History Dependent Unemployment Insurance
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PhD dissertation

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Preface

This dissertation was written in the period from February 2014 to March 2017 while I was enrolled as a PhD student at the Department of Economics and Business Economics at Aarhus University. I am grateful to the department for providing an excellent research environment as well as funding for numerous courses and conferences.

One person to thank stands out a mile: My main supervisor Torben M. Andersen. Torben, you have been my academic muse and mentor for the past almost six years, and I owe you immense appreciation for guiding me through, at times, troubled waters. The countless hours of theoretical discussions we have had have by far been the most educative part of my studies. Latest, we have co-written the third paper of this dissertation. Working this close with you has been extremely inspiring and scholastic. Moreover, you have always managed to find time for me in your (at times) packed schedule. Thank you!

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whenever needed. To my wonderful little daughter, Ebba: Thank you for always, always greeting me with the most amazing smile when I come home. Again and again, you make me realize what is truly important in life.

*Christian Ellermann-Aarslev*

*Aarhus, March 2017*

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**Updated preface**

The predefence meeting was held on April 25, 2017, in Aarhus. I am grateful to the members of the assessment committee consisting of Ann-Sofie Kolm (Professor, Stockholm University), Birthe Larsen (Associate Professor, Copenhagen Business School), and Rune Vejlin (chairman, Associate Professor, Aarhus University) for their careful reading of the dissertation and their many insightful comments and suggestions. Some have already been implemented, while others remain for future research.

*Christian Ellermann-Aarslev*

*Copenhagen, May 2017*
Contents

Preface ....................................................... iii
Updated preface ....................................... iv
Summary ...................................................... viii
Dansk resumé ............................................... x

1 Job Search Responses to Entitlement Conditions in Unemployment Insurance .............................................. 1
  1 Introduction ........................................... 2
  2 The Model ............................................. 5
    2.1 Equilibrium ......................................... 8
    2.2 Model parameterization .......................... 10
  3 Results ................................................ 11
  4 Discussion ............................................. 20
  5 Concluding remarks .................................. 22
  6 References ............................................ 24
A Additional material .................................... 26
B Alternative calibration ................................ 30
C Alternative preferences .............................. 36
D Alternative initial distributions .................. 44

2 General Equilibrium Effects of Entitlement Conditions in Unemployment Insurance ..................................... 52
  1 Introduction ........................................... 53
  2 The model ............................................. 56
    2.1 Updating of the two underlying variables ....... 57
    2.2 The supply side .................................... 59
    2.3 The demand side .................................... 60
    2.4 Matching ........................................... 61
    2.5 Wage determination ............................... 62
    2.6 Steady state worker distribution ............... 63
| 2.7 | Public sector | 65 |
| 3 | Results | 66 |
| 3.1 | Parameterization | 66 |
| 3.2 | Underlying variables | 67 |
| 3.3 | Search effects | 70 |
| 3.4 | Labor market performance | 71 |
| 3.5 | Welfare effects | 73 |
| 4 | Discussion | 75 |
| 4.1 | Wage assumptions | 75 |
| 4.2 | Tax system | 78 |
| 4.3 | Distribution of the unemployed | 79 |
| 4.4 | Optimality | 82 |
| 4.5 | Alternative benefit profile | 84 |
| 4.6 | Parameterization | 85 |
| 5 | Concluding remarks | 86 |
| 6 | References | 88 |
| A | Short introduction to the Danish UIB system | 90 |
| B | General formulation of updating functions | 90 |
| C | A UIB system with no conditions on past employment history | 94 |
| C.1 | Dynamics in line with previous literature | 94 |
| C.2 | Optimal design | 100 |
| D | Additional figures | 105 |

3 Job Duration and History Dependent Unemployment Insurance 110

<p>| 1 | Introduction | 111 |
| 2 | Job duration and unemployment insurance | 114 |
| 2.1 | Reservation duration | 114 |
| 2.2 | Reservation duration and properties of the unemployment insurance scheme | 117 |
| 3 | Worker flows and steady state equilibrium | 119 |
| 4 | Numerical illustration | 122 |
| 4.1 | Parameter values | 123 |
| 4.2 | Employment and unemployment | 124 |
| 4.3 | Benefit duration and (un)employment | 129 |
| 4.4 | Benefit levels and durations | 129 |
| 5 | Concluding remarks | 133 |
| 6 | References | 136 |</p>
<table>
<thead>
<tr>
<th>Section</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>A Proof Proposition 1</td>
<td>137</td>
</tr>
<tr>
<td>A.1</td>
<td>A.1 Second-order condition</td>
<td>138</td>
</tr>
<tr>
<td>A.2</td>
<td>A.2 Threshold duration $\hat{T}$</td>
<td>140</td>
</tr>
<tr>
<td>A.3</td>
<td>A.3 Properties of reservation duration $R(T)$</td>
<td>142</td>
</tr>
<tr>
<td>B</td>
<td>B Proof Corollary 2: Threshold $\hat{T}$</td>
<td>143</td>
</tr>
<tr>
<td>C</td>
<td>C Comparative statics</td>
<td>146</td>
</tr>
<tr>
<td>D</td>
<td>D Constant $R$</td>
<td>150</td>
</tr>
<tr>
<td>E</td>
<td>E Equilibrium employment and unemployment</td>
<td>151</td>
</tr>
<tr>
<td>F</td>
<td>F Employment and benefit design</td>
<td>152</td>
</tr>
</tbody>
</table>
This dissertation comprises three self-contained theoretically founded chapters covering effects of history dependent conditions in unemployment insurance benefit (UIB) systems. UIB systems are in general multi-dimensional as they contain not only the level and duration of benefits, but also job-search criteria, activation requirements, monitoring as well as various eligibility conditions. The latter most often include conditions on past employment record, implying that only workers with a sufficiently strong employment history qualify for UIB. Such conditions presumably mitigate moral hazard issues. The role of employment requirements is interesting from a theoretical point of view as well as in relation to policy implications. While most literature on UIB systems focus on the level and duration of benefits, considerably less attention has been given to entitlement conditions. One reason for this may be that conditions on employment history create history dependence, and thus, state contingency of otherwise identical workers. Keeping track of these differences is clearly an analytical challenge. In this dissertation I offer three different approaches. The first two chapters focus on the search effort of unemployed and how this is affected by entitlement conditions. While the first chapter explicitly models the employment history of workers, the second approximates the employment history by a one variable proxy, but presents results of a general equilibrium analysis of the matter. The third chapter addresses the implication that entitlement conditions might change the direction of job search in terms of expected employment duration. Overall, all three chapters find that introducing history dependence in UIB systems generates state contingent implications for unemployed.

In more details, the first chapter, *Job Search Responses to Entitlement Conditions in Unemployment Insurance*, studies the effects of employment history requirements on job search of unemployed. Specifically, I present an OLG model with endogenous search effort which explicitly tracks the employment history, rather than differentiating workers only with respect to whether one is eligible for UIB or not. To keep the heterogeneity on a manageable level, the model is one-sided, as it considers only the supply side of the labor market and regards wages and taxes as exogenous. Hence, the model describes the impact effects of entitlement conditions. Overall, entitlement conditions are found to introduce endogenous duration dependence, as it creates a discouraged worker effect where long-term unemployed (with a weak employment history) decrease their search effort marginally, while unemployed with a strong employment history intensify their search effort. The differences in job search responses highlight that entitlement conditions may have generational dependent implications;
discouraging the young while encouraging the older. Furthermore, the composition of the workers is affected such that the share of core troops of the labor market is increased, while the share of unemployed with consecutive periods of unemployment is reduced.

In the second chapter, *General Equilibrium Effects of Entitlement Conditions in Unemployment Insurance*, I present a Diamond-Mortensen-Pissarides general equilibrium model with endogenous search effort that explicitly allows UIB to depend on past employment history. The model is able to determine the net effects of the individual job search responses (studied in the first chapter) and takes into account the effects on wages, taxes, and vacancy posting. When calibrated to the Danish economy, unemployment decreases. Hence, stricter conditions on employment history are found to work as a substitute for lowering the generosity of UIB schemes and are, in terms of utilitarian welfare, optimal to introduce. Conditions on past employment history can therefore act as an additional instrument when designing UI systems and, if needed, reduce the overall generosity without reducing the level or the duration.

The third chapter, *Job Duration and History Dependent Unemployment Insurance* (joint work with Torben M. Andersen), develops an analytically tractable matching model capturing the importance of employment history for the design of UIB schemes. We show that the optimal strategy for unemployed is to formulate a reservation duration, i.e. a shortest expected job duration acceptable, and that a stronger employment history leads to higher reservation duration. Consequently, short-term jobs are only acceptable to unemployed with a weak employment history, while unemployed with a stronger employment history have higher reservation durations. This affects the equilibrium distribution of employment according to job durations; fewer short duration jobs are filled, but this also means that fewer are locked into such jobs, and therefore more jobs with long durations are filled, although the net effect generally is to lower employment. Employment history contingencies thus affect both the level and structure of employment. Equilibrium (un)employment depends not only on reservation durations, and it is found that lengthening benefit duration, even when decreasing benefit levels to retain reservation durations, has a negative effect on employment. Lastly, we report a numerical analysis with outset in the role of employment history contingencies in the Danish unemployment insurance scheme to shed further light on the effects of such contingencies.
DANSK RESUMÉ


I det første kapitel, Job Search Responses to Entitlement Conditions in Unemployment Insurance, studerer jeg effekterne af beskæftigelseskrav på arbejdsløses jobsøgning. Specifikt præsenterer jeg en OLG model med endogen søgeindsats, der eksplicit tager højde for beskæftigelseshistorikken i stedet for blot at differentiere personer i forhold til, om de er berettigede til dagpenge eller ej. For at holde heterogeniteten på et håndterbart niveau er modellen ensidet, da den kun betragter udbudssiden af arbejdsmarkedet og anser lønninger og skatter som eksogene. Således beskriver modellen den umiddelbare indvirkning af beskæftigelseskrav. Modellens resultater viser, at beskæftigelseskrav introducerer endogen varighedsafhængighed for arbejdsløse, da de har en demoraliserede effekt på langtidsledige (med en svag beskæftigelseshistorik), der sænker deres søgeindsats marginalt, mens arbejdsløse med en stærkere beskæftigelseshistorik anspores til at øge deres søgeindsats. Disse forskelle understreger, at beskæftigelseskrav
kan tænkes at have generationsmæssige implikationer, hvor yngre demoraliseres, mens ældre generationer opmuntrer. Derudover påvirker de forskellige søgeindsatsreaktioner sammensætningen af arbejderne, således at antallet af arbejdsmarkedets kernetopper øges, mens antallet af arbejdsløse med sammenhængende perioder med arbejdsløshed reduceres.


Most unemployment insurance benefit (UIB) schemes feature entitlement conditions depending on the employment history of the insured. This is potentially important for job search incentives, however, unaddressed by the majority of the literature. This paper studies an OLG model with endogenous search effort in which the workers’ employment history is explicitly accounted for. Strengthening entitlement conditions are found to introduce endogenous duration dependence, as it creates a discouraged worker effect where long-term unemployed (with a weak employment history) decrease their search effort marginally, while unemployed with a strong employment history intensify their search effort. The different response in search effort affects the composition of the workers such that the share of core troops of the labor market is increased, while the share of unemployed with consecutive periods of unemployment is reduced.

JEL Classification: H3, J22, J64, J65

Keywords: Unemployment; Unemployment insurance; UI eligibility; Reentitlement effects; Search
1 Introduction

Entitlement conditions including requirements of a certain connection to the labor market prior to coverage are present in most unemployment insurance benefit (UIB) schemes.¹ Such conditions are potentially of great importance for both the individual worker and for the economy as such, as they imply state contingency of otherwise identical workers. As this is a significant theoretical challenge, it may be part of the explanation why entitlement conditions have not been addressed by the predominant part of the literature. Oppositely, the effects of adjusting the level or the duration of UIB are quite well established empirically as well as theoretically, see e.g. Fredriksson and Holmlund (2006) and Tatsiramos and van Ours (2012) for surveys. The mainstream approach is to assume automatic eligibility; i.e. that workers re-earn entitlement to finite duration benefits by having even a short spell of employment. In reality, this is obviously not the case. The so-called (re-)entitlement effect as such was identified by Mortensen (1977) as an outcome of a two-tier benefit scheme. It increases the incentives of unemployed who have lost entitlement to the highest level of benefits to seek employment. The assumption of automatic eligibility has only been confronted recently. Two approaches exist: Identifying wage differentials between entitled and non-entitled workers and, secondly, analyzing differences in search effort of unemployed. Ortega and Rioux (2010) identify a wage effect of entitlement conditions, and thus, find an entitlement effect even with non-automatic eligibility.² However, they only characterize workers in terms of entitlement and do not take the possible effect on the search effort of the unemployed into account, implying that transitions from both eligible and non-eligible unemployment to employment are equally likely. Andersen et al. (2015) endogenize the search effort in a setting like Ortega and Rioux (2010) and find that if it is harder to get re-entitled to UIB, eligible unemployed search more, while non-entitled unemployed search less. But as Ortega and Rioux (2010), Andersen et al. (2015) do not take the heterogeneity of workers in terms of differences in employment history into account, and workers are only characterized in terms of eligibility. That is, the complex set of potential employment histories are compressed into a binary state variable; either the worker satisfies the requirement or not. In that respect the

¹As an example, the Danish UIB scheme includes 52 weeks of required employment during a reference period of three years. For a comprehensive review of eligibility conditions in general, see Venn (2012).
²Specifically, they set up a Diamond-Mortensen-Pissarides general equilibrium search-matching model, in which workers are either employed or unemployed and either eligible for UIB or not. Their results basically confirm the existence of an entitlement effect in a setting with non-automatic eligibility. Specifically, the effect goes through the wages which differs for entitled and non-entitled workers.
simplification ignores that non-entitled workers with different employment histories due to entitlement conditions have different likelihood of gaining entitlement. Wagner and Jahn (2010) take the complexity of potential employment histories into account and find effects through the wages, which decrease until the unemployed have gained entitlement. However, they do not consider search effort of unemployed as endogenous. Furthermore, concerning wage differentials Coles and Masters (2007) consider how hiring incentives are affected by entitlement conditions. Specifically, they emphasize that conditions on employment history impose an implicit transfer from future potential employers to current employers as entitlement to higher benefits increases the workers’ reservation wage in a future wage bargaining. Hopenhayn and Nicolini (2009) also consider a UIB scheme conditioned on the employment history; however, they take a more normative stand compared to this paper and seek to characterize the optimality of the scheme via a one-sided repeated moral hazard principal-agent model. Their main finding is that it is optimal to let unemployment benefits increase in the employment history if quits cannot be distinguished from layoffs. Contrary to this paper, they do not explicitly consider the state contingency of the search effort of the unemployed. Pan and Zhang (2012) introduce entitlement conditions into the framework of Hopenhayn and Nicolini (2009) and find that they serve to adjust the incentives of workers and mitigate moral hazard quits. Lastly, to give an idea of the potential gains of introducing entitlement conditions, a simulation of Faig and Zhang (2012) shows that the US average unemployment rate would decrease by one percentage point following implementation of optimal entitlement conditions.

In contrast to previous literature, the model considered in this paper explicitly tracks the employment history rather than differentiating the workers only with respect to whether they are eligible for UIB or not. This allows for state contingency in the job search intensity of the unemployed with respect to the previous successes and failures on the job market. To keep the state contingency on a manageable level, the model is one-sided, as it considers only the supply side of the labor market and regards wages as exogenous. Hence, the model describes the impact effects of entitlement conditions on the employment history. These would be the first-order effects in a general equilibrium analysis of the issue, and thus getting a deeper insight is an important starting point. Explicitly tracking the employment history is a challenge, since the number of different combinations of histories evolves exponentially as the extent of the (discretized) past taken into account increases. As a result, the model simplifies employment history by considering only the three previous periods. Restricting the employment history to three periods enables the basics of the state contingency to be laid out without complicating the analysis unnecessarily. Usually, two periods
are sufficient to explore dynamics like the present; however, the timing in this model requires three periods to illustrate the important mechanisms of the issue of interest. In reality, entitlement conditions on employment history are usually restricted to a certain period of the past, i.e. the reference period. This implies that the restriction of the tracking of the employment history in this paper is actually in line with most UIB schemes (see e.g. Venn (2012)), the difference lies in how coarse-grained the period length is.

Specifically, the model is an OLG model where unemployed workers endogenously decide the optimal level of job search effort. As history obviously matters in the analysis at hand, an OLG framework allows me to pay regard to the differences between generations in their employment history, as young enter the labor market with no previous employment and old exit with a more complex employment history.

Intuitively, stricter entitlement conditions could lead to two opposite effects on the search effort of unemployed, depending on their employment history. For unemployed with a weak employment history, the search effort might very well be affected negatively. This is because the potential gain from re-employment is lowered, as one is more unlikely to be entitled to UIB. However, unemployed with a strong employment history, could be induced to increase their search effort, as the value of continuing unemployment is worsened. This intuition is confirmed by the results of the model, as the outcome of the model shows that strengthened entitlement conditions on employment history introduce endogenous duration dependence, as it creates a discouraged worker effect where long-term unemployed (with a weak employment history) decrease their search effort marginally, while unemployed with a strong employment history intensify their search effort. Hence, compared to earlier literature, this paper presents a more detailed description of how search effort of unemployed is affected by entitlement conditions by explicitly accounting for the state contingency following differences in the employment history.

The remaining part of the paper is organized as follows. In section 2 the model is described. I present the results of the model in section 3, discuss some of these in section 4, and finally I conclude in section 5.

---

3The framework of the analysis with finite horizon and new generations entering with a weaker situation than that of older generations is somewhat similar to the literature on the insurance value of UIB schemes, in which arguments of consumption smoothing often tend to neglect the implications for young, who do not have the same possibilities of consumption smoothing as older generations. See e.g. Storesletten et al. (2001).
2 The Model

The agents enter the model as unemployed and are active on the labor market for a predetermined amount of time after which they retire. Hence, when time progresses, one generation of workers retires, while a new generation of equal size enters the labor market (there is no population growth in the model). If unemployed, the agents exert endogenous time-consuming search effort to increase their chances of employment in the following period.

Time is discrete, and to keep track of the employment history I assume the reference period compressing the information of the employment history, i.e. the part of the employment history which determines the eligibility to UIB, to be three periods. The timing of the reference period is such that it covers the three periods prior to the current (i.e. at time $t$ the reference period includes the three periods $t - 1$, $t - 2$, and $t - 3$). Hence, when time progresses one period (from time $t$ to time $t + 1$), the employment status of the last period of the reference period (time $t - 3$) is discarded, while the employment status of the former current period (time $t$) is included into the reference period. Consequently, given the present employment status, the workers know their employment history in the period to come.

Workers can be in two states: employment (indicated by 1) and unemployment (indicated by 0). Since the reference period is three periods long and there are two possible outcomes every period, eight ($2^3$) different employment histories exist. Differentiating workers in terms of these eight different employment histories, I denote the workers as eight different types of workers. The eight types are denoted $a - h$, and are defined and evolve in the following way:

---

4 Using only three periods simplifies the number of different combinations greatly and still captures the basics of the state contingency arising from differences in the employment history.
Table 2.1: The specification of types and the development of these.

When time progresses one period, workers update their type according to the specification above and change employment state in the following way: The transition from employment to unemployment is governed by an exogenous Poisson rate $5$, $\rho$. The transition rate back to employment is equal to the endogenous search effort conducted by the unemployed in the previous period weighted by some exogenous return to search, $\alpha$.  

In every period and employment state, the utility function of the agents is the same and depends only on consumption, $c$, and leisure in the given period. As the time endowment is normalized to one, leisure equals $1 - h$ where $h$ is the (exogenous) share of time devoted to labor when employed, $l_E \in [0, 1]$, and the time spent on job search for unemployed, $s$. Specifically, the utility function is assumed to be logarithmic.

$$u_t(c_t, 1 - h_t) = \log(c_t) + \log(1 - h_t)$$

---

1. Assuming that the termination of an employment spell occurs at a Poisson rate is a common assumption in the literature; see e.g. Fredriksson and Holmlund (2001).

2. Theoretically, for sufficiently high levels of return to search ($\alpha > 1$) the transition rate from unemployment to employment could exceed one. However, when solving the model I continually make sure that this is not the case.

3. For other functional forms of the utility function, see Appendix C.
There are no savings in the model, and so the agents are hand-to-mouth consumers. Any type of worker receives the same (exogenous) real wage, \( w \), if employed. In case the worker is unemployed, the level of benefits depends on their employment history, i.e. their type. The benefit scheme is two-tiered, where UI-covered unemployed receive UIB, replacement rate denoted \( b^{UI} \), while non-covered unemployed get the lower social assistance (SA), replacement rate denoted \( b^{SA} \).

\[
\text{benefits}_{UI} = b^{UI} w, \quad \text{benefits}_{SA} = b^{SA} w, \quad 0 < b^{SA} < b^{UI} < 1
\]

The subjective discount factor is denoted \( \delta \), and both \( w, b^{SA}, b^{UI} \), and \( \delta \) are exogenous. For simplicity, the model considers wages as exogenous and refrain from including an explicit government budget constraint.\(^8\) The costs of running the UI system are, however, briefly discussed.

The lifetime utility (expressed as a value function) of a worker is given by the instantaneous utility from consumption and leisure of the current period plus the discounted expected value of the next period. In the next period two things happen: the employment state changes by the transition rates described earlier and the employment history is updated, leading to the worker-type being updated. The notation of employment states and types is as follows: The value of employment for a type \( a \) worker at time \( t \) is denoted \( V^E_{a,t} \), and, likewise, the value of unemployment is denoted \( V^U_{a,t} \). \( u^E_{a,t} \) is the instantaneous utility from the consumption and leisure of an employed type \( a \) worker at time \( t \), and \( u^U_{a,t} \) is the corresponding for an unemployed type \( a \) worker. Also, the optimal search effort at time \( t \) of a type \( a \) unemployed is denoted \( s_{a,t} \).

\[
V^E_{a,t} = u^E_{a,t} + \delta \left( \rho V^U_{a,t+1} + (1 - \rho) V^E_{a,t+1} \right)
\]
\[
V^U_{a,t} = u^U_{a,t} + \delta \left( \alpha s_{a,t} V^E_{b,t+1} + (1 - \alpha s_{a,t}) V^U_{b,t+1} \right)
\]

Using equivalent notation, the remaining 14 value functions are given in Appendix A. Remember, as can be seen from table 2.1, that the workers with a reference period consisting only of employment (1 1 1) are defined as type \( a \) workers, and the ones who were unemployed in the previous period but employed in the two before that (1 1 0) are defined as type \( b \) workers. Hence, the development of type \( a \) workers is as follows: If they are employed in the current period, they will also be type \( a \) in the period to come. However, if they are unemployed in the current period, they will become type \( b \) in the next period.

The search effort of the unemployed is given by standard utility maximization.

---

\(^8\)In Ellermann-Aarslev (2017a) these assumptions are relaxed.
Below, the optimal level of search effort is given for a type \( a \) unemployed. The level for the remaining types is given in Appendix A.

\[
s_{a,t} = 1 - \frac{1}{\delta \alpha (V^{E}_{b,t+1} - V^{U}_{b,t+1})}
\]

The separability of the utility function implies that at any time of their career, only four distinct levels of search effort exist for each generation, as \( s_{a,t} = s_{e,t}, s_{b,t} = s_{f,t}, s_{c,t} = s_{g,t}, \) and \( s_{d,t} = s_{h,t} \). The reason why the search effort levels coincide pairwise is that as time progresses, the last of the three periods in the reference period is discarded. This implies that for the same outcome in the period to be included (here unemployment), only four different combinations exist. As an example: at time \( t \) an unemployed type \( a \) and an unemployed type \( e \) both know with certainty that they will be type \( b \) in the next period (time \( t + 1 \)). Hence, when they decide how intensely they will search for employment, they take the same possible outcomes into account, namely, the value gain of employment for a type \( b \) worker, \( V^{E}_{b,t+1} - V^{U}_{b,t+1} \). This fact underlines that three periods are needed, as recording just two periods of the past would result in only two different levels of search effort, which is insufficient for a thorough analysis.

The retirement age is fixed and exogenously politically decided. Hence, retirement is an absorbing state and not of particular importance in this analysis. However, it is included to be able to solve for the optimal levels of search effort using backward induction (see below).

2.1 Equilibrium

Due to the complexity of the state space of the model, it is unfeasible to analyze the equilibrium analytically. However, the finite number of periods of labor market participation of each generation of workers makes it possible to solve the model outcome numerically by backward induction. In the period just before retirement, the optimal level of search effort is zero for all types, as in the next period they all end up in the absorbing retirement state independently of how hard they search for a job. Hence, the value of both employment states one period before retirement is known for all types. As types are determined one period ahead, the optimal search effort two periods before retirement is then fully determined for all types. Knowing this makes it possible to calculate the value of both employment states two periods before retirement. This procedure of backward induction is continued to determine the search effort of all generations for all types.

To solve the model in terms of worker flows, a distributional assumption is needed
on the types of the workers as they enter the labor force. It is assumed that young workers enter the labor market as unemployed with no previous employment in their reference period, i.e. as type \( h (0 0 0) \). However, for all possible starting distributions, the distribution of each generation converges as their career unfolds.\(^9\) Of course, looking at the total population, the distribution of the young generations matters. Hence, the assumptions on the initial distribution affect the distribution of the total population. However, the implications for the young generations are, in fact, potentially quite crucial for the analysis and partly the reason for the OLG structure. Hence, the influence of the starting distribution should not be considered a disadvantage of the model but rather a relevant part of interest, as young enter the labor market with no previous employment while older generations have a large employment history but no labor market future.\(^10\) See Appendix D for the key figures of the results in case of different assumptions on the initial distribution.

The notations \( e_{a,t} \) and \( n_{a,t} \) denote the share of each generation who is respectively, employed and non-/unemployed type \( a \) at time \( t \), and likewise for the seven other types. By construction of a normalization, the sum of all 16 shares equals one. As the inflow into employment with a type \( a \) employment history comes from workers who were employed type \( a \) or \( e \) in the previous period and did not lose their job, and the inflow to type \( a \) unemployment comes from those who lost their jobs, the associated flow equations are as follows.

\[
\begin{align*}
  e_{a,t} &= (1 - \rho) (e_{a,t-1} + e_{e,t-1}) \\
  n_{a,t} &= \rho (e_{a,t-1} + e_{e,t-1})
\end{align*}
\]

As the type \( b (1 1 0) \) employment history involves unemployment in the previous period, the inflow into both employment and unemployment of type \( b \) stems from workers who were unemployed in the period before; in particular, from unemployed type \( a \) and \( e \) workers. The workers who find a job (which happens at a probability of \( \alpha s \)) enter employment, the others are obviously still unemployed. Hence, the flow equations for inflow into both employment states for type \( b \) are given by the following:

\[
\begin{align*}
  e_{b,t} &= \alpha s_{a,t-1} n_{a,t-1} + \alpha s_{e,t-1} n_{e,t-1} \\
  n_{b,t} &= (1 - \alpha s_{a,t-1}) n_{a,t-1} + (1 - \alpha s_{e,t-1}) n_{e,t-1}
\end{align*}
\]

\(^9\)For an illustration see figure A1 in Appendix A, where the worker distribution in form of the shares of all types are depicted for all possible starting assumptions (and all four policies). For the youngest generations, the distributions fluctuate depending on starting assumption.

\(^{10}\)Note, however, that the results on the effects of changing the entitlement conditions presented later do not hinge on assumptions on the initial distribution.
The flow equations of the rest of the types are given in Appendix A.

The shares of the total population are given by summing over all generations (and dividing by the number of generations for the sum still to equal one). Note that since there are no growth or aggregate shocks in the model, the size of each generation is the same at any given time. Hence, summing over all generations is equivalent to summing over the time indicator, \( t \), for a career of \( T \) periods. Using the notations \( e_x \) and \( n_x \) to denote the shares of the total population who are employed and unemployed type \( x \), respectively (i.e. skipping the time index indicates that it is the share of the total population and not of one generation, \( t \) periods into their career), these are given as follows:

\[
e_x = \frac{\sum_{i=1}^{T} e_{x;i}}{T}, \quad n_x = \frac{\sum_{i=1}^{T} n_{x;i}}{T}
\]

Total employment, \( e \), and total unemployment, \( n \), are the sum of the eight different types of specific shares. By construction, the total population is constituted of a continuum of mass one, and so \( e + n = 1 \).

\[
e = \sum_{x=a}^{h} e_x, \quad n = \sum_{x=a}^{h} n_x
\]

### 2.2 Model parameterization

The model is calibrated to match the Danish economy and UIB scheme. Hence, the replacement rates are set according to the Danish system.\(^\text{11}\) This implies that the replacement level of social assistance, \( b_{SA} \), is 35.4%, while the replacement level of UIB, \( b_{UI} \), is 62.8%. The period length is one year, while the career length, \( T \), is 40 years\(^\text{12}\), and the real wage rate, \( w \), is numeraire. The subjective discount factor, \( \delta \), equals \( \frac{1}{1+r} \), as it is assumed that the interest rate represents the discount rate of all workers. The yearly interest rate, \( r \), is set to 5% per year (as do Shimer (2005), Rosholm and Svarer (2004), and many others). The workload while employed corresponds to a 37-hour workweek.

---

\(^{11}\)The Danish Unemployment Insurance Commission, Dagpengekommissionen (2015c).

\(^{12}\)The average effective age of retirement in Denmark is 61.8 years according to OECD (2014 data, equal weight of men and women). The time of entrance is harder to measure in average terms, as it depends very much on the level of education and furthermore can be interrupted by continuing education. The labor force participation rate has an increasing concave shape in age, where that of the 15- to 19-year-old is 51.4, 20- to 24-year-old: 71.0, 25- to 29-year-old: 80.1, 30- to 34-year-old: 86.8, and it peaks for the 35- to 39-year-olds where it is 89.9 (also 2014 OECD data for Denmark). Based on this, I use the age of 22 as the age of entrance, since the predominant share of a generation participates in the labor market at that age. Hence, the career length is rounded off to 40 years. Changing this does not affect the results of the model, however.
relative to the 168 hours a week consists of; i.e. $l_E$ is 0.22. As the model considers the demand side of the labor market exogenous, the return to search, $\alpha$, which captures the matching efficiency is somewhat arbitrary. Hence, I set it to one, implying that unemployed end up in employment at a probability equal to their search effort. Note that inducing so much search effort that one reaches employment with certainty will never be optimal due to the time endowment of one and log-utility.

The job destruction rate, $\rho$, is calibrated to fit the very dynamic Danish labor market, which is characterized by short unemployment spells and a low unemployment rate.\textsuperscript{13} 2005 was considered to be a business cycle neutral year for the Danish economy. Hence, I use the unemployment rate of this year (5.1%) to calibrate the job destruction rate to 0.0055. In Appendix B, I present an alternative calibration where both the return to search and the job destruction rate is higher, namely, $\alpha = 1.55$ and $\rho = 0.0178$. The qualitative results are unaffected.

3 Results

The distribution of the population in terms of employment status and employment history is illustrated in figure 3.1 below. For a discussion see section 4.

Figure 3.1: The distribution of the population if the UIB scheme requires one period of employment.

\textsuperscript{13}See e.g. Andersen and Svarer (2007).
Considering the level of benefits received, the unemployed can generally be divided into three groups following changes in the design of the UI system that strengthens the entitlement conditions by requiring a stronger employment history to qualify for UIB. Group one: Unemployed receiving UIB both before and after the change. Group two: Unemployed who receive UIB before the change but social assistance afterwards. Group three: Unemployed receiving social assistance both before and after the change. For the full specification, see table A.1 in Appendix A.

Since the reference period for the employment history is three periods, four different policies on UIB entitlement conditions exist. The required number of periods of previous employment for entitlement to the highest level of benefits, $b_{UI}$, is thus either zero, one, two, or three. I denote the required sum of employment in the reference period $T_{RE}$, and so $T_{RE} = \{0, 1, 2, 3\}$. Hence, three different changes tightening the employment requirements exist: from no requirements to one period of required employment ($T_{RE} = 0 \rightarrow 1$), from one to two required periods ($T_{RE} = 1 \rightarrow 2$), and from two to three periods ($T_{RE} = 2 \rightarrow 3$).

The effects of these three changes on the level of search effort chosen by the three groups of unemployed are the following: Strengthening the entitlement conditions increases the search effort of group one significantly (indicated by the black arrows in figure 3.2 below), while it decreases the search effort of group three marginally (indicated by the dark red arrows in figure 3.2). The effects for group two, who are affected immediately by the changes, is that their search effort increases significantly on four occasions, while it decreases marginally on three (all indicated by the purple arrows in figure 3.2).\(^\text{14}\)

\(^{14}\)Note that the effects for group two depend critically on the utility function, see Appendix C.
CHAPTER 1
JOB SEARCH RESPONSES TO ENTITLEMENT CONDITIONS IN UNEMPLOYMENT INSURANCE

3. RESULTS

Optimal search effort of unemployed depending on type and policy scheme

Optimal search effort of unemployed depending on type and policy scheme

(a) Full illustration

(b) Zoom in at upper levels

Figure 3.2: Search effort for a representative generation halfway through the career for the four possible entitlement policies with respect to the required sum of employment history in the reference period, $T_{RE}$. The black arrows indicate the increase in search effort of group one, the purple arrows indicate the change in search effort of group two, while the dark red arrows indicate the decrease in the search effort of group three.

Hence, the model confirms the expected intuitive results, namely that the potential gain from re-employment is lowered for unemployed with weak employment histories (group three), as strengthened requirements to the employment history makes it less likely to be entitled to UIB. This results in a (marginal) decrease of the search effort.
Oppositely, the value of not finding a job is worsened for unemployed with a strong employment history (group one). Consequently, they are induced to increase their search effort. Thus, strengthening entitlement conditions introduce an endogenous duration dependence as it creates a discouraged worker effect where long-term unemployed (with a weak employment history) decrease their search effort marginally, while unemployed with a strong employment history intensify their search effort.

As a result of the changes in search effort, each of the three possible UI design changes decrease the total unemployment, see figure 3.3 below.\textsuperscript{15}

![Figure 3.3: Total unemployment decreases in the strength of the entitlement conditions.](image)

Concerning the size of the three groups of unemployed, they do, in fact, all decrease for all three possible changes of the required employment history, see figure 3.4 below.\textsuperscript{16} One might expect that the decrease in search effort of group three would result in an increase in the size of this group. However, the development of the types (given

\textsuperscript{15} This is true for all possible starting distributions.

\textsuperscript{16} In the case of a change from two required periods of employment to three periods, group one does actually increase marginally in size. This is because group one in this case consists of unemployed with employment in all three previous periods (type $a$), and only employed workers with employment in the previous two periods can end like as this (by being fired). That type of employed workers increases in size following changes of the (re-)entitlement conditions, making the size of group one increase.

Furthermore, when the utility function is assumed to be non-separable, e.g. CES, and the cross derivative is negative (i.e. consumption and leisure are assumed to be substitutes), the size of group three increases. In fact, it increases so much that total unemployment can potentially increase. For further details, see Appendix C.
in table 2.1) makes the transitions more complicated, as the increase in the search effort of the rest of the unemployed makes the inflow into group three decrease.

Figure 3.4: Changes in the size of the three groups of unemployed following the three possible changes concerning entitlement conditions.

The changes in the composition of the workers following the search responses to the changes in the entitlement conditions are as follows (figure 3.5 below): The shares of the core troops of the labor market (employed workers with an employment history consisting primarily of employment, i.e. employed type \( a (1 1 1) \), \( b (1 1 0) \), \( c (1 0 1) \), and \( e (0 1 1) \)) are increased. The reverse is true for workers who have been unemployed for all three previous periods but have now found employment (employed type \( h (0 0 0) \)), and employed workers who were employed in the previous period but not in the two before that (employed type \( g (0 0 1) \)), as the share of those types decrease. Regarding the unemployed, the shares of unemployed workers with one, two, and three consecutive periods of unemployment in their registered employment history (i.e. unemployed type \( b (1 1 0) \), \( d (1 0 0) \), and \( h (0 0 0) \)) all decrease following all three possible changes in the entitlement conditions. As these unemployed decrease their search effort following (sufficiently tough) changes, one might expect that they would increase in size. However, they do not as the other types of unemployed increase their search effort and thereby decrease the likelihood of ending up as unemployed with less strong employment history. The changes of entitlement conditions do not change the shares of the other types of unemployed (type \( a, -c, -e, -f, \) and \(-g\)) significantly. Summing up, more strict entitlement conditions imply a stronger and more equal dis-
tribution of the workers in terms of employment history, as the share of unemployed with weak employment histories generally decrease in size while the share with a stronger employment history increase in size.

A change from zero to one period of required employment (which is basically a change introducing a two-tiered benefit system) has substantial effects on two types of workers, namely on the share of employed who have been employed in all three previous periods (employed type $a (1 1 1)$), as the share increases drastically, and on the share of unemployed affected immediately by such a change, i.e. long-term unemployed with no registered employment (unemployed type $h (0 0 0)$), as the share decreases even more.

![Graphs showing changes in distribution of employed and unemployed types](image)

Figure 3.5: Changes in the populational distribution following the three possible changes concerning entitlement conditions.

Relative to the share of the population of each type, the changes are more or less the same as described above. However, since the workers who are employed and who have been so for all three previous periods (employed type $a (1 1 1)$) constitute the majority of the population, the changes in their share sizes are not large in relative terms.
Figure 3.6: Relative changes in the populational distribution following the three possible changes concerning entitlement conditions.

Examining the effects of entitlement conditions on aggregated search effort, note that it is the product of the population share and the search effort they conduct. The total aggregated search effort is found to increase with, respectively, 2.9%, 0.3%, and 0.2% following the three changes in required employment history. This is what translates directly into a decrease of the unemployment rate. Focusing on the specific types of unemployed, the effects on their aggregated search effort are as follows: For the majority of types, the level of aggregated search effort increases as a result of changes of the entitlement conditions, see figure 3.7 (a). However, the aggregated search effort of long-term unemployed, type $h (0 \ 0 \ 0)$, decreases following more strict entitlement conditions, see figure 3.7 (b).

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17 See figure A2 in Appendix A.
Figure 3.7: Aggregated search effort of all types for the four possible policies concerning entitlement conditions, $T_{RE}$.

The costs of running the UI scheme are, besides the benefit levels, basically affected by two factors: total unemployment and the distribution of the unemployed in terms of benefit level. As discussed earlier, total unemployment decreases with the strength of the entitlement conditions. Distinguishing the unemployed in terms of their benefit level, more and more unemployed are receiving SA as the conditions on the required employment history are strengthened (figure 3.8 below). Hence, as fewer are receiving
benefits and a larger fraction of them experience a decrease in the level of benefits, the aggregated effect of strengthened entitlement conditions is a reduction of the costs needed to run the UI scheme (figure 3.9 below).\textsuperscript{18}

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{distribution_of_unemployment.png}
\caption{The effect of entitlement conditions on the distribution of the unemployed in terms of benefit level.}
\end{figure}

\textsuperscript{18}Note that in a situation of different preferences, total unemployment might actually increase following changes that strengthen the entitlement conditions (discussed in the next section). However, the costs of running the UI scheme still decrease as a result of sufficient changes in the distribution of the unemployed, where the share who receive SA increases and the share who receive UIB decreases (see figure C10).
4 Discussion

An implication of the results of the model is that entitlement conditions on employment history might have opposite effects on the search effort of young and older generations. As young generations typically have weaker employment histories they are affected by a (marginal) discouraged worker effect of requirements on the employment history. Hence, entitlement conditions might have generational dependent implications; (marginally) discouraging the young while encouraging the older generations.

The preferences of the workers are an interesting focal point as the search response of group two (unemployed who lose the entitlement to UIB as a result of the change in entitlement conditions) depends critically on the assumed utility function; specifically, whether consumption and leisure are assumed to be compliments or substitutes. If
consumption and leisure are compliments, search effort increases significantly for the majority of types. However, if consumption and leisure are substitutes, the search effort decreases drastically for all unemployed in group two. This difference in response is simply because, if consumption and leisure are substitutes, the drop in the level of consumption of the unemployed in group two following an increase of the required employment history will increase the marginal utility of leisure, hence, resulting in a lower level of search effort being optimal. However, if consumption and leisure are compliments, the opposite is, of course, the case. The results of the model when the preferences follow a CES utility function are presented in Appendix C.

Changing the assumption on the distribution of the generations entering the labor market from the current (that young enter the labor market as unemployed having no previous employment, type $h(0 0 0)$), is considered in Appendix D. Here the key results of the model are presented for two alternative starting distributions, namely that all start as unemployed ‘graduates’, rewarded with one period of recorded previous labor market participation (unemployed type $g(0 0 1)$), and that all start with a perfect employment history as unemployed type $a(1 1 1)$. These alternative assumptions do not trigger any significant differences in the results of the model.

Finally, figure 3.1 presented an illustration of the distribution of the workers in the model in terms of employment status and employment history for a UIB scheme like the Danish that requires one period of employment. What might stand out is that, while it seems reasonable that the majority of the population are employed and have been so for the three previous years, the model suggests that 86% of the population are in such a situation (employed type $a(1 1 1)$). This seems quite high. However, when the model mimics a UIB scheme like the Danish, 97.2% of the employed workers are entitled to UIB, and as 91% of all insured and employed Danish workers are entitled to a full duration of UIB in case of unemployment, the model outcome is not that much larger. Furthermore, in the model 81.4% of the unemployed are SA recipients, for Denmark this share is around 15% to 40%. Hence, the model seems to overshoot the population shares of both possible extreme combinations of employment history, i.e. both the strongest (employed type $a(1 1 1)$) and the weakest (unemployed type $h(0 0 0)$) possible employment history. This is most likely due a lack of heterogeneity (e.g. in terms of productivity) and, perhaps more importantly, due to the simplification of

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19 It decreases marginally on three occasions (see the ** comment in fig. C1 in Appendix C).
20 See Fredriksson and Holmlund (2001) for a more thorough discussion of the importance of the preferences on consumption and leisure and the effect of it in connection with job search.
22 SA recipients amounted to 15% of the UIB recipients in Denmark in 2005 (own calculations based on DREAM data for 2005). A more recent estimate (but not in a business neutral year) is 39.75% (own calculations based on Statistic Denmark’s data for November 2016).
having only three periods to cover a period of three years.

5 CONCLUDING REMARKS

By explicitly tracking workers’ employment history in an OLG model of labor market participation with (random) search, I highlight the state contingencies created by requirements on employment history in UIB schemes. Specifically, I find that UIB recipients who maintain their current entitlement will respond to strengthened entitlement conditions by increasing their search effort significantly, while social assistance recipients will respond by decreasing their search effort marginally. Hence, strengthening entitlement conditions are found to introduce endogenous duration dependence, as it creates a discouraged worker effect where long-term unemployed (with a weak employment history) decrease their search effort marginal, while unemployed with a strong employment history intensify their search effort. The differences in job search responses highlight that entitlement conditions may have generational dependent implications; discouraging the young while encouraging the older. Furthermore, the different responses in search effort affect the composition of the workers such that the share of core troops of the labor market is increased, while the share of unemployed with consecutive periods of unemployment is reduced. Regarding the latter, one might expect the opposite as these unemployed decrease their search effort; however, as unemployed with stronger employment histories increase their search effort, the likelihood that they stay unemployed and develop a less strong employment history is reduced. Thus, the overall share of unemployed with a weak employment history is reduced by increasing the required employment history. However, workers who are unemployed after the entitlement conditions have been strengthened are, of course, affected negatively by the change. Moreover, if consumption and leisure are compliments the aggregate result of the changes in search effort is a decline in total unemployment. Additionally, theoretically the results may also be of interest in a business cycle context as a recession will weaken the employment histories of workers. Thus, entitlement conditions can in principal have automatic destabilizing effects since they induce endogenous duration dependence via a discouraged worker effect as long-term unemployed are induced to search less intensely for employment. However, it is worth stressing that the decreases in search effort are only marginal. The model presented is in many dimensions extremely simplified as it does not consider the demand side of the labor market and regards wages and taxes as exogenous. However, the simplifying assumptions make it possible to analyze the search responses of unem-
ployed when explicitly accounting for their employment histories. A relaxation of most of these simplifications is presented in Ellermann-Aarslev (2017a), which presents a GE approach to the issue, however, with the shortcoming that employment history is not explicitly tracked, as in the present paper, but approximated to deal with the multidimensionality of the state space. Furthermore, implementing worker heterogeneity is an interesting addition as differences in employment history and, e.g., heterogeneity of productivity might very well be correlated. Hence, the interaction of the two could be interesting to study. However, this is left for future research.
REFERENCES


APPENDICES

A ADDITIONAL MATERIAL

The remaining 14 value functions:

\[ V_{b,t}^E = u_{b,t}^E + \delta (\rho V_{c,t+1}^U + (1 - \rho) V_{c,t+1}^E) \]
\[ V_{b,t}^U = u_{b,t}^U + \delta (\alpha s_{b,t} V_{d,t+1}^E + (1 - \alpha s_{b,t}) V_{d,t+1}^U) \]
\[ V_{c,t}^E = u_{c,t}^E + \delta (\rho V_{e,t+1}^U + (1 - \rho) V_{e,t+1}^E) \]
\[ V_{c,t}^U = u_{c,t}^U + \delta (\alpha s_{c,t} V_{f,t+1}^E + (1 - \alpha s_{c,t}) V_{f,t+1}^U) \]
\[ V_{d,t}^E = u_{d,t}^E + \delta (\rho V_{g,t+1}^U + (1 - \rho) V_{g,t+1}^E) \]
\[ V_{d,t}^U = u_{d,t}^U + \delta (\alpha s_{d,t} V_{h,t+1}^E + (1 - \alpha s_{d,t}) V_{h,t+1}^U) \]
\[ V_{e,t}^E = u_{e,t}^E + \delta (\rho V_{a,t+1}^U + (1 - \rho) V_{a,t+1}^E) \]
\[ V_{e,t}^U = u_{e,t}^U + \delta (\alpha s_{e,t} V_{b,t+1}^E + (1 - \alpha s_{e,t}) V_{b,t+1}^U) \]
\[ V_{f,t}^E = u_{f,t}^E + \delta (\rho V_{c,t+1}^U + (1 - \rho) V_{c,t+1}^E) \]
\[ V_{f,t}^U = u_{f,t}^U + \delta (\alpha s_{f,t} V_{d,t+1}^E + (1 - \alpha s_{f,t}) V_{d,t+1}^U) \]
\[ V_{g,t}^E = u_{g,t}^E + \delta (\rho V_{e,t+1}^U + (1 - \rho) V_{e,t+1}^E) \]
\[ V_{g,t}^U = u_{g,t}^U + \delta (\alpha s_{g,t} V_{f,t+1}^E + (1 - \alpha s_{g,t}) V_{f,t+1}^U) \]
\[ V_{h,t}^E = u_{h,t}^E + \delta (\rho V_{g,t+1}^U + (1 - \rho) V_{g,t+1}^E) \]
\[ V_{h,t}^U = u_{h,t}^U + \delta (\alpha s_{h,t} V_{h,t+1}^E + (1 - \alpha s_{h,t}) V_{h,t+1}^U) \]

The optimal level of search effort for the remaining types:

\[ s_{b,t} = 1 - \frac{1}{\delta \alpha (V_{d,t+1}^E - V_{d,t+1}^U)} \]
\[ s_{c,t} = 1 - \frac{1}{\delta \alpha (V_{f,t+1}^E - V_{f,t+1}^U)} \]
\[ s_{d,t} = 1 - \frac{1}{\delta \alpha (V_{h,t+1}^E - V_{h,t+1}^U)} \]
\[
s_{e,t} = 1 - \frac{1}{\delta \alpha (V_{b,t+1} - V_{b,t+1}^U)}
\]
\[
s_{f,t} = 1 - \frac{1}{\delta \alpha (V_{d,t+1} - V_{d,t+1}^U)}
\]
\[
s_{g,t} = 1 - \frac{1}{\delta \alpha (V_{f,t+1} - V_{f,t+1}^U)}
\]
\[
s_{h,t} = 1 - \frac{1}{\delta \alpha (V_{h,t+1} - V_{h,t+1}^U)}
\]

The flow equations of the rest of the types:

\[
e_{c,t} = (1 - \rho) (e_{b,t-1} + e_{f,t-1})
\]
\[
e_{d,t} = \alpha s_{b,t-1} n_{b,t-1} + \alpha s_{f,t-1} n_{f,t-1}
\]
\[
e_{e,t} = (1 - \rho) (e_{c,t-1} + e_{g,t-1})
\]
\[
e_{f,t} = \alpha s_{c,t-1} n_{c,t-1} + \alpha s_{g,t-1} n_{g,t-1}
\]
\[
e_{g,t} = (1 - \rho) (e_{d,t-1} + e_{h,t-1})
\]
\[
e_{h,t} = \alpha s_{d,t-1} n_{d,t-1} + \alpha s_{h,t-1} n_{h,t-1}
\]
\[
n_{c,t} = \rho (e_{b,t-1} + e_{f,t-1})
\]
\[
n_{d,t} = (1 - \alpha s_{b,t-1}) n_{b,t-1} + (1 - \alpha s_{f,t-1}) n_{f,t-1}
\]
\[
n_{e,t} = \rho (e_{c,t-1} + e_{g,t-1})
\]
\[
n_{f,t} = (1 - \alpha s_{c,t-1}) n_{c,t-1} + (1 - \alpha s_{g,t-1}) n_{g,t-1}
\]
\[
n_{g,t} = \rho (e_{d,t-1} + e_{h,t-1})
\]
\[
n_{h,t} = (1 - \alpha s_{d,t-1}) n_{d,t-1} + (1 - \alpha s_{h,t-1}) n_{h,t-1}
\]
Figure A1: Convergence of the worker distribution: The shares of all types are constant for all generations who have not entered the labor market recently, independent of the assumption of the how the worker distribution looks for new entrants. For the youngest generations (the far right of the graph), the distributions fluctuate depending on starting assumption (holds for all four policies on the entitlement conditions).

Figure A2: Total aggregated search effort for the four possible policies concerning entitlement conditions, $T_{RE}$. 
<table>
<thead>
<tr>
<th>Type</th>
<th>Changes in the required employment history:</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$T_{RE} = 0 \rightarrow 1$</td>
</tr>
<tr>
<td>a (1 1 1)</td>
<td>Group 1</td>
</tr>
<tr>
<td>b (1 1 0)</td>
<td>Group 1</td>
</tr>
<tr>
<td>c (1 0 1)</td>
<td>Group 1</td>
</tr>
<tr>
<td>d (1 0 0)</td>
<td>Group 1</td>
</tr>
<tr>
<td>e (0 1 1)</td>
<td>Group 1</td>
</tr>
<tr>
<td>f (0 1 0)</td>
<td>Group 1</td>
</tr>
<tr>
<td>g (0 0 1)</td>
<td>Group 1</td>
</tr>
<tr>
<td>h (0 0 0)</td>
<td>Group 2</td>
</tr>
</tbody>
</table>

Table A.1: Members of the three groups of unemployed following the three possible changes of the entitlement conditions. $T_{RE}$ denotes the required sum of periods of employment during the reference period to be entitled to UIB.
B ALTERNATIVE CALIBRATION

Below are presented the key graphs of the model for the alternative calibration of a higher return to search and a higher job destruction rate compared to the main calibration, namely, $\alpha = 1.55$ and $\rho = 0.0178$. The qualitative results of the model are unaffected.

Figure B1: Alternative calibration: Search effort for a representative generation halfway through the career for the four possible entitlement policies with respect to the sum of the employment history in the reference period, $T_{RE}$. The black arrows indicate the increase in search effort of group one, the purple arrows indicate the decrease in search effort of group two, while the dark red arrows indicate the decrease in the search effort of group three.
Figure B2: Alternative calibration: The distribution of the total population in terms of types and employment states for the four possible policies concerning entitlement conditions, $T_{RE}$.

Figure B3: Alternative calibration: Total unemployment for the four possible policies concerning entitlement conditions.
Figure B4: Alternative calibration: Total aggregated search effort for the four possible policies concerning entitlement conditions, $T_{RE}$. Relative change following changes: $T_{RE} = 0 \rightarrow 1$: 3.45%, $1 \rightarrow 2$: 0.92%, $2 \rightarrow 3$: 0.77%.

Figure B5: Alternative calibration: Aggregated search effort of all types for the four possible policies concerning entitlement conditions, $T_{RE}$.
Figure B6: Alternative calibration: Changes in the size of the three groups of unemployed following the three possible changes concerning entitlement conditions.

Figure B7: Alternative calibration: Changes in the populational distribution following the three possible changes concerning entitlement conditions.
Figure B8: Alternative calibration: Relative changes in the popunational distribution following the three possible changes concerning entitlement conditions.
Figure B9: Alternative calibration: The effect of the entitlement conditions in terms of required employment history on the distribution of the unemployed in terms of benefit level and the costs of running the UI scheme.
C  ALTERNATIVE PREFERENCES

The effects of extending the required employment history for entitlement to UIB on
the search effort of unemployed depend on the preferences of the unemployed. The
outline of the differences is given in the table below. Basically, the important thing
is whether consumption and leisure are thought of as compliments or as substitutes.
As noted in the main section of the paper, if the cross derivative is non-negative, i.e.
consumption and leisure are assumed to be compliments, search effort increases sig-
ificantly for the majority of types, while it decreases marginally on three occasions
(see the ** comment in fig. C1). However, if the assumed functional form of the utility
function is such that the cross derivative is negative, i.e. consumption and leisure are
assumed to be substitutes, the search effort decreases drastically for all the unemployed
in group two. This difference in response is simply because, if consumption and leisure
are substitutes, the drop in the level of consumption of the unemployed in group two
following a change of the required employment history will increase the marginal util-
ity of leisure, hence, resulting in a lower level of search effort being optimal. However,
if consumption and leisure are compliments, the opposite is, of course, the case.
<table>
<thead>
<tr>
<th>Utility function</th>
<th>Group 1</th>
<th>Group 2</th>
<th>Group 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\frac{\partial u(c,t)}{\partial d_t} = 0$</td>
<td>Increases significantly</td>
<td>Increases significantly**</td>
<td>Decreases marginally</td>
</tr>
<tr>
<td>LOG</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>ISO-Elastic</td>
<td>Increases marginally</td>
<td>Increases significantly**</td>
<td>Decreases marginally</td>
</tr>
<tr>
<td>More risk av. than LOG ($\lambda = -0.5$)</td>
<td>Increases marginally*</td>
<td>Increases significantly**</td>
<td>Decreases marginally</td>
</tr>
<tr>
<td>Less risk av. than LOG ($\lambda = 0.5$)</td>
<td>Increases marginally*</td>
<td>Increases significantly**</td>
<td>Decreases marginally</td>
</tr>
<tr>
<td>CES ($\gamma + \rho = 1$)</td>
<td>Increases marginally*</td>
<td>Increases significantly**</td>
<td>Decreases marginally</td>
</tr>
<tr>
<td>CES ($\gamma + \rho &lt; 1$)</td>
<td>Increases marginally</td>
<td>Increases significantly</td>
<td>Decreases marginally</td>
</tr>
<tr>
<td>CES ($\gamma + \rho &gt; 1$)</td>
<td>Increases significantly</td>
<td>Decreases drastically</td>
<td>Decreases marginally</td>
</tr>
</tbody>
</table>

* Except the search effort of type $d (1 0 0)$ following a $T_{RE} = 0 \rightarrow 1$ change, of type $b (1 1 0)$ and $c (1 0 1)$ following $T_{RE} = 1 \rightarrow 2$, and of type $a (1 1 1)$ following $T_{RE} = 2 \rightarrow 3$, for which it increases significantly.

** Except the search effort of type $d (1 0 0)$ following $T_{RE} = 1 \rightarrow 2$ and of type $b (1 1 0)$ and $c (1 0 1)$ following $T_{RE} = 2 \rightarrow 3$, they all levels of search effort decrease marginally.

Figure C1: The effect of changes that strengthen the entitlement conditions on the levels of search effort of the three groups of unemployed.
Assuming the utility function to be CES instead of logarithmic:
\[
u(c, l) = \frac{(\bar{\omega}c^\rho + (1 - \bar{\omega}) l^\rho)^{\frac{1-\gamma}{\rho}}}{1 - \gamma}, \quad 0 < \bar{\omega} < 1, \quad \rho < 1, \quad \rho \neq 0, \quad \gamma \geq 0, \quad \gamma \neq 1\]

and so
\[
\frac{\partial^2 u(c, l)}{\partial c \partial l} = \frac{\partial^2 u(c, l)}{\partial l \partial c} = (1 - \gamma - \rho) \bar{\omega}c^{\rho-1}(1 - \bar{\omega}) l^{\rho-1}\left(\bar{\omega}c^\rho + (1 - \bar{\omega}) l^\rho\right)^{\frac{1-\gamma-2\rho}{\rho}} < 0, \text{ for } \gamma + \rho > 1.
\]

The optimal level of search effort, \(s^*\), now depends on the level of consumption:
\[
-(1 - \bar{\omega}) (1 - s^*)^{\rho-1}(\bar{\omega}c^\rho + (1 - \bar{\omega})(1 - s^*)^\rho)^{\frac{1-\gamma-\rho}{\rho}} = 0
\]

Below, the outcome of the model is presented when the preferences of the workers follow a CES utility function instead of the log-utility of the main section of the paper. Changing the preferences changes the levels of the model, and so total unemployment now does not correspond to the Danish 2005-level. The parameter values used in the following are \(\gamma = 10, \rho = 0.66, \text{ and } \bar{\omega} = 0.75\), and so \(\frac{\partial u(c, l)}{\partial c \partial l} < 0\) (i.e. consumption and leisure are assumed to be substitutes). The basic insight of this exercise is that if workers prefer consumption and leisure as substitutes, the search effort of all the unemployed in group two decreases drastically, which has implications for the aggregated outcomes of the model. In the calibration presented below, this decrease of search effort is so large that total unemployment increases, opposed to the situation with log-preferences. Also, group two increases drastically as a result of increasing the required employment history from zero to one year of employment, and likewise for group three following changes that strengthen the requirements even further. However, the costs of running the UI scheme does decrease following such changes of entitlement conditions, as the distribution of the unemployed changes sufficiently such that more receive only SA. Note that for a lower CES coefficient of relative risk aversion, \(\gamma\), the drop in search effort of group two is not large enough to increase the aggregate level of unemployment.
Figure C2: CES utility: Search effort of a representative generation halfway through the career for the four possible entitlement policies with respect to the sum of the employment history in the reference period, $T_{RE}$. The black arrows indicate the increase in search effort of group one, the purple arrows indicate the decrease in search effort of group two (which is opposite to the LOG-case), while the dark red arrows indicate the decrease in the search effort of group three.

Figure C3: CES utility: The distribution of the total population in terms of types and employment states for the four possible policies concerning entitlement conditions, $T_{RE}$. 

39
Figure C4: CES utility: Total unemployment for the four possible policies concerning entitlement conditions.

Figure C5: CES utility: Total aggregated search effort for the four possible policies concerning entitlement conditions, $T_{RE}$. Relative change following changes: $T_{RE} = 0 \rightarrow 1$: 1.94%, $1 \rightarrow 2$: 0.21%, $2 \rightarrow 3$: -0.35%.
CHAPTER 1.
JOB SEARCH RESPONSES TO ENTITLEMENT CONDITIONS IN
UNEMPLOYMENT INSURANCE

C. ALTERNATIVE PREFERENCES

![Chart 1: Aggregate search effort for all types](image1)

**Figure C6:** CES utility: Aggregated search effort of all types for the four possible policies concerning entitlement conditions, $T_{RE}$.

![Chart 2: Changes in the size of the three groups of unemployed](image2)

**Figure C7:** CES utility: Changes in the size of the three groups of unemployed following the three possible changes concerning entitlement conditions.
Figure C8: CES utility: Changes in the populational distribution following the three possible changes concerning entitlement conditions.

Figure C9: CES utility: Relative changes in the populational distribution following the three possible changes concerning entitlement conditions.
Figure C10: CES utility: The effect of the entitlement conditions in terms of required employment history on the distribution of the unemployed in terms of benefit level and the costs of running the UI scheme.
D Alternative initial distributions

Below, the key figures of the results in case of different assumptions on the initial distribution are presented. Note that the utility function is assumed to be logarithmic, and the search effort is not dependent on assumptions on the initial distribution. Comparing to the initial distribution assumed in the paper, where all start as unemployed type $h(0 0 0)$, changing it brings no significant differences to the results of the model.

Assuming that all workers start as ’graduates’, with one period of recorded previous labor market participation, i.e. all start as unemployed type $g(0 0 1)$, the key outcome of the model is as follows:

Figure D1: All start as unemployed type $g(0 0 1)$: The distribution of the total population in terms of types and employment states for the four possible policies concerning entitlement conditions, $T_{RE}$. 
CHAPTER 1.
JOB SEARCH RESPONSES TO ENTITLEMENT CONDITIONS IN
UNEMPLOYMENT INSURANCE

D. ALTERNATIVE INITIAL DISTRIBUTIONS

Figure D2: All start as unemployed type $g(0\ 0\ 1)$: Total unemployment for the four possible policies concerning entitlement conditions.

Figure D3: All start as unemployed type $g(0\ 0\ 1)$: Aggregated search effort of all types for the four possible policies concerning entitlement conditions, $T_{RE}$. 
Figure D4: All start as unemployed type $g$ (0 0 1): Changes in the size of the three groups of unemployed following the three possible changes concerning entitlement conditions.

Figure D5: All start as unemployed type $g$ (0 0 1): Changes in the populational distribution following the three possible changes concerning entitlement conditions.
Figure D6: All start as unemployed type $g \langle 0\ 0\ 1 \rangle$: Relative changes in the populational distribution following the three possible changes concerning entitlement conditions.
The extreme case where all workers start their career as unemployed with a ‘perfect’ employment history, i.e. as unemployed type $a (1 \ 1 \ 1)$, follows below.

Figure D7: All start as unemployed type $a (1 \ 1 \ 1)$: The distribution of the total population in terms of types and employment states for the four possible policies concerning entitlement conditions, $T_{RE}$.

Figure D8: All start as unemployed type $a (1 \ 1 \ 1)$: Total unemployment for the four possible policies concerning entitlement conditions.
CHAPTER 1.

JOB SEARCH RESPONSES TO ENTITLEMENT CONDITIONS IN UNEMPLOYMENT INSURANCE

D. ALTERNATIVE INITIAL DISTRIBUTIONS

Figure D9: All start as unemployed type $a (1\ 1\ 1)$: Aggregated search effort of all types for the four possible policies concerning entitlement conditions, $T_{RE}$.

Figure D10: All start as unemployed type $a (1\ 1\ 1)$: Changes in the size of the three groups of unemployed following the three possible changes concerning entitlement conditions.
Figure D11: All start as unemployed type $a (1 \ 1 \ 1)$: Changes in the populational distribution following the three possible changes concerning entitlement conditions.

Figure D12: All start as unemployed type $a (1 \ 1 \ 1)$: Relative changes in the populational distribution following the three possible changes concerning entitlement conditions.
CHAPTER 1.

JOB SEARCH RESPONSES TO ENTITLEMENT CONDITIONS IN
UNEMPLOYMENT INSURANCE

D. ALTERNATIVE INITIAL DISTRIBUTIONS
Eligibility conditions based on the employment history of the unemployed are part of most unemployment insurance benefit (UIB) schemes and they potentially mitigate moral hazard issues. By explicitly allowing UIB to depend on past employment history in a DMP model, I find that implementing such conditions introduce a discouraged worker effect where long-term unemployed (with a weak employment history) decrease their search effort, while unemployed with a strong employment history intensify their search effort, i.e. an endogenous duration dependence. The model is able to determine the net effects of the individual job search responses and takes into account the effects on wages, taxes, and vacancy posting. When calibrated to the Danish economy, unemployment decreases. Hence, stricter conditions on employment history are found to work as a substitute for lowering the generosity of UIB schemes and are, in terms of utilitarian welfare, optimal to introduce. Conditions on past employment history can therefore act as an additional instrument when designing UI systems and, if needed, reduce the overall generosity without reducing the level or the duration.

**JEL Classification:** H3, J22, J64, J65

**Keywords:** Unemployment; Unemployment insurance; UI eligibility; Reentitlement effects; Search; Matching
1 Introduction

The economic literature on unemployment insurance benefit (UIB) schemes is quite vast and profound\(^1\). However, as exhaustive as it might appear, one aspect of such schemes that is still fairly uncovered is the importance of eligibility conditions. As Fredriksson and Holmlund (2001) phrase it: “A complete welfare analysis of UI policies would also have to consider the eligibility rules and how these affect behavior on both sides of the labor market. Existing systems require that workers have demonstrated some attachment to the labor force, for example, in the form of a minimum number of weeks in employment over the past year. A normative analysis of these issues in an equilibrium framework remains a topic for future research.”\(^2\) While UI eligibility conditions generally include requirements on the search activity, reasons to turn down a job offer, participation in active labor market programs, and monitoring- and sanction conditions the eligibility conditions under study here is conditions on previous employment history. For a comprehensive review of eligibility conditions in the UI systems of all OECD countries, see Venn (2012).

Most UIB schemes include eligibility conditions based on previous employment history. That is, UIB is dependent on past employment history. To give a few examples, the Danish UIB scheme includes 52 weeks of required employment during a reference period of three years.\(^3\) In Germany the minimum required employment is as the Danish system, but the reference period is only two years, and the system is generally more dynamic as the maximum UIB duration increases with the employment history.\(^4\) The Swedish system includes 26 weeks of “part-time” employment during a reference period of 52 weeks, while Norway has a monetary requirement (120,000 DKK of income in the past year, or 80,000 DKK on average in the past three years).\(^5\) Such history dependence introduces state contingency across otherwise identical workers. Accounting for this state contingency is not straightforward. This paper explicitly does so in a setting of a two-tier UI system.

Mortensen (1977) is the first to identify the entitlement effect as an outcome of a UIB scheme with finite benefit duration. This finite nature introduces a support cliff as unemployed losing entitlement will experience a significant drop in consumption possi-

\(^1\)See Fredriksson and Holmlund (2006) for a review of (parts of) this literature.
\(^3\)See Appendix A for a brief introduction to the recent changes in the Danish UIB system.
\(^4\)One year of employment in the two-year reference period results in entitlement to half a year of UIB, while two years of employment (in the reference period) entitle a person to one year of UIB (Bjørn and Høj (2014) p. 13).
bilities. Hence, unemployed who have already lost or are about to lose entitlement will experience an increase in the gain of employment if employment automatically qualifies one to a new full duration of UIB. But what if employment does not automatically qualify one for full UIB entitlement? What if UIB is history dependent? Ortega and Rioux (2010) model non-automatic eligibility in a general equilibrium search-matching model and find entitlement effects through wage differentials as Mortensen (1977). However, they do not take the endogeneity of the search effort of unemployed into account. In a setting like Ortega and Rioux (2010), Andersen et al. (2015) endogenize search effort and find that the more difficult it is to get re-entitled to UIB, the more intensely does eligible unemployed search, while non-entitled unemployed decrease their search effort. However, they (as Ortega and Rioux (2010)) only consider entitlement to UIB as binary; i.e. either you are entitled or not. Wagner and Jahn (2010) relax the binary approach to eligibility but take search effort as exogenous, and find effects through the wages (as Mortensen (1977) and Ortega and Rioux (2010)), which they find decrease until gained benefit entitlement. This paper considers search effort as endogenous and introduces UIB dependency of the employment history, which makes the question of entitlement more realistic as the workers’ different ways of obtaining UIB entitlement is taken into account. That is, if such dependency were not present, two workers with different employment history (and otherwise identical) would have the same chance of obtaining UIB entitlement. Instead, including employment history dependency, the worker with the weakest employment history needs a longer future employment duration than the other to qualify for UIB. Ellermann-Aarslev (2017b) consider an OLG model where the workers’ employment history is explicitly tracked and find that search efforts of unemployed is affected differently depending on the strength of the employment history. This paper builds on the state contingent findings of Ellermann-Aarslev (2017b), but offers a GE approach which allows for an aggregation of the individual state-contingent effects. Other papers worth mentioning include Coles and Masters (2007), who consider how entitlement conditions affect hiring incentives. They show that conditions on employment history can imply an implicit transfer from future potential employers to current employers as eligibility increase a worker’s reservation wage in a future wage bargaining. Hopenhayn and Nicolini (2009) characterize the optimality of a UIB scheme in a one-sided repeated moral hazard principal-agent model where the workers’ employment history is taken into account. They find that it is optimal to let unemployment benefits increase in the

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6This channel of effects is not present in the model presented here. Taking it into account would, however, only amplify the wage effects of this model. Hence, it would strengthen the effects of conditions on employment history.
employment history if quits cannot be distinguished from layoffs. In a similar setting Pan and Zhang (2012) find that entitlement conditions serve to adjust the incentives of workers and mitigate moral hazard quits. The potential benefits of entitlement conditions on workers’ employment history is quite significant as Faig and Zhang (2012) show, in a simulation, that the US average unemployment rate would decrease by one percentage point following implementation of optimal entitlement conditions.

Most UIB systems are formed on a base of two tiers of benefits - in a wide sense: high level benefits attainable for a certain duration under certain conditions, and another lower level of benefits attainable hereafter (given that one is sufficiently needy). Therefore, the UIB system considered in this paper is also two-tiered with certain employment history specific entitlement requirements associated with the highest level of benefits. This basically covers the UI systems in most countries, and so, the differences can be adjusted for in the specific parameterization. The benefits at the lowest level, denoted social assistance (SA) from now on, are unconditional. The high level benefits are denoted unemployment insurance benefits (UIB).

By taking entitlement conditions on employment history into account, two dimensions change as time passes: The likelihood of qualifying to a new full duration of UIB in terms of employment history and the current remaining duration of UIB. That is, as time passes, workers are affected by two forces: The fact that the duration of UIB is finite, i.e. the basic effect of Mortensen (1977), and secondly, the entitlement conditions on past employment. To capture these two effects, the model explicitly accounts for both the employment history of the workers and their remaining duration of UIB.

Obviously, employment history is multidimensional in the sense that the composition of the employment is important; that is, employment in the distant past is most likely less important than employment in the immediate past. In the Danish system, e.g., this is underlined by the fact that only employment in a certain reference period counts in the balance of obtaining eligibility to UIB. Despite of this, the paper compresses the employment history into a one-dimensional variable denoted $h$. However, it takes the stand of the Danish and most other European UIB systems of a reference period.

The structure of the paper is as follows. Below in section 2 I present the primary theoretical model of the paper. In section 3 I solve for the equilibrium outcome of this model, and analyze the key dynamics of it and how these are influenced by the specific design of the UIB system including the key component of the model, namely, the conditions on past employment history. Furthermore, in section 4 I investigate the importance of some key assumptions, and an alternative UIB system with declining benefits is given brief attention. Finally, I conclude in section 5.
2 THE MODEL

The framework of the model is a discrete time Diamond-Mortensen-Pissarides (DMP) model with endogenous search. A two-tiered public benefit system exists, where unemployed entitled to UIB receive a higher fraction of the overall wage level, denoted $b_{UI}$, than the not entitled unemployed, who receive SA, a wage fraction denoted $b_{SA}$.\footnote{It is implicitly assumed that all workers are non-voluntary members of the insurance scheme and that the costs of membership are zero.}

Hence, agents differ with respect to employment and entitlement status as in previous papers, e.g. Fredriksson and Holmlund (2001, 2006), Andersen et al. (2015), and Ortega and Rioux (2010). What is new in this paper, however, is that I take into account the state contingency of otherwise identical agents induced by the entitlement conditions of the UIB scheme. Both the employment history and the duration of remaining UIB are included in the model as state variables, where the employment history is denoted $h$ and the remaining UIB duration is denoted $m$. The updating of the two is considered below in section 2.1. Expressing the benefit replacement rate as a function of the remaining UIB duration, $m$ (where $m$ is influenced by $h$ as described in the second paragraph below):

$$b(m_t) = \begin{cases} 
  b_{UI} & \text{if } m_t > 0 \\
  b_{SA} & \text{if } m_t = 0 
\end{cases}$$

The remaining duration of UIB is naturally limited by the zero lower bound and by the politically decided maximum duration of UIB, denoted $D$, as the upper bound. Thus, $m \in [0, D]$. It is also intuitive that the employment history should be bounded below by zero, when an employment history of zero would indicate no previous relevant employment. The upper bound of the employment history is decided politically just as the upper bound of the remaining duration of UIB. In most UIB schemes the employment history consists of employment in a certain reference period (see the introduction for specific descriptions). Hence, the employment status previous to this reference period does not influence the current entitlement status. This suggests that the employment history should be bounded above, (e.g. by a level corresponding to three years when calibrating the model to the Danish system). Motivated by this, it is assumed that the employment history is bounded above by some level denoted $h_{max}$. Thus, $h \in [0, h_{max}]$. The larger $h_{max}$, the larger is the share of the past employment record taken into account when UI eligibility is evaluated.

Entitlement to a new full duration of UIB in case of unemployment depends on whether the employment history, $h$, is sufficiently strong; i.e. particularly whether
it is below or above a certain threshold level denoted $c$. This is the key parameter of interest denoting the strictness of the entitlement conditions. If the requirement is satisfied, workers are entitled to a (new) full period of UIB of maximum duration, $D$. Thus, the threshold level of employment history can be seen as a goal line which the workers need to cross in order to earn (full) entitlement. Given that a worker experience unemployment and satisfies the requirement, the worker redeems the employment history in exchange for a new full duration of UIB, while the accumulation of employment history restarts as a new reference period is started. That is, the worker’s employment history is subsequently reset to zero in case of unemployment, and the path to re-entitlement starts from scratch. Putting it differently, say an employed worker at time $t$ has an employment history of $h_t$. If that worker loses her job in the next period, she will be entitled to the maximum duration of UIB, $m_{t+1} = D$, irrespectively of her previous amount of remaining UIB duration. Once she enters unemployment, she redeems her strong employment history and automatically starts a new reference period. Hence, her employment history is reset to zero, $h_{t+1} = 0$.

Job destruction occurs at an exogenous rate, denoted $\rho$, as is the common modeling assumption in the literature (see e.g. Fredriksson and Holmlund (2006), Ortega and Rioux (2010), and Andersen et al. (2015)).

2.1 Updating of the two underlying variables

In what follows, the specific assumptions on how the employment history, $h$, and the remaining UIB duration, $m$, evolve are described. For a general formulation of the updating functions, see Appendix B.

The employment history is considered as the history of ‘net’ employment, where $h$ increases in case of employment and decreases in case of unemployment. That is, when time progresses one period, the employment history, $h$, of workers who were employed in the previous period increases to $h + I$, while that of the workers who were unemployed decreases to $h - I$, where $I$ is the period length. If the next period of employment history satisfies the requirements, $h + I \geq c$, the worker will be entitled to UIB. Hence, the worker redeems the employment history and start a new reference period, causing the subsequent employment history to be reset to zero, $h_{t+1} = 0$, in case of unemployment.
2. THE MODEL

CHAPTER 2.

GENERAL EQUILIBRIUM EFFECTS OF ENTITLEMENT CONDITIONS IN UNEMPLOYMENT INSURANCE

\[ h_t = \begin{cases} 
H (h_{t-1}) & \text{if } H (h_{t-1}) < c \\
0 & \text{or no transition to unemp.}
\end{cases} \]

\[ H (h_{t-1}) = \begin{cases} 
h_{t-1} + I & \text{if emp. at } t - 1 \\
h_{t-1} - I & \text{if unemp. at } t - 1
\end{cases} \]

where

The remaining duration of UIB, \( m_t \), stays constant in case of employment, as no UIB is ‘used’, but decreases if the worker becomes unemployed. If the employment history is sufficiently strong \((h \geq c)\), the worker qualifies for a (new) full period of UIB. In this case, \( m \) does not increase to \( D \) until the worker actually moves into unemployment. This is due to technical reasons and has no implications on the outcome of the model.

\[ m_t = \begin{cases} 
M (m_{t-1}) & \text{if } H (h_{t-1}) < c \\
D & \text{or no transition to unemp.}
\end{cases} \]

\[ M (m_{t-1}) = \begin{cases} 
m_{t-1} & \text{if emp. at } t - 1 \\
m_{t-1} - I & \text{if unemp. at } t - 1
\end{cases} \]

Note that the state variables can never evolve outside of the sets they are defined upon, that is, at the boundaries of \( h \) and \( m \) the functions determining their updated value are assumed to return the boundary values, i.e. \( H (h_{\text{max}}) = h_{\text{max}}, H (0) = 0 \), and \( M (D) = D, M (0) = 0 \).

Figure 2.1 illustrates the updating of the two underlying variables by presenting two scenarios: 1) An individual who re-gains entitlement (left figure), and 2) An individual who loses entitlement (center figure). In both scenarios, the individual has just been separated from previous employment and is entitled to a full duration of UIB. Hence, in both scenarios the individual has redeemed the employment history and starts out as unemployed with a maximal remaining duration of UIB, \( m = D \), and an employment history which has been reset to zero as a new reference period has been initiated, i.e. \( h = 0 \). The first spell of unemployment, \( U_1 \), makes \( m \) depreciate, while \( h \) is at its lower bound and thus, unchanged as it cannot decrease below this lower bound.
A short spell of employment, $E_1$, increases $h$, while $m$ is unaffected. The subsequent unemployment spell, $U_2$, decreases both $h$ and $m$. Now, the two scenarios differ in that the following employment spell, $E_2$, differs in length. In scenario 1, $E_2$ is so lengthy that $h$ increases above the requirement for re-entitlement, $c$; i.e. the worker crosses the ‘goal line’. This has the result that any potential future unemployment will begin at the ‘starting point’ of $m = D$ and $h = 0$ (illustrated by the dashed arrow). However, if $E_2$ is not sufficiently long (scenario 2) subsequent unemployment, $U_3$, will exhaust UIB, and the worker will end up on SA. In the first part of the spell, $U_{3.1}$, $m$ is positive, which makes the benefits equal the highest level, $b = b_{UI}$. In the last part, $U_{3.2}$, $m = 0$, and so the individual receives SA, $b = b_{SA}$. The right figure illustrates the situation of automatic entitlement (as in e.g. Mortensen (1977)). Here the first spell of employment, irrespective of the length, qualifies the worker to a new full duration of UIB, i.e. $m = D$. The value of $h$ is unimportant in this situation as no conditions on employment history exist.

Figure 2.1: Illustration of the two scenarios (left and center figure) and the standard case of automatic entitlement (right figure).

### 2.2 The supply side

Workers gain utility from consumption and leisure, and it is assumed that the functional form of the utility function is logarithmic. As time is discrete, the discount factor is $\beta = \frac{1}{1+r}$. Furthermore, employment has a time requirement denoted $l_w$. The income tax level is denoted $\tau$, and $w$ is the real wage of employment.
Generally, a worker’s value of being in a specific state equals the instantaneous utility of this state plus the discounted expected value of the next period. The value of employment while not entitled to a full duration of UIB in case of unemployment is as follows below. Technically, this situation is with an employment history of \( h \) such that \( h + I < c \), and a remaining entitlement to UIB equal to \( m \) such that \( m < D \).

\[
E(h, m) = \log((1 - \tau) w) + \log(1 - l_w) + \beta [\rho U(h + I, m) + (1 - \rho) E(h + I, m)]
\]

If the next period of employment history satisfies the requirements, \( h + I \geq c \), then, in case of unemployment, the workers are entitled to a full duration of UIB, \( m = D \), and they will start a new reference period which causes the subsequent employment history to be reset to zero, \( h = 0 \). In this situation the value of employment is:

\[
E(h, m) = E^{full} \text{ for } h \geq c \text{ and } \forall m
\]

\[
E^{full} = \log((1 - \tau) w) + \log(1 - l_w) + \beta [\rho U(0, D) + (1 - \rho) E^{full}]
\]

The value of unemployment is the following, where \( \alpha (\theta) \) is the return to search.

\[
U(h, m) = \log((1 - \tau) b(m) w) + \log(1 - s(h, m)) + \beta [\alpha (\theta) s(h, m) E(h - I, m - I) + (1 - \alpha (\theta) s(h, m)) U(h - I, m - I)]
\]

The level of search effort is chosen to maximize the value of unemployment. Hence, the optimal level of search effort is given by the following FOC:

\[
s(h, m) = 1 - \frac{1}{\alpha (\theta) \beta [E(h - I, m - I) - U(h - I, m - I)]}
\]

2.3 The demand side

The demand side of the model is standard DMP. Risk-neutral one-worker firms post vacancies which, if filled, produce \( y \) output per period. Flow costs of a vacant job are expressed as a fraction, \( k \), of output. Entry of firms is unhindered; i.e. free entry of firms apply. Like the supply side of the model, the demand side is expressed in value functions. The value of a filled job, \( J \), equals instantaneous profit plus the discounted expected value of the next period.

\[
J = y - w + \beta [\rho V + (1 - \rho) J]
\]
The value of a vacant job position equals the negative effect of the flow costs of posting it plus the discounted expected value of the position in the following period, where \( q(\theta) \) is the vacancy filling rate.

\[
V = -ky + \beta [q(\theta)J + (1 - q(\theta))V]
\]

Free entry of risk-neutral firms implies that the value of a vacant job position is zero, \( V = 0 \). Hence, firms enter the market until the negative outlay of posting the position this period is equalized by the discounted expected value of the position in the subsequent period.

\[
ky = \beta q(\theta)J
\]

Combining this with the expression for the value of a filled job, the classic DMP relationship between the wage rate and labor market tightness, \( \theta \), arrives.

\[
w = \left[ 1 - (r + \rho) \frac{k}{q(\theta)} \right] y \quad (2.1)
\]

### 2.4 Matching

Matching at the labor market is random\(^8\), where the matching function takes total search effort of the unemployed and the number of vacant job positions as inputs and returns the number of matches for each period. The specific assumed functional form is Cobb-Douglas.

\[
m(s,v) = As^{1-\eta}v^{\eta}
\]

This implies that the return to search and the vacancy filling rate are as follows:

\[
\alpha(\theta) = \frac{m(s,v)}{s} = m(1,\theta) = A\theta^n
\]

\[
q(\theta) = \frac{m(s,v)}{v} = m(\theta^{-1},1) = A\theta^{n-1}
\]

where \( \theta = \frac{v}{s} \) is the labor market tightness, \( v \) is the amount of vacancies, \( s \) is the total search effort, and \( A \) is a constant expressing efficiency of the matching technology.

Using this insight in connection with the relationship between the wage rate and labor market tightness derived from the demand side, (2.1), the following equilibrium

\(^8\)As in, e.g., Ortega and Rioux (2010) and Andersen et al. (2015).
expression for labor market tightness arise:

$$\theta = \left( \frac{r + \rho}{A} \frac{ky}{y - w} \right)^{\frac{1}{\tau}}$$  (2.2)

For exogenous wages, (2.2) above implies that the labor market tightness is constant. That is, the free entry of firms makes the number of vacancies adjust to any changes in the search effort of the unemployed. Obviously, endogenous wages imply that labor market tightness is also endogenous.

### 2.5 Wage determination

Considering endogenous wages, the decision about which specific bargaining process to assume is non-trivial. Following Shimer (2005) Nash bargaining might result in too responsive wages. Oppositely, credible bargaining of Hall and Milgrom (2008) might generate more realistic outcomes, cf. Petrosky-Nadeau and Zhang (2013) and Christiano et al. (2015). However, as Nash bargaining is the most commonly used endogenous wage setting, I follow this path. To measure the effect hereof, I continually compare it to a situation of fixed wages. Assuming Nash bargaining, an assumption on the outside option of the workers has to be made. This illustrates that wages, if considered individually, might differ for workers with different employment history as the imposed history dependence of the UI system affect their outside option differently. While such individual wage effects are interesting to analyze, wages are considered non-varying at an individual level for simplicity. For a relaxation of this assumption, see section 4.1. Note that, e.g. in a Danish context, the majority of the population is entitled to UIB for the maximal duration in case of unemployment. Motivated by this, it is assumed that the bargaining outside option of all workers, independent of their employment history and the remaining period of UIB, is the value of unemployment with entitlement to a full duration of UIB, \(U(0, D)\). Hence, by the principles of Nash bargaining, the wage rate is the following:

$$w = \arg \max \left\{ \left( \frac{E^{full} - U(0, D)}{J - V} \right)^{\beta_{Nash}} (J - V)^{1 - \beta_{Nash}} \right\}$$

Solving this yields (taking \(U(0, D)\) as given; i.e. assuming that a potential new

---

9 See e.g. Hall and Milgrom (2008) and Shimer (2005).
10 Rogerson et al. (2005)
11 91% of all insured and employed Danish workers are entitled to a full duration of UIB in case of job loss (data from 2012, De Økonomiske Råd (2014) pp. 238-239).
12 See e.g. Rogerson et al. (2005) pp. 968-969 for a short introduction.
bargaining game involves a new firm):

\[
\beta_{Nash} \frac{\partial E^{full}}{\partial w} \frac{\partial J}{\partial w} + (1 - \beta_{Nash}) \frac{\partial J}{\partial J} = 0
\]

The partial change in the value of a filled job is

\[
\frac{\partial J}{\partial w} = \frac{\partial}{\partial w} \left[ \frac{1 + r}{r + \rho} (y - w) \right] = -\frac{1 + r}{r + \rho}
\]

The partial changes from wage changes in the value of employment and full entitlement are:

\[
\frac{\partial E^{full}}{\partial w} = \frac{1}{w (1 - \beta (1 - \rho))}
\]

Hence, the Nash wage bargaining results in the following wage rate:

\[
w = \frac{y}{(1 - \beta_{Nash}) \frac{1}{(1 - \beta (1 - \rho)) (E^{full} - U (0, D))} + 1}
\]

This results in the following labor market tightness:

\[
\theta = \left( \frac{r + \rho}{A} k \left( 1 + \frac{(1 - \beta_{Nash})}{\beta_{Nash}} \frac{1}{(1 - \beta (1 - \rho)) (E^{full} - U (0, D))} \right) \right)^{\frac{1}{\pi - 1}}
\]

2.6 Steady state worker distribution

The notation is as follows: \( e (h, m) \) denotes the share of workers who are employed and have an employment history of \( h \) and, in case of unemployment in the period to follow, are entitled to unemployment insurance benefits for a remaining duration of \( m \). Similarly, \( u (h, m) \) denotes the corresponding share of the population, who are unemployed. Skipping the subindices, \( e \) and \( u \) denote total employment and total unemployment, respectively. Furthermore, \( e \) and \( u \) denote the \((h \times m)\) matrices covering the entire distribution of employed and unemployed, respectively.

\[
e = \begin{pmatrix}
e (0, 0) & e (0, 1) & e (0, 2I) & \cdots & e (0, D) \\
e (I, 0) & \ddots & \ddots & \vdots \\
e (2I, 0) & \ddots & \ddots & \vdots \\
\vdots & \ddots & \ddots & \vdots \\
e (h_{max}, 0) & \cdots & \cdots & \cdots & e (h_{max}, D)
\end{pmatrix}
\]
Vectorizing $e$ and $u$ and denoting these $\mathbf{ve}$ and $\mathbf{vu}$\(^{13}\), the steady state worker flows of the economy are given as follows, where the $P^e_e$ is the transition matrix denoting the transition rates of all the states of employment into all the different states of unemployment (similarly for $P^u_e$, $P^e_u$ and $P^u_u$):

$$
\mathbf{u} = \\
\begin{pmatrix}
  u(0, 0) & u(0, I) & u(0, 2I) & \cdots & u(0, D) \\
  u(I, 0) & \ddots & \ddots & \vdots \\
  u(2I, 0) & \ddots & \ddots & \ddots \\
  \vdots & \ddots & \ddots & \ddots \\
  u(h_{\text{max}}, 0) & \cdots & \cdots & \cdots & u(h_{\text{max}}, D)
\end{pmatrix}
$$

Writing these systems of equations as one and using the following definitions $\pi \equiv \begin{pmatrix} \mathbf{ve} \\ \mathbf{vu} \end{pmatrix}$, $P \equiv \begin{pmatrix} P^e_e & P^u_e \\ P^e_u & P^u_u \end{pmatrix}$, and $T \equiv I - P$ yield

$$
\pi = P\pi \iff T\pi = 0
$$

Now, to solve the system we need to impose a normalization, because the system exhibits linear dependency as all states of the workers’ state space are connected by transition rates in the interval $[0, 1]$. Specifically, the total population is normalized such that it equals one, $e + u = 1$. This implies that the population share of one state can be derived from the sum of the population in all other states.\(^{14}\)

The transition matrices are defined such that the column defines which state the transition is from, and the row defines which state the transition is to. Denoting $(x, y)$ as the $(h, m)$-pair of the state the transition is to, and likewise $(j, l)$ as the $(h, m)$-pair of the state the transition is from, the transitions are

\(^{13}\)Formally, $\mathbf{ve} = \text{vec}(e)$ and $\mathbf{vu} = \text{vec}(u)$.

\(^{14}\)Technically, the steady state worker distribution is solved relative to one state; i.e. the share of workers in one of the states is held constant. The choice of the particular state is irrelevant to the solution because of the normalization of the total population. Since the particular state is held constant, it does not need to be included in the $\pi$ matrix for which the system is solved. Also, the corresponding equation is not included. This means that the first column of $T$ work as a constant, which in turn has the result that the system has a unique solution if the modified matrix $T$ is non-singular and hence invertible. The equilibrium share of workers in the state held constant can be found subsequent as the total population is adjusted to equal one.
\[
P_{e(j,l)} = \begin{cases} 
1 - \rho & \text{if } j + I = x \text{ and } l = y \\
0 & \text{otherwise}
\end{cases}
\]

\[
P_{u(j,l)} = \begin{cases} 
0 & \text{if } j + I = x \text{ and } l = y \\
1 & \text{otherwise}
\end{cases}
\]

\[
P_{e(j,l)} = \begin{cases} 
\rho & \text{if } j - I = x \text{ and } l - I = y \\
0 & \text{otherwise}
\end{cases}
\]

\[
P_{u(j,l)} = \begin{cases} 
1 - \alpha(\theta)s(j,l) & \text{if } j - I = x \text{ and } l - I = y \\
0 & \text{otherwise}
\end{cases}
\]

Furthermore, there are some modifications of the above due to the boundaries of the two underlying variables, \( h \) and \( m \). Namely, that there is no inflow to \( e(h,D) \) \( \forall h \), to \( u(h,D) \) \( \forall h > 0 \), and (from any \( e \)-state) to \( u(h \geq c, m) \) \( \forall m \). Also, entitled workers start a new reference period and get their employment history reset to zero if they enter unemployment, hence, \( P_{u(h,D)} = \rho \forall m \).

### 2.7 Public sector

The government funds the benefits to the unemployed and other government expenditures, \( G \), by imposing a proportional tax on labor, \( \tau \). Furthermore, the labor tax endogenously adjusts to changes in the expenditures.

\[
\tau w_e + \tau b_{U1}w_{U1} + \tau b_{SA}w_{SA} = b_{U1}w_{U1} + b_{SA}w_{SA} + G
\]

\[
\Downarrow
\]

\[
\tau = \frac{b_{U1}w_{U1} + b_{SA}w_{SA} + \frac{G}{w}}{(e + b_{U1}w_{U1} + b_{SA}w_{SA})}
\]

Note that the tax is non-distorting as both wage income and transfer payments are taxable, and both the endogenous search effort decision and the wage outcome of the Nash bargaining depend on the value gain of employment, i.e. the difference in the utility of employment and that of unemployment. Since utility is assumed to be logarithmic, the tax rate cancels out in this difference. Hence, other government expenditures, \( G \), do not affect the outcome of the model other than the level of the tax rate and the utility.

In section 4.2 an analysis of the model if benefits are non-taxable is presented.
3 Results

As the complexity of the model precludes the possibility of attaining analytical results, I solve the model using numerical solution methods. In terms of calibration strategy, I follow the literature, primarily Ortega and Rioux (2010), and calibrate the model for Denmark.\textsuperscript{15}

The UIB system considered here has three policy instruments: The benefit level, $b_{UI}$, the maximum duration, $D$, and the strictness of the conditions on past employment history, $c$. The literature concerning optimal UI design is rich and diverse in terms of benefit level and maximum duration (see e.g. Fredriksson and Holmlund (2001) and Fredriksson and Holmlund (2006)). The possibility of analyzing the role of the strictness of the conditions on past employment history, however, is what this paper focuses on, with the Danish UIB system used as the basis of the analysis. However, for completeness a UIB system with no conditions on past employment history, i.e. $c = 0$, is analyzed reproducing the findings of previous literature concerning the benefit level and the maximum duration. This is presented in depth in Appendix C.

3.1 Parameterization

The parameter values are as follows: The period length is one quarter, the yearly discount rate, $r$, is set to 5% per year (as do Shimer (2005), Rosholm and Svarer (2004), and many others)\textsuperscript{16}, and the time share devoted to labor when employed, $l_w$, is 0.22 (corresponding to a 37-hour workweek relative to the 168 hours a week consists of). As taxes are non-distorting in the model, other government expenditures, $G$, are set to zero for simplicity. For the job separation rate, $p$, I use the estimate from Rosholm and Svarer (2004) of 0.0019 per week (this is also what is used by Ortega and Rioux (2010)). The elasticity of the matching function, $\eta$, as well as the bargaining power of the workers in the Nash bargaining, $\beta_{Nash}$ is set to 0.5 (as done in Ortega and Rioux (2010)). The vacancy costs of firms are set to 37.03% of the output normalized to one (corresponding to Ortega and Rioux (2010)). The replacement rates are set according to the Danish system (The Danish Unemployment Insurance Commission, Dagpengekommissionen (2015c)). Hence, the replacement level of the lowest benefit level in the two-tier UIB system, $b_{SA}$, is set to 35.4%, while the replacement level of the highest benefit level, $b_{UI}$,

\footnote{Ortega and Rioux (2010) also present a calibration of their model for Denmark.}

\footnote{While Ortega and Rioux (2010) use 1%, adopting this would not change the results of the model. Extreme assumptions on the value of the discount rate are needed for it to have an impact on the results (see section 4.3).}
is equal to 62.8%. The constant in the matching function, $A$, is calibrated to fit the very dynamic Danish labor market, which is characterized by short unemployment spells and a low unemployment rate.\footnote{See e.g. Andersen and Svarer (2007).} 2005 was considered to be a business cycle neutral year for the Danish economy. Hence, I use the unemployment rate of this year (5.1%) to calibrate the match efficiency, such that $A$ is 0.542.\footnote{Note that the Danish UIB system anno 2005 had a duration of four years and a required employment history of half a year. Hence, the match efficiency is calibrated using this UIB scheme.}

### 3.2 Underlying variables

As a descriptive introduction to the model, the search effort as a function of the two underlying variables, $h$ and $m$, is presented below. While the $m$ variable (remaining UIB duration) is closely connected to previous literature, the $h$ variable is novel. Overall, the search effort of the unemployed evolves in the following way when the value of the two underlying variables changes.

When considering how search effort varies in the level of employment history, $h$, it is firstly worth noticing that search effort for unemployed with an employment history larger than the required level, $c$, does not matter as all such unemployed are treated the same way by the UI system.\footnote{For completeness: This artificial search effort decreases in the employment history as it reflects that one is even more comfortable in being entitled to UIB.} This is because such workers redeem their employment history and have it reset upon entering unemployment when they come from employment with full UIB entitlement. Hence, the analysis only considers the cases of $h \leq c$. The results are two-fold: For SA recipients’ search effort increases in their employment history. Thus, if the employment history gets stronger, search effort increases. The intuition is that the stronger the employment history, the closer one is to obtaining full UIB entitlement and, hence, the larger is the gain of employment. This induces these unemployed to search more intensely. However, for UIB recipients the effect of employment history on their search effort depends on the level of their employment history and potentially has an inverse U-shape: If their employment history is sufficiently weak, they increase their search effort as their employment history increases (the intuition is the same as presented above). Oppositely, if their employment history is already sufficiently strong, such unemployed decrease their search effort if their employment history gets even stronger. That is, unemployed with both a sufficiently high remaining duration of UIB and a strong employment history lower their search effort as they obtain an even stronger employment history. The intuition is, basically, that a stronger employment history in this situation makes it more likely of not ending...
up on SA. Thus, the value gain of employment decreases, which induces them to lower their search effort. Moreover, the degree of changes in unemployeds’ search effort as their employment history changes decreases when the remaining duration of UIB increases. This is because higher remaining duration of UIB makes the value of the employment history less important.

These effects are depicted in figure 3.1 below, which illustrates the search effort relative to the level of employment history for four different scenarios of remaining duration of UIB.\textsuperscript{20} Note that the effects on search effort of the level of employment history are quantitatively much smaller than the effects from the length of remaining UIB duration (see below) as the latter affects the value of staying unemployed far more directly.

![Figure 3.1: Search effort increases in the employment history, $h$, for SA recipients as well as unemployed with sufficiently low remaining duration of UIB and weak employment history. For unemployed in a stronger position, search effort decreases.](image)

The more remaining periods of UIB, i.e. the higher level of $m$, the lower is the optimal search effort. Put differently, as the unemployment spell progresses, the remaining duration of UIB decreases (one starts to the right side of the x-axis in figure 3.2 and moves inwards). As a result, the search effort increases. Hence, unemployed increase their search effort, the closer they get to expiration of UIB. In fact, they do so at

\textsuperscript{20}Figure D5 in Appendix D illustrates the effects and the influence of the value of $c$ in terms of ISO-search effort in an $(h, c)$-graph. It highlights that higher $c$ prolongs the threshold value of $h$ after which $\frac{\partial s}{\partial h} < 0$.  

68
an increasing speed; i.e. the relationship is convex. After expiration, when receiving SA, unemployed continue to search at high intensity. Note, as Fredriksson and Holmlund (2006) point out, that “there is overwhelming empirical evidence in favor of the theory’s prediction that exit rates from unemployment increase as workers approach the time when benefits are due to expire.” The model equivalent is exactly what is presented below, namely, that search effort intensifies as the duration of remaining UIB decreases. Hence, this empirical observation is reproduced by the model. The fact that search effort of SA recipients is the highest and the increase in search effort as the remaining duration of UIB decreases is, in fact, the classic entitlement effect of a finite (high level) benefit scheme, as noted by Mortensen (1977). The flat curve for small levels of remaining UIB duration is a result of the period length as the search effort of unemployed with one remaining period of UIB is the same as that of unemployed with no remaining periods of UIB since it is the situation of the following period that determines optimal search effort. This applies to all following figures as well.

![Figure 3.2: Search effort is decreasing in the remaining UIB duration, m.](image)

21 The degree of the convexity depends on the discounting, i.e. the interest rate, r.
23 Proposition 1 of Mortensen (1977) is exactly this: “In the case of a qualified worker who has not yet exhausted his or her unemployment benefits, the escape rate increases with realized unemployment duration”.

69
3.3 Search effects

The responses in search effort to changes in the entitlement conditions are state-contingent: Stricter conditions on employment history decrease the potential gain from re-employment for SA recipients as they are more unlikely to be entitled to UIB. This induces them to search less intensely. On the other hand, the value of staying unemployed decreases significantly for UIB recipients, increasing the gain from re-employment. Hence, they intensify their search effort as conditions are tightened. Basically, the state contingency is due to the difference in the risk of experiencing the effect of the changed requirement on employment history, i.e. the extra employment needed for UIB entitlement. Hence, entitlement conditions introduce endogenous duration dependence, as it create a discouraged worker effect where long-term unemployed (with a weak employment history) decrease their search effort, while unemployed with a strong employment history intensify their search effort. This is in line with the results of Ellermann-Aarslev (2017b), where the employment history is tracked explicitly and not summed in one variable as in this paper.

However, as can be seen from figure 3.3 below, for all parameterizations of the model considered, the positive effects on search effort are quantitatively much larger than the negative effects. The intuition is that the difference in distance to UIB expiration affects the degree to which the unemployed expect to regain entitlement before expiration. Unemployed with longer remaining duration of UIB have higher likelihood of regaining full UIB entitlement. Thus, they are affected less hard by strengthened entitlement conditions that decrease their chances of regaining entitlement than unemployed who are close to UIB expiration and consider their chances of regaining entitlement to be smaller. SA recipients are discouraged by increasing requirements to the employment history, however, they consider their chances of gaining entitlement small already. Hence, strengthened entitlement conditions affect their search effort quantitatively less hard than it affects the search effort of entitled unemployed.
CHAPTER 2. GENERAL EQUILIBRIUM EFFECTS OF ENTITLEMENT CONDITIONS IN UNEMPLOYMENT INSURANCE

3.4 LABOR MARKET PERFORMANCE

For all reasonable calibrations of the model, the net effects of the individual job search responses (taking into account the effects on wages, taxes, and vacancy posting) are the following: Stricter conditions on employment history decrease unemployment. This is the opposite effect of enhancing the generosity of the UIB scheme, as this increases unemployment (see Appendix C). Hence, stricter conditions on employment history are found to work as a substitute for a lower level or maximum duration of UIB.

The underlying mechanism of the effects of changes in the entitlement conditions are as follows: When entitlement conditions are strengthened, i.e. \( c \) increases, the search effort of the unemployed is affected state-contingently as described above. However, unless the parameterization of the model is such that the distribution of the unemployed is very skewed towards unemployed with only a few or even no periods of UIB left, the net effects are dominated by the unemployed who increase their search effort.\(^{24}\) This causes the weighted search effort to increase.\(^{25}\) Also, the strengthened entitlement

\(^{24}\)See section 4.3 for a discussion. The share of SA recipients relative to UIB recipients needed to tip the balance is at least 200%, which is much, much higher than in e.g. Denmark where most unemployment spells are short and SA recipients amount to 15% of the UIB recipients (own calculations based on DREAM data for 2005).

\(^{25}\)See figure D1 in Appendix D. Note that the weighted search effort might actually decreases in the strictness of the entitlement conditions for rather extreme settings of the UI scheme, see Appendix C. However, for a UIB scheme equal to the Danish weighted search effort always increases.
conditions decrease the outside option of the workers, resulting in a lower equilibrium wage. This induces more vacancies to be posted, and since the extra vacancy posting offsets the increase in aggregated search effort, the labor market tightness increases. Because of this, the unemployeds’ return to search also increases. Together with the increase in weighted search effort, this lowers total unemployment. When unemployment is lower, the costs of running the UIB scheme is lower, resulting in a lower tax rate. In a sufficiently generous UIB scheme, below illustrated by a UIB scheme equal to the Danish, the average increase in search effort still decreases the unemployment rate of the economy if the wage effect is ignored; i.e. if the wages are exogenous. Note that the wage effect unambiguously results in a decrease of unemployment.

Figure 3.4: The effects on unemployment, the tax rate, wages, and labor market tightness of changes in the strictness of the entitlement conditions, $c$.

The distributional effects of strengthening the conditions on employment history are quite straightforward. As it gets harder to earn eligibility, fewer are entitled to UIB of any duration, the share of employed who are entitled to a full duration of UIB in case of unemployment drops significantly, while the number of unemployed SA recipients rises. Note that, the wage effect mitigates the negative distributional effects of the policy marginally.

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26 In extreme cases the sign of the wage effect hinges on the specific assumptions concerning wage negotiations (see section 4.1).

27 This share is unrealistically low and is thought to be a result of existing heterogeneity not considered in the model. For a further discussion, see section 4.6.
3.5 Welfare effects

As described above increasing the strictness of the entitlement conditions lowers total unemployment. This results in a positive budget effect as the tax rate needed to fund the UIB system decreases. This budget effect is the positive outcome of entitlement conditions in terms of welfare, and without it, the welfare of all workers decrease following reforms that make entitlement conditions more strict. Including the financing aspect, utilitarian welfare increases following an introduction of entitlement conditions on the employment history expect for UI systems with extremely low generosity. As the entitlement conditions are bounded above by $h_{max}$, we have that if the UIB scheme is sufficiently generous choosing $c = h_{max}$ is optimal, i.e. the upper corner solution. These effects can be seen from figure 3.6 below. For high level and/or duration of benefits, the optimal choice of required employment history is equal to the upper bound, while for very low level and duration of benefits it is not optimal to have a positive required employment history, i.e. $c = 0$. For UIB schemes with a level/duration composition in between there exists an inner solution to the utilitarian welfare optimization problem. Figure 3.7 shows the effect on utilitarian welfare for such a UIB scheme; first increasing and later decreasing. Note that for a UIB scheme equal to the Danish the

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28Except for extreme calibrations of the model, see section 4.3 for a discussion.
optimal level of required employment history is at the upper bound.\(^{29}\)

Figure 3.6: Optimal choice of required employment history, \(c\), given the values of the two other policy variables of the UIB scheme: the maximum duration, \(D\), and the level \(b_{UI}\). Take-away: Requirements on past employment history are optimal if the generosity of the UIB scheme is not extremely low.

\(^{29}\)Furthermore, for the UIB scheme that is optimal if no conditions on employment history exist, where maximum UIB duration is extremely low and the benefit level is high (derived in Appendix C), utilitarian welfare decreases following increased requirements on the employment history; i.e. it is not optimal to introduce entitlement conditions. However, in this UIB scheme the duration is only one period and, thus, de facto a corner solution due to the timing assumptions.
Figure 3.7: The welfare effects of changes in the strictness of the entitlement conditions, $c$, for a UIB system with an inner solution for the optimal level of $c$.

Note that the direct effect of the changes in the distribution of the workers (figure 3.5 above) decreases utilitarian welfare as fewer workers are entitled to UIB. Hence, an increase in utilitarian welfare reflects a genuine increase in utility and not just a change in the distribution of workers.

4 Discussion

4.1 Wage assumptions

The wage negotiations of the model build on Nash bargaining where the outside option of all workers, independent of their employment history and the remaining period of their UIB, is the value of unemployment with entitlement to a full duration of UIB. Relaxing this, two obvious alternatives exist within the framework of Nash bargaining: 1) Keeping the assumption of one wage, but now use an alternative outside option (common to all workers). 2) Abandoning the assumption of one wage and instead assume that wages are negotiated at an individual level.

In the first alternative, the outside option most different from that used up to now is the position of SA recipients with the weakest possible employment history; i.e. to use the value difference $(E(0,0) - U(0,0))$ instead of $(E^\text{full} - U(0,D))$ in the Nash
bargaining solution.\cite{30} By implementing this, the wage effects of the design of the UIB scheme are oppositely directed compared to all above analysis. Instead, the wage level decreases in the generosity of the UIB scheme (both in the maximum duration of UIB and in the benefit level) and increases in the required employment history. Though this is opposed to the previous results, the net results of the model are still valid.

Figure 4.1: Alternative wage assumption: Oppositely directed wage effect while the net effects are unaffected.

The reason behind this change in the sign of the wage effect is rooted in the state-contingent effect on the workers of changes in the design of the UIB scheme: The gain of employment for workers with full entitlement\cite{31} decreases in the UIB generosity (maximum duration and benefit level), while it increases in the required employment history. Oppositely, the gain of employment for workers with the weakest possible situation\cite{32} increases in UIB generosity, while it decreases in the required employment history. Consequently, contrary to previous results, the wage outcome of wage negotiation with this new outside option now decreases in UIB generosity and increases in required employment history.

In the second alternative bargaining situation, where wages are negotiated at an individual level\cite{33}, the wage effects are state-contingent. Wages of workers entitled

\footnote{30This seems to be a rather extreme assumption. As noted earlier De Økonomiske Råd (2014) (pp. 238-239) find that 91\% of all insured and employed Danish workers are entitled to a full duration of UIB in case of unemployment.}

\footnote{31Technically, the value difference \( E^{full} - U(0, D) \)}

\footnote{32\( E(0, 0) - U(0, 0) \)}

\footnote{33This introduces state contingency in the labor demand side, that is, in labor market tightness}
to the full duration of UIB behave as the wage in the main model (since the wage negotiation is identical). That is, the wages increase in the maximum benefit duration and benefit level, while they decrease in the required employment history. The threat point (i.e. the value of staying unemployed) in the wage negotiations of non-entitled workers is affected in the exact opposite way. As a result, these wages decrease in maximum duration and level, while they increase in the required employment history.

![State-contingent wage effects](image)

Figure 4.2: Individual wage negotiations: The effects on wages of the specific UIB design are state-contingent.

Average wages are, however, affected in the same way as the wages of fully entitled workers, i.e. as in the main model. This is because of the overall distribution of workers where fully entitled constitute a clear majority. Furthermore, the net outcome of the model (below represented by a change in total unemployment) is unchanged by introducing individual wage bargaining instead of one wage common to all on the basis of the situation of fully entitled workers as in the main model.

and hereby in the return to search. Furthermore, to keep the UIB scheme as in the main model, benefits are assumed still to be a certain fraction of the wage negotiated with the situation of the fully entitled as the outside option. This has the implication that the tax rate changes to 

\[
\tau = \frac{b_{UI} w_{full} u_{UI} + b_{SA} w_{full} u_{SA} + G}{w_{average} e + b_{UI} w_{full} u_{UI} + b_{SA} w_{full} u_{SA}}. 
\]

34See section 4.3 for a discussion of the importance of the distribution of the unemployed.
Figure 4.3: Individual wage negotiations: Average wages are affected in the same way as in the main model with simplified negotiations of one common wage. The net effects are also the same.

Summing up, as introducing state-contingent wages does not change the outcome and conclusions of the main model, the wage assumption of the main model is not thought to be restrictive.

### 4.2 Tax system

As is the practice of large parts of the literature and the case in the Danish tax system, benefits are taxable in the model. If they were not, the tax rate would increase to

\[
\tau \big|_{\text{unemployed}=0} = \frac{b_{UI}u_{UI} + b_{SA}u_{SA} + \frac{G}{w}}{e}
\]

This decreases the value gain of employment, which in part decreases the optimal level of search effort of the unemployed. Hence, theoretically the assumption matters. However, solving the model for non-taxable benefits reveals practically unchanged results. The effects on the total unemployment, the tax rate, the wage, and the labor market tightness of adjusting the required employment history are depicted below (corresponding to figure 3.4). The effects are virtually unaffected by changing benefits to be non-taxable.

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35E.g. Andersen and Svarer (2014).
4.3 Distribution of the unemployed

For extreme parameterizations of the model, where the distribution of the unemployed is highly dominated by SA recipients, the net effects of the model change from what is described in the preceding. Now, as noted earlier, SA recipients amounted to 15% of the UIB recipients in Denmark in 2005. For the net effects of the individual search responses to change, the share of SA recipients needs to be at least the double of the share of UIB recipients. Hence, in a Danish context such a situation is very extreme. In this section such an extreme situation is illustrated for a generous scheme equal to the Danish. The level shift in the share of SA recipients is driven by an (extremely) high discounting equal to a yearly discount rate of 210%. This induces the SA recipients to search very little and their chances of leaving unemployment are thereby small. Figure 4.5 below illustrates the share of SA recipients relative to UIB recipients as a function of required employment history. Stricter requirements result in an increase in the share of SA recipients as (1) workers are more unlikely to gain entitlement to UIB and thereby more likely to end up on SA if they lose their job, and (2) stricter requirements reduce the search effort of unemployed already on SA. In the main calibration of the model, the

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36 Own calculations based on DREAM data for 2005. A more recent estimate (but not in a business neutral year) is 39.75% (own calculations based on Statistic Denmark’s data for November 2016).
share of SA recipients is much lower than in the alternative calibration. In the case of a UIB scheme equal to the Danish, this share is unrealistically low. This is thought to be a result of existing heterogeneity not considered in the model. For a further discussion, see section 4.6.

![Graph showing the share of SA recipients relative to the share of UIB recipients increases in the required employment history under a UIB scheme equal to the Danish.](image)

Figure 4.5: The share of SA recipients relative to the share of UIB recipients increases in the required employment history under a UIB scheme equal to the Danish.

When SA recipients constitute the overwhelming majority of the unemployed, the search responses of the SA recipients dominate the search responses of the unemployed in a more comfortable situation. Hence, the net effects are dominated by the decrease in search effort of the SA recipients as conditions on the required employment history are strengthened (see section 3.3). This increases total unemployment. Under a UIB scheme equal to the Danish, the tax rate increases. However, the tax rate only increases if the share of SA recipients is not increasing too much. As is the case in the main calibration of the model, wages decrease following stricter entitlement conditions as the workers’ outside option is decreased. Furthermore, labor market tightness increases as the lower wages induce firms to post more vacancies and workers on average search less effectively for employment.
In an extreme situation as described here with a very high share of SA recipients, introducing entitlement conditions does not seem to be optimal even if the UIB scheme is generous. As can be seen below, utilitarian welfare decreases when entitlement conditions on the employment history are strengthened in a UIB scheme equal to the Danish. This is the opposite effect of what is found in the main model (see section 3.5).

Figure 4.6: Extreme worker distribution: The effects on unemployment, the tax rate, wages, and labor market tightness of changes in the strictness of the entitlement conditions.
Figure 4.7: Extreme worker distribution: The welfare effects of changes in the strictness of the entitlement conditions.

4.4 Optimality

As the social planner solution to the level of optimal search effort is complex, optimality is consider via a disproof showing that the decentralized level of search effort is inefficiently low. This is done by comparing the outcome of the model with decentralized search to that of an artificial edition where search effort is exogenously set to a certain share of the endogenous decentralized optimal level. As can be seen from figure 4.8 below, increasing the search effort of unemployed increases utilitarian welfare. Hence, the decentralized level of optimal search effort is not optimal and welfare maximizing. Also, for SA recipients an increase in search effort could, in fact, be beneficial.
CHAPTER 2. GENERAL EQUILIBRIUM EFFECTS OF ENTITLEMENT CONDITIONS IN UNEMPLOYMENT INSURANCE

4. DISCUSSION

This inefficiency is simply due to the unemployed not internalizing the full costs of the UIB scheme when they decide how much search effort to conduct, as they take the taxes funding the scheme as given. This is perhaps best illustrated by the fact that the decentralized level of search effort is optimal in a situation where no benefits are available to the unemployed.\footnote{This holds even if $\beta_{\text{Nash}} \neq \eta$.} See illustration below.\footnote{Technically, the benefit levels are still strictly positive but arbitrarily small. See figure D8 in Appendix D for the situation with a one-tier UIB system at SA level. Here the endogenous decentralized level of search effort is almost optimal (only almost as there are still distorting benefits).}

A natural question is then, whether having a UIB system is even optimal? And furthermore, why is strengthening entitlement conditions that (at an aggregated level) increase the search effort not always optimal? The answer to the first question can basically be seen by comparing the level of welfare in the two situations described above. If no benefits are offered to unemployed, overall welfare is much lower as workers are risk-averse and have no possibility of precautionary saving or borrowing. Hence, some degree of public UI is optimal in spite of the distorted search effort.

Figure 4.8: Utilitarian welfare and welfare of SA recipients for artificial exogenous levels of search effort. The endogenous decentralized level of search effort is inefficiently low.

Figure 4.9: In a situation where no benefits are available to the unemployed, the endogenous decentralized level of search effort is optimal.
Even though strengthening entitlement conditions on past employment history (at an aggregated level) increases the search effort of unemployed, it also results in more SA recipients (cf. figure 3.5). This obviously works in the opposite direction welfare wise.

4.5 Alternative benefit profile

A UIB scheme with two tiers of public unemployment benefit payments is the basic structure of most UI systems. This is why such a scheme has formed the basis of the analysis so far. A cornerstone in such UIB schemes is the finite duration of the highest level of benefits. The decrease in benefits during a sufficiently lengthy unemployment spell introduces the entitlement effect (Mortensen (1977)). Conditioning the highest level of benefits on past employment history is basically an instrument to refine this effect.

Bearing this in mind, a way to increase the incentives of unemployed to search for employment is to adjust the two-tier system to a multi-tier system (as argued in Fredriksson and Holmlund (2001)). Doing so by letting the benefit level decline throughout the unemployment spell lowers unemployment and increases utilitarian welfare with a public budget surplus (illustrated below with a lower tax rate). Hence, a declining profile of UIB seems to dominate a flat two-tier UIB scheme.
4.6 Parameterization

The model is parameterized in line with the literature, primarily Ortega and Rioux (2010), and calibrated to match the Danish labor market characterized by short unemployment spells and a low unemployment rate. This is done by allowing for a very efficient matching technology. While this does the job in terms of calibration, it might not be the whole story, as a distinctive feature of the Danish labor market is the extensive active labor market policies, see e.g. Andersen and Svarer (2007) and Andersen and Svarer (2014). Such policies are not present in the model. Moreover, for a better match of the Danish labor market, and in particular of the distribution of the unemployed in terms of the length of the unemployment spells, it is essential to introduce heterogeneity, e.g., in terms of productivity. However, doing so is left for future research.

Furthermore, assuming UIB to be equal to a certain share of previous wage income is not the case for all in reality as the benefits of high income earners are usually capped, implying that changes in their wages does not change the level of their benefits should they end up in unemployment. Moreover, SA is typically only indirectly connected to
previous wages. However, in the present version of this paper both kind of benefits are expressed as replacement rates as is done in Andersen and Svarer (2014).

5 CONCLUDING REMARKS

Most unemployment insurance benefit (UIB) schemes have eligibility conditions including conditions on past employment history. Such conditions presumably mitigate moral hazard issues. However, they introduce state contingency of otherwise identical workers as workers now differ in their path to UIB eligibility and, hence, in their optimal behavior. In this paper workers’ employment history and their remaining duration of UIB are modeled as state variables in a Diamond-Mortensen-Pissarides GE model, where unemployed conduct endogenous search effort. Using numerical solution methods, I find that unless the unemployed are likely not to end up on SA (strong employment history and long remaining duration of UIB), search effort is increasing in the strength of the employment history due to the UIB conditions thereon. Moreover, search effort is decreasing in the length of the remaining period of UIB. The responses in search effort to changes in the entitlement conditions are state-contingent due to the fact that UIB are conditioned on employment history. Hence, entitlement conditions introduce endogenous duration dependence, as it create a discouraged worker effect where long-term unemployed (with a weak employment history) decrease their search effort, while unemployed with a strong employment history intensify their search effort. The model is able to determine the net effects of the individual job search responses and takes into account the effects on wages, taxes, and vacancy posting. For calibrations of the model with realistic distributions of the unemployed, the net effects are the following: Stricter conditions on employment history decrease unemployment, while increasing the generosity of the UIB scheme (i.e. prolonging the maximum duration of UIB and/or increasing the UIB level) increases unemployment. Hence, stricter conditions on employment history are found to work as a substitute for lowering the generosity of a UIB scheme and are, from a utilitarian welfare perspective, optimal to introduce. Conditions on past employment history can therefore act as an additional instrument when designing UI systems and, if needed, reduce the overall generosity without reducing the level or the duration.

To conduct a full-blown calibration of models with employment history, like the one presented in this paper, one would need to introduce worker heterogeneity to match the worker distribution. One obvious source of heterogeneity is productivity as a correlation between productivity and employment history seems plausible. How-
ever, doing so is left for future work. The implications would likely be that less productive workers will constitute the majority of long-term unemployed as they conduct less search effort as a results of being paid a lower wage. Introducing entitlement conditions on employment history will discourage the long-term unemployed and restrain them in unemployment. Thus, low productive long-term unemployed would be affected negatively by entitlement conditions and this should be a serious issue of consideration when deciding the optimal level of such conditions. Moreover, such implications might call for productivity-contingent entitlement conditions on employment history which are, obviously, not straight forward to introduce in reality.
6 References


Dagpengekommissionen. 2015a. “Dagpengekommissionens Hovedrapport.”


A SHORT INTRODUCTION TO THE DANISH UIB SYSTEM

In a Danish context the role of entitlement conditions is interesting due to massive political interest. In the aftermath of the 2010 reform of the Danish UIB system, which among other things reduced the maximum UIB duration from four to two years and doubled the required employment needed to be re-entitled from half a year of full time employment to one year, much criticism was raised. This led to the establishment of ‘Dagpengekommissionen’, a commission with participants from all parties of the labor market, which delivered its report on how the Danish UIB system could be organized in a more “modern, flexible, and robust”\textsuperscript{39} way in October 2015. Following this, the Danish UIB system was modified to include the possibility of what one could call ‘flexible re-entitlement’, where employment qualifies the worker to an extension of the UIB duration of twice the length.\textsuperscript{40} In a sub-report the commission puts forward a model to analyze the effects of changes in the conditions for re-entitlement.\textsuperscript{41} However, the model does not take all necessary considerations into account. One of its shortcomings is that it only considers the supply side of the labor market and it considers matching to be one-sided. More importantly, the model does not take into account to which extent an unemployed benefits from finding employment in the future. In stead, search effort is optimized based on the overall average value. Hence, the transition rates between the various states of employment and unemployment are given as in the most basic classical DMP model, i.e. by the different shares of population and not via micro level lifetime maximizing rational agents. From a narrow Danish perspective, the limitations of this model combined with the much attention paid to the effects of re-entitlement conditions strengthen the case for a more robust and thorough economic theoretical analysis of the matter.

B GENERAL FORMULATION OF UPDATING FUNCTIONS

Expressing the updating functions of the two underlying variables, $h$ and $m$, in a general formulation:

When time progresses one period the employment history, $h$, of workers who were

\textsuperscript{39}L. 2 in the preface of Dagpengekommissionen (2015a).
\textsuperscript{40}With a maximum of one year of extra UI. The rules became effective as from January 1, 2017.
\textsuperscript{41}“Genoptjeningsmodellen” of Dagpengekommissionen (2015b), section 4.6 pp. 72-84.
employed in the previous period increases to \( k_h(h) \), while that of the workers who were unemployed decreases to \( q_h(h) \). If the next period employment history satisfies the requirements, \( k_h(h) \geq c \), the worker will be entitled to UIB and start a new reference period causing the employment history to be reset to zero, \( h = 0 \), in case of unemployment.

\[
h_{t} = \begin{cases} 
H(h_{t-1}) & \text{if } H(h_{t-1}) < c \\
0 & \text{or no transition to unemp.} \\
H(h_{t-1}) & \text{if } H(h_{t-1}) \geq c \\
0 & \text{and transition to unemp.}
\end{cases}
\]

where

\[
H(h_{t-1}) = \begin{cases} 
k_h(h_{t-1}) > h_{t-1} & \text{if emp. at } t-1 \\
q_h(h_{t-1}) < h_{t-1} & \text{if unemp. at } t-1
\end{cases}
\]

The remaining UIB duration, \( m \), decreases to \( q_m(m) \) for workers who were unemployed in the previous period, oppositely it stays constant while employed. If the employment history is sufficiently strong (\( h \geq c \)), the worker qualifies for a (new) full period of UIB. Note that \( m \) does not increase to \( D \) until the worker moves into unemployment.

\[
m_{t} = \begin{cases} 
M(m_{t-1}) & \text{if } H(h_{t-1}) < c \\
D & \text{or no transition to unemp.} \\
M(m_{t-1}) & \text{if } H(h_{t-1}) \geq c \\
D & \text{and transition to unemp.}
\end{cases}
\]

where

\[
M(m_{t-1}) = \begin{cases} 
k_m(m_{t-1}) = m_{t-1} & \text{if emp. at } t-1 \\
q_m(m_{t-1}) < m_{t-1} & \text{if unemp. at } t-1
\end{cases}
\]

In the boundaries of \( h \) and \( m \), the functions return the boundary values, \( k_h(h_{max}) = h_{max} \), \( q_h(0) = 0 \), and \( k_m(D) = D \), \( q_m(0) = 0 \).

Where the functions \( k_h, q_h, k_m, \) and \( q_m \) describe the development of \( h \) and \( m \) forward in time, their inverse functions describe the development back in time. That is, the inverse functions give the values of the two underlying variables in the period prior
to the current. In other words, the inverted functions of $k_h$, $q_h$, $k_m$, and $q_m$ determine the values of $h$ and $m$, which in the next period lead to specific values of $h$ and $m$, respectively. Hence, when considering the inflow into the different states the inverse functions describe which states the workers come from. Thus, the previous period value of $h$ that leads to $h = \hat{h}$ in case of employment in the previous period is $k_h^{-1}(\hat{h})$, and in case of unemployment in the previous period $q_h^{-1}(\hat{h})$. The corresponding previous period values for $m$ are $k_m^{-1}(\hat{m})$ and $q_m^{-1}(\hat{m})$.

![Diagram of updating functions](image)

**Figure B1:** Illustration of the updating functions.

Comparing this general formulation to the updating of $h$ and $m$ used in the paper, the specific functional forms of the two updating functions of $h$ and $m$, respectively, are the following ($I$ is the period length):

$$k_h(h) = h + I, \quad q_h(h) = h - I$$

$$k_m(m) = m, \quad q_m(m) = m - I$$

Using the general formulation, the value functions of the supply side of the model (section 2.2) are as follows.
\[ E(h, m) = \log ((1 - \tau) w) + \log (1 - l_w) \]
\[ + \beta [\rho U(k_h(h), k_m(m)) + (1 - \rho) E(k_h(h), k_m(m))] \]

\[ E(h, m) = E^{full} \text{ for } h \geq c \text{ and } \forall m \]
\[ E^{full} = \log ((1 - \tau) w) + \log (1 - l_w) + \beta [\rho U(0, D) + (1 - \rho) E^{full}] \]

\[ U(h, m) = \log ((1 - \tau) b(m) w) + \log (1 - s(h, m)) \]
\[ + \beta [\alpha (\theta) s(h, m) E(q_h(h), q_m(m)) + (1 - \alpha (\theta) s(h, m)) U(q_h(h), q_m(m))] \]

\[ s(h, m) = 1 - \frac{1}{\alpha (\theta) \beta [E(q_h(h), q_m(m)) - U(q_h(h), q_m(m))]} \]

The transition matrices of section 2.6 are:
\[
P_{e(j,l)}^{e(x,y)} = \begin{cases} 
1 - \rho & \text{if } k_h(j) = x \text{ and } k_m(l) = y \\
0 & \text{otherwise}
\end{cases}
\]
\[
P_{u(j,l)}^{e(x,y)} = \begin{cases} 
\rho & \text{if } k_h(j) = x \text{ and } k_m(l) = y \\
0 & \text{otherwise}
\end{cases}
\]
\[
P_{u(j,l)}^{u(x,y)} = \begin{cases} 
\alpha (\theta) s(j, l) & \text{if } q_h(j) = x \text{ and } q_m(l) = y \\
0 & \text{otherwise}
\end{cases}
\]
\[
P_{u(j,l)}^{u(x,y)} = \begin{cases} 
1 - \alpha (\theta) s(j, l) & \text{if } q_h(j) = x \text{ and } q_m(l) = y \\
0 & \text{otherwise}
\end{cases}
\]

Note that if the employment history is considered to be a ‘simple sum’ of previous employment such that the employment history is the sum of previous employment, \( h \) does not decrease in case of unemployment, and the two updating functions of \( h \) are assumed to be, \( k_h(h) = h + I, \ q_h(h) = h \).
C A UIB SYSTEM WITH NO CONDITIONS ON PAST EMPLOYMENT HISTORY

C.1 DYNAMICS IN LINE WITH PREVIOUS LITERATURE

If no conditions on past employment history exist, i.e. \( c = 0 \), the model produces dynamics which are in line with previous literature.

The model perfectly reproduces proposition 3 in Mortensen (1977): “In the case of newly laid-off workers, the escape rate decreases with the maximum benefit period”, as the escape rate (in the subsequent denoted exit rate) of unemployed with full remaining UIB duration decreases in the maximum UIB duration. That is, the exit rate in the first period of unemployment of workers who are entitled to a full UIB period \((a(\theta) \cdot s(0, D))\) decreases in the maximum UIB duration (see figure C1 below). The exit rate is a product of return to search, which depends on labor market tightness and the search effort of the unemployed. The decrease in the exit rate described above is due to decreases in the return to search as well as in search effort (see figure D2 in Appendix D). This can also be deduced from the fact that the exit rate decreases even further for the full model compared to when wages are fixed. The intuition behind the result that unemployed with full remaining duration of UIB decrease their search effort as the maximum duration of UIB is prolonged is that the event of benefit expiration is postponed. This increases the value of unemployment and decreases the gain of employment, which lowers the level of optimal search effort.
Figure C1: The exit rate of unemployed with full remaining UIB duration decreases in the maximum UIB duration.

The search responses to changes in the length of maximum UIB duration are, however, generally state-contingent as the search effort increases for unemployed who have less remaining UIB duration than the maximum level. This is because the value of employment increases, which makes the value gain of finding a job higher, and so, optimal search is higher. However, we have that the weighted search effort decreases in the maximum duration of UIB.\footnote{See figure D3 in Appendix D} Moreover, extending the maximum UIB duration increases the outside option of the workers. This results in a higher equilibrium wage, which in turn decreases the labor market tightness via fewer vacancy postings and through this decreases the return to search. Together with the decrease of the weighted search effort, this increases total unemployment.\footnote{Comparing the flat UI system of benefits at SA-level ($D = 0$) to a system of one period of UIB followed by SA ($D = 1$ period), the specific timing of the updating of the underlying history dependent variables ($h$ and $m$) makes the search decisions of the unemployed identical because in the period to come they face benefits at SA-level in both cases. This is why unemployment is constant as the maximum duration of UIB increases from zero to one period (figure C2). The tax rate, however, increases as some unemployed now receive larger benefits. The higher benefits in such a scheme basically work as a lump-sum transfer in the first period of unemployment.} Ignoring the wage effects (and thus the decrease in the return to search), the decrease of average search effort alone increases the unemployment rate of the economy. The wage effect merely amplifies this impact.\footnote{The sign of the wage effect hinges in extreme cases on the specific assumptions concerning wage negotiations (see section 4.1).} The increase in unemployment magnifies the costs of running the UIB
scheme, and thus, results in a budget effect as the tax rate needed to fund the UIB system increases.

Figure C2: The effects on unemployment, the tax rate, wages, and labor market tightness of changes in the maximum duration of UIB, $D$.

A second channel of UIB generosity is the level of UIB. The effects on the search effort of unemployed of changes in the benefit level are, however, different from the effects of changing the maximum UIB duration as the benefit level affects all unemployed currently receiving UIB directly, while the duration only affects newly unemployed entitled to a full duration of UIB directly (even though all workers are of course affected indirectly as they are forward looking). This is why increasing the duration induces the search effort of currently unemployed to increase as the value of employment (and eventually regaining full UIB entitlement) increases. Oppositely, increasing the UIB level results in an increased value of staying unemployed while receiving UIB. Hence, UIB recipients decrease their search effort if the UIB level increases. The effect is larger the longer the remaining UIB duration as, logically, the change in value is increasing in the remaining UIB duration. In other words, the change in benefit level applies for a (in expected terms) longer period. Oppositely to the UIB recipients, SA recipients experience an increase in the value gain of employment as the level of UIB increases since, if they stay employed for sufficiently long and hereby qualify for UIB, the benefits in case of unemployment are increased. This induces them to search more intensely for employment. The effect on SA recipients’ search effort is, however, very moderate compared to that of UIB recipients as the channel is much more indirect.
(they have to become employed and UIB entitled before they can enjoy the higher benefit level). That is, for all reasonable calibrations of the model, the majority of the unemployed respond to a higher level of UIB by decreasing their effort to find a vacant job, and they do this much more forcefully than the SA recipients, who intensify their search effort. Hence, weighted search effort is decreasing in the level of UIB.

Moreover, just as prolonging maximum UIB duration, a higher level of UIB results in a higher outside option of the workers in the wage bargaining, and thus, the equilibrium wage increases. This results in fewer vacancies being posted. Even though weighted search decreases, aggregated search (which is the denominator of the labor market tightness) actually increases (via the increase of unemployed). As the nominator of labor market tightness decreases (vacancies) and the denominator increases (aggregated search effort), the labor market tightness decreases when the level of UIB increases. Hence, the return to search decreases in the UIB level. Both the lower weighted search effort and the lower return to search increase unemployment.

As when the UIB generosity is enhanced by prolonging the maximum UIB duration, the tax rate increases as a result of higher unemployment as well as the higher level of UIB.

\[ b_{UI} \]

Figure C3: The effect on search effort of changes in the UIB benefit level, \( b_{UI} \).

\[ \text{Search effort wrt. UIB level} \]

\[ \text{SA recipients} \]

\[ \text{UI recipients, m low} \]

\[ \text{UI recipients, m medium} \]

\[ \text{UI recipients, m high} \]

Note again that the sign of the wage effect hinges in extreme cases on the specific assumptions concerning wage negotiations (see section 4.1).

\[ ^{45}\text{See section 4.3 for a discussion of the importance of the distribution of the unemployed.} \]

\[ ^{46}\text{See figure D4 in Appendix D.} \]

\[ ^{47}\text{Note again that the sign of the wage effect hinges in extreme cases on the specific assumptions concerning wage negotiations (see section 4.1).} \]
Figure C4: The effects on unemployment, the tax rate, wages, and labor market tightness of changes in the UIB benefit level, $b_{UI}$. 

Overall, the macroeconomic effects of adjusting the benefit level of a UIB scheme is qualitatively the same as adjusting the maximum duration of UIB. Hence, increasing the level of UIB works as a substitute (in terms of total unemployment) to increasing the maximum duration of UIB; i.e. the increase in unemployment following an increase in the level of UIB can be off-set by lowering the maximum duration of UIB and vice versa. This is a well-known interaction of the two policy variables (see e.g. Fredriksson and Holmlund (2006)).
CHAPTER 2.
GENERAL EQUILIBRIUM EFFECTS OF ENTITLEMENT
CONDITIONS IN UNEMPLOYMENT INSURANCE

C. A UIB SYSTEM WITH NO CONDITIONS ON PAST EMPLOYMENT HISTORY

Figure C5: Higher maximum UIB duration and a higher level of UIB are substitutes in terms of unemployment.

The exit rates of unemployment with respect to the UIB level are not in line with Mortensen (1977), who states: “In the case of an unemployed worker not currently receiving benefit payments, an increase in either benefit parameter increases the escape probability.” The reason is the following: Even though the search effort of unemployed “not currently receiving benefit payments”, i.e. SA recipients, increases marginally, the return to search decreases quite significantly (as described above). Hence, the general equilibrium setting of this model changes the result of the partial model of Mortensen (1977). As the return to search is not state contingent, the state-contingent differences in exit rates come from the differences in search effort. Hence, the decrease in the exit rate of unemployment following increases in the benefit level is intensifying in the length of remaining UIB duration.

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49 Mortensen (1977) notes this himself: “A major caveat is in order. The analysis conducted in the paper is partial in the sense that the effects of changes in benefit parameters on the distribution of wage offers and on the probability of being laid off are ignored” (P. 2/506 1. column II. 14-19). Note that the GE effects of the model presented here are not through the wage offer distribution or the probability of being laid off, rather they are through the labor market tightness via the return to the unemployeds’ search effort.
C.2 Optimal design

In a UIB scheme with no entitlement conditions on past employment history (i.e. $c = 0$), the optimal value of maximum duration of UIB and the level decreases in the value of the other. That is, the optimal maximum duration of UIB for a given UIB level decreases in that level, and likewise for the optimal level of UIB given a certain maximum UIB duration. Hence, given that the starting point is optimal, if the generosity of one of the policy variables of the UIB scheme is decreasing and taken as given, it is optimal to increase the other. This is a natural cause of the two policy variables being substitutes in terms of total unemployment. The reason for an inner solution of the optimal UIB generosity is rooted in the classic trade-off of insurance and incentives and confirms the results of many others (see e.g. Fredriksson and Holmlund (2006)): Increasing either the maximum duration of UIB or the level of UIB improves the insurance coverage of a UIB scheme. However, it comes at the price of crumbled incentives since the moral hazard issues of curbed search effort are worsened. At first, the positive insurance effect is dominating, but as the generosity increases then at some point the negative effects of deteriorated incentives equalize the welfare gains from the better insurance coverage. Increasing the generosity further decreases welfare as the incentive issues hereafter dominate.

Below, this is illustrated for two calibrations of the model: (a) The main param-
eterization of the model, calibrated to fit the dynamic Danish labor market, and (b) an alternative less dynamic parameterization of the model. Firstly, figure C7 presents the optimal choice of maximum UIB duration given a certain level of UIB and vice versa. Secondly, figure C8 illustrates the utilitarian welfare of various combinations of maximum duration and the level of UIB.

(a) Main parameterization of the model, calibrated to fit the dynamic Danish labor market

(b) Alternative less dynamic parameterization of the model

Figure C7: Both optimal UIB duration and higher level of UIB, respectively, decrease in the value of the other.
The UIB duration-level-pair that maximizes utilitarian welfare of a UIB scheme with no entitlement conditions is the pair where both policy variables are optimal given that of the other, i.e. the intersection of the two curves in figure C7 above. Comparing to the current Danish UIB system, where the maximum UIB duration is two years and the level UIB is 1.8 times that of SA, the optimal UIB scheme includes a lower maximum UIB duration and a higher UIB level. Below, utilitarian welfare is illustrated for UIB schemes with no conditions on past employment history.\(^50\) Note that when calibrated to the dynamic Danish labor market, the model suggests that optimal UIB is of very short duration as the efficient matching technology makes finding a job relatively easy. Hence, it is better to have a higher level of UIB for a shorter time. As the optimal duration of UIB is just one period this is, basically, a corner solution due to the timing of the model.

\(^{50}\)When the maximum duration of UIB is zero, that is, when the two-tier benefit system basically becomes a flat system of infinite unconditional benefits at the low SA-level, welfare is actually higher than if the maximum duration of UIB is small but positive in the alternative less dynamic parameterization of the model (see figure D6 in Appendix D). This is because, as the duration increases from zero, the needed taxes decrease the consumption possibilities (the budget effect). Doing so for employed workers might be optimal in a utilitarian welfare approach, as concave utility functions call for redistribution. SA recipients, however, are also hurt by the increase in taxes. As the SA recipients are the worst-off workers in the economy, this is obviously not desirable from a utilitarian welfare stand. This effect is present for all UIB schemes funded via taxes on both wage income and benefits. For SA recipients a UI scheme with only SA is actually optimal. If benefits were not taxable, SA recipients would obviously not be directly affected by the increased tax burden following an increase in the maximum duration of UIB. Hence, utilitarian welfare would unambiguously increase when one period of UIB is introduced, as this basically corresponds to a lump-sum redistribution transfer from the best-off workers to (some of) the worst-off.
The welfare effect for unemployed UIB recipients is more or less as the effect on utilitarian welfare. Oppositely, the welfare of SA recipients is overall decreasing in the generosity of UIB, since the resulting increase in the tax rate decreases their disposable income and this dominates the positive effects of a more generous UIB scheme in case of earned UIB eligibility. Even though the increase in wages increases the benefits, this is more than off-set by the increase in the tax rate and the decrease of return to
search. Reversely, utilitarian welfare is increased by the wage effect as this benefits the employed, who constitute the majority of the population.$^{51}$

Note the budget effect of increased UIB generosity: If the financing aspect of increasing the UIB generosity is ignored, welfare is strictly increasing for all. Oppositely, the necessary increase in taxes obviously decreases welfare for all. Hence, the budget effect of increased UIB generosity affects welfare negatively.

$^{51}$See figure D7 in Appendix D for an illustration. Furthermore, note that the sign of the wage effect hinges in extreme cases on the specific assumptions concerning wage negotiations (see section 4.1).
D Additional figures

Figure D1: Weighted search effort increases in the strictness of the entitlement conditions, $c$, for a UIB scheme equal to the Danish.

Figure D2: Both the search effort of unemployed with full remaining UIB, $s(0, D)$, and the return to search, $\alpha(\theta)$, decrease in the maximum duration of UIB, $D$. 
Figure D3: Weighted search effort decreases in the maximum duration of UIB, $D$.

Figure D4: Weighted search effort decreases in the UIB benefit level, $b_{UI}$.
Figure D5: ISO-search effort in an \((h, c)\) illustration. Note that higher \(c\) prolongs the threshold value of \(h\) after which \(\frac{\partial s}{\partial h} < 0\).

Figure D6: Alternative less dynamic parameterization of the model: Utilitarian welfare wrt. the value of UIB duration and level.
Figure D7: Alternative less dynamic parameterization of the model: The welfare effects of changes in the maximum duration of UIB, \( D \).

Figure D8: The endogenous decentralized level of search effort is almost optimal in a situation with a one-tier UIB scheme with low-level benefits.
Unemployment insurance schemes typically include eligibility conditions depending on the employment history of the unemployed. The literature on the design of unemployment insurance schemes has largely ignored this aspect, perhaps due to the implied history dependence and heterogeneity across otherwise identical workers. We develop an analytically tractable matching model permitting an analysis of the consequences of such history contingencies. Unemployed determine reservation durations to the jobs they find acceptable, and stronger employment histories lead to higher reservation durations. Consequently, short-term jobs are only acceptable to unemployed with a weak employment history, while unemployed with a stronger employment history have higher reservation durations. This affects the equilibrium distribution of employment according to job durations; fewer short duration jobs are filled, but this also means that fewer are locked-into such jobs, and therefore more jobs with long durations are filled, although the net effect generally is to lower employment. Employment history contingencies thus affect both the level and structure of employment. Equilibrium (un)employment depends not only on reservation durations, and a lengthening benefit duration combined with a decrease in the benefit level to retain reservation durations, has a negative effect on employment.

JEL Classification: E24, J64, J65

Keywords: Unemployment insurance; Entitlement conditions; Employment history; Job search
1 Introduction

Unemployment insurance schemes have multi-dimensional characteristics including not only the level and duration of benefits, but also job-search criteria, activation requirements, monitoring as well as various eligibility conditions (see e.g. Venn (2012)). The latter include contingencies depending on the past employment record of the unemployed, both in the form of a minimum employment record within some qualifying period to be eligible for benefits, and benefit levels or durations depending positively on the employment history. Such contingencies serve to target unemployment benefits to individuals with a more firm attachment to the labor market, and to make unemployed search for jobs with longer durations. However, this may reduce the incentive for unemployed to accept jobs with shorter durations - An issue which has recently been brought up in policy discussions. A concern is whether strict requirements are a barrier for unemployed to accept short-term jobs. In particular for low-skilled where the issue of short-term jobs is most prevalent, see e.g. Dagpengekommissionen (2015a) and OECD (2016).

Although employment requirements are an important element of unemployment insurance schemes, most research has focused on the role of the level and duration of benefits, see e.g. Fredriksson and Holmlund (2006) and Tatsiramos and van Ours (2012) for surveys. There are, however, a few studies addressing the role of employment requirements. Hopenhayn and Nicolini (2009) consider the moral hazard problem arising when it is not possible to distinguish between layoffs and quits, leaving an opportunistic motive to quit and receive unemployment benefits. They show that employment conditions can reduce this moral hazard problem. Employment conditions may also affect wage formation via an entitlement effect, see Ortega and Rioux (2010) and Faig and Zhang (2012). Unemployed having exhausted their benefits are willing to accept lower wages to regain the right to unemployment benefits. Moreover, the bargaining power of workers may be reduced when unemployment benefits depend on employment history, which therefore makes it more important for the worker to reach an agreement with the firm. Andersen et al. (2015) consider the optimal determination of benefit levels, duration and employment requirements in a DMP search-matching model. However, the issue of job duration has not been explicitly addressed in those

\(^{1}\)Oppositely to the approach of this paper, which builds on a pure ‘search-good’ model where an employment relation is dissolved for exogenous reasons (e.g. arrival of new information), Jovanovic (1979) consider jobs as an ‘experience good’, where the quality of an employment relation is revealed as the match is ‘experienced’. That is, the productivity of a worker in a particular job is not known beforehand, but revealed little by little as the job tenure increases. As worker productivity differ across different firms, workers are laid off if the firm realize that the productivity is not sufficiently strong.
papers, perhaps because requirements based on the employment histories of unemployed introduce state-contingencies or heterogeneities across otherwise identical individuals. The aim of the current paper is to analyze job acceptance across jobs with different (expected) tenure when unemployment benefits and duration depend on the employment history. The primary focus is thus to analyze whether such employment requirements strengthen the incentives of unemployed to accept job offers and, in particular, whether it makes short-term jobs uninteresting.

The analytical challenge is to keep track of the state dependencies arising under an employment history dependent unemployment insurance scheme. We offer an approach that makes it analytically possible to handle these dependencies, and we follow the approach in Fredriksson and Holmlund (2001), where duration is modeled in a stochastic environment as a transition rate from e.g. unemployment to social assistance. Hence, the transition rate determines the expected duration of the benefit period, and analytically this is a manageable proxy for a fixed duration. Similarly, the separation rate determines the expected duration of a job. This approach has the advantage of overcoming problems in handling fixed time periods and allows a specification where the employment history of workers is easily summarized and thus a possible parameter in the unemployment insurance scheme. The framework is a stochastic job-matching model in the spirit of search models but with particular focus on job duration. Clearly, agents would, other things being equal, prefer a longer than a shorter lasting job. This is further strengthened if a long duration of the present job improves the benefit level and duration if becoming unemployed at some later date. A crucial issue is thus whether agents would accept jobs with short durations or turn them down in the hope of finding a job with a longer duration. The latter may in particular be the case for individuals having a strong employment history and thus possibly qualifying for higher benefits and/or longer duration.

In policy debates it is often questioned whether the design of the UIB scheme supports or hinders that unemployed accept jobs with short duration. This may in particular be an issue for unskilled with job possibilities offering wages at or close to the minimum wages; i.e. jobs differ mainly in their duration. This issue is particularly relevant in a flexicurity labor market which on the one hand offers relatively generous unemployment insurance and on the other has lax employment protection legislation implying that job duration can be short. However, the question is more general since it relates to the “quality” of jobs; some jobs are more attractive than others (here simply in terms of expected duration). This is also related to the discussion of temporary vs. per-

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This implies that job tenure differs. The approach in this paper is different but still somewhat related as jobs also differ in tenure, but this is (in expected terms) known before the match is formed.
permanent jobs. In the present setting there is no such explicit distinction between types of job contracts, but the design of the unemployment insurance scheme, in particular the employment history contingencies, create endogenously a distinction between “desirable” and “less desirable” jobs introducing a potentially important margin affecting labor market performance.

We show that the optimal strategy for unemployed is to formulate a reservation duration, i.e. a shortest expected job duration acceptable. If the unemployment insurance scheme includes employment requirements either in the form of higher benefits or longer duration of benefits with a stronger horizon, it follows that the reservation duration is higher, the stronger the past employment history. In a sufficiently generous unemployment scheme, jobs with short duration are not accepted by all unemployed. Individuals with stronger employment histories have higher reservation durations, and therefore experience longer unemployment spells. Benefit levels and durations are shown to be substitutes in determining the reservation durations; i.e. unchanged reservation durations can be achieved by simultaneously increasing the level of benefits and shortening its duration. Increasing UI generosity by increasing the reward to employment history is shown to increase the reservation duration for acceptable jobs.

We show that there is a non-trivial steady-state equilibrium with positive unemployment. Unemployment is not only affected via the reservation duration, and a higher UI duration for given reservation durations leads to lower employment rates (for entitled workers). We report a numerical analysis to supplement the analytical results and use it to discuss some more specific issues on how the design of employment history contingencies affect both the level of (un)employment and its structure.

The paper is organized as follows: Section 2 develops a matching model and works out the optimal reservation duration for unemployed with a given employment history to accept a given job offer. The section includes a detailed discussion of how the reservation duration is determined. Section 3 considers the steady state equilibrium and presents results on how the design of the employment history requirements build into both benefit levels and duration affects employment. Endogenization of wages is considered in Section 4. Section 5 presents a numerical analysis and discusses various policy lessons. Section 6 offers some concluding remarks. All proofs are given in the appendices.
Unemployed workers are randomly matched with employers in a continuous time setting. A job offer is received with a Poisson arrival rate $P$, and it has an expected duration ($T$) drawn from a known distribution, $F$ (pdf denoted $f$) with support on $[\underline{T}, \overline{T}]$. A job with an expected job duration $T$ has a job separation rate $\frac{1}{T}$, see e.g. Mortensen (1977) and Fredriksson and Holmlund (2001). To focus on job duration as a key characteristic of jobs, we assume wages ($w$) to be the same (and exogenous) for all jobs. The population size is assumed constant and normalized to unity.

The UI scheme has two dimensions: the benefit level and its duration. Both may be contingent on the employment history of the unemployed captured by the expected duration ($T$) of the past job possessed by the unemployed. The benefit received by an unemployed is given by the differentiable function $b(T)$ with the property

$$\frac{\partial b(T)}{\partial T} \begin{cases} > 0 & , \ T = \overline{T} \\ \geq 0 & , \ \forall T \in (\underline{T}, \overline{T}] \end{cases}$$

(2.1)

i.e. the level of UIB is non-decreasing in the employment history $T$. It is assumed that $b(T) < w$ for all $T$. The lowest level of benefits, and thus the ultimate support offered by the social safety net (social assistance), corresponds to the benefit received for the lowest possible previous employment spell, $b(\underline{T})$.

Benefits can be claimed for a period with expected duration $D(T)$, $\forall T \in (\underline{T}, \overline{T}]$, and hence, the benefit period expires at the rate $\frac{1}{D(T)}$. $D(T)$ is assumed differentiable. The benefit duration is non-decreasing in the employment history, $\frac{\partial D(T)}{\partial T} \geq 0$.

### 2.1 Reservation duration

If a job offer is received, the job-searcher decides whether to accept or reject the offer. Denote the reservation (expected) duration to acceptable jobs by $R$; i.e. the individual will reject an offer if $T < R$ and accept all offers where $T \geq R$. In determining the reservation duration, the job-searcher faces a trade-off between the immediate gain from accepting a job offer yielding a wage higher than the benefit level and having a

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2For the lowest history $\underline{T}$ corresponding to social assistance, the benefit level can be claimed indefinitely.

3Benefit eligibility is depending on expected job duration rather than the actual duration. Clearly, this assumption is crucial for analytical tractability. Note that the job decision is made ex ante before knowing the exact duration, hence, the difference between a scheme where benefit eligibility depends on the expected rather than the actual job history pertains mainly to the insurance offered if the job duration should turn out to be shorter than expected.
given expected duration or the possibility of waiting in the hope of receiving a better offer with a longer expected duration (higher $T$), which is of direct value, but which also may qualify for higher benefits/duration in future states of unemployment, cf. below.

The value of having a job with job duration $T$, $V(T)$ is given as

$$\rho V(T) = u(w) + \frac{1}{T} [U(T) - V(T)]$$

(2.2)

where $\rho$ is the discount rate, $u(w)$ the flow utility from having a job with wage $w$ ($u(\cdot)$ satisfies all standard assumptions, including $u_w(\cdot) > 0$, $u_{ww}(\cdot) \leq 0$).

The value function $U(T)$ for an unemployed with employment history $T$ is

$$\rho U(T) = u(b(T)) + P [1 - F(R)] [E(V(T') \mid T' \geq R) - U(T)] + \frac{1}{D(T)} [U(T) - U(T)]$$

(2.3)

where $P [1 - F(R)]$ is the rate at which an acceptable offer is received, and $E(V(T') \mid T' \geq R)$ is the expected utility generated by possessing such jobs. We assume that the utility of income is the same for employed and unemployed. If unemployment is associated with some form of stigmatization it is possible that utility of a given income is lower for unemployed than for employed. Modeling this by for instance a fixed transition cost, e.g. associated with transitions from employment to unemployment, does not change the qualitative results.

Rearranging the value functions yields

$$V(T, R, P, w) = \frac{u(w)}{\rho + \frac{1}{T}} + \frac{1}{\rho + \frac{1}{T}} U(T, R, P, w)$$

(2.4)

and

$$U(T, R, P, w) = \frac{u(b(T)) + P [1 - F(R)] E(V(T') \mid T' \geq R) + \frac{U(T)}{D(T)}}{\rho + P [1 - F(R)] + \frac{1}{D(T)}}$$

(2.5)

The above makes explicit which variables influence the value functions. To simplify, below we write the values as $U(T)$ and $V(T)$, suppressing other variables to economize on notation.

Observe for later reference that\footnote{Follows directly from (2.3) by use of $b(T) > b(T)$ for all $T$, and observing that an unemployed with some history $T$ always has the option of choosing the same reservation duration as an unemployed with history $T$.} $U(T) > U(T)$ for all $T > T$. The participation
constraint $V(T) > U(T)$ requires

$$U(T) < \frac{u(w)}{\rho} \quad \text{for all } T \in [T, \bar{T}]$$

The optimal reservation duration maximizes (2.5), i.e.

$$R \equiv \arg \max_R U(T, R, P, w)$$

and the associated first and second order conditions are

$$\frac{\partial U(T, R, P, w)}{\partial R} = 0; \quad \frac{\partial^2 U(T, R, P, w)}{\partial R^2} < 0$$

It can now be established

**Proposition 1.** The reservation duration $R(T)$ is i) for $T \in \left[T, \bar{T}\right)$ given by $R(T) = T$, and ii) for $T \in \left[\hat{T}, \bar{T}\right]$ determined by the condition

$$U(T, R(T), P, w) = V(R(T), R(R(T)), P, w)$$

(2.6)

implying that $T < R(T) < T$ and $0 < \frac{\partial R(T)}{\partial T} < 1$. The threshold duration $\hat{T}$ is uniquely determined by the condition $U(\hat{T}) = V(T)$, and a sufficient condition that $\hat{T} \in [T, \bar{T})$ is $\Theta(b(T), b(T), D(T)) > 0$, where $\Theta(b(T), b(T), D(T)) < 0$ and $\frac{\partial \Theta(b)}{\partial b(T)} > 0$ and $\frac{\partial \Theta(D(T))}{\partial D(T)} > 0$.

**Proof.** See Appendix A. □

The pool of unemployed can thus be divided into two groups depending on their employment history. Those with a weak employment history ($T \leq T \leq \hat{T}$) all have a reservation duration equal to the shortest job duration, i.e. $R(T) = T$. They are at a corner willing to accept jobs of any expected duration, i.e. all jobs. Unemployed with a strong employment history ($\hat{T} < T \leq \bar{T}$) decline job offers which are too short; i.e. they have a reservation duration higher than the lowest job duration, i.e. $R(T) > T$. The reservation duration is determined by the condition that the value of accepting a job with duration $R(T)$ yielding utility $V(R)$ is the same as the value of remaining unemployed given the employment history $T$ yielding utility $U(T)$. This makes intuitive sense; a job is only attractive if the utility when possessing the job is at least as high as remaining unemployed. The reservation duration is always lower than the past employment history, reflecting that jobs offer a higher income than the unemployment benefits ($R(T) < T$). However, agents with a stronger employment history (higher $T$) also have higher reservation durations ($0 < \frac{\partial R(T)}{\partial T} < 1$), showing
that there is an element of persistence in reservation demands. This dependence arises from the role of employment history for benefit levels and duration.

Figure 2.1 illustrates how the reservation duration depends on the employment history of the unemployed. Notice the role of $\tilde{T}$, implying that there potentially is a mass of individuals with a reservation duration $T$. The horizontal line (in dashed) gives the reservation duration in the special case where the employment history is not of importance for benefits ($b(T) = b(\overline{T})$ for all $T$).

The threshold duration of previous employment, $\tilde{T}$, separates unemployed accepting all possible jobs (with previous employment spells of duration $T \in [\underline{T}, \hat{T}]$) from unemployed who do not accept jobs of short duration (with $T \in [\hat{T}, \overline{T}]$). The result above shows that the unemployment insurance scheme has to be sufficiently generous to

\[ \frac{\partial R(T)}{\partial T} = 0 \text{ for all } T \text{ if } b_T(T) = D_T(T) = 0 \text{ for all } T, \text{ implying } R(T) = \overline{T} \text{ for all } T.\]

2.2 Reservation Duration and Properties of the Unemployment Insurance Scheme

The threshold duration of previous employment, $\tilde{T}$, separates unemployed accepting all possible jobs (with previous employment spells of duration $T \in [\underline{T}, \hat{T}]$) from unemployed who do not accept jobs of short duration (with $T \in [\hat{T}, \overline{T}]$). The result above shows that the unemployment insurance scheme has to be sufficiently generous to
generate an interior \( \hat{T} \) (where \( T \leq \hat{T} < T \)). The intuition is that history dependencies in the unemployment insurance scheme rewarding unemployed with a sufficiently strong history make them raise the demands to the duration of jobs they are willing to accept.\(^6\) This may be interpreted as a disincentive effect arising for those who have a strong employment history; see further discussion below. Comparing across schemes with different levels of generosity, we have

**Corollary 2.** A shift from either i) a scheme with benefits \( b(T) \) to a scheme with more generous benefits \( b'(T) \), \( b'(T) > b(T) \), for unchanged \( D(T) \) for all \( T \in [T, \tilde{T}] \), or ii) a scheme with benefit duration \( D(T) \) to a more generous scheme with duration \( D'(T) \), where, \( D'(T) > D(T) \) for \( b(T) \) unchanged \( \forall T \) has a lower threshold duration \( \hat{T}' < \hat{T} \).

**Proof.** See Appendix B. \( \blacksquare \)

A more generous unemployment insurance scheme putting a larger reward to the employment history thus reduces the threshold duration \( \hat{T} \), and this induces more unemployment to have a non-trivial reservation duration.

**Proposition 3.** The reservation duration \( R(T) \) is increasing in \( b(T), D(T), w \) and \( P \) under the sufficient conditions i) \( \frac{\partial f(R)}{\partial R} > -f(R) \forall T \), and ii) \( \lim_{T \to \tilde{T}} \left( \frac{1}{D(T)} \right) < P [f(\tilde{T}) - 1] - \rho \)

**Proof.** See Appendix C. \( \blacksquare \)

If the benefit system becomes more generous (higher benefit level \( b(T) \) or higher duration \( D(T) \)), it has the direct effect of increasing the utility of the unemployed. There is, however, also an indirect effect, since the value of accepting a job today becomes larger since utility in a subsequent unemployment situation becomes higher. The sufficient conditions given in the proposition ensure that the direct effect dominates the indirect effect. The same reasoning applies to the effect of a wage increase, and the job-arrival rate. Note that the wage effect reflects that increased wages increases the value of employment implying that duration of the job becomes more important. Hence, the unemployed increase their reservation duration following increased wages.

Both the benefit level and duration affect the reservation duration, and it can be shown that

**Corollary 4.** Benefit levels and durations are substitutes in the determination of the reservation duration. If \( \{R(T)\}_{T}^{\tilde{T}} \) gives the optimal reservation durations under an unemployment

\(^6\)Note that the system may be so generous that \( \hat{T} = \tilde{T} \), such that all but those with history \( \tilde{T} \) have a reservation duration \( R(T) > \tilde{T} \).
insurance scheme characterized by \( \{ b(T), D(T) \} \) then there exists an alternative scheme \( \{ \tilde{b}(T), \tilde{D}(T) \} \) where \( \tilde{b}(T) > b(T) \) and \( \tilde{D}(T) < D(T) \) for all \( T \) (or vice versa) delivering the same reservation durations \( \{ R(T) \} \) .

**Proof.** See Appendix D. ■

This shows that a system with high benefit levels and short benefit durations may lead to the same reservation durations as a system with low benefits and longer benefit duration; i.e. the two dimensions of the unemployment scheme are substitutes in relation to delivering the same reservation durations for all employment histories. The intuition is that both benefit level and duration increase the utility when unemployed, and hence it is possible to substitute between the two and yet attain the same reservation durations.

## 3 Worker Flows and Steady State Equilibrium

We consider a steady state equilibrium to the model, which in the present setting requires an ergodic distribution of workers across different possible job durations, i.e. that both the share of the unemployed with a given employment history and the share of the population employed with a given expected duration are constant over time. Define \( n(T) \) as the share of the population with employment history \( T \) who are unemployed, and \( e(T) \) as the share of the population employed with employment duration \( T \).

An equilibrium is defined as reservation durations determined by (2.6) and a constant distribution \( \{ e(T), n(T) \} \) of the population across employment and unemployment given all possible job durations \( T \in [\underline{T}, \overline{T}] \).

Consider a given job duration \( T \). The flow into unemployment is \( \frac{1}{T} e(T) \). The inflow into employment depends on who finds this duration acceptable. This is the case for all job-seekers with a reservation duration between \( \underline{T} \) and \( T \). Individuals having a reservation duration \( T \) had a job duration \( R^{-1}(T) \) in their previous job. Hence, the flow into jobs with duration \( T \) is

\[
P f(T) \int_{\underline{T}}^{R^{-1}(T)} n(x) dx
\]
and equal inflows into and outflows from employment of duration $T$ require

\[ P f (T) \int_{\tau}^{R^{-1}(T)} n (x) \, dx = \frac{1}{T} e (T), \forall T \]  

(3.1)

Considering next unemployment. The inflow into unemployment among those with jobs with duration $T_e(T)$ is $\frac{1}{T} e (T)$. There is outflow of unemployed to both employment and social assistance. The number of unemployed with an unemployment history of $T_e(T)$ finding a job is given by $P \{ 1 - F (R (T)) \} n (T)$ and $\frac{1}{D(T)} n (T)$ transit into social assistance (effectively being assigned a past job duration $\hat{T}$). Hence, inflows equal outflows to unemployment with employment history $T_e(T)$ when

\[ \frac{1}{T} e (T) = \left[ P \{ 1 - F (R (T)) \} + \frac{1}{D(T)} \right] n (T), \forall T \in (\tau, \hat{T}] \]  

(3.2)

The similar condition for social assistance recipients (with $T = \hat{T}$) is\(^7\)

\[ \frac{1}{T} e (\hat{T}) + \int_{x \in [\hat{T}, \hat{T}]} \frac{1}{D(x)} n (x) \, dx = P n (\hat{T}) \]  

(3.3)

Combining (3.1) and (3.2) yields

\[ n (T) = \frac{P f (T)}{P \{ 1 - F (R (T)) \} + \frac{1}{D(T)}} \int_{\tau}^{R^{-1}(T)} n (x) \, dx, \forall T \in (\tau, \hat{T}] \]  

(3.4)

and for $\hat{T}$

\[ P f (\hat{T}) \int_{\hat{T}}^{R^{-1}(\hat{T})} n (x) \, dx = P n (\hat{T}) - \int_{x \in [\hat{T}, \hat{T}]} \frac{1}{D(x)} n (x) \, dx \]

From Appendix A we have that the longest previous employment spell for which choosing a reservation job duration equal to the lower bound $\tau$ is denoted $\hat{T}$. Hence, it must be so that $R^{-1}(\hat{T}) = \hat{T}$. This implies that the population share of social assistance recipients is given by

\[ n (\hat{T}) = \frac{P f (\hat{T}) \int_{\hat{T}}^{\hat{T}} n (x) \, dx + \int_{x \in [\hat{T}, \hat{T}]} \frac{1}{D(x)} n (x) \, dx}{P} \]  

(3.5)

\(^7\)Note, as $R (\hat{T}) = \hat{T}$ then by definition $F (R (\hat{T})) = 0.$
From this and (3.3), we have

\[ e(T) = TP f(T) \int_T^T n(x) \, dx \]  \hspace{1cm} (3.6)

Equations (3.4), (3.5) and the function \( R(T) \) give the equilibrium unemployment rate, \( n(T) \), for any \( T \). Given this, employment rates \( e(T) \) can be found from (3.2) and (3.6).

**Proposition 5.** A unique steady-state equilibrium exists with positive (un)employment rates for all durations \( T \in [T, \bar{T}] \).

**Proof.** See Appendix E. \( \blacksquare \)

The prevalence of equilibrium unemployment is a straightforward implication of the exogenous job-separation rate, the stochastic arrival of job offers, and the fact that the offered job duration may fall short of the reservation demand. The fact that \( R(T) < T \forall T \in (T, \bar{T}] \) bounds the relative unemployment rate for a given employment history, since (3.4) implies

\[ \frac{n(T)}{\int_T^{R^{-1}(T)} n(x) \, dx} = \frac{P f(T)}{P [1 - F(R(T))] + \frac{1}{D(T)}} < \frac{P f(T)}{P [1 - F(T)] + \frac{1}{D(T)}}, \forall T \in (T, \bar{T}] \]

i.e. there is an upper bound for the relative unemployment rate at a given employment history \( T \), determined by properties of the job offer distribution function as well as the benefit duration function.

For any job duration \( T > T \), we find from equation (3.2) that the ratio of employed to unemployed is

\[ \frac{e(T)}{n(T)} = T \left[ P [1 - F(R(T))] + \frac{1}{D(T)} \right] \forall T \in (T, \bar{T}] \]  \hspace{1cm} (3.7)

and hence

\[ \frac{\partial e(T)}{\partial T} \left[ \frac{e(T)}{n(T)} \right] = P (1 - F(R(T))) - TP f(R(T)) R_T - \left[ T \frac{D_T(T)}{D(T)} - 1 \right] \frac{1}{D(T)} \]

showing that the employment rate across different job durations can be decomposed into different effects. The duration effect captures that longer job durations directly imply that those holding such jobs will, other things being equal, stay in it longer, which tends to increase the employment rate. The reservation demand works to reduce the employment rate, since a higher job tenure induces a higher reservation duration and
thus lowers the transition out of unemployment. A change in job duration directly increases the employment rate, but via benefit duration it also tends to increase the number of unemployed, and the former effect dominates if the elasticity of benefit duration with respect to job duration is less than one, and vice versa.

Above it was established that the benefit level and duration are substitutes in the determination of the reservation duration. However, the effects on employment rates are different, and we have

**Corollary 6.** A shift from an unemployment scheme \( \{b(T), D(T)\}_{T=0}^{\tilde{T}} \) to a scheme \( \{\tilde{b}(T), \tilde{D}(T)\}_{T=0}^{\tilde{T}} \) leaving all reservation durations unchanged \( \{R(T)\}_{T=0}^{\tilde{T}} \) and the threshold duration \( \tilde{T} \) unchanged, cf. Corollary 4, implies lower employment rates \( \frac{e(T)}{n(T)} \) for all durations \( T > \tilde{T} \), while the employment rate for social assistance claimants \( \frac{e(T)}{n(T)} \) increases if \( \tilde{D}(T) > D(T) \), and vice versa.

**Proof.** See Appendix F. ■

This is a striking result suggesting that a longer benefit duration may be more detrimental to employment than a higher benefit level. The intuition is that a longer duration means that more unemployed retain their benefit level longer, rather than ending up on social assistance upon benefit expiration, which would lead to a lower reservation duration and thus more job acceptance. As a consequence, employment rate falls for all \( T > \tilde{T} \). For social assistance recipients a higher UI duration implies a lower inflow of unemployed losing entitlement. This decreases the population share of social assistance recipients \( \frac{n(T)}{T} \) decreases), which in turn increases the employment share, \( \frac{e(T)}{n(T)} \).

4 Numerical illustration

The preceding has presented analytical results on the effects of employment history contingencies in the eligibility conditions for unemployment insurance. To shed further light on how the specific formulation of such contingencies affect reservation durations as well as the labor market equilibrium we present a numerical analysis. The model is too stylized to warrant a “full” calibration, but we have chosen parameter values and in particular the design of the unemployment insurance scheme to match the key characteristics in Denmark. In addition to the national bias of the authors this is of interest since Denmark is often highlighted as a flexicurity country and because the

\[ \frac{e(T)}{n(T)} \]

\[ \frac{e(T)}{n(T)} \]

\[ \text{If } \tilde{T} = T \text{ then } \frac{e(T)}{n(T)} \] is independent of the policy change.
issue of the design of employment history contingencies has recently been vividly discussed in Denmark from the perspective of strengthening the incentive of unemployed (primarily unskilled) to accept short-term jobs.

4. PARAMETER VALUES

The utility function is assumed to be a time-additive CRRA utility function with an Arrow-Pratt measure of relative risk aversion, $\gamma$, set to 2 to capture an elasticity of intertemporal substitution of $\frac{1}{2}$ well below one as argued by Thimme (2017). However, the specific parameter choice is not at all pivotal for the qualitative insights.

$$u(c) = \frac{c^{1-\gamma}}{1-\gamma}$$

The (yearly) discount rate ($\rho$) is set at 0.05 as in most of the literature. The job offer distribution is assumed to be exponential\(^9\) and truncated above at $T$ and below at $T$.

$$F(T) = \frac{\exp(-\lambda T) - \exp(-\lambda T)}{\exp(-\lambda T) - \exp(-\lambda T)}$$

$$f(T) = \frac{\lambda \exp(-\lambda T)}{\exp(-\lambda T) - \exp(-\lambda T)}$$

The distribution parameter and the value of $T$ are calibrated using OECD tenure data for Denmark, where the average employment spell is 6.9 years.\(^10\) Thus, the exponential distribution parameter, $\lambda$, is set to 0.132, the probability of a match taking place, $P$, is set to 0.99, and $T$ is set to 14.125 years. The lower bound of the job duration interval, $T$, is just above zero at $10^{-16}$. The wage is exogenous and normalized to unity and time is discretized.

The specification of the benefit and duration functions here does not satisfy all the assumptions made in the theoretical part, where the corresponding functions were assumed continuously differentiable to make the analytical analysis tractable. We choose the alternative specification here because it has an empirical motivation and as a robustness check to show that the theoretical results hold under weaker conditions than those made in the theoretical part. The UIB system is parameterized to mimic the Danish two-tier system with a high-level replacement rate of 0.628 and the replacement

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\(^9\)Implying that the density is declining in job-duration. We have no prior empirical knowledge on the distribution function, but it is a well-known fact that job-turnover in Denmark is high, suggesting that many job-offers have a relative short duration.

\(^10\)For 2005, a year usually considered to be business cycle neutral. The observation of a tenure of 10 years or above is assumed to last for 15 years in average when calculating the average employment spell.
rate of social assistance at 0.354.\(^{11}\)

\[
b(T) = \begin{cases} 
0.354 & , \quad T = T \\
0.628 & , \quad \forall T \in ]T, \bar{T}] 
\end{cases}
\quad (4.1)
\]

The duration of high level benefits is a function of the length of the previous employ-
ment spell given by a ‘kink-function’ that increases linearly from zero to the maximum
duration, after which it stays constant. Hence, we relax the theoretical assumption of
differentiability in the point where the maximum duration is earned, denoted \(T_{\text{max}}\).

\[
D(T) = \begin{cases} 
D_{\text{slope}}T & , \quad T < T \leq T_{\text{max}} \\
D_{\text{max}} & , \quad \forall T \in ]T_{\text{max}}, \bar{T}] 
\end{cases}
\quad (4.2)
\]

\(T_{\text{max}}\) is initially set to one year, as in the current Danish system, and the maximum
duration of high-level benefits is two years.\(^{12,13}\) Hence, \(T_{\text{max}} = 1\), \(D_{\text{max}} = 2\), and \(D_{\text{slope}} = \frac{D_{\text{max}}}{T_{\text{max}}} = 2\).

<table>
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<th>Value</th>
<th>Description</th>
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<td>(\rho)</td>
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<td>(\lambda)</td>
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<tr>
<td>(\bar{T})</td>
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<td>Lower bound of job duration interval</td>
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<td>(w)</td>
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<td>Wage level</td>
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<tr>
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<td>High-level replacement rate</td>
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<tr>
<td>(b(T))</td>
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<td>Replacement rate of social assistance</td>
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<td>Req. emp. his. to earn maximum UIB duration</td>
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<tr>
<td>(D_{\text{max}})</td>
<td>2</td>
<td>Maximum duration of high-level benefits</td>
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</table>

Table 4.1: Parameter values

### 4.2 Employment and Unemployment

The properties of the unemployment insurance scheme imply a reservation duration \(R(T)\) as displayed in figure 4.1. The reservation duration is increasing until the point at

\(^{11}\)See Dagpengekommissionen (2015c).

\(^{12}\)Although in the Danish system all employment spells in the last three years are counted instead of just the last spell as is the abstraction in this model.

\(^{13}\)For this parameterization we have \(\Theta(b(T), b(T), D(T)) = 0.8\) which implies that we have interior solutions to the reservation durations.
which the worker is entitled to the maximum duration of benefits, $T_{\text{max}}$, after which it stays constant.\textsuperscript{14} Furthermore, note that parameterization implies that the reservation job duration $R(T)$ is larger than $T$ for all $T > T_r$ reflecting that $\hat{T} = T_r$; i.e. even persons having been employed in jobs with a short duration would not accept jobs with the lowest possible duration. In the parameterization all will accept jobs of a duration of $2/3$ year or longer.

![Reservation job duration $R(T)$ as depending on employment history.](image)

**Figure 4.1:** Reservation job duration $R(T)$ as depending on employment history.

The equilibrium distribution of employment as depending on job duration displays a hump-shaped pattern, figure 4.2.a. This is so even though the underlying distribution of job offers implies that more short than long duration jobs are offered at a given moment in time. The hump-shaped pattern for employment reflects the net effect of two counteracting effects. First, a longer duration implies that more workers are in employment at a given moment, which tends to give an upward sloping relation. Second, since the distribution of job offers is downward sloping in the job duration, it is less likely to be matched with a job having a long duration, which produces a downward sloping relation. For low durations the former effect dominates, and vice versa for long duration. From an empirical point of view it is interesting to note that the employment distribution looks completely different than the underlying job-offer distribution. Note that the inflow into jobs, cf. figure 4.2.b, is at first increasing in duration, reflecting that more are accepting the jobs, and then declining, reflecting that

\textsuperscript{14}Note that the step-wise increase is a result of the numerical discretization of time.
the probability of receiving a job offer of a given duration is declining in the duration. The relative employment to unemployment rate, cf. figure 4.2.c, is increasing in the duration, but has a kink at $T_{\text{max}}$ (where the duration function becomes constant, cf. (4.2)), reflecting that jobs with longer duration are more likely to be filled and that individuals stay in them longer. For $T > T_{\text{max}}$, only the latter effect is at play (and the relation is linear).\textsuperscript{15} Finally, depending on the employment history, the distribution of the unemployed also has a hump-shaped form, but the increasing part arises only for low durations, reflecting low reservation demands and thus more job-matching. For higher duration, unemployment is unambiguously declining in the employment history because a lower outflow into unemployment. Furthermore, the share of the population who receive social assistance is represented by the spike at $T$ in figure 4.2.d.

Figure 4.2 also shows the employment and unemployment levels in the case of no history dependencies in the unemployment insurance scheme\textsuperscript{16}. There are two key differences. First, and unsurprising, the level of employment is lower as a result of a more generous unemployment scheme, as is the case when there is a “reward” in the form of higher benefits to a stronger employment history. Second, the history contingency “cuts” off jobs in the lower end of the duration distribution. This reflects the reservations demands induced by the history contingency. As a result, employment is (slightly) higher at longer duration, capturing the effect discussed above; that is, when job seekers decline jobs with short duration, there is no locking-in effect, and hence they are more likely to receive a job offer with a longer duration. The composition of jobs across the short-long duration dimension thus changes.

\textsuperscript{15}The effects of changes in the probability of receiving a given job offer cancels out as it affects both employment and unemployment (this can directly be seen from (3.7)).

\textsuperscript{16}Specifically, $b(T)$ is independent of $T$ and equal to the level of SA in the system above. In such a system, all unemployed will accept all jobs, i.e. $R(T) = T \forall T$. 

126
Figure 4.2: History and duration dependent employment and unemployment.

How the relative employment to unemployment rate depends, cf. figure 4.2.c, can be decomposed into a duration, reservation demand and benefit duration effect, cf. Section 3. This decomposition is shown in figure 4.3 both for the case with and without history dependent contingencies. Here it can be seen that all three underlying effects are constant when the UI duration function is constant (for \( T \geq T_{\text{max}} \)). The duration effect is positive for all job durations (but decreasing), as longer job durations directly imply that those holding such jobs will, other things being equal, stay in it longer, which increases the relative employment to unemployment rate. The effect of changed reservation demands reduces the relative employment to unemployment rate at an increasing speed as a result of increasing job durations (zero when the UI duration function is constant). As the elasticity of benefit duration with respect to previous job duration under the adopted functional forms is lower than one, the benefit duration effect increases the relative employment to unemployment.
Figure 4.3: The increase in relative employment to unemployment rate decomposed.

Finally, figure 4.4 compares the model generated employment tenures to the observed tenures. The model captures the longer tenures reasonably well, while it falls shorts of matching the jobs with shorter tenures. The latter is noteworthy as the exponential distribution function is “tilted” towards generating many job offers with short durations.

Figure 4.4: Generated employment tenures vs. observed tenures.
4.3 **Benefit duration and (un)employment**

A key result demonstrated analytically is, that changes in benefit levels and durations leaving reservation duration unchanged (Corollary 4) still affect employment and unemployment (Corollary 6). This result is illustrated here for a case where benefit duration is increased, cf. figure 4.5.a, but with corresponding changes in benefit levels leaving the reservation duration unchanged.¹⁷

![Figure 4.5: Policy change increasing the UIB duration while lowering the level leaving all reservation durations unchanged, and employment rates.](image)

Figure 4.5: Policy change increasing the UIB duration while lowering the level leaving all reservation durations unchanged, and employment rates.

This change causes employment to decrease for all durations, cf. figure 4.5.b. This points out that the effects of the unemployment insurance scheme can not be judged solely from the reservation duration. In the experiment shown, for unchanged reservation duration, unemployed with a given employment history obtain a longer benefit duration, and this lowers outflow from unemployment (and inflow into social assistance), which in turn reduces job-matching possibilities and thus employment.

4.4 **Benefit levels and durations**

The history dependence derives from the way the benefit level and duration is linked to employment histories. To see this more clearly we explore various changes in the duration function.¹⁸

---

¹⁷In the present parameterization $\bar{T} = \underline{T}$; i.e. we have that the employment rate of workers with $T = \bar{T}$ is unchanged while the employment rate is decreasing for all $T > \bar{T}$ following a shift to a system $
\left\{ b(T), D(T) \right\}$ leaving all reservation durations unchanged $
\left\{ R(T) \right\}$ and the threshold duration ($\bar{T}$) unchanged as $\bar{D}(T) = D(\bar{T})$. $\bar{D}(T)$ is characterized by $\bar{T}_{\max, D} = 0.5$, $\bar{D}_{\max} = 2.25$.

¹⁸If $D(T) = D_{\max}, \forall T$ and $b(T)$ has two levels, the UIB system is a two-tier scheme with automatic entitlement as is the case of e.g. Mortensen (1977), cf. also (4.1) and (4.2). In such a scheme the $D(T)$ function becomes crucial since it links history to the two benefit levels.
Considering the history dependence built into the $D(T)$-function in (4.2), there are three possible adjustments: (I) Level: Adjusting the maximum UIB duration, $D_{max}$, while keeping the slope, $D_{slope}$, fixed (and let $T_{max}$ adjust accordingly). (II) Slope: Adjusting the required employment duration to earn the maximum UIB duration, $T_{max}$, while keeping the maximum UIB duration, $D_{max}$, fixed and letting $D_{slope}$ adjust accordingly. (III) Level and slope: Adjusting the maximum UIB duration, $D_{max}$, while keeping the required employment duration, $T_{max}$, fixed (and let $D_{slope}$ adjust accordingly). Figure 4.6 shows the changes in the D(T) function as well as the implied reservations durations, and figure 4.7 gives the implied employment relations as well as average employment spells and unemployment rates for the difference specifications.

In case (I), reservation durations increase for workers with such a strong employment history that they are eligible for the highest benefit duration. The improved generosity of the scheme makes them increase their reservation demands. Case (II) increases the required length of previous employment (and decrease the slope accordingly), and this leads to an increase in the reservation job duration of most unemployed. The reason is that unemployed with a sufficiently strong employment history do not encounter any difference in their current UIB duration. However, if they take on a job with a short duration, their expected UIB duration will be lower in a subsequent unemployment situation. This implies that the value of such a job decreases, while their current value of unemployment remains unchanged, and therefore their reservation duration increases. For unemployed who are unentitled to the maximum UIB duration both before and after the adjustment, it is still the case that the value of short duration jobs decreases. However, their current value of unemployment also decreases. Because of discounting and a non-perfect matching technology, the current value decreases more than the future. Hence, the reservation duration for such unemployed decreases (but numerically the effect is tiny). Case (III) does not give as large an increase in the reservation job duration as case (I) due to the counteracting effects of a higher sensitivity of benefit duration to the employment history (the increase $D_{slope}$) and the increase in the maximum UIB duration. This is so since a higher reward to the employment history increases the value of jobs with lower durations, and this goes in the direction of decreasing the reservation duration.

The increased reservation duration in case (I) decreases the share of unemployed willing to take a job with low durations, implying that employment is significantly reduced for low levels of job duration and slightly reduced at higher durations. The longer benefit duration implies that the unemployed maintain their high-level benefits for a longer period of time, which keeps their reservation demands above $T$ for longer. This implies that the average employment spell decreases and total unemployment
SECTION 3. JOB DURATION AND HISTORY DEPENDENT UNEMPLOYMENT INSURANCE

4. NUMERICAL ILLUSTRATION

(I) Level

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Figure 4.6: Possible adjustments of the $D(T)$-function.
Figure 4.7: Implications of adjusted \( D(T) \)-function.
increases (columns in figure 4.7 correspond to the different policies). In case (II) employment decreases for short durations and increases slightly for longer durations, and vice versa for unemployment. Although the reservation durations are largely unaffected, the shorter benefit duration at low duration implies that this group of unemployed end up on social assistance quicker, and this results in an increase in unemployment for this group. As a consequence, employment increases with tenure and unemployment decreases with tenure, and the population share of unemployed receiving social assistance increases. Average employment spells increases and unemployment decreases. Comparing case (III) to case (I), the results are quite similar. However, total unemployment increases slightly more. This is because keeping the required employment duration, $T_{\text{max}}$, constant and letting $D_{\text{slope}}$ increase accordingly implies an increase of UIB duration for all workers not entitled to the maximum duration compared to when the speed of entitlement accumulation was kept constant. In sum, the way employment history is rewarded in terms of benefit duration is important, a high maximum benefit duration is less important than how easily it can be obtained.

5 Concluding remarks

We have analyzed the role of employment history contingencies in unemployment insurance for labor market performance. Such contingencies imply that the job search strategy of unemployed is given by reservation demands to the shortest duration for a job to be acceptable. We develop a search-matching model where jobs differ in their duration. The job-search strategy of unemployed is given by reservation demands to the shortest duration for a job to be acceptable. The reservation duration depends critically of how the unemployment insurance scheme rewards a strong employment history in terms of either benefit level and/or benefit duration. A stronger reward in either dimension tends to increase reservation durations, implying that job acceptance rates come to depend on employment histories. This implies that strong path dependencies arise and otherwise identical agents can end up in very different situations. Unemployed with weak histories are more likely to accept short duration jobs. Thus, unemployed with a low reservation duration are more likely to end up accepting a short duration job. In contrast, unemployed with a stronger history have higher reservation durations and therefore decline short duration jobs (as long as they remain eligible for unemployment benefits). These mechanisms have several important implications, including that the design on history contingencies in the unemployment insurance scheme affects not only the level of (un)employment but also its composition. In a sufficiently generous
scheme, there may be few accepting short duration jobs. On the other hand, acceptance of short duration jobs implies a locking-in effect in the sense that job-opportunities with longer durations are not received. Hence, rewarding employment history may reduce short duration jobs and increase longer duration jobs.

Numerical analyses reported in the paper with outset in the role of employment history contingencies in the Danish unemployment insurance scheme shed further light on the effects of such contingencies. We show that an increase in the maximum UIB duration induces unemployed to be more "picky" in terms of required job duration. This leads to an increase in unemployment. This is in line with predominant parts of the literature on the effect of UIB duration on reservation wages as well as search effort (see e.g. Fredriksson and Holmlund (2001)). Increasing requirements to the employment history (i.e. the duration of previous employment required to earn entitlement to maximum UIB duration) increases the reservation duration of unemployed currently qualified to maximum UIB duration, while unemployed with shorter previous employment spells decrease their reservation duration (slightly). However, in aggregated terms the former effect dominates and unemployment increases. The latter finding somewhat goes in the opposite direction of the effects on search effort, as search effort seems to increase for unemployed qualified for maximum UIB duration, while it decreases for non-entitled unemployed following stricter requirements on the employment history (see e.g. Andersen et al. (2015), Ellermann-Aarslev (2017b), and Ellermann-Aarslev (2017a)). However, a lower reservation duration is not equivalent to a higher level of search effort, as conducting search effort is an investment costly today, whereas deciding on a certain reservation duration is a choice requiring no effort.

Finally, it is worth pointing out that the findings have possible implications in an explicit business cycle model, since employment history contingencies may cause persistence also at the aggregate level. That is, in booms many accumulate a stronger employment history, which implies high reservation demands should a recession cause subsequent unemployment, and this will impair the job recovery.

The model specification has purposeful being stylized to focus on the basic effects of employment histories and allow analytical results. In future work there are a number of interesting extensions and generalization worth exploring. Firstly, moving from a partial to a general equilibrium setting it is obvious to endogenize wage setting and job-offers. This would add additional sources of path dependencies since the wage would depend both on the duration of past jobs and the duration of the job offered. This is so since benefit eligibility depends on the employment history, and since the value of a job depends on its duration. We think of the issues discussed as being
primarily of importance for unskilled (which has also been the motivation in recent policy discussions), but it would be interesting to introduce worker heterogeneity explicitly to explore how employment history contingencies in unemployment insurance affect different segments of the labor market. Employment history contingencies imply that jobs with longer duration get an additional attraction. A frequent argument that short-term jobs should be accepted is that they are an important stepping stone for later obtaining long-term (more valuable) jobs. To analyze this aspect it is necessary to bring in on-the-job-search. Doing so, workers would not be locked in a job of short duration but rather continue to search for more lengthy employment. However, introducing on-the-job-search requires careful consideration of the conditions under which employed workers can search for jobs with longer duration than their current job, and implies that the reservation duration for acceptable jobs depends on the duration of the current job. Finally, introducing on-the-job-search would call for an explicit consideration of the costs of on-the-job search.
6 References


Dagpengekommissionen. 2015a. “Dagpengekommissionens Hovedrapport.”


APPENDICES

A PROOF PROPOSITION 1

First, note that $E(V(T') \mid T' \geq R)$ by use of (2.4) can be written

$$
E(V(T') \mid T' \geq R) = \frac{1}{1 - F(R)} \int_{R}^{T} V(x) f(x) \, dx
$$

$$
= \frac{1}{1 - F(R)} \int_{R}^{T} \left[ \frac{u(w)}{\rho + \frac{x}{\rho}} + \frac{1}{\rho} U(x) \right] f(x) \, dx
$$

$$
= \frac{1}{1 - F(R)} H(R)
$$

where

$$
H(R(T)) = \int_{R(T)}^{T} [u(w) x + U(x)] \frac{f(x)}{\rho x + 1} \, dx
$$

Using this, the value of unemployment can be rewritten as

$$
U(T, R(T), P, w) = \frac{u(b(T)) + PH(R) + \frac{U(T)}{D(T)}}{\rho + P[1 - F(R)] + \frac{1}{D(T)}}, \quad \forall T \in [\underline{T}, \overline{T}] \tag{A.1}
$$

Likewise the value of social assistance is

$$
U(\underline{T}) = \frac{u(b(\underline{T})) + PH(R(\underline{T}))}{\rho + P[1 - F(R(\underline{T}))]}
$$

For unemployed with $T > \underline{T}$ the marginal utility effect of a change in the reservation duration $R$ is

$$
\frac{\partial U(T, R, P, w)}{\partial R} = \frac{P f(R)}{\left[\rho + P(1 - F(R)) + \frac{1}{D(T)}\right]^2} \left[ u(b(T)) + PH(R) + \frac{U(T)}{D(T)} \right] + \frac{1}{\rho + P(1 - F(R)) + \frac{1}{D(T)}} P \frac{\partial H(R)}{\partial R}
$$

$$
= \frac{P f(R)}{\left[\rho + P(1 - F(R)) + \frac{1}{D(T)}\right]^2} \left[ u(b(T)) + PH(R) + \frac{U(T)}{D(T)} + \frac{\rho + P(1 - F(R)) + \frac{1}{D(T)}}{f(R)} \frac{\partial H(R)}{\partial R} \right]
$$

137
Using (A.1), (2.4), and that
\[
\frac{\partial H(R)}{\partial R} = - [(u(w) R + U(R))] \frac{f(R)}{\rho R + 1} < 0
\]
we get
\[
\frac{\partial U(T, R, P, w)}{\partial R} = \frac{P f(R)}{\rho + P (1 - F(R)) + \frac{1}{D(T)}} \left[ \frac{u(b(T)) + PH(R) + \frac{U(T)}{D(T)}}{\frac{1}{D(T)} \frac{f(R)}{f(T)}} \right] - \frac{u(w) R + U(R)}{\rho R + 1}
\]
Define
\[
\Phi(R, T, P, w) \equiv U(T, R(T), P, w) - V(R(T), R(R(T)), P, w)
\]
Hence, the optimal reservation duration, $R$, ensuring $\frac{\partial U(T, R(T), P, w)}{\partial R} = 0$ is given by $R : \Phi(R, T, P, w) = 0$. The optimal reservation duration of a job offer can thus be written as an implicit function $R(T, P, w)$ of the employment history. To economize on notation, we write the value function for unemployed $U(T)$ in the text as a short-hand for $U(T, R(T), P, w)$ and $V(R)$ as short-hand for $V(R(T), R(R(T)), P, w)$.

**A.1 Second-order condition**

Since
\[
\frac{\partial U(T, R(T), P, w)}{\partial R} = \frac{P f(R)}{\rho + P (1 - F(R)) + \frac{1}{D(T)}} \Phi
\]
and
\[
\Phi(R, T, P, w) = \frac{u(b(T)) + PH(R) + \frac{U(T)}{D(T)}}{\frac{1}{D(T)} \frac{f(R)}{f(T)}} - \frac{u(w) R + U(R)}{\rho R + 1}
\]
it follows that
\[ \frac{\partial^2 U(T, R(T), P, w)}{\partial R^2} = \frac{P \partial^2 f(R)}{\partial R^2} \left[ \rho + P (1 - F(R)) + \frac{1}{D(T)} \right] + \left[ P f(R) \right]^2 \Phi \]
\[ \left[ \rho + P (1 - F(R)) + \frac{1}{D(T)} \right]^2 \]
\[ + \frac{P f(R) \partial \Phi}{\rho + P (1 - F(R)) + \frac{1}{D(T)} \partial R} \]

Evaluating this for the optimal reservation duration \((\Phi(\cdot) = 0)\), we get
\[ \frac{\partial^2 U(T, R(T), P, w)}{\partial R^2} \bigg|_{R_{\text{optimal}}} = \frac{P f(R)}{\rho + P (1 - F(R)) + \frac{1}{D(T)} \partial R} \]

From the definition of \(\Phi(\cdot)\) it follows
\[ \frac{\partial \Phi}{\partial R} = \frac{P \partial^2 H(R)}{\partial R^2} \left[ \rho + P (1 - F(R)) + \frac{1}{D(T)} \right] + P f(R) \left[ u(b(T)) + P H(R) + \frac{U(T)}{D(T)} \right] \]
\[ \left[ \rho + P (1 - F(R)) + \frac{1}{D(T)} \right]^2 \]
\[ - \left[ u(w) + \frac{\partial U(R)}{\partial R} \right] [\rho R + 1] - \rho [u(w) R + U(R)] \]
\[ \left[ \rho R + 1 \right]^2 \]

and using that \(U(T) = \frac{u(b(T)) + P H(R) + \frac{U(T)}{D(T)}}{\rho + P (1 - F(R)) + \frac{1}{D(T)}}\) and \(V(R) = \frac{u(w) R + U(R)}{\rho R + 1}\) we get
\[ \frac{\partial \Phi}{\partial R} = \frac{P \left[ \frac{\partial H(R)}{\partial R} + f(R) U(T) \right]}{\rho + P (1 - F(R)) + \frac{1}{D(T)}} - \frac{u(w) + \frac{\partial U(R)}{\partial R} - \rho V(R)}{\rho R + 1} \]

Thus, \(\frac{\partial^2 U(T, R(T), P, w, e)}{\partial R^2} \bigg|_{R_{\text{optimal}}} < 0\) if
\[ \frac{P \left[ \frac{\partial H(R)}{\partial R} + f(R) U(T) \right]}{\rho + P (1 - F(R)) + \frac{1}{D(T)}} - \frac{u(w) + \frac{\partial U(R)}{\partial R} - \rho V(R)}{\rho R + 1} < 0 \]

Inserting the expression for \(\frac{\partial H(R)}{\partial R}\) the condition reads
\[ \frac{P f(R)}{\rho + P (1 - F(R)) + \frac{1}{D(T)}} [U(T) - V(R)] - \frac{u(w) + \frac{\partial U(R)}{\partial R} - \rho V(R)}{\rho R + 1} < 0 \]

Optimal \(R(T)\) implies \(U(T) = V(R(T))\) and \(\frac{\partial U(R)}{\partial R} = 0\). Hence, the condition simplifies to
\[ V(R) < \frac{u(w)}{\rho} \]
which is satisfied as the RHS gives the value of employment with infinite duration and all reservation durations are finite.

A.2 Threshold duration $\hat{T}$

The above presumes that there is an interior solution, i.e. a $R(T) \in [T, \bar{T}]$ solving

$$U(T, R(T), P, w) - V(R(T), R(R(T)), P, w) = 0 \quad (A.2)$$

but this is not necessarily the case for all $T \in [T, \bar{T}]$. To clarify this, note first that any solution to (A.2) implies that $R(T) < T$ for all $T$. To prove this, assume that $R(T) \geq T$ for some $T$. Since the optimal $R$ satisfies $U(T) = V(R(T))$ and $V(\cdot)$ is increasing in $R$, this implies

$$U(T) = V(R(T)) \geq V(T)$$

But this violates the participation constraint, $V(T) > U(T)$, hence we can conclude that $R(T) < T$, $\forall T \in (T, \bar{T}]$ if an interior solution exists. This has the implication that a reservation duration equal to the upper bound of the interval is not optimal for any unemployed, $R(T) < \bar{T}$, $\forall T$. Next, individuals with a history $T$, i.e. social assistance recipients, will always accept any job duration $T \in [T, \bar{T}]$, i.e. $R(T) = T$. To prove this, assume $R(T) > T$. If this was optimal then

$$U(T) = V(R(T))$$

since $\frac{\partial V(R)}{\partial R} > 0$ it follows that $V(R(T)) > V(T)$, but the participation constraint implies that $V(T) > U(T)$. Hence, a contradiction and it follows that $R(T) = T$.

The above rules out that $R(T) \geq \bar{T}$ for any $T \in [T, \bar{T}]$. However, for some unemployed with a weak employment history (small $T > T$) it may be optimal to choose a reservation duration $\bar{T}$. To explore this, define $\hat{T}$ as the duration implying

$$U(\hat{T}) = V(T)$$

i.e. the optimal reservation duration for an unemployed with history $\hat{T}$ is $\bar{T}$, $R(\hat{T}) = T$. Since $\frac{\partial U(T)}{\partial T} > 0$ (see proof below), it follows that for histories $T \in (\hat{T}, \bar{T}]$ there is an interior solution to $R(\cdot)$ determined by (A.2). For unemployed with a history $T \in [T, \hat{T}]$ if follows that $U(T) \leq V(T)$ and hence those unemployed are at the corner where $R(T) = T$. Finally, we need to demonstrate that $\hat{T} < \bar{T}$ to have non-trivial reservation durations ($R(T) > T$) for some employment histories $T$. A sufficient condition is that

140
$U(T) > V(T)$ which implies that individuals with a strong employment history will choose a reservation duration $R(T) > T$.

Define

$$\Theta(b(T), b(T), D(T)) \equiv \frac{u(b(T)) - u(b(T))}{\rho + P[1 - F(T)]} + \frac{1}{D(T)} - \frac{u(w)}{\rho + \frac{1}{T}} + \frac{\rho}{\rho + \frac{1}{T}} U(T)$$

where $U(T) = \frac{u(b(T)) + PH(R(T))}{\rho + P[1 - F(R(T))]}$ and

$$\frac{\partial \Theta(b(T), b(T), D(T))}{\partial b(T)} > 0$$

$$\frac{\partial \Theta(b(T), b(T), D(T))}{\partial D(T)} > 0$$

Moreover, the participation constraint $V(T) > U(T)$ implies

$$\Theta(b(T), b(T), D(T)) < 0$$

Note that

$$\Theta(b(T), b(T), D(T)) > 0$$

requires a sufficiently generous UIB scheme; that is, either a higher benefit generosity (high $b(T)$ relative to $b(T)$) or a long benefit duration ($D(T)$).

We will prove that $\Theta(b(T), b(T), D(T)) > 0$ is a sufficient condition that $U(T) > V(T)$ and hence $R(T) > T$ implying that $T < T$. The proof is by contradiction. Assume that $R(T) = T$ or

$$U(T) \leq V(T)$$

since

$$V(T) = \frac{u(w)}{\rho + \frac{1}{T}} + \frac{1}{\rho + \frac{1}{T}} U(T)$$

the inequality can be written

$$U(T) - U(T) \leq \frac{u(w)}{\rho + \frac{1}{T}} - \frac{\rho}{\rho + \frac{1}{T}} U(T)$$

(A.3)
Using

\[ U(T) = \frac{u(b(T)) + PH(R(T))}{\rho + P[1 - F(R(T))] + \frac{1}{D(T)}} \]

\[ U(T) = \frac{u(b(T)) + PH(R(T))}{\rho + P[1 - F(R(T))] + \frac{1}{D(T)}} \]

it follows, if \( R(T) = T \) then (recall \( R(T) = T \) and \( F(T) = 0 \))

\[ U(T) - U(T) = \frac{u(b(T)) + PH(T) + \frac{U(T)}{D(T)}}{\rho + P[1 - F(T)] + \frac{1}{D(T)}} - U(T) \]

\[ = \frac{u(b(T)) + PH(T) + \frac{U(T)}{D(T)}}{\rho + P[1 - F(T)] + \frac{1}{D(T)}} - U(T) \]

\[ = \frac{\rho + P[1 - F(T)] + \frac{1}{D(T)}}{\rho + P[1 - F(T)] + \frac{1}{D(T)}} \]

Using this in (A.3) we require

\[ \frac{u(b(T)) - u(b(T))}{\rho + P[1 - F(T)] + \frac{1}{D(T)}} \leq \frac{u(w)}{\rho + \frac{1}{T}} - \frac{\rho}{\rho + \frac{1}{T}} U(T) \]

or

\[ \Theta(b(T), b(T), D(T)) \leq 0 \]

which contradicts the assumption that \( \Theta(b(T), b(T), D(T)) > 0 \).

A.3 Properties of reservation duration \( R(T) \)

From (A.2) it follows that \( \frac{\partial R(T)}{\partial T} = \frac{\partial U(T)}{\partial R} \) (using that \( \frac{\partial U(T)}{\partial R} = 0 \)). From (A.1) we have

\[ \frac{\partial^2 R(T)}{\partial T^2} = \frac{\partial^2 U(T)}{\partial T^2} - \frac{\partial U(T)}{\partial R} \frac{\partial V(T)}{\partial R} \left[ \frac{\partial V(T)}{\partial R} \right]^2 \]
where
\[
\frac{\partial U(\cdot)}{\partial T} = \frac{u_c(b(T)) b_T(T) - \frac{U(T)}{D(T)}^2 D_T(T)}{\rho + P [1 - F(R(T))] + \frac{1}{D(T)}} - \left[ u(b(T)) + PH(R(T)) + \frac{U(T)}{D(T)} \right] \left[ - \frac{1}{D(T)^2} D_T(T) \right] \\
\left[ \rho + P [1 - F(R(T))] + \frac{1}{D(T)} \right]^2
\]

Implying
\[
\frac{\partial U(T)}{\partial T} = \frac{u_c(b(T)) b_T(T) + [U(T) - U(T)] \left[ - \frac{1}{D(T)^2} D_T(T) \right]}{\rho + P [1 - F(R(T))] + \frac{1}{D(T)}} > 0
\]

Note that the dependence on T arises solely from the properties of the unemployment insurance scheme, \( b_T(T) \geq 0 \) and \( D_T(T) \geq 0 \). To have a non-trivial situation, we require either that \( b_T(T) > 0 \) or \( D_T(T) > 0 \) for all \( T \in (T, T) \).

From (2.4) we have
\[
\frac{\partial V(R)}{\partial R} = \left[ \frac{u(w) + \frac{\partial U(T)}{\partial T} |_{T=R} [R \rho + 1] - [Ru(w) + U(R)] \rho} {\frac{1}{[R \rho + 1]^2}} \right] \\
= \frac{\rho}{[R \rho + 1]^2} \left[ \frac{\partial U(T)}{\partial T} |_{T=R} \frac{[R \rho + 1]}{\rho} + \frac{u(w)}{\rho} - U(R) \right] > 0
\]

since \( \frac{u(w)}{\rho} - U(T) > 0 \) and \( \frac{\partial U(T)}{\partial T} |_{T=R} > 0 \) from above. This proves that \( \frac{\partial R(T)}{\partial T} > 0 \).

Given this as well as continuity, \( R(T) = T \), and \( R(T) < T \) it follows that \( \frac{\partial R(T)}{\partial T} < 1 \) \( \forall T \in (T, T) \).

B Proof Corollary 2: Threshold \( \hat{T} \)

The threshold for the inner solution of the reservation duration, \( \hat{T} \), is affected by the UI system, i.e. by changes in \( b(T) \) and \( D(T) \). The condition for the determination of \( \hat{T} \) can be written \( U(\hat{T}) = V(T) \) or equivalently as
\[
\Delta(\hat{T}) \equiv \frac{u(b(\hat{T})) + PH(R(\hat{T})) + \frac{U(\hat{T})}{D(\hat{T})}}{\rho + P [1 - F(R(\hat{T}))] + \frac{1}{D(\hat{T})}} - \left[ \frac{u(w)}{\rho + \frac{1}{T}} + \frac{1}{\rho + \frac{1}{T}} \right] U(T) = 0
\]
As noted earlier, \( \frac{\partial \Delta(\hat{T})}{\partial T} = \frac{\partial U(\hat{T})}{\partial T} > 0 \). Expressing \( \Delta(\hat{T}) \) in terms of \( V(\cdot) \) and using that \( R(\hat{T}) = T \) and \( F(\hat{T}) = 0 \) we have

\[
\Delta(\hat{T}) = \frac{u \left( b \left( \hat{T} \right) \right) + \frac{\rho \bar{T}}{P} \int_{\hat{T}}^{T} V(x) f(x) \, dx + \frac{\frac{\partial U(\hat{T})}{\partial b(T)}}{D(T)}}{\rho + P + \frac{1}{D(T)}} - V(T)
\]

The effect of benefits on \( \Delta(\hat{T}) \) is as follows

\[
\frac{\partial \Delta(\hat{T})}{\partial b(T)} = \frac{u_c \left( b \left( \hat{T} \right) \right) + \frac{\rho \bar{T}}{P} \int_{\hat{T}}^{T} \frac{\partial U(x)}{\partial b(T)} f(x) \, dx + \frac{\frac{\partial U(\hat{T})}{\partial b(T)}}{D(T)}}{\rho + P + \frac{1}{D(T)}} - \frac{1}{\rho \bar{T} + 1} \frac{\partial U(T)}{\partial b(T)} > 0
\]

Since \( \frac{\partial U(T)}{\partial b(T)} = \frac{1}{\rho \bar{T} + 1} \frac{\partial U(T)}{\partial b(T)} \) we have that \( \frac{\partial \Delta(\hat{T})}{\partial b(T)} > 0 \) iff.

\[
\frac{u_c \left( b \left( \hat{T} \right) \right) + \frac{\rho \bar{T}}{P} \int_{\hat{T}}^{T} \frac{\partial U(x)}{\partial b(T)} f(x) \, dx + \frac{\frac{\partial U(\hat{T})}{\partial b(T)}}{D(T)}}{\rho + P + \frac{1}{D(T)}} - \frac{1}{\rho \bar{T} + 1} \frac{\partial U(T)}{\partial b(T)} > 0
\]

Rearranging yields

\[
\frac{u_c \left( b \left( \hat{T} \right) \right)}{\rho + P + \frac{1}{D(T)}} + \frac{f(T) D \left( \hat{T} \right) \frac{\rho \bar{T}}{P} \int_{\hat{T}}^{T} \frac{\partial U(x)}{\partial b(T)} f(x) \, dx + \left[ \frac{\rho \bar{T}}{\rho + P} D \left( \hat{T} \right) \right] \frac{\partial U(\hat{T})}{\partial b(T)} f(T)}{\left[ \rho + P \right] D \left( \hat{T} \right) + 1} > 0
\]

Note that \( \frac{\partial U(T)}{\partial b(T)} > 0 \) since

\[
\frac{\partial U(T)}{\partial b(T)} = \frac{u_c \left( b \left( T \right) \right) + \frac{P}{\rho + P} \int_{R(T)}^{T} \frac{\partial U(x)}{\partial b(T)} f(x) \, dx + \frac{P}{\rho + P} \int_{R(T)}^{T} \frac{\partial U(x)}{\partial b(T)} f(x) \, dx}{\rho + P \left[ 1 - F(R) \right] + \frac{1}{D(T)}} + \Omega_b(T)
\]
where
\[
\Omega_b(T) \equiv \frac{P}{\rho + P[1 - F(R)]} + \frac{1}{D(T)} \left[ \int_{T}^{T} \frac{\partial U(x)}{\partial b(T)} f(x) \, dx + \frac{1}{[\rho + P] D(T)} \int_{T}^{T} \frac{\partial U(x)}{\partial b(T)} f(x) \, dx \right]^{-1}
\]

As \( u_c(\cdot) > 0 \), observe at first that \( \frac{\partial U(T)}{\partial b(T)} \neq 0 \forall T \in (T, \hat{T}) \). Furthermore, it implies that \( \frac{\partial U(T)}{\partial b(T)} > \Omega_b(T) \). Now, if \( u_c(\cdot) = 0 \) then there is a solution \( \frac{\partial U(T)}{\partial b(T)} = \Omega_b(T) = 0 \forall T \). Since, as \( u_c(\cdot) > 0 \) we have that \( \frac{\partial U(T)}{\partial b(T)} > 0 \forall T \in (T, \hat{T}) \).

Hence, a sufficient condition for \( \frac{\partial \Delta(T)}{\partial D(T)} > 0 \) is\(^{19}\)
\[
f(T) D(\hat{T}) P > \rho \hat{T} - [\rho + P] D(\hat{T})
\]
Which is equivalent to
\[
D(\hat{T}) > \frac{\rho}{[f(T) + 1]P + \rho \hat{T}}
\]
As the fraction on the RHS is less than one, the condition is satisfied for all \( D(\hat{T}) \in [T, \hat{T}] \) which is the relevant interval. Hence, we conclude that \( \frac{\partial \Delta(T)}{\partial D(T)} > 0 \) and thus \( \frac{\partial \Delta(T)}{\partial b(T)} < 0 \).

Regarding the effect of \( D(T) \) on \( \hat{T} \), we have that
\[
\frac{\partial \Delta(T)}{\partial D(T)} = \frac{P}{\rho + P + \frac{1}{D(T)}} \left[ U(\hat{T}) \frac{\partial D(\hat{T})}{\partial D(T)} \right] - \frac{\partial U(T)}{\partial D(T)} \frac{\partial D(\hat{T})}{\partial D(T)}
\]

\( \frac{\partial U(T)}{\partial D(T)} > 0 \) can be proved by the same procedure used above to prove that \( \frac{\partial U(T)}{\partial b(T)} > 0 \). As \( \frac{\partial V(T)}{\partial D(T)} = \frac{1}{\rho \hat{T} + 1} \frac{\partial U(T)}{\partial D(T)} \) we have that \( \frac{\partial \Delta(T)}{\partial D(T)} > 0 \) iff.
\[
f(T) D(\hat{T}) P \int_{T}^{T} \frac{\partial U(x)}{\partial D(T)} f(x) \, dx + \left[ \rho \hat{T} - [\rho + P] D(\hat{T}) \right] \frac{\partial U(T)}{\partial D(T)} f(T) \frac{1}{\rho \hat{T} + 1}
\]

\[\left[ [\rho + P] D(\hat{T}) + 1 \right] f(T) \]

\(^{19}\)Strictly speaking we need \( \frac{\partial U(T)}{\partial b(T)} f(T) \frac{1}{\rho \hat{T} + 1} \leq \frac{\partial U(T)}{\partial b(T)} f(T) \frac{1}{\rho \hat{T} + 1} \) for at least one \( T \in (T, \hat{T}) \). We assume this is satisfied.
\[ + \frac{U(\hat{T}) - U(T)}{\rho + P} \frac{\partial D(\hat{T})}{\partial D(T)}^2 > 0 \]

Hence, as \( U(\hat{T}) \geq U(T) \) a sufficient condition for \( \frac{\partial \Delta(\hat{T})}{\partial D(T)} > 0 \) is

\[ f(T) D(\hat{T}) P \geq \left[ \rho T - [\rho + P] D(\hat{T}) \right] \]

Which is identical to the condition wrt. the benefit level, which is satisfied for all relevant \( D(\hat{T}) \). Hence, we conclude that \( \frac{\partial \Delta(\hat{T})}{\partial D(T)} > 0 \) and thus \( \frac{\partial \hat{T}}{\partial D(T)} < 0 \).

Hence, a shift to a scheme with more generous benefits, \( b'(T) > b(T) \ \forall T \), or to a scheme with longer benefit, i.e. from \( D(T) \) to \( D'(T) \), where \( D'(T) > D(T) \ \forall T \) has \( \hat{T}' < \hat{T} \).

## C Comparative statics

The optimal \( R \) solves

\[ R \equiv \arg \max_R U(T, R, P, w) \]

and the first order condition reads

\[ U_R(T, R(T), P, w) = 0 \]

and the second order condition

\[ U_{RR}(T, R(T), P, w) < 0 \]

 Totally differentiating the foc we find

\[
\frac{dR(T)}{dz} = -\frac{U_R(z)}{U_{RR}(z)}; \quad z = b(T), D(T), w, P
\]

and hence \( \text{sign} \left[ \frac{dR(T)}{dz} \right] = \text{sign} \left[ U_R(z) \right] \). From Appendix A we have that

\[ U_R(T, R(T), P, w) = Pf(R) \left[ u(b(T)) + PH(R) + \frac{U(T)}{D(T)} + \frac{\rho + P(1 - F(R)) + \frac{1}{D(T)} \partial H(R)}{f(R)} \right] \]

\[ \left[ \rho + P \left( 1 - F(R) \right) + \frac{1}{D(T)} \right]^2 \]
where

\[ H(R(T)) \equiv \int_{R(T)}^{T} \left[ u(w) x + U(x) \right] \frac{f(x)}{\rho x + 1} \, dx \]  
(C.2)

**Wages**

From (C.1) it follows that

\[
U_{Rw} = \frac{P f(R) \left[ \frac{1}{D(T)} \frac{\partial U(T)}{\partial w} + P \frac{\partial H(R)}{\partial w} + \frac{\rho + P(1 - F(R)) + \frac{1}{D(T)} \frac{\partial H(R)}{\partial w}}{f(R)} \right]}{\left[ \rho + P(1 - F(R)) + \frac{1}{D(T)} \right]^2}
\]

Using (C.2) we have

\[
\frac{\partial H(R)}{\partial w} = \int_{R(T)}^{T} \left[ u_c(w) x + \frac{\partial U(x)}{\partial w} \right] \frac{f(x)}{\rho x + 1} \, dx
\]

and

\[
\frac{\partial }{\partial w} \left[ \frac{\partial H(R)}{\partial R} \right] = \left[ u_w(w) R + \frac{\partial U(R)}{\partial w} \right] \frac{f(R)}{\rho R + 1}
\]

It follows that \( \frac{\partial H(R)}{\partial w} > -\frac{\partial U(T)}{\partial w} > 0 \) can be proved by the same procedure used above to prove that \( \frac{\partial U(T)}{\partial w} > 0 \). Since \( \frac{\partial U(T)}{\partial w} = \frac{P \frac{\partial H(T)}{\partial w}}{\rho + P(1 - F(R(T)))} > 0 \), it follows that a sufficient condition that \( U_{Rw} > 0 \) is

\[
P > \frac{\rho + P(1 - F(R)) + \frac{1}{D(T)}}{f(R)}
\]

or

\[
\frac{1}{D(T)} < P f(R) - \rho - P(1 - F(R))
\]

Since \( D(T) \) is non-decreasing in \( T \), the LHS is non-increasing in \( T \). The RHS is increasing in \( R \) and hence \( T \) (recall \( R(T) \) is increasing in \( T \)) if \( \frac{\partial f(R)}{\partial R} > -f(R) \forall T \). Hence, \( \frac{\partial R}{\partial w} > 0 \) if \( \frac{\partial f(R)}{\partial R} > -f(R) \forall T \), and \( \lim_{T \to \infty} \left( \frac{1}{D(T)} \right) < P [f(T) - 1] - \rho \).
Equation (C.1) implies (using $U_R(T, R(T), P, w) = 0$)

$$U_{RP} = P f(R) \left[ H(R) + P \frac{\partial H(R)}{\partial P} + \frac{\partial U(T)}{D(T)} + \frac{(1-F(R)) \partial H(R)}{f(R)} + \rho + P(1-F(R)) + \frac{1}{D(T)} \frac{\partial \rho H(R)}{\partial P} \right]$$

$$\left[ \rho + P (1 - F(R)) + \frac{1}{D(T)} \right]^2$$

From (C.2) it follows that

$$\frac{\partial H(R)}{\partial R} = - [u(w) R + U(R)] \frac{f(R)}{\rho R + 1} < 0$$

and hence

$$\frac{\partial \left( \frac{\partial H(R)}{\partial R} \right)}{\partial P} = - \frac{\partial U(R)}{\partial P} \frac{f(R)}{\rho R + 1}$$

$$\frac{\partial H(R)}{\partial P} = \int_{R(T)}^{T} \frac{\partial U(x)}{\partial P} \frac{f(x)}{\rho x + 1} dx$$

Note also that since $H(R) = [1 - F(R)] E(V(T') \mid T' \geq R)$ it follows that

$$H(R) + \frac{(1 - F(R)) \partial H(R)}{f(R)} = H(R) - \frac{(1 - F(R))}{f(R)} [u(w) R + U(R)] \frac{f(R)}{\rho R + 1}$$

$$= [1 - F(R)] \left[ E(V(T') \mid T' \geq R) - \frac{[u(w) R + U(R)]}{\rho R + 1} \right]$$

$$= [1 - F(R)] [E(V(T') \mid T' \geq R) - V(R)] \geq 0$$

$\frac{\partial U(T)}{\partial P} > 0$ can be proved by the same procedure used above to prove that $\frac{\partial U(T)}{\partial T} > 0$. Hence, $\frac{dR}{dP} > 0$ is ensured if

$$P > \frac{\rho + P(1 - F(R)) + \frac{1}{D(T)}}{f(R)}$$

which is equivalent to the condition for $\frac{dR}{dw} > 0$. 

148
Unemployment benefits \( b(T) \)

From (C.1) follows

\[
U_{Rb(T)} = P f(R) \left[ u_c(b(T)) + P \frac{\partial H(R)}{\partial b(T)} + \frac{\partial U(T)}{\partial b(T)} \frac{1}{D(T)} - \frac{\rho + P(1 - F(R)) + \frac{1}{D(T)}}{f(R)} \frac{\partial^2 H(R)}{\partial R \partial b(T)} \right]
\]

and from (C.2)

\[
\frac{\partial H(R)}{\partial b(T)} = \int_{R(T)} \frac{\partial U(x)}{\partial b(T)} \frac{f(x)}{\rho x + 1} dx
\]

and

\[
\frac{\partial H(R)}{\partial R} = -\left[ u(w) R + U(R) \right] \frac{f(R)}{\rho R + 1} < 0
\]

\[
\frac{\partial^2 H(R)}{\partial R \partial b(T)} = -\frac{\partial U(R)}{\partial b(T)} \frac{f(R)}{\rho R + 1}
\]

Hence,

\[
U_{Rb(T)} = P f(R) \left[ u_c(b(T)) + P \int_{R(T)} \frac{\partial U(x)}{\partial b(T)} \frac{f(x)}{\rho x + 1} dx - \frac{\rho + P(1 - F(R)) + \frac{1}{D(T)}}{f(R)} \frac{\partial U(T)}{\partial b(T)} \frac{1}{\rho R + 1} + \frac{\partial U(T)}{\partial b(T)} \frac{1}{D(T)} \right]
\]

Thus, if \( \frac{\partial U(T)}{\partial b(T)} > 0 \) \( \forall T \) we have \( \frac{dR(T)}{db(T)} > 0 \) if

\[
P > \frac{\rho + P(1 - F(R)) + \frac{1}{D(T)}}{f(R)}
\]

which is the same condition as found above for \( w \).

Benefit duration \( D(T) \)

From (C.1) it follows that (using \( U_R(T, R(T), P, w) = 0 \))

\[
U_{RD(T)} = P f(R) \left[ P \frac{\partial H(R)}{\partial D(T)} + \frac{\partial U(T)}{\partial D(T)} + \frac{1}{f(R)} \frac{\partial H(R)}{\partial R} \frac{1}{[D(T)]} + \frac{1}{f(R)} \frac{\partial^2 H(R)}{\partial R \partial D(T)} \right]
\]

\[
\left[ \rho + P(1 - F(R)) + \frac{1}{D(T)} \right]^2
\]

149
From (C.2) it follows that
\[
\frac{\partial H(R)}{\partial R} = - \left[ u(w)\, R + U(R) \right] \frac{f(R)}{\rho R + 1} < 0
\]

Note first that
\[
U(T) + \frac{1}{f(R)} \frac{\partial H(R)}{\partial R} = U(T) - \frac{[u(w)\, R + U(R)]}{\rho R + 1} < 0
\]

where the sign follows by observing that \( V(T) = \frac{u(w)T + U(T)}{\rho T + 1} \) and the participation constraint requires \( U(T) - V(R(T)) < 0 \).

Next we have
\[
\frac{\partial H(R)}{\partial D(T)} = \int_{R(T)}^{T} \frac{\partial U(x)}{\partial D(T)} \frac{f(x)}{\rho x + 1} dx
\]
\[
\frac{\partial \left( \frac{\partial H(R)}{\partial R} \right)}{\partial D(T)} = - \frac{\partial U(R)}{\partial D(T)} \frac{f(R)}{\rho R + 1}
\]

For \( \frac{\partial U(T)}{\partial D(T)} > 0 \forall T \) we have \( \frac{\partial H(R)}{\partial D(T)} + \frac{\partial \left( \frac{\partial H(R)}{\partial R} \right)}{\partial D(T)} > 0 \), hence, we have \( \frac{dR}{dD(T)} > 0 \) if
\[
P > \frac{\rho + P \left( 1 - F(R) \right) + \frac{1}{D(T)}}{f(R)}
\]

which identical to the condition found above for wages.

## D Constant R

The utility value for unemployed with employment history \( T \) is
\[
U(T) = \frac{u(b(T)) + PH(R(T)) + \frac{U(T)}{D(T)}}{\rho + P \left[ 1 - F(R(T)) \right] + \frac{1}{D(T)}}
\]

For given \( R(T), U(T) \) is increasing in \( b(T) \) and \( D(T) \). Hence, there exists \( \tilde{b}(T) > b(T) \) and \( \tilde{D}(T) < D(T) \) yielding the same value for \( U(T) \) for the given \( R(T) \). If similar adjustments are made for all \( T \), implying that \( U(T) \) is unchanged for all \( T \), and therefore \( H(R(T)) \) and \( V(T) \) are unchanged for all \( T \), it follows that \( R(T) \) is unchanged for all \( T \). Note that since \( U(T) \) is unchanged for all \( T \), it also follows that \( \hat{T} \) is unchanged.
E  EQUILIBRIUM EMPLOYMENT AND UNEMPLOYMENT

The determination of \( R(T) \) and \( \{ e(T), n(T) \}_{T}^{T} \) has a recursive structure, since \( R(T) \) is determined independently of \( e \) and \( n \), but depending on the policy functions \( D(T) \) and \( b(T) \), the distributional characteristics of the job offer and job separation processes as well as the utility function, cf. (2.6). Next, the unemployment levels can be determined from \((3.4)\), and finally employment can be determined from \((3.2)\). Since \( R(T) \) is uniquely determined, it follows that the integral in \((3.4)\) is well defined provided there exists non-trivial solutions to \( n(\cdot) \). To this end, note that the total population is either in employment or unemployment, i.e.

\[
e(T) + n(T) + \int_{\tau}^{T} (e(x) + n(x))dx = 1
\]

Using \((3.2)\), \((3.4)\), \((3.5)\), and \((3.6)\) it follows that

\[
[T_{P} + 1] f(T) \int_{T}^{T} n(x) dx + \int_{\tau}^{T} \left[ x \left[ P [1 - F(R(x))] + \frac{1}{D(x)} \right] + 1 + \frac{1}{PD(x)} \right] n(x)dx = 1
\]

As \([T_{P} + 1] f(T) > 0 \) and \( T \left[ P [1 - F(R(T))] + \frac{1}{D(T)} \right] + 1 + \frac{1}{PD(T)} > 1, \forall T \in (\tau, T] \) it follows that

\[
\int_{\tau}^{T} n(x)dx < 1
\]

Full employment is ruled out, as this would imply (using that population shares are non-negative) \( n(T) = 0 \ \forall T \) which in turn would imply \( e(T) = 0 \ \forall T \), hence, a contradiction. Furthermore, we cannot only have mass at \( \tau \) either, i.e. \( e(T) + n(T) = 1 \), as this would imply \( n(T) = e(T) = 0 \ \forall T \in (\tau, T] \) which would imply \( n(T) = 0 \) and thus \( e(T) = 0 \) again a contradiction.\(^{20}\) Thus, we have

\[
0 < \int_{\tau}^{T} n(x)dx < 1 \quad 0 < n(T) < 1
\]

\[
0 < \int_{\tau}^{T} e(x)dx < 1 \quad 0 < e(T) < 1
\]

\(^{20}\)Technically, in the case where \( \tau = T \) we also require \( f(T) \neq 1 \). We assume this to be the case.
Given continuity, we can conclude that the population share of every state is positive, i.e.

\[ 0 < e(T) < 1 \land 0 < n(T) < 1, \forall T \]

**F Employment and Benefit Design**

It follows directly from (3.7) that \( \frac{e(T)}{n(T)} \) is decreasing in \( D(T) \) for a given reservation duration \( R(T) \) for all \( T > \bar{T} \). A shift in the unemployment insurance scheme between benefit levels and durations, leaving reservation durations \( R(T) \) unchanged for all \( T \), thus reduces \( \frac{e(T)}{n(T)} \) for all \( T > \bar{T} \) and by implication there is a decrease in the overall employment rate for those receiving benefits above the social assistance level. Furthermore, for (un)employed with \( T = \bar{T} \) we have

\[
\frac{e(\bar{T})}{n(\bar{T})} = TPf(\bar{T}) \int_{\bar{T}}^{T} \frac{n(x) dx}{n(\bar{T})} = \frac{TP^2 f(\bar{T}) \int_{\bar{T}}^{T} n(x) dx}{ Pf(\bar{T}) \int_{\bar{T}}^{T} n(x) dx + \int_{x \in [\bar{T}, T]} \frac{1}{D(x)} n(x) dx}
\]

Since the policy change keeps the reservation durations unchanged for all \( T \) it follows from the expression above that \( \frac{e(T)}{n(T)} \) is increasing (if \( \bar{T} > T \)).\(^{21}\)

\(^{21}\)If \( \bar{T} = T \) then \( \frac{e(T)}{n(T)} \) is independent of the policy change.
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<th>Title</th>
</tr>
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<tbody>
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