NONLINEAR FINITE ELEMENT ANALYSIS OF NON-SHEAR REINFORCED CONCRETE BEAMS

Jakob Gren Pedersen, Jakob Fisker and Claus Vestergaard Nielsen

Rambøll Danmark A/S, Bridge department, Aalborg, 9000, Denmark
Aarhus University, Aarhus University School of Engineering Aarhus, 8000, Denmark
Rambøll Danmark A/S, Bridge department, Aalborg, 9000, Denmark

Abstract

This paper investigates the influence of the concrete strength on the shear strength of reinforced concrete beams without shear reinforcement. The investigation is based on an experimental research and nonlinear finite element modelling. The experimental research consists of a parametric investigation of the influence of the concrete strength on the shear strength. The experimental program and the experimental results are shortly summarized followed by an introduction to the complex process of nonlinear finite element modelling. It is described how the reinforced concrete beams from the experimental research are modelled. In this way, it is clarified how every choice in the modelling process affects the results of the nonlinear finite element model. The results of the nonlinear finite element model and the results of the experiments are compared to a mechanical model to draw conclusions regarding the general influence of the concrete strength on the shear strength.

Keywords: Shear strength, nonlinear finite element modelling, reinforced concrete

1 Introduction

Shear failures in structural elements of concrete are usually prevented by shear reinforcement. For some structural elements, such as slabs and retaining walls, shear reinforcement is often omitted. Predicting, modelling and explaining the complex behaviour of reinforced concrete members without shear reinforcement has been the objective for a large number of researchers, i.e. (Fenwick & Paulay 1968). Nevertheless, the design of such elements with respect to shear has throughout history been based on pure empirical or semi-empirical expressions.

Experimental research indicates that shear forces are carried partly by the uncracked concrete in the compression zone, partly by interlocking of aggregate particles located in the surface of cracks, and partly by the flexural reinforcement acting as a dowel (Taylor 1974). The concrete compressive strength is an important parameter since it affects all three mentioned mechanisms. In several codes, it is generally assumed that the shear strength is proportional to the square root of the concrete strength. This assumption is mainly based on beam tests conducted by Moody & al. (1954).

In 2014 J. G. Pedersen & J. O. Eriksen investigated the influence of the concrete strength on the behaviour of reinforced concrete members without stirrups as a part of their Master’s Thesis at the University of Aarhus. The investigations were based on experimental research and nonlinear finite element analysis. Nonlinear finite element analysis is a state-of-the-art numerical calculation method with the aim of predicting the response of a structural element. This method of analysing a structural element imposes high demands on the user since the reliability of nonlinear finite element results, to a great extent, depends strongly on the assumptions made by the user/designer. This is due to the fact that the modelling process involves choices regarding physical and non-physical parameters for which either little is known on the absolute values, or/and for which the influence on the response of the modelled structural element is difficult to predict. Except for specific observations regarding the influence of the concrete strength, the experimental research served as a reference in the evaluation of
the results of the nonlinear finite element model. The experiments involved tests on 12 slender reinforced concrete beams without shear reinforcement cast with four different grades of concrete. This relatively small amount of specimens constitutes a narrow basis for investigating the general influence of the concrete strength on i.e. the shear strength. However, fitting the nonlinear finite element model to the experimental results involved an opportunity to draw general conclusions regarding the influence of concrete strengths that were not investigated in the experiments.

This paper shortly introduces the experimental work conducted by Pedersen & Eriksen (2014b) and summarizes the experimental results. This is followed by a critical review of the applicability of nonlinear finite element analysis on reinforced concrete beams without shear reinforcement. The review is based on the experiences made through the process of modelling the beams from the experiments. Herby, it is clarified why the process of nonlinear finite element analysis is complicated and how the results of a nonlinear finite element model are highly sensitive to the assumptions made by the user. Finally, the paper compares the experimental results and the nonlinear finite element results to the predictions of a mechanical model. This leads to conclusions regarding the influence of the concrete strength on the shear strength and the applicability of the nonlinear finite element modelling for practical design.

2 Experimental Work

The purpose of the experimental research was to investigate the influence of concrete strength on the behaviour of reinforced concrete beams without shear reinforcement. The following sections shortly summarise the experimental program and the most important results. For further information regarding the experiments, the reader is referred to the experimental report by Pedersen & Eriksen (2014b).

2.1 Experimental program

The experimental program consisted of 12 slender reinforced concrete beams divided into 4 series corresponding to 4 values of the uniaxial compressive concrete strength (approximately 20, 30, 40 and 50 MPa). The geometry and the reinforcement design of the beams were identical. All beams measured 450 mm deep x 200 mm wide x 4250 mm long and had a slenderness ratio (a/d) equal to 4.52 and a reinforcement ratio equal to 1.56 %. The type of the aggregates and the maximum aggregate size (8 mm) were held constant as well.

The beams were provided with a sufficient amount of longitudinal ribbed bars to prevent bending failure. Furthermore, stirrups were placed so that the shear failure would happen in the shear span between the applied load and the left support. Stirrups and U-shaped rods prevented anchorage failure at the supports. The measured average yield strength of the reinforcement bars was 578 MPa. The reinforcement design is illustrated in Fig. 1. Due to symmetry of the experimental setup, the shear force was determined from a measured value of the applied load (R=P/2).

The beams were tested 12 to 28 days after casting. The concrete compressive strength for each individual beam were determined based on the mean value of a measured compressive strength of 3 to 4 concrete cylinders with a height of 300 mm and a diameter of 150 mm. The concrete cylinders were cast at the same time as the beams and stored under the same conditions.

**Fig. 1** Reinforcement design of the tested concrete beams

Load = P, Shear force = R, Length of shear span = a (1825 mm), Width of beam = b (200 mm), Height of beam = h (450 mm), Effective height = d (404 mm)
2.2 Experimental results

All beams failed in shear through diagonal cracks. The failures were brittle and the capacity was lost immediately after failure. The concrete compressive strength and the ultimate load for each of the beams are listed in table 1. Fig. 2 displays a diagram of the ultimate load as a function of the concrete strength for each of the beams. Despite of the scatter of the results, it can be seen that an increase of the concrete strength, in general, has a small enhancing effect on the ultimate load. The beams with a measured concrete strength of 50-60 MPa (series 4), however, deviate from this tendency. This could be attributed to the fact that the high concrete strengths caused the cracks in these beams to develop through both cement paste and the aggregates. Investigations of the beams after failure verified this statement (Pedersen & Eriksen 2014b).

![Fig. 2](image)

Fig. 2  Relationship between the concrete strengths and the ultimate loads for individual beams

<table>
<thead>
<tr>
<th>Series</th>
<th>Beams</th>
<th>$f_{c}$ [MPa]</th>
<th>Ultimate load [kN]</th>
<th>Average ultimate load [kN]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Series 1</td>
<td>Beam 1</td>
<td>17.5</td>
<td>154.1</td>
<td>149.5</td>
</tr>
<tr>
<td></td>
<td>Beam 2</td>
<td>19.7</td>
<td>141.9</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Beam 3</td>
<td>21.9</td>
<td>152.5</td>
<td></td>
</tr>
<tr>
<td>Series 2</td>
<td>Beam 4</td>
<td>31.5</td>
<td>176.9</td>
<td>161.7</td>
</tr>
<tr>
<td></td>
<td>Beam 5</td>
<td>31.7</td>
<td>157.0</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Beam 6</td>
<td>32.0</td>
<td>151.2</td>
<td></td>
</tr>
<tr>
<td>Series 3</td>
<td>Beam 7</td>
<td>39.8</td>
<td>172.9</td>
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<tr>
<td></td>
<td>Beam 8</td>
<td>42.0</td>
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<td></td>
<td>Beam 9</td>
<td>40.4</td>
<td>187.2</td>
<td></td>
</tr>
<tr>
<td>Series 4</td>
<td>Beam 10</td>
<td>59.7</td>
<td>174.0</td>
<td>159.3</td>
</tr>
<tr>
<td></td>
<td>Beam 11</td>
<td>51.3</td>
<td>161.6</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Beam 12</td>
<td>51.7</td>
<td>142.2</td>
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</tr>
</tbody>
</table>
3 Nonlinear Finite Element Modelling of Shear Failure

Nonlinear finite element analysis consists, fundamentally, of a complex system of mathematical differential equations that are solved in a number of time steps. This paper deals with the challenges of nonlinear finite element modelling from a structural engineer’s point of view. The mathematical challenges are not treated here.

3.1 The modelling process

The process of nonlinear finite element modelling is not straightforward since a lot of fundamental choices have to be made. Generally, the process can be divided into three main parts: the pre-processing, the analysis and the post processing, see Fig. 4. The first part involves the definition of the basic model. Here choices have to be made regarding the basic geometry, the boundary conditions, the composition of the mesh, and the modelling of the concrete and the reinforcement. These choices are related to the degree of detail of the model, which is an important factor to consider in nonlinear finite element modelling. Even a small increase of the level of detail increases the necessary amount of CPU power massively and complicates the analysis. However, if the model is too simple, the results will become unreliable.

Fig. 4 Nonlinear finite element analysis process diagram (Pedersen & Eriksen 2014a)

When considering the physical behaviour of slender reinforced concrete beams, the choices made in the pre-processing regarding the definition of element types, element shapes and element sizes are not obvious. Unfortunately, these parameters can influence the results of the model. A small element size increases i.e. the level of detail of the model but it does not necessary result in better agreement between the results of the model and the experimental results.

The last part of the pre-processing is the specific modelling. This part involves the definition of the properties of the applied materials such as the compression strength, the tensile strength, Young’s
Modulus, Poisson’s ratio, the fracture energy and the constitutive models. Cracking can either be modelled by use of a discrete cracking or smeared cracking. The discrete crack concept implies that a crack is modelled as a geometrical discontinuity. This means that it is constrained to follow a predefined path along the edges of the elements. The drawback of this method is that the position and the shape of the cracks have to be known in advance. The smeared crack concept implies that a cracked solid is considered to be a continuum. The concept does not impose restrictions with respect to the position and shape of the cracks, thus this concept is useful when the development of the whole system of cracks is to be predicted (Rots & Blauuwendraad 1989). Unfortunately, cracks become non-physical as they are represented by strains in a continuous concrete element instead of separation of concrete elements.

The second part of the process is the analysis. The stress-strain relationship of the applied constitutive models for concrete is nonlinear which means that the result of the analysis becomes path dependent. The load must therefore be applied to the beam during an incremental process. Hence, the analysis is an incremental iterative process where equilibrium is achieved for every load increment. To perform the analysis, the user needs to define how this iterative process is performed. Unfortunately, these choices also have a direct influence on the predicted response of the member.

The third part of the modelling process deals with the results of the analysis. The results of a nonlinear finite element model may be visual images of how cracks develop during the entire loading history, images of how the shear failure develops, images of the distribution of stresses for every load level, and the relationship between the applied load and the deformation of the beams. The validation of the model is based on a comparison between these results and the experimental results. The optimal design of the finite element model is basically found through trial-and-error attempts where the defined parameters are adjusted to achieve as much similarity between the results of the finite element model and the experimental results.

3.2 Modelling of the test beams

The modelling of the reinforced concrete beams were performed using the nonlinear finite element software TNO DIANA 9.4.4. The aim of the modelling process was to build up a model that was consistent with the observations from the experimental research. The basic geometry of the finite element model is illustrated in Fig. 5. As mentioned previously, one of the main considerations in nonlinear finite element modelling is related to the degree of detail and the size of the model. Therefore, due to the symmetry of the experimental setup, only one half of the beams was modelled using nodal horizontal constraints at the centreline of the beam. Additionally, the beams were modelled in 2D. This means that out of plane stresses were assumed to be zero. These choices are rather decisive since it reduces the complexity of the model massively and hence the required amount of CPU power.

The sizes of the finite elements and their shape proved to be of great importance in relation to how cracks develop. This is exemplified in Fig. 6. It is clear that a finer mesh results in narrower crack patterns that are more comparable with the experimental results.
3.3 Crack modelling and material properties

The modelling of the beams from the experiments was based on the smeared crack concept. The experimental results indicated that the orientation and position of the cracks were geometrically fixed after it had developed. To obtain similar behaviour in the nonlinear finite element model, the Total Strain Fixed Crack Model was applied. This model implies the need for a constitutive model for shear subsequent to cracking, owing to the presence of shear stresses along cracks. The applied constitutive model for shear was capable of taking an effect of increasing crack widths on shear transfer across cracks into account by the introduction of a variable retention-factor, inversely related to the crack-width (damage model).

Experimental research regarding reinforced concrete beams without shear reinforcement shows that the general behaviour to a large extent depends on the mechanical properties of the concrete. The definition of the properties of the concrete is hence of great importance when it comes to the results of the finite element model. In the experimental research, the compressive strength of the concrete was determined from compression tests on concrete cylinders. Young’s Modulus, the tensile strength of the concrete and the fracture energy were theoretically determined from the compressive strength through equations defined in the CEB-FIB Model Code 1990. However, the „correct“ value of the fracture energy deviates ± 30 % from the value given by the equation in the code according to the CEB-FIB Model Code 1990. Furthermore the CEB-FIB Model Code 1990 advises a value of Poisson’s ratio between 0.1 and 0.2. The values of the fracture energy have a considerable impact on the results of the finite element model as an increase of the fracture energy, in general, tends to increase the shear strength. The influence of Poisson’s ratio and the fracture energy on the shear strength appears from Fig. 7 and Fig. 8.
4 Nonlinear Finite Element Results

Through an iterative modelling process, the nonlinear finite element model was optimized for the average concrete strengths of the four investigated test series. In this process all assumptions were held constant except for the concrete strength and the material parameters determined from the concrete strength. The results of the model were primarily evaluated on the ultimate load, the development of cracks and the failure mechanisms.

4.1 Results

The visual result of the model is an image of the developed cracks for every load level (see Fig. 6). These images show cracks that develop in a vertical direction initially. As the load is increased, the cracks tend to develop towards the applied load. This is in good agreement with the experimental observations. However, the inclination of the cracks in the finite model was slightly larger than the inclination of the cracks observed in the experiments. The crack images from the finite element model cannot be compared directly to the crack patterns observed in the experiments since the observed crack patterns for tested beams with identical geometry and material properties were not identical. In an overall perspective, though, the crack patterns from the finite element model were comparable with the experimental observations.

Common to both the experimental observations and the finite element results was shear failures through pre-existing inclined cracks. In both the finite element model and in the experiments, the failures developed partly through the uncracked concrete in the compression zone towards the applied load and, in certain cases, along the flexural reinforcement. The capacities were lost immediately after failure. Failure in the finite element model and in the experiments is illustrated by example in Fig. 9 and Fig. 10.
The average shear strength of each of the four test series and the shear strength determined by the nonlinear finite element model for concrete strengths between 10 MPa and 80 MPa appear from Fig. 11. It appears that there is plausible agreement between the nonlinear finite element model and the experimental results. Generally, the relationship between the concrete strength and the shear strength for the nonlinear finite element model and the experimental results are quite similar. In both cases, an increase of the concrete strength, in general, leads to a small increase of the shear strength.

**Fig. 11** Experimental results and finite element results

### 4.2 Discussion

In the experiments, it was observed that the roughness of the crack surfaces was reduced for increasing strengths of the concrete. This is due to the fact that cracks, to a greater extent, develop through both the aggregates and the paste. For low values of the concrete strength, cracks tend to develop through the cement paste around the aggregates which results in rougher crack surfaces.

The roughness of a crack is a very important parameter when considering the ability of a reinforced concrete beam without shear reinforcement to transfer shear across a crack. Generally, the ability to transfer shear across a crack is reduced as the width of the crack is increased, especially for cracks with smooth surfaces, as the contact area between the surfaces on each side of the crack is limited (Walraven 1980). The effect of increasing cracks widths is taken into account in the finite element model by the applied constitutive model for shear. However, the relation between the crack surfaces and the concrete strengths are not incorporated in the constitutive model for shear. The influence of
the reduced roughness of the cracks on the fracture energy was not incorporated as well. This is a plausible explanation of why the nonlinear finite element model, compared to the experimental results, underestimates the shear strength for the concrete strengths below 30 MPa and overestimates the shear strength for concrete strengths above 50 MPa.

In Fig. 12 (right), the result of the finite element model and the tests are compared to the predictions of a mechanical model developed by Fisker (Fisker, 2014), based upon the upper bound theorem of the Theory of Plasticity. As illustrated in Fig. 12 (left) it is assumed that the member fails in shear through the development of a certain pattern of failure lines and a compatible mechanism. As also emphasized in the figure, part of the failure develops as a sliding failure along an already existing crack. In the model, the sliding capacity of such an existing crack is represented by a Mohr-Coulomb-like failure condition taking into account the crack width, the maximum aggregate-size and the direction of the relative displacements along the crack. In Fig. 12 (right), the results of the mechanical model is represented by a graph indicating mean values of the predicted shear capacities and a shaded region representing the level of scatter typically observed in test results related to the influence of the concrete strength, see i.e. Moody & al. (1954).

Fig. 12: left) Assumed failure mechanism in mechanical model (Fisker, 2014), right) Comparison of experimental results, finite element model and mechanical model.

The mechanical model confirms that the nonlinear finite element model tends to underestimate the shear capacity for concrete strengths below 30 MPa, and that it tends to overestimate the capacity when the concrete strength is increased beyond approximately 50 MPa. Additionally, it appears that the shear strength obtained in the test series 4 is lower than the capacity given by the mechanical model. A plausible explanation for this reduction of the capacity may naturally be related to the observed reduced surface-roughness of the cracks for this group of beams compared to series 1 to 3, which is not fully captured by the model. This abrupt change of the crack surface texture may limit transfer of shear across the crack.

The post-processor enabled the opportunity to analyze distributions of stresses before failure. It is clear that shear in the finite element model is carried partly by the uncracked concrete in the compression zone and partly across cracks. However, it is difficult to draw any definite conclusions regarding effects from dowel action. Part of this reason is that the degree of detail of the reinforcement and the surrounding concrete is relatively small. For further details regarding how shear is carried in the finite element model, the reader is referred to the Master’s Thesis (Pedersen & Eriksen 2014a).

Fig. 8, shown previously, displays a considerable variation of the finite element results when varying the fracture energy within the limits proposed by the CEB-FIB Model Code 1990. Additionally, the
choices related to the degree of detail and the choices related to solving the nonlinear equations directly influence the predicted response of the beams. A nonlinear finite element model of reinforced concrete beams without shear reinforcement that involves brittle failure modes and distinct cracking will, consequently, have to be calibrated to known experimental results. For this reason, nonlinear finite element modelling of such failure modes appears to be of limited use for practical design currently.

5 Conclusions

This paper investigates the influence of the concrete compressive strength on the shear strength of reinforced concrete beams without shear reinforcement. The investigation is based on both experimental research and nonlinear finite element analysis.

The main conclusions are:

- The experimental research shows that increasing values of concrete strength, in general, have an enhancing effect on the shear strength. For concrete strengths above 50 MPa a slight reduction of the shear strength is observed due to the development of smoother crack surfaces.
- Nonlinear finite element modelling imposes high demands on the user, since all choices regarding physical and nonphysical parameters effects the results of the model.
- The nonlinear finite element model developed to simulate the behaviour of the beams from the experiments indicates a relationship between the concrete strength and the shear strength very similar to the experimental results.
- The nonlinear model underestimates the shear strength for concrete strengths below 30 MPa and overestimates the shear strength for concrete strengths above 50 MPa. A plausible explanation of this is that the finite element model is not capable of addressing the influence of the concrete strength on the roughness of the developed cracks.
- The shear cracking observed in the tests was similar to that observed from nonlinear finite element modelling with a smeared crack model.
- The model by Fisker (2014) confirms, in general, the relation between the concrete strength and the shear strength given by the experimental results.
- Nonlinear finite element modelling that involves brittle failure modes and distinct cracking is not useful for practical design of reinforced concrete beams without shear reinforcement currently.

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