In the case of a resistance, there is no phase difference, i.e. 0º. Biological tissue is a mix of resistance (from the fluids) and capacitance (from the cell membranes). For this reason, the phase difference is somewhere between 0º and 90º.

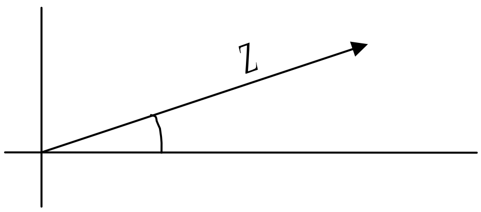
In practice, the phase difference is rather small for biological tissue. The value depends on the frequency, but is typically below 10º for all frequencies. The illustration below shows voltage and current curves for impedance Z = 1 kΩ, phase difference 10º.



The BIS device measures at a range of frequencies. At each frequency the device measures the size of the impedance

$$Z = \frac{U_{max}}{I_{max}} \qquad (3)$$

and the phase difference. We can illustrate the result of a single measurement by drawing an arrow in coordinate system. The arrow is drawn from (0,0) with length Z, and the angle with the x-axis is equal to the phase difference. In this way the result of a single impedance measurement corresponds to a point in a coordinate system, namely the point of the arrow.



The figure below shows the measurements from a spectrum of frequencies: Impedance with phase difference has been measured at 50 different frequencies ranging from 5 to 1000 kHz, and the result of each measurement is shown as a point. The lowest frequencies correspond to the points farthest to the right.

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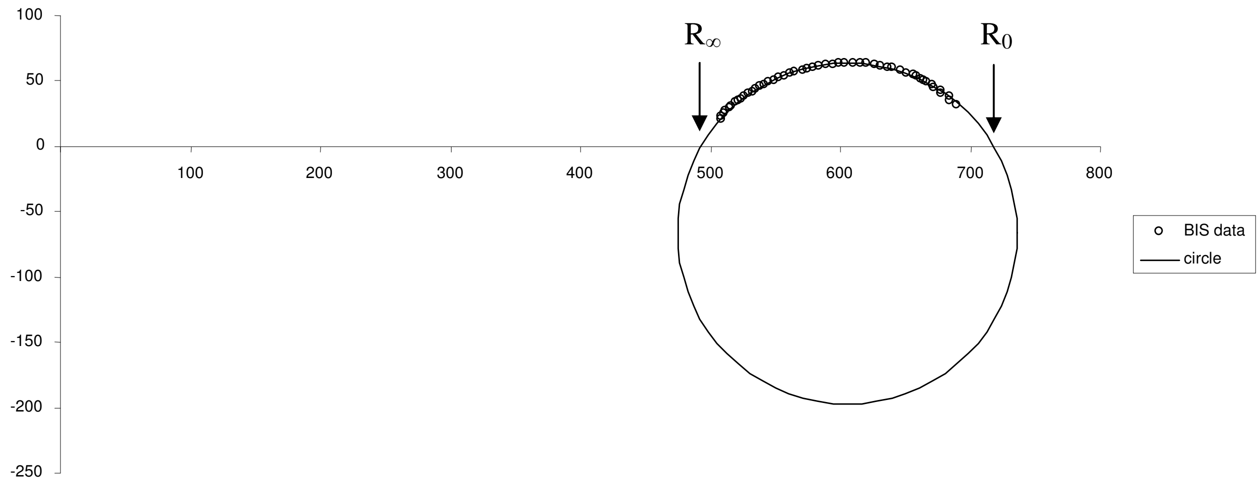
[1] According to standard physics conventions, the phase difference for a capacitance is negative, –90º, because the current comes *before* the voltage. However, in all biological tissue the phase difference has the same sign, making the sign convention less important in this context. So for simplicity the phase difference of biological tissue will consistently be described as a positive number in these notes. The same convention is used in much other literature on BIS.

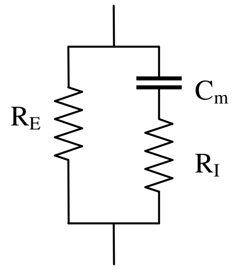


We see that the impedance decreases with increasing frequency. This is consistent with our knowledge that higher-frequency currents can pass the cell membranes and thereby take shorter routes. At very high and very low frequencies the phase difference is close to 0º (i.e. pure resistance, cf. earlier description). The largest phase difference is seen at frequencies of medium size.

As described in the following chapter, it is possible to set up an electric model for biological tissue. Using this model, it is possible (but beyond the scope of these notes) to show mathematically that the points should lie on the arc of a circle. In the case of our test person, we see that the points indeed lie on the arc of a circle, i.e. the measurements are consistent with the model.

Chapter 3: Electrical model for BIS

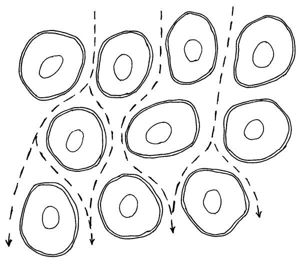
This chapter largely repeats the contents of the preceding chapter, but now from the perspective of the electrical model used in calculations.

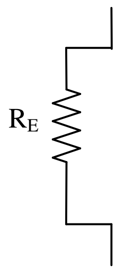


According to the model, the electrical properties of biological tissue can be described by the circuit depicted to the right. The circuit consists of the resistance R_E (from the extra-cellular fluid) wired in parallel with the resistance R_I (from the intra-cellular fluid), where the current must pass the capacitance C_m (from the cell membranes) to run through R_I.

Behaviour at very low frequency

As we saw in Chapter 1, at zero frequency (f = 0) the current cannot pass the capacitance C_m at all. I.e., the current has to run around all cells. In this extreme, it would make no difference if the circuit was drawn without the branch with C_m and R_I:



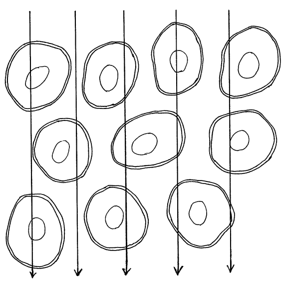


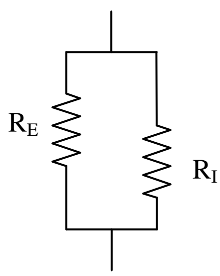
By this reasoning, we see that the resistance at zero frequency is equal to the resistance from the extra-cellular fluid alone:

$$R_0 = R_E \tag{4}$$

Behaviour at very high frequency

At infinite frequency (f = ∞) the current will pass the capacitance as if it had not been there (cf. Chapter 1), and the electric current can run through the cells. In this extreme, the circuit could as well have been drawn without the capacitance C_m:





A circuit with two resistances in parallel acts as a single resistance. We will call the resistance R_∞, since it describes the situation at infinite frequency. R_∞ will be lower than each of the individual resistances R_E and R_I: Two routes will always give a better flow with lower resistance than a single route.

R_∞ can be calculated from R_E and R_I with the formula

$$\frac{1}{R_\infty} = \frac{1}{R_E} + \frac{1}{R_I} \tag{5}$$

which can be re-written as

$$R_\infty = \frac{R_E \cdot R_I}{R_E + R_I} \tag{6}$$

In some of the literature, the name R_{INF} is used instead of R_∞.

Behaviour at intermediate frequencies

In the two extremes of zero or infinite frequency, the circuit of the mathematical model behaved as a pure resistance. I.e., in neither case did we have to include the capacitance in the calculations.

The situation changes when we look at medium-size frequencies, $0 < f < \infty$. In this intermediate range, the circuit responds as an impedance Z with a phase difference. This impedance can be depicted as a length with an angle, as described in the previous chapter. The size of the impedance is somewhere between the resistances found in the two extremes: $R_\infty < Z < R_0$.

It can be shown that for given values of R_E, R_I, and C_m, the points will lie on the arc of a circle. The mathematical proof of this is however considered beyond the scope of these notes.

The frequency that corresponds to the upper point of the circle is called the *resonance frequency*, and is denoted f_c. Calculations on the circle give this formula for the resonance frequency:

$$f_c = \frac{1}{2\pi \cdot C_m \cdot (R_E + R_I)} \tag{7}$$

Summary concerning the Xitron Hydra 4200 device

Here follows a summary of the quantities used in the electrical/mathematical model used by the Xitron Hydra 4200. In parenthesis are shown the names used for these quantities in the printed reports from the device.

R_E (Re): The electrical resistance originating from the extra-cellular fluid with the given distribution of this fluid in the body. $R_0 = R_E$.

For given body dimensions, a *low* amount of extra-cellular fluid will result in a *high* value of R_E: The electric current runs in the fluid, and a narrow space gives a large resistance.

R_I (Ri) The electrical resistance originating from the intra-cellular fluid with the given distribution in the body. Since it is impossible for the current to reach the intra-cellular fluid without passing the extra-cellular fluid, R_I cannot be measured directly. But very high frequency electric current will pass the cell membranes unhindered, and (according to the mathematical/electrical model) a measurement of the total electrical resistance of the body at infinite frequency will give a result R_∞ which corresponds to R_E and R_I being wired in parallel.

Given measurements of R_∞ and $R_E = R_0$, the value of R_I can be calculated by rearranging equation (5):

$$R_I = \frac{R_E \cdot R_\infty}{R_E - R_\infty} \tag{8}$$

With the same of reasoning as for R_E: For given body dimensions, a *low* amount of intra-cellular fluid will result in a *high* value of R_I.

C_m (Cm) Electrical capacitance of the cell membranes. As the capacitance of biological tissue arises from the cell membranes, it may be expected that the more cells being measured, the larger the value of C_m will be.

α (Alpha) Parameter in the mathematical theory, which influences the position of centre of the circle. A value $\alpha = 1$ would place the centre on the horizontal axis. When α is less than 1, the centre of the circle is below the horizontal axis. Typically, $0.6 < \alpha < 0.7$ in body measurements.

T_d (Td) Delay of the electrical signal from running through the body. This parameter corrects for the fact that the electric signal takes a few nanoseconds to pass be body. In practice, this delay affects only the measurements at the highest frequencies. A value $T_d = 0$ corresponds to no correction.

f_c (Fc) Resonance frequency. f_c is the frequency in kHz where the effect of C_m is maximal, corresponding to the top of the circle arc.

Excluding the effect of the time delay, the impedance is mathematically described as a complex number with the value:

$$Z = \frac{R_E}{R_E + R_I} \cdot \left(R_I + \frac{R_E}{1 + [j \cdot \omega \cdot C_m \cdot (R_E + R_I)]^\alpha} \right)$$

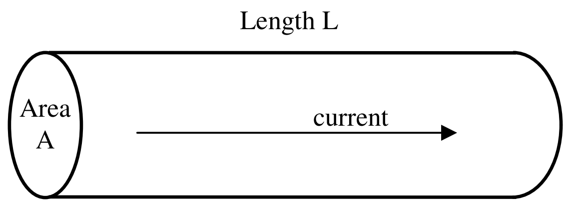
This is the equation that mathematically gives the above-mentioned circle arc. The equation is shown here merely for completeness, and will not be further used in these notes.

(For an equation that includes the time delay, T_d, see the Xitron Manual, Appendix A).

Chapter 4: Resistivity

Definition of resistivity

To connect the results from the BIS measurements with the amounts of body fluid, we have to understand what determines the value of the electrical resistance.



When electric current flows through a wire (or for instance an arm) made of homogeneous material, the resistance depends on three factors:

- The length of the wire, L. The *longer* the wire, the *larger* the resistance.
- The cross-sectional area of the wire. The thicker the wire, the easier current can flow through it, i.e., the *larger* the area, the *smaller* the resistance.
- The material which makes up the wire. For instance, copper wires have less resistance than iron wires of the same dimensions.

We see that to design a wire with a small resistance, we should make it short and thick and use a material that lets the current flow easily. Whereas a long, thin wire will give a larger resistance, especially if it is made of a material less efficient at letting currents flow.

We may express this as an equation:

$$R = \rho \cdot \frac{L}{A} \qquad (9)$$

where ρ (Greek letter: rho) is called the ***resistivity*** of the material. The higher the resistivity, the higher the electrical resistance will be.

Examples of resistivity

From the equation, we see that ρ must be given in units of resistance · area / length. Several possible units exits, for instance

$$[\rho] = \Omega \cdot m^2 / m = \Omega \cdot m$$

or

$$[\rho] = \Omega \cdot cm^2 / cm = \Omega \cdot cm \qquad (1\ \Omega \cdot cm = 0.01\ \Omega \cdot m)$$

or

$$[\rho] = \Omega \cdot mm^2 / m \qquad (1\ \Omega \cdot mm^2 / m = 10^{-6}\ \Omega \cdot m)$$

We may choose our units freely, as long as we remember that the numbers will depend on our choice of unit. The following table shows examples of resistivities.

Table 1. Resistivities for selected materials

Material	*Resistivity ρ*
Lead	$0.192 \cdot 10^{-6}\ \Omega \cdot m = 0.192 \cdot 10^{-4}\ \Omega \cdot cm$
Iron	$0.089 \cdot 10^{-6}\ \Omega \cdot m = 0.089 \cdot 10^{-4}\ \Omega \cdot cm$
Copper	$0.016 \cdot 10^{-6}\ \Omega \cdot m = 0.016 \cdot 10^{-4}\ \Omega \cdot cm$
Silver	$0.015 \cdot 10^{-6}\ \Omega \cdot m = 0.015 \cdot 10^{-4}\ \Omega \cdot cm$
Cell fluids in the human body	in the order of $\Omega \cdot m = 100\ \Omega \cdot cm$

Silver has the lowest resistivity in the table, being even better than copper for conducting electric current (but not so much better that our household wires should be made of silver).

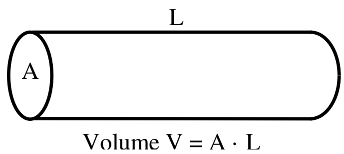
Water and body fluids are also able to conduct electric currents, although they are very much inferior to metals in this respect. I.e., water and body fluids has far larger resistivity than the metals. For these fluids, the precise value of the resistivity depends strongly on the concentrations of ions in the fluid.

Rewritten equation and approximation for human geometry

Our reason for looking at resistivity is that we want to do calculations on the *volume* of a fluid. However, equation (9) uses the length and the area. If we know the length, we can use the relation

volume = area · length

$V = A \cdot L$



and rewrite equation (9) as follows:

$$R = \rho \cdot \frac{L}{A} = \rho \cdot \frac{L \cdot L}{A \cdot L} = \rho \cdot \frac{L^2}{V} \tag{10}$$

This equation is just as valid as the original equation and allows us to calculate the volume of the fluid in which the current flows:

$$R = \rho \cdot \frac{L^2}{V} \quad \Leftrightarrow \quad V = \rho \cdot \frac{L^2}{R} \tag{11}$$

So far, so good. We should, however, be aware that equations (9)-(11) were derived under the assumption that *the cross-sectional area is constant throughout the length* of the object we are working with. The object (the “wire”) may be circular, oval, square, crescent-shaped, or even have a hole in the middle, as long as the cross-sectional area is constant.

Unfortunately, we wish to apply the equations to human beings, who do not satisfy this assumption. Individual body parts, e.g. an upper arm or a thigh, may be approximately cylindrical. But the cross-sectional area of a body on the route from wrist to ankle varies far too much to be considered constant.

To overcome this problem, De Lorenzo *et al* (1997) add a correction factor, K_B, to equation (11):

$$R = K_B \cdot \rho \cdot \frac{L^2}{V} \tag{12}$$

⇕

$$V = K_B \cdot \rho \cdot \frac{L^2}{R} \tag{13}$$

The derivation of K_B is made in Appendix C of the paper by De Lorenzo *et al* (and repeated in Appendix B of the Xitron Manual). The value of K_B depends only on the geometrical shape of the body. Using anthropomorphic phantoms, De Lorenzo *et al* find the value (p. 1548, right column)

$$K_B = 4.3 \tag{14}$$

This value of K_B has been calculated for

- Adult standard persons
- L = the height of the person (also called H)
- Measurement through arm, trunk, and leg, i.e. from wrist to ankle

A person of different shape than "standard" shape possibly ought to be assigned another value for this constant. Since most of the resistance R comes from the thinnest part of the body, the measured resistance (impedance) will depend more on the thickness and length of arms and legs than size and shape of the trunk. This is in line with the Xitron Manual p. 40:

> *Geometry and Fluid Distribution*
>
> A wrist-ankle measurement assumes the body is one perfect cylinder when if fact the body is comprised of five imperfect cylinders (arms, legs and trunk), with the arms an legs contributing 90% to the measurement. As discovered (De Lorenzo 1997), fluid volume tends to be evenly distributed in healthy subjects, thus, a wrist-ankle measurement can provide accurate and useful information (Jaffrin 1996). However, there are conditions and populations where assumed even fluid distribution is invalid. The relationship between a single side wrist-ankle measurement and total ECF and ICF volume is dominated by the arm (typically 250Ω) and leg (typically 200Ω), with the trunk contributing very little to the total impedance measurement (typically 20Ω). As such, any variable causing localized changes in the limbs or trunk can lead to an inaccurate prediction of ECF and ICF volume. Under such conditions, a limb measurement may be useful.

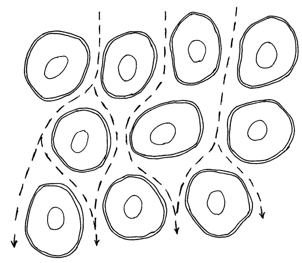
Note that a person who is *evenly* thicker or thinner than the "standard person" will have an *unchanged* value of K_B. E.g., a fat person may have 50% enlarged cross-sectional areas compared to the "standard person", but K_B will be unchanged as long as *all* areas are 50% larger. Likewise, it does not change the value of K_B if the person is shorter or longer than the "standard person", as long as the ratios arm/trunk/leg are unchanged. Thus $K_B = 4.3$ will be a correct value for most persons.

But for some individuals, the true value of K_B will be different than the standard value. According to equation (13), a larger resistance corresponds to a smaller volume if the person height is unchanged. However, a person with long and thin arms *compared to trunk and legs* will have an elevated resistance R, while overall body volume is not much affected by the size of the arms. Thus, the correct K_B for this person will be larger than the standard value of 4.3. Likewise, a person with a large trunk – compared to the arms and legs – is expected to have a larger K_B, because volume V is larger, while resistance R is affected only relatively little by the size of the trunk. Note than in both cases (arms and trunk), the problem is that R and V measured for the whole body are affected individually, not together. Since the legs have a size and shape which will influence *both* R and V of the whole body, "non-standard" legs are not expected to influence K_B as much as "non-standard" arms or trunk.

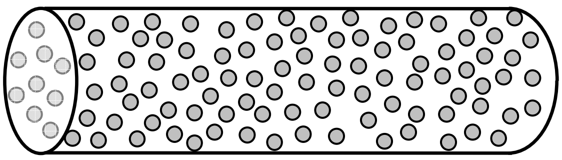
The value $K_B = 4.3$ for whole-body measurements seems to be hard-wired in the Xitron Hydra 4200, but readers prepared to calculate individualized values of K_B can use the equations from the following chapters to calculate fluid volumes (V_{ECF}, V_{ICF}, V_{TBF}) with the individualized K_B.

Hanai mixture theory

So far we have discussed resistivity for homogeneous materials. But tissue is not a homogeneous material. Tissue is a mixture of cells and extra-cellular fluid. For zero frequency ($f = 0$), we have seen that current cannot enter the cells and must flow in the extra-cellular fluid only.



The extra-cellular fluid (ECF) has in itself resistivity ρ_{ECF}. However, the current cannot take the direct route through the fluid but must flow around the cells. For this reason the resistivity of the tissue takes a higher value than ρ_{ECF}.



$$C = \frac{\text{cell volume}}{\text{total volume}}$$

The problem of calculating the resistivity of a conducting fluid mixed with non-conducting spheres has been analyzed by the Japanese researcher Tetsuya Hanai (Hanai 1968). According to his results, if the cells take up the part C of the total volume, then the overall resistivity is:[2]

$$\rho = \frac{\rho_{ECF}}{(1-C)^{3/2}} \qquad \text{(at zero frequency)} \tag{15}$$

Example: If the cells take up 70% of the total volume, then the overall resistivity for low-frequency current is:

$$\rho = \frac{\rho_{ECF}}{(1-0.7)^{3/2}} = \frac{\rho_{ECF}}{(0.3)^{3/2}} = 6.1 \cdot \rho_{ECF}$$

If the cells take up 50% of the volume then resistivity for low-frequency current is

$$\rho = \frac{\rho_{ECF}}{(1-0.5)^{3/2}} = \frac{\rho_{ECF}}{(0.5)^{3/2}} = 2.8 \cdot \rho_{ECF}$$

And if cells take up only 30% of the volume then the resistivity for low-frequency current is

$$\rho = \frac{\rho_{ECF}}{(1-0.3)^{3/2}} = \frac{\rho_{ECF}}{(0.7)^{3/2}} = 1.7 \cdot \rho_{ECF}$$

As expected, the resistivity ρ (and thus the resistance R) is higher for a mixture of fluid and cells than for unmixed extra-cellular fluid. But the less the percentage of volume taken up by cells, the closer ρ is to ρ_{ECF}.

[2] Equation (15) follows from Hanai equation (228) setting $\kappa_l = 1/\rho$ and $\kappa_m = 1/\rho_{ECF}$ and $\Phi = C$.

Chapter 5: Calculation of extra-cellular fluid, ECF

A note on wording

In the literature, the fluid between the cells is often denoted ECW for Extra-Cellular Water. However, the water contains various ions: Na^+, Ca^{2+}, K^+, Cl^-, etc. These ions are important, both for the physiological workings of the cells, and for the electrical resistivity of the fluid. So the fluid is more than just water. Therefore, this text (along with e.g. Matthie 2008) uses the phrase Extra-Cellular Fluid, denoted ECF.

Summary

In this chapter formulas are derived for calculating the volume of extra-cellular fluid, V_{ECF}, on the basis of the bioelectric impedance measurements.

The result of the derivation is that V_{ECF} in liters can be calculated as

$$V_{ECF} = k_{ECF} \cdot \left(\frac{H^2 \cdot \sqrt{W}}{R_E} \right)^{2/3} \qquad \text{(16) = (31) below}$$

where

H = height of the person in cm

W = weight of the person in kg

R_E = measured extra-cellular resistance in Ω

and k_{ECF} has a constant value for a given group, e.g., one value for each sex. The value of this constant can be calculated by the equation

$$k_{ECF} = \frac{1}{100} \cdot \left(\frac{K_B{}^2 \cdot \rho_{ECF}{}^2}{D_b} \right)^{1/3} \qquad \text{(17) = (30) below}$$

where

K_B is a geometry factor described in the section *Rewritten equation and approximation for human geometry* in the preceding chapter

ρ_{ECF} is the resistivity of the extra-cellular fluid, in unit Ω·cm

D_b is the overall body density in kg/liter or g/cm^3

The Xitron Hydra 4200 device comes with default values for these constants, see Table 2. It is possible to adjust the values for ρ_{ECF} in the device, which automatically gives new values of k_{ECF}. However, a change of ρ_{ECF} will change the results of all volume calculations, so the default value should only be changed by persons knowing what they are doing.

Table 2. Default and built-in factors used by the Xitron Hydra 4200 device[3]

	k_{ECF}	ρ_{ECF}	K_B	D_b
Female	0.299	39.0 Ω·cm	4.3	1.05 g/cm³ = 1.05 kg/liter
Male	0.307	40.5 Ω·cm		

Example: Calculation of k_{ECF}

Using the numbers from Table 2, we may calculate the values of k_{ECF} used by Xitron Hydra 4200.

Female:

$$k_{ECF} = \frac{1}{100} \cdot \left(\frac{4.3^2 \cdot 39.0^2}{1.05} \right)^{1/3} = 0.299 \qquad (18)$$

Male:

$$k_{ECF} = \frac{1}{100} \cdot \left(\frac{4.3^2 \cdot 40.5^2}{1.05} \right)^{1/3} = 0.307 \qquad (19)$$

Summary of assumptions

At very low frequencies the electric current flows only in the extra-cellular fluid. This fluid has in itself resistivity ρ_{ECF}, but the presence of non-conducting cells in the fluid increases the overall resistivity to

$$\rho = \frac{\rho_{ECF}}{(1 - C)^{3/2}} \qquad (20) = (15) \text{ above}$$

where C is the share (percentage) of volume taken up by the cells.

The measured resistance is $R = R_E$ because current flows only in the extra-cellular fluid. According to equation (12) the relation between resistance R_E and resistivity ρ is

$$R_E = K_B \cdot \rho \cdot \frac{H^2}{V_{tot}} \qquad (21)$$

for a person with height H and total body volume V_{tot}.

It has been assumed that fluid is evenly distributed throughout the body. For this reason these equations may not be applicable to persons with oedema.

Derivation

If C is the share of cells then (1 – C) is the share of extra-cellular fluid:

---

[3] The values of ρ_{ECF} are found in the Xitron manual p. 22. The values $K_B = 4.3$ and $D_b = 1.05$ g/cm³ = 1.05 kg/liter are found in De Lorenzo (1997) p. 1548, right column; these values are not found in the manual, but they have been controlled by the author by using measured values of R_E to calculate V_{ECF} and compare with the result reported by the Xitron Hydra 4200. The values for k_{ECF} reported in the table has been calculated from the other values – cf. the example following the table.

$$1 - C = \frac{V_{ECF}}{V_{tot}} \tag{22}$$

This may be inserted in the formula (20) for resistivity:

$$\rho = \frac{\rho_{ECF}}{(1-C)^{3/2}} = \frac{\rho_{ECF}}{\left(\frac{V_{ECF}}{V_{tot}}\right)^{3/2}} = \rho_{ECF} \cdot \left(\frac{V_{tot}}{V_{ECF}}\right)^{3/2} \tag{23}$$

Inserting ρ in the equation for R_E (21) we get

$$R_E = K_B \cdot \rho_{ECF} \cdot \left(\frac{V_{tot}}{V_{ECF}}\right)^{3/2} \cdot \frac{H^2}{V_{tot}} \tag{24}$$

$$\Updownarrow$$

$$R_E = K_B \cdot \rho_{ECF} \cdot \frac{(V_{tot})^{1/2}}{(V_{ECF})^{3/2}} \cdot H^2$$

$$\Updownarrow$$

$$(V_{ECF})^{3/2} = \frac{K_B \cdot \rho_{ECF} \cdot \sqrt{V_{tot}} \cdot H^2}{R_E} \tag{25}$$

Measuring body volume V_{tot} for a person is cumbersome, while the body weight W is easily determined. Overall body density D_b can be assumed to be (almost) person independent.[4]

$$D_b = \frac{W}{V_{tot}} \quad \Leftrightarrow \quad V_{tot} = \frac{W}{D_b} \tag{26}$$

We continue the derivation:

$$(V_{ECF})^{3/2} = \frac{K_B \cdot \rho_{ECF} \cdot \sqrt{\frac{W}{D_b}} \cdot H^2}{R_E}$$

$$\Updownarrow$$

$$(V_{ECF})^{3/2} = \frac{K_B \cdot \rho_{ECF}}{\sqrt{D_b}} \cdot \frac{H^2 \cdot \sqrt{W}}{R_E}$$

$$\Updownarrow$$

$$V_{ECF} = \left(\frac{K_B \cdot \rho_{ECF}}{\sqrt{D_b}}\right)^{2/3} \cdot \left(\frac{H^2 \cdot \sqrt{W}}{R_E}\right)^{2/3} \tag{27}$$

$$\Updownarrow$$

$$V_{ECF} = \left(\frac{{K_B}^2 \cdot {\rho_{ECF}}^2}{D_b}\right)^{1/3} \cdot \left(\frac{H^2 \cdot \sqrt{W}}{R_E}\right)^{2/3} \qquad \text{(in SI units)} \tag{28}$$

[4] De Lorenzo *et al* (1997) has a short discussion on variation of D_b on page 1547, in the upper half of the right column.

Now we have a formula for calculating V_{ECF}. The first parenthesis contains the quantities that can be assumed to be constant within large groups of persons (the Xitron Hydra 4200 assumes one value for all females and one value for all males). The second parenthesis contains the quantities we must measure for the individual person: Height, weight, and resistance R_E.

Formula (28) can be used without further ado if we use the SI system of units. However, the SI unit for volume is m^3. More practical units could be volume in liters, height in cm, and resistivity in the unit Ω·cm. Table 3 shows a summary of all quantities and their units.

Table 3. Quantities and units used in the formula for V_{ECF}

Quantity	*SI unit*	*unit used*
V_{ECF} = volume of extra-cellular fluid	m^3	liter = 10^{-3} m^3
W = body weight	kg	kg
H = height	m	cm = 10^{-2} m
R_E = measured extra-cellular resistance	Ω	Ω
K_B = correction factor for body geometry	(none)	(none)
ρ_{ECF} = resistivity of extra-cellular fluid	Ω·m	Ω·cm = 10^{-2} Ω·m
D_b = overall body density	kg/m^3	g/cm^3 = kg/liter = 10^3 kg/m^3

Using these units the formula becomes:[5]

$$V_{ECF} = \frac{1}{100} \cdot \left(\frac{K_B{}^2 \cdot \rho_{ECF}{}^2}{D_b} \right)^{1/3} \cdot \left(\frac{H^2 \cdot \sqrt{W}}{R_E} \right)^{2/3} \tag{29}$$

We do not need calculate the constant quantities more than once, so we may collect them in a single factor

$$k_{ECF} = \frac{1}{100} \cdot \left(\frac{K_B{}^2 \cdot \rho_{ECF}{}^2}{D_b} \right)^{1/3} \tag{30}$$

giving us the more practical formula

$$V_{ECF} = k_{ECF} \cdot \left(\frac{H^2 \cdot \sqrt{W}}{R_E} \right)^{2/3} \tag{31}$$

Values for k_{ECF} can be found in Table 2.

[5] De Lorenzo (1997) and the Xitron Manual has the factor 1/1000, which is wrong with the stated units. In Matthie (2005) this error has been corrected by changing the unit for D_b to kg/cm^3 (the unit for D_b is stated in the Appendix of the paper). With this somewhat unusual choice of unit for density, $D_b = 0.00105$ kg/cm^3, the factor 1/1000 is correct.

Chapter 6: Calculation of total body fluid, TBF, and intra-cellular fluid, ICF

Summary[6]

The body contains fluid both inside and outside the cells. Together, these are called Total Body Fluid, TBF. (In the literature, TBF is also known as TBW, total body water.) The volume of total body fluid can be calculated with the formula

$$V_{TBF} = V_{ECF} \cdot \left(\frac{\rho_{TBF}}{\rho_{ECF}} \cdot \frac{R_E}{R_\infty} \right)^{2/3} \qquad (32) = (42) \text{ below}$$

which may also be written

$$V_{TBF} = V_{ECF} \cdot \left(\frac{\rho_{TBF}}{\rho_{ECF}} \cdot \frac{R_E + R_I}{R_I} \right)^{2/3} \qquad (33) = (43) \text{ below}$$

In these formulas ρ_{TBF} is the resistivity of the mixture of intra- and extra-cellular fluid. The value of ρ_{TBF} will vary from person to person. For a given individual, the value can be calculated with the formula

$$\rho_{TBF} = \rho_{ICF} - (\rho_{ICF} - \rho_{ECF}) \cdot \left(\frac{R_\infty}{R_E} \right)^{2/3} \qquad (34) = (47) \text{ below}$$

which may also be written

$$\rho_{TBF} = \rho_{ICF} - (\rho_{ICF} - \rho_{ECF}) \cdot \left(\frac{R_I}{R_E + R_I} \right)^{2/3} \qquad (35) = (48) \text{ below}$$

Total body fluid is the sum of extra- and intra-cellular fluid. Thus, the volume of intra-cellular fluid (ICF) is equal to the difference between the volume of total body fluid (TBF) and extra-cellular fluid (ECF):

$$V_{ICF} = V_{TBF} - V_{ECF} \qquad (36)$$

It is of course possible to combine the formulas for V_{TBF} and V_{ECF} into a formula for V_{ICF}. See equation (57) on page 40.

Table 4. Default values for extra- and intra-cellular resistivity used by the Xitron Hydra 4200[7]

	ρ_{ECF}	ρ_{ICF}
Female	39.0 Ω·cm	264.9 Ω·cm
Male	40.5 Ω·cm	273.9 Ω·cm

[6] This chapter follows Matthie (2005). Note that Matthie presents *changed* equations for calculation of V_{TBF} and V_{ICF} compared to De Lorenzo *et al* (1997). The Xitron Manual is not updated on this point: In the manual's Appendix B an *obsolete* equation (B10) is presented for calculation of V_{ICF}. The Xitron Hydra 4200 device itself is up to date and gives results following the equations from Matthie, i.e., the equations presented and derived in this chapter.

[7] Xitron Manual p. 22

Example: Calculation of ρ_{TBF}

Measurement on a female subject gives the results

$R_E = 695.1\ \Omega$

$R_I = 1449.3\ \Omega$

As the subject is female, we use the resistivites from the first row of Table 4. We apply the numbers to equation (35) and thereby calculate the overall resistivity for fluids in this person:

$$\begin{aligned}\rho_{TBF} &= 264.9 - (264.9 - 39.0)\cdot\left(\frac{1449.3}{695.1 + 1449.3}\right)^{2/3} \\ &= 264.9 - 225.9\cdot(0.6759)^{2/3} \\ &= 90.9\ \ \Omega\cdot\text{cm}\end{aligned}$$

Note that the number 90.9 Ω·cm is somewhere between the numbers for ρ_{ECF} and ρ_{ICF}. It makes sense to find that a mixture of two components has a value somewhere in the middle of the individual component values.

R_E & R_I versus R_0 & R_∞

The electrical model for BIS was presented in Chapter 3. The model contains two resistances, R_E and R_I, for extra-cellular and intra-cellular fluid. However, a natural focus in the calculations will turn out to be the electrical behaviour at very low (zero) and very high (infinite) frequencies, $f = 0$ and $f = \infty$. This makes the resistances R_0 and R_∞ appear naturally in the equations.

There is no "correct" answer on which resistances to use. Some authors writing on BIS prefer equations with R_E and R_I. Others prefer R_0 (R_E) and R_∞ (also written R_{INF}). In this text, R_E and R_∞ are used in the derivations, but the final equations are also shown with R_E and R_I.

We have:

$$R_0 = R_E \qquad \text{(repetition of (4))}$$

$$R_\infty = \frac{R_E \cdot R_I}{R_E + R_I} \qquad \text{(repetition of (6))}$$

These relationships reflect that at $f = 0$ the current runs only in extra-cellular fluid (so R_0 is the same as R_E), whereas at $f = \infty$ the current runs in a combination of extra- and intra-cellular fluid (so R_∞ is a combination of R_E and R_I).

It will turn out that in the final equations, R_∞ appears only in combination with R_E, either as R_∞/R_E or as R_E/R_∞. Rewriting these ratios we get

$$\frac{R_\infty}{R_E} = \frac{\frac{R_E \cdot R_I}{R_E + R_I}}{R_E} = \frac{R_I}{R_E + R_I} \qquad (37)$$

and conversely:

$$\frac{R_E}{R_\infty} = \frac{R_E + R_I}{R_I} \tag{38}$$

Using these ratios, equations with R_E and R_∞ can be rewritten as equations with R_E and R_I.

Derivation, step 1: Setting up the equations

At very low frequencies (f = 0), where the current flows in the extra-cellular fluid only, we found the following relation between resistance and volume:

$$R_E = K_B \cdot \rho_{ECF} \cdot \left(\frac{V_{tot}}{V_{ECF}}\right)^{3/2} \cdot \frac{H^2}{V_{tot}} \qquad \text{(repetition of (24))}$$

This could be rewritten as:

$$V_{ECF} = \left(\frac{K_B \cdot \rho_{ECF}}{\sqrt{D_b}}\right)^{2/3} \cdot \left(\frac{H^2 \cdot \sqrt{W}}{R_E}\right)^{2/3} \qquad \text{(repetition of (27))}$$

At very high frequencies (f = ∞) the current flows in the total body fluid, TBF = ECF + ICF. In a way, this is very similar to the situation at very low frequencies: We have again an electric current flowing in a conducting fluid mixed with non-conducting material. At the low frequencies, the conducting material was ECF and the non-conducting material was the cells. At high frequencies, the conducting material is TBF and the non-conducting material is the non-fluid parts of the cells.

So we may simply replace R_E with R_∞ and ECF with TBF in the above equations:

$$R_\infty = K_B \cdot \rho_{TBF} \cdot \left(\frac{V_{tot}}{V_{TBF}}\right)^{3/2} \cdot \frac{H^2}{V_{tot}} \tag{39}$$

and

$$V_{TBF} = \left(\frac{K_B \cdot \rho_{TBF}}{\sqrt{D_b}}\right)^{2/3} \cdot \left(\frac{H^2 \cdot \sqrt{W}}{R_\infty}\right)^{2/3} \tag{40}$$

For ECF, we simplified the formula by introducing a constant, k_{ECF}. Unfortunately, we cannot do the same for TBF. The "mixed" resistivity ρ_{TBF} depends on the relative amounts of ECF and ICF, so it varies from person to person.

But we can do something else. Many factors are the same in the formulas for V_{TBF} and V_{ECF}, so dividing the two formulas, these factors are cancelled out:

$$\frac{V_{TBF}}{V_{ECF}} = \frac{\left(\frac{K_B \cdot \rho_{TBF}}{\sqrt{D_b}}\right)^{2/3} \cdot \left(\frac{H^2 \cdot \sqrt{W}}{R_\infty}\right)^{2/3}}{\left(\frac{K_B \cdot \rho_{ECF}}{\sqrt{D_b}}\right)^{2/3} \cdot \left(\frac{H^2 \cdot \sqrt{W}}{R_E}\right)^{2/3}}$$

⇕

(41)

$$\frac{V_{TBF}}{V_{ECF}} = \left(\frac{\rho_{TBF}}{\rho_{ECF}} \cdot \frac{R_E}{R_\infty} \right)^{2/3}$$

(corresponding to Matthie (2005) equation (2)).

Derivation, step 2: Formulas (32) and (33) for V_{TBF}

With equation (41) we are close to having a formula for V_{TBF}. Multiplying with V_{ECF} on both sides of the equality sign we get the first version:

$$V_{TBF} = V_{ECF} \cdot \left(\frac{\rho_{TBF}}{\rho_{ECF}} \cdot \frac{R_E}{R_\infty} \right)^{2/3} \tag{42}$$

Inserting R_E/R_∞ from equation (38) we get the second version of the same equation:

$$V_{TBF} = V_{ECF} \cdot \left(\frac{\rho_{TBF}}{\rho_{ECF}} \cdot \frac{R_E + R_I}{R_I} \right)^{2/3} \tag{43}$$

So far, so good. But to use the result, we also need to know how to calculate the resistivity for TBF, which is a mixture of ECF and ICF.

Derivation, step 3: Formula (34) and (35) for the "mixed" resistivity ρ_{TBF}

(This paragraph shows the most important intermediary results, but parts of the workings have been skipped. Readers, who wish to see all details of the derivation, are invited to find pen and paper and fill in the details themselves. Advice: Keep a close eye on your indices $_{ECF}$, $_{ICF}$, and $_{TBF}$. A link to Matthie's paper can be found in the reference list on page 43 in these notes.)

Using Hanai equation (222), Matthie (2005) reaches his equation (9), which can be written

$$\frac{V_{ECF}}{V_{TBF}} = \left(\frac{\rho_{ICF} - \rho_{TBF}}{\rho_{ICF} - \rho_{ECF}} \right) \cdot \left(\frac{\rho_{ECF}}{\rho_{TBF}} \right)^{2/3} \tag{44}$$

This equation is related to one of our earlier equations. Equation (44) gives V_{ECF}/V_{TBF}, while equation (41) gave the reciprocal value, V_{TBF}/V_{ECF}. That is, (44) = 1/(41):

$$\left(\frac{\rho_{ICF} - \rho_{TBF}}{\rho_{ICF} - \rho_{ECF}} \right) \cdot \left(\frac{\rho_{ECF}}{\rho_{TBF}} \right)^{2/3} = \frac{V_{ECF}}{V_{TBF}} = \left(\frac{\rho_{ECF}}{\rho_{TBF}} \cdot \frac{R_\infty}{R_E} \right)^{2/3} \tag{45}$$

$\Updownarrow$

$$\frac{\rho_{ICF} - \rho_{TBF}}{\rho_{ICF} - \rho_{ECF}} = \left(\frac{R_\infty}{R_E} \right)^{2/3} \tag{46}$$

$\Updownarrow$

$$\rho_{TBF} = \rho_{ICF} - (\rho_{ICF} - \rho_{ECF}) \cdot \left(\frac{R_\infty}{R_E} \right)^{2/3} \tag{47}$$

This is the formula presented in the beginning of this chapter, in its first version. The second version we find by inserting R_∞/R_E from equation (37):

$$\rho_{TBF} = \rho_{ICF} - (\rho_{ICF} - \rho_{ECF}) \cdot \left(\frac{R_I}{R_E + R_I} \right)^{2/3} \tag{48}$$

This concludes the derivation.

Chapter 7: The impact of deviations in ρ_{ECF} and ρ_{ICF}

Since we generally do not have exact values for ρ_{ECF} and ρ_{ICF}, it is important to know how deviations in these parameters affect our calculations of fluid volumes.

The Xitron Hydra 4200 allows the default values of ρ_{ECF} and ρ_{ICF} to be changed. The analysis here can also be used to predict the consequences of such change.

Impact on the calculation of V_{ECF}

Calculation of V_{ECF} is independent on the properties of *intra*-cellular fluid, ICF. Thus, deviations in ρ_{ICF} have no effect on V_{ECF}. We need only consider ρ_{ECF}.

From equations (16)-(17) we can analyse the impact of deviations in extra-cellular resistivity, ρ_{ECF}. V_{ECF} is proportional to $(\rho_{ECF})^{2/3}$. As a rule of thumb, this means that every 3% deviation in the value of ρ_{ECF} corresponds to a 2% deviation in V_{ECF}.

Example: Difference by 3%:

$(1.03)^{2/3} = 1.0199 \approx 1.02$ i.e., 3% increase of ρ_{ECF} corresponds to 2% increase in V_{ECF}

$(0.97)^{2/3} = 0.9799 \approx 0.98$ i.e., 3% decrease of ρ_{ECF} corresponds to 2% decrease in V_{ECF}

Example: Difference by 30%:

$(1.30)^{2/3} = 1.19$ (30% increase corresponds to about 20% increase)

$(0.70)^{2/3} = 0.79$ (30% decrease corresponds to about 20% decrease)

Instead of starting from ρ_{ECF}, we may start from the constant k_{ECF} (see equation (16)). V_{ECF} is proportional to k_{ECF}, so the rule is simple: 1% deviation in k_{ECF} corresponds to 1% deviation V_{ECF}.

Impact on the calculation of V_{TBF}

Total Body Fluid can be calculated with equation (32) or (33).

$$V_{TBF} = V_{ECF} \cdot \left(\frac{\rho_{TBF}}{\rho_{ECF}} \cdot \frac{R_E + R_I}{R_I} \right)^{2/3} \qquad \text{(repetition of (33))}$$

First, we see that any difference from the calculation of V_{ECF} gives a proportional difference in the calculation of V_{TBF}.

Second, the calculation of the "mixed" resistivity, ρ_{TBF}, depends on the intra-cellular resistivity, ρ_{ICF}. It can be shown (see the end of the chapter) that the ratio ρ_{TBF}/ρ_{ECF} in the parenthesis above can be calculated from the ratio ρ_{ICF}/ρ_{ECF}. For this reason, the following constant is introduced:[8]

$$k_\rho = \frac{\rho_{ICF}}{\rho_{ECF}} \qquad (49)$$

Using this constant, the ratio of "mixed" resistivity to extra-cellular resistivity can be written

[8] Following De Lorenzo *et al* (1997), equation (B5).

$$\frac{\rho_{TBF}}{\rho_{ECF}} = k_\rho - (k_\rho - 1) \cdot \left(\frac{R_I}{R_E + R_I} \right)^{2/3} \qquad \text{(50) =(52) below}$$

What is the message of this equation? Along with formula (33) it tells us this: If we know V_{ECF} and the value of k_ρ then we can calculate V_{TBF}. We do not need know ρ_{ICF} itself.

Said another way: To calculate V_{ECF}, we need the measured R_E and R_I along with the value of k_{ECF} (or ρ_{ECF}). To calculate V_{TBF}, we further need the value of $k_\rho = \rho_{ICF}/\rho_{ECF}$. Thus, the value of ρ_{ICF} is not in itself important, only the ratio ρ_{ICF}/ρ_{ECF} is important.

Example

A subject is measured with the Xitron Hydra 4200 device. For one reason or another, this subject has higher resistivities than the default values: As it happens, the default values for both extra-cellular and intra-cellular resistivity are 15% too low. We (the experimenters) do not know this, and simply write down the numbers reported by the device. What are the errors on our values for V_{ECF}, V_{TBF}, and V_{ICF}?

Note first that the resistances are measured on the individual subject, so R_∞, R_E, and R_I are correct. The errors come only from ρ_{ECF} and ρ_{ICF}.

ECF: A 15% deviation on ρ_{ECF} corresponds to approximately ⅔ · 15% = 10% deviation on V_{ECF} (and k_{ECF}). I.e., if calculations are made using a *15% too low value of* ρ_{ECF}, then calculated V_{ECF} *will be 10% too low*.

TBF: In this example, ρ_{ECF} and ρ_{ICF} deviate by the *same* percentage (15%). From the definition of k_ρ we see that *the value for* k_ρ *is correct*, even though the individual resistivities were not correct. Equation (50) tells us that when k_ρ is correct, then the ratio ρ_{TBF}/ρ_{ECF} is also correct. This means that the whole parenthesis in equation (33) has the correct value, so *the percentage error on* V_{TBF} *is equal to the percentage error on* V_{ECF}. In this example: Calculated V_{TBF} will be 10% too low.

ICF: See next paragraph.

Impact on the calculation of V_{ICF}

The volume of intra-cellular fluid is the difference between the volumes of total body fluid and extra-cellular fluid:

$$V_{ICF} = V_{TBF} - V_{ECF} \qquad \text{(repetition of (36))}$$

In the example above, V_{TBF} and V_{ECF} had the same percentage deviations from the true values: Both were 10% too low. In that case, their difference (V_{ICF}) will also be 10% too low. The reason for V_{TBF} to have the same percentage deviation as V_{ECF} was that the value of the constant k_ρ was correct. This leads us to the following rule:

General rule: If $k_\rho = \rho_{ICF}/\rho_{ECF}$ has the *correct value*, then *all* volumes (V_{ECF}, V_{TBF}, and V_{ICF}) will differ by the same percentage.

If, on the other hand, $k_\rho = \rho_{ICF}/\rho_{ECF}$ has an incorrect value then it is hard to say anything general about V_{ICF}.

The impact of deviations in K_B

The geometrical factor K_B (cf. the paragraph *Rewritten equation and approximation for human geometry* starting on page 17) appears in the equations along with ρ_{ECF}, but is otherwise independent of ρ_{ECF} or ρ_{ICF}. For this reason we find:

> A 3% deviation in K_B will result in a 2% deviation on all calculated fluid volumes.

The deviation in K_B is seen through its influence on the value of k_{ECF}.

Summary

V_{ECF}: Calculation of V_{ECF} depends on the value of k_{ECF} and through this dependence on the value of ρ_{ECF} (and K_B). A 3% deviation in ρ_{ECF} gives a 2% deviation in k_{ECF} and V_{ECF}.

V_{TBF} and V_{ICF}: Both of these volumes are calculated as being proportional to V_{ECF}. They further depend on the ratio k_ρ, which is defined in equation (49).

k_{ECF}: 1% deviation in k_{ECF} gives 1% deviation on all calculated volumes.

k_ρ: Deviations in k_ρ affects the calculation of V_{TBF} and V_{ICF}, but not V_{ECF}.

Table 5. Default values of k_{ECF} and k_ρ used in Xitron Hydra 4200 (based on Table 2 and Table 4)

	k_{ECF}	$k_\rho = \frac{\rho_{ICF}}{\rho_{ECF}}$
Female	0.299	6.79
Male	0.307	6.76

Derivation of equation (50)

The derivation is relatively simple. We start with equation (34) for ρ_{TBF} and divide by ρ_{ECF}:

$$\rho_{TBF} = \rho_{ICF} - (\rho_{ICF} - \rho_{ECF}) \cdot \left(\frac{R_\infty}{R_E}\right)^{2/3} \quad \text{(repetition of (34))}$$

$$\Updownarrow$$

$$\frac{\rho_{TBF}}{\rho_{ECF}} = \frac{\rho_{ICF}}{\rho_{ECF}} - \left(\frac{\rho_{ICF}}{\rho_{ECF}} - 1\right) \cdot \left(\frac{R_\infty}{R_E}\right)^{2/3}$$

$$\Updownarrow$$

$$\frac{\rho_{TBF}}{\rho_{ECF}} = k_\rho - (k_\rho - 1) \cdot \left(\frac{R_\infty}{R_E}\right)^{2/3} \quad (51)$$

where k_ρ was defined as

$$k_\rho = \frac{\rho_{ICF}}{\rho_{ECF}} \quad \text{(repetition of (49))}$$

Inserting R_∞/R_E from (37), equation (51) becomes

$$\frac{\rho_{TBF}}{\rho_{ECF}} = k_\rho - (k_\rho - 1)\cdot\left(\frac{R_I}{R_E + R_I}\right)^{2/3} \tag{52}$$

which is the form of the equation shown as (50).

Chapter 8: Calculation of BCM and FFM

Body Cell Mass (BCM)

The Xitron Hydra 4200 also estimates the total mass of body cells (BCM). According to the Xitron Manual page 43 the classic equation of Moore (1963, chapter 2, formula (15)) is used for the estimation:

$$BCM = \frac{ICW}{0.70} \tag{53}$$

Here ICW is the mass of the intra-cellular fluid. Assuming that density of fluid = density of water = 1 kg/liter, we find that ICW in kg = V_{ICF} in liters.

The formula is useful under two assumptions:

1. The average body cell contains 70% water (weight percent)
2. ICW (V_{ICF}) can be determined

This leads us to the following considerations on the accuracy of BCM determined with Xitron Hydra 4200:

1. It is not clear from Moore how accurate and precise the number 70% is. Thus one may ask: How exact is the number 70%, and to what extent individuals vary from the overall value?
2. Since $V_{ICF} = V_{TBF} - V_{ECF}$, the accuracy of V_{ICF} depends on the accuracy of V_{TBF} and V_{ECF}

All in all, the accuracy of BCM reported by the Xitron Hydra 4200 will *at best* be as good as the accuracy of the volume determinations.

Fat-Free Mass (FFM)

The Xitron Hydra 4200 further estimates the total fat-free mass (FFM) of the body. According to the Xitron Manual p. 43, the calculation assumes that fat-free tissue contains 73.2% water, again following Moore (1963, chapter 2, equation (18)).

According to the Xitron Manual equation B8 (manual p. 106), FFM is estimated as

$$FFM = (d_{ECF} \cdot V_{ECF}) + (d_{ICF} \cdot V_{ICF}) \tag{54}$$

where

$d_{ECF} = 1.106$ kg/liter

$d_{ICF} = 1.521$ kg/liter

are the densities of extra/intra-cellular ”water and its associated materials”. It is possible to change these constants in the device.

Since the calculation (54) is an estimate based on V_{ECF} and V_{ICF}, we must expect the accuracy of FFM to be less than the accuracy of V_{ECF} and V_{ICF}. This is in line with the Xitron Manual, where the calculation of FFM and %Fat are called “second order” (manual p. 43).

%Fat

Fat and %fat are calculated with the simple formulas

$$\text{Fat} = \text{W} - \text{FFM} \tag{55}$$

$$\%\text{Fat} = \frac{\text{Fat}}{\text{W}} \cdot 100\% \tag{56}$$

where W is the body weight of the person (as typed on the device interface by the user), and FFM is the estimated Fat Free Mass of formula (54).

Impact on FFM and %Fat of possible estimation errors

From the formula (54) for FFM we see that estimation errors in V_{ECF} and/or V_{ICF} also give errors in estimated FFM. For instance, V_{ECF} and V_{ICF} may be estimated to be larger than they in reality are, in which case FFM will also be estimated to be larger than it really is. As FFM and %Fat are opposing variables, a too high value of FFM will result in a too low value of %Fat.

Chapter 9. Practical considerations on measurements

This chapter gives some practical advice on measurement with the Xitron Hydra 4200. It is the aim to point on details on *how* to measure, explain *why* measurements should be done so, and also *predict* the consequences of doing it wrong.

Positioning of the subject being measured

Whole-body measurements should be made so that the current will flow as wrist → arm → trunk → leg → ankle, or the opposite direction. The calculation of fluid volumes (V_{ECF}, V_{ICF}, and V_{TBF}) assume this route by using the value $K_B = 4.3$. For details on K_B, cf. equation (12)-(14) and the accompanying text.

It is important that current cannot take a shortcut. This means that arms and legs should be free of the body and free of each other. Citing the Xitron Manual p. 37:

> Measurements are best performed with the subject positioned in the supine position (face up), with the legs and arms slightly apart (Figure 3). Extreme abduction and adduction of the limbs, crossing the legs or touching the hands to another body part dramatically effect the results (Kushner 1996). The subject should remain motionless throughout the measurement.

Consequences of incorrect subject position

Example 1: A person is measured with the legs close together. The person is wearing trousers, and there is no skin-to-skin contact.

ECF: Most clothing is electrically insulating, so without skin-to-skin contact the low-frequency current can take no shortcuts. Thus, R_E will be correctly measured, and reported V_{ECF} will be correct (within the estimation error).

TBF and ICF: However, high-frequency currents may pass a thin insulating barrier (cf. the paragraph *Capacitance* on page 7), so these currents may have the possibility of running through both legs instead of just one leg. This will result in the measured value of R_∞ (and R_I) being too small. In this case, the reported values of V_{TBF} and V_{ICF} will be erroneously large: When high-frequency current flows in a too large fluid volume during the measurement (two legs instead of one), then the reported fluid volume will naturally be too large.

For those who prefer equations: According to equation (16) on page 20, the volume V_{ECF} depends only on the resistance R_E, so if R_E is correctly measured then V_{ECF} will be calculated correctly. If the measured value of R_∞ incorrectly is too small, then it follows from equation (32) on page 24 that calculated V_{TBF} will be too large. Since $V_{ICF} = V_{TBF} - V_{ECF}$, also calculated V_{ICF} will be too large.

The report printed by the Xitron Hydra 4200 shows R_E and R_I. The resistance R_∞ is not shown in the report, but it is related to R_I. A low value of R_∞ will be seen as a low value of R_I. For details, cf. equation (8) on page 15.

Example 2: The person being measured has skin-to-skin contact between different body parts, e.g. folded hands or legs touching each other.

The measured values of both R_E and R_∞ will be too small, because the currents can take shortcuts. This results in fluid volumes being reported with incorrectly large values.

Measurement time

The actual measurement lasts only a few seconds, indicated by beeps from the Xitron Hydra 4200 device. Following the measurement, the Xitron Hydra 4200 makes calculations for typically about one minute, after which the results are presented. The patient should lie still during the *measurements* (a few seconds). After that, patient movement, skin contact, etc. have no influence on the results.

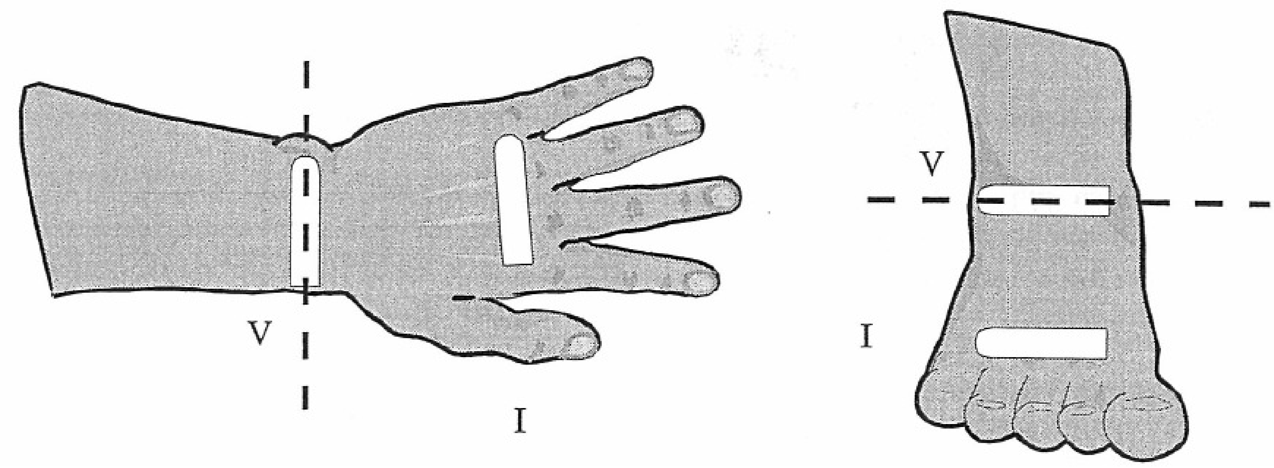
Cables

To avoid interference between currents flowing in the different cables, the cables should be free of each other, the measured subject, metal objects and the earth (cf. the Xitron Manual p. 45). Remember that high-frequency currents may take a shortcut over small distances, despite electric insulation.

In case of such "shortcuts", resistances (primarily R_∞ and thereby R_I) may be measured as incorrectly low values.

Locations of electrodes

Correct locations of electrodes are shown in the Xitron Manual p. 45.



Voltage-detector electrodes V (measurement electrodes) are defined by the <u>red alligator clips</u> and should be located on the <u>midline between the prominent bone ends</u> at ankle and wrist. These positions are anatomically well defined and reproducible, and correspond to the assumptions of the calculation of the constant K_B (cf. page 18).

Current-injection electrodes I are defined by the <u>black alligator clips</u> and should be located at least <u>5 cm distally</u> of the measurement position. Place the electrodes at a <u>good skin surface</u>, i.e. on the hand/foot, not out on the fingers/toes. (In small children, the good skin surface will be more important to the measurement than the full 5 cm distance, see below).

Consequences of various errors in electrode location

Wrong location of measurement electrodes (V): For a given person, the longer the distance between the two measurement points, the larger the measured resistance will be. So, if one or both measurement electrodes are placed too distally, then too large values of the resistances R_E and R_∞ and R_I will be measured, which results in too low estimates on fluid volumes. Conversely: If measurement electrodes are placed too proximally, the measurements will give too low values of the resistances and too high estimates of fluid volumes.

Current-injection electrodes (I) being placed too close to measurement electrodes (V): According to the Xitron Manual, the distance between electrodes V and I should be at least 5 cm. By sufficient distance we achieve a situation where current flows evenly through a cross-section of the hand/foot at the site of measurement. If distance is insufficient, current may flow in only part of the cross-section at the measurement point, which will give slightly too high resistances. Besides, measurement results will be less reproducible, since it is unlikely to make exactly the same error next time.

However: If it is hard to achieve a distance of 5 cm because of small limbs (e.g. on the hand and feet of a baby), then the cross-sectional area over which the current should distribute is also smaller. So current is expected to need less than the 5 cm for achieving even distribution. The means that it is expected to be of no importance for the measurement that distance between electrodes may not be the full 5 cm, when the reason is small limbs.

Swapped red and black alligator clips: Errors happen, so we might ask what will be measured if the electrodes are placed in the correct positions, but red and black are swapped when the alligator clips are applied to the electrodes? Despite of the fact that distance between the measurement points (red alligator clips) is increased, a theoretical consideration[9] comes to conclusion that the measurement is only slightly affected. So the answer to the question is: The effect of the swap is expected to be "small". See also the paragraph below on an experimental test.

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[9] Theoretical consideration from physics:

Assume that black and red electrodes are applied opposite, i.e. current injection (black) at wrist and ankle, measurement (red) at more distal positions. What is measured is the voltage drop, i.e. the difference in electric potential. Almost no current will flow in the distal direction, since the other black electrode is found in the proximal direction. This means that the electric potential will be almost constant distally to a black electrode, with peak value at the position of the black electrode. Thus the *measured* voltage drop between the red electrodes will be almost as high as the *actual* voltage drop from wrist to ankle (the positions of the black electrodes). A slightly too low voltage drop will in itself lead to a slightly too low resistance.
However, the current injected at wrist and ankle will not be evenly distributed through a cross-section until a little distance from the site of injection. This effect will in itself lead to measurement of a slightly too high resistance – see text on *Electrodes being placed too close* in the main text.
Combined, we have effects pulling in opposite directions. This makes their combined effect smaller than their individual effects, which were in themselves small. A quantitative estimate of the effect is however very hard to give, as the result will depend on the precise conditions. So it can only be said that the overall effect of swapping the electrodes will be "small".

Experimental test of errors in electrode location

To test the predictions given above, a subject was measured first correctly, then with swapped red and black alligator clips, and finally with electrodes placed roughly 5 cm too proximally. The subject was male, H = 176 cm, W = 68 kg. The results of the measurements are given in Table 6.

Table 6. Results from test measurements

Measurement description	R_E	R_I	V_{ECF}	V_{ICF}	V_{TBF}
Correct measurement	627.5 Ω	1249.5 Ω	16.85 L	22.43 L	39.28 L
Swapped alligator clips	629.0 Ω	1258.0 Ω	16.82 L	22.31 L	39.13 L
Electrodes too proximally	539.6 Ω	961.0 Ω	18.63 L	27.33 L	45.97 L

The effect of swapping red and black alligator clips is found to be small, as predicted above. In fact, the effect is surprisingly small. This may be taken to mean that such a wrong measurement might be used *with care*, but should not be taken as a sign that the order of the electrodes is unimportant. As described in footnote 9, the result will depend on the precise conditions. Correct application of the electrodes will give the most reproducible results.

Much larger is the effect of a too proximal location of the electrodes: Resistances are markedly reduced, leading to overestimation of fluid volumes.

Chapter 10. Summary of formulas

(All formulas have been derived under the assumption fluids are evenly distributed in the body. For this reason, these formulas may not be applicable for persons with oedemas.)

Formulas for determination of fluid volumes etc.

Measured quantities:

H = height of the measured person (cm)

W = weight of the measured person (kg)

R_E = extra-cellular resistance (Ω)

R_I = intra-cellular resistance (Ω)

Constant values:

ρ_{ECF} = resistivity of extra-cellular fluid (Ω·cm)

ρ_{ICF} = resistivity of intra-cellular fluid (Ω·cm)

K_B = 4.3

D_b = 1.05 kg/liter

Default values of resistivity can be found in Table 4, page 24.

Extra-cellular fluid, ECF (or ECW):

$$V_{ECF} = k_{ECF} \cdot \left(\frac{H^2 \cdot \sqrt{W}}{R_E} \right)^{2/3}$$ formula (16), page 20

where

$$k_{ECF} = \frac{1}{100} \cdot \left(\frac{K_B{}^2 \cdot \rho_{ECF}{}^2}{D_b} \right)^{1/3}$$ formula (17), page 20

Total body fluid, TBF (or TBW):

$$V_{TBF} = V_{ECF} \cdot \left(\frac{\rho_{TBF}}{\rho_{ECF}} \cdot \frac{R_E + R_I}{R_I} \right)^{2/3}$$ formula (33), page 24

where

$$\rho_{TBF} = \rho_{ICF} - (\rho_{ICF} - \rho_{ECF}) \cdot \left(\frac{R_I}{R_E + R_I} \right)^{2/3}$$ formula (35), page 24

Instead of calculating ρ_{TBF} one may calculate the ratio

$$\frac{\rho_{TBF}}{\rho_{ECF}} = k_\rho - (k_\rho - 1) \cdot \left(\frac{R_I}{R_E + R_I} \right)^{2/3}$$ formula (50), page 30

where

$$k_\rho = \frac{\rho_{ICF}}{\rho_{ECF}}$$ definition (49) page 29

Intra-cellular fluid, ICF (or ICW):

$$V_{ICF} = V_{TBF} - V_{ECF}$$ formula (36), page 24

If preferred, V_{ICF} can be calculated directly by combing formula (33) and (36):

$$V_{ICF} = V_{ECF} \cdot \left(\left(\frac{\rho_{TBF}}{\rho_{ECF}} \cdot \frac{R_E + R_I}{R_I} \right)^{2/3} - 1 \right) \tag{57}$$

Body Cell Mass, BCM:

$$BCM = \frac{ICW}{0.70} \quad \text{i.e.} \quad BCM = \frac{ICF}{0.70}$$ formula (53), page 33

Fat Free Mass, FFM:

$$FFM = (1.106 \cdot V_{ECF}) + (1.521 \cdot V_{ICF})$$ formula (54), page 33

Fat and %Fat:

$$Fat = W - FFM$$ formula (55), page 34

$$\%Fat = \frac{Fat}{W} \cdot 100\%$$ formula (56), page 34

Formulas for estimating constants

The preceding formulas assume ρ_{ECF} and ρ_{ICF} to be known constants, while V_{ECF}, V_{ICF}, and V_{TBF} are to be calculated.

The situation may be the opposite: V_{ECF} and V_{TBF} may be known through other measurement techniques, which are independent of the measurement of R_E and R_I, while ρ_{ECF} and ρ_{ICF} are the quantities that should be calculated. Here follows formulas for this purpose. The derivations are left as an exercise to the reader. ☺

Ekstra-cellular resistivity:

$$k_{ECF} = V_{ECF} \cdot \left(\frac{R_E}{H^2 \cdot \sqrt{W}} \right)^{2/3} \tag{58}$$

$$\rho_{ECF} = 1000 \cdot \frac{(k_{ECF})^{3/2} \cdot \sqrt{D_b}}{K_B} = 1000 \cdot \frac{(V_{ECF})^{3/2} \cdot R_E \cdot \sqrt{D_b}}{K_B \cdot H^2 \cdot \sqrt{W}} \tag{59}$$

Intra-cellular resistivity:

The formula for calculating ρ_{ICF} is not simple. For a beginning, we define the ratio

$$r \equiv \frac{R_\infty}{R_0} = \frac{R_I}{R_E + R_I} \tag{60}$$

(this "r" has nothing to do with correlation coefficients). Then, intra-cellular resistivity ρ_{ICF} can be calculated by these formulas:

$$k_\rho = \frac{r \cdot \left(\frac{V_{TBF}}{V_{ECF}} \right)^{3/2} - r^{2/3}}{1 - r^{2/3}} \tag{61}$$

$$\rho_{ICF} = k_\rho \cdot \rho_{ECF} \tag{62}$$

No interpretation of formula (61) is attempted here. It is simply a tool to find a value for k_ρ and thereby ρ_{ICF}.

Alternatively, the software for the Xitron Hydra 4200 allows calculation of new values of ρ_{ECF} and ρ_{ICF} given data on gender, height, weight, R_E, R_I, V_{ECF} (ECF), and V_{TBF} (TBW) for a group of persons. This is described in the Xitron Manual, Addendum Page 5-6 (where ρ_{ECF} and ρ_{ICF} are called Pecf and Picf, respectively).

The software approach may have the advantage that intra- and extra-cellular resistivity can be calculated for a whole set of measurements, instead of a single measurement. It is, however, not described in the manual how the "best" values for the whole data set are determined.

Postscript

In the Greek mythology the Hydra was a kind of sea-serpent, a nine-headed beast which terrorised the coast of Lerna. As one in a series of 12 impossible labours, the hero Hercules was given the task of killing the Hydra. Hercules set out for the battle, but found that for each head he crushed with his club, two new heads grew out! Still, Hercules managed to win the battle. With help from his nephew Iolaus, Hercules used large torches to burn the wound of each destroyed head and thereby prevented new heads from growing out. The last head was immortal, but Hercules killed the body of the Hydra and buried the immortal head under a large rock.

The reasoning in Xitron Technologies when they chose the name HYDRA for their BIS device is not known by this author. But trying to understand the BIS theory in general and the HYDRA device in particular, one can get a feeling somewhat similar to Hercules' experience with the Hydra: Each time one equation has been understood, two new grow out, and each solved problem spawns two new questions.

However, the technique has potential for many applications and much research. It would be a shame if BIS in general and the Xitron Hydra 4200 device in particular were to be treated as a black box for all but the most mathematically oriented researchers. In writing these notes, the author has tried to take the role of Iolaus, who did not kill the beast for Hercules, but brought him the torches that helped him win the battle.

As in all interesting research there will always be at least one issue left which cannot be completely done with, but at best put away under a big rock, where it can lay until the day someone is ready to remove the rock and confront the next Hydra.

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