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Integration of promotion and production decisions in sales and operations planning

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Abstract

This paper presents a new modelling framework for developing a sales and operations plan that integrates promotion and production planning decisions. We adopt a rich demand function that captures the dynamics and heterogeneity of consumer response to price promotions by simulating purchase incidence, consumer choice and quantity decisions, as well as household’s inventory level. Our numerical study reveals interesting findings on the benefits of developing an integrated sales and operations plan as well as the optimal timing and number of promotions, and more importantly, how these findings are influenced by the mutual dependence of marketing and production related factors.

Keywords: marketing-operational interface; aggregate planning; production; promotion; sales and operations planning, forward buying
1. Introduction

Sales and operations planning (S&OP) is a tactical planning process that helps companies to balance demand and supply and to ensure that all the plans of different business functions are synchronized (Wallace, 2006). According to APICS Dictionary (Pittman and Atwater, 2016), S&OP is defined as "... the function of setting the overall level of manufacturing output and other activities to best satisfy the current planned levels of sales, while meeting general business objectives of profitability, productivity, competitive customer lead times, inventory and/or backlog levels, etc". S&OP is also a pillar used for integrating supply chain planning processes and coordinating supply chain members (Affonso et al., 2008). As such, S&OP covers both cross-functional intra-company and supply chain intercompany coordination (Tuomikangas and Kaipia, 2014). Traditionally, the basic S&OP process has sought to facilitate the transfer of information from demand planning to master production planning. However, today, there are both requirements and opportunities to move beyond the mere synchronization of master and demand planning towards integrated planning (Feng et al., 2010, Oliva and Watson, 2011). One important enabler of such integrated planning is the use of advanced planning and scheduling (APS) systems for S&OP. Today, we witness widespread implementation of APS systems in supporting the S&OP processes in practice, and consequently, the use of APS systems has also been extensively discussed in the literature (see e.g. Rudberg and Thulin, 2008, Ivert and Jonsson, 2014). However, as also reported in Ivert and Jonsson (2014), even though the APS systems facilitate better coordination, the integration level in the S&OP processes still seems to be limited to a sequential practice in which the sales/marketing department first prepares a preliminary plan of future sales while the production department then prepares a preliminary production plan, and the possible disagreements of the two plans are resolved through reconciliation meetings. While planners in
the marketing department might be primarily concerned with e.g., the timing and level of promotion in order to maximize revenues, planners in the production department focus on production plans satisfying target sales and minimizing total production-related costs. Such a planning process neglects the mutual dependence of promotions and the use of production resources, and therefore leads to a sub-optimal sales and operations plan. This practice also shows that there seems to be some challenges and barriers in implementing a fully integrated planning system. Lack of appropriate decision support models might be one of the reasons that hinder the development of fully integrated planning (Hinkel et al. 2016).

The central theme of our paper is the development of decision support for S&OP within a single manufacturing firm. We focus particularly on integration of the marketing and production planning functions with respect to coordination of price promotion and production decisions. This theme is undoubtedly of practical relevance as it is discussed extensively in various business reports. Gedenk et al. (2010) reports that 25% of all retail sales is coming from sales during price promotion campaigns. Bursa (2012) reports data from Gartner indicating that an effective S&OP process can increase revenues by 2% to 5% and reduce inventories by 7% to 15%, which is quite significant for many firms. According to Accenture’s Perfect Promotion Study (Samuel and Goldstein, 2013), based on interviews with 350 senior executives at large consumer packaged goods (CPG) companies, 43 percent believe that greater integration is required between the business functions involved in the trade promotion process. In their recent study on the S&OP maturity level in Danish companies, Lund and Raun (2017) reports that although several CEOs sponsor an integrated S&OP, many companies still lack a clear S&OP vision, so that the operations and marketing departments still develop their own separate plans. S&OP is also discussed widely in the academic literature (see e.g. Martinez-Costa et al., 2013 and the references therein).
However, our literature review confirms that previous studies have not explored in depth the benefits of adopting truly integrated planning of the marketing and production functions and therefore have not provided comprehensive insights into the mutual dependence of marketing and production related factors. Most of the existing literature considers demand models that are too simplified so that many factors affecting changes in demand due to promotions, cannot be fully examined. The existing literature also provides little help in understanding how the frequency and timing of promotions are affected by the interaction of production and marketing related factors. We aim to fill this gap in the literature by investigating a more refined modelling framework for generating sales and operations plans that integrate promotion and production planning decisions. In particular, our modelling framework features a rich demand function that captures the dynamics and heterogeneity of consumer responses to price promotions, without which the mutual dependence of marketing and production related factors can only be examined cursorily. We aim to address the following questions: (1) To what extent does integration of production and promotion decisions in an S&OP setting generate benefits for the company, and how are these benefits influenced by the interaction of production and marketing related factors? (2) How is the optimal timing and number of promotions influenced by the interaction of production and marketing related factors?

To this end, we integrate a disaggregate consumer response-based demand model and a mixed integer linear programming (MILP) based aggregate production planning model. Although these two models are well established separately, such an integration framework should also be highlighted as a contribution of this study. None of the existing literature presents a comprehensive model that captures the mutual dependence of production and marketing factors to the extent considered in this paper.
The demand model is able to capture the dynamics and heterogeneity of consumer response to price promotions by simulating purchase incidence, consumer choice and quantity decisions, as well as household’s inventory levels. It allows us to decompose total sales into increased consumption, brand switching and forward buying. Price promotions may attract new or existing customers to increase consumption, and influence the decision of consumers to switch from a competitor’s product, resulting in incremental sales. However, promotion can also result in non-incremental sales, as customers only shift their future purchases to the current period, i.e., they exercise forward buying. The optimal decisions in production planning, i.e., production quantities, number of workers, inventory levels, subcontracts, etc., are determined based on the demand generated as the consequences of the promotion decisions. As the composition of incremental and non-incremental sales observed during promotions will have an influence on the required production capacity, the decisions on production and the timing and level of promotions have to be made jointly. Our suggestion for an integrated decision-support model for S&OP attempts to capture these interaction effects.

The paper proceeds as follows. Section 2 presents our review of relevant literature. Then, in Section 3 we specify the integration of the demand and aggregate production planning models. In Section 4, we present the results of our numerical study. In Section 5, we conclude our study and suggest some directions for future research.

2. Literature Review

Integration in S&OP is a topic widely studied in the literature. We refer the reader to Thome et al. (2012) and Tuomikangas and Kaipia (2014) for comprehensive reviews. This topic is quite broad, covering possible cross-functional coordination between operations, marketing, finance, procurement, etc. More closely related to our paper, is the research stream that focuses particularly
on coordination between operations and marketing. There is also a substantial and growing literature on this topic. Tang (2010) presents an extensive review and suggests that it is important to move from the traditional interface, where marketing and operations just focus on revenues and costs, respectively, to a joint perspective on marketing and operations decisions. This lends support to what we develop in this paper.

Numerous studies in the literature address the coordination of promotion and operational decisions. Many studies, however, focus on inventory rather than production planning decisions. For example, Balcer (1983) studies a dynamic inventory/advertising model in a newsvendor setting. A model considering a joint inventory and promotion planning problem in a periodic review system is presented in Cheng and Sethi (1999). Federgruen and Heching (1999) consider a joint inventory and pricing problem where the price can be dynamically adjusted. Zhang et al. (2008) extend the models of Cheng and Sethi (1999) and Federgruen and Heching (1999) by considering a joint optimization of pricing, promotion, and inventory decisions. Kurata and Liu (2007) study the problem of optimizing promotion depth (discount level) and frequency in a supplier-retailer setting, where the retailer wants to maximize expected revenues and the supplier tries to minimize expected inventory costs. More recently, Albrecht and Steinrüccke (2017) study sales planning of perishable goods where sales price decreases with decreasing quality of goods. All the above studies consider a simple relation between price and demand. Similar to our approach that considers forward buying from customers, Huchzermeier et al. (2002) uses a customer choice model that captures how customers react to retail promotions through stockpiling and package size switching. The authors show that by capturing customer response to price promotions, the retailer can reduce inventory costs. The demand model we adopt in our paper is more comprehensive than theirs. The main distinguishing feature of our paper to the above studies is, in addition to the richer
demand model we adopt, that we also address the aggregate production planning problem of manufacturing firms in which the decisions made are not limited to inventories only.

Most closely related to our study are those articles presenting models that analyse joint promotion and production planning. Martinez-Costa et al. (2013) present a comprehensive review of the literature on the integration of marketing and production decisions in aggregate planning, which is also the central theme of our paper. Leitch (1974) develops a multi-period model to optimize a production and advertising plan. The advertising is mainly intended to lower the production costs by smoothing seasonal product demand. Sogomonian and Tang (1993) present a modelling framework for coordinating promotion and aggregate planning within a firm. In their demand function, demand in each period depends on the time elapsed since the last promotion, the level of the last promotion, and the price at the corresponding period. Ulusoy and Yazgac (1995) present an aggregate multi-product, multi-period production planning model that considers the use of pricing and advertising as tools for smoothing the demand. They use a rather simple demand function, where demand is assumed to be inversely proportional to price and directly proportional to the advertising efficiency. Feng et al. (2008) propose a mixed integer programming model to integrate sales and production, and capture demand uncertainty by assuming that demand and price are normally distributed. Affonso et al. (2008) show the benefit of collaboration in the development of S&OP. In their model, they assume that the forecasted demands from the sales department are given and constant. Since there is no explicit demand function in their model, they perform demand perturbations in their numerical experiment. Other papers that integrate production and marketing decisions are by Gonzales-Ramirez et al. (2011), Lusa et al. (2012), Bajwa et al. (2016), and Mishra et al. (2017). Although we share the basic idea with the above mentioned authors with respect to integration of production and marketing decisions, the simple
demand models adopted in their papers are not really helpful in answering the main research questions postulated in our paper. We extend these previous contributions in two respects. Firstly, the adoption of a rich demand model allows us to examine the interaction of different factors beyond what is covered by the existing literature. We examine the effects of several marketing-related factors such as seasonality, promotion impact, promotion discount level, loyalty rate, and pass through rate together with production-related factors such as flexibility and production costs on the optimal integrated production and promotion plan. Moreover, we examine the possible interaction between those factors. The results of our numerical study provide useful information about, e.g. conditions under which the integration of promotion and production planning gives the most benefits, and how the optimal timing and number of promotions are influenced by the above mentioned factors. As pointed out by Martinez-Costa et al. (2013), there is a lack of research that considers a richer demand model that can be used to develop a decision-support tool for the joint S&OP process. Our research attempts to fill this void in the literature.

Meanwhile, promotion planning is a topic that has been studied extensively in the marketing literature. Promotion is one of the marketing–mix variables that can be used for maintaining sales volume, increasing growth, and attracting consumers’ attention (Kotler, 2013, Steenkamp et al., 2005). Lack of decision-support systems to address the complexity and dynamics of promotion planning has motivated some authors (e.g., Tellis and Zufryden, 1995, Silva-Risso et al., 1999, and Ailawadi et al., 2007) to develop a retail promotion planning model that is based on a disaggregate consumer response model. While such a model is quite useful in capturing the dynamics and heterogeneity of consumer response to price promotions, most studies in the marketing literature focus on maximizing profits viewed only from a marketing perspective. They thereby ignore the interdependence of the resulting promotion plan and production-related (cost)
factors. We contribute to this stream in the marketing literature in two aspects. First, we introduce a framework of how to integrate the disaggregate consumer response model with the aggregate production planning model. Second, our results clearly show the weakness of the sequential (non-integrated) approach. They challenge the standard approach predominantly used in the marketing literature by showing that developing a promotion plan that neglects production related factors may result in profits significantly lower than those obtained using the integrated approach.

Our literature review shows that previous research in each of the operations and marketing literature has its own deficiencies, but that they are complementary to each other. In light of this conclusion, and motivated by the needs for further developments of decision support for S&OP processes, our paper aims to integrate research in these two disciplines.

3. **The Integrated Models Framework**

This section is comprised of three parts. The first part explains the disaggregate consumer response-based demand model used for generating the demand forecast. In the second part, we present the model used to generate the optimal aggregate production plan given the demand forecast. The final part presents the joint optimization problem utilizing the above two models in combination to obtain the optimal promotion and production plan. As a benchmark, we also present the sequential optimization problem, where the promotion plan is first optimized by marketing planners, and the optimal production plan is subsequently optimized by production planners based on this promotion plan.

3.1 **The Demand Model**
Below we explain the main principles used for building the demand model. We use an integrated purchase incidence-brand choice-purchase quantity model that is widely used in the marketing literature (see e.g. Guadagni and Little 1983, Bucklin and Lattin 1992, Silva-Risso et al. 1999, and Ailawadi et al. 2007). The model captures three factors that are related to a customer’s purchase decision: the timing of the purchase, the type of product/brand, and the quantity to purchase. The main idea of the model is that conditional on a store visit, a household (consumer) will decide whether to buy in the product category. For any given purchase decision, the household then chooses a brand alternative, and finally determines the purchase quantity. Given a store visit at time $t$, the expected demand for brand $j$ from household $h$ is given by

$$E(D_{jt}^h) = P_t^h(inc) \times P_t^h(j|inc) \times E(D_{jt}^h|D_{jt}^h > 0)$$  \hspace{1cm} (1)$$

where

- $P_t^h(inc)$ The probability that household $h$ makes a purchase in the product category on a store visit in time period $t$
- $P_t^h(j|inc)$ The probability that household $h$ chooses brand $j$, given that household $h$ decides to make a purchase in the product category in time period $t$
- $E(D_{jt}^h|D_{jt}^h > 0)$ The expected quantity that household $h$ will buy of brand $j$, given that household $h$ decides to purchase brand $j$ in time period $t$

Note that in the above formulation, we consider a particular brand offered by the manufacturer that is in competition with other brands offered by competitors, and assume that the brand has only one size. However, the model can also be adopted in situations where competition occurs between brand sizes or SKUs. In what follows, we present the three main building blocks: the brand choice
model, the purchase incidence model, and the quantity model. Details of the model specifications including the required parameters are presented in the Appendix. In this modelling framework, the purchase incidence and brand choice can be modelled as a nested logit model. Because the decision to purchase is influenced by the parameter value from the brand choice model (Equation A5 in the Appendix), we discuss the brand choice model first and the purchase incidence model afterwards. The brand choice is handled in a multinomial logit framework, which gives the probability that a household will choose the particular brand after deciding to purchase a product. The brand choice probability can be written in the following form:

$$P_t^h (j|inc) = \frac{e^{A_{jt}^h}}{\sum_{j=1}^{J} e^{A_{jt}^h}}$$, \hspace{1cm} (2)

where $A_{jt}^h$ is the deterministic component of utility associated with brand $j$ for household $h$ in time period $t$ that is modelled as a function of price, promotion and consumer-specific variables such as brand and size loyalty (see Appendix for details). The purchase incidence probability is modelled with a binary nested logit model:

$$P_t^h (inc) = \frac{e^{C_t^h}}{1+e^{C_t^h}}$$, \hspace{1cm} (3)

where $C_t^h$ is the deterministic component of utility associated with household $h$ in time period $t$. It is modelled as a function of the proportion of purchase frequency, household inventory, and consumption rate (see Appendix for details). Next, given a purchase of brand $j$, the number of units purchased is captured by a zero-truncated Poisson distribution, which gives the expected number of units purchased by household $h$ at time $t$:

$$E(D_{jt}^h|D_{jt}^h > 0) = \frac{\lambda_{jt}^h}{1-e^{-\lambda_{jt}^h}}$$, \hspace{1cm} (4)
where $\lambda_{jt}^h$ is the purchase rate of household $h$ for brand $j$ at time $t$, which is modelled as a function of the average number of units purchased, household inventory, brand loyalty, and price.

As for model application, parameter values of the model are usually estimated based on scanner-panel data obtained from sources such as Nielsen Consumer Panels for certain product categories, e.g., canned tomato sauce (Silva-Risso et al. 1999) or ketchup and yoghurt (Ailawadi et al. 2007). The store scanner data gives the information of the marketing environment such as price, discount level, retailer’s pass-through, retailer’s mark-up, etc. The panellist data provides information regarding the household shopping trip, loyalty to brand, purchase histories, and consumption rate. Such data are commonly in commercial use today. With the growing importance of business analytics and significant improvements in data-collecting technologies, data like these will become even more widely available in the future. This lends further support to the development of data-driven decision support models like the one presented in this paper (Sanders, 2016). We summarize the various elements of the demand model in Figure 1.
The next step is using Monte Carlo simulation to generate the dynamics of consumer response, which is a common approach in the marketing literature (Ailawadi, 2007, Silva-Risso et al., 1999, Seetharaman, 2004). The simulation generates the dynamics of aggregate demand from the households and is able to capture the effect of stockpiling and repeat purchases.

As an illustration, suppose we have calculated the purchase probability $P_t^{bh}(inc)$ based on the available parameter values. A household purchase occurs in the simulation if the random number generated (between 0 and 1) is less than $P_t^{bh}(inc)$. The logic of the demand model can be seen in Table 1. The output of the model is a demand forecast $D_{jt}$ for brand $j$ in time period $t$, which
represents an aggregate household demand for each period: \( D_{jt} = \sum_{h=1}^{H} E(D_{jt}^h) \), where \( H \) is the number of households simulated. In order to make a fair comparison between different promotion schemes, we employ the same seed number in the simulations, allowing us to use the same set of random numbers (demands). Note that in Table 1 we assume that there are two brands, i.e., \( j = 1, 2 \), and the manufacturer is interested in the demand of its product brand (say, \( j = 1 \)).

---

**Table 1. Algorithm: Demand model**

<table>
<thead>
<tr>
<th>Initialization</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Set</strong> timing and promotion levels (i.e., ( L_{jt} ), discount level for brand ( j ) in time period ( t ))</td>
</tr>
<tr>
<td><strong>For</strong> ( h=1 ) to ( H ) ( \leftarrow ) household</td>
</tr>
<tr>
<td><strong>For</strong> ( r=1 ) to ( R ) ( \leftarrow ) replication</td>
</tr>
<tr>
<td><strong>For</strong> ( t=1 ) to ( T ) ( \leftarrow ) time period</td>
</tr>
<tr>
<td>Calculate ( P_t^h(inc), P_t^h(j</td>
</tr>
<tr>
<td>Generate ( rand1 )</td>
</tr>
<tr>
<td><strong>If</strong> ( rand1 &lt; P_t^h(inc) ) then buy ( \leftarrow ) household</td>
</tr>
<tr>
<td>Generate ( rand2 )</td>
</tr>
<tr>
<td><strong>If</strong> ( rand2 &lt; P_t^h(j = 1</td>
</tr>
<tr>
<td><strong>Else if</strong> ( P_t^h(j = 1</td>
</tr>
<tr>
<td><strong>End if</strong></td>
</tr>
<tr>
<td>Calculate ( E(D_{jt}^h</td>
</tr>
<tr>
<td><strong>Else</strong> set brand = 0, ( D_j^h = 0 )</td>
</tr>
<tr>
<td>( D_{jtr} = D_{jt}^h )</td>
</tr>
<tr>
<td><strong>End for</strong> ( t )</td>
</tr>
<tr>
<td><strong>End for</strong> ( r )</td>
</tr>
<tr>
<td><strong>End for</strong> ( h )</td>
</tr>
<tr>
<td>Calculate total expected demand, ( D_{1t} = D_t = \sum_{h=1}^{H} E(D_{jt}^h) )</td>
</tr>
</tbody>
</table>

The disaggregate consumer response-based demand model described above allows us not only to estimate the direct demand impact of promotion decisions, but also to decompose incremental demand into true incremental demand and demand borrowed from the future, commonly termed
forward buying. In Section 4, we discuss a method used in the literature (Silva-Risso et al., 1999) for decomposing incremental demand.

3.2 Aggregate Production Planning Model

The aggregate production planning is an activity of tactical planning which attempts to balance demand and supply in an effort to streamline operations and increase productivity. Given the demand forecast for each period in the planning horizon, the aggregate production planning model determines the production level, inventory level, and capacity level (internal and outsourced) for each period that minimize the total costs over the planning horizon (Chopra and Meindl, 2016). We follow a standard mixed integer linear programming model to solve the aggregate planning problem (see, e.g., current textbooks like Chopra and Meindl, 2016, Bozarth and Handfield, 2016, Heizer et al. 2017, and Silver et.al. 2017).

Parameters:

- $cp$: Production cost per unit (including materials; excluding labour cost)
- $cl$: Regular labour cost per worker and time period
- $ch$: Hiring cost per worker
- $cf$: Firing cost per worker
- $co$: Overtime cost per hour
- $cInv$: Inventory holding cost per unit per time period
- $cs$: Subcontracting cost per unit
- $LL$: Minimum number of workers
UL  Maximum number of workers

wh\textsubscript{t}  Number of regular working hours available per worker in time period \( t \)

nl\textsubscript{o}  Number of workers at the beginning of the planning horizon

nu  Number of units produced per hour

O\textsubscript{t}  Maximum overtime hours available per worker in time period \( t \)

T  Planning horizon in number of time periods

SS\textsubscript{t}  Safety stock requirement in time period \( t \)

Decision variables:

\( qp\textsubscript{t} \)  Number of units produced during regular time in time period \( t \)

\( qo\textsubscript{t} \)  Number of units produced during overtime in time period \( t \)

\( qs\textsubscript{t} \)  Number of units produced using subcontracting in time period \( t \)

\( nh\textsubscript{t} \)  Number of workers hired at the beginning of time period \( t \)

\( nf\textsubscript{t} \)  Number of workers fired at the beginning of time period \( t \)

Other variables:

\( D\textsubscript{t} \)  Demand forecast for time period \( t \) (index for brand \( j \) is suppressed here, as we are only interested in a single brand for the manufacturer)

\( I\textsubscript{t} \)  Inventory at the end of time period \( t \)

\( nl\textsubscript{t} \)  Number of workers available in period \( t \)
The aggregate production planning problem can now be written as:

\[
\begin{align*}
\text{Min}_{q_{pt},qo_t,qs_t,nh_t,nf_t} & \quad TCOST = \sum_{t=1}^{T} \left( (qp_t + qo_t) \cdot cp + \frac{o_t}{nu} \cdot co + q_s \cdot cs + I_t \cdot cln + nl_t \cdot cl + nh_t \cdot ch + nf_t \cdot cf \right) \\
\text{Subject to:} & \\
I_t = I_{t-1} + qp_t + qo_t + qs_t - D_t & \quad t = 1, \ldots, T \quad (6) \\
SS_t \leq I_t & \quad t = 1, \ldots, T \quad (7) \\
nl_t = nl_{t-1} + nh_t - nf_t & \quad t = 1, \ldots, T \quad (8) \\
nl_0 = nl_T & \quad (9) \\
LL \leq nl_t \leq UL & \quad t = 1, \ldots, T \quad (10) \\
qp_t \leq nl_t \cdot wh_t \cdot nu & \quad t = 1, \ldots, T \quad (11) \\
qo_t \leq nl_t \cdot Ot \cdot nu & \quad t = 1, \ldots, T \quad (12) \\
I_t, qp_t, qo_t, qs_t, nl_t, nh_t, nf_t \geq 0 \quad ; nl_t, nh_t, nf_t \text{ integer} & \quad t = 1, \ldots, T \quad (13)
\end{align*}
\]

The objective function in (5) is minimizing the total cost, consisting of the production cost, overtime cost, subcontracting cost, inventory cost, labour cost, and hiring/firing cost. Demand \( D_t \) as an input for planning the production in time period \( t \) is obtained from the aggregation of household demand simulated in the demand model (see Subsection 3.1). Constraints (6) and (7) represent the inventory balance equations and minimum safety stock level requirements, respectively. The manufacturer’s capacity and resources in terms of the number of workers, maximum production, and overtime are represented by constraints (8)-(12). Constraints (13) represent non-negativity and integer requirements.
3.3 Optimization

We now specify two different optimization approaches. In the first approach, there is no coordination between the marketing and production planning. In this non-coordinated approach, the optimization is done sequentially in two steps. First, the number and timing of promotions are optimized by maximizing the total revenues minus the promotion costs. The second step optimizes the production plan using the demand forecast obtained in the first step. In the second approach, the optimization is carried out jointly by integrating the promotion and production decisions. By comparing the two approaches, we are able to examine the importance of coordination.

Sequential Optimization (the non-coordinated approach)

Let \( L_t \) (\( 0 \leq L_t < 1 \)) denote the level of discount (%) offered in period \( t \) (\( L_t = 0 \) means that there is no promotion offered). We define \( P \in \mathbb{P} \) as a promotion plan, where \( P = (L_1, L_2, \ldots, L_{T-1}, L_T) \) and \( \mathbb{P} \) is the set of all possible promotion plans. The promotion plan is the main input for the demand model, and we define \( D_t|P \) as the resulting demand forecast that corresponds to the promotion plan \( P \). In the first step, the marketing planner solves the following optimization problem:

\[
\text{Max}_{P \in \mathbb{P}} \text{Profit} = \sum_{t=1}^{T} (D_t|P \cdot Rp \cdot (1 - L_t) - Z_t \cdot V)
\]  

(14)

Subject to

\[
L_t \leq Z_t \leq M \cdot L_t \quad t = 1, \ldots, T
\]

(15)

\[
\sum_{t=1}^{T} Z_t \leq K \quad t = 1, \ldots, T
\]

(16)

\[
Z_t \text{ binary} \quad t = 1, \ldots, T
\]

(17)

\[
0 \leq L_t \leq 1 \quad t = 1, \ldots, T
\]

(18)
Where

Parameters:

\( K \) Maximum number of promotions during the planning horizon

\( Rp \) Regular price per unit from manufacturer

\( V \) Promotion cost per promotion event

\( M \) Sufficiently large number

Variables:

\( L_t \) Level of discount (in percent) in time period \( t \)

\( Z_t \) Binary variable: 1 if promotion with discount is offered; 0 otherwise

Suppose \( P^* \) is the optimal promotion plan, and \( MProfit^* \) is the corresponding marketing profit.

In the second step, the production planner solves the production planning problem as formulated in (5)-(13) with \( D_t \) being replaced by \( D_t|P^* \). Letting \( TCOST^* \) denote the minimum total cost obtained, the total manufacturer’s profit is \( Profit(1)^* = MProfit^* - TCOST^* \).

**Joint Optimization (The coordinated approach)**

By integrating the promotion and production planning, the joint optimization problem is formulated as:

\[
Max_{P \in \mathcal{P}, q_{pt}, q_o, n, m} Profit(2)
\]

\[
= \sum_{t=1}^{T} \left[ D_t | P \cdot Rp \cdot (1 - L_t) - Z_t \cdot V - (q_{pt} + q_o) \cdot cp - \frac{q_o}{nu} \cdot co - q_s \cdot cs - L_t \cdot cl - nl_t \cdot cl - nh_t \cdot ch - nf_t \cdot cf \right]
\]

Subject to

(6) – (13); and (15) – (18).
4. Numerical study

This section presents the parameter settings and results of our numerical study. We examine the influence of several factors related to the demand and aggregate production planning models on the optimal profits, as well as the optimal number and timing of promotions. In relation to the optimal profits, our primary interest is to examine the importance of coordination by jointly optimizing promotion and production decisions. This is achieved by making comparisons between the profits obtained by solving the joint optimization problem and the profits obtained when there is no coordination. More specifically, we study the effects of seven experimental factors: flexibility in production capacity, production unit cost, brand loyalty, pass-through rate, demand pattern (seasonality effect), promotion impact, and discount level. To study the effect of flexibility in production capacity, we vary hiring and firing costs, i.e., higher flexibility is represented by lower hiring and firing costs. Indeed, there are other measures that can be used to represent flexibility such as costs associated with the changes in machine capacity or costs of overtime and subcontracting. Our choice can be particularly justified in production settings where production outputs are mainly driven by the number of workers, for example some of the food, apparel and electronics appliances industries. Moreover, it is commonly used in the literature (see e.g. Lusa et al. 2012 and Chopra and Meindl 2016) to represent medium-term, tactical changes of production capacity. Furthermore, we examine the effect of brand loyalty by changing the parameter $B^h$, which represents loyalty to brand and is obtained from the calculation of market share (Bucklin and Lattin 1992). We are also interested in examining the effect of retailer’s pass-through rate, $PT$, which is the proportion of the manufacturer’s discount that the retailer passes on to the consumer. Note, however, that this paper focuses on the coordination between production and marketing functions within a single manufacturing firm, and so exclude the coordination between the
manufacturer and retailer. We adopt the “pay-for-performance” environment commonly used in the marketing literature (e.g. Silva-Risso et al. 1999), where the retailer is paid a promotional allowance based on the volume sold during the promotion period, and this promotional allowance is captured in the demand model in the pass-through response function of the retailer. Seasonality effects in demand are captured through a scale factor on the purchase frequency of households in store visits in the incidence model, $F^h$. This factor represents the proportion of shopping trips in which a household makes a category purchase. We divide the planning horizon into ten time intervals and vary the $F^h$ values in each of the ten intervals to get a demand pattern with seasonality, whereas the $F^h$ scale factor values are set constant to get a demand pattern without seasonality. The expected demand over the planning horizon is the same in both cases. To examine the influence of promotion impact, we use different values for the temporary price reduction coefficient in the choice model, $\theta_6$, which determines the contribution of the temporary price reduction to a household’s utility function. In addition to the six factors mentioned above, we also evaluate how the results are influenced by the promotion discount level. Three levels are evaluated: 10%, 20% and 30%. We limit the choice to these three levels as our primary objective is to examine the effect of discount level as well as its interaction with the other factors rather than to determine an optimal discount level. The three values also cover the range of discount levels often considered in practice and in marketing literature (Van Heerde et al. 2004, Empen et al. 2015). Note that in our numerical study, we only allow one of the three discount levels to be used over the whole planning horizon. Thus, a mix of different discount levels is not allowed. This would allow us to obtain unambiguous results regarding the effect of the discount level.
With all the combinations of the above parameters, there are in total 288 different problem instances evaluated in this numerical study. Table 2 shows the different levels of the factors varied in our numerical experiment. Other parameter values used as inputs are as follows:

\[ cl = 8 \; ; \; co = 12 \; ; \; clnv = 0.092 \; ; \; LL = 35 \; ; \; UL = 140 \; ; \; wh_t = wh = 40 \; ; \; nl_o = 50 \; ; \; nu = 8 \; ; \]  
\[ O_t = 2.5 \; ; SS = 2000 \; ; T = 52 \; ; \; K = 12 \; ; \; RP = 12 \; ; \; V = 1000 \; ; R = 1000 \; ; H = 121350 \]

A time period in the models is interpreted to be represented by a work week. These parameter values have then been calibrated to provide reasonable and non-trivial solutions to the basic production planning problem. The above unit holding cost is directly related to the unit price and scaled to represent the cost per week. Further parameter values used in the demand function are adopted from the model specifications in Silva-Risso et al. (1999). We use the parameter values as presented in Silva-Risso et al. (1999) with the purpose of developing a market demand simulator that is built based on an empirically grounded demand model.

Table 2. Parameter settings for experimental factors

<table>
<thead>
<tr>
<th>Factor</th>
<th># Levels</th>
<th>Parameter</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Flexibility in production capacity</td>
<td>2</td>
<td>ch, cf</td>
<td>Low: 2000, 3000</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>High: 1000, 2000</td>
</tr>
<tr>
<td>Production unit cost</td>
<td>2</td>
<td>cp</td>
<td>Low: 6</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>High: 7</td>
</tr>
<tr>
<td>Seasonality</td>
<td>2</td>
<td>F^n</td>
<td>Low: 0.81 (constant)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>High: 0.83, 0.7, 0.58, 0.48, 0.68, 0.85, 0.92, 0.99, 0.98, 0.92</td>
</tr>
<tr>
<td>Brand loyalty</td>
<td>2</td>
<td>B^n</td>
<td>Low: 0.6</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>High: 0.8</td>
</tr>
<tr>
<td>Pass through rate</td>
<td>2</td>
<td>PT</td>
<td>Low: 0.7</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>High: 0.8</td>
</tr>
<tr>
<td>Promotion impact</td>
<td>3</td>
<td>( \theta_6 )</td>
<td>Low: 0.2</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Medium: 0.5</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>High: 0.8</td>
</tr>
<tr>
<td>Discount level</td>
<td>3</td>
<td>( L_t</td>
<td>L_t &gt; 0 )</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Medium: 20%</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>High 30%</td>
</tr>
</tbody>
</table>
The importance of coordination

To address the first research question, we present the relative increase of the profits obtained using the coordinated approach in comparison to the profits obtained using the non-coordinated approach. Table 3 summarizes the results. The overall averages of the increase of the profits are 1.45%, 6.39%, and 43.26% for the discount levels of 10%, 20% and 30%, respectively. These increases can represent significant improvements. All the seven factors evaluated appear to be influential. As for the production-related factors, the benefits of coordination become more pronounced when the production flexibility level is low and the product has a low profit margin (a high production cost). In the case where changing production capacity is costly, the coordinated approach allows firms to choose a promotion plan that does not lead to too many changes in the production capacity level. This is not possible under the non-coordinated approach, because the non-coordinated approach fails to differentiate promotion plans based on the production related factors. When marketing planners determine the optimal promotion plan solely based on the revenues and cost of promotions, two products that are different in their capacity changing and production costs may be given similar promotion plans. The fact that a higher relative increase of profit is observed in the production system with low flexibility and in the case with the higher production unit cost shows that the motivation for implementing coordination is higher in more costly production environments. In relation to the marketing-related factors, coordination is more beneficial in situations with high demand seasonality, high brand loyalty, high pass-through rate, and high promotion impact. It is also shown that the benefit of coordination is more pronounced as the discount level increases.
### Table 3. The average increase of profits (coordinated vs non-coordinated)

<table>
<thead>
<tr>
<th>Factors</th>
<th>Discount level</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>10%</td>
</tr>
<tr>
<td>Flexibility in production capacity</td>
<td>High</td>
</tr>
<tr>
<td></td>
<td>Low</td>
</tr>
<tr>
<td>Production unit cost</td>
<td>Low</td>
</tr>
<tr>
<td></td>
<td>High</td>
</tr>
<tr>
<td>Seasonality</td>
<td>Low</td>
</tr>
<tr>
<td></td>
<td>High</td>
</tr>
<tr>
<td>Brand loyalty</td>
<td>Low</td>
</tr>
<tr>
<td></td>
<td>High</td>
</tr>
<tr>
<td>Pass-through rate</td>
<td>Low</td>
</tr>
<tr>
<td></td>
<td>High</td>
</tr>
<tr>
<td>Promotion effect</td>
<td>Low</td>
</tr>
<tr>
<td></td>
<td>Medium</td>
</tr>
<tr>
<td></td>
<td>High</td>
</tr>
<tr>
<td>Overall average</td>
<td>1.45%</td>
</tr>
</tbody>
</table>

In addition to examining the effects of varying individual factors, it is also interesting to examine the effects of the interaction of different factors. In Figure 2, we depict the joint effects of two possibly interacting factors on the importance of coordination. The figure provides a better understanding on the importance of the coordinated approach that is not always directly available from intuition. For example, the previous discussion has highlighted the effect of flexibility, i.e. the benefit of coordination is more pronounced when the flexibility is low, but it does not reveal how the achieved benefit is affected by the other factors. Figure 2 shows that even though the flexibility is low, the significance of the benefit of coordination only becomes significant when supported by other factors, e.g. when seasonality is high or promotion impact is high. The interaction between brand loyalty and promotion impact is another example. The importance of brand loyalty is more pronounced when the promotion impact is not too low or too high. The plots in Figure 2 show consistency on the effect of discount level regardless of its interaction with the other factors. That is, the benefit of coordination increases significantly when the discount level is
30%, the highest level evaluated in this study. The advantage of the coordinated approach over the sequential approach is common knowledge in the existing literature. However, to the extent presented here, none of the previous studies has revealed how the advantage is influenced by various specific factors. The results on the joint effect of the different factors can help production and marketing planners to identify groups of products, for which the integrated planning approach is particularly desirable.

Figure 2. Interaction plots (means) for the average increase of profits from coordination (%)  

With the refined demand model we have adopted, it is also possible to examine how the different marketing related factors influence how increase in demand due to a promotion is decomposed into true incremental demand and forward buying, and hence how they influence the profitability. We follow the method used in Silva-Risso et al. (1999) to decompose total incremental demand into true incremental demand and forward buying. An explanation of how the decomposition of
incremental demand is made is presented in the Appendix. The true incremental demand represents the increase of consumption rate due to a promotional event. In Figure 3, we depict how the percentages of forward buying (relative to the total increase in demand) are affected by the interaction of the different marketing factors. The two production-related factors, flexibility and production cost, are excluded, because neither of these factors have an impact on the households’ purchase decisions.

*Figure 3. Interaction plots (means) for the forward buying (%)*

It is clearly shown in Figure 3 that promotion impact is the most influential factor on the percentage of forward buying. The percentage of forward buying is higher when the promotion impact is lower. In the case of high promotion impact, households are more sensitive to the temporary price reduction and react by increasing their purchase quantities. As households have higher inventory levels, their consumptions tend to increase. As a result, true incremental sales due to consumption
is higher than forward buying in the case of high promotion impact, and vice versa. This observation can be traced from the consumption rate function in the demand model. In addition, higher brand loyalty tends to generate a somewhat higher share of forward buying. Information on the percentages of forward buying and true incremental demand is quite helpful in getting a better understanding of the effect of price promotions on the results previously presented. In what follows, we explain in further depth, why there is a significant increase of the benefits of coordination when the discount level changes from 20% to 30% in Table 3. As will be discussed in the next section, the number of promotions in the non-coordinated approach tends to be higher than in the coordinated approach, and since the percentage of forward buying is smaller when the promotion impact is higher, the motivation of offering more promotions becomes stronger when the promotion impact is high. Since the non-coordinated approach does not consider implications of promotions to production related costs, the worst situation that may occur is when offering promotions results in smaller profits than the case of no promotion. And this is particularly likely to happen when the discount level is high. In contrast, the coordinated approach indeed considers the impact of promotions on the production-related costs. In Figure 4 we show how total quantity, sales revenues, production-related costs, and profits are affected by the discount level, for the case with high promotion impact and for both the non-coordinated and coordinated approaches, and for the cases with and without seasonality, respectively. The figure depicts the relative increase or decrease in comparison to the base case with no promotion (i.e., zero discount level). It is shown that the percentage increase of production-related costs under the non-coordinated approach may be higher than the percentage increase of sales revenues. In the case where the discount level is the highest in our experiment (30%), the non-coordinated approach may even result in profits that are lower than in the base case.
Seasonality: Low
Non-coordinated

Seasonality: High
Non-coordinated

Coordinated

* Flexibility: high; Production cost: high; Promotion impact: high
** The percentage increase is relative to the base case with no promotions

Figure 4. The impact of discount level on total quantity, sales revenue, manufacturing costs, and profits

The number of promotions

We now consider the second research question on how the optimal number of promotions is affected by our experimental factors and how the results differ under the two optimization approaches. Table 4 presents the averages of the optimal number of promotions obtained by the
two optimization approaches. We note that there are cases that have the same optimal number of promotions, but as will be discussed in the following sub-section, the time periods where those promotions are carried out are not necessarily the same.

A shown in Table 4, the average number of promotions is higher in the non-coordinated approach than in the coordinated approach. This is understandable as there are more parameters, i.e., costs related to the production, which may limit the number of promotions in the coordinated approach compared to the non-coordinated approach. Note that the optimal number of promotions under the non-coordinated approach is not affected by the changes in the first two factors, i.e. flexibility in production and production unit cost. This is because the optimal promotion plans obtained with the non-coordinated approach do not consider the production factors.
Figures 5 and 6 depict how the averages of the number of promotions are affected by the interaction of the different factors for the cases with the non-coordinated and coordinated approaches, respectively. Contrasting the two figures reveals some interesting insights. First, the joint effect of discount level and brand loyalty on the number of promotions in the non-coordinated approach appears to be quite different from the joint effect observed in the coordinated approach. In the non-coordinated approach, the average number of promotions tends to decrease as the discount level increases, but the effect of brand loyalty is insignificant. In contrast, the effect of brand loyalty in the coordinated approach is more significant, i.e. the number of promotions is reduced as brand loyalty increases. Further, by considering the production related costs, the coordinated approach is able to exploit the benefit of high brand loyalty and reduces the number of promotions. In the case with high discount level (30%) and high brand loyalty, the average number of promotions under the coordinated approach is close to zero. We also see a difference in the joint effect of discount level and seasonality between the two approaches. The effect of seasonality is more pronounced under the coordinated approach, with more frequent promotions resulting from higher seasonality. Likewise, the joint effect of seasonality and promotion impact is also more pronounced under the coordinated approach. Figure 5 clearly shows that the change in the number of promotions under the non-coordinated approach is primarily driven by the promotion impact rather than by the seasonality. This shows that the non-coordinated approach fails to take into account the effect of seasonality on the production-related costs. Under the coordinated approach, there is interaction between the two factors as the number of promotions in the case of high seasonality is larger than that in the case of low seasonality when the promotion impact is at the medium or high level. When the promotion impact is at the low level, the number of promotions is almost the same for both cases with high and low seasonality.
Figure 5: Interaction plots for the average number of promotions with the non-coordinated approach

Figure 6: Interaction plots for the average number of promotions with the coordinated approach
Under the coordinated approach, as the joint optimization model captures the negative effect of promotions on the production-related costs, a higher discount level tends to result in a lower number of promotions. In addition, higher demand seasonality triggers more frequent promotions. The analyses above not only provides managers with insights into the effects of the different production and marketing factors, but also the interplay between those factors. Our study reveals conditions related to production and marketing under which the development of an integrated production and promotion planning generates the most benefits. Furthermore, a better understanding on how the interactions between parameters influence the percentage of forward buying and the optimal number of promotions will help production and marketing planners to focus on the more influential parameters when determining a joint production and promotion plan.

*The timing of promotions*

The final part of our research questions concerns the optimal timing of promotions. As the most noteworthy case, we focus on their optimal timing when demand is seasonal. To illustrate, Figure 7 shows the optimal timing of promotions for one problem instance with the discount level of 30%. It is interesting to see that while the coordinated approach tends to plan the promotions during the low-demand season, the non-coordinated approach, on the contrary, tends to schedule promotions during the high-demand season.

The coordinated approach seeks to smooth the demand and production over the planning horizon by offering more promotions in the low-demand season. In contrast, the non-coordinated approach aims to maximize the total sales by offering more promotions in the high-demand season. Comparing the optimal results of the two approaches for these particular problem instances reveals
that the higher profits obtained by the coordinated approach are mainly due to its lower total inventory, overtime and production costs.

Disc.Level: 30%
Flexibility: low; Production cost: high; Seasonality: high; Promotion impact: low

**Figure 7: The optimal timing of promotions for the discount level of 30%**
5. Conclusions

5.1 Contributions

This paper presents a framework for developing a decision-support model to obtain a sales and operations plan (S&OP) that integrates production and price promotion planning decisions. We integrate a purchase incidence-brand choice-purchase quantity model and a mixed integer linear programming model to determine an optimal promotion and aggregate production plan. To our knowledge, this is the first paper to integrate and apply this kind of advanced demand model with the standard aggregate production planning model. In our numerical study, we examine the benefits of coordination in developing S&OP by comparing the solutions obtained from the coordinated approach to those obtained from the non-coordinated approach. The modelling framework we adopt allows us to examine how the benefits of coordination are affected by various production and marketing related factors, as well as the interaction between these factors. The results show that the average relative improvement of the profit considered can be up to more than 40%, which is a significant improvement. The results on the joint effects of the different factors provide a better understanding on the importance of coordination and its main driving forces. Furthermore, the rich demand model adopted allows us to decompose the total increase in demand due to a promotion into true incremental demand and forward buying. We find that promotion impact is a major influential factor that determines the split between these two types of incremental demand. Information on the percentages of forward buying and true incremental demand enhances the understanding of the effect of promotions. It is also found that the non-coordinated approach tends to generate plans with more promotions than the coordinated approach. Another interesting finding is that the coordinated approach tends to plan promotions during the low-demand season, while the non-coordinated approach tends to schedule them during the high-demand season. All
the above main findings contribute to advancing the literature on the development of decision support for an integrated S&OP.

5.2 Managerial implications

As noted above, when 25% of all retail sales is coming from sales during price promotion campaigns (Gedenk et al., 2010), there is a huge business potential in improving promotion planning. The results of our study provide useful information for production and marketing planners on the importance of developing an integrated S&OP taking into account joint production and promotion decisions. As many firms rely on reconciliation meetings to make adjustments to their joint sales and operations plans (Ivert and Jonsson 2014), the use of an integrated model like ours will help planners reduce the time required for resolving possible disagreements resulting from the sequential approach. Our modelling framework that facilitates a comprehensive examination of various factors and their effects will help production and marketing planners in better understanding the main driving forces of the profitability resulting from a joint production and promotion plan. Production and marketing planners can learn to recognize the characteristics of products in relation to their production and marketing, and thereby identify products for which an integrated production and promotion plan is particularly desirable. As the report from Lund and Raun (2017) suggests, a high degree of silo thinking between business functions and a lack of use of integrated software may represent major barriers in the structural and technological dimensions to achieving a high maturity level of S&OP. The integrated framework presented in this paper can inspire practitioners, including planners and software developers, in taking initiatives to overcome those barriers. Furthermore, our study has clearly shown the advantages of using a rich demand model such as the disaggregate consumer response model adopted in this paper. This should inform
practitioners on the importance of business analytics that take account of market data acquisition and development of data-driven decision support models.

5.3 Future research

We acknowledge some limitations of this paper and would like to suggest several topics for future research. Firstly, the solution space for the promotion decisions in this study is limited since the discount levels are restricted to three values and it is not possible to implement a promotion schedule that contains a mix of more than one discount level. Thus, a possible extension is to develop a heuristic approach to deal with increased combinatorial complexity. Secondly, in this paper we only consider the situation where promotion is planned only for a single product (brand), although the demand model we use can accommodate more than one product. An interesting research avenue is to consider the joint optimization in a multi-product setting. Finally, the results presented in this paper are based on a specific set of problem parameters (although some variations have been considered by varying a number of experimental factors). Our conjecture is that the resulting qualitative insights can be generalized to other problems with a different set of parameters. Further empirical studies are necessary to validate our conjecture and assess the robustness of the results.
Appendix:

Brand choice model

In the brand choice model, we calculate the probability that a household chooses a particular brand given that the household decides to make a purchase decision using a multinomial logit form as presented in (2). This probability depends on the value of the deterministic component of brand utility \( A_{jt}^h \) which is modelled as (see, e.g., Bucklin and Lattin 1992; Silva-Risso et al. 1999; and Ailawadi et al. 2007)

\[
A_{jt}^h = \alpha_b + \alpha_s + \theta_1 B_j^h + \theta_2 LB_j^h + \theta_3 S_j^h + \theta_4 LS_j^h + \theta_5 R_{jt} + \theta_6 TC_{jt} + \theta_7 X_{jt} + \theta_8 Y_{jt} \quad (A1)
\]

where

- \( A_{jt}^h \) The deterministic component of utility related with brand \( j \) for household \( h \) in time period \( t \)
- \( B_j^h \) Brand loyalty of household \( h \) to brand \( j \)
- \( LB_j^h \) 1 if \( j \) was last brand purchased, 0 otherwise
- \( S_j^h \) Size loyalty of household \( h \) to brand \( j \)
- \( LS_j^h \) 1 if \( j \) was last size purchased, 0 otherwise
- \( R_{jt} \) Regular store price for brand \( j \) in time period \( t \)
  \[
  R_{jt} = R_{pjt} (1 + Up) \quad t = 1, ..., T \quad (A2)
  \]
- \( R_{pjt} \) Regular price from manufacturer of brand \( j \) in time period \( t \)
- \( Up \) Store’s markup in percent
- \( TC_{jt} \) Temporary price cut for brand \( j \) in time period \( t \)
\[ TC_{jt} = Rp_{jt}(PT_{jt} \cdot L_{jt}) \quad t = 1, \ldots, T \]  \hspace{1cm} (A3)

\( PT_{jt} \)  
Store’s pass-through in percent

\( L_{jt} \)  
Level of discount in percent for brand \( j \) in time period \( t \)

\( X_{jt} \)  
\( \begin{cases} 1 & \text{if a feature ad is offered for brand } j \text{ in time period } t \\ 0, & \text{otherwise} \end{cases} \)

\( Y_{jt} \)  
\( \begin{cases} 1 & \text{if a display is offered for brand } j \text{ in time period } t \\ 0, & \text{otherwise} \end{cases} \)

\( \alpha_b \)  
Brand constant to be estimated

\( \alpha_s \)  
Size constant to be estimated

\( \{\theta_1, \ldots, \theta_8\} \)  
Parameters to be estimated

**Purchase incidence model**

The buy/no-buy decision is modelled with a binary nested logit model. The purchase incidence model calculates the probability that a household will make a purchase in the product category on a store visit, as presented in (3). The purchase incidence model needs the value of the deterministic component of household utility \( (C^h_t) \) which takes the following form (e.g., Guadagni and Little 1998, Bucklin and Lattin 1992; Silva-Risso et al. 1999; and Ailawadi et al. 2007):

\[ C^h_t = \beta_0 + \beta_1 I^h_t + \beta_2 L^h_t + \beta_3 W^h_t \quad h=1,\ldots, H; \ t=1,\ldots,T \]  \hspace{1cm} (A4)

\[ W^h_t = \ln \sum_{j=1}^{I} e^{A^j_{jt}} \quad h=1,\ldots, H; \ t=1,\ldots,T \]  \hspace{1cm} (A5)

where
The deterministic component of utility related with household $h$ in time period $t$

Proportion of purchase frequency for household $h$ on store visit

Inventory for household $h$ at the end of time period $t$

\[ I_t^h = \text{Max}(0, I_{t-1}^h + \sum_{j=1}^{J} D_{j,t-1}^h - U_{t-1}^h) \quad (A6) \]

Quantity of brand $j$ bought in time period $t-1$ by household $h$

Rate of consumption for household $h$ in time period $t$

\[ U_t^h = I_t^h \left[ \frac{\bar{U}^h}{\bar{U}^h + (I_t^h)^{\pi}} \right] \quad (A7) \]

Mean rate of consumption for household $h$

The expected maximum utility from the brand choice decision for household $h$ in time period $t$.

Parameter to be estimated

Parameters to be estimated

**Quantity model**

The expected quantity purchased by a household is calculated using the expected value of the truncated Poisson distribution as presented in (4). The purchase rate of the household is modelled as (e.g., Bucklin and Lattin 1992; Silva-Risso et al. 1999; and Ailawadi et al. 2007)

\[ \lambda_{jt}^h = \exp(\mu_b + \mu_s + \omega_1 G_t^h + \omega_2 I_t^h + \omega_3 B_t^h + \omega_4 S_t^h + \omega_5 R_{jt} + \omega_6 T C_{jt} + \omega_7 X_{jt} + \omega_8 Y_{jt}) \quad (A8) \]
where

\[ \lambda_{jt}^h \]  
The purchase rate of household \( h \) for the brand alternative \( j \) in time period \( t \)

\[ G^h \]  
Average quantity bought by household \( h \) per purchase trip

\[ I_t^h \]  
Inventory for household \( h \) at the end of time \( t \)

\[ \mu_b \]  
Brand constant to be estimated

\[ \mu_s \]  
Size constant to be estimated

\( \{\omega_1, ..., \omega_B\} \)  
Parameters to be estimated

**The decomposition of incremental demand.**

The expected incremental demand of brand \( j \) sold to household \( h \) in time period \( t \) is obtained by subtracting baseline plus forward buying demand, \( E(BFD_{jt}^h) \), from total demand, \( E(D_{jt}^h) \), and adding back borrowed demand that resulted in incremental consumption \( (\Delta U_t^h = UBF_t^h - UB_t^h) \), as shown in A9 (Silva-Risso et al. 1999)

\[
E(\Delta D_{jt}^h) = E(D_{jt}^h) - E(BFD_{jt}^h) + \Delta U_t^h \tag{A9}
\]

UBF is the consumption rate for the simulated baseline plus forward buying, and UB is the consumption rate for the baseline. Baseline plus forward buying demand are the function of inventory and consumption level for the case that promotions resulted only in forward buying through purchase acceleration and/or stockpiling. Therefore, we remove choice effect in the baseline plus forward buying model. Non-incremental demand (baseline plus forward buying) can be estimated by setting no promotion and no purchased feedback in the choice model, and no-incremental consumption as shown in A10 (Silva-Risso et al. 1999)
\[ E(BFD_{jt}^h) = P_t^h(inc)|_{t_t^hIBF_t^h} \times P_t^h(j|inc)_{\text{No promotion}} \times E(D_t^h|D_t^h > 0)_{|t_t^hIBF_t^h} \]  
\[ (A10) \]

\[ IBF_t^h \] is the household’s inventory given that promotion effect in the choice model is removed, No purchased feedback eliminates carryover effects (last brand purchased) in the choice model.

The expected baseline is given by (Silva-Risso et al. 1999)

\[ E(BD_{jt}^h) = P_t^h(inc)_{\text{No promotion}} \times P_t^h(j|inc)_{\text{No promotion}} \times E(D_t^h|D_t^h > 0)_{\text{No promotion}} \]  
\[ (A11) \]

BD is baseline volume and \( IBF_t^h \) refers to the household’s inventory for the case of no promotions.

<table>
<thead>
<tr>
<th>Parameter for consumer response model</th>
<th>Purchase incidence model</th>
<th>Choice model</th>
<th>Quantity model</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \beta_0 = -5.2562 )</td>
<td>( \alpha_{b1} = 0.4537 )</td>
<td>( \mu_{b1} = 0.0140 )</td>
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<td>( \beta_1 = 5.2590 )</td>
<td>( \alpha_{b2} = 0.8096 )</td>
<td>( \mu_{b2} = -0.1356 )</td>
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<td>( \beta_2 = -0.0201 )</td>
<td>( \alpha_{b3} = 0.7432 )</td>
<td>( \mu_{b3} = -0.2888 )</td>
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<td>( \beta_3 = 0.3338 )</td>
<td>( \alpha_s = -0.4521 )</td>
<td>( \mu_s = -0.0146 )</td>
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<tr>
<td>( \theta_1 = 1.9085 )</td>
<td>( \omega_1 = 0.3153 )</td>
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<tr>
<td>( \theta_2 = 0.9154 )</td>
<td>( \omega_2 = -0.0097 )</td>
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<td>( \theta_3 = 2.5672 )</td>
<td>( \omega_3 = 0.0428 )</td>
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<td>( \theta_4 = 0.3876 )</td>
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<tr>
<td>( \theta_5 = -4.156 )</td>
<td>( \omega_5 = -0.0770 )</td>
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<td>( \theta_6 = 0.4752 )</td>
<td>( \omega_6 = 0.3239 )</td>
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<td>( \theta_7 = 1.2259 )</td>
<td>( \omega_7 = 0.5517 )</td>
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<tr>
<td>( \theta_8 = 1.1042 )</td>
<td>( \omega_8 = -0.0686 )</td>
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Source: Silva-Risso et al. 1999

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References


