International Portfolio Diversification Study from a China’s Perspective

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Abstract

An old idiomatic phrase said: “Don’t put all of your eggs in one basket”; it should be the prototype of the asset allocation theory. Today, the concept of asset allocation has been studied for decades, which means more than the research of the appropriate mixture of currency, bond, and stock and so on. In the wave of the internationalization, China, as one of the fastest developing countries of the world in past decade, seems like a green hand in the area of the international investment compared to the developed country. Strategies should be developed to obtain optimal decision for the investment of Chinese investors. This paper provides a solution to an international diversification problem from a China’s perspective in order to offer the allocation suggestion to the Chinese investors. The main problem is to investigate how much is the gain of international portfolio diversification for China in international investment and how does the home bias effect perform in China. We find that the international diversification strategy is benefit for the Chinese investors. Furthermore, the analysis of the home bias effect shows that the unstable and independent domestic market brings China low-degree home bias effect comparing to the developed countries, which, on the contrary, helps the Chinese investors to achieve more benefits in the international diversified investment. Three methods are used to estimate the efficient frontiers, trace the capital market line and figure intersection point of them. The point of the intersection indicates the optimal portfolio and the proportion of the different assets. The result is objective and demonstrates the weights in the perfect state. However, in the real market the weights show deviation with our results because of the affection of multiple factors and we call the deviation as home bias. Although there are so many reasons for the explanation of the home bias, no sufficient theory is so comprehensive to cover the entire phenomenon of home bias observed. The asymmetrical information, the investment barriers and the purchase power parity indicate that the investors have different attitudes towards the domestic and the international markets. The Tullis and Clarke (1999) model is considered to be used in this paper to illustrate the effect of the
home bias.

Key words: portfolio optimization, China, home bias, international diversification
1. Introduction

Asset Allocation is the most important part of the modern portfolio theory. The investors all over the world are seeking the optimal portfolio for the highest return and the lowest variance at the same time. Such as the investors, the countries also are eager to perform well in whether domestic market or international market. However, most of the developing countries have just realized the importance of the international market, even if others have been initiating into it for decades, like the U.S. In order to enhance to the performance of the national economic, the foreign market cannot be ignored as a part of the country’s potential competitive power.

In China, international diversification was ignored for a long time after the establishment of the People Republic of China (PRC). Since Deng Xiaoping launched the “reform and opening” policies in 1978, foreign investment which bring foreign capital, technology, and advanced management into China has started until now. For example the foreign direct inflow investment has leaded a huge development of more than half a million China’s enterprises and amounts to US$660 billion in asset value since 1978. (Brainard and Fenby, 2007)

Nevertheless, in the past decade, there has been a trend of investment in a reverse direction, which means the development of decades causes China’s enterprises powerful enough to try to make use of foreign resources. Brainard and Fenby (2007) predict that China’s foreign direct investment (FDI)\(^1\) outflow will break even with foreign direct

\(^1\) Foreign direct investment (FDI) is a measure of foreign ownership of productive assets, such as factories, mines and land. Increasing foreign investment can be used as one measure of growing economic globalization. The international investment includes the foreign direct investment and foreign indirect investment. Comparing with the direct investment, the indirect investment means someone will give their money to, for example, an investment fund such as GPT (General property Trust) who take your money and directly invest it in property, or shares, cash, bonds et al. By indirect investment, the investor is not directly associated with the profit and loss of the firm on which the money is ultimately going. So in this paper the indirect investment cannot be taken into consideration because it is hard to evaluate the amount of the indirect investment.
inflow investment as early as 2010.

Table 1 shows a list of the recent FDI outflow by China’s company. These investments are all about the industry of oil and high technology, which is on behalf of the most important part of China’s economic. All the investments indicate that China’s attempt to develop internationally. China’s company Lenovo Group which acquired IBM’s personal computer business for US$1750 million draws the attention all over the world. Furthermore, a series of movements in the industry of oil indicate the competency of China to influence the world energy market.

**Table 1 Recent China’s FDI outflow Investment**

<table>
<thead>
<tr>
<th>Date</th>
<th>Details about the recent China’s FDI outflow</th>
</tr>
</thead>
<tbody>
<tr>
<td>18 January 2002</td>
<td>China National Offshore Oil Corp acquired Repol-YPF (Indonesia oil assets) for US$591.9 million.</td>
</tr>
<tr>
<td>8 December 2004</td>
<td>Lenovo Group acquired IBM’s personal computer business for US$1750 million.</td>
</tr>
<tr>
<td>12 April 2005</td>
<td>CNOOC acquired 17% of Alberta oil-sand concerns of MEG Energy in Canada for US$124.2 million. China’s first acquisition in oil sands.</td>
</tr>
<tr>
<td>14 April 2005</td>
<td>Canadian pipeline operator Enbridge announced plans to develop a US$2.07 billion pipeline with PetroChina International to provide China access to crude oil from Canada’s oil sands.</td>
</tr>
<tr>
<td>31 May 2005</td>
<td>China Petroleum and Chemical, or Sinopec, paid US$124.2 million for a 40% stake in Northern Lights in Canada.</td>
</tr>
<tr>
<td>28 October 2005</td>
<td>China National Petroleum (China’s largest oil producer) acquired Canada-based PetroKazakhstan in Kazakhstan with US$4.18 billion.</td>
</tr>
<tr>
<td>15 December 2006</td>
<td>The Bank of China acquired 100% of the issued share capital of</td>
</tr>
</tbody>
</table>
Singapore Aircraft Leasing Enterprise, the largest aircraft leasing company based in Asia, for US$965 million in cash.

21 May 2007

China’s state investment agency uses US$3 billion to buy 9.9% stake in US private equity firm Blackstone. The deal will help Blackstone to make inroads into China, such as through buying nonvoting shares of Chinese companies. The below 10% stake is not subject to US government scrutiny.

Sources: Carew (2007) and Fung and Liu (2007)

Although, China shows the potential to be global, its foreign investment proportion is still very low in the China’s investor’s equity portfolio. China’s foreign investment as a percentage of gross fixed capital formation remains almost the same from 1990 to 2007: 1%-1.6% (World Investment Report, 2008), showing that the China’s investors are still sticking to the domestic market. The effect of home bias is even much stronger than most of the developed countries. For example, Tesar and Werner (1998) report that in 1996, the home bias for the developed countries have already been very evident: 81.8% of private German investors’ equity portfolios were invested in domestic equities; in the US, 90%; in the UK, 77.5%; in Japan, 94.7%; and in Canada, 88.8%.

Obviously, international diversification is a mixture of opportunity and risk. Then an important problem arises: how much international diversification is an appropriate proportion is optimal for China’s economic? Furthermore, what is the influence about the home bias? This paper provides the constructive solution for both of the questions. For the first question, we analyze the proportion by an empirical study. For the second question, it is required to combine to considerate the preference of the investors and the historical data.

The paper is organized as follows: Section 1 is the introduction of the China’s foreign investment including the history and characters. Section 2 reviews the international diversification about different papers from the multinational researchers and the
explanatory theories about home bias which classified into three main categories: barriers on international investment, deviation from purchase power parity and asymmetric information. Section 3 indicates the sources of the data and the methodology of the research. The black method, the Lagrange’s method and the Solver method are used in this paper to estimate the efficient frontier. Section 4 is the assumptions which make the calculations more feasible. Section 5 provides the analysis and discussion with empirical results. Section 6 is the comprehension of the home bias analysis compared with the former researched and explains why the home bias shows particularity in China. The last section is the conclusion of the paper.

### 1.1 Introduction of the history of the China’s foreign investment.

After the brief overview of the China’s foreign investment above, the annual data and the trend can be concluded as follows. Figure 1 shows the trend of the development of China’s foreign investment.

According to the history of China’s foreign investment, there are four periods. Before 1979, almost no foreign investment activities happened. Due to the situation of the China’s politics, it was impossible to launch efficient investment abroad, especially in the developed countries. For example, China’s foreign investment in 1979 was a negligible amount of 0.8 million yuan (equivalent to about US$0.5 million at the current official foreign exchange rate) (Cai, 1999).

From 1979 to 1989, the preparation period started for China’s foreign investment. The China’s “reform and opening” policy made foreign investment develop. China’s annual foreign investment increased from 1980’s US$35 million to 1984’s US$134 million, an amount of 4 times in four years. Especially, in the 1985 (US$628 million), almost 5 times more had been made compared to the previous year. From 1979, 185 non-trading foreign affiliates were established, mostly in the form of joint venture. By 1990, the
number of the affiliates reached 577. The overseas enterprises spread over 90 countries, most of which were developing countries. As a result, China’s transnational corporations became diversified and numerous. (Cai, 1999)

**Figure 1 China’s foreign investment: 1982 to 2007 (US$ billion)**


Entering the 1990s, China’s foreign investment experienced a booming development. It is so obvious that a growth happened in 1992. Precisely, in 1991 the foreign investment was US$913 million versus in 1992, US$4 billion. The overseas enterprises were more than 6000 and spread over 189 countries.
There comes 21th century. After 20 years’ preparation, China achieved the linear growth since 2003. China acquired the membership of world trade organization at 2001, which marks that China faces more opportunities and challenge. The compounded average growth rate was 25.84% from 1979 to 2006, which coincides with the approximate 25% annual international trade growth since 2001 (Bo, 2006). All these indicate that China’s foreign investment is keeping a favorable situation to development.

1.2 Brief introduction of the China’s foreign investment.

◆ The characteristic

The geographic distribution of China’s foreign investment is widespread. By 2005 China has flowed into more than 189 countries. In 2006, 43.46% and 48.03% of China’s FDI outflow flowed into Asia and Latin America. However, emphasis has been moved to Asia in 2007, with the proportion of 62.6%, which including Hong Kong’s 51.81%. The percentage in Latin America has fallen into 18.5%. The reason is that the investment in Asia has been involved in many countries, but concentrated in three countries in Latin America: Cayman Islands, British Virgin Islands and Bahamas, which are the tax havens there. The amount can be re-invested into other economics at any time. So Asia, especially Hong Kong is the destination of more than half of the China’s foreign investment.

The focus of the sector distribution is the industry of manufacture, with a proportion of 54.66% in 2007. And industry of estate follows by 22.86%. Business services take 5.38%. Others take about 18%.

◆ The motives

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2 The Year Book of Statistic, China, 2008
The motives of the China’s foreign investment are generally similar with the most of the developing or developed countries. However, in view of the basic country situation, China’s foreign investment pays more attention on the different issues. Specifically, following international motives can be described as the main points of the China’s foreign investment:

First, many Chinese transnational corporations (TNCs) invest abroad to seek, maintain or expand export markets. Secondly, Chinese foreign investment is used to acquire a stable supply of resources, primarily in fisheries, forestry and mining. Thirdly, China’s foreign investment is used by some Chinese TNCs as an effective channel to obtain foreign technology and management skills. Fourthly, while some Chinese TNCs rely on the capital rose in the international financial markets for their foreign investment projects, other companies use the foreign investment as an efficient channel to raise capital.

◆ The procedure

The procedure of China’s foreign investment can be shown in Figure 2

**Figure 2 China’s foreign investment procedure**

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3 According to Transnational Corporations and Management Division, Department of Economic and Social Development of the United Nations, there are five primary types of foreign investment undertaken by transactional corporations (TNCs) from developing countries, that is, market-seeking, export-oriented, resource-seeking, technology-seeking and efficiency-seeking foreign investment. UN-TCMD, Transnational Corporations from Developing Countries: Impact on Their Home Countries (New York: United Nations, 1993), pp. 11-14.

1.3 The policy of the China’s foreign investment

China has undergone a process of gradual liberalization of its foreign investment regime since the early 1980s, in line with the country’s overall economic reform. The Chinese government first allowed foreign investment in 1979 as part of its broader “reform and opening” policy. Before 1983, however, the approval process of foreign investment was highly centralized. From that time it has been increasingly decentralized. China’s policies with respect to the administration of foreign investment have been codified in specific rules and regulations, which involve the following important documents; the Approval Procedures and Administration of Non-Trade Joint Ventures Set Up Abroad (1985), the Approval Procedures and Administration of Trade-related Organizations Set up Abroad (1988), the Administration of Foreign Exchange for Outward Investment (1989), and the Approval Procedures and Administration of Overseas Investment (1993).

Entering the new century, the Chinese government has engaged in various initiatives to promote outward investments, as part of its overall ‘Going Global’ strategy, which also address the areas of exports and subcontracting of overseas engineering projects. The State Asset Supervision and Administration Commission (SASAC) was established in April 2003 with the mandate of turning the country’s top state-owned enterprises (SOEs) 

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under its control into 50 global multinational corporations that feature on the global Fortune 500 list.6

Furthermore, the Chinese authorities are taking steps to allow overseas portfolio investment by Chinese companies. For example, in February 2004, the State Council gave approval to the National Social Security Fund to invest their foreign currency funds in overseas markets.

International organizations are also active in China’s ‘Going Global’ strategy. In 2005 the Multilateral Investment Guarantee Agency (MIGA) co-sponsored a workshop with the Chinese Ministry of Finance, China Exim bank and International Finance Corporation (IFC). In addition, this workshop involved the collaboration of the National Development and Reform Commission (NDRC), Sinosure, the State Asset Supervision and Administration Commission (SASAC) the Ministry of Commerce, and Chubb Insurance. The workshop brought together 200 participants, of which two-thirds were from Chinese companies, and the remainder from relevant central and provincial governments, as well as from international organizations, to discuss Chinese foreign investments.7

Similarly, on 6 December 2005, the banking department of China Exim bank and Beijing Municipal Bureau of commerce co-hosted a Briefing on “Financing Products and Policy for Beijing Companies’ ‘Going Global’ Practices”. Mr Li Ruogu, Chairman and President of the Bank, and Mr. Lu Hao, Vice Mayor of Beijing Municipal Government were present at this Brief, during with China Exim Bank signed Master Agreements totaling RMB 10.3 billion with the Beijing Construction Engineering Group Corporation (BCEGC), Founder Group and CGC overseas Constructions Co.Ltd

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(CGCOC), to support these companies’ ‘Going Global’ strategies.8

Besides the establishment of regime of the foreign investment outflow, the government has activated some specific programs to encourage the foreign investment. For example: Qualified Domestic Institutional Investor Program (QDII)

The QDII program allows the domestic investors with foreign currency to invest in overseas financial assets through qualified financial institutions.

**Table 2 Select Qualified Domestic Institutional Investors (QDII) and Their Foreign Exchange Conversion Limits**

<table>
<thead>
<tr>
<th>Name of QDII (US $m)</th>
<th>Foreign exchange conversion limits</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bank of China</td>
<td>2500</td>
</tr>
<tr>
<td>China Construction Bank</td>
<td>2000</td>
</tr>
<tr>
<td>Industrial and Commercial Bank of China</td>
<td>2000</td>
</tr>
<tr>
<td>Bank of Communications</td>
<td>1500</td>
</tr>
<tr>
<td>China Merchants Bank</td>
<td>1000</td>
</tr>
<tr>
<td>Agricultural Bank of China</td>
<td>500</td>
</tr>
<tr>
<td>HSBC</td>
<td>500</td>
</tr>
<tr>
<td>China CITIC Bank</td>
<td>500</td>
</tr>
<tr>
<td>CITI Bank</td>
<td>500</td>
</tr>
<tr>
<td>Standard Chartered Bank</td>
<td>500</td>
</tr>
<tr>
<td>Industrial Bank</td>
<td>500</td>
</tr>
<tr>
<td>Hua-An Asset Management</td>
<td>500</td>
</tr>
<tr>
<td>Bank of East Asia</td>
<td>300</td>
</tr>
<tr>
<td>Hang Seng</td>
<td>300</td>
</tr>
</tbody>
</table>

8 [http://english.eximbank.gov.cn/info/Article.jsp?a_no=1315&col_no=84](http://english.eximbank.gov.cn/info/Article.jsp?a_no=1315&col_no=84)
Between 2001 and 2005, there were only talks about the QDII program because the authorities are afraid that an introduction of the program might exacerbate the already bearish domestic stock markets. However, along with growing pressure from the external imbalances and fast-growing foreign exchange reserves, the government was prompted to formally launch the QDII scheme. The QDII scheme covers three sectors: qualified commercial banks, qualified securities institutions and qualified insurance companies (Hang Seng bank, 2006). As of May 2007, a total of US$15bn in QDII quotas have been distributed among 19 domestic and foreign banks that have been granted QDII licenses, as well as other financial institutions, such as insurance and asset management companies. The Table 2 presented the list of QDII and their foreign exchange conversion limits.

A number of financial products are offered by QDII. Some of the banks such as the Bank of China and the China Construction Bank offer products that are invested in overseas fixed-income securities like money market securities and bonds. In the first half of 2007, these QDII products were not very popular among domestic investors, but the situation is expected to change as the domestic equity markets fluctuates more wildly, and even decline.

2. Theory explanation and literature review

2.1 History of research about the international diversification

9 Note: the above figures are from May 2007. A total of US$15bn of QDII quotas have been distributed among 19 domestic and foreign banks and other financial institutions like insurance and asset management companies.
The discussion of the theoretical model of portfolio selection started from more than 50 years ago. The theoretical models of portfolio selection developed by H. M. Markowitz (1952) and J. Tobin (1952) provide a positive explanation and normative rules for the diversification of risky asset. However, the history of the research about the international diversification is about 40 years old. Most of the research of the international diversification analyzes the benefits from an American standpoint.

In the early period of the research about the international diversification, only the benefits are certified to be existed. In the Levy and Sarnat’s (1970) paper, they recommend the American investor should never restrict his portfolio to developing countries. The gaining of the international diversification is related to the correlation between the U.S. market and the target country market. The high correlation between the two markets may cause little gain when combing them in a portfolio of their own. For example, Belgium, France, Germany and Italy’s markets have very high correlation with the U.S. market, so the American investors should not take the asset of these countries into the portfolio. Conversely, they should invest in the countries which have low even minus correlation with the U.S. market such as Austria, Denmark, Japan and Mexico.

Solnik (1974) shows the degree of the risk reduction when diversify internationally. The primary motivation in holding a diversified portfolio of stocks is to reduce risk. The risk of a portfolio in terms of variability of returns will be less than the risk of its separate parts. The reason is that the greater the number of securities in a portfolio, the less the portfolio is likely to lose as the result of one company’s misfortune. He compared the portfolio diversification in foreign securities with the domestic common stocks and makes a conclusion for American standpoint.

10 The high degree of correlation constitutes impressive evidence of a high degree of economic integration among the capital markets of these countries.
However, the quantities analysis is not applied in the early research. The reason is that such models are difficult to use and tend to result in portfolios that are badly behaved. There are two main barriers: First, expected returns are very difficult to estimate. Investors typically have knowledgeable views about absolute or relative returns in only a few markets. A standard optimization model, however, requires them to provide expected returns for all assets and currencies. Thus investors must augment their views with a set of auxiliary assumptions, and the historical returns they often use for this purpose provide poor guide to future returns. Second, the optimal portfolio asset weights and currency positions of standard asset allocation models are extremely sensitive to the return assumptions used. The two problems compound each other. In practice, despite the obvious conceptual attractions of a quantitative approach, few global investment managers regularly allow quantitative models to play a major role in their asset allocations decisions.  

The first quantitative asset allocation model is presented by Black (1992). This article describes an approach that provides an intuitive solution to the two problems that have plagued quantitative asset allocation models. The key is combining two established tenets of modern portfolio theory—the mean-variance optimization framework of Markowitz and the capital asset pricing model (CAPM) of Sharp and Lintner. They show that at a given level of risk, international diversification enhances portfolio performance whereby in contrast to equity portfolio, bond portfolio substantially benefits from currency hedging. After that, the quantitative models have been applied to the research about the international diversification. At the same time the objects of the research have been developed from the U.S. market to the multinational market.

Bugar and Maurer (2001) study the benefits derived from international diversification of equity portfolios from the German and the Hungarian points of view, considering the

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11 Sources come from “Global Portfolio Optimization” by F. Black and R. Litterman.
currency hedging decision. They find that the international diversification drastically reduced the risk of the domestic stock investment by Hungarian investor. The gains from international diversification for German investors were not as clear-cut as for their Hungarian counterparts, either in terms of risk reduction or performance improvement. What’s more, the hedged strategies performed better than their unhedged counterparts. Their finding on the ex-post mean-standard deviation efficient frontiers confirmed that fully hedging the currency risk is not necessarily worth doing. Although they don't examine the home bias, their findings show that the maximum weight allotted to the German (Hungarian) market is low at approximately 16.6 (0.6) %.

Wolfgang (2005) examines the potential benefits of international portfolio diversification over the last two decades from an ex-post and ex-ante German investor’s perspective considering the effect of a German home bias. The ex-post optimization shows the impressive advantages to be obtained by international diversification. As a matter of fact, it would have been optimal over the whole 20-year period to not invest in the German market at all. A German minimum weight of 20% to 40%, which is considerably higher than the German equity market share of world capitalization, does not have a noticeable negative effect. However, the German weight is indeed employed by institutional German investors. The result of ex-post rolling-window analyses show that a home bias of 20% to 40% is a suitable domestic equity investment range for German investors, who currently implement this percentage. They also find that the ex-post minimum variance portfolios perform well in an out-of-sample analysis. They outperform the benchmark indexes and a sole domestic investment.

Ghilardi and Currie (2006) draw upon the work of Tullis and Clarke¹²-How Much International Exposure is Advantageous in a Domestic Portfolio, but from a Canadian perspective. They found that the degree of how much is appropriate to invest

internationally ultimately depends on the trade-off risk and return an individual investor is willing to take based on long-term investment objectives considering the attitude of the investors towards the risk. Looking at both the S&P 500 \(^{13}\) and MSCI EAFE \(^{14}\), the most risk-averse Canadian investors may consider adding 2% (9%) of their portfolio into MSCI EAFE in the case of nominal (real) returns. For less risk adverse investors, allocating more weight to MSCI EAFE will help to enhance long-term return potential, but similar to the case above, at the price of higher volatility.

The effect of home bias is another important factor to the international diversification; it is discussed in next section.

### 2.2 Theories focusing on home bias

Despite the advantages of international diversification, data show that most developed countries’ investors still prefer the domestic market to the international one. Tesar and Werner (1998) report that in 1996, the home bias for the developed countries has already been very evident: 81.8% of private German investors’ equity portfolios were invested in domestic equities; in the US, 90%; in the UK, 77.5%; in Japan, 94.7%; and in Canada, 88.8%.

There are various explanations for the home bias during last 20 years, which are monetary investment barriers, regulatory investment barriers, political risk, deviation from purchasing power parity, and asymmetric information. For the home bias problem, there is no sufficient explanation which is so comprehensive to cover the entire phenomenon of home bias observed. However, in principle all these effects can induce a home bias in equity portfolios.

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\(^{13}\) Standard & Poor’s (S&P) is a division of McGraw-Hill that publishes financial research and analysis on stock and bonds. It is well known for the stock market indexes, the US-based S&P 500.

\(^{14}\) Morgan Stanley Capital International index (MSCI) in Europe Australia and Far East
Barriers on international investment

One of the earlier explanations of home bias is suggested by Black (1974). The research outlines models of capital market equilibrium when there are explicit barriers to international investment in the form of a tax on holdings of assets in one country by residents of another country. The barriers to international investment are assumed to take the form of taxes on the value of an individual’s holdings of assets in countries other than his own. The effect of these barriers will be the same as the effect of the tax on international investment. The unfavorable taxation leads to the imperfectly diversified portfolio holdings in his opinion.

Then Stulz (1981) improves this model by replacing the taxation factor into a factor called $\beta$. In his opinion, it is shown that all foreign assets with a $\beta$ larger than some $\beta'$ plot on either one of two security market lines. The motivations are the same in both of the papers to present that it is costly for domestic investors to hold foreign assets. However, the assumption and the condition are different. The Stulz’s model holds if barriers to international investment can be represented as taxes on the absolute value of an investor’s holding of risky foreign assets but not the net value for Black model. In the Black model, investors are indifferent between holding their portfolio of risky assets or their portfolio plus one foreign share held simultaneously long and short, whereas in Stulz model, investors offered this choice would never be indifferent. In a word, the Stulz model offers more powerful tests.

However, the barrier theory is argued by Cooper and Kaplanis (1994), they found that the taxes or transaction costs exist only if investors have very high levels of risk tolerance and equity returns are negatively correlated with domestic inflation. Similarly, Tesar and Werner (1995) suggest that due to the high volume of cross-border capital flows and high turnover rate on foreign equity investments relative to turnover on domestic equity markets, the variable transaction costs are unlikely explanation for
Deviations from purchase power parity

Another potential explanation is based on deviations from purchase power parity. Stulz (1981) constructs a new model for international asset pricing which admits different in consumption opportunity sets across countries. The model shows that the real expected excess return on a risky asset is proportional to the covariance of the return of the asset with changes in the world real consumption rate.

Adler and Dumas (1983) suggest that in portfolio theory, to distinguish between the domestic and international settings, one requires an economic concept of nationhood. Two main views lead to the explanation of the dimension of the investment in domestic or international market. The first view stemming from Solnik (1974) has been devoted to models where nations are defined as zone of a common purchasing power (CPP) unit or, more precisely, as subsets of investors who use the same price index in deflating their anticipated, monetary returns. The second view states that national groups of investors are delineated by deviations from purchasing power parity (PPP) which cause them to evaluate differently the returns from the same security.

Like the theory of the barriers of international investment, Cooper and Kaplanis (1994) test whether the home bias in equity portfolio is caused by investors trying to hedge purchasing power parity (PPP) deviations. The empirical evidence is consistent with this motive only if investors have very low levels of risk aversion and equity returns are negatively correlated with domestic inflation. They develop a model to estimate the cost required to generate the observed home bias. These costs are consistent with observable cost such as withholding taxes only if investors have low levels of risk aversion.

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15 Political risk might be an issue for investments in emerging stock markets; however it seems to be negligible.
On the other hand, as Glassman and Riddick point out the three widely-used simplifying assumptions produce optimal portfolio weights that are significantly different from the weights produced by a complicated model. In particular, imposing the valid assumption that PPP holds exaggerates the degree of the home bias.

In a word, the last two effects mentioned so far can induce the home bias in equity portfolios. However, the empirical evidence shows that neither of them seems to be sufficient to explain the magnitude of home bias.

◆ Asymmetric information

The last explanation is the asymmetric information which indicates that the investors have different attitude towards the domestic and the international markets. French and Poterba (1991) think that the lack of diversification appears to be the result of investor choices rather than institutional constrains. The home bias can be explained by domestic investors holding more optimistic expectations about domestic stock than about foreign stocks. Informational asymmetries are normally considered to be the underlying reason for such differences.

Gehrig (1993) points out that the home bias theory presents a major challenge to asset pricing model building on the assumption of symmetrically informed investors. He develops a simply noisy rational model to describe that even in equilibrium, investors remain incompletely informed. It is shown that the domestic bias arises quite naturally when investors are on average better informed about the domestic market.

Similarly, Brenman and Cao (1997) make extensive research to develop a model of international equity portfolio flows that relies on informational differences between foreign and domestic investors using the data on United States equity portfolio flows. The model predicts that if foreign and domestic investors are differentially informed, then portfolio flows between two countries will be a linear function of contemporaneous returns on all national market indices; and if domestic investors have a cumulative
information advantage over foreign investors about domestic securities, the coefficient of the host market return will be positive. If additional symmetry is imposed on the informational endowments, the stronger result is obtained that the portfolio flow will depend only on the host market return.

Levy and Livingston (1995) show in mean-variance portfolio optimization framework that superior information about a specific asset, expressed either by lower expected standard deviation or by higher expected return, may be a strong reason not to diversify.

3. Data description and methodology

3.1 Data description

The analyzable data set in this paper include the bond and stock markets information from three main markets in the world (the China’s market; U.S. market and the European market.). The analysis of the monthly data starts from 31 December 1990 to 31 December 2007.

◆ The time period of the sample

The analysis of the monthly data starts from 31 December 1990 to 31 December 2007. One of the sample data-set in this paper is the Chinese stock market index which is based on the Shanghai Stock Exchange Index. The Shanghai stock market opened since 31 December 1990, although China initiated the international investment from 1978 because of the China’s “reforming and opening” policy. The objection of the analysis ending at 31 December 2007 is trying to minimize the influence of the Economic crisis to the worldwide stock and bond markets.

◆ Sources of the data

The Chinese market secondary data source is from Guo Tai An (GTA) Information Technology’s China database, named China Stock Market Accounting Research
The U.S. market and the European market which supply the compared objections with China can be collected from a database provided by DataStream.

When outputting the data from the DataStream, the total return index item which includes the clean price and dividends should be used as the price of the stock or bond.\(^{16}\)

**The selection of the assets**

This paper helps the Chinese investors to engage into the international portfolio which is reasonable and profitable. The selection of the assets in this paper should follow some instructions:

1. Chinese stock market index must be included.

The Chinese stock market is the main research objection of this paper because the topic is to analyze the Chinese investors’ behavior engage to the international market. The Chinese stock market is the most direct market for Chinese investor. What’s more, the degree of the home bias depends on how much they invest Chinese stock market and how much the asymmetric information from Chinese stock market influences them.

2. The return of the foreign assets should be positive and performing well during the period between 1990 and 2007.

First of all the return of the assets should be positive, or otherwise there is no point to invest the asset as part of the portfolio. The time period (1990-2007) should also be the condition to make the return of the asset to be positive.\(^ {17}\)

---

\(^{16}\) The clean price index should not be used because the excess return calculated is lower than the result when total return index is used. The higher result is benefit for the analysis.

\(^{17}\) After 2007 the economic crisis influences worldwide market. The sample should not be good to be considered.
3. Bond market index problem considering the degree of the Chinese’s investors’ risk aversion

Bonds and stocks get independent returns and variances, although there are so many kinds of the bonds and the stocks. On average, corporate bonds got lower returns and lower variances:

**Table 3 Average annual return and variance of the U.S. small stocks, large stocks (S&P 500) and corporate bonds**\(^{18}\)

<table>
<thead>
<tr>
<th>Investment</th>
<th>Average Annual Return</th>
</tr>
</thead>
<tbody>
<tr>
<td>Small stocks</td>
<td>22.11%</td>
</tr>
<tr>
<td>S&amp;P 500</td>
<td>12.32%</td>
</tr>
<tr>
<td>Corporate bonds</td>
<td>6.52%</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Investment</th>
<th>Return Variance</th>
</tr>
</thead>
<tbody>
<tr>
<td>Small stocks</td>
<td>42.75%</td>
</tr>
<tr>
<td>S&amp;P 500</td>
<td>20.36%</td>
</tr>
<tr>
<td>Corporate bonds</td>
<td>3.18%</td>
</tr>
</tbody>
</table>

Due to the table 3 above, the bonds are safer comparing to the stocks. However, Lilian and F. Wu’s (2008) finding indicate that Chinese institutions have a tendency to demand for assets that have high earnings per share, high idiosyncratic risk, large price, large market capitalization and relatively shorter listing period. The factors that influence the investment decisions of Chinese institutions are broadly similar to those

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\(^{18}\) Sources come from J. Berk, P. DeMarzo (2007) P293-P294.
that affect the decision making of financial institutions in developed markets.\textsuperscript{19}

So for the consideration of the degree of the risk aversion of the Chinese investors, the sample will be the assets which have the higher returns and higher volatility at the same time, which are obviously the stocks rather than the bonds in the main markets of the world. For example, the European stocks and U.S. stocks are available.

4. The diversified assets.

There are so two ways to make the assets diversified:

(1) Appropriate numbers of the asset.
(2) The correlation between the assets should not be large.

At first the numbers of the assets should not be the more the better, because if more assets are collected, it is not evitable to exist that two of the assets have large correlation which make the analysis unnecessary. At the same time, insufficient assets cause the portfolio not diversified enough.

Secondly, the European stocks indexes and U.S. stock indexes are the main indexes worldwide which Chinese investors prefer to engage in. However, they correlate with each other which mean the portfolio may not be diversified. The way to fix this problem in this paper is to add bonds in the portfolio which is totally uncorrelated with any stocks. Considering the preference of the Chinese investors, the number of the bond is limited into one. And the bond should have return which performed well.

Finally, the assets can be chosen due to the principles above. In this paper, five stocks and one bond is picked. They are Shanghai Stock Exchange index, FTSE 100 Stock Index, Dow Jones Industrial Average Stock Index, French Stock Market Index (CAC

40), Deutscher Aktien-Index (DAX) and FTSE Global Government Bonds. Chinese investors are certain to engage in the Shanghai Stock Exchange. FTSE 100 Stock Index, Dow Jones Industrial Average Stock Index, French Stock Market Index (CAC 40) and Deutscher Aktien-Index (DAX) are the representative stock markets in the Europe and U.S. which have appropriate return and volatility. The add-in of the bond index will coordinate the global correlation of the sample assets.

◆ The selection of the risk-free rate

The risk-free rate used in this paper is a complicated problem because of different criteria all over the world. In U.S., the interest rate on a three-month U.S. treasury bill is often used as the yearly risk-free rate. In Europe, the three-month LIBOR rate can be used as risk-free rate. However, there is no general adoption of the market principle in China; it is hard to find a proper rate for risk-free rate. Sometimes the bond rates are used, and sometimes one-year deposit interest rates are used. Viewing the historical data from 1990-2007, the rates fluctuate between 2% and 10% for one-year deposit interest rate. Furthermore, the three-month U.S. Treasury bill rates and three-month LIBOR rates are listed in Figure 3

Figure 3 Three-month U.S. Treasury bill rates and three-month LIBOR rates

20 Sources come from finance.yahoo.com

21 Sources come from the data of People’s Bank of China.

22 Sources come from Financial Forecast Centre in U.S.
The reason why we offer the data from 2007-2009 rather than the research period is that this paper is to help the Chinese investors with the advice of the investment in the future but not past 20 years. Considering that situation, it is better to offer them the more current risk-free rate to make a suggestion. We can see from the Figure 3 that the current risk-free rate fluctuates from 1% to 5% whatever in the U.S. market or the Europe market. If we have to choose one value from 1% to 5% for the risk-free rate, then 3% is pertinent per year for the both markets.

For synthetically thinking, we choose 3% as the risk-free rate in this paper. For the reason that it is convenient for calculation (the monthly risk-free rate should be 0.25%). Then whatever the risk-free rate in the Chinese market is, it is always helpful to consider making an investment from an international point of view, if there are no clear criteria in China about the risk-free rate.
3.2 Methodology

3.2.1 Portfolio Optimization analysis

The theory of the international diversification study is to trace the portfolio optimization set. This section applies the framework of Markowitz (1952) known as Modern Portfolio Theory. At the base of mean-variance theory, the Excel is used which contains a lot of number of functions that makes it relatively easy to model portfolio optimization.

The purpose of the portfolio optimization is to minimize risk (volatility) for a given return (or maximize return for a given level of risk) by constructing a portfolio, that satisfies this criterion. Usually, two steps should be taken in the process: 1) Tracing out the efficient frontier using a given set of risky assets. 2) Deriving the capital market line (CML) that is tangent to the risky-asset frontier and has intersect in the risk free-rate.

In our study, a case is considered on real market data from five stocks and one bond. The purpose of the model is to find the optimal point of the international portfolio from China’s perspective. When the calculation is finished, we can see the proportion of the different assets of investment.

There are three methods for the portfolio optimization, the Black’s method, the Lagrange’s method and the Solver method. The three methods have different levels of the complexity of the calculation and the accuracy of the calculation. Besides, the short sales restriction does not exist in the Black’s and Lagrange’s methods. Because the solution to the problem allows negative portfolio proportions, when $X_i$ (proportion of the assets) < 0, this is equivalent to the following assumptions:

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23. Investors can sell securities they do not own. This type of trade is referred to as a short sale. When an investor short sells a security, a security is physically sold. (Elton, 2003)
The \textit{ith} security is sold short by the investor.

The proceeds from the short sales become immediately available to the investors.

Reality is considerably more complicated than this academic model of short sales. In particular, it is rare for all of the short-sales proceeds to become available to the investor at the time of investment, since brokerage houses typically escrow some or even all the proceeds. It may also be that the investor is completely prohibited from making any short sales (indeed, most small investors seem to proceed on the assumption that short sales are impossible).\footnote{Sources come from Simon Benninga (2000)}

When the short-sales is not allowed, the Solver method can be used. The Solver method is to use the excel tools-\textbf{Solver} which is the most intuitive and as such also the method that has least requirements to the reader’s knowledge of the matrix algebra and econometric techniques. For that reasons, this article will focus on discussing the results of the Black’s and Lagrange’s method and test the results of the Solver method when short-sales is allowed and not allowed.

The objection function of the three models is:

\[
\text{Min}_{(x_1,x_2,\ldots,x_n)} \sigma_p^2 = X' \left( \sigma_{ik} \right) X
\]

And the constraints are:

\[
\text{Min}_{(x_1,x_2,\ldots,x_n)} \sigma_p^2 = X' \left( \sigma_{ik} \right) X \text{ with the constraints: } \sum \sqrt{X_i \cdot \text{E}(R_i)} = \text{E} (R_p)
\]

(portfolio return) and \( \sum X_i = 1 \) (The sum of portfolio weights equals 1)

Especially, when using the Solver method the value \( X_i \) should be between 0 and 1.
Where $X_i$ is the portfolio weights, $X$ is the portfolio weights matrix, $\sigma_{ik}$ is the covariance matrix. The expression states that portfolio variance is minimized by adjusting portfolio weights, the expression is conditioned on the relation between portfolio return and the sum of weighted individual returns and the portfolio weights summing to 1.

**The Black’s method:**

The Black’s method is followed the classical capital asset pricing model (CAPM)-which is based on a risk-free asset and Black’s (1972) zero-beta CAPM. And the following propositions can be proved by the basic calculations of the CAPM.

There are propositions before we discuss the process of the Black’s method:

Proposition 1 Consider that a portfolio of risky assets is a column vector $x$ whose coordinates sum to 1: $x=[x_1, x_2, ..., x_N]$, $x_1 + ... + x_N = 1$; Variable $R$ is the column vector of expected returns of these assets: $R=[E(r_1), E(r_2), ..., E(r_N)]$; $S$ is the $N \times N$ variance-covariance matrix.

Let $c$ be a constant. We use the notation $R-c$ to denote the following column vector: $R-c=[E(r_1)-c, E(r_2)-c, ..., E(r_N)-c]$; let the vector $z$ solve the system of simultaneous linear equation $R-c=Sz$. Then this solution produces a portfolio $x$ on the envelope of the feasible set in the following manner:

$$z = S^{-1}(R-c)$$

$$x = \{ x_1, ..., x_N \}$$

Where: $x_i = z_i / (z_1 + ... + z_N)$.

Proposition 2 By a theorem first proved by Black (1972), any two envelope portfolios are enough to establish the whole envelop. Given any two envelope portfolios $x$ and $y$, ...
all envelope portfolios are convex combinations of x and y. This means given any constant a, the portfolio “ax+(1-a)y” is on the envelope of the efficient frontier.\(^{25}\)

Then follow the proposition we can make the process of the Black’s method:

1. Calculation the basic result such as the return, excess return, covariance and correlation of the market data for the next 3 process.
2. Estimate 2 efficient portfolios (X and Y) by Black’s algorithm.
3. Estimate the efficient frontier (all efficient portfolios) by combining portfolio X and Y.
4. Derive the capital market line (CML).

◆ The calculation of the basic data (available for all the three methods)

For the basic data, it is necessary to calculate the returns at first; this is done using the following equation:

\[
R_{i,t} = \ln \left( \frac{P_{i,t}}{P_{i,t-1}} \right) \quad \text{(continuously compounded returns)} \quad (3.1)
\]

Where \(R_{i,t}\) is the returns, \(P\) is the daily index of the stocks or bonds.

Then the Excel function =AVG ( ) can be used to calculate the average return \(R_i\).

After this Excess return should be figured out in order to calculate the covariance and correlation.

\[
\text{Excess return} = R_{i,t} - R \quad \text{Excess return} = R_{i,t} - R_i \quad (3.2)
\]

Where \(R_{i,t}\) is return on asset i at time t and \(R_i\) is the mean return on asset i.

\(^{25}\) Sources come from Black(1972)
Then we can calculate the covariance to use the formula following:

\[ \sigma_{ik} = E \left[ (R_{it}-\bar{R}_i) (R_{kt}-\bar{R}_k) \right] \tag{3.3} \]

where \( \bar{R}_i \) and \( \bar{R}_k \) are mean returns for asset i and k, respectively, and \( R_{it} \) and \( R_{kt} \) are return at time \( t \) on asset i and k, respectively. Notice that the excess return is part of the formula (3.3).

◆ Estimating the X and Y portfolios

In Black’s model portfolio X and Y are the insignificant instruments for helping to derive the efficient frontier. In order to apply the Black’s model, short-sale should be allowed which will be discussed in the next section (Assumption 2).

From proposition 1, the estimation of efficient portfolios takes the below linear equation system as starting point:

\[ E[R]-c = \sigma_{ik}^* Z \tag{3.4} \]

Where \( E[R]-c \) is the vector of expected returns subtracted by a constant \( c \), \( \sigma_{ik} \) is the covariance matrix and \( Z \) is some vector that solves the equation. Solving for \( Z \) yields:

\[ Z = \sigma_{ik}^{-1} [E[R]-c] \tag{3.5} \]

The vector \( Z \) can be considered the weights of assets in the portfolio. Accordingly, the standardized and meaningful weights \( X_i \), can be computed in the following way:

\[ X_i = Z_i / (z_1 + \ldots + z_n) \]

Alternatively \( Y_i = Z_i / (z_1 + \ldots + z_n) \). \hspace{1cm} (3.6)

Where \( X_i \) and \( Y_i \) are the weights of assets in efficient portfolios X and Y. The corresponding expected portfolio returns \( E(R_p) \) and variance \( \text{Var}(R_p) \) can be calculated by using the Excel function.
Generally, the weights in the efficient portfolios X and Y can now be calculated by using (3.6), setting c equal to 0 and some rate which can be considered the risk-free rate, respectively. In the next section, the value of c will be discussed (Assumption 3).

◆ Estimating the efficient frontier

Before we derive the efficient frontier, four concepts should be illustrated. A feasible portfolio is any portfolio whose proportions sum to one. The feasible set is the set of portfolio means and standard deviation generated by the feasible portfolios; this feasible set is the area inside and to the right of the curved line. A feasible portfolio is on the envelope of the feasible set if for a given mean return it has minimum variance. Finally, a portfolio is an efficient portfolio if it maximizes the return given the portfolio variance. The set of all efficient portfolios is called efficient frontier. For a given portfolio whose proportion of assets sum to one should be feasible but might not be efficient. In this paper we need to find the efficient frontier for the portfolio mixed by five stocks and one bond.

After estimating the mean return and variance of the portfolio X and Y, it is naturally to trace the efficient frontier by changing the weights of the portfolio.

The principle is that given two efficient portfolios, all efficient portfolios will be combinations of these two. Given an arbitrary constant X (the weight in portfolio X) and 2 efficient portfolios X and Y, this means, that the following equation will produce an efficient portfolio: \( w \times X + (1 - w) \times Y \)

All efficient portfolios can now be calculated by adjusting the constant w. this provides a very easy way of tracing out the efficient frontier. When efficient portfolios are generated, the corresponding expected returns \( E[R_p] \) and variances \( \text{Var}(R_p) \) can be calculated using the following equations, respectively:

\[ \text{Sources come from Proposition 2} \]
\[ E[R_p] = w \cdot E[R_x] + (1-w) \cdot E[R_y] \]  
\[ \text{Var}(R_p) = w^2 \cdot \sigma_x^2 + (1-w)^2 \cdot \sigma_y^2 + 2w(1-w) \cdot \text{cov}(x,y) \]

Where \( \text{cov}(x,y) \) is the covariance of the portfolio \( x \) and portfolio \( y \).

- Deriving the capital market line

The results of the portfolio are now helpful to derive the efficient frontier. The efficient frontier curve indicates the property of the portfolio. All the calculated portfolios are now reviewed for their “Reward-To-Variability Ratio”. This indicator measures the risk premium per unit of risk and is calculated by the following formula:

\[ \text{RTVR} = \frac{E[R_p] - R_F}{\sigma_p} \]  

The reason why investors are interested in this indicator is because the RTVR yields the highest possible return given the associated risk. Accordingly, the optimal portfolio can be found where RTVR is greatest, and is called the market portfolio. When a risk-free asset is introduced, one can construct a new efficient frontier consisting of combination of the market portfolio. Imposing a straight line between the risk-free asset and the market portfolio yields the “capital market line”. The capital market line has intersect in \( R_F \) is tangent to the market portfolio (MP or TP for Tangent Portfolio) and has the slope of RVTR. At the same time the proportion of different asset can be calculated by the (3.6) whatever at the tangent point or the whole efficient frontier.

**The Lagrange’s model:**

The optimization problem is similar to the one used in Black’s method. The object function is also \( \min i x_i, x_2, \ldots, x_n | \sigma_p^2 = X'(\sigma_{ii})X, \)  
(3.10). The Lagrange’s method applies an analytical approach to solve (3.10). Here, we input a Lagrange multiplier called \( \lambda \). Thus, the Lagrange’s equation can be written as:
\[ L(X, \lambda) = X' (\sigma_{ik}) X - \lambda (R'X - \mu) \] (3.11)

In this case \( X \) represents a vector of portfolio weights. \((\sigma_{ik})\) represents the covariance matrix, \( \lambda \) is a vector which holds the Lagrange’s multipliers, \( R \) is a \([1*2]\) matrix that contains the expected returns in the first column and “1”s in the second column. The vector \( \mu \) has dimensions \([2*1]\) with mean portfolio return in the first row and one in the second row.

The part \((R'X - \mu)\) equals to zero depending on the assumption given above. So the \( L(X, \lambda) \) equals to the object function.

The basic results of the data are following the Black’s method’s result such as the return, the excess return and the variance, correlation.

Differentiating (3.11) yields

\[ X = [(\sigma_{ik})^{-1} * R * A^{-1} * \mu] \] (3.12)

Where the matrix \( A \) is given by the following \([2*2]\) matrix:

\[ A = [R' (\sigma_{ik})^{-1} R] \] (3.13)

Based on the formulas, the portfolio weights can be computed. To track the efficient frontier it is required to specify the target return in the vector \( \mu \) (for example 0.5, 0.75, 1.00…..) within a suitable range and solve the Lagrange’s equation again. The Capital Market Line and the RTVR is calculated using the same technique as were used in the Black’s Model.

**The Solver method:**

The object function is the same but the constraints changes because of the short sales are not allowed which means that the value of the \( X_i \) should be larger than 0 but cannot exceed 1. And the summary of the \( X_i \) should equal to one.
The basic data are following the Black’s method’s result of the data such as the return, the excess return and the variance, correlation.

Before running the Solver, it is necessary to set the formulation of the variance and the return. Otherwise the Solver cannot run the simulation. When running the Solver, inputs are in accordance with the optimization problem specification in object function \( \min_{x_1, x_2, \ldots, x_n} \sigma_p^2 = X'(\sigma_{ik}) X \) and the constraints \( X_i > 0 \) and \( X_1 + X_2 + \ldots + X_i = 0 \). The Solver is running the simulation of the different weights of the assets to get the different portfolio variances when changing the target return.

Now, a series of portfolios can be constructed to trace out the efficient frontier. The capital market line is estimated as in the Black’s method. It is easy to plot the CML line and the efficient frontier in a scatter plot like demonstrated in Black’s method.

Furthermore, the Solver method can make a calculation when the short sales are allowed. However, the efficient frontier remains the same as the Black method, which can be helpful to check the result of the Lagrange’s method and the Black method.

**Combining optimal portfolio with risk-free assets**

In this paper, there are three different kinds of special points of the portfolio we should compare with the single market return and risk.

The **optimal risky portfolio** is the portfolio tangent to the capital market line which gives the best tradeoff risk against the return which can be derived by the point of the intersection of the capital market line and the efficient frontier.

The **minimum variance portfolio** is the portfolio with the global minimum variance which can be derived by the efficient frontier.

The optimal risky portfolio and the minimum variance portfolio can be derived by the three different methods mentioned above.
The **optimal overall portfolio** combines investments in the risk-free asset and in the risky portfolio. The actual proportion assigned to each of these two investments is determined by the degree of risk aversion characterizing the investor which is the tangent point intercourse by the capital market line and the investors’ indifferent curve.

The market portfolio is the optimal portfolio but an investor might have different risk preference than the market. The final step of the portfolio selection process is choosing the optimal investment for a particular investor to reflect his risk preferences.

The optimal investment choice can be identified by introducing utility theory. In portfolio optimization expected return risk space is considered implying a trade-off between risk and return. Therefore, if the investor is only concerned about maximizing return for a given level of risk, his utility function can easily be drawn in the same expected return risk space. That is, for all levels of utility the investor’s corresponding indifference curve\(^{27}\) can be derived, given a specific level of risk-aversion.

Consider the graphical illusion of the optimization. If the investor’s utility function is quadratic, the indifference curve can be shifted towards and to the west until the capital market line is exactly tangent to the indifference curve. At the tangency point, the optimal portfolio choice can be found.

The utility function is:

\[
U(R_p, \sigma_p) = R_p - \frac{1}{2} \lambda \sigma_p^2 
\]  

(3.14)

\[
R_p = X \times R_x + (1-X) \times R_F 
\]  

(3.15)

\[
\sigma_p = X^2 \times \sigma_{tp}^2 + (1-X) \times 0 = X^2 \times \sigma_{tp}^2 
\]  

(3.16)

\(^{27}\) In microeconomic theory, an indifference curve is a graph showing different bundles of goods, each measured as to quantity, between which a consumer is indifferent. (Wilson, Goddard,2005)
Inserting (3.16) and (3.17) into (3.15) yields:

\[
U(R_p, \sigma_p) = X^2 \lambda X^2 \sigma^2_{tp} + (1-X)^2 \lambda X^2 \sigma^2_{tp} + 1/2 \lambda X^2 \sigma^2_{tp} \quad (3.17)
\]

The object is to maximize the utility by adjusting the weight in the tangent portfolio \( X \) and the risk-free asset \( (1-X) \) respectively:

\[
\text{Max } U(R_p, \sigma_p)
\]

\[
dU/dX = R_x - R_F - 2*1/2*\lambda X^2 \sigma^2_{tp} = 0 \quad (3.18)
\]

Then we got \( X = (R_x - R_F) / \lambda \sigma^2_{tp} \) \quad (3.19)

Using this result \( X, (1-X), R_p, \sigma_p, \) and \( U \) can easily be calculated.

In order to place the utility optimization problem in mean-variance space, the utility function must be rewritten to express return as a function of standard deviation, for a given level of risk aversion and corresponding utility. Rewriting (3.14) yields:

\[
R_p = U(R_p, \sigma_p) + 1/2 \lambda \sigma^2_{tp} \quad (3.20)
\]

Given different value of the \( \sigma_p \), we can derive the different expected return with a given risk-aversion. The indifference curve can be plotted with the capital market line using the scatter plot diagramming function in excel.

However, it is necessary to find an appropriate \( \lambda \) which is the risk-aversion for the calculation. For the risk-neutral investors, \( \lambda = 0 \). Higher levels of risk aversion are reflected in larger value for \( \lambda \).

**3.2.2 Home bias analysis**

In the three methods above, the weights of the different assets can be calculated when the portfolio optimization is reached. However, these weights are idealized which is not existed in the real world.
In this section we will use the relevant set-up and testing framework as devised by Tullis and Clarke (1999) without considering the currency exposure.

The utility function is:

\[ U(R_p, \sigma_p) = R_p - \frac{1}{2} \theta \sigma_p^2 \quad (3.21) \]

\[ R_p = X R_d + (1-X) R_i \quad (3.22) \]

\[ \sigma_p^2 = X^2 \sigma_d^2 + (1-X)^2 \sigma_i^2 + 2X(1-X) \rho_{id} \sigma_d \sigma_i \quad (3.23) \]

Where \( R_d \) and \( \sigma_d \) is the return and the variance of the domestic market which can be calculated from the basic data (the return and the variance of the Shanghai Exchange Stock market). \( R_i \) and \( \sigma_i \) is the return and the variance of the international market which can be calculated from the three methods (the return and the variance of the international portfolio). \( \rho_{id} \) is the correlation of the domestic market and international market. The \( \theta \) is a parameter to show the investors preference. The \( \theta \) could be the reason which can lead to the home bias. The signification of the \( \theta \) should be discussed in the section of the analysis of the empirical results.

Input (3.22) and (3.23) into (3.21), then we got:

\[ U(R_p, \sigma_p) = X R_d + (1-X) R_i - \frac{1}{2} \theta [ X^2 \sigma_d^2 + (1-X)^2 \sigma_i^2 + 2X(1-X) \rho_{id} \sigma_d \sigma_i ] \quad (3.24) \]

In order to maximum \( U(R_p, \sigma_p) \)

We need to find \( \frac{d\theta}{dX} = 0 \)

That is \( R_d - R_i - \frac{1}{2} \theta [ 2X \sigma_d^2 + (2X-2) \sigma_i^2 + 2X(1-X) \rho_{id} \sigma_d \sigma_i ] = 0 \quad (3.25) \)

So \( X = \frac{(R_d - R_i - 0 \rho_{id} \sigma_d \sigma_i + 0 \sigma_i^2)}{(0 \sigma_d^2 + 0 \sigma_i^2 - 2 \rho_{id} \sigma_d \sigma_i)} \) \( (3.26) \)

If the \( R_d, R_i, \sigma_d, \sigma_i \) are the known parameters and the \( \theta \) is unknown the \( X \) can be written as:
\[ X = (R_d - R_i) \left( 0 \sigma_d^2 + 2 \sigma_i^2 - 2 \rho_{id} \sigma_d \sigma_i \right) + \left( \sigma_i^2 - 2 \rho_{id} \sigma_d \sigma_i \right) / \left( \sigma_d^2 + \sigma_i^2 - 2 \rho_{id} \sigma_d \sigma_i \right) \]  \hspace{1cm} (3.27)

When the \( \theta \) is extreme as \(+\infty\), the \( X \) has the minimum value which is \( \left( \sigma_i^2 - 2 \rho_{id} \sigma_d \sigma_i \right) / \left( \sigma_d^2 + \sigma_i^2 - 2 \rho_{id} \sigma_d \sigma_i \right) \) means the intervals of the \( X \) should be \( \left[ \left( \sigma_i^2 - 2 \rho_{id} \sigma_d \sigma_i \right) / \left( \sigma_d^2 + \sigma_i^2 - 2 \rho_{id} \sigma_d \sigma_i \right), 100\% \right] \) \hspace{1cm} (3.28).

\( X \) is the weights of the domestic market, the minimum value of the \( X \) should be constructive to research of the degree of the home bias.

In order to use the asset allocation framework outlined above, the estimation of the expected return and risk of the domestic and international market is required. Furthermore, the correlation of the domestic market and international market is also required.

At first it is really hard to estimate the return and risk of the domestic and international market. However, the return and risk should not be accurate in this paper, because we just afford the description of the home bias in China, and the extreme weight\(^{28}\) of the domestic market is only related to the variance of the markets. What we are required to do is just to assume the Chinese investors equally engaged in the main international stock and bond market. For the domestic market, it is assumed that they invest equally in the stock and bond market too. Again, the transaction cost is ignored.

**Table 4 From international perspective, the return and risk is estimated to be\(^{29}\):**

<table>
<thead>
<tr>
<th></th>
<th>Return</th>
<th>Risk</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\(^{28}\) \( \sigma_i^2 - 2 \rho_{id} \sigma_d \sigma_i \) / \( \sigma_d^2 + \sigma_i^2 - 2 \rho_{id} \sigma_d \sigma_i \)

\(^{29}\) The data we used are from “Modern Portfolio Theory and Investment Analysis” pp.269-272. The Average annual returns and risks from 1990-2000 are listed.
From China’s perspective, the return and risk is estimated to be:\(^30\) :

<table>
<thead>
<tr>
<th></th>
<th>Return</th>
<th>Risk</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stock</td>
<td>24.16%</td>
<td>133.92%</td>
</tr>
<tr>
<td>Bond</td>
<td>11.45%</td>
<td>13.04%</td>
</tr>
</tbody>
</table>

After that we need to combine the stock and bond’s return and risk together will equal weight. When calculating the combined risk, we need to know the correlation between the stock and bond.\(^31\) Thomas (2008) makes a statement there is a very volatile correlation between the stock and the bond markets. The coefficient swings between the positive zone and the negative zone. The average coefficient is around zero, implying the two markets are almost independent. So it is assumed that the correlation of the stock and bond market is zero whatever in the domestic or international market. Then we have \(R_d, R_i, \sigma_d, \sigma_i\) which are 17.81%, 9.04%, 73.48% and 14.76%.\(^32\)

In (3.28), there is one more parameter \(\rho_{id}\) which is required to be figure out. In the portfolio optimization problem, we can see that the all the coefficients between Shanghai Stock Exchange markets and each of the international market:

<table>
<thead>
<tr>
<th>Coefficients</th>
<th>0.021</th>
<th>0.039</th>
<th>-0.004</th>
<th>0.047</th>
<th>0.123</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.045</td>
</tr>
</tbody>
</table>

---

\(^{30}\) The data and calculation are from Appendix C

\(^{31}\) Combined risk= \(1/2*(\sigma_1^2+\sigma_2^2)+1/4\sigma_1\sigma_2\rho_{12}\)

\(^{32}\) The combined return=\(1/2*(R_1+R_2)\), if the correlation of stock and bond is zero the combined risk= \(1/2*(\sigma_1^2+\sigma_2^2)\)
At last, the extreme value of the weights of the domestic market can be calculated. Furthermore, the discussion of the results is in Section 5.

4. Assumptions

Assumption 1: The risk-free rate remains the same.

It is mentioned that in the part of the data description, the monthly risk-free rate is the 0.25% which should remain the same when used in this paper. Obviously, during the 1990-2007 the risk-free rate must fluctuate all the time. However, the result of the paper is regarded as a guidance of the investment for Chinese investor in the future. The constant and realistic risk-free rate provides the persuasion for them when considering the international portfolio they made.

Assumption 2: The transaction cost is ignored in this paper.

The transaction exposure includes the operating exposure, the translation exposure and tax exposure. Operating exposure is the change in expected future cash flows arising from an unexpected change in exchange rates. Translation exposure is the Changes in reported owner’s equity in consolidated financial statements caused by a change in exchange rates. And the tax problem varies by country which brings so many possibilities. But all these transaction is not taken into consideration in this paper. This paper studies international diversification from a China’s point of view. The transaction cost here is supposed to decrease the return for the investment. First, if Chinese investors want to engage into the foreign stock market, the Chinese Yuan they have should be translated into the foreign currency. Then the translation fee generates which can increase the cost. Secondly, the exchange rate might be changed during the investment period, which means appreciation or depreciation. Some of the investors attempt to manage their currency exposure through hedging. Hedging is the taking of a position, acquiring a cash flow, an asset, or a contract (including a forward contract) that will rise (fall) in value and offset a fall (rise) in the value of an existing position.
Hedging therefore protects the owner of the existing asset from the loss.

In this paper, the calculation of the monthly return is based on the various domestic indexes. So average return is measured by their own domestic currencies. For example, if the Dow-Jones index is evaluated from point of view of Chinese investors, the average return might be decreased. For that the Chinese investors are required to pay for the transaction cost and the afford risk caused by the exchange rate. So this might be unfair for international market compared to the Chinese domestic market.

We have to make the assumption that the transaction cost does not exist in this paper, which offers a balance choice and it is much easier when constructing the model.

Assumption 3: Short sale is allowed when using the Black’s method and the Lagrange’s method.

In the result of the research, the negative proportion maybe exists. In the Black model, this is allowed. Because the negative value indicates that the asset should be not kept but sold. This method is more accurate and automatic method of determining the efficient frontier than other methods. And the consideration of the short sale is close to the practice. This method requires thorough knowledge and understanding of matrix algebra and econometrics. It is more accurate and automatic method of determining the efficient frontier comparing with the method denying the short sale.

Assumption 4: A risk-free assets exists.

In the last section, in the Black’s optimization model, when we estimate the efficient portfolio Y, the constant c has some interesting interpretation. The obvious choice for c is the risk-free rate if riskless assets exist. Alternatively, if no riskless asset, c has close resemblance to the zero-beta rate called $R_z$ (from the zero-beta CAPM). In fact, c equals $E[R_z]$ if the estimated market (tangent) portfolio is efficient. In this paper, the case where a risk-free asset exists is considered for simplicity. In the following, c is used in the calculation techniques and $R_f$ is used for interpretation purposes and economic
Assumption 5: The appropriate value of the risk-aversion when combining the optimal portfolio with the risk-free asset.

From the equation $X = \frac{(R_x - R_f)}{\lambda \cdot \sigma^2_{tp}}$ (3.19), we can see that for the given the portfolio and risk-free rate, only the change of the value of the risk-aversion can influence the weights of the assets (risk-free assets and given portfolio).

Inserting different $\lambda$ into the equation with the optimal risk portfolio calculated by the three methods, we can conclude that if $\lambda > 1$, the proportion of the risk-free asset is larger than 90%. However, Lilian and F. Wu’s (2008) finding indicate that Chinese institutions have a tendency to demand for assets that have high earnings per share, high idiosyncratic risk, and large price.\(^{33}\) Due to the tendency, we should not use the $\lambda$ whose value is bigger than 1. For easy calculation, $\lambda = 0.5$ is used in this paper.

5. **Analysis and discussion with empirical results**

5.1 Report of the basic data

Table 5 The monthly returns and variances of the single market are:

<table>
<thead>
<tr>
<th>Asset Description</th>
<th>Return</th>
<th>Variance</th>
</tr>
</thead>
<tbody>
<tr>
<td>Shanghai Stock Exchange (SSE)</td>
<td>1.82%</td>
<td>205.68%</td>
</tr>
<tr>
<td>FTSE 100</td>
<td>0.54%</td>
<td>14.66%</td>
</tr>
<tr>
<td>Dow Jones Industrial Average (DJIA)</td>
<td>0.79%</td>
<td>15.68%</td>
</tr>
<tr>
<td>CAC 40</td>
<td>0.64%</td>
<td>27.07%</td>
</tr>
<tr>
<td>DAX</td>
<td>0.74%</td>
<td>28.70%</td>
</tr>
<tr>
<td>FTSE Global Government Bonds (FTSE GG)</td>
<td>0.40%</td>
<td>2.44%</td>
</tr>
</tbody>
</table>

**Covariance and correlation matrix:**

<table>
<thead>
<tr>
<th></th>
<th>SSE</th>
<th>FTSE100</th>
<th>DJIA</th>
<th>CAC</th>
<th>DAX</th>
<th>FTSE GG</th>
</tr>
</thead>
<tbody>
<tr>
<td>SSE</td>
<td>205.68%</td>
<td>1.13%</td>
<td>2.26%</td>
<td>-0.36%</td>
<td>3.64%</td>
<td>2.76%</td>
</tr>
<tr>
<td>FTSE100</td>
<td>1.13%</td>
<td>14.66%</td>
<td>10.90%</td>
<td>14.89%</td>
<td>13.68%</td>
<td>0.17%</td>
</tr>
<tr>
<td>DJIA</td>
<td>2.26%</td>
<td>10.90%</td>
<td>15.69%</td>
<td>13.44%</td>
<td>13.47%</td>
<td>-0.12%</td>
</tr>
<tr>
<td>CAC</td>
<td>-0.36%</td>
<td>14.89%</td>
<td>13.44%</td>
<td>27.07</td>
<td>22.00%</td>
<td>-0.40</td>
</tr>
<tr>
<td>DAX</td>
<td>3.64%</td>
<td>13.68%</td>
<td>13.47%</td>
<td>22.00%</td>
<td>28.71%</td>
<td>0.05%</td>
</tr>
<tr>
<td>FTSE GG</td>
<td>2.76%</td>
<td>0.17%</td>
<td>-0.12%</td>
<td>-0.40%</td>
<td>0.05%</td>
<td>2.44%</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>SSE</th>
<th>FTSE100</th>
<th>DJIA</th>
<th>CAC</th>
<th>DAX</th>
<th>FTSE GG</th>
</tr>
</thead>
<tbody>
<tr>
<td>SSE</td>
<td>1</td>
<td>0.020624</td>
<td>0.039752</td>
<td>-0.00479</td>
<td>0.047351</td>
<td>0.123285</td>
</tr>
<tr>
<td>FTSE100</td>
<td>0.020624</td>
<td>1</td>
<td>0.719067</td>
<td>0.747577</td>
<td>0.666791</td>
<td>0.028453</td>
</tr>
<tr>
<td>DJIA</td>
<td>0.039752</td>
<td>0.719067</td>
<td>1</td>
<td>0.652415</td>
<td>0.634598</td>
<td>-0.02016</td>
</tr>
<tr>
<td>CAC</td>
<td>-0.00479</td>
<td>0.747577</td>
<td>0.652415</td>
<td>1</td>
<td>0.789368</td>
<td>-0.04876</td>
</tr>
<tr>
<td>DAX</td>
<td>0.047351</td>
<td>0.666791</td>
<td>0.634598</td>
<td>0.789368</td>
<td>1</td>
<td>0.006242</td>
</tr>
<tr>
<td>FTSE GG</td>
<td>0.123285</td>
<td>0.028453</td>
<td>-0.02016</td>
<td>-0.04876</td>
<td>0.006242</td>
<td>1</td>
</tr>
</tbody>
</table>
5.2 Benefit from international diversification

From the test results of the sample data, it is certain that Chinese investors could receive benefit from the international diversification. They should engage in multiple international markets rather than the single domestic market.

In the following table, we compared that the single market’s data with the portfolio combined by multiple markets data.

Table 6 The result of the monthly returns and variances of the combined portfolio from multiple methods\(^\text{34}\) are:

<table>
<thead>
<tr>
<th></th>
<th>The optimal risky portfolio</th>
<th>Portfolio with global minimum variance</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Return</td>
<td>Variance</td>
</tr>
<tr>
<td>The black model</td>
<td>0.65%</td>
<td>4.03%</td>
</tr>
<tr>
<td>The Lagrange model</td>
<td>0.65%</td>
<td>4.03%</td>
</tr>
<tr>
<td>The Solver (short sales are not allowed)</td>
<td>0.63%</td>
<td>3.80%</td>
</tr>
</tbody>
</table>

\(^{34}\) The efficient frontier calculated by the three methods are shown in the Appendix A
The tangent portfolio is the portfolio with the highest Sharpe ratio. Investments on the capital market line connecting the risk-free investment and the tangent portfolio provide the best risk and return tradeoff available to an investor: As a result, the tangent portfolio is also referred to as the efficient portfolio. Furthermore, the efficient frontier shows the global minimum variance, but the point does not offer the best tradeoff, we can see from the result that all the GMVs (global minimum variance) offer the lower variance (2.04%-2.07% which is lower than all the variances of the single market) but pertinent return (0.45%-0.67% which is higher than half of the return of the single market) which means the Chinese investors should engage into the international portfolio rather than stay in the single domestic portfolio.

In conclusion, the international portfolio will be benefit for the Chinese investors because of the lower risk they take when receiving the same return. That is why in recent years; the Chinese securities are becoming more and more interested in the international market when the transaction cost (including the factor about tax, exchange rate and operating cost) is lower than before after China’s entering into the World Trade Organization.

5.3 Consider the international portfolio with the investor’s preference

One’s allocation decision will influence by his degree of risk aversion. Now we have optimal risky portfolio, we can use the concept of the optimal overall portfolio allocation funds between risky portfolio and risk-free asset.

Table 7 The monthly returns and risks of the optimal overall portfolio calculated by multiple methods with a risk-aversion

| The Solver (short sales are allowed) | 0.65% | 4.03% | 0.45% | 2.04% |
equals to 0.5 and risk-free rate equals to 0.25% are:

<table>
<thead>
<tr>
<th>Method</th>
<th>Return</th>
<th>Variance</th>
</tr>
</thead>
<tbody>
<tr>
<td>The black model</td>
<td>0.33%</td>
<td>0.40%</td>
</tr>
<tr>
<td>The Lagrange model</td>
<td>0.33%</td>
<td>0.40%</td>
</tr>
<tr>
<td>The Solver (short sales are not allowed)</td>
<td>0.33%</td>
<td>0.40%</td>
</tr>
<tr>
<td>The Solver (short sales are allowed)</td>
<td>0.33%</td>
<td>0.40%</td>
</tr>
</tbody>
</table>

The proportion of the international portfolio and risk-free assets are:

<table>
<thead>
<tr>
<th>Method</th>
<th>International portfolio</th>
<th>Risk-free asset</th>
</tr>
</thead>
<tbody>
<tr>
<td>The black model</td>
<td>20%</td>
<td>80%</td>
</tr>
<tr>
<td>The Lagrange model</td>
<td>20%</td>
<td>80%</td>
</tr>
<tr>
<td>The Solver (short sales are not allowed)</td>
<td>20%</td>
<td>80%</td>
</tr>
<tr>
<td>The Solver (short sales are allowed)</td>
<td>20%</td>
<td>80%</td>
</tr>
</tbody>
</table>

5.4 Explanation about the three methods.

5.4.1 The same result derived from the three methods when
short-sales is allowed.

From the result which is stated above, we can see that the three methods can derive the same result even if they track the efficient frontier from different angles. This proves that all the three methods are unbiased.

It is clear that the Lagrange’s method and the Solver method are unbiased due to the given process of these two methods in the section of “Methodology”.

The Lagrange’s method inputs the Lagrange’s multiplier \( \lambda \) to transform the object function \( \min_{x_1,x_2,\ldots,x_n} \sigma_p^2 = x' \sigma x \) into \( L(X, \lambda) = x' \sigma x - \lambda (R'X - \mu) \). Where the part \( (R'X - \mu) \) equals to zero. To track the efficient frontier it is required to specify the target return in the vector \( \mu \) (for example 0.5, 0.75, 1.00…..) within a suitable range and solve the Lagrange’s equation again. After that different tracked points on the efficient frontier can make up the whole efficient frontier. During the process of the Lagrange’s method, we can see that only the matrix algebra tricks is use to differentiate the transformation of the objection function and then find the weights of different assets in the object function.

Another way to compute points along the efficient frontier we use Excel Solver. When using the Solver, it is necessary to enter the cell of the target function and the all the constraints: first that the sum of the weight equals to 1; and second, that the portfolio expected return equals target return. The Solver beeps when it has found a solution and automatically alters the portfolio weight to show the makeup of the efficient portfolio. It adjusts the entries in the border-multiplied covariance matrix to reflect the multiplication by these new weights, and it shows the mean and variance of the optimal portfolio-the minimum variance portfolio with the target mean. The Solver method is an excel function to help to solve the differential algebraic equation problem.

From the explanation above, we can see that the Lagrange’s method and the Solver method are just differentiating the object function. However, the Black’s method is
based on the Capital Asset Pricing Model. It tracks the efficient frontier from another point of view. In the next section we will discuss that the proof of proposition for the Black’s method which makes itself unbiased.

**The Black’s method is unbiased**

Proposition 1 Consider that a portfolio of risky assets is a column vector \( x \) whose coordinates sum to 1: \( x = [x_1, \ldots, x_N] \), \( x_1 + \ldots + x_N = 1 \); Variable \( R \) is the column vector of expected returns of these assets: \( R = [E(r_1), E(r_2), \ldots, E(r_N)] \); \( S \) is the \( N \times N \) variance-covariance matrix.

Let \( c \) be a constant. We use the notation \( R-c \) to denote the following column vector: \( R-c = [E(r_1)-c, E(r_2)-c, \ldots, E(r_N)-c] \); let the vector \( z \) solve the system of simultaneous linear equation \( R-c = Sz \). Then this solution produces a portfolio \( x \) on the envelope of the feasible set in the following manner:

\[
Z = S^{-1}\{R-c\}
\]

\[
x = \{x_1, \ldots, x_N\}
\]

Where: \( x_i = z_i / (z_1 + \ldots + z_N) \).

Proof: A portfolio \( x \) is on the envelope of feasible set of portfolio if and only if it lies on the tangency of a line connecting some point \( c \) on the y-axis to the feasible set. Such a portfolio must either maximize or minimize the object function:

\[
\theta = R_p \cdot c / \sigma_p ,
\]

Subject to the constraint: \( x_1 + \ldots + x_N = 1 \)

This is a constrained maximization problem. It can be solved by the Lagrangian multipliers. The constraint could be substituted into the objective function and the objective function maximized as in an unconstrained problem. We can write \( c \) as \( c \) times
1. Thus we have:

\[ c = 1 \cdot c = (\sum_{i=1}^{N} X_i) \cdot c = \sum_{i=1}^{N} (X_i \cdot c) \]

Making this substitution \( c \) in the objective function \( (\theta = \frac{R_p - c}{\sigma_p}) \) and stating the expected return and standard deviation of return in the general form yields:

\[ \theta = \left[ \sum_{i=1}^{N} X_i \cdot (R_p - c) \right] \cdot \left[ \sum_{i=1}^{N} X_i^{2} \cdot \sigma_i^2 + \sum_{i=1}^{N} \sum_{j \neq i}^{N} X_i \cdot X_j \cdot \sigma_{ij} \right]^{1/2} \]

In calculus it is shown that to find the maximum of a function you take the derivative with respect to each variable and set it equal to zero. The solution to the maximum problem involves finding the solution of the equation: \( \frac{d\theta}{dx_n} = 0 \).

In the Appendix B we show that:

\[ \frac{d\theta}{dx_n} = -(\rho x_1 \sigma_{11} + \rho x_2 \sigma_{21} + \ldots + \rho x_i \sigma_{1i} + \ldots + \rho x_{N-1} \sigma_{N-1,i} + \rho x_N \sigma_{Ni}) \cdot \frac{R_p - c}{\sigma_p} = 0 \]

Where \( \rho \) is a constant.\(^{35}\) Noted that each \( x_i \) is multiplied by a constant \( \rho \). Define a new variable \( z_i = \rho x_i \). Substituting the \( z_i \) for \( \rho x_i \) and moving the variance covariance terms to the right-hand side of the equality yields:

\[ R - c = z_1 \sigma_{11} + z_2 \sigma_{21} + \ldots + z_i \sigma_{1i} + \ldots + z_{N-1} \sigma_{N-1,i} + z_N \sigma_{Ni} \]

Which is \( R - c = Sz \)

The \( x_i \) are the fraction to invest in each security, and the \( z_i \) are proportional to this

\(^{35}\) The constant is equal to \( \sum_{i=1}^{N} X_i \cdot (R_p - c) / \sum_{i=1}^{N} X_i^{2} \cdot \sigma_i^2 + \sum_{i=1}^{N} \sum_{j \neq i}^{N} X_i \cdot X_j \cdot \sigma_{ij} \)
faction. So the optimum proportion to invest in the asset k is \( x_i \), where

\[
x_i = \frac{z_i}{z_1 + z_2 + \ldots + z_n}
\]

Proposition 2 By a theorem first proved by Black (1972), any two envelope portfolios are enough to establish the whole envelope. Given any two envelope portfolios x and y, all envelope portfolios are convex combinations of x and y. This means given any constant a, the portfolio “ax+(1-a)y” is on the envelope of the efficient frontier.

Proof: Let x and y be portfolio on the envelope. By proposition 1, it follows that there exist two vectors \( z_x \) and \( z_y \) and two constant \( c_x \) and \( c_y \) such that

\[
x \text{ is the normalized to unity vector of } z_x, \text{ that is } x_i = \frac{z_i}{z_1 + z_2 + \ldots + z_n} \text{ and } y \text{ is the normalized-to-unity vector of } z_y.
\]

\[
R_p - c_x = S z_x \quad \text{and} \quad R_p - c_y = S z_y
\]

Furthermore, since z maximizes the ratio \( z(R-c)/\sigma^2(z) \), it follows that any normalization of z also maximizes this ratio. With no loss in generality, therefore, we can assume that z sums to 1. It follows that for any real number “a” the portfolio \( az_x+(1-a)z_y \) solves the system \( R- [ac_x+(1-a)c_y] = S z \). This result proves our claim.

5.4.2 The different results derived by the Solver method (when the short-sales is allowed or not)

◆ Why the Black’s method cannot be used when the short-sales is not allowed?

Recall the Black’s method discussed in the methodology section, it was enough to find two efficient portfolios in order to determine the whole efficient frontier. When we impose short-sales restrictions, this statement is no longer true. In the case of the short-sales is not allowed, the determination of the efficient frontier required the
plotting of a large number of points. This is the reason why a VBA program is required to repeatedly apply the Solver and put the solutions in a table.

◆ The relationship between the two efficient frontiers

Figure 4 Comparing two efficient frontiers (short-sales allowed and not allowed)\textsuperscript{36}

![Portfolio Optimization](image)

It is obvious that in the figure 4, the efficient frontier with short-sales dominates the efficient frontier without short-sales, since the short-sales restriction imposes an extra constraint on the maximization problem. That is to say some points on the two efficient frontiers coincide when the weights of the asset are larger than zero at the same time. And the reason is when the weights are all larger than zero, the Solver function is not necessary to coordinate the results with the limitation that $X_i > 0$ which makes the limitation the same as the short-sales exist.

\textsuperscript{36} All calculated by the Solver.
6. Home bias Analysis

6.1 The weights calculated by the portfolio optimization method

Table 8 The proportions of the different assets when “the optimal risky portfolio” is reached are:

<table>
<thead>
<tr>
<th></th>
<th>The black method</th>
<th>The Lagrange’s method</th>
<th>The Solver method (short sales not allowed)</th>
<th>The Solver method (short sales allowed)</th>
</tr>
</thead>
<tbody>
<tr>
<td>SSE</td>
<td>-5.92%</td>
<td>-5.92%</td>
<td>6.4%</td>
<td>-5.92%</td>
</tr>
<tr>
<td>FTSE100</td>
<td>19.61%</td>
<td>19.61%</td>
<td>0%</td>
<td>19.61%</td>
</tr>
<tr>
<td>DJIA</td>
<td>-13.55%</td>
<td>-13.55%</td>
<td>34.04%</td>
<td>-13.55%</td>
</tr>
<tr>
<td>CAC</td>
<td>5.25%</td>
<td>5.25%</td>
<td>0%</td>
<td>5.25%</td>
</tr>
<tr>
<td>DAX</td>
<td>-6.74</td>
<td>-6.74</td>
<td>0.23%</td>
<td>-6.74</td>
</tr>
<tr>
<td>FTSE GG</td>
<td>100.35%</td>
<td>100.35%</td>
<td>59.33%</td>
<td>100.35%</td>
</tr>
<tr>
<td>Sum</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

The proportions of the different assets when “Portfolio with global minimum variance” is reached are:

<table>
<thead>
<tr>
<th></th>
<th>The black method</th>
<th>The Lagrange’s method</th>
<th>The Solver method (short sales not allowed)</th>
<th>The Solver method (short sales allowed)</th>
</tr>
</thead>
<tbody>
<tr>
<td>SSE</td>
<td>-1.06%</td>
<td>-1.06%</td>
<td>0%</td>
<td>-1.06%</td>
</tr>
<tr>
<td>FTSE100</td>
<td>7.13%</td>
<td>7.13%</td>
<td>4.55%</td>
<td>7.13%</td>
</tr>
<tr>
<td>DJIA</td>
<td>5.62%</td>
<td>5.62%</td>
<td>8.63%</td>
<td>5.62%</td>
</tr>
<tr>
<td>CAC</td>
<td>3.68%</td>
<td>3.68%</td>
<td>1.94%</td>
<td>3.68%</td>
</tr>
<tr>
<td>DAX</td>
<td>-3.03</td>
<td>-3.03</td>
<td>0%</td>
<td>-3.03</td>
</tr>
</tbody>
</table>
6.2 The estimated proportion from the Tullis and Clarke model

Input value of the $R_d$, $R_i$, $\sigma_d$, $\sigma_i$ into (3.28), we get the intervals of the $X$ which should be $(15.57\%, 100\%)$, and the intervals of the $(1-X)$ should be $(0, 84.43\%)$.

Then input the $R_d$, $R_i$, $\sigma_d$, $\sigma_i$ and $\rho_{id}$ into (3.26), the different weights can be calculated when the value of the $\theta$ is changed, this is shown in the following column.

Table 9 The domestic weights when $\theta$ is changing:

<table>
<thead>
<tr>
<th>$\theta$</th>
<th>Domestic Weights</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.25</td>
<td>0.5671</td>
</tr>
<tr>
<td>0.3</td>
<td>0.4985</td>
</tr>
<tr>
<td>0.5</td>
<td>0.3614</td>
</tr>
<tr>
<td>1</td>
<td>0.2585</td>
</tr>
<tr>
<td>5</td>
<td>0.1763</td>
</tr>
<tr>
<td>50</td>
<td>0.1578</td>
</tr>
<tr>
<td>10000</td>
<td>0.1557</td>
</tr>
</tbody>
</table>

6.3 Comprehension about the results of home bias analysis

From Table 8, it is obvious that when the portfolio optimization is reached, what weights of the assets should be. However, these weights are very hard to be actualized
in the real world for there are so many barriers and much asymmetric information which can make great influence on the investors’ choice. For example, when the short-sales are allowed, the weight of domestic market is negative according to our calculation. Of course, this result may drive Chinese investors pay more attention in the international markets and put nothing in the domestic market. This situation will never happen, the investors are always having better attitude to the native stock market because they think they got more information about domestic market than the international one. This argument can be proved by the Tullis and Clarke model.

In the Tullis and Clarke model, the weights of the domestic market should be between 15.57% and 100%. The signification of the $\theta$ can help us to know the result better. In (3.21), the return is changed to the utility when the investors considering the risk at the same time. The $\theta$ is known as the investor’s preference. The preference may be caused by so many reasons such as the investment barriers, the asymmetric information, the various parity purchase power and so on.

Comparing the domestic market with the international market, the China’s market has higher returns but riskier. The different value of $\theta$ stands for the different preference the investors have. The lower $\theta$ shows that the investors are risk-neutral ones. The larger $\theta$ shows that the investors are risk-aversion. From the signification of the $\theta$, we can see that if an investor has a large $\theta$, he may prefer the international market which is less risky.

However, our analysis shows that the extreme value of the domestic weight is not zero but 15.57%. In the case $\theta=10000$, which means the investor is extremely risk-aversion, he still puts 15.57% into the domestic market. The home bias of the China’s market does exist.

This can explain that why our calculation about the weight of the domestic market is negative but will never happen in the real world.
6.4 Compare the home bias effect with China and developed countries, good news or bad one?

Tesar and Werner (1998) report that in 1996, the home bias for the developed countries have already been very evident: 81.8% of private German investors’ equity portfolios were invested in domestic equities; in the US, 90%; in the UK, 77.5%; in Japan, 94.7%; and in Canada, 88.8%.

However, in our analysis, the range of the China’s home bias should be between around 17.63-37.14 (with the range of $\theta$ between 0.5-5 from table 9). It is clear that as a developing country, China does not show a strong home bias effect as the developed country. This phenomenon shows that the China’s investors are more like to be internationally diversified. This might be caused by two reasons following:

At first, the China’s economy system is different from the developed countries. China is claimed to be a big, growing and centrally planned economic clout in the world. From the historical data, the high volatility drops a hint that the stock market in China is immature. Formula (3.28) shows that the extreme weight of the domestic asset is only related to the domestic risk and international risk the investors take. The extreme weight $= (\sigma_i^2 - \rho_{id}\sigma_d\sigma_i) / (\sigma_d^2 + \sigma_i^2 - 2\rho_{id}\sigma_d\sigma_i)$, we can see clearly that if the $\sigma_d$ is really larger than $\sigma_d$ the extreme weight becomes smaller. Only if the $\sigma_d$ is smaller than $\sigma_d$ the extreme weight becomes bigger (ignoring the $\rho_{id}$ factor because it is only between -1 and 1). Generally speaking, from a China’s perspective, the domestic risk is much bigger than the international risk, which causes the extreme domestic weight smaller. However, among the developed countries, the domestic risk is equal to the international risk and even smaller than the international risk, which may cause the extreme domestic weight bigger. This is the first reason why the China’s market shows the weaker home bias effect than the developed countries.
Secondly, the correlation between the China’s market and international market is tiny and even ignorable. In our data, the average correlation $\rho_{id} = 0.045$, which means the China’s market and international market are almost independent from China’s perspective which is impossible among the develop countries. In table 5, we can see the correlations among the developed countries are around 0.7. 

$$\frac{(\sigma_i^2 - \rho_{id}\sigma_d\sigma_i)}{(\sigma_d^2 + \sigma_i^2 - 2\rho_{id}\sigma_d\sigma_i)}$$

shows that the $\rho_{id}$ can influence the value of the formula but not strong. From China’s perspective, the $\rho_{id}$ is almost ignorable, let us make it 0, the formula becomes $(\sigma_i^2) / (\sigma_d^2 + \sigma_i^2)$. Among develop countries, $\rho_{id}$ is strong, let us make it 1, the formula becomes $(\sigma_i^2 - \sigma_d\sigma_i) / (\sigma_d^2 + \sigma_i^2 - 2\sigma_d\sigma_i)$. Obviously, $(\sigma_i^2) / (\sigma_d^2 + \sigma_i^2)$ is smaller than $(\sigma_i) / (\sigma_d - \sigma_i)$. Of course, due to the value of $\rho_{id}$ is between -1 and 1, so it won’t influence the result so much.

In conclusion, the riskier and independent market both comes from characters of China’s economy. Nevertheless, this phenomenon can help the Chinese investors to grab more benefits in the international diversification. Since in our analysis, when the optimal portfolio is reached, the weight of domestic asset should be small (Table 8). That is to say, the low-degree home bias effect might help China to approach the optimal portfolio and take higher return and lower risk in the international investment instead.
7. Conclusion

During the past twenty years, the waves of internationalization became stronger and stronger. It is almost impossible to ignore the benefits from the international diversification. The imposing of the limited resources on the unlimited economy system has triggered a debate on the management of the strategic assets all over the world. However, most of the argument deals about the higher return and lower risk from a developed country perspective; although it is hard to achieve most of the time. When applying to China, such a growing, big, and centrally planned player with a distinct economic structure in the global economy, conventional analysis might fail to tackle the reality. It has been more than fifty years since people started the asset allocation discussion. Gradually, people pay more attention on the internationally diversified strategy, especially the developed countries. As one of the fastest growing economy in past twenty years, China should keep step with the developed countries.

In this paper, firstly we present a portfolio optimization case to show the benefit gained by the investors from a China’s perspective in order to offer the allocation suggestion to the Chinese investors. At the same time the optimal weights of the different assets are also presented. The most important markets all over the world are combined in this case for imitating an objective investment environment, including the main stock markets from U.S., UK, France, Germany, China, and one bond index. Three methods are used to estimate the portfolio return and risk when optimization is reached when short-sales is allowed and it is proved that the result of the three methods is identical. However, when the short-sales is not allowed, the Black method and Lagrange’s method does not work any longer. Only the Solver method can satisfy the limitation the all the weights of the assets are larger than zero. Furthermore, comparing the two efficient frontiers (when the short-sales is allowed or not), we found that the efficient frontier with short sales dominates the one without short sales, just because the short-sales restriction imposes an extra constraint on the maximization problem. That is to say some points on the two efficient frontiers coincide when the weights of the asset are larger than zero at the same
time. And the reason is when the weights are all larger than zero, the Solver function is not necessary to coordinate the results with the limitation that \( X_i > 0 \) which makes the limitation the same as the short-sales exist.

Previous research is an ideal test, the results presents the benefit to be internationally diversified. The case study is just a suggestion to the investors, but will never happen in the real world because the home bias effect exists. When the optimal portfolio reached, the weight of domestic asset should be zero even negative. Tullis and Clarke (1999) offer a model which can help to investigate the home bias effect considering the investors preference. The preference is influence by the asymmetric information, the investment barriers, and even the purchase power parity. Whatever the influence is, it causes the investors to reconsider the return.

To apply the Tullis and Clarke model, the estimation of return and risk of the domestic and international markets are required. According to the research, we found that even if the extreme risk-aversion investors should allocate 15.57% in the domestic market. For the normal investors, they allocate 30%-50% in the domestic market.

The results are very different from the developed countries. Tesar and Werner (1998) report that in 1996, the home bias for the developed countries have already been very evident: 81.8% of private German investors’ equity portfolios were invested in domestic equities; in the US, 90%; in the UK, 77.5%; in Japan, 94.7%; and in Canada, 88.8%. The analysis of the Tullis and Clarke model turn out that this is caused by two reasons. At first, the domestic risk of the China’s stock market is several times larger than the international stock market. However, among the developed countries, the difference seems to be not so large. Secondly, the correlation between China’s market and international market is smaller than the correlation among the developed countries. The indication is that the China’s market is independent of the international market.

During the last 20 years, China’s “reform and opening” policies bring the booming economic development. The new policy made China become one of the fastest growing
countries all over the world. However, the Micro-economic control of the China’s government affects the China’s stock and bond market. This makes the immature capital market even more unstable. The correlation with the international market is still so tiny even if the centrally situation improves a lot these years.

Is it good or bad news? As far as we know, the home bias problem of China is not as serious as the developed countries. In the analysis of the portfolio optimization, we can see that when the optimal portfolio is reached, the weight of the domestic assets should be zero even negative. Therefore, on the contrary, China’s low-degree home bias effect might help the Chinese investors to achieve more benefits during international diversification. Too much Micro-economic control brings the unstable market in China but help with the investors who want to be internationally diversified.
Appendix A

Graphical Optimization:

The Black’s method:

The Lagrange’s method:
The Solver method (short sales not allowed):

![Graph showing Portfolio Optimization with Capital Market Line and Efficient Frontier.]

The Solver method (short sales allowed):

![Graph showing Portfolio Optimization with Capital Market Line and Efficient Frontier.]

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Appendix B

\[ \theta = \left[ \sum_{i=1}^{N} X_i (R_p - c) \right] \left[ \sum_{i=1}^{N} X_i^2 \sigma_i^2 + \sum_{j=i}^{N} \sum_{j \neq i}^{N} X_i X_j \sigma_{ij} \right]^{1/2} \]

In order to maximize the \( \theta \), \( \frac{d\theta}{dX} \) should equal to 0. Then \( \frac{d\theta}{dX} = ? \)

Two rules from calculus are needed:

1. **The product rule:** \( \theta \) is the product of two functions. The product rule states that the derivative of the product of two functions is the first function times the derivative of the second function plus the second times the derivative of the first. In symbols,

\[
\frac{d}{dx} \left[ (F_1(x))(F_2(x)) \right] = F_1(x) \frac{dF_2(x)}{dx} + F_2(x) \frac{dF_1(x)}{dx} \tag{B.1}
\]

And \( F_1(x) = \left[ \sum_{i=1}^{N} X_i (R_p - c) \right], \ F_2(x) = \left[ \sum_{i=1}^{N} X_i^2 \sigma_i^2 + \sum_{j=i}^{N} \sum_{j \neq i}^{N} X_i X_j \sigma_{ij} \right]^{1/2} \) (B.2) and (B.3)

Obviously \( \frac{dF_1(x)}{dx} = R_p-c \) \( \tag{B.4} \)

Now consider the derivative of \( F_2(x) \)

2. **The Chain rule:** \( F_2(x) \) involves a term in brackets to a power (the power -1/2). The chain rule states that its derivative is the power times the expression in parentheses to the power minus one, times the derivative of what is inside the brackets. Thus,

\[
\frac{dF_2(x)}{dx} = (-1/2) \left[ \sum_{i=1}^{N} X_i^2 \sigma_i^2 + \sum_{j=i}^{N} \sum_{j \neq i}^{N} X_i X_j \sigma_{ij} \right]^{-3/2} \cdot (2X_k \sigma_k^2 + 2 \sum_{j \neq i}^{N} X_j \sigma_{jk}) \tag{B.5}
\]

Substituting (B.2)(B.3)(B.4)(B.5) into (B.1) yields:
\[
\frac{d\theta}{dX} = \left[ \sum_{i=1}^{N} X_i \left( R_p - c \right) \right] \left\{ (-1/2) \left[ \sum_{i=1}^{N} X_i^2 \sigma_i^2 + \sum_{i=1}^{N} \sum_{j=1, j \neq i}^{N} X_i X_j \sigma_{ij} \right]^{3/2} \times (2X_k \sigma_k^2 + 2 \sum_{j=1, j \neq k}^{N} X_j \sigma_{jk}) \right\} + \left[ \sum_{i=1}^{N} X_i^2 \sigma_i^2 + \sum_{i=1}^{N} \sum_{j=1, j \neq i}^{N} X_i X_j \sigma_{ij} \right]^{1/2} (R_p - c) = 0
\]

Multiplying the derivative by \[ \left[ \sum_{i=1}^{N} X_i^2 \sigma_i^2 + \sum_{i=1}^{N} \sum_{j=1, j \neq i}^{N} X_i X_j \sigma_{ij} \right]^{1/2} \]

And rearranging yields:

\[
\left[ \sum_{i=1}^{N} X_i \left( R_p - c \right) \right] \left\{ (-1/2) \left[ \sum_{i=1}^{N} X_i^2 \sigma_i^2 + \sum_{i=1}^{N} \sum_{j=1, j \neq i}^{N} X_i X_j \sigma_{ij} \right]^{3/2} \times (2X_k \sigma_k^2 + 2 \sum_{j=1, j \neq k}^{N} X_j \sigma_{jk}) \right\} + \left[ \sum_{i=1}^{N} X_i^2 \sigma_i^2 + \sum_{i=1}^{N} \sum_{j=1, j \neq i}^{N} X_i X_j \sigma_{ij} \right]^{1/2} (R_p - c) = 0\
\]

Defining \( \rho \) as \[ \sum_{i=1}^{N} X_i \left( R_p - c \right) / \left\{ \sum_{i=1}^{N} X_i^2 \sigma_i^2 + \sum_{i=1}^{N} \sum_{j=1, j \neq i}^{N} X_i X_j \sigma_{ij} \right\} \]

Yields \( -\rho \left( X_k \sigma_k^2 + \sum_{j=1, j \neq k}^{N} X_j \sigma_{jk} \right) + (R_p - c) = 0 \) where, \( (X_k \sigma_k^2 + \sum_{j=1, j \neq k}^{N} X_j \sigma_{jk}) \) is the covariance of the assets.

then \[ \frac{d\theta}{dX} = -\left( \rho x_1 \sigma_1^2 + \rho x_2 \sigma_2^2 + \cdots + \rho x_i \sigma_i^2 + \cdots + \rho x_{N-1} \sigma_{N-1}^2 + \rho x_N \sigma_N^2 \right) + R_p - c = 0 \]
Appendix C

<table>
<thead>
<tr>
<th></th>
<th>Return</th>
<th>Risk</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stock</td>
<td>24.16%</td>
<td>133.92%</td>
</tr>
<tr>
<td>Bond</td>
<td>11.45%</td>
<td>13.04%</td>
</tr>
</tbody>
</table>

It is a little complicated when calculating the China’s domestic return and risk; let us calculate one by one:

1. Stock return:

   From the historical data, during 1990-2000, the average monthly return is 1.85%. So the yearly return is $1.0185^{12}-1=24.16\%$

2. Stock risk:

   **Table 1 the volatility of Shanghai Stock Exchange market 1990-2000**

<table>
<thead>
<tr>
<th></th>
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<th></th>
<th></th>
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<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>SSE</td>
<td>178.9</td>
<td>388.1</td>
<td>97.5</td>
<td>223.1</td>
<td>76.1</td>
<td>141.6</td>
<td>73.6</td>
<td>36.4</td>
<td>67.6</td>
<td>55.7</td>
</tr>
</tbody>
</table>

   Average is 133.92%

3. Bond return and bond risk:

   It is claimed that there is no officially China’s bond index before 2002, so we collect the historical data from 2002-2007, before the financial crisis.

   And found yearly return and risk are 11.45%, 13.04% according to the data.

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37 Lei, 2005, The Statistic analysis of the Volatility in China’s Stock Market

Reference A


Gregory C. Chow, 1994, “The Lagrange Method of Optimization with Applications to


Reference B


   The data for 2008 GDP are available through the following link at their website: [http://www.stats.gov.cn/tjsj/ndsj/2008/indexeh.htm](http://www.stats.gov.cn/tjsj/ndsj/2008/indexeh.htm).


   Visit the website of the World Investment Reports at [www.unctad.org/wir](http://www.unctad.org/wir).


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