Coordination and Integration in Supply Chain Planning: Studies on Sales and Operations Planning and Supply Chain Network Design

PhD dissertation

Agus Darmawan

Supervisor: Hartanto Wijaya Wong
Co-supervisor: Anders Thorstenson

Aarhus BSS, Aarhus University
Department of Economics and Business Economics

2018
Acknowledgements

This dissertation was prepared during my PhD study at Cluster for Operations Research, Analytics and Logistics (CORAL), Department of Economics and Business Economics, School of Business and Social Sciences, Aarhus University in the period from 2015 to 2018. This dissertation would not have been possible without the support of many people. Therefore, it is a pleasure to be able to express my gratitude to everyone who has given support and assistance.

First, I would like to express my deepest gratitude to my supervisor Associate Professor Hartanto Wijaya Wong for his support, advice, and insights during my PhD and the writing up of this dissertation. His understanding, encouraging and personal guidance, especially during my difficult moments have made this long journey possible. Also, I would like to thank my co-supervisor Professor Anders Thorstenson for his advice, guidance and support. His wide knowledge has been great value to me in completing this research work.

I am also grateful to the assessment committee members Associate Professor Marcel Turkensteen, Associate Professor Julia Pahl, and Associate Professor Renzo Akkerman for their suggestions and comments.

I want to express my gratitude to Directorate General of Resources for Science, Technology and Higher Education (DGRSTHE) of the Republic of Indonesia for the funding during my study. My gratitude also goes to Aarhus University for supporting me in numerous courses, conference trips as well as my stay abroad.

I would like to thank Professor El-Houssaine Aghezzaf for hospitality and his valuable suggestions during my research visit at Department of Industrial Systems Engineering and Product Design, Ghent University, Belgium.

I express my sincere thanks to the Department of Economics and Business Economics especially to the Cluster for Operations Research, Analytics and Logistics (CORAL) for providing a great research environment. I am also thankful to the administrative staff for their support and assistance during my PhD, especially: Ingrid Lautrup, Betina Sørensen, Susanne Christensen, Christel Mortensen, Malene Vindfeldt Skals and Anne Arnfeldt Källberg. I am also grateful to all my
colleagues at CORAL, current and former PhD students: Ata Jalili Marand, Maryam Ghoreishi, Lone Kiilerich, Hani Zbib, Parisa Bagheri Tookanlou, Maria Elbek, Samira Mirzaei, Viktoryia Buhayenko, Sune Lauth Gadegaard, and Reza Pourmoayed for a friendly work environment and insightful discussions.

I am greatly indebted to my parents, brothers, and sister for their encouragement and support. I also want to give my special thanks to my wife Sriyanti, and my sons Aptadika Darmawan, Aldriandika Darmawan for their love, patience, understanding and everlasting support. Finally, I would like to thank everyone else who provided me with advice, support and assistance throughout my study.

Agus Darmawan
Aarhus, 2018
Summary

The central theme of this dissertation is coordination and integration in supply chain planning, focusing on sales and operations planning and supply chain network design. The dissertation consists of four self-contained papers that address different issues related to internal coordination in sales and operations planning (Paper 1 and Paper 2) and external coordination in supply chain network design (Paper 3 and Paper 4).

In the first paper, we address coordination between the marketing and operations functions in the development of an integrated promotion and production plan in sales and operations planning (S&OP). Contrary to most of the existing studies that use a simple demand model, this paper considers an advanced demand model that takes into account how demand is affected by not only price but also other marketing factors such as customer loyalty, purchase frequency, consumption, and purchase rate. Such a rich demand model allows for a more comprehensive assessment of the benefits of an integrated approach in S&OP. In particular, the paper demonstrates that coordination can generate significant improvements in profits. The results also reveal the effects of various marketing and production-related factors, as well as their interactions, on the benefits of coordination, which are not possible to explore in the existing previous studies.

The second paper is an extension of the first paper where we consider coordination in S&OP in the case where the manufacturing firm sells a family of similar products. The existence of product substitution/cannibalization becomes an essential issue when determining the number and timing of promotions as well as the production plan. Although a promotion eventually increases the demand for the promoted product, its effect on the demand of the non-promoted product cannot be neglected since some of the customers divert their choice to the promoted product. The adoption of a demand model that captures consumption, brand switching and forward buying in this paper supports the development of an integrated S&OP for multiple products. Due to the large solution space, we develop and evaluate heuristic approaches for generating a good promotion plan to be integrated with the mixed integer aggregate production planning model. In relation to the promotion of multiple products, our numerical results show no evidence of strong preference for implementing simultaneous promotions in general.
In the third paper, we study the effect of external coordination in the context of supply chain network design (SCND). The network considered consists of retailers located downstream in the supply chain that are supplied by warehouses which get supplies from a supplier located upstream in the supply chain. The design problem consists of the location-transportation and inventory sub problems, and the objective is to minimise the expected total cost that is the sum of the costs for opening and operating warehouses, the transportation cost, and the inventory-related cost while meeting the requirement that a target fill rate must be met at all the retailers. Allowing coordination between the warehouses and retailers in making joint inventory decisions represents an important addition to the existing studies in the literature. Due to the complexity of the optimisation problem in an integrated approach, we develop a heuristic method based on a Genetic Algorithm. The numerical results show that significant cost savings can be obtained by implementing coordinated inventory control, thereby affirming the importance of enhancing integration in supply chain network design.

The fourth paper is an extension of the third paper and considers supply chain network design in the presence of disruption risks. We show that a network design neglecting disruption risks may lead to poor service levels. Therefore, choosing an appropriate recovery policy is important for maintaining an adequate service level. In this paper, we introduce a two-stage approach where we first solve the SCND problem by ignoring the presence of disruption. In the second stage, we refine the decisions in the first stage using simulation-optimisation. We consider a combination of a proactive strategy and a reactive strategy, where the former corresponds to the design solution before the disruptions, and the latter corresponds to the reconfigured plan after the disruptions. The results show that the proposed approach is able to make the network more resilient.

The specific modelling frameworks presented in this dissertation to address some of the challenges in the development of a more integrated supply chain plan have not been studied before. The results obtained rearticulate the importance of coordination in supply chain planning.
Det centrale tema for denne afhandling er koordinering og integration i planlægningen af forsyningskæden med fokus på *sales and operations planning* (S&OP) og netværksdesign. Afhandlingen består af fire selvstændige artikler, der behandler forskellige problemstillinger i forbindelse med intern koordinering af S&OP (artikel 1 og 2) og ekstern koordinering af netværksdesignet i forsyningskæden (artikel 3 og 4).


De specifikke modeller, der præsenteres i denne afhandling og som omhandler nogle af udfordringerne med udviklingen af en integreret plan for forsyningskæden, er ikke blevet undersøgt tidligere. De fundne resultater understreger betydningen af koordinering i planlægningen af forsyningskæden.
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Chapter 1

Introduction
1.1 Background and motivation

Most organisations including manufacturing companies, wholesalers, and retailers implement supply chain planning - the administration of supply-facing and demand-facing activities - as an attempt to minimise mismatches and thus create and capture value (Oliva and Watson, 2011). In other words, the main purpose of supply chain planning is to improve performance in operations and to better align operations and supply chain partners with business strategy (Jonsson and Holmström, 2016). Supply chain planning is a critical component of any business's supply chain management, which is a broader concept that covers the planning, executing, monitoring, and controlling activities by a firm or a group of firms in order to maximise supply chain performance and achieve sustainable competitive advantage (Stadtler, 2005; Bozarth and Handfield, 2016). Excellent supply chain planning has contributed to the success of numerous companies such as Amazon, Apple, Cisco, Coca-Cola, Dell, H&M, Intel, Nike, Procter & Gamble, Starbucks, and Wall-Mart (Kozlenkova et al., 2015; Chopra and Meindl, 2016).

The organisational perspective of supply chain planning covers both cross functional (internal) and supply chain (external) coordination (Tuomikangas and Kaipia, 2014). Since different functional areas such as marketing, operations, and finance are interrelated, supply chain planning requires a cross-functional effort. When the goals of the functional areas are conflicting, every decision within a firm becomes a territorial battle (Tate et al., 2015) so that matching supply and demand becomes more challenging, and one often sees poor performances reflected in operational inefficiencies and unsatisfied customers. This internal coordination idea can be generalized to external coordination where different organisations (e.g., manufacturers, wholesalers, and retailers) work together to maximise the overall supply chain performance. In the operations management literature, this type of coordination is also often referred to as integration. Integration ensures seamlessly linking relevant business processes and reducing unnecessary parts of the processes within and across firms (Barratt, 2004; Chen et al., 2009; Oliva and Watson, 2011). While the two terms – coordination and integration - can be used interchangeably in general, in some instances in this dissertation, the terms are used to distinguish between organisational decision making format, on the one hand, and certain model specifications, on the other hand.

In supply chain planning, there are three planning levels differentiated by the time horizon: strategic, tactical, and operational. The decisions at the strategic level have long-lasting effects, which may include decisions in, for example, designing a supply chain network configuration that
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involves the number, location, capacity, and technology of the facilities. On the tactical level, the decisions may include the aggregate production quantities, stock levels, material flows for processing and distribution of products. Finally, short-term decisions are executed on the operational level which may involve production volume of an item, transportation orders, and purchase orders (Van Landeghem and Vanmaele, 2002; Sodhi, 2003; Santoso et al., 2005).

Strategic supply chain planning should combine aspects of business-strategy formulation and aspects of tactical and operational planning to attain the most valuable planning effort (Sodhi, 2003). Coordination between the different planning levels would be valuable in achieving the overall corporate goal of maximising shareholder value – which in supply chain planning, is often translated into minimising cost subject to satisfying a target customer satisfaction level – more effectively than using either planning level in isolation.

Globalisation, market uncertainty and increasing supply chain complexity force companies to have a distinct supply chain capability (Bleda et al., 2017) where joint improvements of intra- and inter-organisational processes and supply chain partnership should be given high priorities (Zhao et al., 2008; Flynn et al., 2010; Hu and Monahan, 2015). On the positive side, the recent developments in information technologies and business analytics open up many opportunities for fostering the coordination (Boulton, 2015; Sanders, 2016). The right technology with a decision support system that accommodates information sharing and joint decisions will coordinate the supply chain more effectively. The exchange of demand information and supply visibility within and across organisations will drive the supply chain integration (Barratt and Barratt, 2011; Wallace and Choi, 2011; Turkulainen et al., 2017; Vanpoucke et al., 2017; Chaudhuri et al., 2018). Intrigued by the above challenges and opportunities, this PhD dissertation presents new approaches and insights about how enhanced internal or external coordination as well as coordination between the different planning levels can address some of the challenges in current supply chain planning practices.

1.2 Focus of this study

To limit the scope of the research, this dissertation focuses on two main topics related to supply chain planning. The first topic is sales and operations planning (S&OP), and the emphasis within this topic is the internal coordination between the marketing and operations functions within a firm. The second topic is supply chain network design that involves coordinated inventory control
in the supply chain. Since the decisions in a network design are strategic while the inventory
decisions are tactical, we also consider coordination between two different planning levels.

**Coordination and integration in sales and operations planning (S&OP)**

Functional areas such as marketing and operations should bring the supply-facing and demand-
facing activities together to gain an opportunity for efficiency and value creation (Oliva and
Watson, 2011; Tate et al., 2015). While marketing is the creation of customer demand, operations
management is the supply and fulfilment of that demand (Ho and Tang, 2004). There has been
growing attention in the literature on the coordination between marketing and operations. Tang
(2010), in his extensive review, advocates the need to move from the traditional interface where
marketing and operations just focus on revenues and costs, respectively, to an integrated
perspective on marketing and operations decisions.

Sales and operations planning (S&OP) is one of the core business processes in supply chain
planning to balance customer demand with supply capabilities in the medium term, and it certainly
accommodates coordination between marketing and operations as outlined above. Such
coordination should help firms develop, for example, a joint promotion and production plan that
maximises profitability. Though the advent of advanced planning and scheduling systems (APS)
facilitates enhanced coordination in S&OP, most of the current practices observed today are still
far from the fully integrated planning. Each functional area typically only prepares a preliminary
plan and uses reconciliation meetings for resolving possible disagreements (Ivert and Jonsson,
2014; Hinkel et al., 2016).

Our literature review suggests that there have been numerous studies proposing models for an
integrated promotion and production planning (e.g., Sogomonian and Tang, 1993; Ulusoy and
Yazgac, 1995; Affonso et al., 2008; Feng et al., 2008; González-Ramírez et al., 2011; Lusa et al.,
2012; Bajwa et al., 2016; Mishra et al., 2017). However, one major drawback of these studies is
that they use simple demand models to capture the relationship between demand and price such
that there are many aspects related to the joint promotion and production decisions that remain
unexplored. To overcome this drawback, we adopt a richer demand model widely used in the
marketing literature that captures the dynamics and heterogeneity of consumer response to price
promotions by simulating purchase incidence, consumer choice, and quantity decisions as well as
household’s inventory level. We integrate such a rich demand model with a mixed integer linear
programming based aggregate production planning model. This integration allows us to develop a market demand simulator that facilitates the examination of the effect of marketing-related factors (e.g., seasonality, promotion impact, discount level, loyalty, and pass through rate) and production-related factors (e.g., flexibility and production cost), as well as the mutual dependence between them. The model is also capable of identifying the potential competition and cannibalization between products.

Coordination and integration in supply chain network design

As a supply chain is a network of entities (suppliers, manufacturers, warehouses, and retailers), supply chain network design (SCND) is a good example of supply chain planning activities where coordination and integration need to be extended beyond a single organisation. SCND decisions may involve e.g., the assignment of facility, location of manufacturing, storage, or transportation-related facilities, and the allocation of capacity and markets to each facility (Chopra and Meindl, 2016). External collaboration between entities is needed to achieve high efficiency and adequate customer service level. There has been growing attention in the past decades toward the optimisation of strategic and tactical planning. In the context of network design, while the location and capacity of facilities may represent strategic decisions, the inventory deployment in the network can be considered a tactical decision.

Several studies addressing inventory deployment in network design are available in existing literature. Some of them only consider the inventory problem at a single echelon (see e.g., Miranda and Garrido, 2004; Croxton and Zinn, 2005; Sourirajan et al., 2009; Yao et al., 2010; Liao et al., 2011; Shahabi et al., 2013). Another stream considers inventory deployment in more than one echelon (see e.g., Farahani and Elahipanah, 2008; Kang and Kim, 2012; Tancrez et al., 2012; Kumar and Tiwari, 2013; Askin et al., 2014; Manatkar et al., 2016; Taxakis and Papadopoulos, 2016; Puga and Tancrez, 2017). Common in all the aforementioned studies is that the inventory control parameters are optimised independently without taking into account any coordination between different stages in the network. Hence, there is a void in the literature addressing coordinated inventory control in supply chain network design that this PhD dissertation will try to fill. Considering coordinated inventory control in supply chain network design would require a more sophisticated model but is expected to generate further cost savings.
Introduction

Nowadays, resilience has become an interesting topic along with the existence of disturbances or disruptions in the supply chain (Snyder et al., 2016; Ivanov et al., 2017; Dolgui et al., 2018). Tukamuhabwa et al. (2015) define resilience as the adaptive capability of a network to respond to disruptions in a timely and cost effective manner. Disruption can be caused by natural disasters (e.g., floods, earthquakes, hurricanes, fires) or man-made disasters (e.g., industrial accidents, labour strikes). Simchi-Levi et al. (2008) refer to such disruption as the “unknown-unknown” type of risks that are hard to quantify due to the lack of data. The other type of risks caused by, for example, forecast accuracy or operational problems, is referred to as the “known-unknown”. Of course, due to their nature, dealing with the “unknown-unknown” is more difficult than dealing with the “known-unknown”. Using supply chain network design with coordinated inventory control as a departure point, our further interest is to identify recovery strategies in dealing with disruption such that the supply chain can become more resilient. A recovery (mitigation) strategy includes a set of possible actions that should be taken to reduce the impact of unexpected events in supply chain operations (Chopra and Sodhi, 2004).

1.3 Structure of dissertation

This dissertation consists of four self-contained papers, two of which have been presented in international conferences. The first paper has been published in the International Journal of Production Research (IJPR), and the other three papers are under review, and/or in the final editing stage for possible publications in international journals. All the papers address and highlight the importance of coordination and integration in supply chain planning. While the first two papers (Paper 1 and Paper 2) focus on the first topic concerning coordination and integration in S&OP, the other two papers (Paper 3 and Paper 4) are related to the second topic concerning coordination and integration in supply chain network design. Figure 1.1 summarises the main issues addressed in each of the four papers and highlights how each paper differs from the other papers. The four papers can be differentiated based on: (a) organisational aspect of coordination – whether it is internal or external; (b) planning level – whether it is only tactical or strategic and tactical; (c) number of products – whether we deal with a single product or multiple products; and (d) type of risks – whether we deal with the “known-unknown” or both the “known-unknown and “unknown-unknown”. 
Introduction

Coordination in SC planning

<table>
<thead>
<tr>
<th>Organisational</th>
<th>Planning level</th>
<th>Number of products</th>
<th>Type of risks</th>
</tr>
</thead>
<tbody>
<tr>
<td>Internal (S&amp;OP)</td>
<td>Tactical</td>
<td>Single</td>
<td>Known-unknown</td>
</tr>
<tr>
<td>External (SCND)</td>
<td>Strategic &amp; Tactical</td>
<td>Multiple</td>
<td>Known-unknown &amp; unknown-unknown</td>
</tr>
</tbody>
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Figure 1.1 The dissertation positioning

1.4 Highlights of each paper and main contributions

In this section, we present the most important highlights of each paper and summarise how the paper advances the current literature.

**Paper 1: Integration of promotion and production decisions in sales and operations planning**

In this paper, we address tactical supply chain planning through sales and operations planning (S&OP). In balancing supply and demand, we develop an integrated framework that enables the development of a joint promotion and production plan aiming at maximising profitability. We adopt a rich demand model from the marketing literature that takes into account purchase incidence, brand choice, and purchase quantity. Depending on marketing information (e.g., price, advertisement) and shopping histories (e.g., consumption and purchase rate, loyalty, and purchase frequency), the demand model captures the effect of stockpiling and repeat purchases from the customer. As a result, the model allows us to decompose the total increase in demand due to a promotion into true incremental demand and forward buying.
We compare two different approaches, namely the non-coordinated and the coordinated approaches. In the non-coordinated approach, decisions are made sequentially where we first determine the best promotion decisions and then solve the aggregate production planning problem. Whereas in the coordinated approach, we jointly optimise the promotion and production decisions. Through an extensive numerical study, we examine the effect of marketing-related factors (e.g. seasonality, promotion impact, discount level, loyalty, and pass through rate) and production-related factors (e.g., flexibility and production cost).

The main contribution of this paper is that we propose a new framework for developing a sales and operations plan that allows for joint price promotion and production decisions. To our knowledge, this is the first paper that integrates an advanced demand model with the standard aggregate production model in S&OP. The proposed modelling framework is able to show the effect of marketing and production related factors as well as their interactions.

**Paper 2: Integrated sales and operations planning with multiple products: Jointly optimising the number and timing of promotions and production decisions**

In this study, we extend Paper 1 by considering a situation where a firm sells multiple products manufactured using the same production resources. In addition to external competition, the firm must also consider internal competition or potential cannibalization of its own products. The rich demand model adopted allows us to capture the presence of such internal competition.

We are primarily interested in studying whether promotions for the different products sold by the firm should be carried out simultaneously or in different periods. As we deal with multiple products, the optimisation problem becomes harder to solve. We develop and compare three heuristic approaches for solving the joint optimisation problem. The performances of the heuristics are promising especially when considering the development of decision support tools for an integrated S&OP.

This paper advances the existing literature by presenting a framework for an integrated S&OP with multiple products that considers product substitution or cannibalization. The possibility of examining the effects of promotion in one product on the demand of other products and the consequences on the use of common production resources will help firms make better decisions.
Paper 3: Supply chain network design with coordinated inventory control

In this paper, we study supply chain network design (SCND) with inventory deployment. We consider a network consisting of a supplier, multiple warehouses, and multiple retailers as shown in Figure 1.2.

We develop models and solution methods for supply chain network design (SCND) that not only integrate the location-transportation and inventory problems but also consider the implementation of coordinated inventory control in the network. In contrast to most related studies in the literature where inventory decisions at the different echelons are made separately without synchronisation, the coordinated approach adopted in this paper ensures that the expected total inventory costs are close to minimum while maintaining a predetermined service level for end customers.

We examine how the different levels of coordination influence the expected total costs. There are two approaches of how we solve the location-transportation and inventory problem. In the sequential approach, we first solve the location-transportation problem and then solve the inventory optimisation problem whereas the two problems are jointly optimised in the integrated approach. Furthermore, we compare two approaches in solving the inventory optimisation problem differentiated by whether or not coordinated inventory control is adopted such that the value of coordination can be assessed.
Introduction

This paper extends the existing literature on SCND considering inventory deployment by implementing coordinated inventory control in the network, resulting in enhanced coordination and integration. This study reveals that significant benefits can be gained when a holistic view of inventory optimisation in the network is embedded in supply chain network design. The modelling framework developed in this study can be very useful when one considers developing a decision support system for developing a more integrated supply chain network design. Our finding should motivate supply chain planners to implement this kind of integration and coordination in (re-)designing their supply chain networks.

Paper 4: Exploring proactive and reactive strategies in supply chain network design with coordinated inventory control in the presence of disruptions

In this paper, we study supply chain network design (SCND) for a single product with multiple plants (suppliers), multiple warehouses, and multiple retailers. We extend Paper 3 by introducing disruptions that may occur at any echelon in the network. The implementation of coordinated inventory control in SCND results in relatively lower stock levels at the upper echelon (warehouses) when compared to the case where the inventory decisions at every echelon are determined without coordination. This is perfectly reasonable when one considers situations where only the “known-unknown” type of risks are present. However, anecdotal examples indicate that the “unknown-unknown” type of risky events or unexpected disruptions may occur from time to time. Although it rarely happens, this kind of disruption may lead to serious consequences, unless a recovery plan is well prepared. To this end, in this paper we examine the effects of disruption and provide an evaluation of several strategies in responding to disruption.

We propose a two-stage optimisation approach in dealing with disruption. In the first stage, we solve the SCND problem with the baseline approach of ignoring the disruption. In the second stage, we use discrete event simulation to introduce disruption and fine-tune the inventory decisions at the warehouses and retailers.

In managing disruption, we combine proactive and reactive strategies. In a proactive strategy, we allow multiple sourcing in the network. We also fortify the echelons by allocating a relatively high inventory level. In a reactive strategy, we adjust the flow in the network to avoid the disrupted facility. To the best of our knowledge, this is the first paper that considers SCND with coordinated
inventory control in the presence of disruption. With the increase of the level of risk that many companies are exposed to, the results from this study provide valuable insights to supply chain practitioners into the importance of embedding risk mitigation strategies in the supply chain planning.

1.5 Limitations of the study

There are several limitations of the study presented in this PhD dissertation, which are worth addressing in future studies. In all the four papers, the strategic behaviour of the players considered in the supply chain has been neglected. The focus in the first two papers is on the coordination between production and marketing functions within a single manufacturing firm. The study can be extended by considering the interaction between a manufacturing firm and a retailer. In practice, there are scenarios where the retailer makes a decision on pass-through rate, defined as the proportion of the manufacturer’s discount that the retailer passes on to the consumer, in the interest of maximising her profit. In Paper 3 and Paper 4, we consider external coordination in the supply chain network design problems and show that coordination may result in significant cost savings. However, it is assumed that information about demands and costs is readily available to all players. Furthermore, the issue of how cost savings gained through coordination should be distributed among the different players in the supply chain network has not been examined. This issue is prevalent especially in scenarios where different facilities in the network are owned by different firms. To address this issue, future studies need to consider contractual aspects, e.g. between warehouses and retailers, such that the outcomes for all parties are clarified and the possibilities for reaching a Pareto (‘win-win’) solution are explored.

While this study has shown the importance of coordination in supply chain planning, more specifically in the contexts of sales and operations planning and supply chain network design, the extent of coordination can be extended further. It is indisputable that fostering coordination further would generate higher benefits. Future studies could include coordination with other functions such as finance, in addition to marketing and operations. The consideration of cash flow, for example, is certainly beneficial when firms develop an integrated S&OP. Likewise, considering
budget constraints in supply chain network design in the presence of disruption will result in more cost effective recovery strategies.

Finally, as the main insights in this dissertation are generated from numerical studies that mostly use hypothetical parameter values, it would be useful to conduct empirical studies to assess the applicability and generalisability of the results. Such empirical studies would also be very valuable for the development of decision tools for supply chain planners.
Chapter 2

Paper 1: Integration of promotion and production decisions in sales and operations planning

History: This chapter has been prepared in collaboration with Hartanto Wong and Anders Thorstenson. It has been published in International Journal of Production Research, 56(12): 4186–4206, 2018. The chapter has been presented at The 23rd EurOMA conference, June 2016, Trondheim, Norway.
Integration of promotion and production decisions in sales and operations planning

Agus Darmawan, Hartanto Wong and Anders Thorstenson

CORAL - Cluster for Operations Research, Analytics and Logistics

Department of Economics and Business Economics, Aarhus University

Fuglesangs Allé 4, 8210 Aarhus, Denmark

Abstract

This paper presents a new modelling framework for developing a sales and operations plan that integrates promotion and production planning decisions. We adopt a rich demand function that captures the dynamics and heterogeneity of consumer response to price promotions by simulating purchase incidence, consumer choice and quantity decisions, as well as household’s inventory level. Our numerical study reveals interesting findings on the benefits of developing an integrated sales and operations plan as well as the optimal timing and number of promotions, and more importantly, how these findings are influenced by the mutual dependence of marketing and production related factors.

Keywords: marketing-operations interface, aggregate planning, production, promotion, sales and operations planning, forward buying
2.1 Introduction

Sales and operations planning (S&OP) is a tactical planning process that helps companies to balance demand and supply and to ensure that all the plans of different business functions are synchronised (Wallace, 2006). According to APICS Dictionary (Pittman and Atwater, 2016), S&OP is defined as "... the function of setting the overall level of manufacturing output and other activities to best satisfy the current planned levels of sales, while meeting general business objectives of profitability, productivity, competitive customer lead times, inventory and/or backlog levels, etc". S&OP is also a pillar used for integrating supply chain planning processes and coordinating supply chain members (Affonso et al., 2008). As such, S&OP covers both cross-functional intra-company and supply chain intercompany coordination (Tuomikangas and Kaipia, 2014). Traditionally, the basic S&OP process has sought to facilitate the transfer of information from demand planning to master production planning. However, today, there are both requirements and opportunities to move beyond the mere synchronisation of master and demand planning towards integrated planning (Feng et al., 2010; Oliva and Watson, 2011). One important enabler of such integrated planning is the use of advanced planning and scheduling (APS) systems for S&OP. Today, we witness widespread implementation of APS systems in supporting the S&OP processes in practice, and consequently, the use of APS systems has also been extensively discussed in the literature (see e.g. Rudberg and Thulin, 2009; Ivert and Jonsson, 2014). However, as also reported in Ivert and Jonsson (2014), even though the APS systems facilitate better coordination, the integration level in the S&OP processes still seems to be limited to a sequential practice in which the sales/marketing department first prepares a preliminary plan of future sales while the production department then prepares a preliminary production plan, and the possible disagreements of the two plans are resolved through reconciliation meetings. While planners in the marketing department might be primarily concerned with e.g., the timing and level of promotion in order to maximise revenues, planners in the production department focus on production plans satisfying target sales and minimising total production-related costs. Such a planning process neglects the mutual dependence of promotions and the use of production resources, and therefore leads to a sub-optimal sales and operations plan. This practice also shows that there seems to be some challenges and barriers in implementing a fully integrated planning system. Lack of appropriate decision support models might be one of the reasons that hinder the development of fully integrated planning (Hinkel et al., 2016).
The central theme of our paper is the development of decision support for S&OP within a single manufacturing firm. We focus particularly on integration of the marketing and production planning functions with respect to coordination of price promotion and production decisions. This theme is undoubtedly of practical relevance as it is discussed extensively in various business reports. Gedenk et al. (2010) reports that 25% of all retail sales is coming from sales during price promotion campaigns. Bursa (2012) reports data from Gartner indicating that an effective S&OP process can increase revenues by 2% to 5% and reduce inventories by 7% to 15%, which is quite significant for many firms. According to Accenture’s Perfect Promotion Study (Samuel and Goldstein, 2013), based on interviews with 350 senior executives at large consumer packaged goods (CPG) companies, 43 percent believe that greater integration is required between the business functions involved in the trade promotion process. In their recent study on the S&OP maturity level in Danish companies, Lund and Raun (2017) reports that although several CEOs sponsor an integrated S&OP, many companies still lack a clear S&OP vision, so that the operations and marketing departments still develop their own separate plans. S&OP is also discussed widely in the academic literature (see e.g., Martínez-Costa et al., 2013 and the references therein).

However, our literature review confirms that previous studies have not explored in depth the benefits of adopting truly integrated planning of the marketing and production functions and therefore have not provided comprehensive insights into the mutual dependence of marketing and production related factors. Most of the existing literature considers demand models that are too simplified so that many factors affecting changes in demand due to promotions, cannot be fully examined. The existing literature also provides little help in understanding how the frequency and timing of promotions are affected by the interaction of production and marketing related factors. We aim to fill this gap in the literature by investigating a more refined modelling framework for generating sales and operations plans that integrate promotion and production planning decisions. In particular, our modelling framework features a rich demand function that captures the dynamics and heterogeneity of consumer responses to price promotions, without which the mutual dependence of marketing and production related factors can only be examined cursorily. We aim to address the following questions:

1. To what extent does integration of production and promotion decisions in an S&OP setting generate benefits for the company, and how are these benefits influenced by the interaction of production and marketing related factors?
How is the optimal timing and number of promotions influenced by the interaction of production and marketing related factors?

To this end, we integrate a disaggregate consumer response-based demand model and a mixed integer linear programming (MILP) based aggregate production planning model. Although these two models are well established separately, such an integration framework should also be highlighted as a contribution of this study. None of the existing literature presents a comprehensive model that captures the mutual dependence of production and marketing factors to the extent considered in this paper.

The demand model is able to capture the dynamics and heterogeneity of consumer response to price promotions by simulating purchase incidence, consumer choice and quantity decisions, as well as household’s inventory levels. It allows us to decompose total sales into increased consumption, brand switching and forward buying. Price promotions may attract new or existing customers to increase consumption, and influence the decision of consumers to switch from a competitor’s product, resulting in incremental sales. However, promotion can also result in non-incremental sales, as customers only shift their future purchases to the current period, i.e., they exercise forward buying. The optimal decisions in production planning, i.e., production quantities, number of workers, inventory levels, subcontracts, etc., are determined based on the demand generated as the consequences of the promotion decisions. As the composition of incremental and non-incremental sales observed during promotions will have an influence on the required production capacity, the decisions on production and the timing and level of promotions have to be made jointly. Our suggestion for an integrated decision-support model for S&OP attempts to capture these interaction effects.

The paper proceeds as follows. Section 2.2 presents our review of relevant literature. Then, in Section 2.3 we specify the integration of the demand and aggregate production planning models. In Section 2.4, we present the results of our numerical study. In Section 2.5, we conclude our study and suggest some directions for future research.


2.2 Literature review

Integration in S&OP is a topic widely studied in the literature. We refer the reader to Thomé et al. (2012) and Tuomikangas and Kaipia (2014) for comprehensive reviews. This topic is quite broad, covering possible cross-functional coordination between operations, marketing, finance, procurement, etc. More closely related to our paper, is the research stream that focuses particularly on coordination between operations and marketing. There is also a substantial and growing literature on this topic. Tang (2010) presents an extensive review and suggests that it is important to move from the traditional interface, where marketing and operations just focus on revenues and costs, respectively, to a joint perspective on marketing and operations decisions. This lends support to what we develop in this paper.

Numerous studies in the literature address the coordination of promotion and operational decisions. Many studies, however, focus on inventory rather than production planning decisions. For example, Balcer (1983) studies a dynamic inventory/advertising model in a newsvendor setting. A model considering a joint inventory and promotion planning problem in a periodic review system is presented in Cheng and Sethi (1999). Federgruen and Heching (1999) consider a joint inventory and pricing problem where the price can be dynamically adjusted. Zhang et al. (2008) extend the models of Cheng and Sethi (1999) and Federgruen and Heching (1999) by considering a joint optimisation of pricing, promotion, and inventory decisions. Kurata and Liu (2007) study the problem of optimising promotion depth (discount level) and frequency in a supplier-retailer setting, where the retailer wants to maximise expected revenues and the supplier tries to minimise expected inventory costs. More recently, Albrecht and Steinrücke (2017) study sales planning of perishable goods where sales price decreases with decreasing quality of goods. All the above studies consider a simple relation between price and demand. Similar to our approach that considers forward buying from customers, Huchzermeier et al. (2002) uses a customer choice model that captures how customers react to retail promotions through stockpiling and package size switching. The authors show that by capturing customer response to price promotions, the retailer can reduce inventory costs. The demand model we adopt in our paper is more comprehensive than theirs. The main distinguishing feature of our paper to the above studies is, in addition to the richer demand model we adopt, that we also address the aggregate production planning problem of manufacturing firms in which the decisions made are not limited to inventories only.
Most closely related to our study are those articles presenting models that analyse joint promotion and production planning. Martínez-Costa et al. (2013) present a comprehensive review of the literature on the integration of marketing and production decisions in aggregate planning, which is also the central theme of our paper. Leitch (1974) develops a multi-period model to optimise a production and advertising plan. The advertising is mainly intended to lower the production costs by smoothing seasonal product demand. Sogomonian and Tang (1993) present a modelling framework for coordinating promotion and aggregate planning within a firm. In their demand function, demand in each period depends on the time elapsed since the last promotion, the level of the last promotion, and the price at the corresponding period. Ulusoy and Yazgac (1995) present an aggregate multi-product, multi-period production planning model that considers the use of pricing and advertising as tools for smoothing the demand. They use a rather simple demand function, where demand is assumed to be inversely proportional to price and directly proportional to the advertising efficiency. Feng et al. (2008) propose a mixed integer programming model to integrate sales and production, and capture demand uncertainty by assuming that demand and price are normally distributed. Affonso et al. (2008) show the benefit of collaboration in the development of S&OP. In their model, they assume that the forecasted demands from the sales department are given and constant. Since there is no explicit demand function in their model, they perform demand perturbations in their numerical experiment. Other papers that integrate production and marketing decisions are by González-Ramírez et al. (2011), Lusa et al. (2012), Bajwa et al. (2016), and Mishra et al. (2017). Although we share the basic idea with the above mentioned authors with respect to integration of production and marketing decisions, the simple demand models adopted in their papers are not really helpful in answering the main research questions postulated in our paper. We extend these previous contributions in two respects. Firstly, the adoption of a rich demand model allows us to examine the interaction of different factors beyond what is covered by the existing literature. We examine the effects of several marketing-related factors such as seasonality, promotion impact, promotion discount level, loyalty rate, and pass through rate together with production-related factors such as flexibility and production costs on the optimal integrated production and promotion plan. Moreover, we examine the possible interaction between those factors. The results of our numerical study provide useful information about, e.g. conditions under which the integration of promotion and production planning gives the most benefits, and how the optimal timing and number of promotions are influenced by the above mentioned factors. As pointed out by Martínez-Costa et al. (2013), there is a lack of research that considers a richer
demand model that can be used to develop a decision-support tool for the joint S&OP process. Our research attempts to fill this void in the literature.

Meanwhile, promotion planning is a topic that has been studied extensively in the marketing literature. Promotion is one of the marketing–mix variables that can be used for maintaining sales volume, increasing growth, and attracting consumers’ attention (Steenkamp et al., 2005; Kotler and Armstrong, 2013). Lack of decision-support systems to address the complexity and dynamics of promotion planning has motivated some authors (e.g., Tellis and Zufryden, 1995; Silva-Risso et al., 1999; Ailawadi et al., 2007) to develop a retail promotion planning model that is based on a disaggregate consumer response model. While such a model is quite useful in capturing the dynamics and heterogeneity of consumer response to price promotions, most studies in the marketing literature focus on maximising profits viewed only from a marketing perspective. They thereby ignore the interdependence of the resulting promotion plan and production-related (cost) factors. We contribute to this stream in the marketing literature in two aspects. First, we introduce a framework of how to integrate the disaggregate consumer response model with the aggregate production planning model. Second, our results clearly show the weakness of the sequential (non-integrated) approach. They challenge the standard approach predominantly used in the marketing literature by showing that developing a promotion plan that neglects production related factors may result in profits significantly lower than those obtained using the integrated approach.

Our literature review shows that previous research in each of the operations and marketing literature has its own deficiencies, but that they are complementary to each other. In light of this conclusion, and motivated by the needs for further developments of decision support for S&OP processes, our paper aims to integrate research in these two disciplines.

### 2.3 The integrated models framework

This section is comprised of three parts. The first part explains the disaggregate consumer response-based demand model used for generating the demand forecast. In the second part, we present the model used to generate the optimal aggregate production plan given the demand forecast. The final part presents the joint optimisation problem utilizing the above two models in combination to obtain the optimal promotion and production plan. As a benchmark, we also present the sequential optimisation problem, where the promotion plan is first optimised by marketing
planners, and the optimal production plan is subsequently optimised by production planners based on this promotion plan.

2.3.1 The demand model

Below we explain the main principles used for building the demand model. We use an integrated purchase incidence-brand choice-purchase quantity model that is widely used in the marketing literature (see e.g., Guadagni and Little, 1983; Bucklin and Lattin, 1992; Silva-Risso et al., 1999; Ailawadi et al., 2007). The model captures three factors that are related to a customer’s purchase decision: the timing of the purchase, the type of product/brand, and the quantity to purchase. The main idea of the model is that conditional on a store visit, a household (consumer) will decide whether to buy in the product category. For any given purchase decision, the household then chooses a brand alternative, and finally determines the purchase quantity. Given a store visit at time $t$, the expected demand for brand $j$ from household $h$ is given by

$$E(D_{jt}^h) = P_t^h(inc) \times P_t^h(j|inc) \times E(D_{jt}^h|D_{jt} > 0)$$  \hspace{1cm} (2.1)$$

where

- $P_t^h(inc)$: The probability that household $h$ makes a purchase in the product category on a store visit in time period $t$
- $P_t^h(j|inc)$: The probability that household $h$ chooses brand $j$, given that household $h$ decides to make a purchase in the product category in time period $t$
- $E(D_{jt}^h|D_{jt} > 0)$: The expected quantity that household $h$ will buy of brand $j$, given that household $h$ decides to purchase brand $j$ in time period $t$

Note that in the above formulation, we consider a particular brand offered by the manufacturer that is in competition with other brands offered by competitors, and assume that the brand has only one size. However, the model can also be adopted in situations where competition occurs between brand sizes or SKUs. In what follows, we present the three main building blocks: the brand choice model, the purchase incidence model, and the quantity model. Details of the model specifications including the required parameters are presented in the Appendix. In this modelling framework, the purchase incidence and brand choice can be modelled as a nested logit model. Because the
decision to purchase is influenced by the parameter value from the brand choice model (Equation A2.5 in the Appendix), we discuss the brand choice model first and the purchase incidence model afterwards. The brand choice is handled in a multinomial logit framework, which gives the probability that a household will choose the particular brand after deciding to purchase a product. The brand choice probability can be written in the following form:

\[ P_t^h(j|\text{inc}) = \frac{e^{A_{jt}^h}}{\sum_j e^{A_{jt}^h}} \]  

(2.2)

where \( A_{jt}^h \) is the deterministic component of utility associated with brand \( j \) for household \( h \) in time period \( t \) that is modelled as a function of price, promotion and consumer-specific variables such as brand and size loyalty (see Appendix for details). The purchase incidence probability is modelled with a binary nested logit model:

\[ P_t^h(\text{inc}) = \frac{e^{C_t^h}}{1 + e^{C_t^h}} \]  

(2.3)

where \( C_t^h \) is the deterministic component of utility associated with household \( h \) in time period \( t \). It is modelled as a function of the proportion of purchase frequency, household inventory, and consumption rate (see Appendix for details). Next, given a purchase of brand \( j \), the number of units purchased is captured by a zero-truncated Poisson distribution, which gives the expected number of units purchased by household \( h \) at time \( t \):

\[ E(D_{jt}^h|D_{jt}^h > 0) = \frac{\lambda_{jt}^h}{1 - e^{-\lambda_{jt}^h}} \]  

(2.4)

where \( \lambda_{jt}^h \) is the purchase rate of household \( h \) for brand \( j \) at time \( t \), which is modelled as a function of the average number of units purchased, household inventory, brand loyalty, and price.

As for model application, parameter values of the model are usually estimated based on scanner-panel data obtained from sources such as Nielsen Consumer Panels for certain product categories, e.g., canned tomato sauce (Silva-Risso et al., 1999) or ketchup and yoghurt (Ailawadi et al., 2007). The store scanner data gives the information of the marketing environment such as price, discount level, retailer’s pass-through, retailer’s mark-up, etc. The panellist data provides information regarding the household shopping trip, loyalty to brand, purchase histories, and consumption rate. Such data are commonly in commercial use today. With the growing importance of business analytics and significant improvements in data-collecting technologies, data like these will become
even more widely available in the future. This lends further support to the development of data-driven decision support models like the one presented in this paper (Sanders, 2016). We summarise the various elements of the demand model in Figure 2.1.

![Figure 2.1 The elements of the demand model](image)

The next step is using Monte Carlo simulation to generate the dynamics of consumer response, which is a common approach in the marketing literature (Silva-Risso et al., 1999; Seetharaman, 2004; Ailawadi et al., 2007) The simulation generates the dynamics of aggregate demand from the households and is able to capture the effect of stockpiling and repeat purchases.

As an illustration, suppose we have calculated the purchase probability $P_t^h(inc)$ based on the available parameter values. A household purchase occurs in the simulation if the random number generated (between 0 and 1) is less than $P_t^h(inc)$. The logic of the demand model can be seen in Table 2.1. The output of the model is a demand forecast $D_{jt}$ for brand $j$ in time period $t$, which
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represents an aggregate household demand for each period: \( D_{jt} = \sum_{h=1}^{H} E(D_{jt}^h) \), where \( H \) is the number of households simulated. In order to make a fair comparison between different promotion schemes, we employ the same seed number in the simulations, allowing us to use the same set of random numbers (demands). Note that in Table 2.1 we assume that there are two brands, i.e., \( j = 1, 2 \), and the manufacturer is interested in the demand of its product brand (say, \( j = 1 \)).

Table 2.1 Algorithm: Demand model

<table>
<thead>
<tr>
<th>Initialisation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Set timing and promotion levels (i.e., ( L_{jt} ), discount level for brand ( j ) in time period ( t ))</td>
</tr>
<tr>
<td>For ( h=1 ) to ( H ) ( \leftarrow ) household</td>
</tr>
<tr>
<td>For ( r=1 ) to ( R ) ( \leftarrow ) replication</td>
</tr>
<tr>
<td>For ( t=1 ) to ( T ) ( \leftarrow ) time period</td>
</tr>
<tr>
<td>Calculate ( P_t^h(inc) ), ( P_t^h(j</td>
</tr>
<tr>
<td>Generate rand1</td>
</tr>
<tr>
<td>If ( \text{rand1} &lt; P_t^h(inc) ) then buy ( \leftarrow ) household</td>
</tr>
<tr>
<td>Generate rand2</td>
</tr>
<tr>
<td>If ( \text{rand2} &lt; P_t^h(j = 1</td>
</tr>
<tr>
<td>Else if ( P_t^h(j = 1</td>
</tr>
<tr>
<td>End if</td>
</tr>
<tr>
<td>Calculate ( E(D_{jt}^h</td>
</tr>
<tr>
<td>Else set brand=0, ( D_{jt}^h = 0 )</td>
</tr>
<tr>
<td>( D_{jt}^h = D_{jt}^h )</td>
</tr>
<tr>
<td>End for ( t )</td>
</tr>
<tr>
<td>End for ( r )</td>
</tr>
<tr>
<td>Calculate ( \sum_{r=1}^{R} D_{jt}^h )</td>
</tr>
<tr>
<td>End for ( h )</td>
</tr>
<tr>
<td>Calculate total expected demand, ( D_{1t} = D_t = \sum_{h=1}^{H} E(D_{jt}^h) )</td>
</tr>
</tbody>
</table>

The disaggregate consumer response-based demand model described above allows us not only to estimate the direct demand impact of promotion decisions, but also to decompose incremental demand into true incremental demand and demand borrowed from the future, commonly termed forward buying. In Section 2.4, we discuss a method used in the literature (Silva-Risso et al., 1999) for decomposing incremental demand.
2.3.2 Aggregate production planning model

The aggregate production planning is an activity of tactical planning which attempts to balance demand and supply in an effort to streamline operations and increase productivity. Given the demand forecast for each period in the planning horizon, the aggregate production planning model determines the production level, inventory level, and capacity level (internal and outsourced) for each period that minimise the total costs over the planning horizon (Chopra and Meindl, 2016). We follow a standard mixed integer linear programming model to solve the aggregate planning problem (see e.g., current textbooks like Bozarth and Handfield, 2016; Chopra and Meindl, 2016; Heizer et al., 2016; Silver et al., 2017).

Parameters:

- $c_p$: Production cost per unit (including materials; excluding labour cost)
- $c_l$: Regular labour cost per worker and time period
- $c_h$: Hiring cost per worker
- $c_f$: Firing cost per worker
- $c_o$: Overtime cost per hour
- $c_{Inv}$: Inventory holding cost per unit per time period
- $c_s$: Subcontracting cost per unit
- $D_t$: Demand forecast for time period $t$ (index for brand $j$ is suppressed here, as we are only interested in a single brand for the manufacturer)
- $L_L$: Minimum number of workers
- $U_L$: Maximum number of workers
- $w_{ht}$: Number of regular working hours available per worker in time period $t$
- $n_{lo}$: Number of workers at the beginning of the planning horizon
- $n_{u}$: Number of units produced per hour
- $O_t$: Maximum overtime hours available per worker in time period $t$
- $T$: Planning horizon in number of time periods
- $S_{St}$: Safety stock requirement in time period $t$
- $I_o$: Beginning inventory

Decision variables:

- $q_{pt}$: Number of units produced during regular time in time period $t$
- $q_{ot}$: Number of units produced during overtime in time period $t$
- $q_{st}$: Number of units produced using subcontracting in time period $t$
- $n_{ht}$: Number of workers hired at the beginning of time period $t$
- $n_{ft}$: Number of workers fired at the beginning of time period $t$

Other variables:
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\[ \text{Min}_{q_p, q_o, q_s, n_t, n_f} \quad TCOST = \sum_{t=1}^{T} \left( (q_p + q_o) \cdot cp + \frac{q_o}{nu} \cdot co + q_s \cdot cs + l_t \cdot clnv + n_t \cdot cl + nh_t \cdot ch + nf_t \cdot cf \right) \]  

Subject to:

\[ I_t = I_{t-1} + q_p + q_o + q_s - D_t \quad t = 1, ..., T \]  

\[ SS_t \leq I_t \quad t = 1, ..., T \]  

\[ nl_t = nl_{t-1} + nh_t - nf_t \quad t = 1, ..., T \]  

\[ nl_0 = nl_T \]  

\[ LL \leq nl_t \leq UL \quad t = 1, ..., T \]  

\[ qp_t \leq nl_t \cdot wh_t \cdot nu \quad t = 1, ..., T \]  

\[ qo_t \leq nl_t \cdot Ot \cdot nu \quad t = 1, ..., T \]  

\[ l_t, q_p, q_o, q_s, nl_t, nh_t, nf_t \geq 0 ; \quad nl_t, nh_t, nf_t \text{ integer} \quad t = 1, ..., T \]

The aggregate production planning problem can now be written as

The objective function in (2.5) is minimising the total cost, consisting of the production cost, overtime cost, subcontracting cost, inventory cost, labour cost, and hiring/firing cost. Demand \( D_t \) as an input for planning the production in time period \( t \) is obtained from the aggregation of household demand simulated in the demand model (see Subsection 2.3.1). Constraints (2.6) and (2.7) represent the inventory balance equations and minimum safety stock level requirements, respectively. The manufacturer’s capacity and resources in terms of the number of workers, maximum production, and overtime are represented by constraints (2.8)-(2.12). Constraints (2.13) represent non-negativity and integer requirements.
2.3.3 Optimisation

We now specify two different optimisation approaches. In the first approach, there is no coordination between the marketing and production planning. In this non-coordinated approach, the optimisation is done sequentially in two steps. First, the number and timing of promotions are optimised by maximising the total revenues minus the promotion costs. The second step optimises the production plan using the demand forecast obtained in the first step. In the second approach, the optimisation is carried out jointly by integrating the promotion and production decisions. By comparing the two approaches, we are able to examine the importance of coordination.

**Sequential Optimisation (the non-coordinated approach)**

Let $L_t$ $(0 \leq L_t < 1)$ denote the level of discount (%) offered in period $t$ ($L_t = 0$ means that there is no promotion offered). We define $P \in P$ as a promotion plan, where $P = (L_1 L_2 \cdots L_{T-1} L_T)$ and $P$ is the set of all possible promotion plans. The promotion plan is the main input for the demand model, and we define $D_t|P$ as the resulting demand forecast that corresponds to the promotion plan $P$. In the first step, the marketing planner solves the following optimisation problem:

$$\max_{P \in P} M\text{Profit} = \sum_{t=1}^{T} (D_t|P \cdot Rp \cdot (1 - L_t) - Z_t \cdot V)$$  \hspace{1cm} (2.14)

Subject to

$$L_t \leq Z_t \leq M \cdot L_t \hspace{2cm} t = 1, ..., T$$  \hspace{1cm} (2.15)

$$\sum_{t=1}^{T} Z_t \leq K \hspace{2cm} t = 1, ..., T$$  \hspace{1cm} (2.16)

$$Z_t \text{ binary} \hspace{2cm} t = 1, ..., T$$  \hspace{1cm} (2.17)

$$0 \leq L_t \leq 1 \hspace{2cm} t = 1, ..., T$$  \hspace{1cm} (2.18)

Where

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K$</td>
<td>Maximum number of promotions during the planning horizon</td>
</tr>
<tr>
<td>$Rp$</td>
<td>Regular price per unit from manufacturer</td>
</tr>
<tr>
<td>$V$</td>
<td>Promotion cost per promotion event</td>
</tr>
<tr>
<td>$M$</td>
<td>Sufficiently large number</td>
</tr>
</tbody>
</table>
Suppose $P^*$ is the optimal promotion plan, and $MProfit^*$ is the corresponding marketing profit. In the second step, the production planner solves the production planning problem as formulated in (2.5)-(2.13) with $D_t$ being replaced by $D_t|P^*$. Letting $TCOST^*$ denote the minimum total cost obtained, the total manufacturer’s profit is $Profit(1)^* = MProfit^* - TCOST^*$.

Joint Optimisation (The coordinated approach)

By integrating the promotion and production planning, the joint optimisation problem is formulated as:

$$\max_{p \in \mathbb{P}, q_p, q_o, q_s, nh_t, nf_t} \text{Profit}(2)$$

$$= \sum_{t=1}^{T} \left[ D_t|P \cdot R_p \cdot (1 - L_t) - Z_t \cdot V - (q_p + q_o) \cdot cp - \frac{q_o}{nu} \cdot co - q_s \cdot cs - l_t \cdot cl\text{Inv} - nl_t \cdot cl - nh_t \cdot ch - nf_t \cdot cf \right] \quad (2.19)$$

Subject to

(2.6) – (2.13); and (2.15) – (2.18).

In the joint optimisation, for each promotion plan $P$ and the resulting demand forecast, we solve the aggregate production planning problem. The total manufacturer’s profit for each plan is $Profit(2)$, as defined in (2.19). We perform complete enumeration of all the possible promotion plans to get the optimal solution.

2.4 Numerical study

This section presents the parameter settings and results of our numerical study. We examine the influence of several factors related to the demand and aggregate production planning models on the optimal profits, as well as the optimal number and timing of promotions. In relation to the optimal profits, our primary interest is to examine the importance of coordination by jointly optimising promotion and production decisions. This is achieved by making comparisons between
the profits obtained by solving the joint optimisation problem and the profits obtained when there is no coordination. More specifically, we study the effects of seven experimental factors: flexibility in production capacity, production unit cost, brand loyalty, pass-through rate, demand pattern (seasonality effect), promotion impact, and discount level. To study the effect of flexibility in production capacity, we vary hiring and firing costs, i.e., higher flexibility is represented by lower hiring and firing costs. Indeed, there are other measures that can be used to represent flexibility such as costs associated with the changes in machine capacity or costs of overtime and subcontracting. Our choice can be particularly justified in production settings where production outputs are mainly driven by the number of workers, for example some of the food, apparel and electronics appliances industries. Moreover, it is commonly used in the literature (see e.g., Lusa et al., 2012 and Chopra and Meindl, 2016) to represent medium-term, tactical changes of production capacity. Furthermore, we examine the effect of brand loyalty by changing the parameter $B^h$, which represents loyalty to brand and is obtained from the calculation of market share (Bucklin and Lattin, 1992). We are also interested in examining the effect of retailer’s pass-through rate, $PT$, which is the proportion of the manufacturer’s discount that the retailer passes on to the consumer. Note, however, that this paper focuses on the coordination between production and marketing functions within a single manufacturing firm, and so exclude the coordination between the manufacturer and retailer. We adopt the “pay-for-performance” environment commonly used in the marketing literature (e.g., Silva-Risso et al., 1999) where the retailer is paid a promotional allowance based on the volume sold during the promotion period, and this promotional allowance is captured in the demand model in the pass-through response function of the retailer. Seasonality effects in demand are captured through a scale factor on the purchase frequency of households in store visits in the incidence model, $F^h$. This factor represents the proportion of shopping trips in which a household makes a category purchase. We divide the planning horizon into ten time intervals and vary the $F^h$ values in each of the ten intervals to get a demand pattern with seasonality, whereas the $F^h$ scale factor values are set constant to get a demand pattern without seasonality. The expected demand over the planning horizon is the same in both cases. To examine the influence of promotion impact, we use different values for the temporary price reduction coefficient in the choice model, $\theta_6$, which determines the contribution of the temporary price reduction to a household’s utility function. In addition to the six factors mentioned above, we also evaluate how the results are influenced by the promotion discount level. Three levels are evaluated: 10%, 20% and 30%. We limit the choice to these three levels as our primary objective is to examine
the effect of discount level as well as its interaction with the other factors rather than to determine an optimal discount level. The three values also cover the range of discount levels often considered in practice and in marketing literature (van Heerde et al., 2004; Empen et al., 2015). Note that in our numerical study, we only allow one of the three discount levels to be used over the whole planning horizon. Thus, a mix of different discount levels is not allowed. This would allow us to obtain unambiguous results regarding the effect of the discount level.

With all the combinations of the above parameters, there are in total 288 different problem instances evaluated in this numerical study. Table 2.2 shows the different levels of the factors varied in our numerical experiment. Other parameter values used as inputs are as follows:

\[
\begin{align*}
cl &= 8; \quad co = 12; \quad cl\text{inv} = 0.092; \quad LL = 35; \quad UL = 140; \quad wh_t = wh = 40; \quad nl_o=50; \quad nu = 8; \\
O_t &= 2.5; \quad I_o=4000; \quad SS_t = 2000; \quad T = 52; \quad K = 12; \quad R_p = 12; \quad V = 1000; \quad R = 1000; \quad H = 121350.
\end{align*}
\]

A time period in the models is interpreted to be represented by a work week. These parameter values have then been calibrated to provide reasonable and non-trivial solutions to the basic production planning problem. The above unit holding cost is directly related to the unit price and scaled to represent the cost per week. Further parameter values used in the demand function are adopted from the model specifications in Silva-Risso et al. (1999). We use the parameter values as presented in Silva-Risso et al. (1999) with the purpose of developing a market demand simulator that is built based on an empirically grounded demand model.
Table 2.2 Parameter settings for experimental factors

<table>
<thead>
<tr>
<th>Factors</th>
<th># Levels</th>
<th>Parameter</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Flexibility in production capacity</td>
<td>2</td>
<td>$ch, cf$</td>
<td>Low: 2000, 3000</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>High: 1000, 2000</td>
</tr>
<tr>
<td>Production unit cost</td>
<td>2</td>
<td>$cp$</td>
<td>Low: 6</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>High: 7</td>
</tr>
<tr>
<td>Seasonality</td>
<td>2</td>
<td>$F^h$</td>
<td>Low: 0.81 (constant)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>High: 0.83, 0.7, 0.58, 0.48, 0.68, 0.85, 0.92, 0.99, 0.98, 0.92</td>
</tr>
<tr>
<td>Brand loyalty</td>
<td>2</td>
<td>$B^h$</td>
<td>Low: 0.6</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>High: 0.8</td>
</tr>
<tr>
<td>Pass through rate</td>
<td>2</td>
<td>$PT$</td>
<td>Low: 0.7</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>High: 0.8</td>
</tr>
<tr>
<td>Promotion impact</td>
<td>3</td>
<td>$\theta_6$</td>
<td>Low: 0.2</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Medium: 0.5</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>High: 0.8</td>
</tr>
<tr>
<td>Discount level</td>
<td>3</td>
<td>$L_t</td>
<td>L_t&gt;0$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Medium: 20%</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>High 30%</td>
</tr>
</tbody>
</table>

The importance of coordination

To address the first research question, we present the relative increase of the profits obtained using the coordinated approach in comparison to the profits obtained using the non-coordinated approach. Table 2.3 summarises the results. The overall averages of the increase of the profits are 1.45%, 6.39%, and 43.26% for the discount levels of 10%, 20% and 30%, respectively. These increases can represent significant improvements. All the seven factors evaluated appear to be influential. As for the production-related factors, the benefits of coordination become more pronounced when the production flexibility level is low and the product has a low profit margin (a high production cost).
Table 2.3 The average increase of profits (coordinated vs non-coordinated)

<table>
<thead>
<tr>
<th>Factors</th>
<th>Discount level</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>10%</td>
</tr>
<tr>
<td>Flexibility in production capacity</td>
<td>High</td>
</tr>
<tr>
<td></td>
<td>Low</td>
</tr>
<tr>
<td>Production unit cost</td>
<td>Low</td>
</tr>
<tr>
<td></td>
<td>High</td>
</tr>
<tr>
<td>Seasonality</td>
<td>Low</td>
</tr>
<tr>
<td></td>
<td>High</td>
</tr>
<tr>
<td>Brand loyalty</td>
<td>Low</td>
</tr>
<tr>
<td></td>
<td>High</td>
</tr>
<tr>
<td>Pass-through rate</td>
<td>Low</td>
</tr>
<tr>
<td></td>
<td>High</td>
</tr>
<tr>
<td>Promotion effect</td>
<td>Low</td>
</tr>
<tr>
<td></td>
<td>Medium</td>
</tr>
<tr>
<td></td>
<td>High</td>
</tr>
<tr>
<td>Overall average</td>
<td>1.45%</td>
</tr>
</tbody>
</table>

In the case where changing production capacity is costly, the coordinated approach allows firms to choose a promotion plan that does not lead to too many changes in the production capacity level. This is not possible under the non-coordinated approach, because the non-coordinated approach fails to differentiate promotion plans based on the production related factors. When marketing planners determine the optimal promotion plan solely based on the revenues and cost of promotions, two products that are different in their capacity changing and production costs may be given similar promotion plans. The fact that a higher relative increase of profit is observed in the production system with low flexibility and in the case with the higher production unit cost shows that the motivation for implementing coordination is higher in more costly production environments. In relation to the marketing-related factors, coordination is more beneficial in situations with high demand seasonality, high brand loyalty, high pass-through rate, and high promotion impact. It is also shown that the benefit of coordination is more pronounced as the discount level increases.

In addition to examining the effects of varying individual factors, it is also interesting to examine the effects of the interaction of different factors. In Figure 2.2, we depict the joint effects of two possibly interacting factors on the importance of coordination. The figure provides a better
understanding on the importance of the coordinated approach that is not always directly available from intuition. For example, the previous discussion has highlighted the effect of flexibility, i.e. the benefit of coordination is more pronounced when the flexibility is low, but it does not reveal how the achieved benefit is affected by the other factors. Figure 2.2 shows that even though the flexibility is low, the significance of the benefit of coordination only becomes significant when supported by other factors, e.g. when seasonality is high or promotion impact is high. The interaction between brand loyalty and promotion impact is another example. The importance of brand loyalty is more pronounced when the promotion impact is not too low or too high. The plots in Figure 2.2 show consistency on the effect of discount level regardless of its interaction with the other factors. That is, the benefit of coordination increases significantly when the discount level is 30%, the highest level evaluated in this study. The advantage of the coordinated approach over the sequential approach is common knowledge in the existing literature. However, to the extent presented here, none of the previous studies has revealed how the advantage is influenced by various specific factors. The results on the joint effect of the different factors can help production and marketing planners to identify groups of products, for which the integrated planning approach is particularly desirable.

With the refined demand model we have adopted, it is also possible to examine how the different marketing related factors influence how increase in demand due to a promotion is decomposed into true incremental demand and forward buying, and hence how they influence the profitability. We follow the method used in Silva-Risso et al. (1999) to decompose total incremental demand into true incremental demand and forward buying. An explanation of how the decomposition of incremental demand is made is presented in the Appendix. The true incremental demand represents the increase of consumption rate due to a promotional event. In Figure 2.3, we depict how the percentages of forward buying (relative to the total increase in demand) are affected by the interaction of the different marketing factors. The two production-related factors, flexibility and production cost, are excluded, because neither of these factors have an impact on the households’ purchase decisions.
Figure 2.2 Interaction plots (means) for the average increase of profits from coordination (%)

Figure 2.3 Interaction plots (means) for the forward buying (%)

It is clearly shown in Figure 2.3 that promotion impact is the most influential factor on the percentage of forward buying. The percentage of forward buying is higher when the promotion impact is lower. In the case of high promotion impact, households are more sensitive to the temporary price reduction and react by increasing their purchase quantities. As households have higher inventory levels, their consumptions tend to increase. As a result, true incremental sales due to consumption is higher than forward buying in the case of high promotion impact, and vice versa. This observation can be traced from the consumption rate function in the demand model. In addition, higher brand loyalty tends to generate a somewhat higher share of forward buying. Information on the percentages of forward buying and true incremental demand is quite helpful in getting a better understanding of the effect of price promotions on the results previously presented. In what follows, we explain in further depth, why there is a significant increase of the benefits of coordination when the discount level changes from 20% to 30% in Table 2.3. As will be discussed in the next section, the number of promotions in the non-coordinated approach tends to be higher than in the coordinated approach, and since the percentage of forward buying is smaller when the promotion impact is higher, the motivation of offering more promotions becomes stronger when the promotion impact is high. Since the non-coordinated approach does not consider implications of promotions to production related costs, the worst situation that may occur is when offering promotions results in smaller profits than the case of no promotion. And this is particularly likely to happen when the discount level is high. In contrast, the coordinated approach indeed considers the impact of promotions on the production-related costs.

In Figure 2.4 we show how total quantity, sales revenues, production-related costs, and profits are affected by the discount level, for the case with high promotion impact and for both the non-coordinated and coordinated approaches, and for the cases with and without seasonality, respectively. The figure depicts the relative increase or decrease in comparison to the base case with no promotion (i.e., zero discount level). It is shown that the percentage increase of production-related costs under the non-coordinated approach may be higher than the percentage increase of sales revenues. In the case where the discount level is the highest in our experiment (30%), the non-coordinated approach may even result in profits that are lower than in the base case.
**Seasonality: Low**

<table>
<thead>
<tr>
<th>Non-coordinated</th>
<th>Coordinated</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image1.png" alt="Graph" /></td>
<td><img src="image2.png" alt="Graph" /></td>
</tr>
</tbody>
</table>

**Seasonality: High**

<table>
<thead>
<tr>
<th>Non-coordinated</th>
<th>Coordinated</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image3.png" alt="Graph" /></td>
<td><img src="image4.png" alt="Graph" /></td>
</tr>
</tbody>
</table>

* Flexibility: high; Production cost: high; Promotion impact: high
** The percentage increase is relative to the base case with no promotions

Figure 2.4 The impact of discount level on total quantity, sales revenue, manufacturing costs, and profits
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The number of promotions

We now consider the second research question on how the optimal number of promotions is affected by our experimental factors and how the results differ under the two optimisation approaches. Table 2.4 presents the averages of the optimal number of promotions obtained by the two optimisation approaches. We note that there are cases that have the same optimal number of promotions, but as will be discussed in the following sub-section, the time periods where those promotions are carried out are not necessarily the same.

Table 2.4 The average optimal number of promotions

<table>
<thead>
<tr>
<th>Factors</th>
<th>Discount level</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>10%</td>
</tr>
<tr>
<td></td>
<td>C</td>
</tr>
<tr>
<td>Flexibility in production capacity</td>
<td>High</td>
</tr>
<tr>
<td></td>
<td>Low</td>
</tr>
<tr>
<td>Production unit cost</td>
<td>Low</td>
</tr>
<tr>
<td></td>
<td>High</td>
</tr>
<tr>
<td>Seasonality</td>
<td>Low</td>
</tr>
<tr>
<td></td>
<td>High</td>
</tr>
<tr>
<td></td>
<td>High</td>
</tr>
<tr>
<td>Pass-through rate</td>
<td>Low</td>
</tr>
<tr>
<td></td>
<td>High</td>
</tr>
<tr>
<td>Promotion effect</td>
<td>Low</td>
</tr>
<tr>
<td></td>
<td>Medium</td>
</tr>
<tr>
<td></td>
<td>High</td>
</tr>
<tr>
<td>Overall average</td>
<td></td>
</tr>
</tbody>
</table>

C: Coordinated
NC: Non-coordinated

As shown in Table 2.4, the average number of promotions is higher in the non-coordinated approach than in the coordinated approach. This is understandable as there are more parameters, i.e., costs related to the production, which may limit the number of promotions in the coordinated approach compared to the non-coordinated approach. Note that the optimal number of promotions under the non-coordinated approach is not affected by the changes in the first two factors, i.e. flexibility in production and production unit cost. This is because the optimal promotion plans obtained with the non-coordinated approach do not consider the production factors.
Figure 2.5 and Figure 2.6 depict how the averages of the number of promotions are affected by the interaction of the different factors for the cases with the non-coordinated and coordinated approaches, respectively. Contrasting the two figures reveals some interesting insights. First, the joint effect of discount level and brand loyalty on the number of promotions in the non-coordinated approach appears to be quite different from the joint effect observed in the coordinated approach. In the non-coordinated approach, the average number of promotions tends to decrease as the discount level increases, but the effect of brand loyalty is insignificant. In contrast, the effect of brand loyalty in the coordinated approach is more significant, i.e. the number of promotions is reduced as brand loyalty increases. Further, by considering the production related costs, the coordinated approach is able to exploit the benefit of high brand loyalty and reduces the number of promotions. In the case with high discount level (30%) and high brand loyalty, the average number of promotions under the coordinated approach is close to zero. We also see a difference in the joint effect of discount level and seasonality between the two approaches. The effect of seasonality is more pronounced under the coordinated approach, with more frequent promotions resulting from higher seasonality. Likewise, the joint effect of seasonality and promotion impact is also more pronounced under the coordinated approach. Figure 2.5 clearly shows that the change in the number of promotions under the non-coordinated approach is primarily driven by the promotion impact rather than by the seasonality. This shows that the non-coordinated approach fails to take into account the effect of seasonality on the production-related costs. Under the coordinated approach, there is interaction between the two factors as the number of promotions in the case of high seasonality is larger than that in the case of low seasonality when the promotion impact is at the medium or high level. When the promotion impact is at the low level, the number of promotions is almost the same for both cases with high and low seasonality.
Figure 2.5 Interaction plots for the average number of promotions with the non-coordinated approach

Figure 2.6 Interaction plots for the average number of promotions with the coordinated approach
Under the coordinated approach, as the joint optimisation model captures the negative effect of promotions on the production-related costs, a higher discount level tends to result in a lower number of promotions. In addition, higher demand seasonality triggers more frequent promotions.

The analyses above not only provides managers with insights into the effects of the different production and marketing factors, but also the interplay between those factors. Our study reveals conditions related to production and marketing under which the development of an integrated production and promotion planning generates the most benefits. Furthermore, a better understanding on how the interactions between parameters influence the percentage of forward buying and the optimal number of promotions will help production and marketing planners to focus on the more influential parameters when determining a joint production and promotion plan.

**The timing of promotions**

The final part of our research questions concerns the optimal timing of promotions. As the most noteworthy case, we focus on their optimal timing when demand is seasonal. To illustrate, Figure 2.7 shows the optimal timing of promotions for one problem instance with the discount level of 30%. It is interesting to see that while the coordinated approach tends to plan the promotions during the low-demand season, the non-coordinated approach, on the contrary, tends to schedule promotions during the high-demand season.

The coordinated approach seeks to smooth the demand and production over the planning horizon by offering more promotions in the low-demand season. In contrast, the non-coordinated approach aims to maximise the total sales by offering more promotions in the high-demand season. Comparing the optimal results of the two approaches for these particular problem instances reveals that the higher profits obtained by the coordinated approach are mainly due to its lower total inventory, overtime and production costs.
Disc.Level: 30%
Flexibility: low; Production cost: high; Seasonality: high; Promotion impact: low

Coordinated

Non.coordinated

Figure 2.7 The optimal timing of promotions for the discount level of 30%
2.5 Conclusions

Contributions
This paper presents a framework for developing a decision-support model to obtain a sales and operations plan (S&OP) that integrates production and price promotion planning decisions. We integrate a purchase incidence-brand choice-purchase quantity model and a mixed integer linear programming model to determine an optimal promotion and aggregate production plan. To our knowledge, this is the first paper to integrate and apply this kind of advanced demand model with the standard aggregate production planning model. In our numerical study, we examine the benefits of coordination in developing S&OP by comparing the solutions obtained from the coordinated approach to those obtained from the non-coordinated approach. The modelling framework we adopt allows us to examine how the benefits of coordination are affected by various production and marketing related factors, as well as the interaction between these factors. The results show that the average relative improvement of the profit considered can be up to more than 40%, which is a significant improvement. The results on the joint effects of the different factors provide a better understanding on the importance of coordination and its main driving forces. Furthermore, the rich demand model adopted allows us to decompose the total increase in demand due to a promotion into true incremental demand and forward buying. We find that promotion impact is a major influential factor that determines the split between these two types of incremental demand. Information on the percentages of forward buying and true incremental demand enhances the understanding of the effect of promotions. It is also found that the non-coordinated approach tends to generate plans with more promotions than the coordinated approach. Another interesting finding is that the coordinated approach tends to plan promotions during the low-demand season, while the non-coordinated approach tends to schedule them during the high-demand season. All the above main findings contribute to advancing the literature on the development of decision support for an integrated S&OP.

Managerial implications
As noted above, when 25% of all retail sales is coming from sales during price promotion campaigns (Gedenk et al., 2010), there is a huge business potential in improving promotion planning. The results of our study provide useful information for production and marketing planners on the importance of developing an integrated S&OP taking into account joint production
and promotion decisions. As many firms rely on reconciliation meetings to make adjustments to their joint sales and operations plans (Ivert and Jonsson, 2014), the use of an integrated model like ours will help planners reduce the time required for resolving possible disagreements resulting from the sequential approach. Our modelling framework that facilitates a comprehensive examination of various factors and their effects will help production and marketing planners in better understanding the main driving forces of the profitability resulting from a joint production and promotion plan. Production and marketing planners can learn to recognise the characteristics of products in relation to their production and marketing, and thereby identify products for which an integrated production and promotion plan is particularly desirable. As the report from Lund and Raun (2017) suggests, a high degree of silo thinking between business functions and a lack of use of integrated software may represent major barriers in the structural and technological dimensions to achieving a high maturity level of S&OP. The integrated framework presented in this paper can inspire practitioners, including planners and software developers, in taking initiatives to overcome those barriers. Furthermore, our study has clearly shown the advantages of using a rich demand model such as the disaggregate consumer response model adopted in this paper. This should inform practitioners on the importance of business analytics that take account of market data acquisition and development of data-driven decision support models.

Future research

We acknowledge some limitations of this paper and would like to suggest several topics for future research. Firstly, the solution space for the promotion decisions in this study is limited since the discount levels are restricted to three values and it is not possible to implement a promotion schedule that contains a mix of more than one discount level. Thus, a possible extension is to develop a heuristic approach to deal with increased combinatorial complexity. Secondly, in this paper we only consider the situation where promotion is planned only for a single product (brand), although the demand model we use can accommodate more than one product. An interesting research avenue is to consider the joint optimisation in a multi-product setting. Finally, the results presented in this paper are based on a specific set of problem parameters (although some variations have been considered by varying a number of experimental factors). Our conjecture is that the resulting qualitative insights can be generalized to other problems with a different set of parameters. Further empirical studies are necessary to validate our conjecture and assess the robustness of the results.
Appendix:

Brand choice model

In the brand choice model, we calculate the probability that a household chooses a particular brand given that the household decides to make a purchase decision using a multinomial logit form as presented in (2.2). This probability depends on the value of the deterministic component of brand utility ($A_{jt}^h$) which is modelled as (see e.g., Bucklin and Lattin, 1992; Silva-Risso et al., 1999; Ailawadi et al., 2007)

$$A_{jt}^h = \alpha_b + \alpha_s + \theta_1 B_j^h + \theta_2 L B_j^h + \theta_3 S_j^h + \theta_4 L S_j^h + \theta_5 R_{jt} + \theta_6 T C_{jt} + \theta_7 X_{jt} + \theta_8 Y_{jt}$$

(A2.1)

where

$A_{jt}^h$ The deterministic component of utility related with brand $j$ for household $h$ in time period $t$
$B_j^h$ Brand loyalty of household $h$ to brand $j$
$L B_j^h$ 1 if $j$ was last brand purchased, 0 otherwise
$S_j^h$ Size loyalty of household $h$ to brand $j$
$L S_j^h$ 1 if $j$ was last size purchased, 0 otherwise
$R_{jt}$ Regular store price for brand $j$ in time period $t$

$$R_{jt} = R p_{jt} (1 + U p) \quad t = 1, ..., T$$

(A2.2)

$R p_{jt}$ Regular price from manufacturer of brand $j$ in time period $t$
$U p$ Store’s markup in percent
$T C_{jt}$ Temporary price cut for brand $j$ in time period $t$

$$T C_{jt} = R p_{jt} (P T_{jt} \cdot L_{jt}) \quad t = 1, ..., T$$

(A2.3)

$P T_{jt}$ Store’s pass-through in percent
$L_{jt}$ Level of discount in percent for brand $j$ in time period $t$
$X_{jt}$ \{1 if a feature ad is offered for brand $j$ in time period $t$
\{0, otherwise
$Y_{jt}$ \{1 if a display is offered for brand $j$ in time period $t$
\{0, otherwise
$\alpha_b$ Brand constant to be estimated
$\alpha_s$ Size constant to be estimated
$\{\theta_1, ..., \theta_8\}$ Parameters to be estimated
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**Purchase incidence model**

The buy/no-buy decision is modelled with a binary nested logit model. The purchase incidence model calculates the probability that a household will make a purchase in the product category on a store visit, as presented in (2.3). The purchase incidence model needs the value of the deterministic component of household utility \( C_t^h \) which takes the following form (e.g., Guadagni and Little, 1983; Bucklin and Lattin, 1992; Silva-Risso et al., 1999; Ailawadi et al., 2007):

\[
C_t^h = \beta_0 + \beta_1 F_t^h + \beta_2 I_t^h + \beta_3 W_t^h \quad h=1,\ldots, H; \quad t=1,\ldots, T \tag{A2.4}
\]

\[
W_t^h = \ln \sum_{j=1}^J e^{A_{jt}^h} \quad h=1,\ldots, H; \quad t=1,\ldots, T \tag{A2.5}
\]

where

- \( C_t^h \): The deterministic component of utility related with household \( h \) in time period \( t \)
- \( F_t^h \): Proportion of purchase frequency for household \( h \) on store visit
- \( I_t^h \): Inventory for household \( h \) at the end of time period \( t \)
  \[
  I_t^h = \text{Max}(0, I_{t-1}^h + \sum_{j=1}^J D_{j,t-1}^h - U_{t-1}^h) \tag{A2.6}
  \]
- \( D_{j,t-1}^h \): Quantity of brand \( j \) bought in time period \( t-1 \) by household \( h \)
- \( U_t^h \): Rate of consumption for household \( h \) in time period \( t \)
  \[
  U_t^h = I_t^h \left[ \frac{U^h}{{\bar{U}^h} + (I_t^h)^\pi} \right] \tag{A2.7}
  \]
- \( \bar{U}^h \): Mean rate of consumption for household \( h \)
- \( W_t^h \): The expected maximum utility from the brand choice decision for household \( h \) in time period \( t \).
- \( \pi \): Parameter to be estimated
- \( \{\beta_0, \ldots, \beta_3\} \): Parameters to be estimated

**Quantity model**

The expected quantity purchased by a household is calculated using the expected value of the truncated Poisson distribution as presented in (2.4). The purchase rate of the household is modelled as (e.g., Bucklin and Lattin, 1992; Silva-Risso et al., 1999; Ailawadi et al., 2007)

\[
\lambda_{jt}^h = \exp\left( \mu_b + \mu_s + \omega_1 G_t^h + \omega_2 I_t^h + \omega_3 B_j^h + \omega_4 S_j^h + \omega_5 R_{jt} + \omega_6 TC_{jt} + \omega_7 X_{jt} + \omega_8 Y_{jt} \right) \tag{A2.8}
\]
where

\[ \lambda_{jt}^h \] The purchase rate of household \( h \) for the brand alternative \( j \) in time period \( t \)

\( G^h \) Average quantity bought by household \( h \) per purchase trip

\( I_t^h \) Inventory for household \( h \) at the end of time \( t \)

\( \mu_b \) Brand constant to be estimated

\( \mu_s \) Size constant to be estimated

\( \{\omega_1, ..., \omega_8\} \) Parameters to be estimated

The decomposition of incremental demand.

The expected incremental demand of brand \( j \) sold to household \( h \) in time period \( t \) is obtained by subtracting baseline plus forward buying demand, \( E(BFD_{jt}^h) \), from total demand, \( E(D_{jt}^h) \), and adding back borrowed demand that resulted in incremental consumption (\( \Delta U_t^h = UBF_t^h - UB_t^h \)), as shown in A2.9 (Silva-Risso et al., 1999)

\[ E(\Delta D_{jt}^h) = E(D_{jt}^h) - E(BFD_{jt}^h) + \Delta U_t^h \quad (A2.9) \]

UBF is the consumption rate for the simulated baseline plus forward buying, and UB is the consumption rate for the baseline. Baseline plus forward buying demand are the function of inventory and consumption level for the case that promotions resulted only in forward buying through purchase acceleration and/or stockpiling. Therefore, we remove choice effect in the baseline plus forward buying model. Non-incremental demand (baseline plus forward buying) can be estimated by setting no promotion and no purchased feedback in the choice model, and non-incremental consumption as shown in A2.10 (Silva-Risso et al., 1999)

\[ E(BFD_{jt}^h) = P_t^h(inc)|_{l_t^h=IBF_t^h} \times P_t^h(j|inc)|_{No promotion \; No purchased feedback} \times E(D_{jt}^h|D_{jt}^h > 0)|_{l_t^h=IBF_t^h} \quad (A2.10) \]

IBF\(_t^h\) is the household’s inventory given that promotion effect in the choice model is removed, No purchased feedback eliminates carryover effects (last brand purchased) in the choice model. The expected baseline is given by (Silva-Risso et al., 1999)

\[ E(BD_{jt}^h) = P_t^h(inc)|_{l_t^h=IB_t^h} \times P_t^h(j|inc)|_{No promotion \; No purchased feedback} \times E(D_{jt}^h|D_{jt}^h > 0)|_{l_t^h=IB_t^h} \quad (A2.11) \]

BD is baseline volume and IB\(_t^h\) refers to the household’s inventory for the case of no promotions.
### Parameter for consumer response model

<table>
<thead>
<tr>
<th>Purchase incidence model</th>
<th>Choice model</th>
<th>Quantity model</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta_0 = -5.2562$</td>
<td>$\alpha_{b1} = 0.4537$</td>
<td>$\mu_{b1} = 0.0140$</td>
</tr>
<tr>
<td>$\beta_1 = 5.2590$</td>
<td>$\alpha_{b2} = 0.8096$</td>
<td>$\mu_{b2} = -0.1356$</td>
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<tr>
<td>$\beta_2 = -0.0201$</td>
<td>$\alpha_{b3} = 0.7432$</td>
<td>$\mu_{b3} = -0.2888$</td>
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<td>$\beta_3 = 0.3338$</td>
<td>$\alpha_s = -0.4521$</td>
<td>$\mu_s = -0.0146$</td>
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<tr>
<td></td>
<td>$\theta_1 = 1.9085$</td>
<td>$\omega_1 = 0.3153$</td>
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<tr>
<td></td>
<td>$\theta_2 = 0.9154$</td>
<td>$\omega_2 = -0.0097$</td>
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<tr>
<td></td>
<td>$\theta_3 = 2.5672$</td>
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<td></td>
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<td>$\theta_5 = -4156$</td>
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<td></td>
<td>$\theta_8 = 1.1042$</td>
<td>$\omega_8 = -0.0686$</td>
</tr>
</tbody>
</table>

Source: Silva-Risso et al. 1999
Chapter 3

Paper 2: Integrated sales and operations planning with multiple products: Jointly optimising the number and timing of promotions and production decisions

History: This chapter has been prepared in collaboration with Hartanto Wong and Anders Thorstenson. It has been presented at The 47th International Conference on Computers & Industrial Engineering, October 2017, Lisbon, Portugal, and has been submitted to Applied Mathematical Modelling.
Integrated sales and operations planning with multiple products: Jointly optimising the number and timing of promotions and production decisions

Agus Darmawan, Hartanto Wong and Anders Thorstenson

CORAL - Cluster for Operations Research, Analytics and Logistics

Department of Economics and Business Economics, Aarhus University

Fuglesangs Allé 4, 8210 Aarhus, Denmark

Abstract

This paper presents a modelling framework for sales and operations planning (S&OP) that considers the integration of price promotion and production planning for multiple products. Such a modelling framework takes into account the potential competition and cannibalization between products as well as the allocation of shared production resources. The demand model that we adopt combines purchase incidence, consumer choice and purchase quantity in a sequential framework to obtain the dynamics and heterogeneity of consumer response to promotions. Due to large problem sizes, we develop heuristic approaches for solving the resulting joint optimisation problem. The results of our numerical study show interesting findings on the optimal number and timing of promotions that take into account the mutual dependence of marketing and production related factors.

Keywords: demand model, forward buying, product substitution, cannibalization, promotion
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3.1 Introduction

Nowadays, cross-functional intra-company and supply chain inter-company coordination have become important requirements to obtain competitive advantages. In some industries, these are requirements just to stay in the market. One of the typical planning activities that serve to synchronise different business functions within firms as well as to integrate supply chain planning processes is sales and operations planning (S&OP). According to APICS Dictionary (Pittman and Atwater, 2016), S&OP is defined as "… the function of setting the overall level of manufacturing output and other activities to best satisfy the current planned levels of sales, while meeting general business objectives of profitability, productivity, competitive customer lead times, inventory and/or backlog levels, etc".

Bursa (2012) reports data from Gartner indicating that a demand-driven S&OP process can increase revenues by 2% to 5% and reduce inventories by 7% to 15%, which is quite significant for many firms. He also points out the need for technology and skill sets that share a common set of terms for all business functions. Hinkel et al. (2016) report that there is still a widespread use of rough approaches and lack of coordination in S&OP. They emphasise the need for enhanced coordination among cross-functional teams and to improve operational plan stability through a more advanced and integrated planning procedure. Trade promotions play an important role as a marketing-mix tool. According to a study by Nielsen (2014), spending on trade promotions in the consumer packaged goods sector is approximately $1 trillion annually. Gomez et al. (2007) report that trade promotions play an extremely important role in the U.S. supermarket industry, as well as in many consumer packaged goods industries. Gedenk et al. (2010) report that price promotion may increase total sales by 12-25% for retailers in Europe, thereby playing an important role in generating revenue.

Motivated by the fact that balancing supply and demand is the key to success for S&OP, in this paper we present a modelling framework for S&OP that considers the integration of price promotion and production planning and supports joint decision making. Although many studies have advocated the importance of coordination between marketing and production decisions, it is still a common practice today that sales and operations plans are developed sequentially without using a common objective shared by the marketing and production planners. While planners in the marketing department are primarily concerned with the timing and level of promotions to
maximise revenues, planners in the production department focus on production plans satisfying target sales and minimising total production-related costs. Such a planning process neglects the impact of promotions on the use of production resources, and therefore leads to a sub-optimal sales and operations plan. Lack of appropriate decision support models might be one of the reasons for this deficiency of the planning process. The integration of promotion and production planning is particularly relevant when considering manufacturing firms that offer a family of similar products or the same product with different package sizes. This is so not only because these products consume the same production resources, but also because there is internal competition in the selling of these products. The existing literature offers little help in developing an integrated promotion and production plan that takes into account product substitution, and in understanding the combined effects of marketing and production-related factors. Our study aims to fill this void in the literature.

In our modelling framework, we integrate an existing econometric-based demand model and a standard mixed integer linear programming based aggregate production planning model. Such a demand model is able to capture the dynamics and heterogeneity of consumer response by simulating purchase incidence, consumer choice and quantity decisions and household’s inventory level. This allows us to decompose total sales into consumption, brand switching and forward buying. The increase in demand observed during product price promotions can be attributed to increased consumption, brand switching and forward buying. Promotions may attract new or existing customer in increasing consumption, and also influence the decision of consumers to switch from a competitor’s product, resulting in incremental sales. However, promotions can also result in non-incremental sales as when customers only shift their future purchases to the current period, i.e., they exercise forward buying. Furthermore, such a demand model is able to capture the potential competition and cannibalization between products offered by the same manufacturer. One relevant issue in relation to multiple products offered by the same manufacturer is whether the promotion timing of these products should be the same or different. Addressing this question requires simultaneous consideration of both demand and production related factors. Due to the large size of the solution space of the integrated optimisation problem, we develop and evaluate three heuristic approaches that can be used to determine near optimal or good promotion and production plans without requiring excessive computation times.
The paper proceeds as follows. Section 3.2 presents our review of relevant literature. Section 3.3 consists of two parts. In the first part, we introduce the main elements of the demand model that include purchase incidence, brand choice, and purchase quantity. The second part of this section specifies the integration of the demand and aggregate production planning models. In Section 3.4, we present the three heuristic approaches and the results of our numerical study for the evaluation of the heuristics. In Section 3.5, we study the integrated promotion and production decisions in the case of multiple products. In particular, we focus on assessing the timing of promotions of the products. Section 3.6 summarises the conclusions of our study and provide some directions for future research.

### 3.2 Literature review

We first highlight the results of our literature review on the coordination of marketing and production decisions in aggregate planning. Martínez-Costa et al. (2013) present a comprehensive review on the importance of integration in aggregate planning that coordinates marketing and production decisions. Based on the fact that advertising can be used to smooth seasonal product demand, Leitch (1974) presents an optimisation model for production and advertising planning in a multi-period setting. Sogomonian and Tang (1993) develop a modelling framework for joint optimisation of promotion and aggregate planning within a firm. They use a simple demand function to generate demand in each period, which depends on price, time and level of last promotion. They test the model for both the sequential and integrated planning approaches. Ulusoy and Yazgac (1995) consider pricing and advertising in their aggregate planning within a multi-product and multi-period setting. In their simple demand function, demand is assumed proportional to the advertising level but inversely proportional to the price. Feng et al. (2008) present a model to integrate sales and production under the assumption that demand and price are both normally distributed. Affonso et al. (2008) perform demand perturbations to show the importance of coordination in S&OP. González-Ramírez et al. (2011), Lusa et al. (2012), and Bajwa et al. (2016) discuss the integration of production and marketing decisions using a simple relation between demand and price. Sodhi and Tang (2011) present a stochastic programming model for S&OP that determines the production requirement while optimally trading off risks of unmet demand, excess inventory, and inadequate liquidity in the presence of demand uncertainty. Although they consider an effective unit price that are realized after various discounts off the list price, the effects of
discounts on demand are not considered in their model and promotion is not part of the decision variables.

We concur with Feng et al. (2010) and Martínez-Costa et al. (2013) in concluding that there is a lack of research in the literature that considers richer demand models that can be used to develop decision support tools for the joint planning process. We extend the previous studies by adopting such a rich demand model that captures the possible effects of various factors such as price discount level, seasonality, promotion impact, brand loyalty, etc. Moreover, the demand model adopted in this paper is able to capture the existence of product substitution or cannibalization that is necessary when considering integrated production and promotion planning in a multi-product setting. More recently, Darmawan et al. (2018a) develop a modelling framework for an integrated S&OP that considers joint promotion and production decisions using the same demand model. However, they only consider the case with a single product. We extend their modelling framework by considering a multi-product setting and developing heuristic approaches for solving the considerably more complex optimisation problem. Such an extension gives us the opportunity to investigate the benefits of an integrated S&OP more fully.

Our literature review suggests that research that considers S&OP in a multiple-product setting is very scant. Taskın et al. (2015) develop a mathematical programming-based S&OP based on a real case at a television manufacturer. They minimise production and procurement costs with respect to the resource constraints in a dynamic environment, where sales forecasts are regularly updated. Lim et al. (2017) use a simulation-optimisation approach for solving the S&OP problem for a case in the automotive industry with multiple parts and distant sourcing. However, these papers consider neither promotions nor how sales of one product might have an impact on other products. Thus, there is certainly a gap in the literature to be filled. Our model in this paper takes into account the potential competition and cannibalization between products, as well as the allocation of shared production resources.

Promotion planning is a topic that is extensively studied in the marketing literature. Promotion is one of the marketing–mix variables that can be used to maintain sales volume, increase growth, and attract consumers’ attention (Steenkamp et al., 2005; Kotler and Armstrong, 2013). A number of authors, e.g., Silva-Risso et al. (1999), and Ailawadi et al. (2007), present a promotion planning model that is based on a disaggregate consumer response model. This is also the approach adopted
in our paper. Fok et al. (2012) study the purchase-timing behaviour of households. Promotion may result in shortened inter-purchase times, but may also result in longer inter-purchase times due to stockpiling. To incorporate the dynamic effect of marketing strategies, they suggest combining purchase incidence with brand choice and purchase quantity decisions. Simester (1997) studies the characteristics of promotion strategies in a multiple-product setting. His study suggests that retailers should offer deeper promotions in the case where customers are highly price sensitive and the impact of promotion on product switching is strong. He also points out that deeper promotions are suitable for products that savour complementary relationships and low substitution effects. Srinivasan et al. (2005) discuss the effect of cannibalization on the marketing strategies. Promotions not only increase the sales of the promoted product but also have an impact on a substituted product due to diverting mechanisms. Ignoring this phenomenon leads to a sub-optimal promotion plan. Gumus et al. (2016) further justify that a deeper price promotion is common in the case of expensive products with a low degree of substitution. In relation to promotion timing, their finding is that simultaneous schedules of promotions are more suitable in the case of high substitution effects, while sequential schedules are more suitable in the case of low substitution effects. All the above mentioned studies published in the marketing literature seem to disregard the impact of promotions on production-related factors and costs. Hence, even though supported by rich demand models, the resulting promotion plans are most likely sub-optimal when viewed from an integrated planning perspective.

To the best of our knowledge, this paper is the first attempt to integrate existing research in the operations and marketing literature for the development of integrated S&OP that considers multiple products. We adopt the disaggregate consumer response model widely used in the marketing literature, where promotions are embedded in a model that considers purchase incidence, consumer choice and purchase quantity. The resulting demand forecasts that correspond to a promotion plan become the main inputs for the development of an aggregate production plan. Our integrated approach allows us to capture the mutual-dependence of both marketing and production related factors and thereby permits joint optimisation of promotion and production plans. Moreover, in the case of multiple products, as the demand model facilitates the potential substitution (cannibalization) effects between products, the integrated approach allows us to take into account the fact that net demand of these products also has an impact on the use of production resources at the manufacturer.
3.3 The models

In this section, we discuss two parts. The first part presents an introduction to the demand model and the second part describes the joint optimisation of sales and operations planning by considering promotion and production decisions simultaneously.

3.3.1 Demand model

The demand model adopted in this paper is based on the incidence-brand choice-purchase quantity model that is widely used in the marketing literature (see e.g., Guadagni and Little, 1983; Bucklin and Lattin, 1992; Silva-Risso et al., 1999; Ailawadi et al., 2007). We refer to Darmawan et al. (2018) for the similar model that only considers a single product offered by the manufacturer. The model captures consumers’ purchase timing, product or brand choice, and quantity decisions, and it takes into account household specific variables (e.g. brand loyalty, consumption, and purchase rates), as well as environment variables (e.g. retailer pass through, mark-up, and competition). We summarise the various elements of the demand model in Figure 3.1 (Darmawan et al. 2018). See the Appendix for a more detailed presentation of the model.

![Figure 3.1 The elements of the demand model](image-url)
To generate the dynamics of consumer response, we use the Monte Carlo technique to simulate the purchase probabilities of a panel of households (Seetharaman, 2004; Ailawadi et al., 2007). This technique allows us to capture the effect of stockpiling and repeat purchases. The output is a demand forecast $D_{jt}$ for brand $j$ in time period $t$, which represents the aggregate household demand for each period, i.e., $D_{jt} = \sum_{h=1}^{H} E\left(D_{jt}^h\right)$, where $H$ is the number of households simulated.

### 3.3.2 The integrated optimisation model

Let $L_{jt}$ ($0 \leq L_{jt} < 1$) denote the level of discount (%) offered in period $t$ for product $j$ ($L_{jt} = 0$ means that there is no promotion offered). We define $P \in \mathcal{P}$ as a promotion plan for the products offered by the manufacturer, where $P = (L_{11} \ldots L_{1T} \ldots L_{j1} \ldots L_{jT})$ and $\mathcal{P}$ is the set of all possible promotion plans. The promotion plan is the main input for the demand model, and we define $D_{jt}|P$ as the resulting demand forecast for product $j$ at period $t$ that corresponds to the promotion plan $P$.

<table>
<thead>
<tr>
<th>Parameters:</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$c_p j$</td>
<td>Production unit cost (including materials; excluding labour cost) for product $j$</td>
</tr>
<tr>
<td>$cl$</td>
<td>Regular labour cost per worker and time period</td>
</tr>
<tr>
<td>$ch$</td>
<td>Hiring cost per worker</td>
</tr>
<tr>
<td>$cf$</td>
<td>Firing cost per worker</td>
</tr>
<tr>
<td>$co$</td>
<td>Overtime cost per hour</td>
</tr>
<tr>
<td>$clnvj$</td>
<td>Inventory holding cost per unit-time for product $j$</td>
</tr>
<tr>
<td>$cs_j$</td>
<td>Subcontracting cost per unit for product $j$</td>
</tr>
<tr>
<td>$LL$</td>
<td>Minimum number of workers</td>
</tr>
<tr>
<td>$UL$</td>
<td>Maximum number of workers</td>
</tr>
<tr>
<td>$wh_t$</td>
<td>Number of regular working hours available per worker in time period $t$</td>
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<tr>
<td>$nl_o$</td>
<td>Number of workers at the beginning of the planning horizon</td>
</tr>
<tr>
<td>$nu_j$</td>
<td>Number of units produced per hour for product $j$</td>
</tr>
<tr>
<td>$O_t$</td>
<td>Maximum number of overtime hours per worker in period $t$</td>
</tr>
<tr>
<td>$T$</td>
<td>Planning horizon in number of time periods</td>
</tr>
<tr>
<td>$SS_{jt}$</td>
<td>Safety stock requirement for product $j$ at the end of time period $t$</td>
</tr>
<tr>
<td>$K_j$</td>
<td>Maximum number of promotions during the planning horizon for product $j$</td>
</tr>
<tr>
<td>$Rp_{jt}$</td>
<td>Regular price per unit from manufacturer for product $j$ in time period $t$</td>
</tr>
<tr>
<td>$V_j$</td>
<td>Promotion cost per promotion event</td>
</tr>
<tr>
<td>$M$</td>
<td>Sufficiently large number</td>
</tr>
<tr>
<td>$R$</td>
<td>Number of replications in the simulation</td>
</tr>
<tr>
<td>$H$</td>
<td>Number of households simulated</td>
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<table>
<thead>
<tr>
<th>Decision variables:</th>
<th></th>
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<tr>
<td>$q_{Pr jt}$</td>
<td>Number of units produced during regular time for product $j$ in period $t$</td>
</tr>
<tr>
<td>$q_{o jt}$</td>
<td>Number of units produced during overtime for product $j$ in period $t$</td>
</tr>
</tbody>
</table>
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*qs*<sub>jt</sub> Number of units produced using subcontracting for product *j* in period *t*

*nh*<sub>t</sub> Number of workers hired at the beginning of time period *t*

*nf*<sub>t</sub> Number of workers fired at the beginning of time period *t*

*L*<sub>jt</sub> Level of discount (in percent) in time period *t* for product *j*

*Z*<sub>t</sub> Binary variable: 1 if promotion with discount is offered in time period *t*, 0 otherwise

Consequential variables:

<table>
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<tr>
<th>Variable</th>
<th>Description</th>
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<tr>
<td><em>D</em>&lt;sub&gt;jt&lt;/sub&gt;</td>
<td>Demand forecast for product <em>j</em> in time period <em>t</em></td>
</tr>
<tr>
<td><em>I</em>&lt;sub&gt;jt&lt;/sub&gt;</td>
<td>Inventory of product <em>j</em> at the end of time period <em>t</em></td>
</tr>
<tr>
<td><em>nl</em>&lt;sub&gt;t&lt;/sub&gt;</td>
<td>Number of workers available in period <em>t</em></td>
</tr>
</tbody>
</table>

By integrating the promotion and production planning, the joint optimisation problem is formulated as:

\[
\max_{P \in \mathbb{P}, a_{jt}, a_{jt}, ns_{jt}, nh,t, nf,t} \text{Profit} = \sum_{t=1}^{T} \sum_{j=1}^{J} D_{jt} \cdot P \cdot R_{jt} \cdot (1 - L_{jt} \cdot P) - \sum_{t=1}^{T} \sum_{j=1}^{J} \left( (qp_{jt} + qo_{jt}) \cdot cp_{j} + \frac{qo_{jt}}{nu_{j}} \cdot co + \right.

\left. qs_{jt} \cdot cs_{j} + l_{jt} \cdot cln_{jt} \right) - \sum_{t=1}^{T} (nl_{t} \cdot cl + nh_{t} \cdot ch + nf_{t} \cdot cf + Z_{t} \cdot V_{t}) \quad (3.1)
\]

Subject to:

\[ l_{jt} = l_{jt-1} + qp_{jt} + qo_{jt} + qs_{jt} - D_{jt} \cdot P \quad t = 1, ..., T; \quad j = 1, ..., J \quad (3.2) \]

\[ SS_{jt} \leq l_{jt} \quad t = 1, ..., T; \quad j = 1, ..., J \quad (3.3) \]

\[ nl_{t} = nl_{t-1} + nh_{t} - nf_{t} \quad t = 1, ..., T \quad (3.4) \]

\[ nl_{0} = nl_{T} \quad (3.5) \]

\[ LL \leq nl_{t} \leq UL \quad t = 1, ..., T \quad (3.6) \]

\[ \sum_{j=1}^{J} \frac{qp_{jt}}{nu_{j}} \leq nl_{t} \cdot wh_{t} \quad t = 1, ..., T \quad (3.7) \]

\[ \sum_{j=1}^{J} \frac{qo_{jt}}{nu_{j}} \leq nl_{t} \cdot O_{t} \quad t = 1, ..., T \quad (3.8) \]

\[ 0 \leq L_{jt} \cdot P < 1 \quad t = 1, ..., T; \quad j = 1, ..., J \quad (3.9) \]

\[ L_{jt} \cdot P \leq Z_{jt} \leq M \cdot L_{jt} \cdot P \quad t = 1, ..., T \quad (3.10) \]
\[ Z_t \leq \sum_{j=1}^{J} Z_{jt} \leq J_t \quad t = 1, \ldots, T \]  \hfill (3.11)

\[ \sum_{t=1}^{T} Z_{jt} \leq K_j \quad j = 1, \ldots, J \]  \hfill (3.12)

\[ Z_{jt} \text{ binary} \quad t = 1, \ldots, T; \quad j = 1, \ldots, J \]  \hfill (3.13)

\[ Z_t \text{ binary} \quad t = 1, \ldots, T \]  \hfill (3.14)

\[ l_{jt}, qp_{jt}, qo_{jt}, qs_{jt} \geq 0 \quad t = 1, \ldots, T; \quad j = 1, \ldots, J \]  \hfill (3.15)

\[ nl_t, nh_t, nf_t \geq 0 \text{ and integer} \quad t = 1, \ldots, T \]  \hfill (3.16)

In the above optimisation problem, the objective is to maximise the profit, obtained by subtracting all the costs for material, overtime, subcontracting, inventory, labour, hiring and firing, and promotion from the sales revenue that is affected by price promotions. Constraints (3.2) and (3.3) are the inventory balance equations and target safety stock levels, respectively. The number of resources and capacity in terms of the size of the work force, and the units produced in regular time and overtime are represented by Constraints (3.4) - (3.8). The range of the promotion discount levels is given by (3.9), and constraints in relation to the number of promotions are given in (3.10) - (3.14). Constraints (3.15)-(3.16) are the usual non-negativity and integer constraints.

Note that in the above formulation, we capture the economies of scale in implementing a joint promotion by considering a scenario where a fixed promotional cost is paid per promotional time period regardless of whether the promotional event is only for a single product or for several products. In this paper, we will focus on a scenario where there are several possible discrete discount levels that the manufacturer may choose, and once a discount level is selected for product \( j \), this level is used throughout the whole planning horizon. Applying the same discount level to a particular product is commonly observed in practice, and besides reducing the decision space, it will allow us to get a clearer understanding regarding the effect of changing the discount level.

### 3.4 Heuristic approaches

The integrated optimisation problem specified in Section 3.3.2 is a non-linear mixed integer problem. When considering all the possible combinations of promotion levels and promotion timing in realistic settings, the solution space can be quite large so that complete enumeration is
not feasible. Therefore, in this paper we apply and test heuristic approaches that may help obtain good solutions with acceptable computation times. The main purpose of developing the heuristics in this paper is not computational efficiency but the possibility to produce good quality solutions for comparisons of the solution characteristics. We compare three types of heuristics: simulated annealing, a genetic algorithm, and particle swarm optimisation. These three heuristics are well-known methods for solving complex optimisation problems. In solving the optimisation problem, the heuristics are used to generate a promotion plan, based on which an aggregate production plan is optimised by solving the corresponding mixed integer linear programming problem. In what follows, we present short introductions to the three heuristics. We refer to the appendix for more detailed algorithmic specifications of the three heuristics applied. Following the introductions, the next subsection presents our evaluations of the performance of the heuristics.

*Simulated Annealing (SA)*

Simulated annealing (Kirkpatrick et al., 1983) considers random moves in a solution’s neighbourhood. If a move results in a better objective function value, then SA will always accept it. However, to avoid premature convergence to a local optimum, SA will also accept a worse solution based on an acceptance function. SA works with one solution at a time and, in our study, a solution is represented by a promotion plan \( P \in \mathbb{P} \).

*A Genetic Algorithm (GA)*

GA is a well-known and widely applied method for solving complex optimisation problems. Besides its capability in handling large search spaces easily, GA explores a population of solution points in parallel rather than a single solution point. Hence, it has relatively good performance in terms of speed (Chaudhry and Luo, 2005; Fahimnia et al., 2012; Wari and Zhu, 2016). GA adopts a mechanism that is an analogy of natural selection by introducing concepts like population, selection, crossover and mutation. Using a directed random search procedure, GA attempts to find a near-optimal solution in multi-dimensional search spaces. As GA is sensitive to parameterization, there are a few approaches to prevent premature convergence (Pham and Karaboga, 2000; Pandey et al., 2014). In this study, we use rank selection and new generation process for the selection procedure and two-point crossover. Different from SA that works with a single solution at a time, GA evaluates multiple chromosomes at each generation, where a chromosome represents a promotion plan \( P \in \mathbb{P} \).
**Chapter 3: Paper 2**

*Particle Swarm Optimisation (PSO)*

Inspired by the behaviour of bird flocking or fish schooling, Eberhart and Kennedy (1995) developed PSO as a technique for solving optimisation problems. Similar to GA, PSO evaluates multiple solutions (particles) at a time. Unlike in GA, though, there are no evolution operators such as crossover and mutation. Particles in PSO keep track of their best performance so far as their personal best. In addition, the best solution among all the particles is stored as the global best. The mechanism to update the particle’s position is based on the velocity of each particle in relation to its personal best and the global best.

### 3.4.1 Evaluation of heuristics

In this section, we first present the numerical results for the performance evaluation of the three heuristics. As we intend to use the optimal solution obtained by complete enumeration as a benchmark, we limit the optimisation problem in this evaluation by considering a single product and by restricting it to the case where promotions may only be undertaken in the first week of each month. This keeps the solution space small enough such that the optimal solutions can be obtained by complete enumeration. In the next subsection, we consider a more general problem with no restriction on the promotion timing. Note that in our numerical study, the time periods specified in the optimisation model presented in Section 3.3 are interpreted to be represented by work weeks. This interpretation is motivated by our observation that the typical length of a promotion event is a week.

We generate 72 problem instances differentiated by five experimental factors: flexibility in changing production capacity, production unit cost, seasonality effect, promotion impact, and promotion discount level. The flexibility in changing the production capacity is represented by the levels of the hiring and firing costs. Seasonality effects are captured through the scale factor, $F^h$, in the purchase incidence model. We divide the planning horizon into ten segments of equal length (5-6 weeks) and vary the scale factor values across these segments to obtain a demand pattern with seasonality. If the scale factor value is set at a constant value, we obtain a demand pattern without seasonality. The total expected demand over the entire planning horizon is assumed to be the same with and without seasonality. We vary the temporary price reduction coefficient in the choice model, $\theta_6$, to capture the effect of different levels of the promotion impact. Furthermore, we
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evaluate the results based on three price discount levels: 10%, 20%, and 30%. Table 3.1 presents the parameter values used in our study. Note that the parameter values for the demand model are adopted from Silva-Risso et al. (1999). This approach allows us to develop a market demand simulator that is built based on empirical data and has been tested in the literature. The other base-case parameters used are as follows:

\[ cl = 8; \] \[ co = 12; \] \[ clnv_j = 0.092; \] \[ LL = 35; \] \[ UL = 140; \] \[ wh_t = 40; \] \[ nl_o = 50; \] \[ nu_j = 8; \] \[ O_t = 2.5; \] \[ lfo = 4000; \] \[ SS_{jt} = 2000; \] \[ T = 52; \] \[ K_j = 12; \] \[ Rp_{jt} = 12; \] \[ V_t = 1000; \] \[ R = 1000; \] \[ H = 121,350. \]

Table 3.1 Parameter settings for experimental factors

<table>
<thead>
<tr>
<th>Factors</th>
<th># Levels</th>
<th>Parameter</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Flexibility in production capacity</td>
<td>2</td>
<td>\textit{ch, cf}</td>
<td>Low: 2000, 3000 \hspace{1cm} High: 1000, 2000</td>
</tr>
<tr>
<td>Production unit cost</td>
<td>2</td>
<td>\textit{cp_j}</td>
<td>Low: 6                             \hspace{1cm} High: 7</td>
</tr>
<tr>
<td>Seasonality</td>
<td>2</td>
<td>\textit{F^h}</td>
<td>Low: 0.81 (constant) \hspace{1cm} High: 0.83, 0.7, 0.58, 0.48, 0.68, 0.85, 0.92, 0.99, 0.98, 0.92</td>
</tr>
<tr>
<td>Promotion impact</td>
<td>3</td>
<td>\textit{\theta_6}</td>
<td>Low: 0.2 \hspace{1cm} Medium: 0.5 \hspace{1cm} High: 0.8</td>
</tr>
<tr>
<td>Promotion discount level</td>
<td>3</td>
<td>\textit{L_t</td>
<td>L_t&gt;0}</td>
</tr>
</tbody>
</table>

In order to make fair comparisons of all the problem instances, we use the same set of random numbers to simulate the forecasted demands in the demand model. For all the 72 problem instances, Table 3.2 presents the performances of the three heuristics measured by the relative gaps (in %) of the profits obtained by each of the three heuristics compared to the optimal profit. Each instance in the table is represented by the levels of the four experimental factors. For example, the first instance is represented by H-L-L-L, i.e., the instance with high flexibility, low production cost, low seasonality and low promotion impact.
In the comparison of the three heuristics, we set the same stopping criterion for all three heuristics by allowing a maximum of 70 iterations in total. Preliminary experiments revealed that this stopping criterion allows all three heuristics to run long enough to reach convergence. As indicated in Table 3.2, the three heuristics all perform reasonably well, as indicated by the small relative gaps. In general, GA performs better than both SA and PSO. The average gaps for SA, GA, and PSO are 0.09%, 0.02%, and 0.46%, in the case with a discount level of 20%, whereas the average gaps are 0.13%, 0.00%, and 1.03%, in the case with a discount level of 30%. In addition to the gaps, we also recorded the number of promotions suggested, as presented in Table 3.3. The table shows that in general the heuristics result in a number of promotions that is close to or the same as

<table>
<thead>
<tr>
<th>Instance</th>
<th>Discount level: 10%</th>
<th>Discount level: 20%</th>
<th>Discount level: 30%</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>SA</td>
<td>GA</td>
<td>PSO</td>
</tr>
<tr>
<td>H-L-H-L</td>
<td>0.00</td>
<td>0.00</td>
<td>0.23</td>
</tr>
<tr>
<td>H-L-L-M</td>
<td>0.19</td>
<td>0.03</td>
<td>0.00</td>
</tr>
<tr>
<td>H-L-H-H</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>H-L-H-L</td>
<td>0.32</td>
<td>0.00</td>
<td>0.58</td>
</tr>
<tr>
<td>H-L-H-M</td>
<td>0.00</td>
<td>0.00</td>
<td>1.35</td>
</tr>
<tr>
<td>H-L-H-H</td>
<td>0.00</td>
<td>0.00</td>
<td>0.17</td>
</tr>
<tr>
<td>H-H-L-L</td>
<td>0.00</td>
<td>0.00</td>
<td>0.83</td>
</tr>
<tr>
<td>H-H-L-M</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>H-H-H-H</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>H-H-H-M</td>
<td>0.13</td>
<td>0.01</td>
<td>0.00</td>
</tr>
<tr>
<td>L-L-L-L</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>L-L-L-M</td>
<td>0.42</td>
<td>0.00</td>
<td>0.32</td>
</tr>
<tr>
<td>L-L-H-L</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>L-L-H-M</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>L-L-H-H</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>L-L-H-L</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>L-L-L-L</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>L-L-H-H</td>
<td>0.00</td>
<td>0.03</td>
<td>0.00</td>
</tr>
<tr>
<td>L-L-H-M</td>
<td>0.75</td>
<td>0.00</td>
<td>0.39</td>
</tr>
<tr>
<td>L-L-H-H</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>L-L-L-L</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
</tbody>
</table>

Table 3.2 The profit gap (in %) between the heuristic and the optimal solutions

* SA: Simulated annealing;  GA: Genetic algorithm;  PSO: Particle swarm optimisation
the optimal number obtained by complete enumeration. However, the same number of promotions
does not necessarily imply the same timing of the promotions, which explains why we observe
positive gaps in the profits in Table 3.2.

Table 3.3 The number of promotions for the discount level of 10%, 20% and 30%

<table>
<thead>
<tr>
<th>Instances</th>
<th>Discount level:10%</th>
<th>Discount level:20%</th>
<th>Discount level:30%</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>*Opt   SA   GA   PSO</td>
<td>*Opt   SA   GA   PSO</td>
<td>*Opt   SA   GA   PSO</td>
</tr>
<tr>
<td>H-L-L-L</td>
<td>8   8   8   8</td>
<td>3   3   3   3</td>
<td>1   1   1   1</td>
</tr>
<tr>
<td>H-L-L-M</td>
<td>4   4   4   4</td>
<td>2   2   2   2</td>
<td>1   1   1   1</td>
</tr>
<tr>
<td>H-L-L-H</td>
<td>11  11  11  11</td>
<td>11  11  11  11</td>
<td>0   0   0   0</td>
</tr>
<tr>
<td>H-L-H-L</td>
<td>7   7   7   6</td>
<td>4   4   4   4</td>
<td>6   4   4   4</td>
</tr>
<tr>
<td>H-L-H-M</td>
<td>9   8   9   8</td>
<td>9   9   9   5</td>
<td>3   2   3   2</td>
</tr>
<tr>
<td>H-L-H-H</td>
<td>8   8   8   8</td>
<td>8   8   8   7</td>
<td>6   5   6   5</td>
</tr>
<tr>
<td>H-H-H-L</td>
<td>8   8   8   8</td>
<td>3   3   3   3</td>
<td>1   1   1   1</td>
</tr>
<tr>
<td>H-H-H-M</td>
<td>4   4   4   4</td>
<td>2   2   2   2</td>
<td>1   1   1   1</td>
</tr>
<tr>
<td>H-H-L-H</td>
<td>11  11  10  11</td>
<td>2   2   2   2</td>
<td>0   0   0   0</td>
</tr>
<tr>
<td>H-H-H-L</td>
<td>9   7   7   9</td>
<td>4   4   4   3</td>
<td>6   4   6   4</td>
</tr>
<tr>
<td>H-H-H-M</td>
<td>8   8   8   8</td>
<td>5   5   5   5</td>
<td>3   2   3   2</td>
</tr>
<tr>
<td>H-H-H-H</td>
<td>8   8   8   8</td>
<td>7   6   7   7</td>
<td>5   2   5   5</td>
</tr>
<tr>
<td>L-L-L-L</td>
<td>8   8   8   8</td>
<td>3   3   3   3</td>
<td>1   1   1   1</td>
</tr>
<tr>
<td>L-L-L-M</td>
<td>11  10  11  11</td>
<td>2   2   2   3</td>
<td>1   1   1   1</td>
</tr>
<tr>
<td>L-L-L-H</td>
<td>10  10  10  10</td>
<td>10  10  10  10</td>
<td>0   0   0   0</td>
</tr>
<tr>
<td>L-L-H-L</td>
<td>7   7   7   7</td>
<td>4   4   4   5</td>
<td>6   5   6   4</td>
</tr>
<tr>
<td>L-L-H-M</td>
<td>8   8   8   8</td>
<td>9   9   9   5</td>
<td>2   2   2   2</td>
</tr>
<tr>
<td>L-L-H-H</td>
<td>8   8   8   7</td>
<td>8   8   8   7</td>
<td>6   5   6   6</td>
</tr>
<tr>
<td>L-H-L-L</td>
<td>8   8   8   8</td>
<td>3   3   3   3</td>
<td>1   1   1   1</td>
</tr>
<tr>
<td>L-H-L-M</td>
<td>4   4   4   4</td>
<td>2   2   2   2</td>
<td>1   1   1   1</td>
</tr>
<tr>
<td>L-H-L-H</td>
<td>10  10  10  10</td>
<td>1   1   1   3</td>
<td>0   0   0   0</td>
</tr>
<tr>
<td>L-H-H-L</td>
<td>7   7   7   8</td>
<td>4   4   4   5</td>
<td>6   3   6   4</td>
</tr>
<tr>
<td>L-H-H-M</td>
<td>8   8   8   8</td>
<td>5   5   5   3</td>
<td>2   2   2   2</td>
</tr>
<tr>
<td>L-H-H-H</td>
<td>8   8   8   8</td>
<td>7   7   7   7</td>
<td>5   5   5   4</td>
</tr>
</tbody>
</table>

*Opt: Optimal solution; SA: Simulated annealing; GA: Genetic algorithm; PSO: Particle swarm optimisation

The average computation time to run the implementation of the heuristic with the above stopping
criterion is 2.37 hours for one instance where the complete enumeration needs 5.82 hours. The
experiments were run on a computer with the following technical specifications: Intel(R)
Core(TM) i5-5200U CPU, 8.00GB (RAM), 64bit, Windows10. In the heuristics, we embed
What’sBest®15.0.1.2 for solving the production planning problem for each promotion plan, so
that we can determine the promotion and production decisions simultaneously. The numerical
study shows that standard heuristics like the three tested in this paper can be useful for developing a decision support system that integrates promotion and production planning in S&OP. As GA was the best performing heuristic, we choose to use GA to solve the optimisation problems in the next numerical study, where we integrate promotion and production decisions in S&OP in the case of two products.

The stopping criterion for GA

Further numerical evaluation revealed that the performance of GA is still satisfactory even if the maximum number of iterations is reduced. This finding is particularly useful when considering the idea of developing a decision support model for integrating promotions and production decisions in S&OP, where long computation times should be avoided in order to allow for alternative scenarios to be explored. We tested several values of $n$, where $n$ is the number of consecutive iterations that do not result in solution improvements before the heuristic is stopped. In Figure 3.2, we show the average profit gap and the average computation time for several values of $n$. Based on these observations, $n = 10$ appears to be a good choice for the stopping criterion, as the resulting profit gap is still below 0.1% while the average computation time is less than 30 minutes. This stopping criterion will be used in Section 3.5 for the numerical study that considers multiple products.

Figure 3.2 The performance of GA with different threshold values for the stopping criterion
3.4.2 Sensitivity analysis of promotion timing

Before we present the results of the second numerical study, we discuss some modifications in relation to the promotion timing decisions and the stopping criterion of the GA heuristic. Recall that in the first numerical study, we restricted promotions to be carried out only in the first week of each month. One could question whether such a restriction may generate promotion plans that are far from optimal. Therefore, we have undertaken a sensitivity analysis to examine the effect of this promotion timing restriction. Removing the restriction on the promotion timing means that promotions may be carried out in any week of the months. This results in a significant increase of the solution space from $2^{12}$ to $2^{52}$ possible promotion plans (1 year = 52 weeks).

We use the same problem instances as in the first numerical study and apply GA to solve the corresponding optimisation problem. Table 3.4 presents the average relative increase of profits for the 24 problem instances as the consequence of relaxing the restricted timing of promotions.

<table>
<thead>
<tr>
<th>Factors</th>
<th>Level</th>
<th>Promotion discount level</th>
<th>10%</th>
<th>20%</th>
<th>30%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Flexibility in production capacity</td>
<td>High</td>
<td>3.36</td>
<td>3.06</td>
<td>1.82</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Low</td>
<td>3.44</td>
<td>3.07</td>
<td>1.97</td>
<td></td>
</tr>
<tr>
<td>Production unit cost</td>
<td>Low</td>
<td>3.41</td>
<td>3.31</td>
<td>1.94</td>
<td></td>
</tr>
<tr>
<td></td>
<td>High</td>
<td>3.39</td>
<td>2.81</td>
<td>1.86</td>
<td></td>
</tr>
<tr>
<td>Seasonality</td>
<td>Low</td>
<td>1.44</td>
<td>1.38</td>
<td>0.33</td>
<td></td>
</tr>
<tr>
<td></td>
<td>High</td>
<td>5.37</td>
<td>4.74</td>
<td>3.46</td>
<td></td>
</tr>
<tr>
<td>Promotion impact</td>
<td>Low</td>
<td>2.35</td>
<td>1.98</td>
<td>1.45</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Medium</td>
<td>2.96</td>
<td>2.92</td>
<td>1.99</td>
<td></td>
</tr>
<tr>
<td></td>
<td>High</td>
<td>4.90</td>
<td>4.29</td>
<td>2.25</td>
<td></td>
</tr>
<tr>
<td>Overall average</td>
<td>3.40</td>
<td>3.06</td>
<td>1.90</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The overall averages of the profit increase after relaxing the restricted promotion timing are 3.40%, 3.06% and 1.90% for the discount levels of 10%, 20% and 30%, respectively. These results show the benefits of having higher flexibility in the promotion timing. The relative benefits decrease when the discount level is higher. Relaxing the restriction on timing of promotions also tends to generate more frequent promotions, as shown in Table 3.5. Based on the results of this sensitivity
analysis, we relax the promotion timing constraint in the numerical study with multiple products presented in Section 3.5.

Table 3.5 The average number of promotions based on GA1 and GA2 for the discount level of 10%, 20% and 30%

<table>
<thead>
<tr>
<th>Factors</th>
<th>Level</th>
<th>10%</th>
<th>20%</th>
<th>30%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Flexibility in production capacity</td>
<td>High</td>
<td>7.50</td>
<td>9.25</td>
<td>2.75</td>
</tr>
<tr>
<td></td>
<td>Low</td>
<td>7.42</td>
<td>9.08</td>
<td>2.58</td>
</tr>
<tr>
<td>Production unit cost</td>
<td>Low</td>
<td>8.00</td>
<td>9.67</td>
<td>2.75</td>
</tr>
<tr>
<td></td>
<td>High</td>
<td>6.92</td>
<td>8.67</td>
<td>2.58</td>
</tr>
<tr>
<td>Seasonality</td>
<td>Low</td>
<td>7.17</td>
<td>8.83</td>
<td>0.67</td>
</tr>
<tr>
<td></td>
<td>High</td>
<td>7.75</td>
<td>9.50</td>
<td>4.67</td>
</tr>
<tr>
<td>Promotion impact</td>
<td>Low</td>
<td>7.50</td>
<td>7.63</td>
<td>3.50</td>
</tr>
<tr>
<td></td>
<td>Medium</td>
<td>6.63</td>
<td>7.88</td>
<td>1.75</td>
</tr>
<tr>
<td></td>
<td>High</td>
<td>8.25</td>
<td>12.00</td>
<td>2.75</td>
</tr>
<tr>
<td>Overall average</td>
<td></td>
<td>7.46</td>
<td>9.17</td>
<td>4.92</td>
</tr>
</tbody>
</table>

GA1: Promotion only in the first week of each month
GA2: Relaxed promotion timing

3.5 Numerical study: integrated S&OP in the case of multiple products

In this section, we present the setup and results of our numerical study on the integrated promotion and production decisions in the case of multiple products. We consider a scenario where there are two products in the same product family. In such a scenario, in addition to the competition with products offered by competitors, product substitution or cannibalization within the same product family also occurs. This could be due to differences in customer preference, prices and marketing strategies (Srinivasan et al., 2005). To maximise profit, the existence of product substitution should be recognised when determining the ultimate promotion and production plan.

We extend the setting used in Section 3.4 by considering the manufacturing and selling of two different products (products A and B) in the same product family, while there is also a related product (product C) offered by a competitor. Table 3.6 shows the extended parameter setting for this numerical study.
Table 3.6 Parameter setting for experimental factors with product substitution

<table>
<thead>
<tr>
<th>Factors</th>
<th># Levels</th>
<th>Parameter</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Flexibility in production</td>
<td>2</td>
<td>$ch, cf$</td>
<td>High: 1000, 2000</td>
</tr>
<tr>
<td>capacity</td>
<td></td>
<td></td>
<td>Low: 2000, 3000</td>
</tr>
<tr>
<td>Margin gap</td>
<td>2</td>
<td>$R_pjt, cp_j$</td>
<td>Low: $R_pA = 12; R_pB = 12; cp = 7; R_pC = 10$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>High: $R_pA = 12; R_pB = 11; cp = 7; R_pC = 10$</td>
</tr>
<tr>
<td>Seasonality</td>
<td>2</td>
<td>$F^k$</td>
<td>Low: 0.81 (constant)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>High: 0.83, 0.7, 0.58, 0.48, 0.68, 0.85, 0.92, 0.99, 0.98, 0.92</td>
</tr>
<tr>
<td>Loyalty gap</td>
<td>2</td>
<td>$B^h_j$</td>
<td>Low: $B_A = 0.4; B_B = 0.3$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>High: $B_A = 0.6; B_B = 0.1$</td>
</tr>
<tr>
<td>Promotion impact</td>
<td>3</td>
<td>$\theta_6$</td>
<td>Low: 0.2</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Medium: 0.5</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>High: 0.8</td>
</tr>
<tr>
<td>Promotion discount level</td>
<td>3</td>
<td>$L_t</td>
<td>L_t &gt; 0$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Medium: 20%</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>High: 30%</td>
</tr>
</tbody>
</table>

We use the same demand and base-case parameters as in the first numerical study but make the following two modifications. First, we add two levels of gap between product A’s and product B’s profit margins. This is achieved by differentiating the selling price for product B, while keeping the same unit production cost. All other parameters are also the same for products A and B. Second, we add two levels of loyalty gap between the two products. Note that the brand loyalty parameter ($B_j$) is set in the brand choice model (see the Appendix). It represents loyalty to brand and is obtained from the calculation of market share (Bucklin and Lattin, 1992). Loyalty gap is the difference of loyalty parameter of the two products. Thus, in total there are 144 problem instances evaluated in this numerical study. Note that the size of the solution space for the case of two products now becomes $2^{2 \times 52}$ promotion plans, and the average computation time required by the GA heuristic is 2.62 hours.

Table 3.7 presents the average number of promotions for each of the two products, as well as the average number of simultaneous promotions. In general, a higher discount level tends to result in a lower average number of promotions. The motivation to offer promotions of the two products at the same time could be supported by the saving in the promotion costs, but hindered by the possible cannibalization between the two products. Furthermore, carrying out promotions at the same time...
may result in higher production costs, especially when the flexibility is low. A higher number of promotions tends to give a higher chance of getting the same promotion timing for product A and product B. This explains why we observe that the average number of promotions with the same timing is higher in the case of a low discount level than in the case of a high discount level.

Table 3.7 The average number of promotions for two products with substitution effects

<table>
<thead>
<tr>
<th>Factors</th>
<th>Level</th>
<th>Disc: 10%</th>
<th>Disc: 20%</th>
<th>Disc: 30%</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>A</td>
<td>B</td>
<td>Same timing</td>
</tr>
<tr>
<td>Flexibility in production capacity</td>
<td>H</td>
<td>3.79</td>
<td>2.88</td>
<td>0.71</td>
</tr>
<tr>
<td></td>
<td>L</td>
<td>3.25</td>
<td>2.17</td>
<td>0.25</td>
</tr>
<tr>
<td>Margin gap</td>
<td>L</td>
<td>3.67</td>
<td>3.83</td>
<td>1.00</td>
</tr>
<tr>
<td></td>
<td>H</td>
<td>3.38</td>
<td>1.21</td>
<td>0.01</td>
</tr>
<tr>
<td>Seasonality</td>
<td>L</td>
<td>2.54</td>
<td>2.25</td>
<td>0.33</td>
</tr>
<tr>
<td></td>
<td>H</td>
<td>4.50</td>
<td>2.79</td>
<td>0.62</td>
</tr>
<tr>
<td>Loyalty gap</td>
<td>L</td>
<td>3.71</td>
<td>2.42</td>
<td>0.42</td>
</tr>
<tr>
<td></td>
<td>H</td>
<td>3.33</td>
<td>2.63</td>
<td>0.34</td>
</tr>
<tr>
<td>Promotion impact</td>
<td>L</td>
<td>1.25</td>
<td>0.81</td>
<td>0.06</td>
</tr>
<tr>
<td></td>
<td>M</td>
<td>3.19</td>
<td>2.06</td>
<td>0.63</td>
</tr>
<tr>
<td></td>
<td>H</td>
<td>6.13</td>
<td>4.69</td>
<td>1.25</td>
</tr>
<tr>
<td>Overall average</td>
<td></td>
<td>3.52</td>
<td>2.52</td>
<td>0.51</td>
</tr>
</tbody>
</table>

L: Low, M: Medium, H: High

Table 3.7 also shows that high flexibility (low cost of hiring/firing) tends to yield more promotions than low flexibility (high cost of hiring/firing). This seems reasonable, because more frequent promotions imply that more frequent adjustments of production capacity are necessary. The number of promotions with the same timing is also higher in the case of high flexibility than in the case of low flexibility. As product A and product B are produced using the same production resources, the same timing of promotions should particularly be avoided when the flexibility is low.

The profit margin of a product also affects the chosen number of promotions. The motivation to offer promotions could be reduced when the product’s profit margin is low, and this is especially true when promotions generate a high number of units of forward buying. Fewer promotions observed in Table 3.7 in the case of high margin gap are mainly due to the lower profit margin of
product B.

The average number of promotions with the same timing is higher in the case of low margin gap than in the case of high margin gap. Offering promotions at the same time for products A and B is less preferable when product B has a narrower profit margin. Figure 3.3 shows the number of promotions with the same timing for the three discount levels and for low and high margin gap, respectively. Table 3.7 also shows that the average number of promotions with the same timing is higher in the case of low loyalty gap than in the case of high loyalty gap. In the case of high loyalty gap, cannibalization may become more severe such that offering promotions at the same time will not be beneficial for the product with a low loyalty parameter. These findings provide useful information for production and marketing planners about aspects they need to consider when deciding on whether or not joint promotions for products within a product family should be carried out.

Our results also show that higher demand seasonality seems to trigger more frequent promotions. In the case of high demand seasonality, offering promotions in the low-demand season will help smooth the demand and production over the planning horizon. As the two products A and B are assumed to have identical expected demand patterns, there is also a higher likelihood to find the same promotion timing for the two products when there is seasonality. Finally, as expected, increasing the promotion impact seems to generate more frequent promotions during the planning horizon.
In what follows, we present a more detailed discussion on the effect of the discount level. In Figure 3.4, we plot the profits for all problem instances differentiated by the three discount levels. The figure shows that in most cases a discount level of 20% gives a higher average profit compared to what the other two discount levels provide. In our numerical experiments, we observe that a higher discount level will generate a higher total demand for a particular promotion event. However, since we also have to consider the lower price per unit due to the higher discount level and the promotion and production-related costs, focusing solely on higher total demand for a particular promotion may not necessarily be a good approach.
Figure 3.4 Profit for each problem instance differentiated by the discount level

Table 3.8 The average of incremental demand (in %) for two products with substitution effects

<table>
<thead>
<tr>
<th>Factors</th>
<th>Level</th>
<th>Disc: 10%</th>
<th>Disc: 20%</th>
<th>Disc: 30%</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Inc.D</td>
<td>Percentage of Inc.Demand</td>
<td>Inc.D</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Cons.</td>
<td>BS</td>
<td>FB</td>
</tr>
<tr>
<td>Flexibility in production capacity</td>
<td>H</td>
<td>3.1</td>
<td>22.9</td>
<td>46.2</td>
</tr>
<tr>
<td></td>
<td>L</td>
<td>2.8</td>
<td>27.0</td>
<td>46.4</td>
</tr>
<tr>
<td>Margin gap</td>
<td>L</td>
<td>4.1</td>
<td>23.5</td>
<td>49.0</td>
</tr>
<tr>
<td></td>
<td>H</td>
<td>1.9</td>
<td>26.3</td>
<td>43.3</td>
</tr>
<tr>
<td>Seasonality</td>
<td>L</td>
<td>2.7</td>
<td>24.1</td>
<td>46.9</td>
</tr>
<tr>
<td></td>
<td>H</td>
<td>3.3</td>
<td>25.6</td>
<td>45.7</td>
</tr>
<tr>
<td>Loyalty gap</td>
<td>L</td>
<td>3.0</td>
<td>26.7</td>
<td>43.8</td>
</tr>
<tr>
<td></td>
<td>H</td>
<td>2.9</td>
<td>22.8</td>
<td>48.9</td>
</tr>
<tr>
<td>Promotion impact</td>
<td>L</td>
<td>0.2</td>
<td>12.3</td>
<td>45.6</td>
</tr>
<tr>
<td></td>
<td>M</td>
<td>1.7</td>
<td>27.1</td>
<td>45.9</td>
</tr>
<tr>
<td></td>
<td>H</td>
<td>7.0</td>
<td>33.5</td>
<td>47.3</td>
</tr>
</tbody>
</table>

Overall average: 3.0 24.7 46.3 29.0 4.2 32.5 39.8 27.7 3.6 41.8 31.8 26.4

L: Low, M: Medium, H: High
Inc.D/Inc.Demand: Incremental demand; Cons.: Consumption; BS: Brand switching; FB: Forward buying

Table 3.8 presents the averages of total incremental demand for the discount levels of 10%, 20% and 30%, and shows that these averages are higher when the discount level is 20%. Note that the total incremental demand for each discount level is also dependent on the number of promotions carried out throughout the planning horizon. This number is shown in Table 3.7.
As stated above, the demand model we adopt allows us to divide the incremental demand into consumption, brand switching and forward buying. As for illustration, in the problem instances with high flexibility and discount level 20%, the average total incremental demand is 4.3 %, while the averages of increase of consumption, brand switching and forward buying are 32.9%, 40.3%, and 26.8%, respectively. In Figure 3.5 we depict incremental demands for products A, B, and C due to promotion for the problem instance with high-flexibility, low-margin gap, high-seasonality, high-promotion effect, high-loyalty gap, and a 20% discount level. The figure also shows how the incremental demand is obtained from increase of consumption, brand switching, and forward buying. An illustrative explanation is provided in the figure to show how a promotion event for product A, e.g. in week 37, results in a decrease in demand for products B and C due to brand switching, which contributes to the total increase in demand for product A.
3.6 Conclusions

We have integrated a rich econometric-based demand model and an aggregate production planning model to generate a joint promotion and production plan in an environment, where manufacturers sell a family of separate products that substitute each other and are produced using the same production resources. This multi-product setting constitutes a relevant framework for a sales and operations planning process. The demand model that we have adopted captures the dynamics and heterogeneity of consumer response by combining purchase incidence, consumer choice and purchase quantity, and it also allows for taking into account possible substitution among the products. Due to the large problem sizes, we have developed and evaluated three commonly used heuristics for solving the integrated promotion and production planning problem, namely simulated annealing, a genetic algorithm, and particle swarm optimisation. Our numerical results show that while all the three heuristics seem to perform quite satisfactorily, the implementation of the genetic algorithm appears to be the best performing heuristic.

We have also conducted a numerical study to examine how different factors related to marketing and production affect the overall profitability of the promotion and production plan. In particular, we have been interested in understanding if simultaneous promotion events are preferred to sequential promotions in case the manufacturing firm offers multiple products. Our results show no evidence of strong preference for implementing simultaneous promotions in general. However, in the cases with high flexibility, narrow margin gaps, small loyalty gaps, and low discount levels, the frequency of simultaneous promotions is higher relative to the other cases.

The modelling framework we have developed can help production and marketing planners to better understand some of the driving forces of their planning results. In particular, the possibility of decomposing incremental sales into true incremental sales and forward buying that is facilitated by the demand model helps to provide insightful explanations. This would be very difficult to obtain if one uses a rather simple demand model, as has been the case in most of the previous literature on this topic. The optimal decision on discount level, for example, depends on many inter-related factors. Although our numerical study shows that applying a moderate discount level seems to give the largest net profit in general, there are also some cases, where applying a lower or higher discount level is preferable. These cases can be difficult to identify without an integrated decision-support tool for sales and operations planning. Furthermore, the possibility of examining
the effects of promotion for one product on the demand of other products will most likely help guiding planners to make better decisions.

We acknowledge some limitations of this paper and therefore suggest a few topics for future research. Firstly, no strategic interaction between the manufacturer and the retailer is considered in this study. For example, the retailer’s pass through rate in a promotion event is assumed to be given. In many realistic settings, however, retailers may respond strategically to the manufacturer’s promotion plan by choosing pass through rates that maximise their own benefits. The existence of such strategic behaviour may have an effect on the optimal production and promotion plans of the manufacturer. Secondly, the model developed in this paper considers a fixed planning horizon where demand uncertainties and forecast inaccuracies have not been incorporated. An interesting research avenue is to extend the modelling framework presented in this paper so that it works based on a rolling instead of fixed planning horizon. This would allow us to further address planning issues in practice, especially in relation to demand forecast updating.
Appendix:

The demand model

The expected of household demand, given a store visit, is given by

\[ E(D_{jt}^h) = P_t^h(inc) \times P_t^h(j|inc) \times E(D_{jt}^h|D_{jt}^h > 0) \]  \hspace{1cm} (A3.1)

where

- \( P_t^h(inc) \) is the probability that household \( h \) makes a purchase in the product category on a store visit in time period \( t \).
- \( P_t^h(j|inc) \) is the probability that household \( h \) chooses product \( j \), given that household \( h \) decides to make a purchase in the product category in time period \( t \).
- \( E(D_{jt}^h|D_{jt}^h > 0) \) is the expected quantity that household \( h \) will buy of product \( j \), given that household \( h \) decides to purchase product \( j \) in time period \( t \).

In what follows, we present the purchase incidence model, the product choice model, and the quantity model. The purchase incidence probability is modelled with a binary nested logit model:

\[ P_t^h(inc) = \frac{e^{C_t^h}}{1 + e^{C_t^h}} \]  \hspace{1cm} (A3.2)

where \( C_t^h \) is the deterministic component of utility associated with household \( h \) in time period \( t \), and is modelled as a function of the proportion of purchase frequency, household inventory, and consumption rate. The probability that a household will choose brand \( j \) is handled in a multinomial logit framework in the following form:

\[ P_t^h(j|inc) = \frac{e^{A_{jt}^h}}{\sum_{j=1}^{J} e^{A_{jt}^h}} \]  \hspace{1cm} (A3.3)

where \( A_{jt}^h \) is the deterministic component of utility associated with brand \( j \) for household \( h \) in time period \( t \) that is modelled as a function of price, promotion and consumer-specific variables such as brand loyalty.

Next, given a purchase of brand \( j \), the number of units purchased is captured by a Poisson distribution with truncation of the zero outcome, which gives the expected number of units purchased by household \( h \) at time \( t \):

\[ E(D_{jt}^h|D_{jt}^h > 0) = \frac{\lambda_{jt}^h}{1 - e^{-\lambda_{jt}^h}} \]  \hspace{1cm} (A3.4)
where $\lambda_{jt}^h$ is the purchase rate of household $h$ for brand $j$ at time $t$, which is modelled as a function of the average number of units purchased, household inventory, brand loyalty, and price.

As for model application, parameter values of the model are usually estimated based on scanner-panel data obtained from sources such as Nielsen Consumer Panels for certain product categories, e.g., canned tomato sauce (Silva-Risso et al., 1999) or ketchup and yoghurt (Ailawadi et al., 2007). In our numerical study, we use the parameter values as presented in Silva-Risso et al. (1999) for the purpose of forming the basis of a market simulator that allows us to examine how the integrated promotion and production decisions are affected by various production and marketing related factors in a multiple product setting.

**Purchase incidence model**

The decision that a household will make a purchase in the product category on a store visit is modelled with a binary nested logit model, as presented in (A3.2). The value of the deterministic component of household utility ($C_t^h$) in the purchase incidence model takes the following form (e.g., Guadagni and Little, 1983; Bucklin and Lattin, 1992; Silva-Risso et al., 1999; Ailawadi et al., 2007):

\[
C_t^h = \beta_0 + \beta_1 F_t^h + \beta_2 I_t^h + \beta_3 W_t^h \\
W_t^h = \ln \sum_{j=1}^{J} e^{A_j t^h} \\
\text{where}
\]

- $C_t^h$ The deterministic component of utility related with household $h$ in time period $t$
- $F_t^h$ Proportion of purchase frequency for household $h$ on store visit
- $I_t^h$ Inventory for household $h$ at the end of time period $t$
- $W_t^h = \ln \sum_{j=1}^{J} e^{A_j t^h}$

\[
I_t^h = \text{Max}(0, I_{t-1}^h + \sum_{j=1}^{J} D_{j,t-1}^h - U_{t-1}^h) \\
D_{j,t-1}^h \text{ Quantity of brand } j \text{ bought in time period } t-1 \text{ by household } h \\
U_{t}^h \text{ Rate of consumption for household } h \text{ in time period } t \\
\bar{U}^h \text{ Mean rate of consumption for household } h \\
W_t^h \text{ The expected maximum utility from the brand choice decision for household } h \text{ in time period } t.
\]
Parameter to be estimated
{\beta_0, ..., \beta_3} Parameters to be estimated

**Brand choice model**

In the brand choice model, we use a multinomial logit form as presented in (A3.3) to calculate probability that a household chooses a particular brand. The value of the deterministic component of brand utility \(A_{jt}^h\) in (A3.3) is modelled as (see e.g., Bucklin and Lattin, 1992; Silva-Risso et al., 1999; Ailawadi et al., 2007)

\[
A_{jt}^h = \alpha_b + \alpha_s + \theta_1 B_j^h + \theta_2 LB_j^h + \theta_3 S_j^h + \theta_4 Ls_j^h + \theta_5 R_{jt} + \theta_6 TC_{jt} + \theta_7 X_{jt} + \theta_8 Y_{jt}
\] (A3.9)

where

- \(A_{jt}^h\) The deterministic component of utility related with brand \(j\) for household \(h\) in time period \(t\)
- \(B_j^h\) Brand loyalty of household \(h\) to brand \(j\)
- \(LB_j^h\) 1 if \(j\) was last brand purchased, 0 otherwise
- \(S_j^h\) Size loyalty of household \(h\) to brand \(j\)
- \(LS_j^h\) 1 if \(j\) was last size purchased, 0 otherwise
- \(R_{jt}\) Regular store price for brand \(j\) in time period \(t\)
  \[
  R_{jt} = R_{pj}(1 + Up) \quad t = 1, ..., T
  \] (A3.10)
- \(R_{pj}\) Regular price from manufacturer of brand \(j\) in time period \(t\)
- \(Up\) Store’s markup in percent
- \(TC_{jt}\) Temporary price cut for brand \(j\) in time period \(t\)
  \[
  TC_{jt} = R_{pj} \left( PT_{jt} \cdot L_{jt} \right) \quad t = 1, ..., T
  \] (A3.11)
- \(PT_{jt}\) Store’s pass-through in percent
- \(L_{jt}\) Level of discount in percent for brand \(j\) in time period \(t\)
- \(X_{jt}\) \(1\) if a feature ad is offered for brand \(j\) in time period \(t\)
- \(Y_{jt}\) \(0\) otherwise
- \(X_{jt}\) \(1\) if a display is offered for brand \(j\) in time period \(t\)
- \(0\), otherwise
- \(\alpha_b\) Brand constant to be estimated
- \(\alpha_s\) Size constant to be estimated
- \{\theta_1, ..., \theta_8\} Parameters to be estimated
Quantity model

The expected value of the truncated Poisson distribution as presented in (A3.4) is used to calculate the expected quantity purchased by a household. The purchase rate of the household takes the following form (e.g., Bucklin and Lattin, 1992; Silva-Risso et al., 1999; Ailawadi et al., 2007)

$$\lambda_{jt}^h = \exp(\mu_b + \mu_s + \omega_1 G^h + \omega_2 l_t^h + \omega_3 B_j^h + \omega_4 S_j^h + \omega_5 R_{jt} + \omega_6 TC_{jt} + \omega_7 X_{jt} + \omega_8 Y_{jt})$$  (A3.12)

where

- $\lambda_{jt}^h$: The purchase rate of household $h$ for the brand alternative $j$ in time period $t$
- $G^h$: Average quantity bought by household $h$ per purchase trip
- $l_t^h$: Inventory for household $h$ at the end of time $t$
- $\mu_b$: Brand constant to be estimated
- $\mu_s$: Size constant to be estimated
- $\{\omega_1, ..., \omega_8\}$: Parameters to be estimated

The decomposition of incremental demand.

The expected incremental demand of brand $j$ sold to household $h$ in time period $t$ is obtained by subtracting baseline plus forward buying demand, $E(BFD_{jt}^h)$, from total demand, $E(D_{jt}^h)$, and adding back borrowed demand that resulted in incremental consumption ($\Delta U_t^h = UBF_t^h - UB_t^h$), as shown in A9 (Silva-Risso et al., 1999)

$$E(\Delta D_{jt}^h) = E(D_{jt}^h) - E(BFD_{jt}^h) + \Delta U_t^h$$  (A3.13)

UBF is the consumption rate for the simulated baseline plus forward buying, and UB is the consumption rate for the baseline. In the baseline plus forward buying model, we remove choice effect such that promotions resulted only in forward buying through purchase acceleration and/or stockpiling by setting no promotion and no purchased feedback in the choice model, and no-incremental consumption as shown in A10 (Silva-Risso et al., 1999)

$$E(BFD_{jt}^h) = P_t^h(inc)|_{l_t^h=IBF_t^h} \times P_t^h(j|inc)_{No\ promotion\ feedback} \times E(D_{jt}^h|D_{jt}^h > 0)|_{l_t^h=IBF_t^h}$$  (A3.14)

$IBF_t^h$ is the household’s inventory given that promotion effect in the choice model is removed, No purchased feedback eliminates carryover effects (last brand purchased) in the choice model. The expected baseline is given by (Silva-Risso et al., 1999)
\[
E(BD^h_{jt}) = P^h_t(\text{inc})_{\text{No promotion}} x P^h_j(\text{inc})_{\text{No promotion}} x E(D^h_{jt}|D^h_{jt} > 0)_{\text{No promotion}} \quad (A3.15)
\]

BD is baseline volume and \(IB^h_t\) refers to the household’s inventory for the case of no promotions.

Parameter for consumer response model

<table>
<thead>
<tr>
<th>Purchase incidence model</th>
<th>Choice model</th>
<th>Quantity model</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\beta_0 = -5.2562)</td>
<td>(\alpha_A = 0.4537)</td>
<td>(\mu_A = 0.0140)</td>
</tr>
<tr>
<td>(\beta_1 = 5.2590)</td>
<td>(\alpha_B = 0.8096)</td>
<td>(\mu_B = -0.1356)</td>
</tr>
<tr>
<td>(\beta_2 = -0.0201)</td>
<td>(\alpha_C = 0.7432)</td>
<td>(\mu_C = -0.2888)</td>
</tr>
<tr>
<td>(\beta_3 = 0.3338)</td>
<td>(\alpha_S = -0.4521)</td>
<td>(\mu_S = -0.0146)</td>
</tr>
<tr>
<td></td>
<td>(\theta_1 = 1.9085)</td>
<td>(\omega_1 = 0.3153)</td>
</tr>
<tr>
<td></td>
<td>(\theta_2 = 0.9154)</td>
<td>(\omega_2 = -0.0097)</td>
</tr>
<tr>
<td></td>
<td>(\theta_3 = 2.5672)</td>
<td>(\omega_3 = 0.0428)</td>
</tr>
<tr>
<td></td>
<td>(\theta_4 = 0.3876)</td>
<td>(\omega_4 = -0.3135)</td>
</tr>
<tr>
<td></td>
<td>(\theta_5 = -4.156)</td>
<td>(\omega_5 = -0.0770)</td>
</tr>
<tr>
<td></td>
<td>(\theta_6 = 0.4752)</td>
<td>(\omega_6 = 0.3239)</td>
</tr>
<tr>
<td></td>
<td>(\theta_7 = 1.2259)</td>
<td>(\omega_7 = 0.5517)</td>
</tr>
<tr>
<td></td>
<td>(\theta_8 = 1.1042)</td>
<td>(\omega_8 = -0.0686)</td>
</tr>
</tbody>
</table>

Source: Silva-Risso et al., 1999

The algorithms of the three heuristic approaches

The common parameters and variables for the three heuristics:

- \(N\): Population size / number of particles
- \(g\): Index for generation (\(g = 1, \ldots, G\))
- \(P_{\text{best}}\): The best promotion plan so far / personal best solution
- \(I\text{Profit}(P)\): Objective function value when using promotion plan \(P\)
- \(I\text{Profit}_\text{best}\): The best objective function value so far
- \(I\text{Profit}_\text{lowest}\): The lowest objective function value in each generation

Simulated Annealing

The specific parameters and variables are:

- \(Temp^0\): Initial temperature
- \(Temp^f\): Final temperature
- \(\alpha\): Decreasing rate of temperature
- \(MaxIt\): Maximum number of iterations at each temperature

The algorithm consists of the following steps:

Step 1: Choose an initial promotion plan \(P_0\), and assign \(P_{\text{best}} = P_0\), \(Temp = Temp^0\); Calculate the corresponding demand forecasts \(D_{jt}|P_0\) \((j=1, \ldots, J; t=1, \ldots, T)\) and solve the resulting aggregate production planning problem; Calculate the objective function value \(I\text{Profit}(P_0)\); Assign \(I\text{Profit}_\text{best} = I\text{Profit}(P_0)\).
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Step 2: Generate a neighbourhood solution, promotion plan $P''$.

Step 3: Calculate demand forecasts $D_{jt}|P''$ and solve the aggregate production planning problem; Calculate the objective function value $IPrfit(P'')$.

Step 4: If $IPrfit(P') \geq IPrfit(P_{best})$, then $P_{best} = P'$ and $IPrfit_{best} = IPrfit(P')$ and go to Step 6; otherwise go to Step 5.

Step 5: Generate $y \leftarrow U(0,1)$. If $y < e^{-\frac{|IPrfit(P') - IPrfit(P_{best})|}{Temp}^\text{T}}$, then $IPrfit_{best} = IPrfit(P')$, $P_{best} = P'$.

Step 6: Is the number of iterations in temperature $Temp < MaxI$? If yes, then go to Step 2; otherwise go to Step 7.

Step 7: $Temp = \alpha \cdot Temp$.  

Step 8: If $Temp = Temp^f$, then Stop; else go to Step 2.

**Genetic Algorithm**

The specific parameters and variables are:

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>CrossRate</td>
<td>Crossover rate</td>
</tr>
<tr>
<td>MutRate</td>
<td>Mutation rate</td>
</tr>
<tr>
<td>FV</td>
<td>The fitness value</td>
</tr>
</tbody>
</table>

The algorithm consists of the following steps:

Step 1: Choose an initial generation that consists of $N$ chromosomes: $P^{(1)}, P^{(2)}, ..., P^{(N)}$ Set $g = 0$.

Step 2: Calculate the corresponding demand forecasts $D_{jt}|P^{(i)}$ ($i=1,...,N; j=1,...,J; t=1,...,T$);

Step 3: For each chromosome, solve the aggregate production planning problem; Calculate the objective function value $IPrfit(P^{(i)})$, ($i=1,...,N$).

Step 4: Find the lowest and best objective function

\[
IPrfit_{lowest} = \min \{IPrfit(P^{(i)})\}
\]

If $IPrfit_{best} = \max \{IPrfit(P^{(i)})\}$ then $P_{best} = P^{(i)}$

Step 5: If $g = G$ then select $P_{best}$ and $IPrfit_{best}$; otherwise go to Step 6.

Step 6: Calculate the fitness value for each chromosome

\[
FV^{(i)} = IPrfit(P^{(i)}) - IPrfit_{lowest}
\]

Step 7: Selection procedure (combining three ways of picking chromosomes).

7.1 Select the best solution for next generation, $P^{(1)} = P_{best}$

7.2 For each of the existing old chromosomes, calculate the probability

\[
Prob^{(i)} = \frac{FV^{(i)}}{\sum_{i=1}^{N} FV^{(i)}}
\]

Generate a random variate $r_i \leftarrow U(0, 1)$ and pick the chromosome that corresponds to the c.d.f of the fitness value. Repeat this procedure until we pick $n_1$ chromosomes.

7.3 Generate $N - 1 - n_1$ new chromosomes (to avoid premature convergence)

Step 8 Crossover (two points of crossover)

Form $N/2$ pair of chromosomes. For each pair of chromosomes, e.g.

\[
P^{(1)} = (L_{11}^{(1)} ... L_{1T}^{(1)} ... L_{j1}^{(1)} ... L_{jT}^{(1)}) \text{ and } \\
P^{(2)} = (L_{11}^{(2)} ... L_{1T}^{(2)} ... L_{j1}^{(2)} ... L_{jT}^{(2)})
\]
Generate \( r_2 \leftarrow U(0, 1) \); if \( r_2 < \text{CrossRate} \) then undergo the following crossover; Otherwise no crossover

Set \( Lnew_{jt}^{(1)} = L_{jt}^{(1)} \) and \( Lnew_{jt}^{(2)} = L_{jt}^{(2)} \) \((j=1,\ldots,J; t=1,\ldots,T)\);

Generate the borders of cross-over range \( x_1 \leftarrow U(0, T) \) and \( x_2 \leftarrow U(0, T) \) with \( x_1 < x_2 \)

Set \( L_{jt}^{(1)} = Lnew_{jt}^{(1)} \) and \( L_{jt}^{(2)} = Lnew_{jt}^{(2)} \) \((j=1,\ldots,J; t=1,\ldots,T)\);

Step 9: Mutation (swap mutation)

For each chromosome \( p^{(i)} = (L_{11}^{(i)} \ldots L_{1T}^{(i)} \ldots L_{j1}^{(i)} \ldots L_{jT}^{(i)}) \)

Generate \( r_3 \leftarrow U(0, 1) \); if \( r_3 < \text{MutRate} \) then undergo mutation; Otherwise no mutation

Set \( Lnew_{jt}^{(i)} = L_{jt}^{(i)} \)

Generate \( y_1 \leftarrow U(0, T) \) and \( y_2 \leftarrow U(0, T) \)

Set \( L_{jy_1}^{(i)} = Lnew_{jy_1}^{(i)} \) and \( L_{jy_2}^{(i)} = Lnew_{jy_2}^{(i)} \)

Step 10: Set \( g = g + 1 \). Go to Step 2.

**Particle Swarm Optimisation**

The specific parameters and variables are:

- \( k \) Index for previous generation
- \( \mathcal{V}_g \) Velocity for generation \( g \)
- \( C_1, C_2 \) Learning rates
- \( \omega \) Inertia weight
- \( P_{G\text{best}} \) Global best solution

The steps of PSO algorithm that we use in this study are as follows.

Step 1: Choose an initial generation \((g = 1)\) that consists of \( N \) particles: \( p^{(g,1)}, p^{(g,2)}, \ldots, p^{(g,N)} \).

\[ p^{(g,i)} = (L_{11}^{(g,i)} \ldots L_{1T}^{(g,i)} \ldots L_{j1}^{(g,i)} \ldots L_{jT}^{(g,i)}) \] Set personal best \( P_{i}^{(g)} = p^{(g,i)} \); For each particle, set initial velocity \( \mathcal{V}^{(g,i)} = (V_{11}^{(g,i)} \ldots V_{1T}^{(g,i)} \ldots V_{j1}^{(g,i)} \ldots V_{jT}^{(g,i)}) \)

Step 2: Calculate the corresponding demand forecasts \( D_{it}^{(g,i)} \) \((i=1,\ldots,N; j=1,\ldots,J; t=1,\ldots,T)\);

Step 3: For each particle, solve the aggregate production planning problem; Calculate the objective function value \( IProfit(p^{(g,i)}), (i=1,\ldots,N)\).

Step 4: Find the objective function values for the personal best and global best

\[ IProfit_{i}^{(g)} = \max \{ IProfit(p^{(k,i)}); k = 1 \ldots g \} \]

If \( IProfit_{i}^{(g)} = \max \{ IProfit_{i}^{(k)} \} \) then \( P_{G\text{best}} = p^{(k,i)} \)

Step 5: If \( g = G \) then stop; select \( P_{G\text{best}} \) and \( IProfit_{i}^{(g)} \); otherwise go to Step 6.

Step 6: Update velocity and position.

\[
V_{jt}^{(g+1,i)} = \omega V_{jt}^{(g,i)} + C_1 r_1 (L_{jt}^{(g,i)} | P_{best} - L_{jt}^{(g,i)} | p^{(g,i)}) + C_2 r_2 (L_{jt}^{(g,i)} | P_{G\text{best}} - L_{jt}^{(g,i)} | p^{(g,i)})
\]

where \( r_1 \leftarrow U(0, 1) \) and \( r_2 \leftarrow U(0, 1) \).

\[
L_{jt}^{(g+1,i)} | p^{(g+1,i)} = L_{jt}^{(g,i)} + V_{jt}^{(g+1,i)}
\]

Step 7: Set \( g = g + 1 \). Go to Step 2.
Chapter 4

Paper 3: Supply chain network design with coordinated inventory control

History: This chapter has been prepared in collaboration with Hartanto Wong and Anders Thorstenson. It has been submitted to Transportation Research Part E: Logistics and Transportation Review.
Supply chain network design with coordinated inventory control

Agus Darmawan, Hartanto Wong and Anders Thorstenson

CORAL - Cluster for Operations Research, Analytics and Logistics

Department of Economics and Business Economics, Aarhus University

Fuglesangs Allé 4, 8210 Aarhus, Denmark

Abstract

In this paper, we present models and solution methods for supply chain network design that do not only integrate the location-transportation and inventory problems, but also consider the implementation of coordinated inventory control in the network. The network we consider has a divergent structure where retailers located downstream of the supply chain are supplied by warehouses that get supplies from a supplier located upstream of the supply chain. The objective function in our network design problem is to minimise the expected total cost that is the sum of the costs for opening and operating warehouses, the transportation cost, and the inventory-related cost, while meeting the requirement that a target fill rate must be met at all the retailers. We develop and compare four different approaches to solve the network design problem, differentiated by whether an integrated approach is used as opposed to a sequential approach, and whether or not coordinated inventory control is implemented. Due to the complexity of the optimisation problem in the integrated approach, we develop a heuristic method based on a Genetic Algorithm to solve the problem. The numerical results show that significant cost savings can be obtained by implementing coordinated inventory control in supply chain network design.

Keywords: supply chain network design, inventory control, coordination, heuristics, stochastic models
4.1 Introduction

A supply chain is a network of entities consisting of suppliers, plants, warehouses, and distribution channels organised to acquire and transform raw material to end-products, and deliver the products to customers (Santoso et al., 2005; Chopra and Meindl, 2016). An effective and responsive supply chain network design (SCND) is a vital component of high performance characterized by high efficiency and adequate customer service level. SCND decisions include the assignment of facility, location of manufacturing, storage, or transportation-related facilities, and the allocation of capacity and markets to each facility (Chopra and Meindl, 2016). It is widely acknowledged that the supply chain network design heavily influences the efficiency and customer service. As more organisations dynamically change their operations and engage in global sourcing, coupled with shorter product life cycles, today’s executives are forced to revisit and redesign their distribution networks more frequently than ever. Companies such as BMW (Fleischmann et al., 2006), 3M (Hagerty, 2012) and PepsiCo (Rice and Rowell, 2016) have reported significant cost savings and improved operations by redesigning their supply chain networks.

Basic theoretical approaches for SCND have been available for a long time. The literature on the topic is extensive and sophisticated software packages are also available (e.g., Llamasoft, Logility and JDA). Today, the goal of network design is similar to but much larger in scope compared to what we saw in the past. The main issue remains to deal with the tradeoffs between service levels and the cost of transportation, storage, and inventory. However, today’s companies are faced with a more complex, more global and more unpredictable business world. A recent study based on an online survey conducted by Supply Chain Insights (2017) reports that out of the respondents whose companies already have a network design process in place, only 19% would rate it as "very effective", which gives an indication that there is motivation for enhancing existing SCND models and tools. The lack of skilled resources is cited in the study as one of the most common challenges faced by companies with and without an SCND process in place, thus hindering the progress from using simple to more refined models that is necessary for obtaining an optimal network design.

Inventory deployment in SCND has become an important issue in past decades. In the competitive environment with uncertain demand, companies have to manage their inventory to achieve adequately high service levels and save costs at the same time. The motivation for including inventory deployment in SCND is high as there is certainly mutual dependence between design
decisions such as warehouse locations and capacities, and inventory decisions concerning where to deploy inventories along the supply chain and at what levels. There are several studies in the literature that deal with inventory deployment in SCND (see e.g., Farahani et al., 2015 for a comprehensive review). However, our literature review suggests that most of the existing studies, despite the explicit consideration of inventory, do not pay attention to the possible adoption of a fully coordinated inventory control in the supply chain. This is clearly indicated by, for example, a common assumption used in those studies that all stages in the supply chain maintain the same and relatively high service levels. It is true that companies often report high inventory service levels for their stores, warehouses and distribution centers as a sign of excellence in performance. However, an intriguing question is whether these relatively high service levels at all stages really represent what should be the appropriate targets for an efficiently operated supply chain. According to the standard SCM paradigm, the overall objective of the supply chain should be to deliver a sufficiently high end customer service at minimum cost. Hence, end customer service at the most downstream part of the supply chain is the primary constraint and therefore a key performance indicator (see e.g., Hausman, 2005).

The multi-echelon (or multi-stage/multi-level) inventory control theory reveals that, if inventory control is properly coordinated across the whole supply chain or across some parts of it (see e.g., Neale et al., 2005), total costs can be reduced without hurting end customer service. This should be obtained by effective coordination accomplished through collaboration and information sharing between the individual companies at the different echelons in the supply chain. Applying a fully coordinated inventory control often results in the service measures at the upstream stage being lower compared to the service measures for end customers applied at the downstream stage. Thus, we foresee potential cost savings that can be generated from applying a fully coordinated inventory control in SCND. Integrating SCND and fully coordinated inventory control is quite topical since today, collaboration and information sharing initiatives such as VMI (Vendor Managed Inventory), CPFR (Collaborative Planning, Forecasting and Replenishment), CR (Consumer Response), etc, become more widely implemented in practice. A recent study ("2016: The Future Value Chain" available in www.capgemini.com) conducted by Global Commerce Initiative in conjunction with Capgemini and Intel underlines the importance of improved collaboration between all parties in the supply chain in order to enhance the efficiency and effectiveness while better serving the needs of the consumer, thereby lending support to our study.
In this paper, we extend the literature on inventory deployment in SCND by considering full coordination that aims at minimising the total cost while assuring that the end customer service level is above a prespecified target. More specifically, we consider a network consisting of multiple warehouses and retailers that all use continuous review ($R, nQ$) policies for their inventory controls. In addition, our model uses a fill rate as the performance indicator for end customer service, whereas most of the existing studies only use a cycle service level. This is motivated by our observation that the use of fill rate is widespread in practice, as well as the conclusion in the standard literature that the fill rate is a more appropriate measure for inventory performance. Clearly, embedding fully coordinated inventory control in SCND with fill rate as the end customer service measure makes the optimisation problem more complex and difficult to solve. Hence, developing an easily implementable solution method is necessary. The model and solution methods presented in this paper contribute to enhancing the progress from using simple to more refined models in SCND, thereby addressing one of the challenges outlined above. They are intended to be used as building blocks for the development of decision support tools that address the complexity in the integration of coordinated inventory control and SCND.

The rest of the paper is organised as follows. Section 4.2 presents our survey of the relevant literature. In Section 4.3, we first formulate the SCND problem with coordinated inventory control. The section continues with two principle approaches for solving the problem, the sequential and the integrated approaches, respectively. In the sequential approach, we decompose the optimisation problem into two sequential stages. In the first stage, we solve the location-transportation problem. Next, for the network structure obtained in the first stage, we solve the inventory problem at the warehouses and retailers in the second stage. Whereas in the integrated approach, we solve the location-transportation and inventory problems simultaneously. We also discuss the use of the non-coordinated and coordinated inventory control in each approach. Section 4.4 presents the specific solution methods used in this paper. Section 4.5 presents the setup and results of the numerical study. In Section 4.6, we summarise the main findings of the paper and give some directions for future research.
4.2 Literature review

The literature on SCND is extensive, and the topic of SCND has been the subject of several recent review papers (e.g., Melo et al., 2009; Farahani et al., 2015 and Govindan et al., 2017). Most relevant to our paper is the literature on SCND that includes inventory deployment. In general, the papers that consider inventory deployment in SCND can be categorized based on a number of factors such as the number of echelons where inventory decisions need to be made, demand characteristics (deterministic or stochastic), and solution techniques. We summarise the existing studies that examine inventory deployment in SCND in Table 4.1.

There are several papers on SCND that consider inventory decisions in a single echelon. The primary focus of these papers is on examining the benefit of risk pooling as a result of consolidating inventories from multiple locations into a single location. Miranda and Garrido (2004, 2009) propose models for SCND that take into account inventory decisions at warehouses. They show that the benefit of risk pooling is increasing in demand variance, holding cost and service level. Croxton and Zinn (2005) compare the integrated approach with the non-integrated approach, and the two approaches are differentiated by whether or not inventory costs are included in SCND. They use the simplistic square root law to analyse inventory costs at warehouses. One of their main findings is that the inclusion of inventory in network design generally reduces the optimal number of warehouses in the network. Sourirajan et al. (2007, 2009) analyse the network design problems with lead-time and safety-stock considerations. They include the non-linear relationships between the flows in the network, lead times, and safety stocks and capture the trade-off between risk-pooling benefits and congestion costs. They consider a two-stage supply chain with a production facility that replenishes a single product to a given set of retailers. Their optimisation problem is to locate distribution centers that minimise the sum of location and inventory (pipeline and safety stock) costs. Javid and Azad (2010) present a heuristic approach for simultaneously optimising location, allocation, capacity, inventory, and routing decisions where warehouses maintain a certain level of safety stock. Fleischmann (2016) rejects the square root law and studies the relationship between the number of warehouses and the total inventory while considering the replenishment in a full truck load (FTL) mode and a fill rate constraint. Utilizing a queueuing approach, Diabat et al. (2017) present a model of network design with inventory decisions at the warehouse. Some authors, e.g. Park et al. (2010), Yao et al. (2010), Reza Nasiri et
al. (2010), Liao et al. (2011), and Shahabi et al. (2013) consider SCND with more than two echelons. However, like the previously mentioned studies, they all consider inventories in just one echelon (i.e. at warehouses or distribution centers).

Maintaining inventory in only one echelon may not represent a realistic situation in practice. This is particularly true when considering retailers that actually face demand directly from end customers. This has motivated some studies to consider inventory decisions not only at warehouses but also at retailers. Farahani and Elahipanah (2008), Keskin and Üster (2012), Tancrez et al. (2012), Taxakis and Papadopoulos (2016) present models for SCND that consider inventory costs at more than one echelon. However, those papers neglect the uncertainty of demand at retailers. Kang and Kim (2012) argue that incorporating inventory decisions at retailers is important because the risk pooling strategy may increase the lead-time for the retailers due to longer distances to the designated warehouses, which may eventually result in increased inventory and transportation costs. They consider a two-echelon SCND where demand at each retailer follows a normal distribution and develop a heuristic algorithm based on Lagrangian relaxation for solving a non-linear mixed integer programming problem. Tsao et al. (2012) propose a new continuous approximation modelling technique for a three-echelon supply chain where demand at each retailer follows a Poisson distribution. Kumar and Tiwari (2013), Askin et al. (2014), Nasiri et al. (2014), and Manatkar et al. (2016) present multi-product location-distribution planning problems with inventory considerations at both warehouses and retailers. Puga and Tancrez (2017) study a location–inventory problem for the design of supply chain networks with uncertain demand and present a heuristic algorithm based on a linear approximation.

All the aforementioned studies have in common that inventory decisions at the different echelons are made separately without coordination. This is reflected in their use of the same service levels across all echelons. Our hypothesis is that the resulting network design may be sub-optimal due to an excessive stock at the upper echelons (i.e. warehouses) as the consequence of maintaining unnecessary high service levels. This has motivated us to develop a model for SCND that addresses the impact of coordinated inventory control between echelons (e.g. warehouses and retailers) in the network design. In addition, most of the previous studies use a cycle service level as the service measure for the inventory problem. Motivated by its wide adoption in practice, as well as its dominance in the literature, we opt to use a fill rate as the service measure in this paper. Furthermore, a multiple sourcing policy is also commonly applied nowadays by retailers, but this
policy only receives little attention in the existing models. This paper is developed to address all these challenges. To the best of our knowledge, our paper is the first to consider SCND with a fully coordinated inventory control, where fill rate is used as the service performance measure, and a multiple sourcing policy is applied in the network.

Table 4.1 Review summary of literature on SCND with inventory consideration

<table>
<thead>
<tr>
<th>Papers</th>
<th>Echelon</th>
<th>Inventory Location</th>
<th>Demand</th>
<th>Service level</th>
<th>Coordinated Inventory</th>
<th>Sourcing (at retailer)</th>
<th>Method and notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Miranda and Garrido (2004)</td>
<td>2</td>
<td>WH</td>
<td>Normal distribution</td>
<td>CSL</td>
<td>No</td>
<td>SS</td>
<td>Lagrangian relaxation</td>
</tr>
<tr>
<td>Croxton and Zinn (2005)</td>
<td>3</td>
<td>WH</td>
<td>Normal distribution</td>
<td>CSL</td>
<td>No</td>
<td>MS</td>
<td>Mixed integer programming</td>
</tr>
<tr>
<td>Sourirajan et al. (2007)</td>
<td>2</td>
<td>WH</td>
<td>Normal distribution</td>
<td>CSL</td>
<td>No</td>
<td>SS</td>
<td>Lagrangian relaxation</td>
</tr>
<tr>
<td>Farahani and Elahipanah (2008)</td>
<td>3</td>
<td>WH+RT</td>
<td>Deterministic</td>
<td>-</td>
<td>-</td>
<td>MS</td>
<td>Genetic algorithm</td>
</tr>
<tr>
<td>Sourirajan et al. (2009)</td>
<td>2</td>
<td>WH</td>
<td>Normal distribution</td>
<td>CSL</td>
<td>No</td>
<td>SS</td>
<td>Genetic algorithm</td>
</tr>
<tr>
<td>Miranda and Garrido (2009)</td>
<td>2</td>
<td>WH</td>
<td>Normal distribution</td>
<td>CSL</td>
<td>No</td>
<td>SS</td>
<td>Heuristic algorithm</td>
</tr>
<tr>
<td>Javid and Azad (2010)</td>
<td>2</td>
<td>WH</td>
<td>Normal distribution</td>
<td>CSL</td>
<td>No</td>
<td>SS</td>
<td>Tabu search and Simulated annealing</td>
</tr>
<tr>
<td>Park et al. (2010)</td>
<td>3</td>
<td>WH</td>
<td>Normal distribution</td>
<td>CSL</td>
<td>No</td>
<td>SS</td>
<td>Tabu search and Lagrangian relaxation</td>
</tr>
<tr>
<td>Yao et al. (2010)</td>
<td>3</td>
<td>WH</td>
<td>Normal distribution</td>
<td>CSL</td>
<td>No</td>
<td>SS</td>
<td>Heuristic algorithm</td>
</tr>
<tr>
<td>Reza Nasiri et al. (2010)</td>
<td>3</td>
<td>WH</td>
<td>Normal distribution</td>
<td>CSL</td>
<td>No</td>
<td>SS</td>
<td>Sub-gradient search and Lagrangian relaxation</td>
</tr>
<tr>
<td>Liao et al. (2011)</td>
<td>3</td>
<td>WH</td>
<td>Normal distribution</td>
<td>CSL</td>
<td>No</td>
<td>SS</td>
<td>Continuous approximation</td>
</tr>
<tr>
<td>Tsao et al. (2012)</td>
<td>3</td>
<td>WH + RT</td>
<td>Poisson distribution</td>
<td>CSL</td>
<td>No</td>
<td>SS</td>
<td>Continuous approximation</td>
</tr>
<tr>
<td>Tancrez et al. (2012)</td>
<td>3</td>
<td>Plant + WH+RT</td>
<td>Deterministic</td>
<td>-</td>
<td>-</td>
<td>MS</td>
<td>Heuristics</td>
</tr>
<tr>
<td>Keskin and Uster (2012)</td>
<td>3</td>
<td>WH+RT</td>
<td>Deterministic</td>
<td>-</td>
<td>-</td>
<td>SS</td>
<td>Simulated annealing</td>
</tr>
<tr>
<td>Kang and Kim (2012)</td>
<td>2</td>
<td>WH + RT</td>
<td>Normal distribution</td>
<td>CSL</td>
<td>No</td>
<td>SS</td>
<td>Lagrangian relaxation</td>
</tr>
<tr>
<td>Authors</td>
<td>Year</td>
<td>Type</td>
<td>Policy</td>
<td>Demand Distribution</td>
<td>Service Level</td>
<td>Sourcing</td>
<td>Methodology</td>
</tr>
<tr>
<td>----------------------</td>
<td>------</td>
<td>------</td>
<td>--------</td>
<td>---------------------</td>
<td>--------------</td>
<td>---------</td>
<td>----------------------------------------------------------------------------</td>
</tr>
<tr>
<td>Silva and Gao</td>
<td>2013</td>
<td>WH</td>
<td>Deterministic</td>
<td>-</td>
<td>-</td>
<td>SS</td>
<td>Greedy randomized adaptive search procedure</td>
</tr>
<tr>
<td>Shahabi et al.</td>
<td>2013</td>
<td>WH</td>
<td>Normal distribution</td>
<td>CSL</td>
<td>No</td>
<td>SS</td>
<td>Conic integer programming</td>
</tr>
<tr>
<td>Kumar and Tiwari</td>
<td>2013</td>
<td>WH+RT</td>
<td>Normal distribution</td>
<td>CSL</td>
<td>No</td>
<td>SS</td>
<td>Lagrangian relaxation</td>
</tr>
<tr>
<td>Askim et al.</td>
<td>2014</td>
<td>WH+RT</td>
<td>Normal distribution</td>
<td>CSL</td>
<td>No</td>
<td>SS</td>
<td>Genetic algorithm</td>
</tr>
<tr>
<td>Nasiri et al.</td>
<td>2014</td>
<td>WH+RT</td>
<td>Normal distribution</td>
<td>CSL</td>
<td>No</td>
<td>SS</td>
<td>Genetic algorithm and Lagrangian relaxation</td>
</tr>
<tr>
<td>Zhang and Xu</td>
<td>2014</td>
<td>RT</td>
<td>(Exponential, Uniform and Normal dist)</td>
<td>-</td>
<td>No</td>
<td>MS</td>
<td>Particle swarm optimisation, Genetic algorithm</td>
</tr>
<tr>
<td>Manatkar et al.</td>
<td>2016</td>
<td>WH+RT</td>
<td>Normal distribution</td>
<td>CSL</td>
<td>No</td>
<td>SS</td>
<td>Genetic algorithm</td>
</tr>
<tr>
<td>Taxakis and Papadopoulos</td>
<td>2016</td>
<td>Plant+RT</td>
<td>Deterministic</td>
<td>-</td>
<td>-</td>
<td>MS</td>
<td>Genetic algorithm</td>
</tr>
<tr>
<td>Fleischmann 2016</td>
<td></td>
<td>RT</td>
<td>Normal and Gamma distribution</td>
<td>Fill rate</td>
<td>No</td>
<td>SS</td>
<td>Simulated annealing and direct search method</td>
</tr>
<tr>
<td>Diabat et al.</td>
<td>2017</td>
<td>WH</td>
<td>Poisson distribution</td>
<td>-</td>
<td>No</td>
<td>SS</td>
<td>Heuristics</td>
</tr>
<tr>
<td>Puga and Tancrez</td>
<td>2017</td>
<td>WH+RT</td>
<td>Normal distribution</td>
<td>CSL</td>
<td>No</td>
<td>MS</td>
<td>Heuristics</td>
</tr>
<tr>
<td>This paper</td>
<td>2017</td>
<td>WH+RT</td>
<td>Normal distribution</td>
<td>Fill Rate</td>
<td>Yes</td>
<td>MS</td>
<td>Genetic algorithm</td>
</tr>
</tbody>
</table>

*WH: warehouse, RT: retailer, CSL: cycle service level, SS: single sourcing, MS: multiple sourcing

Our paper is also related to the literature on coordinated inventory control, especially the studies that consider a divergent multi-echelon setting with stochastic demand. This setting is commonly seen in SCND, where goods are distributed from a central warehouse to a number of local warehouses and/or retailers. For overviews, see, for example, Zipkin (2000), van Houtum (2006), Axsäter and Marklund (2008), Marklund and Rosling (2012), and Axsäter (2015). Most studies in this literature present either exact solution procedures or approximation techniques and heuristics that are developed for various inventory systems under different assumptions about demand and control policies. In this paper, we consider a continuous review setting, where all locations in the network use \((R, nQ)\) policies. Some authors, e.g. Axsäter (1993, 1998), Chen and Zheng (1997), and Forsberg (1997) present exact cost analysis under Poisson demand. A major limitation of these
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studies is that they cannot deal with the computational complexity found in real-life systems. Several papers develop approximation techniques to reduce the required computational effort. One of the techniques that is commonly used is based on a decomposition approach. That is, under centralised control (e.g., a VMI arrangement), a system of one-warehouse and $n$ retailers is decomposed into $n+1$ single echelon problems, and an induced backorder cost is used at the central warehouse to represent the expected cost incurred by the retailers due to shortages at the central warehouse (Andersson et al., 1998; Andersson and Marklund, 2000; Axsäter, 2003 and 2005; Berling and Marklund, 2006 and 2014). In particular, Berling and Marklund (2014) present a method for determining near-optimal reorder points that is simple enough to be implemented in practice with reasonable computation time and good solution quality. We adopt their method in this paper for solving the coordinated inventory problem as part of the integrated SCND problem. In the above papers, as well as in our paper, the demand during the lead-time is modelled using a normal distribution. Axsäter (2013) indicates that the normal approximation works well for a high backorder cost or, equivalently, a high service level.

In their extensive literature reviews, Melo et al. (2009), Farahani et al. (2015), and Govindan et al. (2017) emphasise the importance of research that integrates strategic and tactical/operational decisions, and they also highlight the need for accommodating uncertainty in network design. Our integrated approach that considers coordinated inventory control in SCND contributes to advancing the literature along their suggested direction.

4.3 Problem formulation

4.3.1 Problem description

The SCND problem we consider is the following. There is a supply chain network for a single product consisting of a single supplier (plant), multiple warehouses and multiple retailers, as shown in Figure 4.1. We use index $i$ ($i = 1, 2, ..., I$) for the warehouses and $j$ ($j = 1, 2, ..., J$) for the retailers. Demand at retailer $j$ is assumed to be normally distributed with mean $\mu_j$ and variance $\sigma_j^2$. The retailers are supplied by the warehouses, and each warehouse has a finite capacity. Note that we allow multiple sourcing for retailers in our model, i.e., a retailer can be supplied by more than one warehouse in the network. All the warehouses are supplied by the single plant that is
assumed to have an unlimited capacity. Thus, even though we have a three-echelon network, we deal with a two-echelon network from the inventory perspective. However, unlike the standard two-echelon inventory system that considers only one warehouse, there are multiple warehouses available in our network. All the stocking points, i.e. warehouses and retailers, use continuous review \((R_i, nQ_i)\) policies for their inventory control. Demand at each warehouse is satisfied on a first-come-first-served (FCFS) basis, and we assume complete backordering of unsatisfied demand at all stocking points. The transportation times from the warehouses to the retailers are assumed constant, but the lead times are stochastic due to possible shortages at the warehouses.

The SCND problem involves the following decisions: (a) Which warehouses to open? (b) Which warehouse(s) should supply which retailer(s) and by how much? and (c) Which inventory control policy parameter values should be used by each stocking point? For future reference, we term problems (a) and (b) as the location-transportation problem and problem (c) as the inventory problem.

The objective is to minimise the system wide costs while attaining the target fill rates for the end-customer demand. One of the objectives of our study is to compare a number of approaches for solving the above SCND problem. We will compare four approaches that are differentiated by whether a sequential or an integrated approach is used, and by whether or not coordinated

![Diagram of supply chain network design](image)
inventory control is implemented. In the sequential approach, we first solve the location-transformation problem based on which we then solve the inventory problem, whereas in the integrated approach we solve the two problems simultaneously. When implementing coordinated inventory control, we determine the best (near-optimal) inventory control policy parameter values by ensuring that the target fill rates for the end-customer demand (at retailers) are achieved. In contrast, without coordinated inventory control, all the warehouses set the same target fill rates as the retailers and determine their own inventory control policy parameter values accordingly.

### 4.3.2 Model formulation

Notation for the mathematical model of the proposed problem is as follows:

<table>
<thead>
<tr>
<th>Index</th>
<th>Parameters</th>
<th>Decision variables</th>
</tr>
</thead>
<tbody>
<tr>
<td>$i$</td>
<td>Index for warehouse</td>
<td>$Q_j^r$ Replenishment order quantity at retailer $j$</td>
</tr>
<tr>
<td>$j$</td>
<td>Index for retailer</td>
<td>$Q_i^w$ Replenishment order quantity at warehouse $i$</td>
</tr>
<tr>
<td>$A_j^r$</td>
<td>Ordering cost at retailer $j$</td>
<td>$Q_j^r$ Replenishment order quantity at retailer $j$</td>
</tr>
<tr>
<td>$A_i^w$</td>
<td>Ordering cost at warehouse $i$</td>
<td>$Q_i^w$ Replenishment order quantity at warehouse $i$</td>
</tr>
<tr>
<td>$b_j^r$</td>
<td>Backorder cost per unit and time unit at retailer $j$</td>
<td>$R_j^r$ Reorder point at retailer $j$</td>
</tr>
<tr>
<td>$b_i^w$</td>
<td>Backorder cost per unit and time unit at warehouse $i$</td>
<td>$R_i^w$ Reorder point at warehouse $i$</td>
</tr>
<tr>
<td>$C_i$</td>
<td>Truck capacity at warehouse $i$</td>
<td>$R_j^r$ Reorder point at retailer $j$</td>
</tr>
<tr>
<td>$C_j$</td>
<td>Truck capacity at retailer $j$</td>
<td>$R_i^w$ Reorder point at warehouse $i$</td>
</tr>
<tr>
<td>$Cap_i$</td>
<td>Capacity at warehouse $i$</td>
<td></td>
</tr>
</tbody>
</table>
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X_i = 1 if warehouse i is opened; 0 otherwise
\( \mu_{ij} \) Expected demand at retailer j supplied by warehouse i
\( \mu_{i}^w \) Expected demand at warehouse i

Other variables:

\( IL_j^r \) Inventory level at retailer j
\( IL_i^w \) Inventory level at warehouse i
\( \mu_{D_j(l_j)}^r \) Expected lead-time demand at retailer j
\( \mu_{D_i(l_i)}^w \) Expected lead-time demand at warehouse i
\( \sigma_i^w \) Standard deviation of demand at warehouse i
\( \sigma_{D_j(l_j)}^r \) Standard deviation of lead-time demand at retailer j
\( \sigma_{D_i(l_i)}^w \) Standard deviation of lead-time demand at warehouse i

4.3.2.1 The sequential approach

In this approach, we decompose the optimisation problem into two sequential stages. In the first stage, we solve the location-transportation problem. Hence, we decide the location of warehouses and the allocation of warehouses that serve retailers by taking into account the total accrued fixed and variable costs for opening and operating warehouses and the total transportation costs from the supplier to the warehouses and from the warehouses to the retailers. Next, in the second stage, for the resulting network structure from the first stage, we solve the inventory problem at the warehouses and retailers.

The expected total cost for the sequential approach is equal to

\[
TC_1 = TRC + IRC, \quad (4.1)
\]

where \( TRC \) is the expected total cost for the location-transportation problem and \( IRC \) is the expected inventory-related cost for the inventory problem.

a. The location-transportation problem

The location-transportation problem in the first stage can be described as

\[
\min_{X_i, \mu_i^w, \mu_{ij}} TRC = \sum_{i=1}^{I} \left( X_i M_i + V_i \mu_i^w + \left[ \frac{\mu_i^w}{C_i} \right] P_{0i} + \sum_{j=1}^{J} \left[ \frac{\mu_{ij}}{C_j} \right] P_{ij} \right) \quad (4.2)
\]
Subject to

\[
\begin{align*}
\mu_j^r &= \sum_{i=1}^I \mu_{ij} \quad \forall j \quad (4.3) \\
\mu_i^w &= \sum_{j=1}^J \mu_{ij} \quad \forall i \quad (4.4) \\
\mu_i^{lw} &\leq X_i Cap_i \quad \forall i \quad (4.5) \\
X_i &\text{ binary} \quad \forall i \quad (4.6) \\
\mu_{ij} &\geq 0 \quad \forall i, j \quad (4.7)
\end{align*}
\]

The objective function in (4.2) is to minimise the total cost that consists of the accrued fixed and variable costs for opening and operating warehouses, transportation costs from the supplier to warehouses and transportation costs from warehouses to retailers. We round up the required number of trucks to the nearest integer to account for the possibility of running less than full truck loads for the given value of demand and truck capacity. This round up approach is common in the literature (see e.g., Askin et al., 2014). Constraint (4.3) represents the average demand at each retailer that can be satisfied by some of the warehouses. A positive value of \( \mu_{ij} \) indicates that retailer \( j \) is supplied by warehouse \( i \). Constraint (4.4) represents the average demand supplied by each warehouse. The average demand satisfied by each warehouse is limited by the warehouse capacity (constraint (4.5)). Constraint (4.6) represents a binary decision variable associated with whether or not a warehouse is opened. The optimal network structure obtained in this stage will be used for solving the inventory problem specified below.
b. **Inventory problem – Non-coordinated**

In the second stage, we need to solve the following non-coordinated inventory problem:

\[
\begin{align*}
    \min_{Q_i^w, R_i^w; Q_j^r, R_j^r} & \quad IRC = \sum_{i=1}^I \left( A_i^w \frac{h_i^w}{Q_i^w} + h_i^w E(I_{L_i}^w)^+ \right) + \sum_{j=1}^J \left( \frac{A_j^r}{Q_j^r} \sum_{i=1}^I \mu_{ij} + h_j^r E(I_{L_j}^r)^+ \right) \\
    & \quad = \sum_{i=1}^I \left( A_i^w \frac{\mu_i^w}{Q_i^w} + h_i^w \left( R_i^w + Q_i^w/2 - \mu_{D_i(L_i)}^w \right) \right) \\
    & \quad + h_i^w \left( \frac{\sigma_{D_i(L_i)}^w}{Q_i^w} \right)^2 \left[ H \left( \frac{R_i^w - \mu_{D_i(L_i)}^w}{\sigma_{D_i(L_i)}^w} \right) - H \left( \frac{R_i^w + Q_i^w - \mu_{D_i(L_i)}^w}{\sigma_{D_i(L_i)}^w} \right) \right] \\
    & \quad + \sum_{j=1}^J \left( \frac{A_j^r}{Q_j^r} \sum_{i=1}^I \mu_{ij} + h_j^r \left( R_j^r + Q_j^r/2 - \mu_{D_j(L_j)}^r \right) \right) \\
    & \quad + h_j^r \left( \frac{\sigma_{D_j(L_j)}^r}{Q_j^r} \right)^2 \left[ H \left( \frac{R_j^r - \mu_{D_j(L_j)}^r}{\sigma_{D_j(L_j)}^r} \right) - H \left( \frac{R_j^r + Q_j^r - \mu_{D_j(L_j)}^r}{\sigma_{D_j(L_j)}^r} \right) \right] \\
\end{align*}
\]

Subject to

\[
\begin{align*}
    FillRate_i &= 1 - P(\text{IL}_{L_i}^w \leq 0) \geq FillRate_i^{\text{min}} \quad (4.9) \\
    FillRate_j &= 1 - P(\text{IL}_{L_j}^r \leq 0) \geq FillRate_j^{\text{min}} \quad (4.10)
\end{align*}
\]

The objective function in (4.8) is to minimise the total inventory cost, which consists of the ordering and holding costs for both the warehouses and the retailers. See Appendix 2 for details of this standard cost structure. Note that even though the pipeline inventory cost is not included in the above formulation, its relevance is, to some extent, captured in the location-transportation problem through the parameter \( P_{ij} \), which depends on the transportation lead time between facilities (see e.g., Fichtinger et al., 2017). The decisions on order quantity and reorder point at each stocking location should ensure that the target fill rates specified in (4.9) and (4.10) are satisfied (\( P(\cdot) \) in (4.9) and (4.10) is a probability and the right-hand side of the two inequalities represent the fill-rate targets). The fill-rate specification is based on the fact, that for normally distributed demand the fill rate is equal to the ready rate (Axsäter, 2015). In this non-coordinated approach, inventory decisions at the warehouses and retailers are made separately, and hence, it is a common observation that all the warehouses and retailers set high fill-rate targets for themselves.
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In the case where backorder costs per unit and time unit are used instead of target fill rates, we can remove constraints (4.9)-(4.10) and include the backorder costs in the objective function. For given order quantities, the relationship between fill rate (fillrate) and backorder cost (b) takes the form, \( b = \text{fillrate} \times h / (1 - \text{fillrate}) \) (see e.g., Axsäter, 2015).

To simplify, assume that the order quantity at each warehouse and each retailer are determined based on the deterministic economic order quantity (EOQ) formula so that

\[
Q_j^r = \sqrt{\frac{2A_j^r \mu_j^r \mu_j^r}{h_j^r}}; \quad Q_i^w = \sqrt{\frac{2A_i^w \mu_i^w \mu_i^w}{h_i^w}}
\]

(4.11)

Thus, we follow a common approach adopted in the multi-echelon literature by pre-determining the order quantities using the deterministic model and then optimise the reorder points using the stochastic demand model. Using a deterministic EOQ-type model in a stochastic environment has a small effect on the expected total inventory costs, as long as the reorder points are optimised accordingly (Zheng, 1992; Axsäter, 1996). In fact, this assumes that the backorder version of the EOQ formula is used; however, when the target fill rate is high, it does not matter much if the standard formula is used instead.

The reorder points are determined as

\[
R_j^r = \min\{R_j^r: (1 - P(IL_j^r \leq 0) \geq \text{FillRate}_j^\text{min}\}
\]

(4.12)

\[
R_i^w = \min\{R_i^w: (1 - P(IL_i^w \leq 0) \geq \text{FillRate}_i^\text{min}\}
\]

(4.13)

Since we allow for a multiple sourcing policy in our model, some further assumptions need to be made. We assume that a retailer supplied by multiple warehouses uses a single reorder point, and incurs a single ordering cost for each replenishment event. When its inventory position reaches the reorder point, the retailer determines the replenishment order to each supplying warehouse based on the average flow of units from the supplying warehouse to the retailer, determined in the location-transportation problem. We also make the following approximation for calculating the variance of lead-time demand for the order made by retailer \( j \) to warehouse \( i \). In our approximation,

\[
\sigma_{ij}^2 = \frac{\mu_i}{\sum_{i=1}^2 \mu_i} \sigma_f^2,
\]

which reflects the situation where demand at retailer \( j \) is split into multiple independent streams of demand, and the variance of demand stream at retailer \( j \) for which the replenishment is from warehouse \( i \) is assumed to be proportional to the average demand at retailer
to a numerical study where the results from a simulation are used as the benchmark. We set the reorder point values at the retailers equal to the values calculated based on the approximation, and run the simulation to estimate the expected fill rate. The results show that the approximation is reasonably accurate, which is evidenced by limited gaps between the target fill rates used for determining the reorder point values and the fill rates achieved in the simulations. The results of this numerical study are presented in Appendix 3.

Note also that in calculating the lead-time demand at each warehouse (for the purpose of determining the reorder points at the warehouse), we take into account the fact that the warehouse receives a stream of $Q^r_j$ demands from retailers. To calculate the true distribution of the lead-time demand at the warehouse, we would need successive convolutions of the probability mass functions for the different retailers. These calculations are demanding. Most other studies considering inventory in SCND simplify the problem by assuming that the warehouse and retailers receive precisely the same stream of demand, which may lead to inaccurate results. We refer to Appendix 4, Eqs. (A4.10-A4.13) for the approximate calculation in our model of a more appropriate lead-time demand at the warehouse.

Furthermore, we consider the possible existence of undershoot in calculating the mean and variance of lead-time demand (Tempelmeier, 2006; Berling and Marklund, 2014, 2017). This is relevant, because in a continuous review model, it is assumed that the replenishments can occur at any time on a continuous time axis. In practice, however, the replenishments are restricted to discrete points in time (e.g. the trucks arrive on a daily basis within a small time window). Hence, there could be situations where an order is placed when the inventory position is below the reorder point. The adjusted mean and variance of lead-time demand are, respectively

$$\mu_{D_j(L_j)} = \sum_{i=1}^l \mu_{ij}L_{ij} + \mu_{u_j}$$  \hspace{1cm} (4.14)

$$\sigma_{D_j(L_j)} = \sqrt{\sum_{i=1}^l \sigma_{ij}^2 L_{ij} + \sigma_{u_j}^2},$$  \hspace{1cm} (4.15)

Where the mean and variance of the undershoot are, respectively (Tempelmeier 2006)
\[ \mu_{u_j} = \frac{\mu_j^r + (\sigma_j^r)^2}{2} \]  
\[ \sigma_{u_j}^2 = \frac{1}{2} (\sigma_j^r)^2 \left( 1 - \frac{1}{2} (\mu_j^r)^2 \right) + \frac{(\mu_j^r)^2}{12} \]  

The expressions \( P(\mathcal{I}L_j^r \leq 0) \) and \( P(\mathcal{I}L_i^w \leq 0) \) in (4.12) and (4.13) are then calculated as.

\[ P(\mathcal{I}L_j^r \leq 0) = \frac{\sigma_{r_j}^r}{\sigma_{r_j}^w} \left[ G \left( \frac{R_j^r - \mu_{D_j(L_j)}}{\sigma_{D_j(L_j)}} \right) - G \left( \frac{R_j^r + Q_j^r - \mu_{D_j(L_j)}}{\sigma_{D_j(L_j)}} \right) \right] \]  
\[ P(\mathcal{I}L_i^w \leq 0) = \frac{\sigma_{D_i(L_i)}}{\sigma_{D_i(L_i)}} \left[ G \left( \frac{R_i^w - \mu_{D_i(L_i)}}{\sigma_{D_i(L_i)}} \right) - G \left( \frac{R_i^w + Q_i^w - \mu_{D_i(L_i)}}{\sigma_{D_i(L_i)}} \right) \right] \]

where \( G(x) \) is the normal loss function (see Appendix 1 for details). As a note, since our focus is on the target fill rates at the retailers, we consider the undershoot and make the adjustment only for the retailers.

c. Inventory problem – coordinated

The inventory problem with coordination has the same objective function and target fill rate constraints at the retailers as in the inventory problem with no coordination, as defined in (4.8) and (4.10). The main difference lies in the target fill rate constraints at the warehouses that are now removed. Information sharing between the warehouses and retailers is the main idea behind the coordinated inventory control. The reorder points at the warehouses and retailers are interdependent, which is not captured in the non-coordinated inventory approach. If the warehouse fill-rate targets are set too high, this interdependence is suppressed.
We use the solution procedure based on the so-called induced backorder cost to determine the reorder points for the warehouses and retailers (Berling and Marklund, 2014, 2017). The induced backorder cost at a warehouse represents the cost associated with the inability of the warehouse to deliver the units requested, and serves as a means for coordination between the warehouse and the retailers it supplies. However, this is not exactly true in the case with multiple sourcing, because a demand variation on one warehouse link may be compensated for by another warehouse link. Nevertheless, the systematic approach is used here to achieve some coordination and test its effect on the overall performance. In Figure 4.2 and specified in Appendix 4, we show that there are five steps in the implementation of the coordinated inventory control. In Step 1, we determine the near-
optimal induced backorder cost at each warehouse. Step 2 calculates the lead-time demand at the warehouses. For given values of induced backorder cost and lead-time demand calculated in Steps 1 and 2, in Step 3 we calculate the reorder point and expected delay at each warehouse. Then, we adjust the lead time at retailers by adding the expected delay at the warehouse, as in the METRIC approach (Axsäter, 2015). To simplify the procedure, we exclude the variability of delay at the warehouse. In their experiment, Berling and Marklund (2014) found that the adjustment for the lead-time variability has less impact on the obtained fill rate than the adjustment for undershoot. The methods for finding the lead-time demand and reorder points at the retailers (Steps 4 and 5) are the same as in the non-coordinated approach.

### 4.3.2.2 The integrated approach

In this approach, we solve the location-transportation and inventory problems simultaneously. The main principle is that we search over the solution space, defined as the set of all possible network structures that represent solution candidates for the location-transportation problem, and for each solution candidate, we solve the inventory problem. The evaluation for each solution candidate is made based on the expected total costs of the solutions to the two problems. We define the following objective function:

\[
\min_{X_i, \mu^w_i, \mu_{ij}, Q_i^w, R_i^w; Q_j^r, R_j^r} TC_2
\]

\[
= \sum_{i=1}^{I} \left( X_i M_i + V_i \mu^w_i \left[ \frac{\mu^w_i}{C_i} P_{oi} + \sum_{j=1}^{J} \left[ \frac{\mu_{ij}}{C_j} P_{ij} \right] \right) \right.
\]

\[
+ \sum_{i=1}^{I} \left( \frac{A_i^w}{Q_i^w} \mu^w_i + h_i^w \left( R_i^w + Q_i^w / 2 - \mu^w_{D_i(L_i)} \right) \right)
\]

\[
+ h_i^w \left( \frac{\sigma^w_{D_i(L_i)}}{Q_i^w} \right)^2 \left[ H \left( \frac{R_i^w - \mu^w_{D_i(L_i)}}{\sigma^w_{D_i(L_i)}} \right) - H \left( \frac{R_i^w + Q_i^w - \mu^w_{D_i(L_i)}}{\sigma^w_{D_i(L_i)}} \right) \right]
\]

\[
+ \sum_{j=1}^{J} \left( \frac{A_j^r}{Q_j^r} \sum_{i=1}^{I} \mu_{ij} + h_j^r \left( R_j^r + Q_j^r / 2 - \mu^r_{D_j(L_j)} \right) \right)
\]

\[
+ h_j^r \left( \frac{\sigma^r_{D_j(L_j)}}{Q_j^r} \right)^2 \left[ H \left( \frac{R_j^r - \mu^r_{D_j(L_j)}}{\sigma^r_{D_j(L_j)}} \right) - H \left( \frac{R_j^r + Q_j^r - \mu^r_{D_j(L_j)}}{\sigma^r_{D_j(L_j)}} \right) \right] \quad (4.20)
\]

Subject to:

(4.3)-(4.7).
In the approach with the non-coordinated inventory problem we also include constraints (4.9)-(4.11). In the approach with the coordinated inventory problem we only use constraints (4.10)-(4.11), but apply the 5-step procedure outlined in Fig. 4.2 (and specified in Appendix 4).

4.4 Solution methods

As described in Section 4.3, we compare four approaches that are differentiated by whether the problem is solved using the sequential or integrated approach, and by whether the non-coordinated or coordinated inventory control is adopted in the inventory problem. We use What’sBest®15.0.1.2, LINDO System, Inc. to solve the location-transportation problem in the sequential approach. The inventory models are coded as macros in Visual Basic for Application (VBA) within Microsoft®Excel 2016. Due to the simultaneous optimisation of the location-transportation and inventory problems, the optimisation problem in the integrated approach becomes more complex than in the sequential approach. We develop a heuristic method based on Genetic Algorithms (GA) to solve this complex optimisation problem.

Genetic Algorithms

Genetic Algorithms are a well-known set of meta-heuristic techniques for solving optimisation problems with large search spaces. In solving our optimisation problem, we use the heuristic to generate a network flow structure, based on which we solve the inventory optimisation problem. GA evaluates multiple chromosomes at each generation, where a chromosome represents a network flow structure in our implementation. In the integrated approach, we use priority-based encoding (Gen et al., 2006) for defining a network flow structure that represents a possible solution to the location-transportation problem. Figure 4.3 shows an example of a network with one plant, two warehouses, and three retailers, as well as the information regarding demand at each retailer, capacity at each warehouse and transportation costs between the warehouses and retailers.
With priority-based encoding, we generate priorities $V(s)$ for the warehouses and retailers randomly, where $s$ represents a node. The highest priority value given to a node implies that we allocate demand/capacity in this node first by considering the lowest transportation cost from/to this node. In Figure 4.4, we give an example to illustrate the priority-based encoding mechanism. In this example, Retailer 1 has the highest priority value. Since it is cheapest to supply Retailer 1 from Warehouse 1, we allocate a flow of 100 units ($= \min(150; 100)$) from Warehouse 1 to Retailer 1 so that demand at Retailer 1 is satisfied. Next, we look at the second highest priority value at Warehouse 1. Since it is cheapest to distribute units from Warehouse 1 to Retailer 3, we allocate a flow of 50 units ($= \min(50; 120)$) from Warehouse 1 to Retailer 3. We continue this procedure until demand at all the retailers is fulfilled. We refer to Darmawan et al. (2018b) for the complete algorithmic specification of the GA heuristic.
In the numerical study, we evaluate the performance of the four different approaches, namely the sequential with non-coordinated inventory control (SNC), the sequential approach with coordinated inventory control (SC), the integrated approach with non-coordinated inventory control (INC) and the integrated approach with coordinated inventory control (IC). We adopt the case study of network design in Chopra and Meindl (2016; pp.151). We make some modifications...
to the cost structure, such that non-trivial differences can be distinguished in the performances of the four approaches compared. The network we consider consists of one supplier, 10 potential warehouses and 30 retailers. We generate 128 problem instances that represent the combinations of two levels of the following factors: ordering cost, unit holding cost, target fill-rate, variance of demand, transportation time, warehouse capacity, and transportation cost. We generate each retailer’s average daily demand, $\mu_j^r$, from the uniform distribution $U(100,160)$. The average total demand per year is 1,425,000 for 30 retailers. Table 4.2 summarises the other parameter values.

Table 4.2 Values for the experimental factors

<table>
<thead>
<tr>
<th>Factor</th>
<th>Low value</th>
<th>High value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ordering cost ($A$)</td>
<td>U(150, 190)</td>
<td>U(300, 380)</td>
</tr>
<tr>
<td>Holding cost ($h$)</td>
<td>Retailer: U(1, 1.25)</td>
<td>Retailer: U(2, 2.5)</td>
</tr>
<tr>
<td></td>
<td>Warehouse: U(0.5, 0.6)</td>
<td>Warehouse: U(1, 1.25)</td>
</tr>
<tr>
<td>Fill rate</td>
<td>90 %</td>
<td>95 %</td>
</tr>
<tr>
<td>Standard deviation of demand ($\sigma$)</td>
<td>$\sqrt{\mu_j^r/4}$</td>
<td>$\sqrt{\mu_j^r}$</td>
</tr>
<tr>
<td>Transportation time ($L$)</td>
<td>U(1, 2)</td>
<td>U(2, 4)</td>
</tr>
<tr>
<td>Warehouse capacity ($Cap$)</td>
<td>$4 \cdot \frac{\sum_{j=1}^I \mu_j^r}{I}$</td>
<td>$7 \cdot \frac{\sum_{j=1}^I \mu_j^r}{I}$</td>
</tr>
<tr>
<td>Accrued fixed cost ($M$)</td>
<td>U(975, 1212)</td>
<td>U(1950, 2424)</td>
</tr>
<tr>
<td>Variable cost ($V$)</td>
<td>0.3</td>
<td>0.6</td>
</tr>
<tr>
<td>Transportation cost ($P$)</td>
<td>U(250, 690)</td>
<td>U(500, 1380)</td>
</tr>
<tr>
<td>Truck capacity ($C$)</td>
<td>50</td>
<td>50</td>
</tr>
</tbody>
</table>
Table 4.3 Comparison of costs and number of warehouses with the four approaches

<table>
<thead>
<tr>
<th>Factor</th>
<th>Level</th>
<th>Total cost reduction</th>
<th>Inventory cost reduction</th>
<th>Number of Warehouses</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>SNC</td>
<td>SC</td>
<td>INC</td>
</tr>
<tr>
<td>Ordering cost</td>
<td>Low</td>
<td>2.58%</td>
<td>6.93%</td>
<td>9.14%</td>
</tr>
<tr>
<td></td>
<td>High</td>
<td>3.35%</td>
<td>7.07%</td>
<td>9.73%</td>
</tr>
<tr>
<td>Holding cost</td>
<td>Low</td>
<td>2.60%</td>
<td>5.41%</td>
<td>7.44%</td>
</tr>
<tr>
<td></td>
<td>High</td>
<td>3.34%</td>
<td>8.59%</td>
<td>11.43%</td>
</tr>
<tr>
<td>Fill rate</td>
<td>Low</td>
<td>2.89%</td>
<td>6.67%</td>
<td>9.05%</td>
</tr>
<tr>
<td></td>
<td>High</td>
<td>3.04%</td>
<td>7.33%</td>
<td>9.82%</td>
</tr>
<tr>
<td>Std. dev. of demand</td>
<td>Low</td>
<td>3.07%</td>
<td>6.99%</td>
<td>9.48%</td>
</tr>
<tr>
<td></td>
<td>High</td>
<td>2.97%</td>
<td>7.01%</td>
<td>9.38%</td>
</tr>
<tr>
<td>Transportation time</td>
<td>Low</td>
<td>2.98%</td>
<td>7.01%</td>
<td>9.43%</td>
</tr>
<tr>
<td></td>
<td>High</td>
<td>2.96%</td>
<td>7.00%</td>
<td>9.44%</td>
</tr>
<tr>
<td>Warehouse capacity</td>
<td>Low</td>
<td>3.22%</td>
<td>6.32%</td>
<td>8.98%</td>
</tr>
<tr>
<td></td>
<td>High</td>
<td>2.72%</td>
<td>7.69%</td>
<td>9.89%</td>
</tr>
<tr>
<td>Transportation cost</td>
<td>Low</td>
<td>3.47%</td>
<td>9.42%</td>
<td>12.21%</td>
</tr>
<tr>
<td></td>
<td>High</td>
<td>2.47%</td>
<td>4.58%</td>
<td>6.66%</td>
</tr>
<tr>
<td>Overall average</td>
<td></td>
<td>2.98%</td>
<td>7.00%</td>
<td>9.43%</td>
</tr>
</tbody>
</table>
In our comparison, we use the result from SNC as the baseline for the other three approaches. Table 4.3 shows the average percentages of total cost reduction and inventory cost reduction as well as the optimal number of warehouses. In Table 4.3, for each of the experimental factors, the value in each cell represents the average result over 64 (i.e. 128/2) cases. The overall averages of total cost reduction for SC, INC and IC are 2.97%, 7.00%, 9.43 %, respectively. Clearly, the adoption of the integrated approach with coordinated inventory control in SCND generates promising results, which is reflected by the highest overall percentage of total cost reduction. The results of our study also reinforce the importance of using the integrated approach instead of the sequential approach. This is reflected by the average cost reduction of 7% obtained by using INC instead of SNC, i.e. the integration reduces cost even in the absence of coordinated inventory control. The overall averages of inventory cost reduction for SC, INC and IC are 9.98%, 22.68% and 30.66%, respectively, which highlights the importance of incorporating coordinated inventory control in SCND.

All the experimental factors except for standard deviation of demand and transportation time seem to be influential on the percentage of total cost reduction. The percentages of total cost reduction in all the three approaches increase when the ordering cost, holding cost and fill rate are higher, which shows that there is stronger motivation to consider the integrated approach as well as coordinated inventory control in designing the supply chain network in the settings characterized by high inventory-related costs and service level requirement.

When looking at the effect of warehouse capacity, there are differences in how the percentage of total cost reduction changes as a result of increasing warehouse capacity for each of the three approaches. When the warehouse capacity is larger, the percentage of total cost reduction decreases under the SC approach but increases under the integrated approaches, INC and IC. Note that both SC and SNC have the same network resulting from the first stage of the sequential approach, where only the location-transportation related costs are considered. The difference between SNC and SC lies in the inventory allocations across all the retailers and warehouses. We use Figure 4.5 to help explain the results. In Figure 4.5 (and also in Table 4.3), the average percentage of inventory cost reduction for SC is decreasing in warehouse capacity. Enlarging the warehouse capacity will increase the fixed (leasing) cost, which results in the lower percentage of total cost reduction. However, we see the opposite effect in the integrated approaches (INC and
IC) as the average percentage of total cost reduction is increasing in the warehouse capacity. These two approaches are able to exploit the benefit of increasing the warehouse capacity from the integration of the location-transportation and inventory decisions. In particular, the integrated approach is able to capture the advantage of the risk pooling effect that is more pronounced in the setting with larger warehouse capacity. As shown in Figure 4.5, both the total transportation and inventory costs in these two approaches are reduced significantly when the warehouse capacity is high.

![Figure 4.5](image)

Figure 4.5 The increase/decrease of cost for different approaches relative to the baseline (SNC) differentiated by warehouse (WH) capacity

The percentage of total cost reduction seems to be negatively affected by the transportation cost as the percentage of total cost reduction becomes lower when the transportation cost is higher. Even though we still see cost reduction as a result of using the integrated approach and/or coordinated inventory control, increasing the transportation cost results in higher total cost so that the relative cost saving tends to diminish.

The two factors that does not seem to have an impact on the percentage total cost reduction are the standard deviation of demand and the transportation time. These two factors certainly have direct impact on the expected inventory level, and hence the total inventory cost. However, from our numerical study, there is no evidence that changing these two factors influences the percentage of either total cost or inventory cost reduction.
Table 4.3 also presents the optimal number of warehouses for the four approaches. On average, the sequential approaches will use more warehouses than the integrated approaches. The optimal number of warehouses for SNC and SC are only affected by the warehouse capacity and the transportation cost. This is because these two approaches neglect inventory when finding an optimal network design. In the case of both integrated approaches, i.e. INC and IC, we observe more diverse effects of the different experimental factors.

Figure 4.6 Interaction plot (data means) for number of warehouses with IC approach

Figure 4.6 depicts the interaction plot of two experimental factors on the number of warehouses for the IC approach. In order to increase the inventory turnover, more warehouses are preferable when the holding cost is higher. Nonetheless, the figure shows that the effect of the holding cost may also be influenced by the other factors. For example, the effect of the holding cost is less pronounced in the setting with low warehouse capacity and high transportation cost than in the setting with high warehouse capacity and low transportation cost. We can also explain the joint effect of warehouse capacity and transportation cost on the optimal number of warehouses. The effect of changing warehouse capacity is more significant when the transportation cost is low, and less so when the transportation cost is high. Understanding the joint effects of different factors as
presented above helps in revealing more specific conditions under which using the integrated approach and coordinated inventory control give the maximum benefits.

As previously stated, on average, the sequential approach results in more warehouses than the integrated approach (see Table 4.3). However, caution should be used when interpreting the effect of implementing coordinated inventory control as the use of coordinated inventory control does not always result in fewer warehouses. There are some cases where the number of warehouses in the IC approach is larger than in the SNC approach. In Figure 4.7 we plot all the possible number of warehouses and the corresponding total costs for the problem instance with low ordering cost, high holding cost, high target fill rate, high standard deviation of demand, high lead time, high warehouse capacity and low transportation cost. It can be seen in Figure 4.7 that the optimal solution for the SNC approach is achieved when the number of warehouses is 3, whereas the optimal number of warehouses in the IC approach is 4. Under the SNC approach, the optimal number of warehouses resulting from the first stage is obtained without considering the inventory cost. In the IC approach, the simultaneous optimisation allows for exploring the possibility to reduce the transportation cost by opening more warehouses, and together with the reduction in the inventory cost, to compensate the higher cost for opening and operating more warehouses. As a contrast, in Figure 4.8 we plot all the possible number of warehouses and the corresponding total costs for another specific problem instance with high ordering cost, low holding cost, high target fill rate, high standard deviation of demand, high lead time, low warehouse capacity and high transportation cost. In this problem instance, the optimal number of warehouses in the IC approach (=4) is smaller than the optimal number of warehouses in the SNC approach (=5). As shown in Figure 4.8, the total transportation cost in this problem instance accounts for the major part of the total cost. Compared to the previous problem instance (Figure 4.7), we observe a different role played by the IC approach. The IC approach allows for the reduction of the number of warehouses such that the reduction of fixed cost for opening warehouses together with the saving in inventory cost can compensate for the increase in the transportation cost.
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Figure 4.7 An example with more warehouses opened in the integrated approach (IC) than in the sequential approach (SNC).

Figure 4.8 An example with fewer warehouses opened in the integrated approach (IC) than in the sequential approach (SNC).
**Multiple sourcing vs single sourcing**

As we discussed in Section 4.2, most of the previous studies of SCND assume a single sourcing policy to simplify their models. Under this policy, a retailer is only supplied by one warehouse, and hence, has no choice of sourcing from more than one warehouse. This restriction may lead to suboptimal results. In this subsection, we analyse the impact of retailers’ sourcing policy and compare the results between the single sourcing and multiple sourcing policies. We focus our comparison on the integrated approaches, namely INC and IC.

<table>
<thead>
<tr>
<th>Factors</th>
<th>Level</th>
<th>The average number of warehouses</th>
<th>Cost reduction (%) using Multiple</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>INC</td>
<td>Multiple</td>
</tr>
<tr>
<td>Ordering cost</td>
<td>Low</td>
<td>4.0</td>
<td>3.8</td>
</tr>
<tr>
<td></td>
<td>High</td>
<td>4.0</td>
<td>3.6</td>
</tr>
<tr>
<td>Holding cost</td>
<td>Low</td>
<td>3.8</td>
<td>3.5</td>
</tr>
<tr>
<td></td>
<td>High</td>
<td>4.2</td>
<td>3.8</td>
</tr>
<tr>
<td>Fill rate</td>
<td>Low</td>
<td>3.9</td>
<td>3.6</td>
</tr>
<tr>
<td></td>
<td>High</td>
<td>4.0</td>
<td>3.7</td>
</tr>
<tr>
<td>Std. dev. of demand</td>
<td>Low</td>
<td>3.9</td>
<td>3.6</td>
</tr>
<tr>
<td></td>
<td>High</td>
<td>4.0</td>
<td>3.7</td>
</tr>
<tr>
<td>Retailer transportation time</td>
<td>Low</td>
<td>4.0</td>
<td>3.7</td>
</tr>
<tr>
<td></td>
<td>High</td>
<td>3.9</td>
<td>3.6</td>
</tr>
<tr>
<td>Warehouse capacity</td>
<td>Low</td>
<td>4.4</td>
<td>3.9</td>
</tr>
<tr>
<td></td>
<td>High</td>
<td>3.6</td>
<td>3.5</td>
</tr>
<tr>
<td>Transportation cost</td>
<td>Low</td>
<td>3.7</td>
<td>3.4</td>
</tr>
<tr>
<td></td>
<td>High</td>
<td>4.3</td>
<td>3.9</td>
</tr>
<tr>
<td>Overall average</td>
<td></td>
<td>4.0</td>
<td>3.7</td>
</tr>
</tbody>
</table>

As shown in Table 4.4, with both integrated approaches the single sourcing policy tends to open more warehouses than the multiple sourcing policy. In the INC approach, the average numbers of opened warehouses are 4.0 and 3.7 for the single and multiple sourcing policies, respectively. In
the IC approach, the results are very similar and the average numbers of warehouses are 4.0 and 3.8. The multiple sourcing policy certainly gives the opportunity to maximise the utilization of the available warehouse capacity.

Table 4.4 also shows that relaxing the assumption of sourcing policy from single to multiple sourcing slightly reduces the total cost with average percentage values equal to 0.86% and 0.60% for INC and IC, respectively. The fact that the IC approach seems marginally less sensitive to the sourcing policy than the INC approach also shows the role played by the coordinated inventory control in dealing with restrictions in the system, for example regarding sourcing policy.

Although the overall percentages of the cost reduction seem to be relatively small in our numerical study, one cannot dismiss the potential benefits of adopting a multiple sourcing policy. In more complex settings where e.g. multiple products and supply disruption risk exist, the motivation for using a multiple sourcing policy would become stronger.

### 4.6 Conclusions

In this study, we have developed models and solution methods for supply chain network design (SCND) that not only integrate the location-transportation and inventory problems, but also consider the implementation of coordinated inventory control in the network. The network we consider has a divergent structure where retailers located downstream of the supply chain are supplied by warehouses that get supplies from a supplier located upstream of the supply chain. The objective in our network design problem is to minimise the expected total cost that is the sum of the costs for opening and operating warehouses, the transportation costs, and the inventory-related cost, while meeting the requirement that a target fill rate must be met at all the retailers. The most distinguishing feature of our model, compared to models in the literature, is the implementation of coordinated inventory control. As a consequence, service to end customers represented by the target fill rate at the retailers serves as the primary and only service constraint. This implies that the warehouses should not necessarily use the same target fill rate as the retailers. By contrast, the existing literature on SCND imposes the same (unnecessarily) high service levels on all stocking points in the supply chain.
We develop and compare four different approaches to solve the network design problem. These approaches are differentiated by whether an integrated solution approach is used as opposed to a sequential approach, and whether or not coordinated inventory control model is implemented. Due to the complexity of the optimisation problem in the integrated approach, we develop a heuristic method based on a Genetic Algorithm to solve the problem. We implement an induced backorder cost model to solve the inventory optimisation problem with coordinated inventory control. The models and solution methods presented in this paper can serve as building blocks for developing a decision support system of a fully integrated supply chain network design.

The results of our numerical study reveal several interesting findings. First, our results reinforce and substantiate the claims made in the existing literature on the importance of using the integrated approach rather than the sequential approach. More importantly, we show the additional benefits of considering coordinated inventory control in designing the network. The coordinated inventory control reduces the inventory cost significantly, particularly at the warehouses. On average, the inclusion of coordinated inventory control in the network design results in a saving of 9.43% in the total cost and 30.66% in the inventory cost, compared to the sequential approach with no coordinated inventory control. The cost saving of such a magnitude is significant, especially for firms that operate in very competitive business environments with narrow profit margins. Our results give impetus to the idea of network (re-)design that emphasises integration and coordination.

On average, the integrated approach results in fewer warehouses than the non-integrated approach. This may not sound too surprising, as supply chains benefit from the reduction of costs for opening and operating warehouses, as well as from the greater risk pooling effect resulting from fewer warehouses. However, this is not always the case when looking further into individual instances. In some instances, especially when the warehouse capacity is high and the transportation cost is low, the integrated approach may also result in more warehouses in the supply chain in comparison to the non-integrated approach. The simultaneous optimisation in the integrated approach with coordinated inventory control offers the possibility of compensating the increase of costs for opening and operating warehouses by the reduction of transportation and inventory costs.

We are aware of some assumptions in this paper that suggest directions for extending the research. First, we only consider a single product in this study, while supply chains typically need to carry
and deal with multiple products. Further research that addresses SCND with coordinated inventory control in the case of multiple products could represent an interesting research avenue. In such an extension, the possible application of joint replenishments of multiple products is an important issue to consider. Another possible extension is to study the impact of risks in SCND with coordinated inventory control. Our study has shown that applying coordinated inventory control in network design reduces the inventory at the warehouses significantly. The robustness of the resulting network design becomes questionable when considering unexpected disruption in the supply chain. For example, any disruptions that occur at the warehouse echelon could lead to severe problems for the retailers. Therefore, a model that considers coordinated inventory control and disruption effects could be very helpful for designing more robust networks.
Appendix

Appendix 1: The normally distributed demand

Let $\phi(x)$ and $\Phi(x)$ represent the density and distribution function of demand.

The normal loss function $G(x)$ is obtained as

$$G(x) = \int_x^{\infty} (v - x) \varphi(v) dv = \varphi(x) - x(1 - \Phi(x)) \quad (A4.1)$$

where

$$\varphi(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}} \quad \Phi(x) = \int_{-\infty}^{x} \frac{1}{\sqrt{2\pi}} e^{-\frac{v^2}{2}} dv \quad (A4.2)$$

Moreover,

$$G'(x) = \phi(x) - 1; \quad G(\infty) = 0 \quad (A4.3)$$

$$H(x) = \int_x^{\infty} G(v) dv = \frac{1}{2} [(x^2 + 1)(1 - \phi(x)) - x\varphi(x)]; \quad H'(x) = -G(x) \quad (A4.4)$$

Appendix 2: The inventory cost

We break down warehouse and retailer inventory cost which consists of ordering cost and holding cost subject to the target fill rate as follows.

1. Warehouse $i$

$$\min_{Q_i^{w}, R_i^{w}} C_{WH} = \tilde{C}_i(R_i) = \text{ordering cost} + \text{holding cost}$$

$$= A_i^{w} \frac{H_i^{w}}{Q_i^{w}} + h_i^{w} E(I_{L_i}^{w})^+ = A_i^{w} \frac{\mu_i^{w}}{Q_i^{w}} + h_i^{w} E(I_{L_i}^{w}) + h_i^{w} E(I_{L_i}^{w})^-$$

$$= A_i^{w} \frac{H_i^{w}}{Q_i^{w}} + h_i^{w} \left( R_i^{w} + Q_i^{w}/2 - \mu_{D_i(L_i)}^{w} \right) + h_i^{w} E(I_{L_i}^{w})^-$$

$$= A_i^{w} \frac{H_i^{w}}{Q_i^{w}} + h_i^{w} \left( R_i^{w} + Q_i^{w}/2 - \mu_{D_i(L_i)}^{w} \right) + h_i^{w} \int_{-\infty}^{0} F(x) dx$$

$$= A_i^{w} \frac{H_i^{w}}{Q_i^{w}} + h_i^{w} \left( R_i^{w} + Q_i^{w}/2 - \mu_{D_i(L_i)}^{w} \right)$$

$$+ h_i^{w} \int_{-\infty}^{0} \frac{1}{Q_i^{w}} \int_{R_i^{w}}^{R_i^{w} + Q_i^{w}} \left[ -G'\left( \frac{u - x - \mu_{D_i(L_i)}^{w}}{\sigma_{D_i(L_i)}^{w}} \right) \right] du dx \quad (A4.5)$$
\[ A_i^w \frac{h_i^w}{Q_i^w} + h_i^w \left( R_i^w + \frac{Q_i^w}{2} - \mu_{D_i(l_i)} \right) + h_i^w \sigma_{D_i(l_i)} \int_{R_i^w}^{R_i^w + Q_i^w} G \left( \frac{u - \mu_{D_i(l_i)}}{\sigma_{D_i(l_i)}} \right) \, du \]

\[ = A_i^w \frac{h_i^w}{Q_i^w} + h_i^w \left( R_i^w + \frac{Q_i^w}{2} - \mu_{D_i(l_i)} \right) + h_i^w \sigma_{D_i(l_i)} \frac{(\sigma_{D_i(l_i)})^2}{Q_i^w} \left[ H \left( \frac{R_i^w - \mu_{D_i(l_i)}}{\sigma_{D_i(l_i)}} \right) - H \left( \frac{R_i^w + Q_i^w - \mu_{D_i(l_i)}}{\sigma_{D_i(l_i)}} \right) \right] \]

Subject to

\[ \text{FillRate}_i = 1 - P(\text{ILL}_i^w \leq 0) \geq \text{FillRate}_i^{\text{min}} \]  

(A4.6)

2. Retailer \( j \)

\[ \min_{Q_j^r, R_j^r} C_R = \bar{C}_j(R_j) = \text{ordering cost} + \text{holding cost} \]

\[ = A_j^r \frac{\mu_j^r}{Q_j^r} + h_j^r \left( R_j^r + \frac{Q_j^r}{2} - \mu_{D_j(l_j)} \right) \]

\[ + h_j^r \frac{(\sigma_{D_j(l_j)})^2}{Q_j^r} \left[ H \left( \frac{R_j^r - \mu_{D_j(l_j)}}{\sigma_{D_j(l_j)}} \right) - H \left( \frac{R_j^r + Q_j^r - \mu_{D_j(l_j)}}{\sigma_{D_j(l_j)}} \right) \right] \]  

(A4.7)

Subject to

\[ \text{FillRate}_j = 1 - P(\text{ILL}_j^r \leq 0) \geq \text{FillRate}_j^{\text{min}} \]  

(A4.8)

**Appendix 3: The model validation for multiple sourcing policy**

<table>
<thead>
<tr>
<th>Target fill rate</th>
<th>90%</th>
<th>95%</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \mu )</td>
<td>100</td>
<td></td>
</tr>
<tr>
<td>( \sigma )</td>
<td>5</td>
<td>10</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Avg.fill rate and deviation</th>
<th>(simulation)</th>
</tr>
</thead>
<tbody>
<tr>
<td>3WH10RT</td>
<td>90.65 0.65 90.97 0.97 93.20 -1.80 93.48 -1.52</td>
</tr>
<tr>
<td>3WH15RT</td>
<td>89.92 -0.08 90.15 0.15 93.70 -1.30 93.88 -1.12</td>
</tr>
<tr>
<td>3WH20RT</td>
<td>90.96 0.96 91.28 1.28 93.15 -1.85 93.57 -1.43</td>
</tr>
<tr>
<td>3WH25RT</td>
<td>90.99 0.99 91.34 1.34 93.42 -1.58 93.96 -1.04</td>
</tr>
<tr>
<td>4WH10RT</td>
<td>89.78 -0.22 89.95 -0.05 93.53 -1.47 93.80 -1.20</td>
</tr>
<tr>
<td>4WH15RT</td>
<td>90.70 0.70 90.92 0.92 92.97 -2.03 93.40 -1.60</td>
</tr>
<tr>
<td>4WH20RT</td>
<td>90.44 0.44 90.61 0.61 93.70 -1.30 93.99 -1.01</td>
</tr>
<tr>
<td>4WH25RT</td>
<td>90.86 0.86 91.17 1.17 93.13 -1.87 93.69 -1.31</td>
</tr>
</tbody>
</table>

| Avg deviation | 0.54 0.80 -1.65 -1.28 |

Simulation time: 7300; Warmup time: 365; Replications: 4
Appendix 4: The coordinated inventory control

In the centralised inventory, we take into account the mutual interdependence between warehouse and retailer in the process of finding the appropriate reorder point for given batch quantities (Axsäter, 2003; Berling and Marklund, 2006, 2014). There are five steps:

1. Induced backorder cost at warehouse

\[
b_i^w = \frac{\sum_{j=1}^J \mu_{ij} b_j^r}{\sum_{j=1}^J \mu_{ij}}
\]

Where

\[
b_j^r = h_j g\left(\frac{100Q_j^r}{\mu_{ij}L_{ij}}, \frac{b_j^r}{h_j}\right) \quad g_b = \max(-1.2, -2(b_j^r/h_j^r)^{-0.25})
\]

\[
g\left(\frac{100Q_j^r}{\mu_{ij}L_{ij}}, \frac{b_j^r}{h_j}\right) = \min\left\{g_a, \left(\frac{100Q_j^r}{\mu_{ij}L_{ij}}\right) g_b, G\right\}
\]

\[
g_a = \min(0.015 \frac{b_j^r}{h_j^r}, \max(0.65/\sqrt{b_j^r/h_j^r} , 0.05))
\]

\[
G = \min(0.015, 0.005(b_j^r/h_j^r)^{0.2})
\]

\[
k\left(\frac{100Q_j^r}{\mu_{ij}L_{ij}}, \frac{b_j^r}{h_j}\right) = \max\left\{1, \min(k_a\left(\frac{100Q_j^r}{\mu_{ij}L_{ij}}\right) k_b, K\right\}
\]

\[
k_a = \max(0.7, \min(0.9, 0.6(b_j^r/h_j^r)^{0.075})
\]

\[
k_b = \min(0.2, 0.4(b_j^r/h_j^r)^{-0.35})
\]

\[
K = \max(1.3, \min(2, 2.5(b_j^r/h_j^r)^{-0.15})
\]

2. Lead time demand at warehouse

\[
\mu_{D_i(L_i)} = \sum_{j=1}^J \mu_{ij} L_i
\]

\[
\sigma_{D_i(L_i)}^w = \sqrt{\sum_{j=1}^J \sum_{n=0}^\infty \left(\mu_{D_i(L_i)} - nQ_j^r\right)^2 f_j(nQ_j^r)}
\]

\[
f_j(nQ_j^r) = \gamma_j(n) - \gamma_j(n - 1)
\]

The probability of retailer \( j \) orders at most \( n \) times during \( L_i \) is

\[
\gamma_j(n) = \frac{1}{Q_j^r} \phi\left(\frac{(n + 1)Q_j^r - \mu_{ij}L_i}{\sqrt{\sigma_{ij}^2 L_i}}\right)
\]

\[
+ \sqrt{\frac{1}{\sigma_{ij}^2 L_i}} \left(\phi\left(\frac{(n + 1)Q_j^r - \mu_{ij}L_i}{\sqrt{\sigma_{ij}^2 L_i}}\right) - \phi\left(\frac{nQ_j^r - \mu_{ij}L_i}{\sqrt{\sigma_{ij}^2 L_i}}\right)\right)
\]

\[
+ (nQ_j^r - \mu_{ij}L_i) \left(\phi\left(\frac{(n + 1)Q_j^r - \mu_{ij}L_i}{\sqrt{\sigma_{ij}^2 L_i}}\right) - \phi\left(\frac{nQ_j^r - \mu_{ij}L_i}{\sqrt{\sigma_{ij}^2 L_i}}\right)\right)\right)
\]

(A4.10)
3. Reorder point and the expected delay at warehouse

\[ R_i^w = \text{argmin}\{\bar{C}_i(R_i^w)\} \]

\[ \bar{C}_i(R_i^w) = A_i^w \frac{\mu_i^w}{Q_i^w} + h_i^w \left( R_i^w + \frac{Q_i^w}{2} - \mu_i^w \right) + (h_i^w + b_i^w) \frac{\sigma_i^w}{Q_i^w} \left[ H \left( \frac{R_i^w - \mu_i^w}{\sigma_i^w D_i(L_i)} \right) - H \left( \frac{R_i^w + Q_i^w - \mu_i^w}{\sigma_i^w D_i(L_i)} \right) \right] \]

\[ \frac{\partial \bar{C}_i(R_i^w)}{\partial R_i^w} = h_i^w + (h_i^w + b_i^w) \frac{\sigma_i^w D_i(L_i)}{Q_i^w} = G \left( \frac{R_i^w + Q_i^w - \mu_i^w}{\sigma_i^w D_i(L_i)} \right) - G \left( \frac{R_i^w - \mu_i^w}{\sigma_i^w D_i(L_i)} \right) = 0 \]

\[ G \left( \frac{R_i^w - \mu_i^w}{\sigma_i^w D_i(L_i)} \right) = \frac{h_i^w + b_i^w}{h_i^w + b_i^w} \frac{Q_i^w}{\sigma_i^w D_i(L_i)} \]

\[ E(Delay) = \frac{E(II_i^w)}{\mu_i^w} = \frac{\sigma_i^2}{Q_i^w} \left[ H \left( \frac{R_i^w - \mu_i^w}{\sigma_i^w D_i(L_i)} \right) - H \left( \frac{R_i^w + Q_i^w - \mu_i^w}{\sigma_i^w D_i(L_i)} \right) \right] \]

4. Lead time demand at retailer

Adjusted by considering expected delay at warehouse and undershoot that might happen.

\[ L_i^j = L_{ij} + E(Delay) \]

\[ \mu_i^r(D_j(L_j)) = \sum_{i=1}^{l} \mu_{ij} L_i^j + \mu_u \]

\[ \sigma_i^2 = \sum_{i=1}^{l} \sigma_{ij}^2 L_i^j + \sigma_u^2 \]

5. Reorder point at retailer

\[ R_i^r = \min \{ R_i^r : FillRate_j = 1 - P(II_j^r \leq 0) \geq FillRate_j^{min} \} \]

\[ P(II_j^r \leq 0) = \frac{\sigma_{D_i(L_j)}^2}{Q_j^r} \left[ G \left( \frac{R_j^r - \mu_{D_i(L_j)}}{\sigma_{D_i(L_j)}} \right) - G \left( \frac{R_j^r + Q_j^r - \mu_{D_i(L_j)}}{\sigma_{D_i(L_j)}} \right) \right] \]
Chapter 5

Paper 4: Exploring proactive and reactive strategies in supply chain network design with coordinated inventory control in the presence of disruptions

History: This chapter was initiated during a research visit at Ghent University in March-May, 2018. It has been prepared for possible future publication.
Exploring proactive and reactive strategies in supply chain network design with coordinated inventory control in the presence of disruptions

Agus Darmawan

CORAL - Cluster for Operations Research, Analytics and Logistics
Department of Economics and Business Economics, Aarhus University
Fuglesangs Allé 4, 8210 Aarhus, Denmark

Abstract
In this paper, we study the supply chain network design (SCND) problem with multiple suppliers, multiple warehouses and multiple retailers in the presence of random disruptions. We consider coordination between the warehouses and retailers in determining the inventory control parameters. The SCND problem is to minimise the expected total costs that consist of the costs for opening and operating warehouses, transportation costs, inventory costs and disruption costs subject to the target fill rates at the retailers. The existence of disruption and coordination between the warehouses and retailers makes the problem hard to solve analytically. We propose a two-stage approach to solve the problem. In the first stage, we address the SCND problem without disruption and solve it using a genetic algorithm based heuristic, integrated with the induced backorder approach for inventory coordination. In the second stage, we use simulation to evaluate several reactive strategies whilst introducing disruptions in the network. We also use simulation to adjust the inventory control parameters obtained in the first stage as part of a proactive strategy. The numerical results show that neglecting the risk of disruptions may lead to supply chain network designs with low fill rates, and that the proposed approach is able to make the supply chain network more resilient in the event of disruptions.

Keywords: supply chain network design, inventory control, coordination, heuristics, stochastic models, disruption, mitigation strategy
5.1 Introduction

Supply chain network design (SCND) is one of the strategic supply chain planning activities that deals with the assignment of facilities, transportation between facilities and the allocation of capacity and markets to each facility (Chopra and Meindl, 2016). The typical trade-off in the standard SCND problem is usually between transportation and fixed facility costs. When there are few warehouses in the network, for example, fixed warehousing costs are low, but transportation costs are high. The average distance travelled is reduced when there are more warehouses in the network (Croxton and Zinn, 2005) leading to lower transportation costs, but at the expense of increased fixed facility costs.

What is missing in this standard SCND problem is the inclusion of inventory costs. This has motivated some authors to initiate studies that examine the inclusion of inventory in SCND (see e.g., Croxton and Zinn, 2005; Shapiro and Wagner, 2009). The main conclusions that can be drawn from these studies are that including inventory considerations often significantly affects the optimal design of the network. Hence, managers should account for inventory when designing supply chain networks.

Today, the literature on SCND with inventory considerations is growing. Farahani et al. (2015) present a comprehensive review of studies that consider inventory deployment in SCND. Although the existing studies include inventory in their models, most of them ignore the possibility of implementing a fully coordinated inventory control in the supply chain. Without coordinated inventory control, all echelons in the supply chain (e.g., plants, warehouses, and retailers) are typically forced to set equally high service levels. This practice, however, is not in line with the spirit adopted in literature on integrated multi-echelon inventory optimisation. Hausman (2005) emphasise that in relation to the service provision, the primary concern in supply chain management should be how to provide a sufficiently high service level to the end customers. This implies that only the players directly serving the end customers (e.g., retailers) need to maintain a high service level while the players at the middle and upper echelons (e.g., plants and warehouses) do not necessarily need to provide the same. When the inventory control is coordinated in the supply chain, satisfying the target service level for the end customers should be achieved at the minimum expected total inventory costs. The willingness to collaborate and to share the information between players at the different echelons are prerequisites for the successful
implementation of coordinated inventory control. In a recent work, Darmawan et al. (2018b) study the problem of designing a supply chain network that involves coordinated inventory control. Their results are motivating for this study, as they show that significant cost savings can be obtained when coordinated inventory control is embedded in the SCND problem.

The increase in the level of risks, especially those associated with disruptions, represents one of the new trends and challenges that will affect supply chain design. There are two types of risks, namely the “known-unknown” and the “unknown-unknown” (Simchi-Levi et al., 2008). The first type of risks, also referred to as operational risks such as demand and lead-time uncertainties, are, to a great extent, controllable as we may have a lot of data available. In the last few years, however, we also observe the impact of the second type of risks caused by unexpected rare events or disruptions such as natural disasters (e.g., floods, earthquakes, hurricanes) and man-made disasters (e.g., industrial accidents, labour strikes) for which there are hardly any data. Consequently, the occurrence of such an event is very difficult to predict. Even though a disruption rarely happens, its impact can be substantial. The strike at General Motors in 1998 forced the company to shutdown 100 parts plants and reduced earnings by $2.83 billion (Simison, 1998). Hendricks and Singhal (2005) report that, on average, firms experience 107% decrease in operating income, 6.92% lower sales and 10.66% higher cost following the disruptions. In 2011, the earthquake in Japan forced 80% of the automobile plants to suspend their production (Schmidt and Simchi-Levi, 2013). This may explain why the concept of supply chain resilience has gained much attention in recent years. Tukamuhabwa et al. (2015) define resilience as the adaptive capability of a network to respond to disruptions in a timely and cost effective manner. To make the supply chain more resilient, firms must develop a recovery or mitigation strategy, so that the impact of disruptions can be reduced (Chopra and Sodhi, 2004).

Designing the supply chain network while taking into account coordinated inventory control contributes to managing the first type of risks more effectively. However, as the presence of the second type of risks cannot be ignored, the question is how the resulting design will be affected when disruptions are considered in the network design. The answer to this question will help managers in developing an effective recovery strategy to deal with disruptions. To this end, this paper aims to develop a decision model for SCND with coordinated inventory control in the presence of disruptions. Due to the complexity of the problem, we propose a two-stage
optimisation approach. In the first stage, we solve the SCND problem without disruptions. We consider inventory deployment in SCND with full coordination between warehouses and retailers to achieve the pre-specified target service levels for the end customers. We employ a Genetic Algorithm based heuristic to solve the problem in the first stage. In the second stage, we develop a simulation model and use the solution obtained in the first stage for fine-tuning the inventory control parameters (reorder point and order quantity) while introducing probabilistic disruptions at the supply echelons. In this study, we evaluate a number of recovery strategies to enhance the network’s resilience in the presence of disruptions.

The rest of the paper is organised as follows. Section 2 presents relevant literature on SCND with inventory deployment and disruption in the supply chain. In Section 3, we present the problem description, the model, and the solution method. In this section, we also specify the two-stage approach for solving the problem. Section 4 presents and discusses the results of our numerical study. In Section 5, we summarise the main findings of this study and propose some directions for future research.

5.2 Literature review

There are a number of papers that discuss and classify the existing studies related to SCND (see e.g., Melo et al., 2009; Farahani et al., 2015; and Govindan et al., 2017). Particularly relevant to our paper is the research stream that considers the inclusion of inventory deployment in SCND. Several papers consider inventory decisions in a single echelon and focus on examining the risk pooling effect (e.g., Miranda and Garrido, 2004; Croxton and Zinn, 2005; Sourirajan et al., 2009; Yao et al., 2010; Liao et al., 2011 and Shahabi et al., 2013). In practice, a supply chain maintains inventory in more than one echelon, which motivates some authors to consider multi-echelon inventory deployment in SCND (see e.g., Farahani and Elahipanah, 2008; Tancrez et al., 2012 and Taxakis and Papadopoulos, 2016). However, those papers assume that the demand at retailers is deterministic. Kang and Kim (2012), Kumar and Tiwari (2013), Askin et al. (2014), Manatkar et al. (2016), and Puga and Tancrez (2017) study the SCND problems while considering inventory at retailers and warehouses under stochastic demand. Nevertheless, all those studies consider inventory decisions at the different echelons separately, i.e. without synchronisation. Motivated
by this shortcoming, Darmawan et al. (2018b) present models where the inventory decisions at the retailers and warehouses are synchronised simultaneously. One of their findings is that employing coordinated inventory control in SCND can reduce the inventory costs significantly compared to the network design with non-coordinated inventory control. However, none of the aforementioned studies take into account the possible occurrence of disruptions in the network. In Darmawan et al. (2018b), the authors suggest considering the unexpected events (disruptions) in the supply chain network for future research so that the resulting network design can become more resilient. This paper is a response to that suggestion.

Disruptions and their impact on the supply chain have been investigated extensively (see e.g., Chopra and Sodhi, 2014; Tang et al., 2014; Ivanov et al., 2016; Khalili et al., 2017). In their review, Snyder et al. (2016), Ivanov et al. (2017), and Dolgui et al. (2018) all emphasise the importance of a resilient supply chain in a competitive and volatile environment. Maintaining a high service level at a reasonable cost are still crucial issues when the disruption occurs. In the literature, there are two different strategies for managing disruptions in the supply chain, namely the proactive and the reactive strategies (Klibi et al., 2010; Snyder et al., 2016). A proactive strategy provides some level of protection before the disruption occurs but without recovery considerations (Snyder and Daskin, 2005; Wilson, 2007; Ravindran et al., 2010; Baghalian et al., 2013; Sawik, 2016). Multiple sourcing and increased inventory to provide an extra buffer are examples of the proactive strategy (e.g., Qi et al., 2010; Campbell and Jones, 2011; Snyder, 2014). Whereas in a reactive strategy, we only make some adjustments to the supply chain network structure after the disruption occurs as part of the recovery process (Knemeyer et al., 2009; Ivanov et al., 2016).

Several modelling approaches have been proposed to deal with disruptions. Lim et al. (2010) and Paul et al. (2014) present mixed-integer programming models in studying SCND with disruptions. Their approaches consider disruption as a static event without taking into account the disruption duration and the recovery policies. In order to reduce the impact of unexpected events, Snyder and Daskin (2005) present a model that incorporates reliability into the facility location problem where each facility has a probability of failing. In their model, Lim et al. (2013) show that misestimating the probability of disruption, particularly in the case of underestimating, could have serious consequences on the supply chain. They suggest an investment in additional capacity when the disruption probability is high. Rezapour et al. (2017) develop a resilient topology of the supply chain.
chain by considering multiple sourcing, backup capacity and emergency stock at retailers. Even though these measure might create a more costly supply chain structure, they argue that mitigation policies are important to maintain the market share (service level) when a disruption occurs. Ghavamifar et al. (2018) propose a bi-level multi-objective programming model for designing a supply chain network under disruption risks. In their model, each retailer is supplied by only one warehouse. Under the assumption that disruptions may occur at warehouses, their study shows the importance of considering a proactive strategy to reduce the impact of disruptions. All the papers discussed above are common in that they do not consider the disruption as a dynamic event and exclude the duration of the disruption in their models.

The inclusion of disruptions can also be captured through the scenario approach in stochastic programming. Although this approach is intuitively appealing, the problem size increases exponentially with the number of facilities (e.g., Snyder, 2006; Shen et al., 2011; Sawik, 2016). Baghalian et al. (2013) develop a model to protect the supply chain with respect to disruption while considering both the demand-side and supply-side uncertainties in a single period horizon. They show that changing disruption rate in the model may affect the supply chain network structure. Torabi et al. (2015) propose a two-stage stochastic programming model for supplier selection and order allocation in a single period horizon. Their numerical results show that there is a direct relation between the likelihood of disruptive events and the selection of supply portfolio. Fattahi et al. (2017) present a multi-stage stochastic program for SCND under known-known (operational) and unknown-known (disruption) risks. In their model, each customer zone (retailer) uses a single-sourcing policy where this customer zone should be supplied only from one warehouse. Disruption may occur at any warehouse with a certain probability but without information regarding its duration. They implement scenario reduction to make the problem tractable. Zhalechian et al. (2018) present a model of network design under operational and disruption risks. Their study suggests the importance of considering resilience strategies and recovery budget for anticipating the losses during the disruptions. None of the aforementioned papers considers reactive strategies in their models. In addition, they consider a rather simple network structure with a limited number of nodes and disruption scenarios in order to make their optimisation problems tractable.
Simulation can be used as an alternative approach for capturing the randomness of disruption, dynamic recovery policies, as well as the ripple effect in the supply chain (Gao and Chen, 2017; Ivanov, 2017; Schmitt et al., 2017). Wilson (2007) uses simulation to mitigate the impact of disruption in a five-echelon supply chain subject to transportation disruptions. She finds that the disturbances near the middle echelons create a ripple effect downstream and upstream, and it has the greatest impact on the supply chain. Schmitt and Singh (2012) construct a discrete event simulation model to analyse the impact of disruption at the supply chain echelons. Under the assumption of a base-stock inventory control policy applied in a consumer packaged goods supply chain, they consider different backup capacity at the echelons and show that the network becomes more resilient if it is sufficiently protected. According to their result, the speed of providing a backup resource is more important than the volume of the backup capacity. Although all the papers discussed above consider disruption as a dynamic event, they use the simulation model only as an evaluation tool that facilitates the what-if analysis without exploring the use of simulation for optimising the decisions in the supply chain. In this paper, we perform simulation for the purpose of finding a good solution in the presence of disruptions.

In their extensive literature reviews, Melo et al. (2009), Farahani et al. (2015), Snyder et al. (2016), Govindan et al. (2017), Ivanov et al. (2017), and Dolgui et al. (2018) highlight the importance of studies focusing on the development of robust network design that take uncertainty and disruptions into account. More specifically, they point out the importance considering a disruption as a dynamic event. Considering both the rate and the duration of a disruption will enable us to thoroughly examine the expected recovery time and disruption costs, which are comprised not only the setup costs for the backup facilities but also of the potential lost sales during the disruption. Moreover, there is also a lack of research that considers both the proactive and reactive strategies in an attempt to find the best mitigation strategy dealing with disruptions. All these settings also require the synchronisation of inventory decisions between the echelons (e.g., warehouse and retailers). This synchronisation has not been explored in the previous models of disruptions. In line with the suggested research directions, the analyses in this paper contributes to filling the gaps identified in the literature.
5.3 Model

5.3.1 Problem description and assumptions

This paper considers a three-echelon supply chain network for a single product. The network consists of multiple plants, multiple warehouses and multiple retailers as shown in Figure 5.1. We assume that the locations of the potential plants, the potential warehouses, and the retailers are fixed and known in advance. Demand at retailer \( j \) \((j = 1,2, ..., J)\) is stochastic and follows a normal distribution with mean \( \mu_j \) and variance \( \sigma_j^2 \). There are \( I \) potential warehouses that can supply the product to the retailers, and each warehouse \( i \) \((i = 1,2, ..., I)\) has a finite capacity. There are \( N \) potential plants that serve the demand from the warehouses. We assume that each plant \( n \) \((n = 1,2, ..., N)\), has an unlimited capacity. The transportation times from the plants to the warehouses and also from the warehouses to the retailers are assumed to be constant.

We assume that the retailers and warehouses use continuous review \((R, Q)\) policies for their inventory control and a first-come-first-served \((FCFS)\) rule for the delivery mechanism. We also assume that unsatisfied demand at all stocking points is backordered. An exception is that the end customers buying the product at the retailers are assumed to have a maximum tolerable waiting time in getting the product. Demand that cannot be satisfied within this maximum waiting time is assumed to be lost. Note that this assumption becomes relevant only in the event of disruption, and is introduced to capture the potential lost sales as one of the costly consequences of the disruption. In the normal situation (without disruption), this maximum waiting time is not of relevance because the actual waiting time in the case of stock out is, most likely, going to be shorter than the maximum waiting time.

As in Darmawan et al. (2018), we allow multiple sourcing for the retailers, i.e., each retailer can be supplied by more than one warehouse. Several previous studies (e.g., Hendricks and Singhal, 2005; Tomlin, 2006; Ivanov, 2017; Rezapour et al., 2017; Dolgui et al., 2018) have advocated the important use of multiple plants to ensure that there is a backup facility for recovery in the occurrence of a disruption. We follow their advices in our model by imposing the restriction that there should be more than one warehouse and one plant in the network. Unlike the retailers that may get supplies from more than one warehouse even in the period without disruption, we assume that a warehouse sources the product only from a single plant in our network design. However, as
will be further elaborated on later, when a disruption occurs at the supplying plant, the supplied warehouses may get the product from one of the other plants as part of a recovery strategy.

Disruptions can occur at any echelon in the supply chain network. Randomness in a disruption event is represented by its probabilistic frequency and duration. In this paper, we use a Poisson distribution with mean disruption rate $\lambda$ to model the occurrences of disruptions, and use a uniform distribution to model the duration of disruptions $D$. We assume that all the network facilities have the same disruption rate and duration. We define the disruption-related cost as the combination of the cost for setting up a backup facility in the event of a disruption and the cost of lost sales. As stated earlier, a lost sale occurs when a customer’s waiting time exceeds a prespecified threshold. The lost sales-cost may represent the opportunity cost of losing the profit contribution and deteriorating the firm’s reputation in the long term.

The objective of the SCND problem in this paper is to minimise the expected total costs which include the costs for opening and operating facilities, transportation, inventory, and the costs in relation to the disruption while satisfying the target fill rates for the end-customers.

Figure 5.1 The supply chain network design
5.3.2 The solution approach

The inclusion of coordinated inventory control in SCND already increases the complexity of the problem compared to the standard SCND problem without coordination. Thus, introducing the disruptions makes the problem even more complex and harder to solve analytically. Therefore, in this paper, we propose a two-stage approach to solve the problem. In this approach, we first solve the SCND problem in the case of no disruption. In the second stage, using the solution obtained in the first stage, we fine-tune the inventory control parameters through discrete event simulation (DES)-optimisation while introducing the disruptions and a recovery plan in the event of disruptions.

5.3.2.1 The first stage: The SCND with coordinated inventory control but without disruption

In the first stage, we ignore the disruption and model the SCND problem with coordinated inventory control as a mixed integer nonlinear programming problem. The parameters and decision variables are as follows.

<table>
<thead>
<tr>
<th>Index</th>
<th>Parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>$n$</td>
<td>Index for plant</td>
</tr>
<tr>
<td>$i$</td>
<td>Index for warehouse</td>
</tr>
<tr>
<td>$j$</td>
<td>Index for retailer</td>
</tr>
<tr>
<td>$A_j$</td>
<td>Ordering cost at retailer $j$</td>
</tr>
<tr>
<td>$A_i$</td>
<td>Ordering cost at warehouse $i$</td>
</tr>
<tr>
<td>$b_j$</td>
<td>Backorder cost per unit and time unit at retailer $j$</td>
</tr>
<tr>
<td>$b_i$</td>
<td>Backorder cost per unit and time unit at warehouse $i$</td>
</tr>
<tr>
<td>$C_i$</td>
<td>Truck capacity at warehouse $i$</td>
</tr>
<tr>
<td>$C_j$</td>
<td>Truck capacity at retailer $j$</td>
</tr>
<tr>
<td>$Cap_i$</td>
<td>Capacity at warehouse $i$</td>
</tr>
<tr>
<td>$h_j$</td>
<td>Holding cost per unit and time unit at retailer $j$</td>
</tr>
<tr>
<td>$h_i$</td>
<td>Holding cost per unit and time unit at warehouse $i$</td>
</tr>
<tr>
<td>$L_{ni}$</td>
<td>Transportation time from plant $n$ to warehouse $i$</td>
</tr>
<tr>
<td>$L_{ij}$</td>
<td>Transportation time from warehouse $i$ to retailer $j$</td>
</tr>
<tr>
<td>$M_i$</td>
<td>Accrued fixed cost of opening warehouse $i$</td>
</tr>
<tr>
<td>$P_{ni}$</td>
<td>Transportation cost/truck from plant $n$ to warehouse $i$</td>
</tr>
<tr>
<td>$P_{ij}$</td>
<td>Transportation cost/truck from warehouse $i$ to retailer $j$</td>
</tr>
<tr>
<td>$V_i$</td>
<td>Variable cost of operating warehouse $i$</td>
</tr>
</tbody>
</table>
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\( \mu_j^r \) Expected demand at retailer \( j \)
\( \sigma_j^r \) Standard deviation of demand at retailer \( j \)

\( \text{FillRate}_j^{min} \) The target fill rate

\( CS \) Setup cost for a backup facility

\( CL \) Cost of lost sales per unit

Decision variables:

\( Q_j^r \) Replenishment order quantity at retailer \( j \)
\( Q_i^w \) Replenishment order quantity at warehouse \( i \)
\( R_j^r \) Reorder point at retailer \( j \)
\( R_i^w \) Reorder point at warehouse \( i \)
\( X_i \) 1 if warehouse \( i \) is opened; 0 otherwise
\( Y_{ni} \) 1 if warehouse \( i \) is supplied by plant \( n \); 0 otherwise
\( Z_n \) 1 if plant \( n \) is selected; 0 otherwise
\( \mu_{ij}^w \) Expected demand at retailer \( j \) supplied by warehouse \( i \)
\( \mu_{ni}^p \) Expected demand at warehouse \( i \) supplied by plant \( n \)

Other variables:

\( IL_j^r \) Inventory level at retailer \( j \)
\( IL_i^w \) Inventory level at warehouse \( i \)
\( \mu_{D_j(L_j)}^r \) Expected lead-time demand at retailer \( j \)
\( \mu_{D_i(L_i)}^w \) Expected lead-time demand at warehouse \( i \)
\( \mu_i^w \) Expected demand at warehouse \( i \)
\( \sigma_i^w \) Standard deviation of demand at warehouse \( i \)
\( \sigma_{D_j(L_j)}^r \) Standard deviation of lead-time demand at retailer \( j \)
\( \sigma_{D_i(L_i)}^w \) Standard deviation of lead-time demand at warehouse \( i \)

\( \text{FillRate}_j \) Expected fill rate at retailer \( j \)

\( E(\text{Lost}_j^r) \) Expected number of lost sales at retailer \( j \)
\( E(OR_i^w) \) Expected number of orders made by warehouse \( i \)
\( E(OR_j^r) \) Expected number of orders made by retailer \( j \)

\( E(\text{ST}_i) \) Expected number of setups at warehouse \( i \) for backup facility
\( E(\text{ST}_j) \) Expected number of setups at retailer \( j \) for backup facility

\( E(TD_{ni}) \) Expected number of truck-delivery from plant \( n \) to warehouse \( i \)
\( E(TD_{ij}) \) Expected number of truck-delivery from warehouse \( i \) to retailer \( j \)
The optimisation problem can be written as

\[
\begin{align*}
\min_{y_i(z_n, x_i, \mu_i^{w}, \mu_i^{wr}, q_i^{w}, q_j^{r}, R_i^{w}, R_j^{r})} & \quad TC = TRC + IRC \\
= & \sum_{i=1}^{l} \left( X_i M_i + V_i \mu_i^{w} + \sum_{n=1}^{N} E(TD_{ni}) Y_{ni} P_{ni} + \sum_{j=1}^{J} E(TD_{ij}) P_{ij} \right) \\
& + \sum_{i=1}^{l} \left( A_i^{w} E(OR_i^{w}) + h_i^{w} E(IL_i^{w})^{+} \right) + \sum_{j=1}^{J} \left( A_j^{r} E(OR_j^{r}) + h_j^{r} E(IL_j^{r})^{+} \right) \\
= & \sum_{i=1}^{l} \left( X_i M_i + V_i \mu_i^{w} + \sum_{n=1}^{N} \left[ \frac{\mu_{ni}^{w}}{C_i} \right] Y_{ni} P_{ni} + \sum_{j=1}^{J} \left[ \frac{\mu_{ij}^{wr}}{C_j} \right] P_{ij} \right) \\
& + \sum_{i=1}^{l} \left( A_i^{w} \frac{\mu_i^{w}}{Q_i^{w}} + h_i^{w} \left( R_i^{w} + Q_i^{w}/2 - \mu_i^{w}_{D_i(L_i)} \right) \\
& \quad + h_i^{w} \frac{(\sigma_{D_i(L_i)}^{w})^2}{Q_i^{w}} \left[ H \left( \frac{R_i^{w} - \mu_i^{w}_{D_i(L_i)}}{\sigma_{D_i(L_i)}^{w}} \right) - H \left( \frac{R_i^{w} + Q_i^{w} - \mu_i^{w}_{D_i(L_i)}}{\sigma_{D_i(L_i)}^{w}} \right) \right] \right) \\
& + \sum_{j=1}^{J} \left( A_j^{r} \frac{\mu_j^{r}}{Q_j^{r}} \sum_{i=1}^{l} \mu_{ij} + h_j^{r} \left( R_j^{r} + Q_j^{r}/2 - \mu_j^{r}_{D_j(L_j)} \right) \\
& \quad + h_j^{r} \frac{(\sigma_{D_j(L_j)}^{r})^2}{Q_j^{r}} \left[ H \left( \frac{R_j^{r} - \mu_j^{r}_{D_j(L_j)}}{\sigma_{D_j(L_j)}^{r}} \right) - H \left( \frac{R_j^{r} + Q_j^{r} - \mu_j^{r}_{D_j(L_j)}}{\sigma_{D_j(L_j)}^{r}} \right) \right] \right) \right) \\
\end{align*} \tag{5.1}
\]

Subject to:

\[
\begin{align*}
\mu_j &= \sum_{i=1}^{l} \mu_{ij}^{wr} \quad \forall j \tag{5.2} \\
\sum_{n=1}^{N} \mu_{ni}^{w} Y_{ni} &= \mu_i^{w} = \sum_{j=1}^{J} \mu_{ij}^{wr} \quad \forall i \tag{5.3} \\
\sigma_{ij}^{2} &= \frac{\mu_{ij}^{wr}}{\sum_{l=1}^{l} \mu_{il}^{wr}} \sigma_{r}^{2} \quad \forall i, j \tag{5.4} \\
\mu_i^{w} &\leq X_i Cap_i \quad \forall i \tag{5.5} \\
Q_j^{r} &= \frac{2A_j^{r} \mu_j^{r}}{h_j^{r}} ; \quad Q_i^{w} = \frac{2A_i^{w} \mu_i^{w}}{h_i^{w}} \quad \forall i, j \tag{5.6} \\
R_i^{w} &= \mu_{D_i(L_i)}^{w} + G^{-1} \left( \frac{h_i^{w} Q_i^{w}}{h_i^{w} + b_i^{w} \sigma_{D_i(L_i)}^{w}} \right) \sigma_{D_i(L_i)}^{w} \quad \forall i \tag{5.7}
\end{align*}
\]
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\[ R_j^r = \mu^r_{D_j(l_j)} + G^{-1}(\frac{h_j^r}{h_j^w + b_j^w} \sigma^r_{D_j(l_j)}) \sigma^r_{D_j(l_j)} \quad \forall j \]  

(5.8)

\[ 1 - P(IL_j^r \leq 0) \geq \text{FillRate}_j^{\text{min}} \quad \forall j \]

(5.9)

\[ \sum_{l=1}^{I} Y_{ni} \geq Z_n \quad \forall n \]

(5.10)

\[ Z_n \sum_{l=1}^{I} Y_{ni} = \sum_{l=1}^{I} Y_{ni} \quad \forall n \]

(5.11)

\[ \sum_{n=1}^{N} Z_n \geq 2 \]

(5.12)

\[ \sum_{i=1}^{I} X_i \geq 2 \]

(5.13)

\[ Y_{ni}, Z_n, X_i \text{ binary} \quad \forall n, i \]

(5.14)

\[ \mu_{ij}^{wr} \geq 0; \mu_{ni}^{pw} \geq 0; \quad \forall n, i, j \]

(5.15)

The optimisation problem in (5.1) can be decomposed into two inter-related (sub) problems, namely the location-transportation and inventory problems. The expected total cost \( (TC) \) is the sum of the expected total cost for the location-transportation problem \( (TRC) \) and the expected inventory-related cost for the inventory problem \( (IRC) \). The first term in the first large bracket in (5.1) consists of the fixed and variable costs for opening and operating warehouses, transportation costs from plants to warehouses, as well as from warehouses to retailers. We assume that the plants belong to external suppliers, and hence, we exclude the fixed costs for choosing the plants in the above objective function. The second and third large brackets represent the total inventory costs for the warehouses and retailers, respectively, consisting of the ordering and holding costs. Appendix 2 presents a more detailed derivation of the inventory cost structure.

Constraint (5.2) represents the multiple sourcing policy at the retailers, i.e., more than one warehouse can supply the demand at a retailer. Constraint (5.3) represents the expected total demand supplied by each warehouse, which must be supplied by a dedicated plant. The cost structure will ensure that only one plant (the cheapest) will be used to supply a particular warehouse. As in Darmawan et al. (2018), in (5.4) we assume that the variance of demand from retailer \( j \) to warehouse \( i \) is proportional to the expected demand at retailer \( j \) supplied by warehouse \( i \). The expected demand satisfied by each warehouse is limited by the warehouse capacity in (5.5). The order quantities at the stocking points are calculated based on the economic order quantity (EOQ) formula specified in (5.6). Constraints (5.7) and (5.8) specify the reorder point as the sum
of the expected lead-time demand and safety stock. The decisions on order quantity and reorder point at each stocking point should satisfy the target fill rate at each retailer specified in (5.9). The left term in (5.9) represents the ready rate, which is equal to the probability of having positive inventory level. For normally distributed demand, the fill rate is equal to the ready rate (Åxsäter, 2015) (see Appendix 3, Equation (A5.19) for details). Constraints (5.10) and (5.11) will ensure that supplies from a plant are only possible and must be used if the plant is selected. Constraints (5.12) and (5.13) are required to impose the multiple plant and warehouse policy. Constraint (5.14) are the binary decision variables that represent the decisions whether or not a warehouse is supplied by a particular plant, and whether or not a plant and a warehouse are selected. Constraint (5.15) shows the non-negativity requirement for the expected flows between facilities in the network.

Following Darmawan et al. (2018), we use a genetic algorithm based heuristic to explore the solutions of the location-transportation problem and integrate it with the induced-backorder approach for solving the inventory problem with coordination between the retailers and warehouses.

*The genetic algorithm based heuristic*

We use a priority-based Genetic Algorithm for solving the location-transportation problem. A set of priorities represents a possible solution to the location-transportation problem. Figure 5.2 shows an example of a network with two plants, two warehouses, and three retailers. Let $s (s=1, 2, \ldots, S)$ represent a node for a warehouse or a retailer. We generate unique priorities $V(s)$ from 1 to $S$ randomly. The process of allocating demand/capacity in a node starts from the node with the highest priority value by finding the lowest transportation cost to/from this node.

Figure 5.3 shows an illustration of how we obtain a solution through the priority-based encoding. We start from Retailer 1 that has the highest priority value and choose Warehouse 1 that has the lowest transportation cost to Retailer 1. We allocate a flow of 100 units ($= \min(150; 100)$) from Warehouse 1 to Retailer 1. Since it is cheapest to supply Warehouse 1 from Plant 1, we then also allocate 100 units from Plant 1 to Warehouse 1. We continue this procedure based on the priority values until all the demand at the retailers is satisfied. Appendix 4 documents the algorithm in more details.
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Figure 5.2 A network example

Figure 5.3 The priority-based encoding
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*Coordinated inventory control – the induced backorder approach*

For a given network flow explored in the genetic algorithm based heuristic, the next step is to solve the inventory problem at the warehouses and retailers subject to the target fill rate at the retailers. We consider coordinated inventory control in the network where the retailers and warehouses are willing to share information and minimise the expected total costs while ensuring that the target fill rates for the end customers are achieved. Under this coordination, the primary concern should be the fill rates at the retailers.

We use the solution procedure based on the so-called induced backorder cost to determine the reorder points for the warehouses and retailers (Berling and Marklund, 2014, 2017). There are five steps in the solution procedure. In Step 1, we find the near-optimal backorder cost at the warehouses for the given values of the retailers' data and inventory control parameters. Step 2 calculates the lead-time demand at the warehouses. In Step 3, we then calculate the reorder point and expected delay at each warehouse. Finally, Step 4 and Step 5 calculate the lead-time demand and reorder points at the retailers, respectively (see Appendix 3 for details). After solving the coordinated inventory problem, we get the total cost of the network design by adding the total inventory cost to the cost that corresponds to the location-transportation problem.

Table 5.1. Main steps for solving the location-transportation and inventory optimisation problem

<table>
<thead>
<tr>
<th>Step 0: Initialisation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Step 1: Generate a number of chromosomes as possible solutions (network flows) using priority-based encoding.</td>
</tr>
<tr>
<td>Step 2: For each possible solution, determine the inventory control parameters using the induced backorder cost procedure</td>
</tr>
<tr>
<td>Step 3: Calculate the value of the objective function in (5.1) (facilities, transportation and inventory cost)</td>
</tr>
<tr>
<td>Step 4: Selection procedure (combining three ways)</td>
</tr>
<tr>
<td>- Select the best solution for next generation</td>
</tr>
<tr>
<td>- Select some of the other chromosomes based on a probability (calculated based on the fitness value)</td>
</tr>
<tr>
<td>- Generate new chromosomes to avoid premature convergence</td>
</tr>
<tr>
<td>Step 5: Crossover (weight mapping crossover)</td>
</tr>
<tr>
<td>Step 6: Mutation (swap mutation)</td>
</tr>
</tbody>
</table>

If the maximum number of consecutive iterations without improvement is reached, then stop. Otherwise, go to Step 2.
We run the optimisation procedure for solving the SCND without disruption in Visual Basic for Application (VBA) within Microsoft® Excel 2016, with the following computer specifications: Intel(R) Core(TM) i5-5200U CPU, 8.00GB (RAM), Windows10, 64bit. Table 5.1 shows the main steps that integrate the inventory optimisation procedure with the genetic algorithm (see Appendix 4 for details).

### 5.3.2.2 The second stage: Reactive and proactive strategies with disruptions

With the presence of disruptions, the analytical model used in the first stage for estimating the expected total costs and fill rates can no longer be used. Moreover, there are additional costs related to the disruption that need to be included. Naturally, the solutions obtained in the first stage are likely to be far from optimal when implemented in the settings where disruptions may occur. In those solutions, we have not devised any recovery strategy should a disruption occur. Thus, in the second stage of the procedure, we use discrete event simulation (DES) to analyse the impact of disruptions and a recovery strategy on the network design performance, aiming at minimising the expected total costs subject to meeting the target fill rates for the end customers.

We include both reactive and proactive strategies to deal with disruptions. By reactive strategy, we mean a strategy that is executed after the disruption occurs, by for example establishing a modified network flow during the disruption. A proactive strategy, on the other hand, represents a recovery plan that is prepared prior to the occurrence of disruption.

#### Reactive strategy

We consider three different reactive strategies, namely the passive-reactive strategy, the joint-reactive strategy and the localized-reactive strategy. In all three strategies, we assume that a disrupted facility can still receive or place an order but the delivery of the product ordered can only take place when the disruption is over (see e.g., Wilson, 2007; Snyder et al., 2016). In the passive-reactive strategy, as the term implies, there is no reconfiguration in the existing network in the event of disruption. Essentially, this strategy can also be considered as a ‘do-nothing’ strategy, and it relies heavily upon the inventory levels maintained in the network.

We use Figure 5.4 to illustrate the joint-reactive and localized-reactive strategies at the retailers. Under normal conditions (no disruption), Retailer 1 is supplied by Warehouse 1. When the
disruption occurs at Retailer 1, the other non-disrupted retailer(s) that get supplies from Warehouse 1 (e.g. Retailer 3) is listed as candidate(s) for establishing a backup facility to supply customers at Retailer 1. We assume that the disrupted retailer (Retailer 1) can still process demand and place orders to the supplying warehouse but uses Retailer 3 for keeping its inventory. If there is more than one candidate, we choose the retailer with the shortest transportation time from the supplying warehouse. Ties are broken in favour of the retailer with the lower transportation cost. The joint-reactive strategy and the localized-reactive strategy differ in the way of treating the demand from the disrupted retailer (Retailer 1) and placing orders at the new dedicated Retailer. In the joint-reactive strategy, we combine the demand from customers at Retailer 1 and Retailer 3. Then, Retailer 3 uses the new reorder point and order quantity for placing orders to Warehouse 1. The available inventory at Retailer 3 is used to cover the demand from customers at Retailer 1 and Retailer 3 jointly. In contrast, we treat the demand from customers at both retailers independently in the localized-reactive strategy. We only use Retailer 3 as a transit point for the stock that belongs to Retailer 1.

![Diagram](image)

Figure 5.4 Reactive strategy against disruption at retailer: (a) joint-reactive strategy; (b) localized-reactive strategy
Figure 5.5 Reactive strategy against disruption at warehouse: (a) joint-reactive strategy; (b) localized-reactive strategy

These three reactive strategies are also applicable to the warehouses. The assumption and the mechanism in establishing a reconfigured flow in the event of disruption are more or less the same as at the retailers. Figure 5.5a shows an example where Warehouse 1 is used as the backup facility that supplies additional retailers (e.g., Retailer 1, Retailer 3, and Retailer 4) during the disruption at Warehouse 1 (- in the normal situation, Retailers 1, 3 and 4 are supplied by Warehouse 1). We are aware of the fact that a warehouse has a finite capacity. Nevertheless, since disruption is a rare event, we assume that the warehouse used as backup facility can still handle the additional demand by e.g., utilizing some available space or establishing a temporary warehouse in the same area for keeping the additional stock.

Since the plants are assumed to have infinite capacity, one of the other plants can be used as backup facility in the event of disruption at a plant. Therefore, in the event of disruption at a plant, we only consider the passive-reactive strategy. Table 5.2 shows the algorithm for implementing a reactive strategy in our simulation model.
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Table 5.2 The algorithm of the reactive strategy

For \( j = 1 \) to \( J \) \( \leftarrow \) retailer

\[ RT\text{demand} \sim N(\mu, \sigma) \]

If retailer \( j \) \( \rightarrow \) disrupted

Find alternative retailer and update for replacing retailer \( j \)

Update \( R_j^r \) and \( Q_j^r \) \( \leftarrow \) e.g., based on reactive strategy

End if

Calculate RTfill-rate

Update inventory position

If \( IP_j^r \leq R_j^r \) \( \leftarrow \) check inventory position

Order \( Q_j^r \)

For \( i = 1 \) to \( I \) \( \leftarrow \) split order to dedicated warehouses (multiple-sourcing policy)

If retailer \( j \) is supplied by warehouse \( i \)

\[ WH\text{demand}=Q_j^r \times \frac{\mu_i}{\mu_j} \quad \leftarrow \text{e.g., proportional to the demand allocation} \]

If warehouse \( i \) \( \rightarrow \) disrupted

Find alternative warehouse and update for replacing warehouse \( i \)

Update \( R_i^w \) and \( Q_i^w \) \( \leftarrow \) e.g., based on reactive strategy

End if

Update inventory position

If \( IP_i^w \leq R_i^w \)

Order \( Q_i^w \)

For \( n = 1 \) to \( N \) \( \leftarrow \) split order to dedicated plants (multiple-sourcing policy)

If warehouse \( i \) is supplied by plant \( n \)

\[ SP\text{demand}=Q_i^w \times \frac{\mu_i}{\mu_n} \quad \leftarrow \text{e.g., proportional to the demand allocation} \]

If plant \( n \) \( \rightarrow \) disrupted

Find alternative plant and update for replacing plant \( n \)

End if

End if

End for \( n \)

End if

End for \( i \)

End if

End for \( j \)

Calculate total cost (facilities, transportation, inventory, and disruption cost)

Proactive strategy

As part of the proactive strategy, in the first stage, we already use the multiple-sourcing policy for the retailers and impose sourcing from multiple warehouses and plants in the network such that backup supplies can be delivered in the event of disruption. Furthermore, one could also employ
fortification through increased inventory levels at each echelon, which will help maintain the service level in the event of disruptions. We accommodate such fortification by refining the inventory control parameters obtained in the first stage.

More specifically, for a given reactive strategy, we refine the inventory control parameters for the warehouses and retailers \( (Q^w_i, R^w_i, Q^r_j, R^r_j) \). For a given set of inventory control parameters, simulation in Flexsim is used to estimate the expected total costs and fill rates. The optimisation tool OptQuest in Flexsim is then used to find near-optimal inventory control parameters. OptQuest is built on a set of metaheuristics such as evolutionary algorithms, tabu search, and scatter search. It is widely used in the literature for optimisation or benchmarking purposes (see e.g., Kleijnen and Wan, 2007; Becerril-Arreola et al., 2013; Melouk et al., 2013; Kim et al., 2018).

For a given reactive strategy and first stage’s solution, the fortification problem in the second stage can be formulated as follows

\[
\begin{align*}
\min_{Q^w_i, R^w_i, Q^r_j, R^r_j} & \quad TC_2 = TRC + IRC + DISC \\
= & \sum_{i=1}^{I} \left( X_i M_i + V_i \mu^w_i + \sum_{n=1}^{N} E(TD_{ni}) Y_{ni} P_{ni} + \sum_{j=1}^{J} E(TD_{ij}) P_{ij} \right) \\
+ & \sum_{i=1}^{I} \left( A^w_i E(OR^w_i) + h^w_i E(IL^w_i) \right) + \sum_{j=1}^{J} \left( A^r_j E(OR^r_j) + h^r_j E(IL^r_j) \right) \\
+ & CS \left( \sum_{i=1}^{I} E(ST_i) + \sum_{j=1}^{J} E(ST_j) \right) + CL \sum_{j=1}^{J} E(Lost^r_j) \\
\text{Subject to:} & \quad Fillrate_j \geq FillRate^m_j \quad \forall j
\end{align*}
\]

(5.16)

Compared to the objective function in (5.1), the new objective function in (5.16) contains the additional disruption cost \( DISC \), which is the sum of setup costs for backup facilities and lost sales costs. Note that since we use the network design obtained in the first stage, the term \( \sum_{i=1}^{I} X_i M_i \) is fixed. All the other measures including the expected values of demand at warehouses, number of truck-delivery, number of orders, inventory levels, attained fill rates as well as number of setups
for backup facilities and lost sales, need to be estimated in the simulation. The decisions should satisfy the target fill-rate constraint in (5.17). In the simulation model, we ensure that the flow balance and warehouse capacity constraints are respected.

Simulation-optimisation is a very expensive method for problems with large solution space, and we may end up with poor results if starting from bad initial solutions. Therefore, in the second stage (when considering disruption), we reduce the solution space by setting lower and upper bounds for the reorder-point and order quantity values. We argue that a network becomes more resilient against disruption when there is a higher inventory level maintained. Hence, we use the reorder point and order quantity values obtained in the first stage as the lower bounds. To reduce the solution space, we set the upper bound equal to $c (>1)$ times the lower bound, where $c$ is an arbitrary number deemed sufficient to find good solutions within a reasonable computation time.

### 5.4 Numerical study

In this section, we present the results of our numerical study that consists of two parts. In the first part, we use the simulation model for two purposes. Firstly, we validate the analytical model used in the first stage for the baseline problem with no disruption by comparing the target fill rates used in determining the optimal inventory control parameters to those achieved in the simulation model. Secondly, we carry out a preliminary evaluation of the three reactive strategies to select the best strategy that will be used in the next analysis showing how the use of the proactive and reactive strategies can make the supply chain more resilient in the presence of disruptions. As the simulation optimisation for fine-tuning the inventory control parameters is time-consuming, we only select the best performing reactive strategy to be combined with the proactive strategy. The second part of this section presents the performance evaluation of the proposed approach when combining the proactive and reactive strategies.

We consider several problem instances differentiated by the number of plants (SP), the number of warehouses (WH), and the number of retailers (RT). Table 5.3 shows the parameter values used in our numerical study. A disruption is modelled as a dynamic event that is assumed to follow a Poisson process, and its duration follows a uniform distribution (see e.g., Snyder et al., 2016; Schmitt et al., 2017).
Table 5.3 Parameters in the numerical study

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Distribution Range</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ordering cost ((A))</td>
<td>U (150, 190)</td>
</tr>
<tr>
<td>Holding cost ((h)) per unit and day</td>
<td>Retailer: U(1, 1.25)</td>
</tr>
<tr>
<td></td>
<td>Warehouse: U(0.5, 0.6)</td>
</tr>
<tr>
<td>RT’s avg. daily demand ((\mu))</td>
<td>U (100, 160)</td>
</tr>
<tr>
<td>Std. dev. of demand ((\sigma))</td>
<td>(\sqrt{\frac{\mu^T_j}{4}})</td>
</tr>
<tr>
<td>Lead time ((L))</td>
<td>U(1, 3) days</td>
</tr>
<tr>
<td>Warehouse capacity ((Cap))</td>
<td>(4 \cdot \frac{\sum_{j=1}^{I} \mu^T_j}{I})</td>
</tr>
<tr>
<td>Accrued fixed cost ((M)) per day</td>
<td>U(975, 1212)</td>
</tr>
<tr>
<td>Variable cost ((V)) per unit</td>
<td>0.3</td>
</tr>
<tr>
<td>Transportation cost ((P))</td>
<td>U(250, 690)</td>
</tr>
<tr>
<td>Truck capacity ((C))</td>
<td>50 units</td>
</tr>
<tr>
<td>Disruption rate ((\lambda)) per day</td>
<td>0.005</td>
</tr>
<tr>
<td>The duration of disruption ((D))</td>
<td>U(28, 35) days</td>
</tr>
<tr>
<td>Disruption cost</td>
<td></td>
</tr>
<tr>
<td>- Setup temporary backup facility ((CS))</td>
<td>1000h</td>
</tr>
<tr>
<td>- Lost sales ((CL))</td>
<td>(\frac{365h}{0.35})</td>
</tr>
</tbody>
</table>

5.4.1 Model validation and the performance of recovery policy

For the purposes of model validation in the case without disruption and performance evaluation of the reactive strategies in the case with disruption, we consider two values for the target fill rate for the end customers, namely 90% and 95%. Table 5.4 shows the average fill rates for all the problem instances from the simulation. Since it is expected (and will be confirmed later) that the solution in the first stage is not able to satisfy the target fill rates in the presence of disruptions, even when a reactive strategy is also implemented, the performance evaluation of the reactive strategies will be made based on the attained expected fill rate.

In the case without disruption, the model presented in Section 5.3.2.1 performs reasonably well, as indicated by the relatively small gaps between the target fill rates and the fill rates attained in the simulation. The table also shows that, in the presence of disruption, the passive-reactive strategy eventually yields low fill rates that fall far below the target. This reflects the risk of
neglecting the possible occurrence of disruptions, supporting the need for a reactive strategy. We use the problem case with 2 plants, 3 warehouses and 10 retailers (2SP3WH10RT) and the target fill rate 90% for an illustration. When the disruptions occur only at the plants, the average fill rates for the passive and reactive strategies are 67.53% and 82.87 %, respectively. The impact of implementing the reactive strategy at the plants appears to be significant, reflected in a much higher average fill rate. Nevertheless, the average fill rate achieved is still below the target, which suggests the need for considering a proactive strategy as well. When the disruptions occur at the warehouses, our numerical study show that the localized-reactive strategy consistently outperforms the joint-reactive strategy.

Table 5.4 The average end customer fill rates under different strategies

<table>
<thead>
<tr>
<th>Problem case</th>
<th>Disrupted echelon</th>
<th>Target fill rate = 90%</th>
<th>Target fill rate = 95%</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>NoDis</td>
<td>PA</td>
<td>RE</td>
</tr>
<tr>
<td>2SP3WH10RT</td>
<td>89.97</td>
<td>67.53</td>
<td>82.87</td>
</tr>
<tr>
<td>Plant</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Warehouse</td>
<td>75.50</td>
<td>85.03</td>
<td>31.80</td>
</tr>
<tr>
<td>Retailer</td>
<td>75.20</td>
<td>87.13</td>
<td>87.63</td>
</tr>
<tr>
<td>3SP5WH15RT</td>
<td>91.20</td>
<td>71.58</td>
<td>80.79</td>
</tr>
<tr>
<td>Plant</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Warehouse</td>
<td>77.04</td>
<td>87.49</td>
<td>28.60</td>
</tr>
<tr>
<td>Retailer</td>
<td>76.00</td>
<td>89.60</td>
<td>90.93</td>
</tr>
<tr>
<td>4SP8WH20RT</td>
<td>91.72</td>
<td>73.88</td>
<td>77.90</td>
</tr>
<tr>
<td>Plant</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Warehouse</td>
<td>77.28</td>
<td>78.78</td>
<td>37.20</td>
</tr>
<tr>
<td>Retailer</td>
<td>76.32</td>
<td>90.22</td>
<td>89.38</td>
</tr>
<tr>
<td>5SP10WH30RT</td>
<td>91.97</td>
<td>72.24</td>
<td>77.68</td>
</tr>
<tr>
<td>Plant</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Warehouse</td>
<td>77.16</td>
<td>82.82</td>
<td>59.73</td>
</tr>
<tr>
<td>Retailer</td>
<td>76.03</td>
<td>90.77</td>
<td>90.81</td>
</tr>
</tbody>
</table>

NoDis= no disruption; PA= passive; RE= reactive; LR= localized-reactive; JR= joint-reactive

Simulation time= 7300 days; warmup time = 365 days; Replications= 4

The impact of disruption on the fill rate depends on the inventory level when disruption hits the network. As a backup facility, the warehouse used as the backup facility has to cover additional demand diverted from the disrupted warehouse. More retailers compete for the available stock at this warehouse. Consequently, the disruption at a warehouse not only influences the fill rate of the
retailers supplied by the disrupted warehouse but also the fill rate of the retailers supplied by the non-disrupted warehouse used as the backup facility. The impact of disruption spreads widely in the joint-reactive strategy. On the other hand, the localized-reactive strategy ensures that the effect of disruption is localized such that it will not spread to the other facilities.

Unlike at the warehouses, the two reactive strategies applied at the retailers do not seem to be significantly different, as reflected in nearly the same average fill rates. When the disruption occurs at a retailer, the non-disrupted retailer used as the backup facility supports fewer retailers than those supplied by the backup warehouse when the disruption occurs at one of the warehouses. In addition, the implementation of coordinated inventory control reduces the inventory level significantly, particularly at the warehouse (see e.g., Darmawan et al., 2018b). Hence, we see a greater impact when implementing the joint-reactive strategy at the warehouse.

Figure 5.6 shows the fluctuation of expected fill rates (over all the retailers) in specific time periods (weeks 475-510) for the problem instance with 2 plants, 3 warehouses, and 10 retailers (2SP3WH10RT), for the different strategies and disrupted echelons. In Figure 5.7a, we can see that under the passive strategy, the fill rate can fall to 0% in weeks 498-502. This means that none of the retailers have any stock during those time periods. Figure 5.7b shows the worse performance of the joint-reactive strategy when the disruptions occur at the warehouses. Whereas in Figure 5.7c, we see the insignificant differences between the localised-reactive strategy and the joint-reactive strategy in the case where the disruptions occur at the retailers. In general, as shown in Table 5.4 and Figure 5.6, the decisions from the first stage are not good enough to deal with the disruptions. In most of the cases, one cannot reach the target fill rate even though the reactive strategy is applied. In the next section, we discuss how the implementation of both reactive and proactive strategies help protect the network against disruptions.
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Figure 5.6 The dynamics of average fill rate under different strategies and disrupted echelons
5.4.2 The results of the proposed approach

In this section, we evaluate the proposed approach for combining the proactive and reactive strategies while considering possible disruptions at all echelons (plants, warehouses, and retailers). We use the localized-reactive strategy for the recovery policy since it provides better performances in relation to the obtained fill rate, as discussed in Section 5.4.1. In the second stage of the proposed approach, we perform the simulation optimisation by utilizing OptQuest in Flexsim 7 with a predefined stopping criterion (i.e. stopping time).

Figure 5.7 shows the comparison of the total costs for different stopping criteria in the case of 2 plants, 3 warehouses, and 10 retailers (2SP3WH10RT). The retailers who fail to fulfil the backordered units within a pre-specified time length (the customer’s maximum waiting time) will suffer from lost sales. We also compare how the approach works with and without the lower bounds in the second stage. According to the results in Figure 5.7, using the lower bound constraints for the decision variables gives lower total costs. As a note, we limit the simulation running time to 24 hours for the two approaches. With the large solution space and limited computation time, utilising the solutions obtained in the first stage as the lower bounds appears to be useful. Further, running the simulation for two hours seems to be sufficient in finding reasonably good solutions (after which no real improvements are obtained). Hence, we use the 2 hour running time as the stopping criterion for the other problem instances.

![Figure 5.7 The stopping criterion](image)

Table 5.5 presents the results of the proposed approach for all the problem instances tested. In the presence of disruption, the solution of the baseline approach (the first stage’s solution) combined
with the reactive strategy cannot reach the target fill rate. In contrast, the proposed approach is capable of anticipating the impact of disruption by adjusting the inventory control parameters such that the target fill rate can eventually be attained. In the case 3SP5WH15RT, for example, with a two-week customer’s maximum waiting time and 90% target fill rate, the cost saving of the proposed approach is 27.76%, which is quite significant. For comparison, in the special case with no lost sales, i.e., the maximum waiting time is larger than the duration of the disruption, the proposed approach is more costly than the baseline approach (with a cost saving of -15.72%, i.e., a cost increase). However, although the proposed approach leads to a higher cost in this special case, the expected fill rate is still higher than that achieved by the baseline approach. As keeping the market share through satisfactory service provisions is important for many firms, costs should not be the primary consideration, and neglecting the potential lost sales may not represent a recommendable approach.

Table 5.5 The average fill rates and cost savings (Baseline approach vs Proposed approach)

<table>
<thead>
<tr>
<th>Problem case</th>
<th>Customer’s maximum waiting time without lost sales</th>
<th>Target fill rate = 90%</th>
<th></th>
<th>Target fill rate = 95%</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Baseline approach</td>
<td>Proposed approach</td>
<td>Cost saving (%)</td>
<td>Baseline approach</td>
</tr>
<tr>
<td>2SP3WH10RT</td>
<td>2 weeks</td>
<td>75.60</td>
<td>92.30</td>
<td>31.40</td>
<td>78.10</td>
</tr>
<tr>
<td></td>
<td>3 weeks</td>
<td>75.30</td>
<td>92.70</td>
<td>8.88</td>
<td>78.10</td>
</tr>
<tr>
<td></td>
<td>4 weeks</td>
<td>75.30</td>
<td>90.40</td>
<td>0.76</td>
<td>77.60</td>
</tr>
<tr>
<td></td>
<td>No lost sales</td>
<td>75.90</td>
<td>90.10</td>
<td>-11.22</td>
<td>78.10</td>
</tr>
<tr>
<td>3SP5WH15RT</td>
<td>2 weeks</td>
<td>75.00</td>
<td>92.33</td>
<td>27.76</td>
<td>77.67</td>
</tr>
<tr>
<td></td>
<td>3 weeks</td>
<td>75.00</td>
<td>90.53</td>
<td>7.34</td>
<td>77.20</td>
</tr>
<tr>
<td></td>
<td>4 weeks</td>
<td>75.13</td>
<td>90.40</td>
<td>0.54</td>
<td>77.60</td>
</tr>
<tr>
<td></td>
<td>No lost sales</td>
<td>74.53</td>
<td>92.13</td>
<td>-15.72</td>
<td>77.07</td>
</tr>
<tr>
<td>4SP8WH20RT</td>
<td>2 weeks</td>
<td>69.60</td>
<td>92.35</td>
<td>17.75</td>
<td>71.55</td>
</tr>
<tr>
<td></td>
<td>3 weeks</td>
<td>69.55</td>
<td>91.75</td>
<td>4.99</td>
<td>71.15</td>
</tr>
<tr>
<td></td>
<td>4 weeks</td>
<td>70.00</td>
<td>90.45</td>
<td>0.72</td>
<td>71.30</td>
</tr>
<tr>
<td></td>
<td>No lost sales</td>
<td>69.65</td>
<td>90.95</td>
<td>-12.18</td>
<td>71.30</td>
</tr>
<tr>
<td>5SP10WH30RT</td>
<td>2 weeks</td>
<td>72.50</td>
<td>93.60</td>
<td>13.55</td>
<td>75.10</td>
</tr>
<tr>
<td></td>
<td>3 weeks</td>
<td>72.70</td>
<td>91.47</td>
<td>5.13</td>
<td>75.30</td>
</tr>
<tr>
<td></td>
<td>4 weeks</td>
<td>72.50</td>
<td>90.67</td>
<td>0.05</td>
<td>75.20</td>
</tr>
<tr>
<td></td>
<td>No lost sales</td>
<td>72.67</td>
<td>90.70</td>
<td>-11.23</td>
<td>75.30</td>
</tr>
</tbody>
</table>
5.4.3 The effect of disruption rate

We also examine the effect of the rate and duration of disruptions on the performance of the supply chain. In addition to the case described in Table 5.5 with $\lambda=0.005$ and $D = U(4,5)$, we consider an additional case with a higher disruption rate but shorter duration ($\lambda=0.01$ and $D = U(2,3)$). Figure 5.8 depicts how the total costs are affected by the customers’ maximum waiting time differentiated by the disruption rate for two specific network configurations (3SP5WH15RT and 5SP10WH30RT). The target fill rate 90% is used in the figure.

The figure shows that the total cost decreases as the maximum tolerable waiting time increases and is relatively constant when the maximum customer waiting time is higher than the duration of disruption. When the maximum waiting time is 2 or 3 weeks, the total cost for the case with the lower disruption rate and longer duration is higher than the cost for the case with the higher disruption rate and shorter duration. This can be attributed to the cost caused by lost sales, and this cost is higher than the setup cost for backup facilities. In contrast, when the maximum waiting time is higher than 3 weeks, we observe the opposite as the case with the higher disruption rate and shorter duration incurs a higher total cost. In the latter case, the higher frequency of disruptions directly increases the setup cost so that it dominates the lost sales cost. This study shows that the characteristics of a disruption, for example represented by its frequency and duration, as well as the maximum waiting time of the customers are important for the supply chain performance. The proposed approach is able to reach the target fill rate 90%.

Figure 5.8 The effect of disruption rate and duration
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5.5 Conclusions

This paper presents models to study the supply chain network design problem where coordinated inventory control is applied and disruptions may occur in the network. The objective in our network design problem is to minimise the expected total cost that is the sum of the costs for opening and operating warehouses, the transportation costs, the inventory-related costs, and the disruption-related costs while meeting the requirement that a target fill rate must be met at all the retailers. The disruption-related costs are incurred for setting up the backup facility in the event of disruption and for taking account of the lost sales. The solution to the problem with possible disruptions requires the consideration of a recovery plan to minimise the negative consequences of the disruption, and this characterises the main difference compared to the solution for the standard problem that neglects the possible occurrence of disruptions.

We propose a two-stage approach to solve the problem. In the first stage, we solve the supply chain network design problem ignoring disruptions. We develop a heuristic based on a Genetic Algorithm for exploring the network design solutions with respect to the choice of facilities and capacity allocation, combined with the induced backorder approach for enforcing coordination between the warehouses and retailers in determining the optimal inventory control parameters. In the second stage, we use discrete event simulation to facilitate the evaluation of recovery plans in anticipation of disruptions. We use the simulation model for evaluation of the reactive strategies and for fine-tuning the inventory control parameters obtained in the first stage as part of the proactive strategy.

Our numerical study reveals some interesting findings. Firstly, this study concurs with previous studies on the importance of devising a recovery plan in the presence of disruptions. Although its probability is small, a disruption may lead to the situation where the fill rate falls to a very low level if no reactive and proactive strategies are in place. This finding is quite useful, especially when one considers implementing coordinated inventory control in the supply chain network that usually reduces the inventory level at the upper echelons. This study emphasises the importance of using both reactive and proactive strategies to make the supply chain more resilient in the event of disruption.
Secondly, from our evaluation on three different reactive strategies, the localised reactive strategy appears to be the best. It is worth noting that, in some cases where disruptions occur at the warehouses, the joint reactive strategy can even perform worse than the passive strategy. This shows that one should choose the reactive strategy carefully, and that the echelon where disruptions occur is very important. Our results also show that, although it helps to avoid a low fill rate, the reactive strategy alone is not sufficient for satisfying the target fill rate.

The proposed approach of combining the reactive and proactive strategies is able to achieve the target fill rate. The proposed approach can generate substantial cost savings compared to the approach using only the reactive strategy, especially when the customers’ maximum waiting time is less than the duration of the disruption. Our numerical experiment demonstrates the significant effects of disruption rate and duration, as well as the customers’ maximum waiting time. We are aware, however, of the fact that finding good estimates for those parameters, remain one of the major challenges when designing a good recovery strategy.

There are some limitations in this paper and we suggest several directions for future work. First, the number of examples used in the numerical study is rather limited. Testing more numerical examples is required to enhance the generality of the results presented in this paper. For example, increasing the range of disruption rates would allow us to conduct a more thorough assessment of when a recovery plan is critical or less critical. Second, in this paper, we assume that when a disruption occurs, we can immediately implement the reactive strategy. In practice, there are situations where it takes some time to set up a backup facility or to reconfigure the flow in the supply chain network. It would be interesting to examine the effect that this non-instantaneous setup or reconfiguration time has on, for example, the increase of inventory levels required in the proactive strategy. Finally, even though the solution method proposed in this paper is useful for the evaluation of proactive and reactive strategies, the computation time is still too long if one considers developing a decision support model with the capability of evaluating more proactive and reactive strategies in a larger supply chain network. Therefore, developing approximation methods to replace the simulation is also an interesting topic for future research.
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Appendix

Appendix 1: The normally distributed demand

Let $\varphi(x)$ and $\phi(x)$ represent the density and distribution function of demand. The normal loss function $G(x)$ is obtained as

$$G(x) = \int_{x}^{\infty} (v - x) \varphi(v) dv = \varphi(x) - x(1 - \phi(x)) \quad (A5.1)$$

where

$$\varphi(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}} \quad \phi(x) = \int_{-\infty}^{x} \varphi(v) dv = \int_{-\infty}^{x} \frac{1}{\sqrt{2\pi}} e^{-\frac{v^2}{2}} dv \quad (A5.2)$$

Moreover,

$$G'(x) = \phi(x) - 1; \quad G(\infty) = 0 \quad (A5.3)$$

$$H(x) = \int_{x}^{\infty} G(v) dv = \frac{1}{2} [(x^2 + 1)(1 - \phi(x)) - x\varphi(x)]; \quad H'(x) = -G(x) \quad (A5.4)$$

Appendix 2: The inventory cost

We break down warehouse and retailer inventory cost which consists of ordering cost and holding cost subject to the target fill rate as follows.

1. Warehouse $i$

$$\min_{Q_i, R_i} C_{WH} = \tilde{C}_i(R_i) = \text{ordering cost + holding cost}$$

$$= A_i^w \frac{H_i^w}{Q_i^w} + h_i^w E(I_{L_i}^w)^+ = A_i^w \frac{\mu_i^w}{Q_i^w} + h_i^w E(I_{L_i}^w)^-$$

$$= A_i^w \frac{H_i^w}{Q_i^w} + h_i^w \left( R_i^w + Q_i^w / 2 - \mu_{D_i(L_i)} \right) + h_i^w E(I_{L_i}^w)^-$$

$$= A_i^w \frac{H_i^w}{Q_i^w} + h_i^w \left( R_i^w + Q_i^w / 2 - \mu_{D_i(L_i)} \right) + h_i^w \int_{-\infty}^{0} F(x) dx$$

$$= A_i^w \frac{H_i^w}{Q_i^w} + h_i^w \left( R_i^w + Q_i^w / 2 - \mu_{D_i(L_i)} \right)$$

$$+ h_i^w \int_{-\infty}^{0} \frac{1}{Q_i^w} \int_{R_i^w}^{R_i^w + Q_i^w} \left[ -G' \left( \frac{u - x - \mu_{D_i(L_i)}}{\sigma_{D_i(L_i)}} \right) \right] du$$

$$= A_i^w \frac{H_i^w}{Q_i^w} + h_i^w \left( R_i^w + Q_i^w / 2 - \mu_{D_i(L_i)} \right) + h_i^w \frac{\sigma_{D_i(L_i)}}{Q_i} \int_{R_i^w}^{R_i^w + Q_i^w} G \left( \frac{u - \mu_{D_i(L_i)}}{\sigma_{D_i(L_i)}} \right) du \quad (A5.5)$$
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\[ = A_i^w \mu_i^w + h_i^w \left( R_i^w + \frac{Q_i^w}{2} - \mu_i^w (L_i) \right) + h_i^w \left( \frac{(\sigma_{D_i(L_i)})^2}{Q_i^w} \right) \]

\[ H \left( \frac{R_i^w + Q_i^w - \mu_i^w (L_i)}{\sigma_{D_i(L_i)}} \right) \]

Subject to

\[ \text{FillRate}_i = 1 - P(1L_i^w \leq 0) \geq \text{FillRate}_i^{\text{min}} \quad (A5.6) \]

2. Retailer \( j \)

\[ \min_{Q_j^r, R_j^r} C_R = \tilde{C}_j(R_j) = \text{ordering cost + holding cost} \]

\[ = A_j^r \frac{\mu_j^r}{Q_j^r} + h_j^r \left( R_j^r + \frac{Q_j^r}{2} - \mu_j^r (L_j) \right) \]

\[ + h_j^r \left( \frac{(\sigma_{D_j(L_j)})^2}{Q_j^r} \right) \]

\[ H \left( \frac{R_j^r - \mu_j^r (L_j)}{\sigma_{D_j(L_j)}} \right) - H \left( \frac{R_j^r + Q_j^r - \mu_j^r (L_j)}{\sigma_{D_j(L_j)}} \right) \quad (A5.7) \]

Subject to

\[ \text{FillRate}_j = 1 - P(1L_j^r \leq 0) \geq \text{FillRate}_j^{\text{min}} \quad (A5.8) \]

**Appendix 3: The coordinated inventory control**

In the centralised inventory, we assume that retailer and warehouse can share information. The near-optimal backorder cost at warehouse synchronise the decisions of reorder point at warehouse and retailer (Axsäter, 2003; Berling and Marklund, 2006, 2014). There are five steps as follows (see also Figure A5.1).
1. Induced backorder cost at warehouse

\[ b_i^w = \frac{\sum_{j=1}^{I} \mu_{ij} b_j^{r'}}{\sum_{j=1}^{I} \mu_{ij}} \]  

Where

\[ b_j^{r'} = h_j^r g\left(\frac{100Q_j^r}{\mu_i h_j^r}, \frac{b_j^r}{h_j^r}\right) \]

\[ g\left(\frac{100Q_j^r}{\mu_i L_{ij}}, \frac{b_j^r}{h_j^r}\right) = \min\left\{ g_a, \left(\frac{100Q_j^r}{\mu_i L_{ij}}\right) g_b \right\} \]

\[ g_a = \min(0.015 \frac{b_j^r}{h_j^r}, \max(0.65/ \sqrt{b_j^r / h_j^r}, 0.05)) \]

\[ g_b = \max(-1.2, -2(b_j^r / h_j^r)^{-0.25}) \]
2. Lead time demand at warehouse

\[ \begin{align*}
\mu_{D_i(L_i)}^w &= \sum_{j=1}^{J} \mu_{ij} L_i \\
\sigma_{D_i(L_i)}^w &= \sqrt{\sum_{j=1}^{J} \sum_{n=0}^{\infty} \left( \frac{\mu_{D_i(L_i)}^w - nQ_j^r}{Q_j^r} \right)^2 f_j(nQ_j^r)} \\
f_j(nQ_j^r) &= \gamma_j(n) - \gamma_j(n-1)
\end{align*} \]  

(A5.10)

(A5.11)

(A5.12)

The probability of retailer j orders at most n times during \( L_i \) is

\[ \gamma_j(n) = \frac{1}{Q_j^r} Y_{ij} \left( Q_j^r \phi \left( \frac{(n+1)Q_j^r - \mu_{ij} L_i}{\sqrt{\sigma_{ij}^2 L_i}} \right) + \sqrt{\sigma_{ij}^2 L_i} \left( \phi \left( \frac{(n+1)Q_j^r - \mu_{ij} L_i}{\sqrt{\sigma_{ij}^2 L_i}} \right) - \phi \left( \frac{nQ_j^r - \mu_{ij} L_i}{\sqrt{\sigma_{ij}^2 L_i}} \right) \right) + (nQ_j^r - \mu_{ij} L_i) \left( \phi \left( \frac{(n+1)Q_j^r - \mu_{ij} L_i}{\sqrt{\sigma_{ij}^2 L_i}} \right) - \phi \left( \frac{nQ_j^r - \mu_{ij} L_i}{\sqrt{\sigma_{ij}^2 L_i}} \right) \right) \right) \]  

(A5.13)

3. Reorder point and the expected delay at warehouse

\[ \begin{align*}
R_i^w &= \text{argmin}\{ \tilde{C}_i(R_i^w) \} \\
\tilde{C}_i(R_i^w) &= A_i^w \frac{\mu_{D_i(L_i)}^w}{Q_i^w} + h_i^w \left( R_i^w + Q_i^w/2 - \mu_{D_i(L_i)}^w \right) + (h_i^w + b_i^w) \frac{(\sigma_{D_i(L_i)}^w)^2}{Q_i^w} \left[ H \left( \frac{R_i^w - \mu_{D_i(L_i)}^w}{\sigma_{D_i(L_i)}^w} \right) - H \left( \frac{R_i^w + Q_i^w/2 - \mu_{D_i(L_i)}^w}{\sigma_{D_i(L_i)}^w} \right) \right] \\
\frac{\partial \tilde{C}_i(R_i^w)}{\partial R_i^w} &= h_i^w + (h_i^w + b_i^w) \frac{\sigma_{D_i(L_i)}^w}{Q_i^w} \left[ G \left( \frac{R_i^w + Q_i^w/2 - \mu_{D_i(L_i)}^w}{\sigma_{D_i(L_i)}^w} \right) - G \left( \frac{R_i^w - \mu_{D_i(L_i)}^w}{\sigma_{D_i(L_i)}^w} \right) \right] = 0 \\
G \left( \frac{R_i^w - \mu_{D_i(L_i)}^w}{\sigma_{D_i(L_i)}^w} \right) - G \left( \frac{R_i^w + Q_i^w/2 - \mu_{D_i(L_i)}^w}{\sigma_{D_i(L_i)}^w} \right) &= \frac{h_i^w}{h_i^w + b_i^w} \frac{Q_i^w}{\sigma_{D_i(L_i)}^w} 
\end{align*} \]  

(A5.14)
\[ G \left( \frac{R_i^w + Q_i^w - \mu_{D_i(L_i)}}{\sigma_{D_i(L_i)}} \right) = 0 \text{ for big } Q_i^w \]
\[ G \left( \frac{R_i^w - \mu_{D_i(L_i)}}{\sigma_{D_i(L_i)}} \right) = \frac{h_i^w}{h_i^w + b_i^w \sigma_{D_i(L_i)}} Q_i^w \]

The expected delay at warehouse is given by
\[ E(\text{Delay}) = \frac{E(IL_i^w)^2}{\mu_i^w} = \frac{\sigma_{D_i(L_i)}^2}{\mu_i^w} \left[ H \left( \frac{R_i^w - \mu_{D_i(L_i)}}{\sigma_{D_i(L_i)}} \right) - H \left( \frac{R_i^w + Q_i^w - \mu_{D_i(L_i)}}{\sigma_{D_i(L_i)}} \right) \right] \]  
(A5.15)

4. Lead time demand at retailer

Adjusted by considering expected delay at warehouse and undershoot that might happen.

\[ L_i' = L_i + E(\text{Delay}) \]  
(A5.16)
\[ \mu_{D_j(L_j)} = \sum_{i=1}^{l} \mu_{ij} L_i' + \mu_{uj} \]  
(A5.17)
\[ \sigma_{D_j(L_j)} = \sqrt{\sum_{i=1}^{l} \sigma_{ij}^2 L_i' + \sigma_{uj}^2} \]  
(A5.18)

Where the mean and variance of undershoot are (Tempelmeier, 2006)

\[ \mu_{uj} = \frac{\mu_j + \left(\sigma_j\right)^2 / \mu_j}{2} \]
\[ \sigma_{uj}^2 = \frac{1}{2} \left( \sigma_j \right)^2 \left( 1 - \frac{1}{2} \left( \sigma_j \right)^2 / \left( \mu_j \right)^2 \right) + \frac{\left( \mu_j \right)^2}{12} \]

5. Reorder point at retailer

\[ R_j^r = \min \{ R_j^r : \text{FillRate}_j = 1 - P(\text{IL}_j^r \leq 0) \geq \text{FillRate}^{min}_j \} \]
\[ P(\text{IL}_j^r \leq 0) = \frac{\sigma_{D_j(L_j)}^r}{Q_j^r} \left[ G \left( \frac{R_j^r - \mu_{D_j(L_j)}^r}{\sigma_{D_j(L_j)}^r} \right) - G \left( \frac{R_j^r + Q_j^r - \mu_{D_j(L_j)}^r}{\sigma_{D_j(L_j)}^r} \right) \right] \]  
(A5.19)
Appendix 4: Genetic Algorithm

We define \( Z \in \mathbb{Z} \) as a possible network that corresponds to a set of priority values, i.e., \( Z \sim (v(1), v(2) \ldots v(I + J)) \) and \( Z \) is the set of all possible networks. The parameters and variables for GA in this study are as follows:

- \( N \) Population size
- \( G \) Index for generation \((g = 1, \ldots, G)\)
- \( CrossRate \) Crossover rate
- \( MutRate \) Mutation rate
- \( FV \) The fitness value
- \( Z_{best} \) The best network structure so far
- \( TC_2(Z) \) Objective function value when using network structure \( Z \)
- \( TC_{2\,\text{best}} \) The best objective function so far
- \( TC_{2\,\text{highest}} \) The highest objective function in each generation

The necessary steps of our GA are the following.

1. **Step 1**: Choose an initial generation that consists of \( N \) chromosomes: \( Z^{(1)}, Z^{(2)}, \ldots, Z^{(N)} \). Set \( g = 0 \).
2. **Step 2**: For each chromosome, calculate objective function \( TC_2(Z^{(n)}) \), \((n = 1, 2, \ldots, N)\). (Call Procedure 1)
3. **Step 3**: Find the highest and the best objective function value
   - \( TC_{2\,\text{highest}} = \max \{ TC_2(Z^{(n)}) \} \)
   - \( TC_{2\,\text{best}} = \min \{ TC_2(Z^{(n)}) \} \)
   - If \( TC_{2\,\text{best}} < TC_{2\,\text{best}} \) then \( TC_{2\,\text{best}} = TC_{2\,\text{best}} ; Z_{best} = Z^{(n)} \)
4. **Step 4**: If \( g = G \) then select \( Z_{best} \) and \( TC_{2\,\text{best}} \); otherwise go to Step 5.
5. **Step 5**: Calculate the fitness value for each chromosome \( FV^{(n)} = TC_{2\,\text{highest}} - TC_2(Z^{(n)}) \)
6. **Step 7**: Selection procedure (combining three ways of picking chromosomes).
   - **7.1** Select the best solution for next generation, \( Z^{(1)} = Z_{best} \)
   - **7.2** For each of the existing old chromosomes, calculate the probability \( Prob^{(n)} = \frac{FV^{(n)}}{\sum_{n=1}^{N} FV^{(n)}} \)
     - Generate a random variate \( r_1 \leftarrow U(0, 1) \) and pick the chromosome that corresponds to the c.d.f. of the fitness value. Repeat this procedure until we pick \( n_1 \) chromosomes.
   - **7.3** Generate \( N - 1 - n_1 \) new chromosomes (to avoid premature convergence)
7. **Step 8**: Crossover (a cut-point of crossover)
   - Form \( N/2 \) pair of chromosomes. For each pair of chromosomes, e.g.
     - \( Z^{(1)} \sim (v(1)^{(1)} \ldots v(I + J)^{(1)}) \)
     - \( Z^{(2)} \sim (v(1)^{(2)} \ldots v(I + J)^{(2)}) \)
   - Generate \( r_2 \leftarrow U(0,1); \) if \( r_2 < CrossRate \) then undergo the following crossover; otherwise no crossover
     - Set \( v(s)_{\text{new}}^{(1)} = v(s)^{(1)} \) and \( v(s)_{\text{new}}^{(2)} = v(s)^{(2)} \) \((s = 1, \ldots, I + J)\)
     - Generate the border of cross-over \( x_1 \leftarrow U(1, I + J) \)
     - Set \( v(s)_{\text{new}}^{(1)} = v(s)^{(2)} \) and \( v(s)_{\text{new}}^{(2)} = v(s)^{(1)} \) \((s = x_I + 1, \ldots, I + J)\)
   - Weight mapping crossover (avoiding the same priority after crossover)
Chapter 5: Paper 4

Sort_{s}(. ) \leftarrow \text{sorting } v(s)^{(n)} \text{, the right segment of the border } (n=1, 2; s=x_{i} + 1, ..., I+J);

If v(x_{i} + k)_{new}^{(1)} = \text{Sort}_{2}(m) \text{ then } v(x_{i} + k)_{new}^{(1)} \leftarrow \text{Sort}_{1}(m) \text{ (} k, m = 1, ..., I+J-x_{i}) ;

If v(x_{i} + k)_{new}^{(2)} = \text{Sort}_{1}(m) \text{ then } v(x_{i} + k)_{new}^{(2)} \leftarrow \text{Sort}_{2}(m) \text{ (} k, m = 1, ..., I+J-x_{i}) ;

Set v(s)^{(1)} = v(s)_{new}^{(1)} \text{ and } v(s)^{(2)} = v(s)_{new}^{(2)} \text{ (} s=1, ..., I+J) ;

Step 9: Mutation (swap mutation)

For each chromosome \( Z^{(n)} = (v(1)^{(n)} \cdots v(I + J)^{(n)}) \)

Generate \( r_{3} \leftarrow U(0,1) \); if \( r_{3} < \text{MutRate} \) then undergo mutation; Otherwise no mutation

Set \( v(s)_{new}^{(n)} = v(s)^{(n)} \)

\( (s = 1, 2, ..., I + J) \)

Generate \( y_{1} \leftarrow U(1, I + J) \) and \( y_{2} \leftarrow U(1, I + J) \)

Set \( v(y_{2})^{(n)} = v(y_{1})^{(n)}_{new} \) and \( v(y_{1})^{(n)} = v(y_{2})^{(n)}_{new} \)

Step 10: Set \( g = g + 1 \). Go to Step 2.

---------------------------------------------------------------------------------------------------------------

Procedure 1: Calculate total cost in priority-based encoding

Input: \( \mu_{j}^{r} \): mean demand at retailer \( j \) \( j = 1, 2, ..., J \)

\( \text{Cap}_{i} \): capacity at warehouse \( i \) \( i = 1, 2, ..., I \)

\( P_{ij} \): transportation cost from warehouse \( i \) to retailer \( j \)

\( P_{ni} \): transportation cost from plant \( n \) to warehouse \( i \)

\( v(s) \): priority value \( s = 1, 2, ..., I+J \)

Output: \( \mu_{ij} \): mean demand at retailer \( j \) supplied by warehouse \( i \)

\( \mu_{ni} \): mean demand at warehouse \( i \) supplied by plant \( n \)

Step 1 : \( \mu_{ij} \leftarrow 0; \mu_{ni} \leftarrow 0, \forall i, j, n \)

Step 2 : \( s \leftarrow \text{arg max} \{ v(s) \} \); select a node

Step 3 : if \( s \leq I \), then \( i \leftarrow s \); select a warehouse

\( j \leftarrow \text{arg min} \{ P_{ij} | v(s) \neq 0, I < s \leq I + J \} \); select a retailer with the lowest transportation cost

\( n \leftarrow \text{arg min} \{ P_{ni} \} \); select a plant with the lowest transportation cost

else \( j \leftarrow s \); select a retailer

\( i \leftarrow \text{arg min} \{ P_{ij} | v(s) \neq 0, s \leq I \} \); select a warehouse with the lowest transportation cost

\( n \leftarrow \text{arg min} \{ P_{ni} \} \); select a plant with the lowest transportation cost

Step 4 : \( \mu_{ij} \leftarrow \text{min} (\mu_{j}^{r}, \text{Cap}_{i}) \)

\( \mu_{ni} = \mu_{ni} + \text{min} (\mu_{j}^{r}, \text{Cap}_{i}) \)

Update available capacity at warehouse (\( \text{Cap}_{i} \)) and uncovered demand at retailer (\( \mu_{j} \))

\( \text{Cap}_{i} = \text{Cap}_{i} - \mu_{ij} \); \( \mu_{j}^{r} = \mu_{j}^{r} - \mu_{ij} \)

Step 5 : Update priority (\( v(s) \))

If \( \mu_{j}^{r} = 0 \) then \( v(I + j) = 0 \)

If \( \text{Cap}_{i} = 0 \) then \( v(i) = 0 \)

Step 6 : If \( v(I + j) = 0 \), \( \forall j \) then there is no uncovered demand;

Calculate the total cost of the network defined in (5.1) that includes the fixed costs and variable cost of opening the warehouses, ordering cost, transportation cost and holding cost.

else go to Step 2
Bibliography


Bibliography


Declaration of co-authorship

Full name of the PhD student: Agus Darmawan

This declaration concerns the following article/manuscript:

<table>
<thead>
<tr>
<th>Title:</th>
<th>Integration of promotion and production decisions in sales and operations planning</th>
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<tr>
<td>Authors:</td>
<td>Agus Darmawan, Haranto Wong and Anders Thorstenson</td>
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The article/manuscript is: Published ☑ Accepted □ Submitted □ In preparation □


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<tr>
<td>21/08/18</td>
<td>ANDERS THORSTENSON</td>
<td></td>
</tr>
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Date: 23/08/18

Signature of the PhD student:
Declaration of co-authorship

Full name of the PhD student: Agus Darmawan

This declaration concerns the following article/manuscript:

| Title: | Integrated sales and operations planning with multiple products: Jointly optimizing the number and timing of promotions and production decisions |
| Authors: | Agus Darmawan, Hartanto Wong and Anders Thorstenson |

The article/manuscript is: Published ☐ Accepted ☐ Submitted ☑ In preparation ☐

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Signature of the PhD student
Declaration of co-authorship

Full name of the PhD student: Agus Darmawan

This declaration concerns the following article/manuscript:

Title: Supply chain network design with coordinated inventory control
Authors: Agus Darmawan, Hartanto Wiyono and Anders Thorstenson

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