DIDACTICAL MODELLING

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Abstract: *The purpose of this paper is to introduce Didactical Modelling as a research methodology in mathematics education. We compare the methodology with other approaches and argue that Didactical Modelling has its own specificity. We discuss the methodological "why" and explain why we find it useful to construct this approach in mathematics education research.*

INTRODUCTION

Research articles often describe "how" the research was done but more rarely what influenced the researcher to use a particularly approach. Burton (2002) stresses the point that a researcher has to justify "why" he/she uses a particular research method. There are many ways to conduct research in mathematics education, so it is important to clarify and discuss the reasons for choosing a certain method to research into a given problematique. One such problematique is the potentials and hindrances of using a set of mathematical competencies as a developmental tool in mathematics education. We will refer to two research projects relating to this problematique. The first is Tomas' PhD study: *The Allerød Project.* The second research project is Rune's ongoing PhD study (with Tomas as the supervisor): *The MKM Project.* Both projects are longitudinal developmental research projects.

DIDACTICAL MODELLING

Didactical Modelling (DM) has been developed as a systematic research approach to different processes in making sense of mathematical teaching and learning. We will describe the components in the didactical modelling process in order to let the reader gain insight into the constructive dimension of the modelling process.

Why a modelling approach?

The idea of didactical modelling emerged during The Allerød Project. Tomas was and still is involved in discussions of the concepts of model and modelling in the mathematics education research community. Inspired by the mathematical modelling process (Blomhøj & Jensen, 2007) he began to contemplate if the modelling approach somehow could be transferred to the research process. There was an offer to think in terms of models and modelling that seemed meaningful for a researcher.

The didactical modelling process

In DM we try to incorporate the reality that research is often carried out in a cyclic process, in which we consider and reconsider different aspects of the research. The visual model of the modelling process in figure 1 reveals the complexity in the research process. Here the mathematical modelling process has been translated to a didactical context.

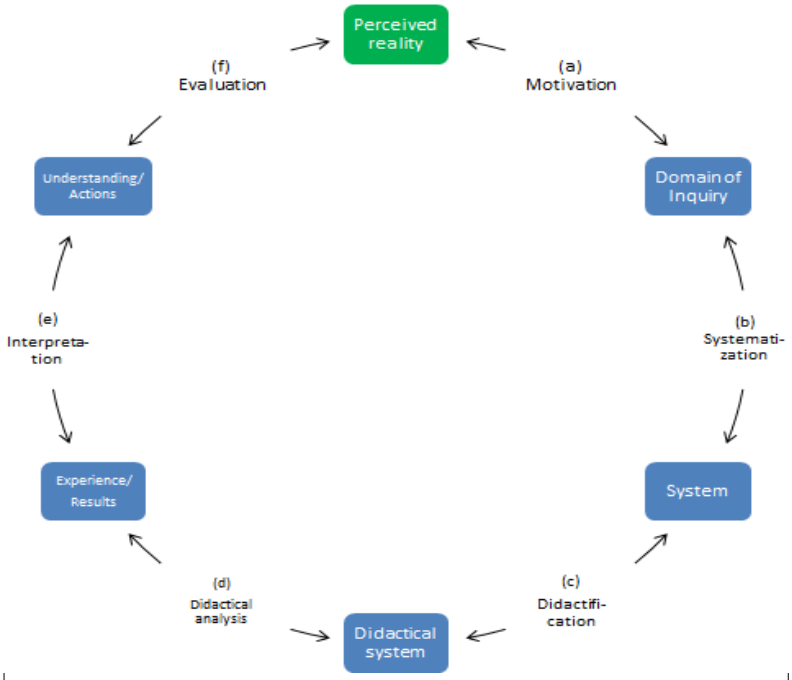

Fig. 1 A visual model of the modelling process.

Motivation: The modelling process begins with some kind of challenge in an educational context. As researchers we begin by describing our *motivation* (a) as a way to establish the basis for our research. In The Allerød Project Tomas was motivated by a belief that teaching the students to apply mathematics in non-school contexts ought to the crux of general mathematics education (Jensen, 2007, pp. 3-24). In The MKM Project Rune is motivated by a seemingly disparity between the intentions of the official curriculum program and how many teachers are teaching in the mathematical classroom.

Systematization: Subsequently we begin to explore the motivating challenge by considering its most important features and components. With the *systematization* (b) the complexity of the domain of inquiry is reduced based on assumptions about what is important to make a model of the challenge. Through a priori analysis of relevant research a focus is established. This leads to formulating and defining specific research questions to which answers should be sought. The motivation (a) and systematization (b) takes place in order to conceptualize the perceived reality in which the challenge is seen and understood in certain ways. The Allerød Project consisted of making a systematic enquiry structured around the question: *Why is mathematical modelling not the hub of mathematics education?* In the MKM Project a systematic enquiry is structured around the question: *Why are teachers not setting goals as a focal point for their teaching in mathematics?*

The initial analysis of the questions rely on more specific research questions guiding the systematization process (Jensen, 2007).

Didactification: Now a *system* identifying and highlighting central aspects of the situation have been established. With the *didactification* (c) we contemplate if and how a specific educational context can help us address the questions to which we seek answers. Following what we consider to be the defining characteristic of a *didactical* system, it requires us to be specific in relation to a teaching practice. Based on our didactical choices we need to translate our abstract analysis into a set of related variables capturing the chosen essential aspects of an educational setting. In the process a set of didactical headings are developed and they will all affect the teaching and learning situation in some way. We create a *didactical system*, which allows us to perform operations on the identified didactical variables and the identified relationships.

In The Allerød Project the didactification process was guided by the following research question (Jensen, 2007, p. vi): *What organizational characteristics of the way mathematical modelling can potentially be integrated into the teaching can I defend as being central based on theoretical*

analyses, if the goal is to develop the pupils mathematical modelling competence as much as possible?

In the MKM Project the didactification lead to developing four headings regarding a question about *organizational characteristics*. One of the four headings in The MKM Project is: *In dialogue with the students the teacher must focus on clarifying learning goals for each module and students reflections shall be supported and promoted through evaluation.*

Didactical analysis: The analysis of the developed didactical model (understood as the relationship between the *system, didactification* and *didactical system*) can be carried out in different ways. One option is to continue with further desk research based on existing literature, now focusing on exploiting and challenging the model rather than on constructing it initially. Another option is empirical research, where some of the main features of the model are confronted with empirical data of some kind. Hence, a main point in DM is that the researcher has to justify if he/she believes empirical research can contribute to the understanding and development of the model.

In both The Allerød Project and The MKM Project the didactical model has been analyzed by means of experimental teaching in a mathematics classroom, where the set of didactical headings meets genuine teachers and students, supported by further theoretical desk research. In both projects this process is guided by research question related to identifying *hindrances* in a specific case.

Creating the didactical system is a researcher driven process, but putting it under "empirical pressure" supports co-operation between researchers and practitioners. Often the experimental teaching is organized in the phases establishment, implementation and processing the results. The establishment creates a frame for the teaching situation that includes the didactical headings deemed important. This results in a "deformation" of the frame for the teaching situation. During and after the experimental teaching the researcher can "put a magnifying glass" on selected issues, analyzing relations between the deformations imposed on the framing of the teaching situation and what actually takes place in the classroom. This is visualized in Figure 2.

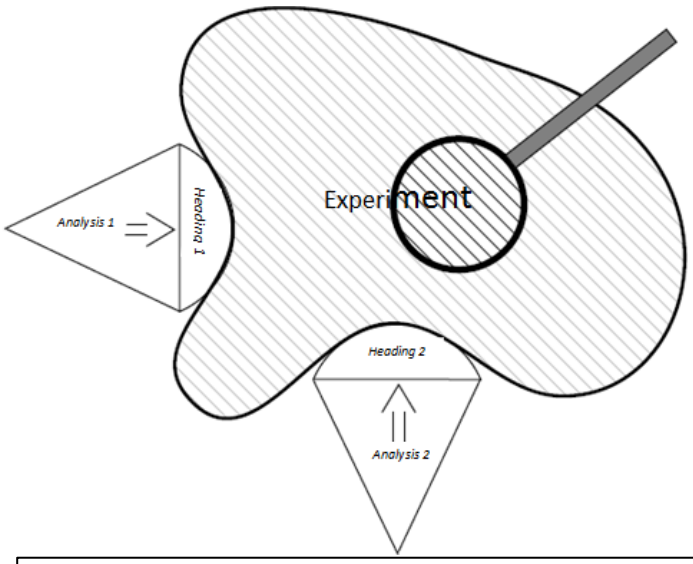

Fig. 2 A visual model of experimental teaching used as part of didactical modelling (Jensen, 2007, p. 177).

Interpretation: It is important to highlight that as part of the modelling process it is the didactical model that is the object of analysis, not the possible experimental teaching in itself. So experiences and results should be interpreted in relation to the didactical model, which can lead to *understanding* and/or *actions* related to the perceived reality. Regarding the interpretation it is crucial to acknowledge that the didactical model is a representation regarding the perceived reality. All results must be considered with appropriate caution as they reflect the choices the researcher has

made throughout the modelling process. In The Allerød Project a list of *successful elements* with respect to the established ideals were pointed out, and subsequently four *hindering matters* were brought to light and analyzed (Jensen, 2007, p. vi).

Evaluation: As part of the research process the modelling process – modelled by the sub processes (a) to (e) in figure 1 – have to be *evaluated* (f). Here the components and the qualities of the modelling process in its entirety are assessed. In this phase the researcher has to take a step back and re-evaluate his/her methodological choices. Did it fulfil the researcher's methodological ambitions and needs and was it a relevant methodological choice for gaining insight into the problematique?

METHODOLOGICAL AMBITIONS AND NEEDS

We have tried to outline the framework for DM, but it is important to notice that we did not begin with the method. The starting point was our methodological ambitions and needs that lead us to DM as a research approach. In order to address the research questions we will stress the point, that researchers must develop a strategy regarding conducting research in mathematics education (Bryman, 2012). As a starting point the researcher has to ask: What do you want to know? What are your ambitions? Then he/she can proceed to clarify the methodological needs before identifying a specific method. One approach to select and/or develop a method is to write down your methodological ambitions and needs. Referring to the two research projects mentioned above it looks as follows: 1) the need for creating a systematic approach to the research process, guided by the research question and fostering an internal validation, 2) the ambition to focus a complex problematique through *a priori* analysis, 3) the need for an approach that is not rooted in a specific didactical theory but requires the researcher to be open about his values, beliefs and theoretical stance in the approach to the problematique, 4) the ambition of creating a cyclical and iterative process to understand and explain the problematique, 5) the ambition to gain insight into and understand the problematique and not necessarily seeking to improve it, 6) the need for a methodology to mediate between theory and practice, allowing us to experiment with educational settings in order to foster our understanding of the problematique, 7) the ambition of creating researcher driven processes that support co-operation with practitioners.

"TEMPTING" EXISTING RESEARCH APPROACHES

As part of the research strategy the elaboration of the methodological needs and ambitions help us selecting and/or developing a method. Both projects required us to systematically consider the relationship between theory and practice as well as the relationship between researchers and practitioners. Hence, it seems natural to contemplate existing approaches to educational research addressing these two relationships. In this paper we have chosen Design-Based Research and Didactical Engineering.

Design-Based Research

Design-Based Research (DBR) combines empirical educational research with a theory-driven didactical design. The methodology was introduced in the early 1990s and is focused on developing, testing, implementing and refining innovative didactical designs in an educational setting (Collins, Joseph, & Bielaczyc, 2004). The ambition of DBR is bilateral both to generate new scientific knowledge and to design innovative didactical designs. DBR is not only driven by a need

for understanding an educational problem but also an aspiration to improve the educational environment (Cobb & Gravemeijer, 2008; Collins et al., 2004).

DBR can be characterized by the following characteristics: 1) DBR intervene in educational practices by infusing design prototypes on educational settings. 2) DBR entails iterative cycles of design, testing, analysis and redesign. 3) DBR should lead to designs or innovation that can be communicated to and applied by other researchers and practitioners. 4) DBR involves collaboration where teachers can function as codesigners. The didactical design is contextualized in authentic educational settings, so the participation from teachers and students are essential and of great value to the design process. Build into DBR is a need to improve a didactical situation as the design has a pragmatic aim of developing didactical solutions (Cobb & Gravemeijer, 2008; Collins et al., 2004).

Didactical Engineering

Didactical Engineering (DE) emerged as a methodology aligned with the scientific field of the didactics of mathematics. Based on the theory of didactical situations DE seeks to develop didactical situations informed by theory and using classroom practices to investigate and analyze the theoretical ideas. Often DE focus on examining the learning of a specific mathematical content (Artigue, 2015; Artigue & Perrin-Glorian, 1991).

DE is structured into different phases. The first phase is preliminary analyses which consist of three dimensions: 1) an epistemological analysis of the mathematical content in search for finding fundamental situations. It provides the researcher with a distance to the educational practices and creates a reference point, 2) an institutional analysis of the context in which the DE occurs, 3) a didactical analysis of the research on the specific mathematical content.

The next phase is conception and *a priori* analysis. Here the research hypotheses are made explicit and through *a priori* analysis the main didactical variables are identified and reflected upon in creating the didactical situation. It builds a reference point before the classroom implementation.

The third phase is the realization phase, where data are collected for the *a posteriori* analysis. The *a posteriori* analysis offers a form for internal validation between the *a priori* and *a posteriori* analysis. In this phase the researcher analyze the didactical system by comparing the data with the reference from the *a priori* analysis looking for similarities and differences (Artigue, 2015).

DISCUSSION

In our opinion a method should never be discussed in its own rights, it should always be relative to the connection between the perceived reality and the researcher's ambition. In this section we will discuss the three presented methods regarding our research ambitions.

Ambition: Design, understanding or possibilities?

Didactical Modelling, Design-Based Research and Didactical Engineering aim to focus a complex didactical situation through *a priori* analysis and all creates a systematic approach to the research process. Thus all three approaches correspond with the systematic approach (item 1) using an *a priori* analysis (item 2) described under our methodological ambitions and needs. Where DM is systematized in regard to understanding the perceived reality both DBR and DE are structured concerning the design of some didactical tool (Artigue, 2015). DE employs an internal validation

relating *a priori* and *a posteriori analyses* of the didactical situation. In DBR there should be a detailed design protocol that allows outsiders to evaluate the credibility of the design decisions but there is also a focus on transferability that take in account local contingencies Build into DBR is a need to improve a didactical situation as the design has a pragmatic aim of developing didactical solutions (Collins et al., 2004). In our opinion it can provide an unbalanced emphasis on the design process at the expense of gaining insight into and understanding the didactical problematique. Consequently both DE and DBR are not in line with the ambition to understand and not necessarily improve a problematique from our methodological ambitions (item 5).

Methodological transparency

In DM there is an aspiration of clarity and transparency in term of internal validation and credibility. The strength of the modelling process is that the relationship between the various elements becomes explicit. You cannot use the model without having described why you choose to emphasize some didactical variable at the expense of others. In our opinion models are very valuable as a method to conduct and present the research. The model can provide the researchers and the research audience with a particular view on the didactical situation that is open for discussion. The acceptability of the didactical choices can be questioned not only by critics but also by the researcher him- or herself. In DBR the research must be reported so it can be retraced or virtual replicated (Gravemeijer & Cobb, 2006). This makes transparency an essential dimension. To make sure the readers will be able to follow the research process, the researcher has to report in such a way it leaves a detailed audit trail that others can track (Akkerman, Admiraal, Brekelmans, & Oost, 2008). In DE the methodological transparency is deeply embedded into the research process. The deductive approach follows the path of logic. It begins by developing a hypothesis based on the *a priori* analysis of a didactic situation. Then the hypothesis is tested in a classroom setting that leads to the *a posteriori* analysis. The transparency relies on the reader's ability to follow the deductive reasoning (Artigue & Perrin-Glorian, 1991).

A cyclic process?

DE often uses a noncyclic approach concerning controlled teaching experiments based on a specific didactical theory. Michèle Artigue even stresses the point, that DE is not a cyclic process. DE is "a tool for answering didactical questions for identifying, analyzing and producing didactical phenomena through the controlled organization of teaching experiments" (Artigue, 2015, p. 477).

Both DM and DBR create a cyclical and iterative process built on a more diverse theoretical foundation. As described the methods differ regarding research aims which influences the cyclical nature. In DBR the successive cycles is focused on testing, analysis and redesign as a way to produce a didactical design (Collins et al., 2004). In DM the cyclic process is focused on understanding and explaining a didactical phenomenon. It is a reflection of the notion that research is a complex process that cannot be captured in a linear way. New perceptions or understandings may instigate new ways of thinking of the problem or the didactical system. The iterative dimension takes in account that the pathway from motivation to evaluation is a complex process (Blomhøj & Jensen, 2007). Here both DM and DBR (but not DE) complies with our methodological ambitions as they instigate an open didactical approach (item 3) in a cyclic and iterative process (item 4).

The role of empirical work in the classroom

All three methods seek to mediate between theory and practice (item 6) with some kind of co-operation between researcher and practitioners (item 7). Both DBR and DE have a need for empirical experimenting with educational settings in order to reach their research objectives. DM differs from the other approaches as the researcher has to justify why it is relevant to experiment with an educational setting as a way to gain an understanding of the didactical system. This perceptive rests on the premise mentioned above that DM is orientated towards understanding the didactical situation and not necessarily seeks to improve it.

A schematic overview

To give an overview we have, inspired by Guba and Lincoln (2005) and Creswell (2007), depicted the similarities and differences between the three approaches as a brief summary of the methodological ambitions and needs.

Issue	Didactical Modelling	Design-Based Research	Didactical Engineering
Inquiry aim (item 5)	Understanding of a complex didactical problematique	Transformation and understanding of a didactical problem	Transformation and understanding of a specific mathematical content
Validity (item 1)	Internal validity focusing on the credibility of the model	Internal and external validity by using detailed design protocols	Internal validity relating *a priori* and *a posteriori* analyses of a didactical situation
Research approach (item 2,3,4)	Cyclical process that use new understandings to reconsider the didactical model	Iterative cycles of design, testing, analysis and redesign	A noncyclic approach concerning controlled organization of teaching experiments
Empirical research (item 6)	Only if it can contribute to the understanding and development of the model	Testing in a classroom context is necessary.	Testing in a classroom context is necessary.
Relationship with practitioners (item 7)	Researcher driven process supporting co-operation between the researcher and practitioners	Researcher driven process supporting co-operation between the researcher and practitioners	Researcher driven process supporting co-operation between the researcher and practitioners
Product (item 5)	Producing a didactical model as an answer to a problematique	Developing didactical solutions	Identifying and producing didactical phenomena

Table 1. Differences and similarities between DM, DBR and DE.

CONCLUDING REMARKS

In this paper, we have introduced Didactical Modelling as a research methodology. We have attempted to describe its main characteristics and provide the readers with a comparison to other approaches. A main aspiration for DM is to keep the research process on track. Although the purpose of the paper has been to introduce DM it was not the only objective. We have also taken the opportunity to articulate how we believe researchers can and should decide on a particular research method. Articulating one's ambitions and needs creates an overview that can help guiding the researcher in selecting a research method. If you can find an existing well-described method that meets your requirements it seems like an obvious choice. But the process of writing down the research ambitions and needs helps the researcher answering the methodological "why". Why does the method provide a relevant approach to answering the research question? If you cannot find an existing method, you need to create a new one that can live up to the conglomerate of your ambitions and needs. That was what happened in the development of didactical modelling as a research methodology.

REFERENCES

Akkerman, S. F., Admiraal, W., Brekelmans, M., & Oost, H. (2008). Auditing quality of research in social sciences. *Quality and Quantity, 42*, 257-274.

Artigue, M. (2015). Perspectives on Design Research: The Case of Didactical Engineering. In A. Bikner-Ahsbahs, C. Knipping, & N. Presmeg (Eds.), *Approaches to Qualitative Research in Mathematics Education* (pp. 467-496): Springer Netherlands.

Artigue, M., & Perrin-Glorian, M.-J. (1991). Didactic Engineering, Research and Development Tool: Some Theoretical Problems Linked to This Duality. *For the Learning of Mathematics, 11*(1), 13-18.

Blomhøj, M., & Jensen, T. (2007). What's all the Fuss about Competencies? In W. Blum, P. Galbraith, H.-W. Henn, & M. Niss (Eds.), *Modelling and Applications in Mathematics Education* (Vol. 10, pp. 45-56): Springer US.

Bryman, A. (2012). *Social Research Methods*: OUP Oxford.

Burton, L. (2002). Methodology and Methods in Mathematics Education Research: Where Is "The Why"? In S. Goodchild & L. English (Eds.), *Researching mathematics classrooms: a critical examination of methodology* (pp. 1-10). Westport CT: Praeger Publishers.

Cobb, P., & Gravemeijer, K. (2008). Experimenting to Support and Understand Learning Processes. In A. E. Kelly, R. A. Lesh, & J. Y. Baek (Eds.), *Handbook of design research methods in education: innovations in science, technology, engineering, and mathematics learning and teaching* (pp. 68-95). New York, NY: Routledge.

Collins, A., Joseph, D., & Bielaczyc, K. (2004). Design Research: Theoretical and Methodological Issues. *Journal of the Learning Sciences, 13*(1), 15-42.

Creswell, J. W. (2007). *Qualitative inquiry & research design: choosing among five approaches* (2. ed.). Thousand Oaks, Calif.: SAGE Publications.

Gravemeijer, K., & Cobb, P. (2006). Design research from a learning design perspective. In J. Van den Akker, K. Gravenmeijer, S. McKenney, & N. Nieveen (Eds.), *Educational design research* (pp. 45-85). London Routledge.

Guba, E. G., & Lincoln, Y. S. (2005). Paradigmatic Controversies, Contradictions, and Emerging Confluences. In N. K. Denzin & Y. S. Lincoln (Eds.), *The SAGE handbook of qualitative research* (3 ed., pp. 191-215). Thousand Oaks, Calif.: Sage Publications.

Jensen, T. H. (2007). *Udvikling af matematisk modelleringskompetence som matematikundervisningens omdrejningspunkt - hvorfor ikke?* Roskilde: IMFUFA.